HW2

February 5, 2019

```
In [1]: from mxnet import nd, autograd, gluon
        import seaborn as sns
        import matplotlib.pyplot as plt
        import numpy as np
```

0.1 1

```
In [2]: def sampler(probs, shape):

            length = nd.prod(nd.array(shape, dtype=int)).asscalar()

            t = nd.zeros(length)

            for i in range(length):

                cum_sum = 0

                p = nd.random.uniform()

                for j, prob in enumerate(probs):

                    cum_sum += prob

                    if p < cum_sum:

                        t[i] = j
                        break

            return nd.reshape(t, shape)
```

```
In [3]: sampler(nd.array([0, .1, .2, .3, .4]), (2,5,10))
```

```
Out[3]:
        [[[3. 3. 4. 4. 4. 4. 3. 4. 3. 4.]
          [4. 3. 3. 2. 4. 1. 4. 2. 3. 3.]
          [4. 4. 3. 3. 3. 3. 4. 4. 1. 3.]
          [1. 4. 1. 3. 4. 4. 4. 2. 4. 4.]
          [4. 3. 4. 4. 3. 3. 4. 4. 2. 4.]]
```

```
 [[4. 3. 2. 3. 4. 4. 3. 2. 3. 3.]
  [2. 2. 4. 4. 3. 2. 3. 2. 1. 3.]
  [4. 2. 4. 2. 4. 3. 4. 4. 4. 3.]
  [3. 4. 3. 4. 4. 1. 1. 4. 4. 4.]
  [4. 2. 2. 3. 2. 4. 3. 4. 3. 3.]]]
<NDArray 2x5x10 @cpu(0)>
```

0.2 2

```
In [4]: filename = gluon.utils.download('https://www.gutenberg.org/files/84/84-0.txt')

        with open(filename) as f:
            book = f.read()

        print(book[0:100])
```

```
Project Gutenberg's Frankenstein, by Mary Wollstonecraft (Godwin) Shelley

This eBook is for the u
```

```
In [5]: # split by space
        book = book.split()
```

```
In [6]: def clean(data):

            import re

            words = []

            for word in data:

                # if word has punctuation, skip
                if re.search(r'[^\w]', word):

                    continue

                else:

                    words.append(word.lower())

            return words
```

```
In [7]: words = clean(book)
```

```
In [8]: n = len(words)
```

```
        counts = nd.zeros((5, n))
        totals = nd.zeros(5)
```

```
In [9]: lst = ['a', 'and', 'the', 'is', 'i']

        for i, w in enumerate(words):
            if w in lst:
                totals[lst.index(w)] += 1
            counts[:, i] = totals
```

```
In [10]: totals /= n
```

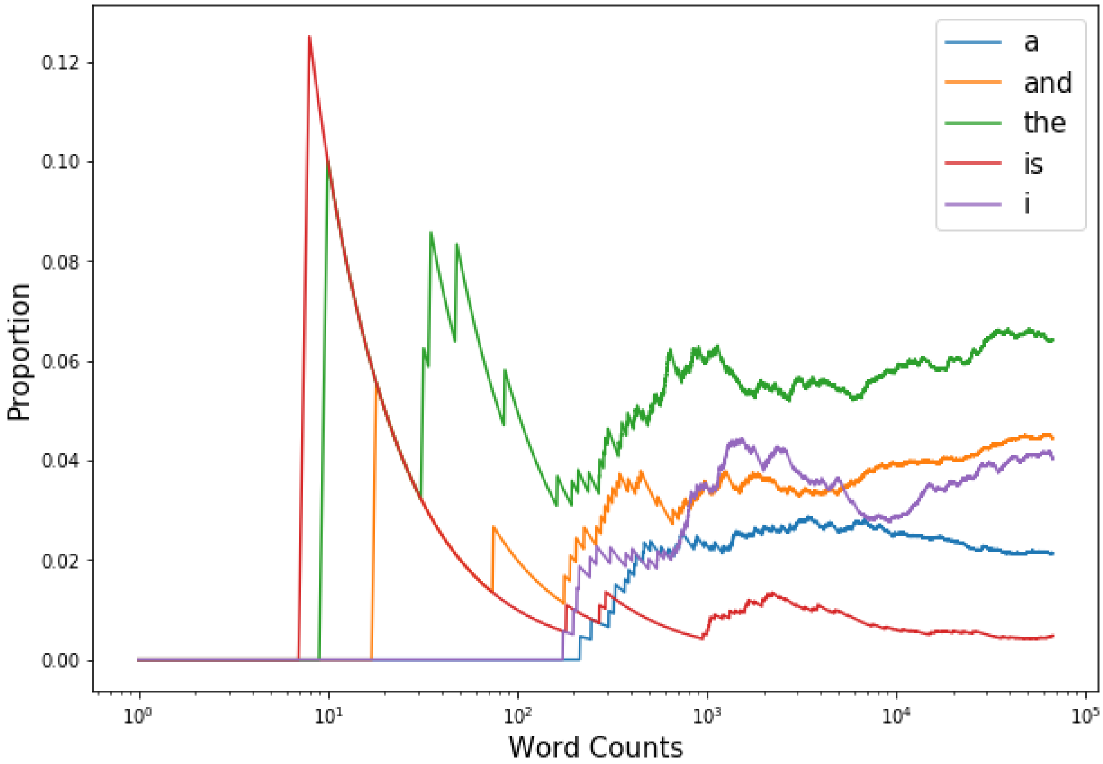
```
In [11]: totals
```

```
Out[11]:
         [0.02128573 0.04445135 0.06404962 0.00481075 0.0402475 ]
         <NDArray 5 @cpu(0)>
```

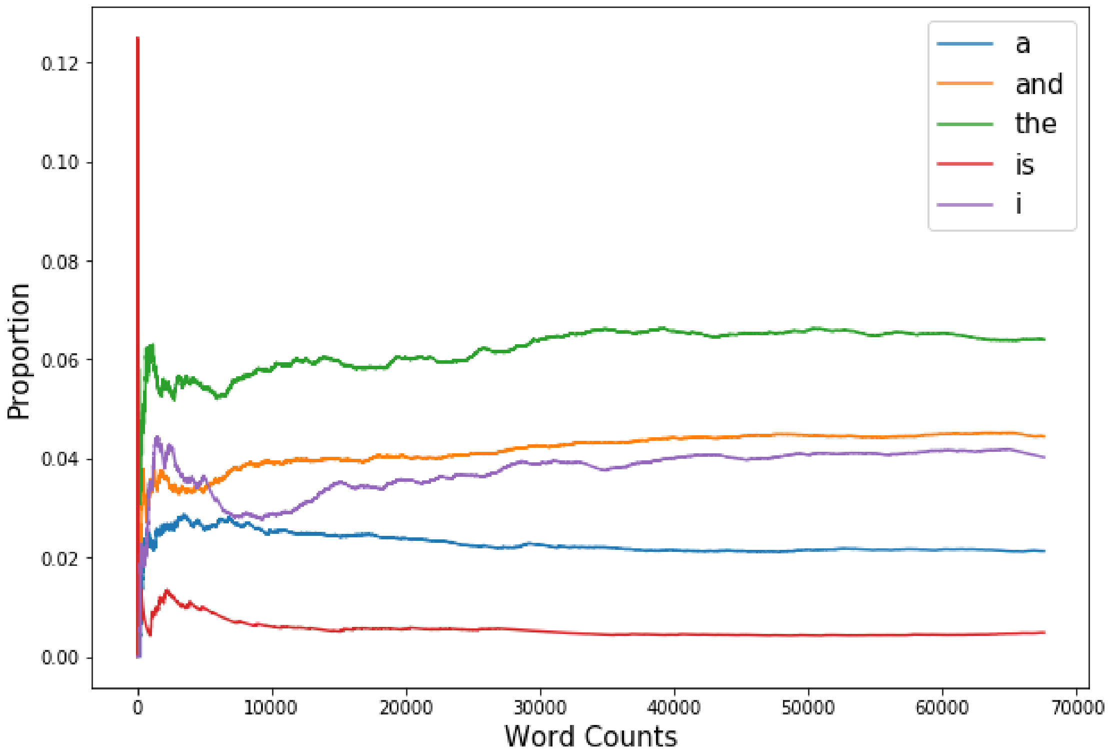
```
In [12]: estimates = nd.arange(n).reshape(1, -1) + 1
         estimates = counts / estimates
```

```
In [13]: y = np.arange(1, n+1).reshape(-1, 1)
```

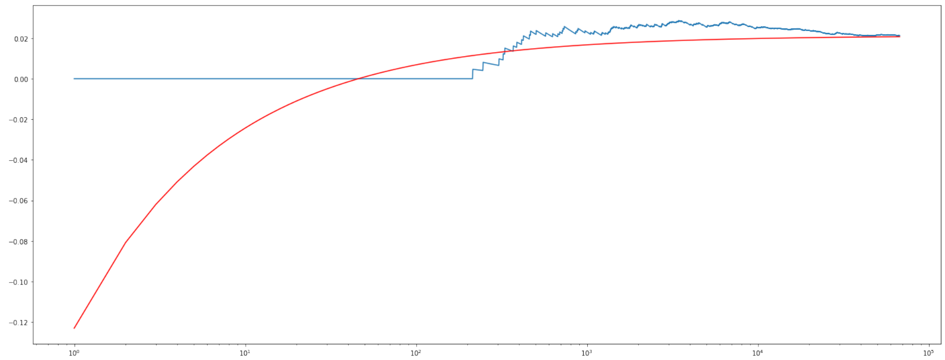
```
In [14]: plt.figure(figsize=(10,7))

         for i in range(5):
             plt.semilogx(y, estimates[i,:].asnumpy(), label=lst[i])

         plt.xlabel('Word Counts', size=15)
         plt.ylabel('Proportion', size=15)
         plt.legend(prop={'size':15})
         plt.show()
```



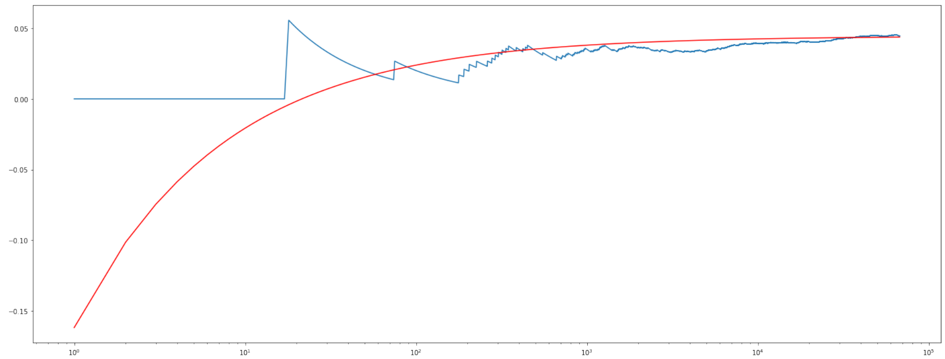
```
In [15]: plt.figure(figsize=(10,7))

         for i in range(5):
             plt.plot(estimates[i,:].asnumpy(), label=lst[i])

         plt.xlabel('Word Counts', size=15)
         plt.ylabel('Proportion', size=15)
         plt.legend(prop={'size':15})
         plt.show()
```



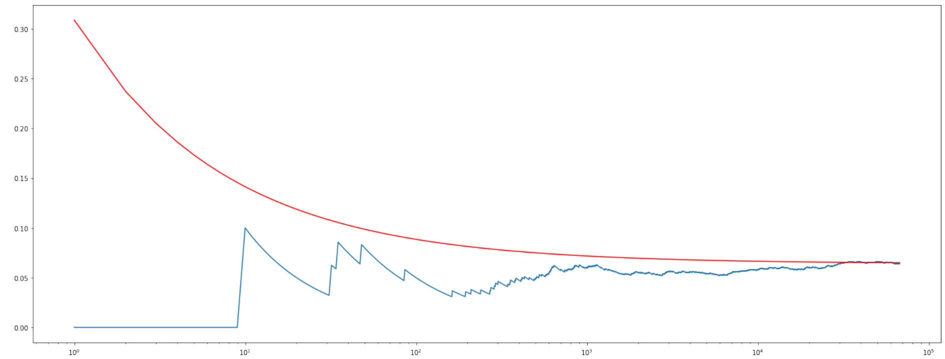
```
In [16]: mean = .0235
         variance = .005
         y = np.arange(1,n+1).reshape(-1,1)

         fig, ax = plt.subplots(5, 1, figsize=(20, 40))
         mean = totals.asnumpy()
         variance = mean - mean **2


         for i in range(5):
             ax[i].semilogx(y, estimates[i, :].asnumpy())
             if i in [2, 3]:
                 ax[i].semilogx(y, (variance[i]**0.5) * np.power(y,-0.5) + mean[i],'r')
             else:
                 ax[i].semilogx(y, -(variance[i]**0.5) * np.power(y,-0.5) + mean[i],'r')
             ax[i].set_xlabel(lst[i], size=25)

         fig.tight_layout()
         plt.show();
```


0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
10⁰
10¹
10²
10³
10⁴
10⁵

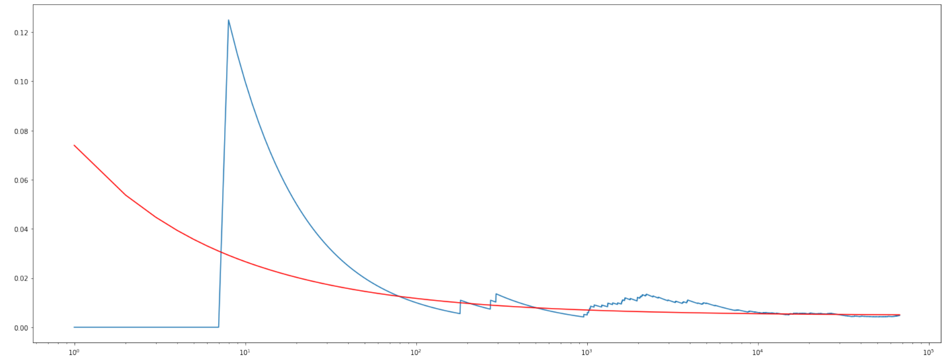
a


0.05
0.00
−0.05
−0.10
−0.15
10⁰
10¹
10²
10³
10⁴
10⁵

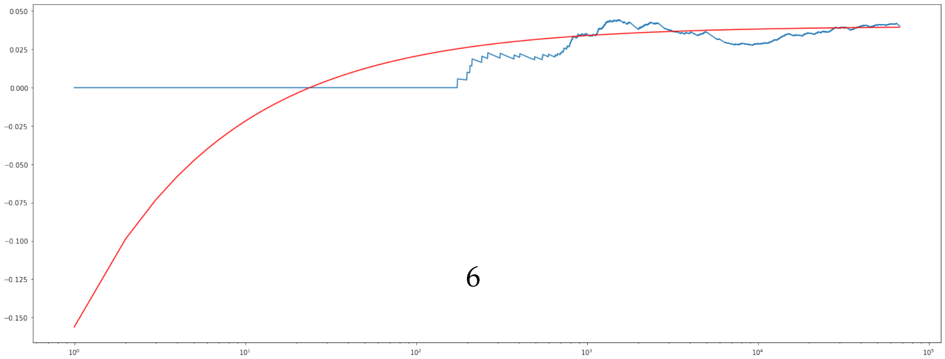
and


0.30
0.25
0.20
0.15
0.10
0.05
0.00
10⁰
10¹
10²
10³
10⁴
10⁵

the


0.12
0.10
0.08
0.06
0.04
0.02
0.00
10⁰
10¹
10²
10³
10⁴
10⁵

is


0.050
0.025
0.000
−0.025
−0.050
−0.075
−0.100
−0.125
−0.150
10⁰
10¹
10²
10³
10⁴
10⁵

i

We cannot use CLT on this problem directly since all the probabilites of each word are not equal that they don't converge to one point. To make CLT happen, the fequencies of them should be equal. However, we can still apply them here on each word that as the number counts (total words counts) increase, each of their probabilities converge to one point.

0.3 3

1. $\frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \frac{\partial \mathbf{y}}{\partial \mathbf{u}} \frac{\partial \mathbf{u}}{\partial \mathbf{x}}$
2. $\frac{\partial z}{\partial \mathbf{w}} = 2X^T Xw - 2X^T y$

0.4 4

```
In [17]: def log_exp(x, y):

             return - nd.log(nd.exp(x) / (nd.exp(x) + nd.exp(y)))
```

```
In [18]: x, y = nd.array([2]), nd.array([3])
         z = log_exp(x, y)
         z
```

```
Out[18]:
         [1.3132617]
         <NDArray 1 @cpu(0)>
```

```
In [19]: def grad(forward_func, x, y):

             x.attach_grad()
             y.attach_grad()

             with autograd.record():

                 z = forward_func(x, y)

             z.backward()

             print('x.grad =', x.grad)
             print('y.grad =', y.grad)

         grad(log_exp, x, y)
```

```
x.grad =
[-0.7310586]
<NDArray 1 @cpu(0)>
y.grad =
[0.7310586]
<NDArray 1 @cpu(0)>
```

```
In [20]: x, y = nd.array([50]), nd.array([100])

         grad(log_exp, x, y)

x.grad =
[nan]
<NDArray 1 @cpu(0)>
y.grad =
[nan]
<NDArray 1 @cpu(0)>
```

As we can see from the above print, exponential of y is too big to fit into the memory resulting in too large and infinity values. Because it cannot fit, the grad of the functions will return weird value (or nan) values. As covered in the lecture slides, we can put data into mantissa instead of exponents to avoid this problem.

```
In [21]: def stable_log_exp(x, y):

             a = nd.array([max(x.asscalar(), y.asscalar())])

             denom = a + nd.log(nd.exp(x - a) + nd.exp(y - a))

             return -1 * nd.log(nd.exp(x) / denom)

         stable_log_exp(nd.array([50]), nd.array([100]))

Out[21]:
         [-45.39483]
         <NDArray 1 @cpu(0)>

In [22]: grad(stable_log_exp, x, y)

x.grad =
[-1.]
<NDArray 1 @cpu(0)>
y.grad =
[0.01]
<NDArray 1 @cpu(0)>
```