Work and Fluid Force
Section 6.5

Idea: Work = Force × Distance (when the force is constant)

Example #1: A car jack applies a constant force of 100 lb (but you only need to apply 30 lb of force). How much work does the jack perform to lift the side of the car 1.25 ft? How much work did you perform?

$W_j = 70(1.25) = 87.5$ ft lb

$W_p = 30(1.25) = 37.5$ ft lb

Question: What happens when the force isn't constant? Then the work it takes to move an object from $x = a$ to $x = b$ is

$$W = \int_a^b F(x)\,dx$$

SI units
J or ft lb

Hooke's Law (for springs):

$F = kx$ where k is the spring constant and x is how far the spring is displaced from its natural resting position.

Example #2: A spring has a natural length of 10 inches. An 800-lb force stretches the spring to 14 inches.

(a) Find the spring constant.

$x = 4 \quad F = 800$

$800 = 4k$

$k = 200$ lb/in

(b) How much work is done in stretching the spring from 10 inches to 12 inches? From 12 inches to 14 inches?

$x = 2 \quad k = 200$

$\Delta F = \int_0^2 200x\,dx$

$100x^2\Big|_0^2 = 400$ in lb

$100x^2\Big|_2^4 = 1200$ in lb

Work to move an object/fluid
Work = weight × distance = density × volume × distance

Example #3: A mountain climber is about to haul up a 50 meter length of hanging rope. How much work will it take if the rope weighs 0.5 N/m?

$F = .5(50-x)$

$W = \int_0^{50} 0.5(50-x)\,dx = 625\text{ J}$

Example #4: Compute the work (against gravity) required to build a concrete column of height 5 meters and square base with sides that are 2 meters. Assume that concrete has a density of 1500 kg/m^3.

$\Delta V = 4\,\Delta y$

force $= 1500 \cdot 4\,\Delta y \cdot 9.8$

distance $= y$

work on one slab $= 1500 \cdot 4\Delta y \cdot 9.8y$

$W = \int_0^5 1500 \cdot 2^2 \cdot 9.8y\,dy = 735000\text{J} = 735\text{ kJ}$

Example #5: A truncated conical container is full of strawberry milkshake that weighs 4/9 oz/in^3. The container is 7 inches deep, 2.5 inches across at the base, and 3.5 inches across the top. The straw sticks up an inch above the top. How much work does it take to drink the milkshake through the straw (neglecting friction)?

$$m = \frac{7-0}{1.75-1.25} = 14$$

$$y - 0 = 14(x - 1.25)$$

$$y = 14x - 17.5$$

$$x = \frac{y+17.5}{14}$$

$$\pi x^2 h = \pi\left(\frac{y+17.5}{14}\right)^2 \Delta y$$

$$W = \int_0^7 \frac{4}{9}\pi\left(\frac{y+17.5}{14}\right)^2 (8-y)\,dy = 91.32 \text{ in oz}$$

Example #6: Water is pumped into a spherical tank of radius 2 feet from a source 1 foot below the bottom. Fresh water has a weight-density of 62.4 lb/ft^3. Find the work required to fill the tank.

$V = \pi x^2 h \quad x^2 = 4 - y^2$

$\pi(4 - y^2)h$

Weight $= 62.4\pi(4 - y^2)h\,\Delta y$

Dist $= 3 + y$

$W = \int_{-2}^{2} 22.4\pi(4 - y^2)(3 + y)\,dy = 1996.8\pi$ ft lb