Matrix Multiplication
Section 2.2 (**Hartman**)

The Matrix product:
Let A be an $m \times r$ matrix and B be an $r \times n$ matrix. The matrix product of A and B, denoted by AB is the $m \times n$ matrix M whose entry in the i-th row and j-th column is the product of the i-th row of A and the j-th column of B.

NOTE:

- In order to multiply A and B, the number of columns of A must be the same as the number of rows of B.
- The resulting matrix has the same number of rows as A and columns as B.

Important: In general, AB $\neq$ BA.

Example #1: Given $A = \begin{pmatrix} 2 & -3 & 0 \\ 3 & 1 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 1 & -1 \\ 2 & 0 \end{pmatrix}$. and $C = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}$. If possible, find;

(a) AB =

$$\underset{2\times 3}{\begin{pmatrix} 2 & -3 & 0 \\ 3 & 1 & 5 \end{pmatrix}} \cdot \underset{3\times 2}{\begin{pmatrix} 1 & 0 \\ 1 & -1 \\ 2 & 0 \end{pmatrix}} = \begin{pmatrix} 2(1)-3(1)+0(2) & 2(0)-3(-1)+0(0) \\ 3(1)+1(1)+5(2) & 3(0)+1(-1)+5(0) \end{pmatrix}$$

$$= \underset{2\times 2}{\begin{pmatrix} -1 & 3 \\ 14 & -1 \end{pmatrix}}$$

(b) BA =

$$\underset{3\times 2}{\begin{pmatrix} 1 & 0 \\ 1 & -1 \\ 2 & 0 \end{pmatrix}} \cdot \underset{2\times 3}{\begin{pmatrix} 2 & -3 & 0 \\ 3 & 1 & 5 \end{pmatrix}} = \underset{3\times 3}{\begin{pmatrix} 2 & -3 & 0 \\ -1 & -4 & -5 \\ 4 & -6 & 0 \end{pmatrix}}$$

(c) AC = Not possible, columns in A $\neq$ rows in C

d. $CA = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2 & -3 & 0 \\ 3 & 1 & 5 \end{pmatrix} = \begin{pmatrix} 2 & -3 & 0 \\ -1 & -4 & -5 \end{pmatrix}$

Identity Matrix:
The $n \times n$ square matrix with 1's on the diagonal and zeros else-where is the $n \times n$ identity matrix, and it is denoted by I_n.

e.g. $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\quad I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Example #2: Given $A = \begin{pmatrix} 2 & -3 & 0 \\ 3 & 1 & 5 \end{pmatrix}$. Prove AI = IA = A.

AI $= \begin{pmatrix} 2 & -3 & 0 \\ 3 & 1 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -3 & 0 \\ 3 & 1 & 5 \end{pmatrix}$

IA $= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & -3 & 0 \\ 3 & 1 & 5 \end{pmatrix} = \begin{pmatrix} 2 & -3 & 0 \\ 3 & 1 & 5 \end{pmatrix}$

Properties:
Let A, B, C be matrices with dimensions so that the following operations make sense, and let k be a scalar. Then the following equalities hold:

- $(AB)C = A(BC)$
- $A(B + C) = AB + AC$
- $k(AB) = (kA)B = A(kB)$
- $AI = IA = A$

NOTE: For matrices, $A^{-1} \neq \frac{1}{A}$. In fact, $\frac{1}{A}$ makes no sense. We will learn about A^{-1} under inverse matrices.