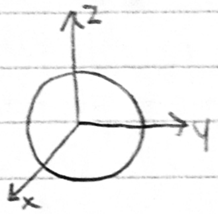
Torque and Newton's Second Law of Rotational Motion PH 112

$\vec{\tau} = \vec{r}_\perp \vec{F} = \vec{r}\vec{F}\sin\theta = \vec{r}\times\vec{F}$

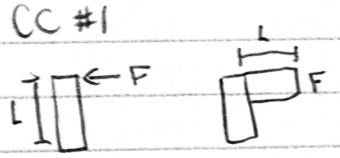
$\vec{\tau} \perp \vec{r}, \vec{F}$

Cross Product

$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$ $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$



CC #1



τ is the same C

CC #2

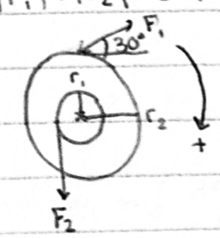
Large lever arm = higher τ

Example #1

$r_1 = 30cm = 0.3m$ Looking for τ

$r_2 = 50cm = 0.5m$

$|F_1| = |F_2| = 50N$

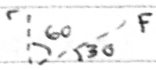


$\tau_{Net} = \tau_2 - \tau_1$

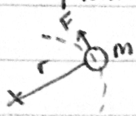
$= F_1 r_1 - F_2 r_2 \sin 60$

$= 50(.3) - 50(.5)\sin 60$

$= \boxed{6.7 \text{ Nm}}$



Rotational dynamics



$\vec{F}_{Net} = m\vec{a}_{tan} = mr\alpha$

$r\vec{F}_{net} = mr^2\vec{\alpha}$

$\vec{\tau} = I\vec{\alpha}$

net torque = moment of inertia × angular acceleration

$I = mr^2$ in kgm^2

Resistance to change in angular velocity

$\tau_{net} = \sum_{i=1}^{N} \tau_i \qquad I = \sum_{i=1}^{N} m_i r_i^2$

Example #2

$\omega_0 = 80 rev/s \cdot \frac{2\pi}{1rev} = 16\pi \, rad/s = 503 \, rad/s$

$\theta = 240 \, rev \cdot \frac{2\pi}{1rev} = 480\pi \, rad = 1508 \, rad$

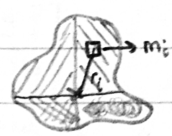
$I = 1.41 \times 10^{-3} \, kgm^2$ Looking for τ

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$\alpha = \frac{\omega^2}{2\theta} = \frac{503^2}{2(1508)} = 84 \, rad/s^2$

$\tau = I\alpha = 1.41 \times 10^{-3}(84) = \boxed{0.12 \, Nm}$

Moment of inertia for rigid bodies



$I_{total} = \sum_{i=1}^{N} m_i r_i^2$

$I = \lim_{\Delta m \to 0} \sum_{i=1}^{N} \Delta m_i r_i^2 = \int r^2 dm$

$dm = \rho dV$

$I = \int \rho r^2 dV$

density

linear density of the mass

$\lambda = \frac{M}{L} = \frac{dm}{dL} = \frac{dm}{dx}$

$dm = \lambda dx$

$I = \int r^2 \lambda dx$

surface density

$\sigma = \frac{M}{Area} = \frac{dm}{dxdy}$

$dm = \sigma dxdy$

$I = \iint r^2 \sigma dxdy$

volume density

$\rho = \frac{M}{V} = \frac{dm}{dxdydz}$

$dm = \rho dxdydz$

$I = \iiint r^2 \rho dxdydz$

Parallel axis theorem

$I = I_G + md^2$

center of mass