Getting ready for 10.4

We skip section 10.3 until we finish section 8.8. We will define the **p-series** now and learn the proof later in section 10.3.

<u>Definition:</u> The **p-series**, given by

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

is convergent if $p > 1$ and divergent if $p \leq 1$.

Example #1: Determine whether the following series are convergent or divergent:

(a) $\sum_{n=1}^{\infty} \frac{1}{n^5}$ $p = 5 > 1$

Converges

(b) $\sum_{n=1}^{\infty} \frac{1}{n^{0.25}}$ $p = .25 \leq 1$

Diverges

(c) $\sum_{n=2}^{\infty} \frac{\sqrt{5}}{(n-1)^{2/3}}$

$\sum_{n=1}^{\infty} \frac{\sqrt{5}}{n^{2/3}}$ $p = \frac{2}{3} \leq 1$

Diverges

Harmonic series: $\sum_{n=1}^{\infty} \frac{1}{n}$.

We already know this is divergent using the definition of p-series. We can see that alternatively, using partial sums.
Consider partial sums with index 2^k.

$s_1 = \frac{1}{1} = 1$

$s_2 = \frac{1}{1} + \frac{1}{2} = 1 + \frac{1}{2}$

$s_4 = 1 + \frac{2}{2}$

$s_8 = 1 + \frac{3}{2}$

$s_{16} = 1 + \frac{4}{2}$

$S_{2^k} = 1 + \frac{k}{2}$

$(1 + \frac{k}{2}) \to \infty$ as $k \to \infty$

S_{2^k} diverges, so $\sum_{n=1}^{\infty} \frac{1}{n}$ also diverges