SHM Dynamics

PH 112

$F_{sp} = ma = -kx$

$a = \frac{d^2x}{dt^2} = \frac{-km}{x} = -\omega^2 x$

$\omega = \sqrt{\frac{k}{m}}$

$x = A\cos\omega t$

$x' = -A\omega\sin\omega t \quad x'' = -A\omega^2\cos\omega t = -\omega^2 x$

General solution: $x = B\sin(\omega t) + C\cos(\omega t)$

$x' = \omega B\cos(\omega t) - \omega C\sin(\omega t)$

$x'' = -\omega^2 B\sin(\omega t) - \omega^2 C\cos(\omega t) = -\omega^2(B\sin(\omega t) + C\cos(\omega t))$

$= -\omega^2 x$

Angular Frequency

$x = A\cos(\omega t + \phi)$

Displacement to the left

$\omega = 2\pi f = \frac{2\pi}{T}$

$T = \frac{2\pi}{\omega}$

Example #1

$x = 2\cos(\pi t)$

$A = ?\quad f = ?\quad T = ?$

$x(\frac{1}{3}) = ?\quad v(\frac{1}{3}) = ?\quad a(\frac{1}{3}) = ?$

$\boxed{A = 2m}\quad \omega = \pi\quad \phi = 0$

$f = \frac{\omega}{2\pi} = \frac{\pi}{2\pi} = \boxed{\frac{1}{2}\ Hz}$

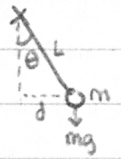
$T = \frac{1}{f} = \frac{1}{1/2} = \boxed{2s}$

$x(\frac{1}{3}) = 2\cos(\frac{\pi}{3}) = \boxed{1\ m}$

$v(\frac{1}{3}) = -2\pi\sin(\frac{\pi}{3}) = \boxed{-\pi\sqrt{3}\ m/s}$

$a(\frac{1}{3}) = -2\pi^2\cos(\frac{\pi}{3}) = \boxed{-\pi^2\ m/s^2}$

Simple pendulum



When θ is small, $\sin\theta = \theta$ and $\cos\theta = 1$

$\tau = I\alpha \Rightarrow -mgd = mL^2\frac{d^2\theta}{dt^2} \qquad \omega = \sqrt{\frac{g}{L}}$

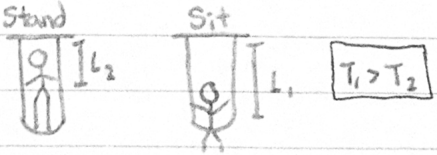
$\frac{d^2\theta}{dt^2} = \frac{-gd}{L^2} = \frac{-gL\theta}{L^2} = \frac{-g\theta}{L} = -\omega^2\theta$

$\theta = \Theta_0\cos(\omega t + \phi)$

Example #2

$\omega = \sqrt{\frac{g}{L}} = \frac{2\pi}{T} \qquad T = \frac{2\pi}{\omega}$

$T_{sit} = 2\pi\sqrt{\frac{Lcm}{g}} \qquad T_{stn} = 2\pi\sqrt{\frac{Lcm}{g}}$



$\boxed{T_1 > T_2}$

Rod pendulum

θ is small

$\tau = I\alpha \Rightarrow -mgd = \frac{1}{3}mL^2\frac{d^2\theta}{dt^2}$

$d = \frac{L}{2}\sin\theta = \frac{L\theta}{2}$

$-\frac{1}{2}mgL\theta = \frac{1}{3}mL^2\frac{d^2\theta}{dt^2} \qquad \frac{d^2\theta}{dt^2} = -\frac{3g\theta}{2L} \qquad \omega = \sqrt{\frac{3g}{2L}}$

$\frac{d^2\theta}{dt^2} = -\omega^2\theta \qquad \theta = \Theta_0\cos(\omega t + \phi)$

Example #3

$T_S = \frac{2\pi}{\omega_S}$ $\quad T_R = \frac{2\pi}{\omega_R}$

$2\pi\sqrt{\frac{L_S}{\frac{2}{3}g}} = 2\pi\sqrt{\frac{2L_R}{3g}}$

$\sqrt{L_S} = \sqrt{\frac{2}{3}L_R}$

$\boxed{L_S = \frac{2}{3}L_R}$