Velocity and Acceleration Worksheet MA 113

1. $r(x) = \langle x, x^2 \rangle$

$\frac{dr}{dt} = \frac{dr}{dx}\frac{dx}{dt} = \langle 1, 2x \rangle \frac{dx}{dt} = \langle \frac{dx}{dt}, 2x\frac{dx}{dt} \rangle$ $\quad \frac{dx}{dt} = 2$

$\hookrightarrow \langle 2, 4x \rangle$ $\quad \hat{T} = \frac{r'(x)}{\|r'(x)\|} = \frac{1}{\sqrt{1+4x^2}} \langle 1, 2x \rangle$

$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = 2\langle 0, 4 \rangle = \langle 0, 8 \rangle = a_T \hat{T} + a_N \hat{N}$

$a_T = a \cdot \hat{T} = \langle 0, 8 \rangle \cdot \hat{T}(1) = \langle 0, 8 \rangle \cdot \frac{1}{\sqrt{5}} \langle 1, 2 \rangle = \boxed{\frac{16}{\sqrt{5}}}$

$a_N \hat{N} = \langle 0, 8 \rangle - \frac{16}{\sqrt{5}} \frac{\langle 1, 2 \rangle}{\sqrt{5}} = \langle \frac{-16}{5}, \frac{8}{5} \rangle$

$\|a_N \hat{N}\| = a_N = \boxed{\frac{8\sqrt{5}}{5}}$

2. $\|v\| = 3 \times 10^5$ $\quad a_T = 10$ $\quad a = a_T \hat{T} + a_N \hat{N} = 10\hat{T} + K(3\times 10^5)^2 \hat{N}$ $\quad \frac{1}{r} = 1$

$\hat{N}$ ↙ ↘ $\hat{T}$ $\quad \hat{N} = a\hat{r} = a\langle \frac{3}{5}, \frac{4}{5} \rangle$ $\quad a < 0$

$\boxed{\hat{N} = -\langle \frac{3}{5}, \frac{4}{5} \rangle}$

$0 = \hat{T} \cdot \hat{N} = -\frac{3}{5}a - \frac{4}{5}b$ $\quad b = -\frac{3}{4}a$

$\|\langle a, -\frac{3}{4}a \rangle\| = 1$ $\quad a = \pm\frac{4}{5}$

$\boxed{\hat{T} = \langle \frac{4}{5}, -\frac{3}{5} \rangle}$

$\boxed{a = 10\langle \frac{4}{5}, -\frac{3}{5} \rangle - (3\times 10^5)^2 \langle \frac{3}{5}, \frac{4}{5} \rangle}$