Matrix Addition and Scalar Multiplication

Section 2.1 (**Hartman**)

What is a matrix?

A matrix is a rectangular array of numbers. The horizontal lines of numbers form rows and the vertical lines of numbers form columns. A matrix with m rows and n columns is said to be an $m \times n$ matrix ("an m by n matrix"). If $m = n$ we say that the matrix is square.

Example #1:

$\begin{pmatrix} 2 & -3 & 0 \\ 3 & 1 & 5 \end{pmatrix}$ 2×3

$\begin{pmatrix} 3 & 4 \\ 1 & 5 \end{pmatrix}$ 2×2

Square matrix

$\begin{pmatrix} 3 & 4 & -1 & 0 & 4 \end{pmatrix}$ 1×5

Row matrix

$\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$ 3×1

Column matrix

Matrix Equality:

Two $m \times n$ matrices A and B are equal if their corresponding entries are equal.

Example #2: Find the values of a, b, c and d.

$$\begin{pmatrix} 2 & -3 & 0 \\ 3 & 1 & 5 \end{pmatrix} = \begin{pmatrix} a & c+d & d \\ -c & b & a-c \end{pmatrix}$$

$a = 2$
$b = 1$
$c = -3$
$d = 0$

Matrix Addition and Scalar Multiplication:

Let A and B are $m \times n$ matrices and k be any scalar. Then;

$$A + B = \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} + b_{m1} & a_{m2} + b_{m2} & \cdots & a_{mn} + b_{mn} \end{pmatrix}$$

$$kA = \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix}$$

When all the entries of a matrix are zero, it is called a ***zero matrix*** and denoted by $\mathbf{0}$ or $\mathbf{0}_{m\times n}$.

Example #3: Given $A = \begin{pmatrix} 2 & -3 & 0 \\ 3 & 1 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 3 & 0 \\ 2 & 1 & 1 \end{pmatrix}$. Find;

(a) $A - B$

$$\begin{pmatrix} 2 & -6 & 0 \\ 1 & 0 & 4 \end{pmatrix}$$

(b) $5A + B$

$$\begin{pmatrix} 10 & -12 & 0 \\ 17 & 6 & 26 \end{pmatrix}$$

Properties:
For any $m \times n$ matrices A, B, C and a scalar k;

- $A + B = B + A$
- $(A + B) + C = A + (B + C)$
- $k(A + B) = kA + kB$
- $kA = Ak$
- $A + \mathbf{0} = \mathbf{0} + A = A$, where $\mathbf{0}$ is the $m \times n$ zero matrix.
- $0A = \mathbf{0}$