The Slope Field (Graphical Methods)
Section 2.2 (Noonburg)

Idea: We can describe a solution to a DE without actually solving it. This is really helpful with the DEs which cannot be solved easily, or explicitly.

For first order DEs, we can write $\frac{dx}{dt} = f(t, x)$. If we think back to calculus, $\frac{dx}{dt}$ tells us the slope of the tangent line to the solution.

At each value of t and x we can solve for $\frac{dx}{dt}$.

The slope of the tangent line to the solution $x(t)$ at the point (t_0, x_0) is given by $f(t_0, x_0)$.

Example #1: Make a hand sketch of a slope field for $x' = x + t/2$ in the region $-3 \leq t \leq 3$, $-3 \leq x \leq 3$.

	3	1.5	2	2.5	3	3.5	4	4.5
	2	0.5	1	1.5	2	2.5	3	3.5
	1	-0.5	0	0.5	1	1.5	2	2.5
x	0	-1.5	-1	-0.5	0	0.5	1	1.5
	-1	-2.5	-2	-1.5	-1	-0.5	0	0.5
	-2	-3.5	-3	-2.5	-2	-1.5	-1	-0.5
	-3	-4.5	-4	-3.5	-3	-2.5	-2	-1.5
		-3	-2	-1	0	1	2	3
					t			

This process is tedious if done by hand. The DEplot command in **Maple** is useful here. To create a slope field for Example 1, we will type:

with(*DEtools*)
$de := x'(t) = x(t) + t/2$
$DEplot(de, x(t), t = -3..3, x = -3..3)$

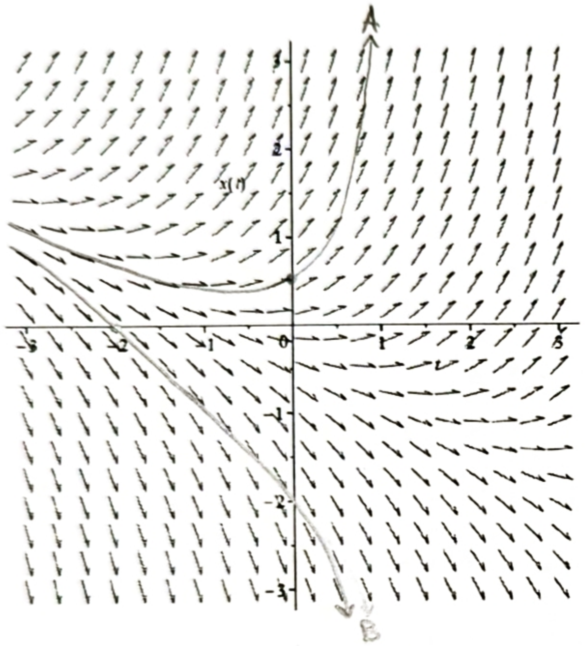
$x' - x = \frac{t}{2}$

$p(t) = e^{\int -dt} = e^{-t}$

$\frac{d}{dx} e^{-t} - xe^{-t} = \frac{t}{2} e^{-t}$

$\int d(e^{-t}x) = \int \frac{t}{2} e^{-t} dt$

$x = \frac{-t}{2} - \frac{1}{2} + Ce^{t}$



Identify the solution curves when;

A: $x(0) = 0.5$

B: $x(-2) = 0$