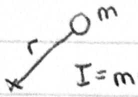
Moment of Inertia

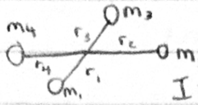
PH 112

Mass represents resistance to change in linear velocity

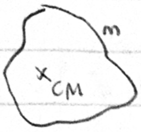
$\tau = I\alpha$



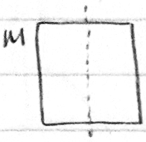
$I = mr^2$



$I = \sum_{i=1}^{n} m_i r_i^2$

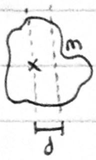


$I_{cm} = \int r^2 dm = \int_0^L x^2 \lambda dx$ $\quad dm \to r \quad \lambda = \frac{M}{L} = \frac{dm}{dx} \Rightarrow dm = \lambda dx$



$\sigma = \frac{M}{A} = \frac{dm}{dxdy}$

$I = \iint r^2 \sigma dxdy$



Parallel axis theorem

$I = I_{CM} + md^2$

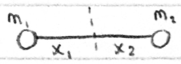
Example #1

$m_1 = 5kg$

$m_2 = 7kg$

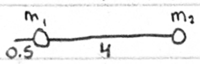
$x = 4m$

a. $I = ?$



$I = m_1x_1^2 + m_2x_2^2 = 5(2)^2 + 7(2)^2 = \boxed{48\ kgm^2}$

b. $I = ?$



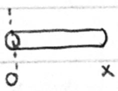
$I = 5(0.5)^2 + 7(4.5)^2 = \boxed{143\ kgm^2}$

Example #2

M

L

$I = ?$



$I = \int r^2 dm \quad \lambda = \frac{M}{L} \Rightarrow dm = \lambda dx$

$I = \int_0^L x^2 \lambda dx$

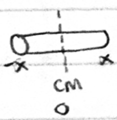
$= \frac{1}{3}x^3\lambda \Big|_0^L = \frac{\lambda}{3}(L^3 - 0^3) = \frac{1}{3}\lambda L^3 = \frac{1}{3}\left(\frac{M}{L}\right)L^3 = \boxed{\frac{ML^2}{3}\ kgm^2}$

Example #3

M

L

$I_{cm} = ?$



$I = \int r^2 dm \quad dm = \lambda dx$

$I = \int_{-L/2}^{L/2} r^2 \lambda dx = \frac{1}{3}\lambda r^3 \Big|_{-L/2}^{L/2} = \frac{1}{3}\lambda\left(\left(\frac{L}{2}\right)^3 + \left(\frac{L}{2}\right)^3\right) = \frac{1}{3}\left(\frac{M}{L}\right)\left(\frac{L^3}{8} + \frac{L^3}{8}\right) = \frac{2ML^2}{24}$

$= \boxed{\frac{ML^2}{12}\ kgm^2}$

Example #4

M $\quad I = ?$

R

$I = \int r^2 dm = r^2 \underbrace{\int dm}_{m} = \boxed{mr^2\ kgm^2}$

Example #5

R

M

L

$I=?$

$I=\int r^2\,dm$ $\quad \rho=\frac{M}{V}\Rightarrow dm=\rho\,dV$ $\quad V=\pi R^2 L$

$dm=\rho 2\pi r L\,dr$ $\quad dV=2\pi r\,dr\,L$

$I=\int_0^R 2\pi r^3\rho L\,dr=2\pi\rho L\left(\frac{r^4}{4}\right)\Big|_0^R$

$=2\pi L\left(\frac{M}{\pi R^2 L}\right)\left(\frac{R^4}{4}\right)=\boxed{\frac{MR^2}{2}\ kgm^2}$

Hoop $=MR^2$

Rod $=\frac{1}{12}MR^2$ through center

Cyl $=\frac{1}{2}MR^2$

Sphere $=\frac{2}{5}MR^2$

Hollow cyl $=\frac{1}{2}M(R_1^2+R_2^2)$

Rod $=\frac{1}{3}ML^2$ at end

Plate $=\frac{1}{12}M(a^2+b^2)$

Sphere $=\frac{2}{3}MR^2$ hollow

Example #6

$\theta=45^\circ$

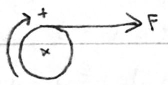
$\ell=50cm=.5m$

$F=200N$ $\quad \tau=F\ell\sin\theta=200(.5)\sin 45=\boxed{70.71\ Nm}$

$m=3kg$ $\quad \tau=I\alpha \quad \alpha=\frac{\tau}{I}=\frac{3\tau}{ML^2}=\frac{3(70.71)}{3(.5)^2}=\boxed{282.8\ rad/s^2}$

$\tau=?\ \alpha=?$

Example #7



$F=50N$

$m=4kg$

$r=33cm=.33m$ $\quad \tau_{net}=I\alpha$ $\quad \omega=\omega_0+\alpha t$

$\omega_0=0$ $\quad \tau_F-\tau_f=I\alpha$ $\quad \alpha=\frac{\omega}{t}=\frac{30}{3}=10\ rad/s^2$

$\omega=30\ rad/s$ $\quad Fr-\tau_f=I\alpha$

$t=3s$ $\quad I=\frac{Fr-\tau_f}{\alpha}$

$\tau_f=1.1\ Nm$ $\quad I=\frac{50(.33)-1.1}{10}=\boxed{0.385\ kgm^2}$

$I=?$

*

Example #8

b. $\omega=\omega_0+\alpha t=4.69(3)=\boxed{14.07\ rad/s}$

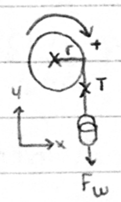
$v=r\omega=.33(14.07)=\boxed{4.64\ m/s}$

$m=4kg$ $\quad \tau_f=1.1Nm$

$r=33cm=.33m$

$F_W=15N$ $\quad t=3s$

$\alpha=?\ a=?\ \omega=?\ v=?$



a. $\tau_{net}=I\alpha$ $\quad F_{net}=ma$ $\quad a_{tan}=r\alpha=.33(4.69)=\boxed{1.55\ m/s^2}$

$\tau_T-\tau_f=I\alpha$ $\quad T-W=-ma$

$\alpha=\frac{Tr-\tau_f}{I}$ $\quad T=W-ma$

$\alpha=\frac{(W-ma)r-\tau_f}{I}=\frac{(W-mr\alpha)r-\tau_f}{I}=\frac{(Wr-\tau_f)/I}{1+\frac{mr^2}{I}}=\boxed{4.69\ rad/s^2}$