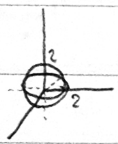
Polar Integration

MA 113

 $\frac{1}{2}r^2\theta$  $\frac{1}{2}\theta\left((r+\Delta r)^2-r^2\right)$

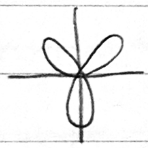
$\Delta A = \frac{1}{2}\Delta\theta\Delta r(2r+\Delta r)$ $\quad dA = r\,dr\,d\theta$

Volume of a sphere of radius 2



$$\int_0^{2\pi}\int_0^2 2\sqrt{4-r^2}\, r\,dr\,d\theta = \int_0^{2\pi} -\sqrt{4-r^2}\Big|_0^2 d\theta = \int_0^{2\pi} 2\,d\theta = \boxed{4\pi}$$

Find the area of $r=2\sin(3\theta)$



$$A = \iint_R 1\,dA = 3\int_0^{\pi/3}\int_0^{2\sin(3\theta)} r\,dr\,d\theta = 6\int_0^{\pi/3}\sin^2(3\theta)\,d\theta = \boxed{\pi}$$

$I = \int_0^\infty e^{-x^2}dx = \int_0^\infty e^{-y^2}$ $\quad I^2 = \int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dx\,dy = \lim_{a\to\infty}\int_0^{\pi/2}\int_0^a e^{-r^2}r\,dr\,d\theta$

$I = \boxed{\frac{\sqrt{\pi}}{2}}$ $\quad = \lim_{a\to\infty}\int_0^{\pi/2} -\frac{1}{2}(e^{-a^2}-1)\,d\theta = \lim_{a\to\infty} -\frac{\pi}{4}(e^{-a^2}-1) = \frac{\pi}{4}$