10.2 Series MA 112

1. $\sum_{n=1}^{4} a_n = a_1 + a_2 + a_3 + a_4$

$= (1+\frac{1}{1}) + (1+\frac{1}{2}) + (1+\frac{1}{3}) + (1+\frac{1}{4}) = 5 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{73}{12}$

* To raise replace n with n-h

To lower replace n with n+h

2. a. $\sum_{n=1}^{19} \frac{2^{(n+1)}}{n}$

b. $\sum_{n=0}^{18} \frac{2^{(n+2)}}{n+1}$

c. $\sum_{n=5}^{23} \frac{2^{(n-3)}}{n-4}$

3. a. $S_3 = a_1 + a_2 + a_3 = 1+3+5 = 9$

b. $S_5 = 1+3+5+7+9 = 25$

c. $S_N = a_1 + a_2 + a_3 + \cdots + a_N = N^2$

Look for a pattern

If Σa_n converges and Σb_n diverges, then $\Sigma(a_n \pm b_n)$ will diverge

All terms except first and last cancel

4. a. $S_1 = a_1 = \frac{1}{1} - \frac{1}{1+1} = 1 - \frac{1}{2}$

$S_2 = a_1 + a_2 = (1-\frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) = 1 - \frac{1}{3}$

$S_N = a_1 + a_2 + \cdots + a_N = (1-\frac{1}{2}) + (\frac{1}{2}-\frac{1}{3}) + \cdots + (\frac{1}{N-1} - \frac{1}{N}) + (\frac{1}{N} - \frac{1}{N+1}) = 1 - \frac{1}{N+1}$

$\lim_{N\to\infty} 1 - \frac{1}{N+1} = 1$ Converges to 1

b. $\frac{A}{n} + \frac{B}{n+2} \Rightarrow A = \frac{1}{2},\ B = -\frac{1}{2}$ (Partial fractions)

$\frac{1/2}{n} - \frac{1/2}{n+2} = \frac{1}{2}(\frac{1}{n} - \frac{1}{n+2}) = \frac{1}{2}\sum_{n=1}^{\infty}(\frac{1}{n} + \frac{1}{n+2})$

$S_N = \frac{1}{2}((\frac{1}{1} - \frac{1}{3}) + (\frac{1}{2} - \frac{1}{4}) + (\frac{1}{3} - \frac{1}{5}) + \cdots + (\frac{1}{N-2} - \frac{1}{N}) + (\frac{1}{N-1} - \frac{1}{N+1}) + (\frac{1}{N} - \frac{1}{N+2})) = \frac{1}{2}(1 + \frac{1}{2} - \frac{1}{N+1} - \frac{1}{N+2})$

$\lim_{N\to\infty} \frac{1}{2}(1 + \frac{1}{2} - \frac{1}{N+1} - \frac{1}{N+2}) = \frac{3}{4}$ Converges to $\frac{3}{4}$

5. a. Converges to 3

b. $5\sum_{n=0}^{\infty}(\frac{-1}{4})^{n+2} = 5\sum_{n=0}^{\infty}(\frac{-1}{4})^n(\frac{-1}{4})^2 = \frac{5}{16}\sum_{n=0}^{\infty}(\frac{-1}{4})^n$

Converges to $\frac{1}{4}$

6. a. $\lim_{n \to \infty} 1 + \frac{1}{n} = 1 \neq 0$ Diverges

b. $\lim_{n \to \infty} \cos(n\pi) = (-1)^n = DNE$ Diverges

c. $\lim_{n \to \infty} \frac{n}{e^n} = \lim_{n \to \infty} \frac{1}{e^n} = 0$ The test is inconclusive