Interference of Waves

Example #1

$\lambda = 0.3\,m$

$l = 300\,m$ $\mu = \frac{m}{l} = \frac{15}{300} = 0.05\,kg/m$

$m = 15\,kg$ $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{1000}{0.05}} = \boxed{141.42\,m/s}$

$T = 1000\,N$

$v = ?$

Example #2
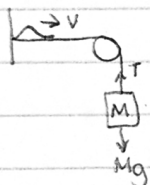
$m = 0.3\,kg$ $T - Mg = 0$ $T = Mg = 2(9.8) = 19.6\,N$

$l = 6\,m$ $\mu = \frac{m}{l} = \frac{0.3}{6} = 0.05\,kg/m$

$M = 2\,kg$ $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{19.6}{0.05}} = \boxed{19.8\,m/s}$

$v = ?$ $t = ?$ $x = vt$ $t = \frac{x}{v} = \frac{5}{19.8} = \boxed{0.25\,s}$
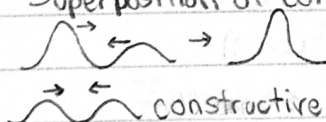
Superposition   Coherent

adding waves $\lambda_1 = \lambda_2,\ \omega_1 = \omega_2,\ f_1 = f_2,\ \Delta\phi_0 \leq \frac{\pi}{2}$

Superposition of coherent waves is interference

constructive   destructive

$$y(x,t) = a\sin(\underbrace{kx_1 - \omega t + \phi_{o1}}_{\phi_1}) \quad y_2(x,t) = a\sin(\underbrace{kx_2 - \omega t + \phi_{o2}}_{\phi_2})$$

$D(x,t) = y_1 + y_2 = a\sin\phi_1 + a\sin\phi_2 = 2a\cos\frac{\Delta\phi}{2}\sin\phi_{avg}$

$\phi_{avg} = kx_{avg} - \omega t + \phi_{oavg}$ $A = 2a\cos\frac{\Delta\phi}{2}$

$D = A\sin(kx_{avg} - \omega t + \phi_{oavg})$

Conditions for constructive: destructive:

$A_{max} = 2a\cos\frac{\Delta\phi}{2} = 2a$ $A_{min} = 0 \Rightarrow \cos\frac{\Delta\phi}{2} = 0$

$\frac{\Delta\phi}{2} = m\pi$ $\frac{\Delta\phi}{2} = (m + \frac{1}{2})\pi$

$\Delta\phi = 2m\pi$ $\Delta\phi = 2(m + \frac{1}{2})\pi$

$I = A^2 = 4a^2\cos^2\frac{\Delta\phi}{2} = 4I_0\cos^2\frac{\Delta\phi}{2}$ W/m^2
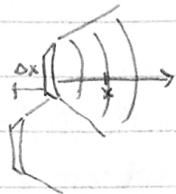
Example #3

$f = 500$ Hz

$a = 0.1$ mm $= 10^{-4}$ m

$\Delta x = 1$ m

$\Delta \phi_0 = \frac{\pi}{2}$ rad

$x = 2$ m

$A = ?$

$A = 2a\cos\frac{\Delta\phi}{2}$

$\Delta\phi = \phi_1 - \phi_2 = (kx_1 - \omega t + \phi_{01}) - (kx_2 - \omega t + \phi_{02})$

$\Delta\phi = k(x_1 - x_2) + \Delta\phi_0 = k\Delta x + \Delta\phi_0 = \frac{2\pi}{\lambda}\Delta x + \Delta\phi_0$

$\lambda = \frac{v}{f} = \frac{340}{500} = 0.68$ m

$\frac{2\pi}{0.68}(1+2) + \frac{\pi}{2} = 29.3$ rad

$A = 2(10^{-4})\cos\frac{29.3}{2} = -0.09$ mm $= \boxed{-0.09 \times 10^{-4} \text{ m}}$

Destructive because $A < a$
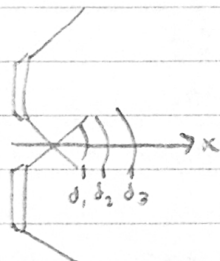
Example #4

$f = 170$ Hz

$v = 340$ m/s

$d_{1,2,3} = ?$

$\Delta\phi = k\Delta x + \Delta\phi_0 = k\Delta x = \frac{2\pi}{\lambda}d = 2(m + \frac{1}{2})\pi$
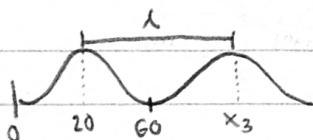
$d = \lambda(m + \frac{1}{2})$

$d_0 = \frac{\lambda}{2} = 1$ m $d_1 = \frac{3}{2}\lambda = 3$ m $d_2 = \frac{5}{2}\lambda = 5$ m

Example #5

$x_1 = 20$ cm

$x_2 = 60$ cm

$\lambda = ?$ $x_3 = ?$

$\frac{\lambda}{2} = 40$ cm $\boxed{\lambda = 80 \text{ cm}}$ $x_3 = 60 + 40 = \boxed{100 \text{ cm}}$