Kinematics and Dynamics of Rotational Motion PH 112

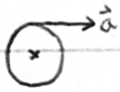
Step-by-step process

- Read the example a few times
- Write what is given or known
- Write what you are looking for
- Convert units
- Draw a diagram
- Solve in equations and find a final equation
- Plug in numbers to find a final result
- Check units
- Understand final result

Example #1

Given: $r = 0.4m$, $t_0 = 0$, $a = 4 m/s^2$, $t = 10s$

Looking for: number of revolutions N



$N = \frac{\Delta\theta}{2\pi r}$ $\alpha = \frac{a}{r} = \frac{4}{0.4} = 10 \ rad/s^2$

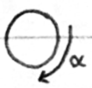
$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + 0(10) + \frac{1}{2}(10)(10)^2 = 500 \ rad$

$N = \frac{500}{2\pi(.4)} = \boxed{80 \ rev}$

Example #2

Given: $\omega_0 = 500 \ rad/s$, $\alpha = -0.5 \ rad/s^2$, $\omega = 0$

Looking for: time to stop, t



$\omega = \omega_0 + \alpha t$ $0 = 500 - 0.5t$ $t = \frac{\omega - \omega_0}{\alpha}$

$t = \boxed{1000s}$

Example #3

Given: $\omega_0 = 2 \ rad/s$, $t = 2s$, $\theta = 5 \ rev \cdot \frac{2\pi \ rad}{1 \ rev} = 10\pi \ rad$

Looking for: angular acceleration α

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $\alpha = \frac{2(\theta - \omega_0 t)}{t^2} = \frac{2(10\pi - 2(2))}{2^2} = \boxed{13.7 \ rad/s^2}$

Example #4

Given: $\omega_0 = 8$ rad/s, $t = 2.5$ s, $\omega = 0$

Looking for: angular displacement θ

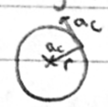
$\omega = \omega_0 + \alpha t \qquad \alpha = -\frac{\omega_0}{t} = -\frac{8}{2.5} = -3.2 \text{ rad/s}^2$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \qquad \theta = 8(2.5) + \frac{1}{2}(-3.2)(2.5)^2 = \boxed{10 \text{ rad}}$

Example #5

Given: $\alpha = 2$ rad/s², $\omega_0 = 0$, $r = 0.1$ m, $t = 0.6$ s

Looking for: accelerations a_{tan}, a_c



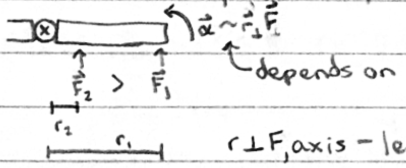
$a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2 = 0.1(1.2)^2 \quad \boxed{a_c = 0.14 \text{ m/s}^2}$

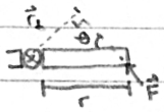
$\omega = \omega_0 + \alpha t = 2(0.6) = 1.2$ rad/s

$a_{tan} = r\alpha = 0.1(2) \quad \boxed{a_{tan} = 0.2 \text{ m/s}^2}$

Torque Concept

Mechanics < Kinematics – x, v, a, how? – θ, ω, α / Dynamics – F, why?, τ





$r_\perp = r\sin\theta$

$r \perp F$, axis – lever arm

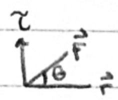
$\vec{\tau} = \vec{F}\vec{r}\sin\theta = \vec{r} \times \vec{F}$ (cross product)

$\vec{r}_\perp \vec{F} = \tau$ torque (Nm)

$|\tau| = rF\sin\theta \qquad \theta_\tau$ – right hand rule

curl fingers starting at r to F, thumb shows direction

(order is important!)



$\theta = 0 \Rightarrow$ no torque

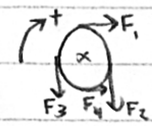
$\theta = 90° \Rightarrow$ max torque

Rotational analogy of the force

Produces angular acceleration, which depends on F and r

If several torques, α is the net torque

Can be positive or negative



$\alpha \sim \tau_{net}$

$\tau_{net} = \sum_{i=0}^{n} \tau_i$ Choose direction to determine +/-

$\tau_{net} = \tau_1 + \tau_2 - \tau_3 - \tau_4$