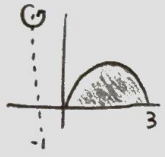
Cylindrical Shell Method
Section 6.2

Discussion: How to find the volume of the solid generated by revolving the region enclosed by $y = 3x - x^2$ and $y = 0$ about the vertical line $x = -1$?



$V = 2\pi(1+x)\,y\,\Delta x = 2\pi(1+x)(3x-x^2)\,\Delta x$

$\int_0^3 2\pi(1+x)(3x-x^2)\,dx = \frac{45\pi}{2}$

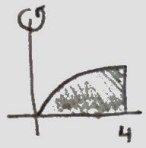
Shell formula for revolution about the vertical line:

The volume of the solid generated by revolving the region between the x-axis and the graph of a continuous function $y = f(x) \geq 0$, about a vertical line $x = L$, where $L \leq a \leq x \leq b$ is

$$V = \int_a^b 2\pi \text{ (shell radius) (shell height) } dx.$$

A similar formula hold for rotation about the y-axis.

Example #1: Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$ about the y-axis.

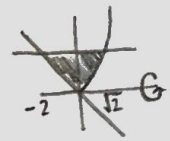


$V = 2\pi(x)(\sqrt{x})\,\Delta x$

$\int_0^4 2\pi x^{3/2}\,dx = \frac{128\pi}{5}$

Example #2: Find the volumes of the solids generated by revolving the regions bounded by

(a) $x = \sqrt{y}$, $y = -x$, and $y = 2$ about the x-axis.



$x = \sqrt{y}$

$x = -y$

~~$V = 2\pi y(-y) + 2\pi y(\sqrt{y})$~~

~~$\int_0^2 2\pi y^{3/2}\,dy - \int_0^{\sqrt{2}} 2\pi y^2\,dy =$~~

~~$2\pi\left[\frac{2}{5}y^{5/2}\right]_0^2 \quad 2\pi\left[\frac{y^3}{3}\right]_0^{\sqrt{2}}$~~

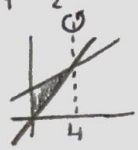
~~$\frac{4\sqrt{2}\pi}{3}$~~

$\int_0^2 2\pi y(\sqrt{y} + y)\,dy$

$\frac{16\pi\sqrt{2}}{5} + \frac{16\pi}{3}$

(b) $2y = x + 4$, $y = x$, and $x = 0$ about the line $x = 4$.

$y = \frac{x}{2} + 2$



$\frac{x}{2} + 2 = x$

$x + 4 = 2x$

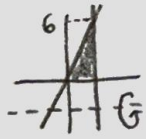
$4 = x$

$V = 2\pi(4 - x)(4 - \frac{x}{2} - 2)$

$\int_0^4 2\pi(4-x)(2-\frac{x}{2})\,dx = \int_0^4 \pi(4-x)^2\,dx$

$\frac{64\pi}{3}$

(c) $y = 3x$, $y = 0$, and $x = 2$ about the line $y = -2$.



$V = 2\pi(2 + y)(2 - \frac{y}{3})$

$\int_0^6 2\pi(2+y)(2-\frac{y}{3})$

48π

1. Draw the region and sketch a line segment across it parallel to the axis of rotation
2. Label the segment's height and distance from the axis of rotation
3. Find the limits of integration
4. Integrate 2π(shell radius)(shell height) with respect to the thickness variable

rotation	dimensions
vertical	$R(x), h(x)$
horizontal	$R(y), h(y)$

(d) $y = 3x$, $y = 0$, and $x = 2$ about the line $x = 2$

$R = (2 - x)$

$h = 3x$

$V = 2\pi(2 - x)3x$

$$\int_0^2 2\pi(2 - x)3x$$

8π

Which variable to integrate: