Rotational Kinematics

PH112

Rigid body doesn't change shape

Angular velocity - how fast the body rotates

$$\text{avg} = \bar{\omega} = \Delta\theta/\Delta t \text{ rad/s}$$

$$\text{inst} = \vec{\omega} = \lim_{\Delta t \to 0} \Delta\vec{\theta}/\Delta t = \frac{d\vec{\theta}}{dt} \text{ rad/s}$$

Does not depend on radius

Right hand rule to find direction - curl fingers in direction of rotation

Angular acceleration

$$\text{avg} = \bar{\alpha} = \Delta\omega/\Delta t \text{ rad/s}^2$$

$$\text{inst} = \vec{\alpha} = \lim_{\Delta t \to 0} \Delta\vec{\omega}/\Delta t = \frac{d\vec{\omega}}{dt} \text{ rad/s}^2 = \frac{d^2\theta}{dt^2} \text{ rad/s}^2$$

Does not depend on radius

Direction: $\alpha > 0$, $\vec{\alpha} \uparrow\uparrow \vec{\omega}$ same direction
$\alpha < 0$, $\vec{\alpha} \downarrow\uparrow \vec{\omega}$ opposite direction

Frequency - revolutions per second

$$f = \frac{\omega}{2\pi} \text{ Hz}$$

Period - time for one revolution

$$T = \frac{1}{f} \text{ s}$$

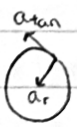
Relationship between v and ω

$$v = \frac{\Delta l}{\Delta t} \quad \theta = \frac{l}{r} \Rightarrow \Delta\theta = \frac{\Delta l}{r} \Rightarrow \Delta l = r\Delta\theta$$

$$\hookrightarrow r\frac{\Delta\theta}{\Delta t} = r\omega \qquad \vec{v} = r\vec{\omega}$$

Linear velocity depends on radius

Acceleration relationship



$$\vec{a}_r = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

$$\vec{a}_{tan} = \frac{\Delta v}{\Delta t} = \frac{r\Delta\omega}{\Delta t} = r\alpha$$

Linear acceleration depends on radius

Linear = rotational · radius

Uniformally accelerated motion	$\vec{a}$ = constant = $\vec{\alpha}$	Uniformally accelerated rotational motion
$v = v_0 + at$	$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\omega = \omega_0 + \alpha t$
$\bar{v} = \frac{v_0 + v}{2}$	$v^2 = v_0^2 + 2a\Delta x$	$\bar{\omega} = \frac{\omega_0 + \omega}{2}$
	$x \to \theta$, $v \to \omega$, $a \to \alpha$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
		$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$