5.4 FTC MA 112

$\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}e^{ax} = ae^{ax}$ $\frac{d}{dx}\sec x = \sec x \tan x$

$\frac{d}{dx}\cos x = -\sin x$ $\frac{d}{dx}\ln x = \frac{1}{x}$ $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$

$\frac{d}{dx}\tan x = \sec^2 x$

Ex: $\ln x$ is an antiderivative of $\frac{1}{x}$

2 is an antiderivative of $2x$

1a. $\int_0^{\pi/4} \sec x \tan x \, dx = \sec x \Big|_0^{\pi/4} = \sec\frac{\pi}{4} - \sec 0 = \sqrt{2} - 1$

b. $\int_0^1 \frac{1}{1+x^2} dx = \tan^{-1}(x)\Big|_0^1 = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

$\rightarrow \int_0^{\pi/3} \sec^2 x \, dx = \tan x \Big|_0^{\pi/3} = \tan\frac{\pi}{3} - \tan 0 = \sqrt{3} - 0 = \sqrt{3}$

c. $\int_{-2}^{2} |x| \, dx = \int_{-2}^{0} -x \, dx + \int_0^2 x \, dx = -\frac{x^2}{2}\Big|_{-2}^{0} + \frac{x^2}{2}\Big|_0^2 = 4$

In general $F'(x) = \frac{d}{du}\int_a^u f(t)\,dt \cdot \frac{du}{dx} = f(u)\cdot\frac{du}{dx}$; u is a function of x

2a. $f(t) = \frac{t}{t^2+1}$ $\frac{x}{x^2+1}$

b. $f(\theta) = \sin 3\theta$ $-\sin 3x$

c. $f(z) = 2z - 4$ $u = x^2 + x$ $du = 2x + 1$

$(2(x^2+x)-4)(2x+1) = 4x^3 + 6x^2 - 6x - 4$

d. $\int_1^{3x} (\frac{1}{t} + 2)\,dt$ $f(t) = \frac{1}{t} + 2$ $u = 3x$ $du = 3$

$3(1/3x + 2) = \frac{1}{x} + 6$

3. $g(c) = 3 \Rightarrow \frac{1}{2-0}\int_0^2 x^3 - x + 2 \, dx = \frac{1}{2}\Big| \frac{x^4}{4} - \frac{x^2}{2} + 2x \Big|_0^2 = \frac{1}{2}(\frac{2^4}{4} - \frac{2^2}{2} + 2(2))$

$= \frac{1}{2}(4 - 2 + 4) = 3$ ✓