The Matrix Transpose
Section 3.1 (**Hartman**)

Transpose
Let A be an $m \times n$ matrix. The transpose of A, denoted A^T, is the $n \times m$ matrix whose columns are the respective rows of A.

Example #1: Given $A = \begin{pmatrix} 2 & -3 & 0 \\ 3 & 1 & 5 \end{pmatrix}_{2\times 3}$. Find A^T. $\begin{pmatrix} 2 & 3 \\ -3 & 1 \\ 0 & 5 \end{pmatrix}_{3\times 2}$

The Diagonal, a Diagonal Matrix, Triangular Matrices
Let A be an $m \times n$ matrix. The diagonal of A consists of the entries $a_{11}, a_{22}, \ldots$ of A.

A diagonal matrix is an $n \times n$ matrix in which the only nonzero entries lie on the diagonal.

An upper (lower) triangular matrix is a matrix in which any nonzero entries lie on or above (below) the diagonal.

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{pmatrix} \qquad \begin{pmatrix} 2 & 0 & 0 \\ 3 & -1 & 0 \\ 5 & 1 & 5 \end{pmatrix} \qquad \begin{pmatrix} 2 & 4 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{pmatrix}$$

Diagonal | Lower Triangular | Upper Triangular

Example #2: For each of the following matrices A, what can we say about A^T?

(a) A is a diagonal matrix:
A^T is also diagonal
$A^T = 0$

(b) A is an upper triangular matrix:
A^T is a lower triangular

(c) A is a lower triangular matrix:
A^T is an upper triangular