Applications of Linear systems
Section 1.5 (**Hartman**)

Many problems that are addressed by engineers, business people, scientists and mathematicians can be solved by properly setting up systems of linear equations. In this section we highlight only a few of the wide variety of problems that matrix algebra can help us solve.

Example #1: A jar contains 100 blue, green, red and yellow marbles. There are twice as many yellow marbles as blue; there are 10 more blue marbles than red; the sum of the red and yellow marbles is the same as the sum of the blue and green. How many marbles of each color are there?

b = # of blue marbles
g = # of green marbles
r = # of red marbles
y = # of yellow marbles

$$b + g + r + y = 100$$
$$y = 2b$$
$$b = r + 10$$
$$r + y = b + g$$

$$\begin{array}{c} \begin{array}{cccc} b & g & r & y \end{array} \\ \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 100 \\ -2 & 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 10 \\ -1 & -1 & 1 & 1 & 0 \end{array}\right) \end{array} \xrightarrow{\text{reduce}} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 & 30 \\ 0 & 0 & 1 & 0 & 10 \\ 0 & 0 & 0 & 1 & 40 \end{array}\right)$$

$b = 20$
$g = 30$
$r = 10$
$y = 40$

Curve Fitting:

Example #2: Find the equation of the quadratic function that goes through the points $(-1, 6)$, $(1, 2)$ and $(2, 3)$.

$f(x) = ax^2 + bx + c$

$6 = a - b + c$

$2 = a + b + c$

$3 = 4a + 2b + c$

$a \neq 0$

$$\begin{array}{ccc} a & b & c \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 1 & 1 & 1 & 2 \\ 4 & 2 & 1 & 3 \end{array}\right) \xrightarrow{\text{reduce}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$a = 1$

$b = -2$

$c = 3$

$f(x) = x^2 - 2x + 3$

When there are more variables, we can use Maple to find the reduced row echelon form.

<u>Maple Commands:</u>

- To insert a matrix in Maple you can use the Matrix palette on the left of your worksheet and select the number of rows and columns.
- To find the reduced row echelon form, type;

  $with(LinearAlgebra)$:

  $A :=$ (type your Matrix here!)

  $ReducedRowEchelonForm(A)$

Balancing Chemical Reactions:
When balancing a chemical reaction, there must be equal number of atoms on both the sides of the equation. It needs to be balanced from both sides, due to the Law of the Conservation of Mass. Additionally, balanced equation is necessary in determining how much reactant you would need to have, for making the specific product. This simply means that the correct products will not be formed unless you add the right amount of reactants.

Example #3: Balance the following chemical reaction so that there is the same amount of each element (N, H, P, O and Pb) on both sides of the equation.

$$^{4}(NH_4)_3PO_4 + {}^{3}Pb(NO_3)_4 \to Pb_3(PO_4)_4 + {}^{12}NH_4NO_3$$

$$x_1(NH_4)_3PO_4 + x_2Pb(NO_3)_4 = x_3Pb_3(PO_4)_4 + x_4NH_4NO_3$$

$N: 3x_1 + 4x_2 = 2x_4$
$H: 12x_1 = 4x_4$
$P: x_1 = 4x_3$
$O: 4x_1 + 12x_2 = 16x_3 + 3x_4$
$Pb: x_2 = 3x_3$

$$\left(\begin{array}{cccc|c} 3 & 2 & 0 & -2 & 0 \\ 12 & 0 & 0 & -4 & 0 \\ 1 & 0 & -4 & 0 & 0 \\ 4 & 12 & -16 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array}\right) \xrightarrow{\text{reduce}} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & 1 & -1/12 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 = \frac{1}{3}x_4$
$x_2 = \frac{1}{4}x_4$ $\quad x_4$ is free
$x_3 = \frac{1}{12}x_4$

$x_1 = 4$
$x_2 = 3$
$x_3 = 1$
$x_4 = 12$