

Initial Value Problems

Section 1.2 (Noonburg)

Idea: When using DE's to model real-world phenomena, we want to have only one solution. we do this by adding constraints to the DE.

Definition: An n -th order differential equation with n initial conditions specified is called an **initial-value problem** (IVP). The solution of an IVP is called a particular solution of the DE.

Although we didn't learn any formal method to solve DE's, there are some simple equations we can solve by inspection.

Example #1:

(a) Solve $y' = 2x$ by inspection.

$$y = x^2 + C$$

(b) If $y(1) = 5$, what is the particular solution for part (a)?

$$5 = 1 + C$$

$$C = 4$$

$$y = x^2 + 4$$

Example #2: Solve the IVP, $y'' = 2x$, $y(0) = 4$, $y(3) = 7$.

$$y' = x^2 + C$$

$$y = \frac{1}{3}x^3 + Cx + D$$

$$4 = D$$

$$7 = \frac{1}{3}x^3 + Cx + 4$$

$$3 = \frac{1}{3}(27) + 3C = 9 + 3C$$

$$3C = -6 \quad C = -2$$

$$y = \frac{1}{3}x^3 - 2x + 4$$