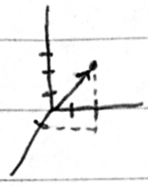
Vectors

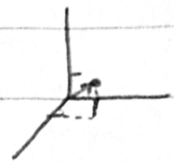
MA113



$v = \langle 1,2,3\rangle$

$= [1,2,3]$

$= \hat{x} + 2\hat{y} + 3\hat{z}$



Unit vector - a vector with one unit of length

$\hat{x}\ \hat{y}\ \hat{z}$

$\hat{i}\ \hat{j}\ \hat{k}$ $\hat{x}_1\ \hat{x}_2\ \hat{x}_3\ \hat{x}_4 \dots$

Length of $\langle 1,2,3\rangle = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

distance from the point to the origin

Magnitude $\|v\|$

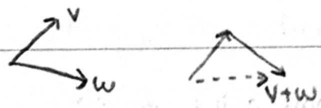
Almost every vector can be described uniquely by its magnitude and direction

$\langle 1,2,3\rangle = \underbrace{\sqrt{14}}_{\text{Magnitude}}\underbrace{\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\rangle}_{\text{Direction (unit vector)}}$ $v = \|v\|\,\hat{v}$

The zero vector doesn't have a direction

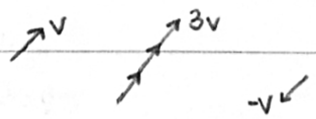
Addition

$\langle a,b,c\rangle + \langle d,e,f\rangle = \langle a+d, b+e, c+f\rangle$



Scalar multiplication

$s\langle v_1, v_2, \dots, v_n\rangle = \langle sv_1, sv_2, \dots, sv_n\rangle$



Dot product (scalar product)

$\langle v_1, v_2, v_3\rangle \cdot \langle w_1, w_2, w_3\rangle = v_1w_1 + v_2w_2 + v_3w_3$

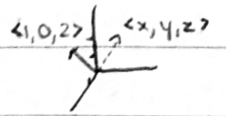
Find all vectors $\perp$ to $\langle 1,0,2\rangle$

$v\cdot w = \|v\|\,\|w\|\cos\theta$ ← between the vectors

$\perp \Rightarrow \theta = \pm\frac{\pi}{2} \Rightarrow \cos\theta = 0 \Rightarrow v\cdot w = 0$

If v and w are nonzero vectors, then they are perpendicular if $v\cdot w = 0$

↑ orthogonal



$\langle x,y,z\rangle \cdot \langle 1,0,2\rangle = 0$

$x + 2z = 0$ $\langle -2z, y, z\rangle$ Every vector of this form (except $y=z=0$) is orth. to $\langle 1,0,2\rangle$

$y\langle 0,1,0\rangle + z\langle -2,0,1\rangle$ A plane orth. to $\langle 1,0,2\rangle$