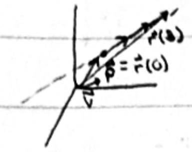
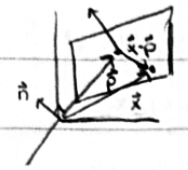
Lines and Planes

MA 113



Starting point (represented by $\vec{p}$)
Direction (represented by $\vec{v}$)

$\vec{r}(t) = \vec{p} + t\vec{v}$



$\vec{n}$ - normal to the plane
$\vec{x}$ - represents any point on the plane

$(\vec{x}-\vec{p})\cdot\vec{n}=0 \qquad (\langle x,y,z\rangle - \langle x_0,y_0,z_0\rangle)\cdot\langle a,b,c\rangle = 0$

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$ Standard form of a plane

Find an equation for the line normal to the plane $x+y+2z=1$ which passes through the origin

$\vec{n} = \langle 1,1,2\rangle \qquad \vec{r}(t) = t\langle 1,1,2\rangle = \boxed{\langle t,t,2t\rangle}$

Where does this line intersect the plane?

$t+t+4t=6t=1 \quad t=\frac{1}{6} \quad \boxed{\langle \frac{1}{6}, \frac{1}{6}, \frac{2}{3}\rangle}$

An object moves along the line $\vec{r}(t)=\langle t, 2t+1, -t\rangle$ where t=time. How fast is the object moving and what is the direction of its motion?

At what time is the object 5 units from its start point?

$\vec{r}(0)=\langle 0,1,0\rangle \leftarrow$ start point $\qquad \langle t,2t+1,-t\rangle = \langle 0,1,0\rangle + t\vec{v}$

$\vec{v}=\langle 1,2,-1\rangle \leftarrow$ velocity

$\boxed{\sqrt{6}\langle \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\rangle} \qquad \|\vec{v}\| = \sqrt{1^2+2^2+(-1)^2} = \sqrt{6} \qquad \text{direction} = \langle \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\rangle$

$\sqrt{6}t=5 \Rightarrow t=\boxed{\frac{5}{\sqrt{6}}}$ or $\|\vec{r}(t)-\vec{p}\|=5$

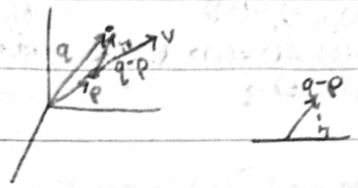
$\|\langle t,2t,-t\rangle\|=5$

$\pm t\sqrt{6}=5 \Rightarrow t=\boxed{\frac{\pm 5}{\sqrt{6}}}$

Distance from a point to a line

line $\vec{r}(t)=\vec{p}+t\vec{v}$

point $\vec{q}=\langle a,b,c\rangle$

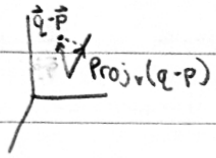


distance $=\|\vec{r}-\vec{q}\|=d(t)$

$\sqrt{(q_1-p_1-v_1t)^2+(q_2-p_2-v_2t)^2+(q_3-p_3-v_3t)^2}$

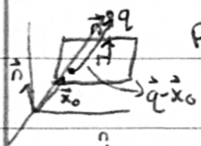
$\|\vec{q}-\vec{p}\|\sin\theta=\dfrac{\|(\vec{q}-\vec{p})\times\vec{v}\|}{\|\vec{v}\|}$

$\|(\vec{q}-\vec{p})\times\vec{v}\|$



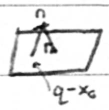
$\|\vec{q}-\vec{p}-Proj_v(\vec{q}-\vec{p})\|$

A particle travels along $\langle t, t+2, 0\rangle$, $t\geq 10$. How close does it come to a detector defined by the plane $x+y-z=2$?



Point $p=(x_0,y_0,z_0)$ Normal vector $\vec{n}=\langle a,b,c\rangle$

$\|Proj_n(q-x_0)\|$



a point $(1,1,0)$ a normal $\langle 1,1,-1\rangle$

$\langle t-1, t+1, 0\rangle \Leftarrow Proj_n$ $\qquad Proj_w(u)=\dfrac{u\cdot w}{w\cdot w}w$

$\dfrac{2t}{3}\langle 1,1,-1\rangle \qquad \dfrac{2t}{3}\|\langle 1,1,-1\rangle\|=\dfrac{2t\sqrt{3}}{3}$

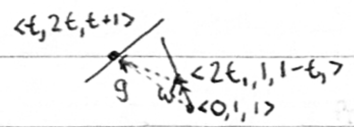
distance min @ $t=10$ $\boxed{\dfrac{20\sqrt{3}}{3}}$

* Line 1 $\langle t,2t,t+1\rangle$ $t\in\mathbb{R}$ Line 2 $\langle 2t,1,1-t\rangle$ $t\in\mathbb{R}$ Do these intersect?

$\langle t,2t,t+1\rangle=\langle 2t_1,1,1-t_1\rangle$ $\quad t=2t_1 \quad 2t=1 \quad t+1=1-t_1$

$t_1=\frac{1}{4} \quad t=\frac{1}{2} \quad t_1=-\frac{1}{2}$ Contradiction $\emptyset$

The lines do not intersect



How close do the lines come to one another?

$\langle 0,1,1\rangle+t_1\langle 2,0,-1\rangle$

$g=\langle t,2t-1,t\rangle$

$\|\langle t-2t_1, t-1, t+1-1+t_1\rangle\|=F(t,t_1)$ $\qquad \|g-Proj_w(g)\|=F(t)=Dist$