Integration by Parts
Section 8.2

<u>Main Idea:</u> Integration by parts is used to "undo" the product rule.

$\int u\,dv = uv - \int v\,du$

Example #1: $\displaystyle\int xe^{5x}\,dx$

$u = e^{5x}$ $\quad dv = x\,dx$

$du = 5e^{5x}\,dx$ $\quad v = \frac{x^2}{2}$ ← More complicated compared to the original

$e^{5x} \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot 5e^{5x}\,dx$

$\frac{e^{5x}}{5}\left(x - \frac{1}{5}\right) + C$

What do I pick for u/v'?

u should be differentiable repeatedly

dv should be integratable repeatedly

We expect to have $\int v\,du$ as an easy integral

Examples: Evaluate the following integrals:

1. $\displaystyle\int_1^e y^3 \ln(y)\, dy$

$u = \ln y \quad dv = y^3 dy$

$du = \frac{1}{y} dy \quad v = \frac{y^4}{4}$

$\ln y \cdot \frac{y^4}{4}\Big|_1^e - \int_1^e \frac{y^4}{4} \cdot \frac{1}{y} dy$

$\frac{e^4}{4} - \frac{e^4}{16} + \frac{1}{16}$

2. $\displaystyle\int \sqrt{x} e^{\sqrt{x}}\, dx = 2\int t^2 e^t\, dt$

$t = \sqrt{x}$

$dt = \frac{1}{2\sqrt{x}} dx$

$2t\, dt = dx$

$u = 2t^2 \quad dv = e^t\, dt$

$du = 4t\, dt \quad v = e^t$

$2t^2 e^t - \int 4te^t\, dt$

$u_1 = 4t \quad dv_1 = e^t\, dt$

$du_1 = 4dt \quad v_1 = e^t$

$2t^2 e^t - (4te^t - \int 4e^t\, dt)$

$2t^2 e^t - 4te^t + 4e^t + C$

$2xe^{\sqrt{x}} - 4\sqrt{x}\, e^{\sqrt{x}} + 4e^{\sqrt{x}} + C$

$e^{\sqrt{x}}(2x - 4\sqrt{x} + 4) + C$

3. $\int \ln(x)\,dx$

$u = \ln x \quad dv = dx$

$du = \frac{1}{x}dx \quad v = x$

$x\ln x - \int x \cdot \frac{1}{x}\,dx$

$x\ln x - \int dx$

$x(\ln x - 1) + C$

4. $\int e^{-x}\sin(x)\,dx = -e^{-x}\cos x - e^{-x}\sin x - \int e^{-x}\sin x\,dx$

$u = e^{-x} \qquad dv = \sin x\,dx$

$du = -e^{-x}dx \quad v = -\cos x$

$-e^{-x}\cos x - \int -\cos x(-e^{-x})\,dx$

$-e^{-x}\cos x - \int \cos x\, e^{-x}\,dx$

$u_1 = e^{-x} \quad dv_1 = \cos x\,dx$

$du_1 = -e^{-x}dx \quad v_1 = \sin x$

$-e^{-x}\cos x - (-e^{-x}\sin x + \int \sin x\, e^{-x}\,dx)$

$2\int e^{-x}\sin x\,dx = -e^{-x}\cos x - e^{-x}\sin x$

$\int e^{-x}\sin x\,dx = \dfrac{-e^{-x}(\cos x + \sin x)}{2} + C$