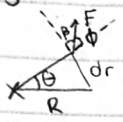
Work and Energy
Angular Momentum

PH112



$dW = F dr \cos\beta = F r d\theta \cos\beta = F r d\theta \cos(90-\phi) = F r d\theta \sin\phi$

$d\theta = \frac{dr}{r}$ $\quad dr = r d\theta$ $\quad \beta = 90-\phi$

$dW = \tau d\theta \Rightarrow W = \tau\theta$ J

$W_{net} = \Delta KE = \frac{1}{2} I(\omega^2 - \omega_0^2)$

Example #1

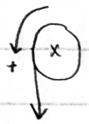
$\theta = 10$ rev $= 20\pi$ rad

$m = 40g = 0.04$ kg

$r = 10cm = 0.1$ m

$F = 10N$

$\omega = ?$



$W_{net} = \frac{1}{2} I(\omega^2 - \omega_0^2) = \frac{1}{2} I\omega^2$ ($\omega_0 = 0$)

$\omega = \sqrt{\frac{2W}{I}}$ $\quad I = \frac{1}{2}mr^2$ $\quad W = \tau\theta = Fr\theta$

$\omega = \sqrt{\frac{2Fr\theta}{\frac{1}{2}mr^2}} = \sqrt{\frac{4(10)(20\pi)}{0.04(0.1)}} = \boxed{792.7 \text{ rad/s}}$

$\vec{p} = m\vec{v}$

$\Sigma p_i = \Sigma m_i v_i$ $\quad v_i = r_i\omega$

$\Sigma m_i r_i \omega$ $\quad \Sigma p_i r_i = \Sigma m_i r_i^2 \omega \Rightarrow L = I\omega$

When $\vec{F}_{net} = 0$, $\vec{p}_i = \vec{p}_f$

When $\vec{\tau}_{net} = 0$, $\vec{L}_i = \vec{L}_f \Rightarrow I_1\vec{\omega}_1 = I_2\vec{\omega}_2$

$\tau \frac{dL}{dt}$

Example #2

$L_1 = I_1\omega_1$ $\quad L_2 = I_1\omega_2 + I_1\omega_2$

$I\omega_1 = 2I\omega_2$ $\quad \omega_2 = \boxed{\frac{\omega_1}{2}}$

Example #3

$L_1 = I_1\omega_1$ $\quad L_2 = I_2\omega_2$

$I_1\omega_1 = I_2\omega_2$ $\quad \boxed{\frac{I_1}{I_2} = \frac{\omega_2}{\omega_1}}$ Kinetic energy increases $\quad K = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$