Local Maximums and Minimums

MA113

$D_u f = \hat{u} \cdot \nabla f$

When $D_u f = 0$ for all u, $\nabla f = \vec{0}$

A point p is called a local minimum if $f(p) \geq f(x)$ for all x in some
maximum $f(p) \leq f(x)$
neighborhood of p

If at some interior point of a domain $\nabla f = 0$ or undefined, then the point is a critical point

If p is a critical point of f and in every neighborhood of p there are points x and y with $f(x) > f(p)$ and $f(y) < f(p)$, then p is neither a local min nor max; it is a saddle point

$f(x,y) = x^2 + y^2$ $\quad f \geq 0$ for all x and y $\quad f(0,0) = 0$

Local min at $(0,0)$

$\nabla f = \langle 2x, 2y \rangle$ $\quad f, \nabla f$ are defined for all x and y

$\vec{0} = \langle 2x, 2y \rangle = \langle 0, 0 \rangle \Rightarrow (x,y) = (0,0)$ $\quad f$ has only 1 critical point: $(0,0)$

$f(x,y) = 1 - (x-1)^2 - 3|y|$ $\quad f \leq 1$ for all x and y $\quad f(1,0) = 1$

Local max at $(1,0)$

$$\nabla f = \begin{cases} \langle -2(x-1), -3 \rangle, & y > 0 \\ \langle -2(x-1), 3 \rangle, & y < 0 \end{cases} \quad \text{undefined @ } y = 0, \quad \nabla f \neq \vec{0}$$

∞ many critical points

$\left.\begin{array}{l} f(x,y) = 2 + x^2 - y^2 \\ \nabla f = \langle 2x, -2y \rangle \end{array}\right\}$ Defined for all x and y.

$\vec{0} = \langle 2x, -2y \rangle \Rightarrow (x,y) = (0,0)$ $\quad$ 1 critical point

$f(0,0) = 2$

$f(x,0) = 2 + x^2 > 2$ for $x \neq 0$

∞ many values of $f > 2$

$f(0,y) = 2 - y^2 < 2$ for $y \neq 0$

∞ many values of $f < 2$

$(0,0)$ is a saddle point