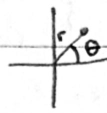
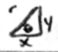
Polar Coordinates



r - distance from point to origin

θ - angle from positive horizontal axis in ccw direction



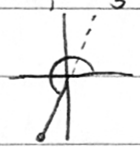
$x = r\cos\theta \quad y = r\sin\theta \quad r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x}$

Express $(x,y) = (-1,1)$ as a polar coordinate

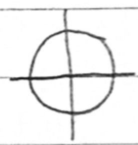
$r = \sqrt{2} \quad \phi = \frac{\pi}{4} \quad \theta = \frac{3\pi}{4}$

$(r, \theta) = (\sqrt{2}, \frac{3\pi}{4})$

$r = -2, \theta = \frac{\pi}{3} \Rightarrow r = 2, \theta = \frac{4\pi}{3}$



$x^2 + y^2 = 9$ - set of all points exactly 3 units from the origin



$r = 3$

$(r\cos\theta)^2 + (r\sin\theta)^2 = 9 \quad r^2(\cos^2\theta + \sin^2\theta) = 9$

$r^2 = 9 \quad r = \pm 3$

$y = x^2 \quad r\sin\theta = (r\cos\theta)^2 = r^2\cos^2\theta$

$\sin\theta = r\cos^2\theta$

$r = \frac{\sin\theta}{\cos^2\theta}$

$r = 0$ or $r = \frac{\sin\theta}{\cos^2\theta}$

Case 1: $r = 0$

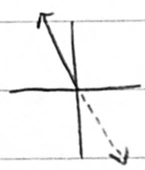
Case 2: $r \neq 0$

Case 2a: $\cos^2\theta = 0 \quad \emptyset$

Case 2b: $\cos^2\theta \neq 0$

$\theta = \frac{5\pi}{8}, \vec{r} \geq 0$

$-\infty < \overleftrightarrow{r} < \infty$



$a \leq r \leq b$ annulus

$\alpha \leq r \leq B$ annular sector

$y = mx + b$

$r\sin\theta = mr\cos\theta + b$

$r(\sin\theta - m\cos\theta) = b$

$r = \dfrac{b}{\sin\theta - m\cos\theta}$, $\sin\theta - m\cos\theta \neq 0$

$r = \dfrac{3}{2\cos\theta + \sin\theta}$ Case: $r = 0$ $\emptyset$

$r = \dfrac{3r}{2r\cos\theta + r\sin\theta}$, $r \neq 0$

$r = \dfrac{3r}{2x+y}$

$1 = \dfrac{3}{2x+y}$, $r \neq 0$

$2x + y = 3$

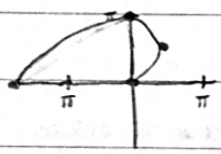
$r^2 - 3r + 2 = 0$

$(r-1)(r-2) = 0$

$r = 1, 2 \Rightarrow x^2 + y^2 = 1$, $x^2 + y^2 = 4$

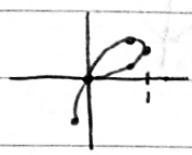
$r = 2\theta$

θ	r
0	0
$\frac{\pi}{4}$	$\frac{\pi}{2}$
$\frac{\pi}{2}$	π
π	2π



$r = \sin(3\theta)$

θ	r
0	0
$\frac{\pi}{12}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{6}$	1
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	0
$\frac{5\pi}{3}$	$-\frac{\sqrt{2}}{2}$



* Exact domain for one full rotation is important

Find the local maxima for $r=1+10\cos(4\theta),\ \theta\in[0,2\pi)$

$r'=-40\sin(4\theta)=0$

$\theta=\frac{n\pi}{4}\quad \theta=(0,1,2,3,4,5,6,7)\frac{\pi}{4}$

$1\pm10=\boxed{11,-9}$

Find the slope at $\theta=\frac{\pi}{3}$

$y=r\sin\theta=(1+10\cos4\theta)\sin\theta \qquad \frac{dy}{d\theta}=\frac{dy}{dx}\cdot\frac{dx}{d\theta}$

$\frac{dy}{d\theta}=\frac{dy}{d\theta}\Big/\frac{dx}{d\theta}$

$\frac{dy}{dx}=\frac{7\sqrt{3}}{9}\approx\boxed{1.347}$ (using Maple)