Understanding Instruments in Macroeconomics - A Study of High-Frequency Identification

Pooyan Amir-Ahmadi Christian Matthes
Mu-Chun Wang
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Abstract

The effects of monetary policy shocks are regularly estimated using high-frequency surprises in asset prices around central bank meetings as an instrument. These studies assume a constant relationship between the instrument and the monetary policy shock. By allowing for time variation in this relationship, we show that only a few distinct periods are informative about monetary policy shocks. We thus build a narrative for instrument-based identification and ameliorate weak identification problems. For the instrument in Gertler & Karadi (2015), the effect of monetary policy shocks on the (log) price level is almost 50 percent larger than the standard specification would suggest.

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1 Introduction

Identifying impulse responses via external instruments has become commonplace in empirical macroeconomics over the last decade (Stock & Watson 2012, Mertens & Ravn 2013, Gertler & Karadi 2015). These external instruments are interpreted as imperfect measurements of unobserved structural shocks. An instrument-based approach mitigates issues that can arise when using standard sign restrictions to identify monetary policy shocks, as highlighted by Wolf (2020).

A key assumption of the studies that use this approach is that there is a fixed, time-invariant relationship between the instrument and the shock of interest. However, in this paper we present evidence that for a common application of external instruments—the study of monetary policy shocks using high-frequency variation in asset prices around central bank announcements—there is actually substantial time variation in this relationship. To see this, Figure 1 plots the surprises in the three-month-ahead Fed Funds futures (FF4) in a 30-minute window around meetings of the Federal Open Market Committee (FOMC), an instrument popularized by Gertler & Karadi (2015) that we use as well. The figure shows there are periods where the dynamics of this instrument are substantially different from the rest of the sample, mainly the early 1990s, 2001, and during the Great Recession.

Building on this finding, we construct vector autoregressions (VARs) that explicitly capture this time variation, using the Bayesian approach for VARs with instruments (commonly called proxy VARs).[1] The pattern observed in Figure 1 can be explained parsimoniously by moving

[1]Time-varying identification strength is also a feature of the non-parametric framework in Rambachan & Shephard (2021).

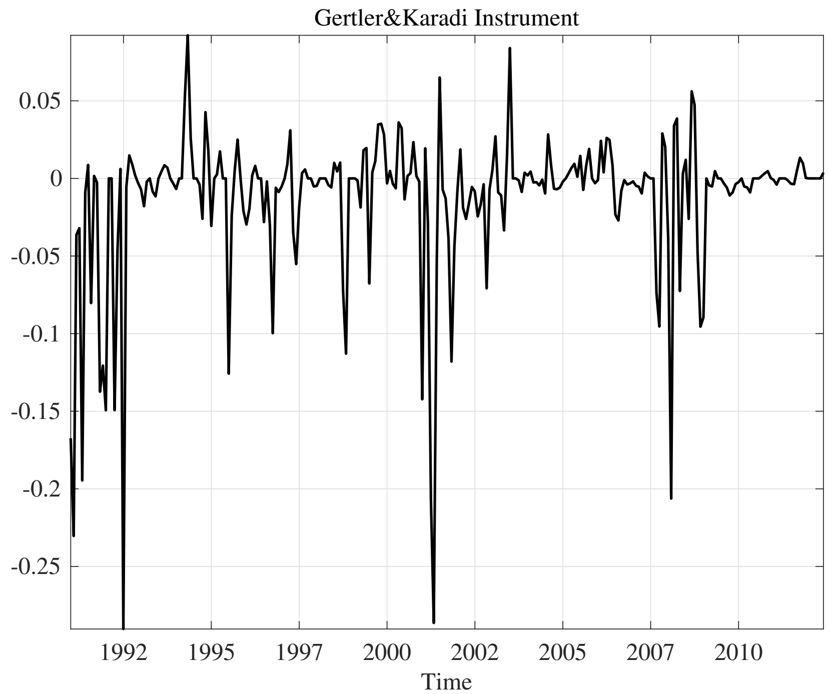


Figure 1: Surprise in 3-month-ahead Fed Funds futures (Gertler & Karadi 2015).

to a nonlinear measurement equation linking the instrument and the structural shock of interest, while maintaining a linear and Gaussian structure for the VAR itself to be comparable to the bulk of the literature.[2] We introduce this nonlinearity by allowing either changes in the volatility of the noise term or changes in the parameter multiplying the unobserved shock of interest.[3] These two assumptions translate changes in the volatility of the instrument into time variation in identification strength in opposite ways - volatile realizations of the instrument are deemed informative when we use the time-varying parameter approach, but are considered uninformative (i.e. a source of weak identification) when we estimate a model with stochastic volatility in the noise term. We

[2]In theory, time-varying volatility of monetary policy shocks could also lead to the pattern described here. We show in a Monte Carlo exercise in Section 4 that such a data-generating process would lead to estimates of time-varying parameters that are qualitatively very different to those obtained using U.S. data.

[3]Our approach comes at negligible additional computational cost relative to the previous literature.

show that for our specific application, assuming changes in the volatility of the noise term leads to results most economists will find questionable. First, a standard VAR estimated via ordinary least squares (OLS) delivers forecast errors that are highly correlated with the instrument during periods where volatility is high, but not otherwise. Second, the implied impulse responses in the case of stochastic volatility in the noise term are estimated with a large amount of uncertainty and have the wrong sign for the response of prices. We thus interpret changes in the volatility of the instrument through the lens of a model with time variation in the parameter linking instrument and monetary policy shock instead.

We discuss what the implications of various types of misspecification[4] are on estimates obtained using our preferred specification. These implications are at odds with our estimates using US data, which we confirm using Monte Carlo experiments.

Our approach yields two important insights. First, we can infer periods where the instrument is most informative about monetary policy shocks, thus helping to answer the question as to where identification is coming from and allowing us to develop a narrative for identification. As such, our approach can be seen as complementing the narrative sign restrictions approach of Antolín-Díaz & Rubio-Ramírez (2018), who impose identification via sign restrictions (and related restrictions) for certain periods only. In fact, as shown by Plagborg-Møller & Wolf (2021) and highlighted by Giacomini et al. (2022), narrative sign restrictions can be recast as binary instruments. Our approach instead identifies informative periods for a given instrument. We find, for a standard US instrument, that high-frequency-based instruments for monetary policy shocks

[4]One source of misspecification we study is stochastic volatility in the monetary policy shock itself.

are only relevant for a small number of distinct periods. We show that even when we set 90 percent of the instrument observations for the standard Gertler & Karadi (2015) instrument to zero (while keeping those periods our approach estimates to be the most informative), we can recover the same impulse responses as when we use all available observations. Second, because inference about the monetary policy shock is no longer contaminated by periods where the instrument is not actually informative (our algorithm discounts information contained in the instrument from these periods), we can gain a clearer picture of the effects of monetary policy shocks. Using the same instrument as Gertler & Karadi (2015) in our application yields, for example, effects on prices that are almost 50 percent larger after four years.
Error bands for impulse responses are generally *not* wider than their fixed coefficient counterparts. Even in applications where our approach yields similar impulse responses to the benchmark fixed coefficient approach (which is something that is not known a priori), the sharpening of the identification narrative can be crucial for interpreting the results.

The use of instruments in macroeconomics to identify the effects of monetary policy shocks was pioneered by Romer & Romer (2004), who estimate a sophisticated monetary policy rule using real-time data and obtain their instrument as the residual in that estimated monetary policy rule. More recently, the focus has shifted toward using instruments that are based on high-frequency variation in asset prices, first in event studies (Kuttner 2001, Gürkaynak & Wright 2013, Faust et al. 2007) and later as an instrument incorporated into time series models (Gertler & Karadi 2015, Jarociński & Karadi 2020, Caldara & Herbst 2019, Miranda-Agrippino & Ricco 2020), building on the work of Stock (2008) and Mertens & Ravn

(2013), who introduced the proxy VAR framework.[5] Other papers directly use information from high-frequency variation in asset prices around monetary policy decisions as a right-hand side variable for regressions to estimate the effects of monetary policy shocks (Campbell et al. 2016, Nakamura & Steinsson 2018).

Miranda-Agrippino & Ricco (2020) develop an instrument that is also based on high-frequency-based asset price variation around FOMC meetings, but further controls for information that the Federal Reserve had at the time of its meeting as well as possible autocorrelation in the instrument. We show in Section 3.4 that with this instrument, we also find relatively rare spikes in instrument relevance. The differences between the impulse responses using the standard approach and our method are substantially smaller with this instrument than with the Gertler & Karadi (2015) instrument. In fact, the impulse responses obtained using this instrument are similar to those obtained with our approach and the Gertler & Karadi (2015) instrument. However, our approach does not require central bank forecasts to clean the original instrument and thus provides a general purpose technology that can also be used for other instruments and shocks.

Section 2 lays out our framework and discusses various modeling choices. Section 3 discusses the results for the US, robustness with respect to the instrument used, and the modeling assumptions linking the instrument and the monetary policy shock. Section 4 discusses sources and consequences of misspecification in our model using Monte Carlo experiments, and Section 5 concludes.

[5] The use of this type of identification is becoming more common. For example, Känzig (2021) uses a high-frequency-based identification to identify oil shocks.

2 A VAR Model to Study Changes in Instrument Relevance

We set out to study the response of an n dimensional vector of observables $\mathbf{y}_t$ to a monetary policy shock e_t^{MP}, which is one element of the n dimensional vector of structural shocks $\mathbf{e}_t$.[6] To estimate said response, we use a structural vector autoregression (SVAR) in equation (1):

$$\mathbf{y}_t = \mathbf{c} + \sum_{\ell=1}^{\mathcal{L}} \mathbf{A}_\ell \mathbf{y}_{t-\ell} + \mathbf{\Sigma} \mathbf{e}_t, \tag{1}$$

where $\mathbf{e}_t \sim_{iid} N(\mathbf{0}, \mathbf{I})$.

The well-known identification problem in Gaussian structural VARs (Canova 2011, Baumeister & Hamilton 2015, Kilian & Luetkepohl 2018) implies that we need additional information to identify the column of the response matrix $\mathbf{\Sigma}$, which tells us how the elements of $\mathbf{y}_t$ respond to the monetary policy shock e_t^{MP}. The additional information that we exploit, following a substantial fraction of the recent literature in empirical macroeconomics, is an instrument m_t for the monetary policy shock e_t^{MP}. There are various frequentist (Mertens & Ravn 2013, Stock & Watson 2018) and Bayesian (Arias et al. 2021, Caldara & Herbst 2019, Drautzburg 2020) approaches to incorporating such information in an SVAR analysis.[7] Since our ultimate goal is to study possible changes in the relationship between the observable instrument m_t[8] and the unobserved monetary policy shock, we explicitly model the relationship between the

[6] We use boldface for vectors and matrices.

[7] Frequentist inference using proxies in dynamic factor models was introduced by Stock & Watson (2012).

[8] Instead of one scalar instrument, we could use multiple instruments. In that case we would, for example, need to make a decision about possible correlation in the error terms of the measurement equations.

instrument and the structural shock of interest. Within the standard homoskedastic VAR framework in the literature (Mertens & Ravn 2013, Caldara & Herbst 2019), there are two diametrically opposite assumptions that can generate the patterns observed in Figure 1.[9] We introduce these cases in Equations (2) and (3):

$$m_t = \beta_t e_t^{MP} + \sigma_v v_t. \tag{2}$$

$$m_t = \beta e_t^{MP} + \sigma_{v,t} v_t. \tag{3}$$

v_t is distributed independently and identically over time as $N(0,1)$ in either equation. Equation (2) accounts for the observed patterns in the instrument by allowing the coefficient on the monetary shock to change, while Equation (3) allows for changes in the noise variance. Although these alternatives look deceivingly similar, they lead to very different interpretations of the observed data, as we show below. Measurement equations of the kind we use are standard in the literature going back to Mertens & Ravn (2013), but the common assumption is that $\beta_t = \beta \forall t$ and $\sigma_{v,t} = \sigma_v \forall t$.[10]

The key identification assumptions are twofold. First, we assume that v_t is independent of all other shocks in our model, both the vector of structural shocks e_t and any shocks determining the evolution of β_t or $\sigma_{v,t}$, generally denoted as w_t.[11] Second, the instrument is informative for the

[9]Mumtaz & Petrova (2021) estimate time-varying parameter VARs with external instruments, but in their application the relationship between the instrument and the shock of interest is time invariant.

[10]One exception is Mertens & Ravn (2013), where the authors allow for censoring of the instrument (so the entire right hand side is multiplied by an indicator function), which is conceptually distinct from the type of time variation we study.

[11]As is common in the literature on time-varying VAR (Cogley & Sargent 2002, Primiceri 2005), we also assume that the structural shocks e_t are independent of the innovations to parameters w_t.

monetary policy shock, meaning that at least for some periods, $\beta_t \neq 0$. Our assumptions then imply

$$E\left[e_{j,t} m_t\right] = 0 \quad \text{for } j = 2, \ldots, n, \qquad \text{(exogeneity)} \qquad (4)$$

$$E\left[v_t \mathbf{e}_t\right] = 0 \quad \text{and} \quad E\left[v_t w_t\right] = 0, \qquad (5)$$

$$E\left[e_t^{MP} m_t\right] = \beta_t \neq 0 \quad \text{for some } t, \qquad \text{(relevance)} \qquad (6)$$

where $e_{j,t}$ denotes the jth element of $\mathbf{e}_t$.

Our Bayesian estimation approach is still valid even if $\beta_t = 0 \quad \forall t$ in Equation (2) or $\beta = 0$ in Equation (3). Our approach automatically approximates the posterior distribution of all time-varying parameters and the associated instrument reliability for each time period t. If those are always small (i.e., standard posterior bands include zero), we can infer that the instrument is weak.[12] We borrow the approach of directly estimating the parameters of this measurement equation from Caldara & Herbst (2019). Unlike in that paper, we allow for changes in parameters governing the systematic relationship between instruments and shocks.[13]

A useful summary statistic for assessing the strength of the instrument in different periods is a time-varying version of the common relia-

[12]Our approach also assumes invertibility of the monetary policy shock. For our monetary policy application, this seems to be a widely accepted assumption (Wolf 2020). For recent work on the link between inference using instruments and invertibility, see Miranda-Agrippino & Ricco (2022).

[13]Following Caldara & Herbst (2019), we normalize the relevant column of $\mathbf{\Sigma}$ so that the monetary policy shock increases interest rates on impact. Such a sign normalization is necessary for any structural VAR identification scheme. In our specific application, it furthermore allows us to center the prior for β_t or β at zero while still maintaining a standard interpretation of the estimated monetary policy shock.

bility (or relevance) statistic ρ_t:

$$\rho_t \equiv \begin{cases} \frac{\beta_t^2}{\beta_t^2+\sigma_v^2} & \text{if Equation (2) holds} \\ \frac{\beta^2}{\beta^2+\sigma_{v,t}^2} & \text{if Equation (3) holds} \end{cases} \tag{7}$$

This statistic represents the squared correlation between the instrument and the structural shock at time t and measures time-varying identification strength.

Equations (2) and (3) provide different theories for changes in volatility of the instrument m_t. These different theories have opposite effects on ρ_t: If an increase in volatility of m_t is driven by an increase in β_t this will lead to an increase in ρ_t, whereas an increase in $\sigma_{v,t}$ will lead to a decrease in ρ_t. The first specification hence interprets periods where the instrument is volatile as more informative for the identification of the effects of monetary policy shocks. The stochastic volatility specification instead discounts these periods because it attributes these fluctuations in the instrument to noise and instead identifies the effects of monetary policy shocks using periods with low instrument volatility.

In Section 3.1, we first compare three different specifications for time variation in parameters:[14]

$$\text{Constant: } \beta_t = \beta, \sigma_{v,t} = \sigma_v \tag{8}$$

$$\text{Time variation in Equation (2): } \beta_t = \beta_{t-1} + \sigma_\beta w_t \tag{9}$$

$$\text{Time variation in Equation (3): } \log(\sigma_{v,t}^2) = \log(\sigma_{v,t-1}^2) + \sigma_u w_t \tag{10}$$

[14]We focus on these diametrically opposite cases here. One could entertain stochastic volatility $\sigma_{v,t}$ and time-varying β_t jointly, but in that case the prior in the relative variability on β_t and $\log(\sigma_{v,t})$ would be crucial. More importantly, our choice of diametrically opposite cases helps us to interpret our findings.

In Equations (9) and (10), we assume that $w_t \sim_{iid} N(0,1)$. The first specification is a constant parameter specification reminiscent of Caldara & Herbst (2019) as a benchmark (equation (8)), the second specification is a Gaussian random walk specification for β_t in the tradition of the literature on time-varying parameters in state space models and VARs (Cogley & Sargent 2002, Primiceri 2005, Stock & Watson 2007), and the third specification is a standard specification for stochastic volatility, where log volatility follows a random walk (Kim et al. 1998, Cogley & Sargent 2002, Primiceri 2005). We choose these specification not only because they are common in the literature, but, more importantly, because estimates obtained using these specifications can capture many patterns of time variation even if the random walk specifications are misspecified (see, for example, Amir-Ahmadi et al. (2020)).

To approximate the posterior of our model, which consists of equations (1), (2), and one of the equations (8), (9), or (10), we modify the Metropolis-within-Gibbs sampling framework of Caldara & Herbst (2019) (the specification with equation (8) is exactly their specification). One important feature of our algorithm is that we do not require the same number of observations for the instrument m_t as for the macro variables collected in $\mathbf{y}_t$. Details about the algorithm can be found in Appendix A. In Appendix B, we show how our approach is related to, but distinct from, identification based on heteroskedasticity (Rigobon 2003).

3 Effects of Monetary Policy Shocks Identified Through High-Frequency Variation in Asset Prices

Section 3.1 studies the effects of allowing for time-varying reliability using the specifications outlined above. Crucially, we will make our case for modeling time variation in β_t *for this specific application* and analyze further variations of that specification. Section 3.2 uses a Markov-switching approach to model time variation in β_t for reasons that will become obvious once we study the posterior of β_t in our benchmark random-walk specification. In Section 3.3, we highlight that indeed only a few periods are informative for the effects of monetary policy shocks by estimating our model with an instrument that is set to zero except for the most informative periods. Finally, Section 3.4 estimates the effects of monetary policy shocks with the Miranda-Agrippino & Ricco (2020) instrument, the Bauer & Swanson (2022) instrument, and an alternative version of the Gertler & Karadi (2015) instrument.

3.1 The Effects of Time-Varying Reliability

We first contrast the constant parameter specification with the random walk specification for β_t and the stochastic volatility in noise specification. Our application uses US data:[15] y_t consists of the log of the Consumer Price Index (CPI), the log of industrial production (IP), the interest rate on one-year government bonds i, and the excess bond premium (EBP) (Gilchrist & Zakrajsek 2012). As Caldara & Herbst (2019) high-

[15]In Appendix F, we apply our preferred specification to UK data, using the high-frequency instrument of Cesa-Bianchi et al. (2020).

light, including a measure of financial conditions like the EBP in our VAR is crucial in order to get the effects of monetary policy right. The sample for y_t runs from July 1979 to June 2012. We follow Gertler & Karadi (2015) in our choice of the instrument m_t and use the surprise in the three-month-ahead Fed Funds futures around FOMC meetings (the series depicted in Figure 1). The sample for m_t is January 1991 to June 2012. We use 12 lags in all VARs estimated on US data in this paper.

To motivate our preference for the assumption of variation in β_t, it is instructive to study the relationship between the instrument and various forecast errors implied by a VAR. We estimate a version of our benchmark VAR via OLS to make sure the estimation does not use any information on the instrument (as it would when we estimate our benchmark model using Bayesian methods). We then compute the correlation between the OLS-based one-step-ahead forecast errors for the variables in our VAR and the Gertler & Karadi (2015) instrument, for both periods where, according to our models, the instrument is volatile and periods where it is not.[16] Identification schemes for structural VARs generally posit a linear relationship between these forecast errors and the structural shocks of interest. Thus, these are key correlations that are exploited whenever researchers use an instrument for identification in a structural VAR (as discussed before, standard proxy VARs assume a time-invariant relationship, so this correlation should be constant across subsamples).

Table 1 shows the correlation for each forecast error in a different column and each period in a different row. In periods where our approach identifies the instrument as not volatile, the absolute value of the corre-

[16]More specifically, we use the periods that our model with time variation in β_t identifies as informative as described in Section 3.3. Using the model with stochastic volatility to identify these volatile periods leads to very similar results.

lation between the instrument and the (one-step-ahead) forecast errors decreases by at least 70 percent and by as much as 82 percent, depending on the variable.[17] Furthermore, the absolute decrease in correlation is meaningful (a fall in correlation of 0.34 for inflation and EBP, for example). Not only is the correlation between the instruments and forecast errors stronger when the instrument is volatile, in periods where the instrument is volatile, the *signs* of the correlation between the instrument and the forecast errors in the VAR are more in line with correlations implied by standard New Keynesian theories - a contractionary shock raises interest rates, but lowers prices. The sign of the correlation for IP might seem unusual, but we will see a small initial positive impulse response of IP to a monetary policy shock (confirming the sign of the correlation) that quickly turns negative in our preferred specification below.

Table 1: Correlation between instrument and forecast errors in OLS version of our VAR.

	i	CPI	IP	EBP
High volatility in m_t	0.48	-0.34	0.10	0.46
Low volatility in m_t	0.14	0.06	0.03	0.12
Percent reduction in (absolute) correlation	0.71	0.82	0.70	0.74

For the rest of the analysis, we use a Bayesian approach. The priors we use throughout are standard in the literature and are described in detail in Appendix A. We make the priors as comparable as possible across the different specifications: The same parameters always have the same priors. Furthermore, the prior for β in the fixed coefficient variant is the same as the prior for β_0 in the random walk specification. Estimation results for models with time-varying coefficients can often be somewhat

[17] The slightly awkward use of the absolute value of the correlation is necessary because the correlation of the instrument with CPI forecast errors becomes positive when the instrument is not informative.

sensitive to the choice of prior for the innovation standard deviations σ_β and σ_{vol} in the law of motion for the parameter. This parameter governs the amount of time variation. Sensitivity is less of an issue here because (i) we only have one time-varying parameter (in contrast to papers where all VAR parameter can vary, such as Cogley & Sargent 2002 and Primiceri 2005), and (ii) we only have *either* time-varying parameters or stochastic volatility in our models, which helps sharpen inference. Nonetheless, to make sure that this is not an issue, we follow some of our previous work (Amir-Ahmadi et al. 2020) and estimate the hyperparameters that enter the priors for σ_β and σ_{vol}. Details on the priors can be found in Appendix A.

We first analyze the case of time variation in β. Figure 2 shows the posterior path of β_t and ρ_t. We plot the corresponding elements of the fixed coefficient version in gray. We show the posterior median as well as 68 percent equal-tail posterior bands.[18] It is striking that there are few short periods of high instrument relevance when allowing for time variation in β.[19] Three periods stand out, which we now discuss in turn. The first period is the first half of the 1990s. It is useful to point out that the large posterior value of β_t at the beginning of the sample for the instrument is not driven by our prior as our prior for the initial value of β_t is centered at zero. Instead, the first half of the 1990s was characterized by relatively high inflation at the beginning as well as a (mild) recession. Our model highlights the period coming out of the 1990s recession when annual CPI inflation was still high in 1991 (4.2 percent) as a period where

[18]All posterior bands in this paper are 68 percent posterior bands.

[19]The posterior median reliability of our time-varying parameter specification is almost always larger than its fixed coefficient counterpart because the estimated variance of the noise part is substantially larger in the fixed coefficient version—in the fixed coefficient case part of the time variation is soaked up in the noise term. This is also evident from the posterior of β_t.

the Federal Reserve was surprisingly accommodative (see Figure 1).

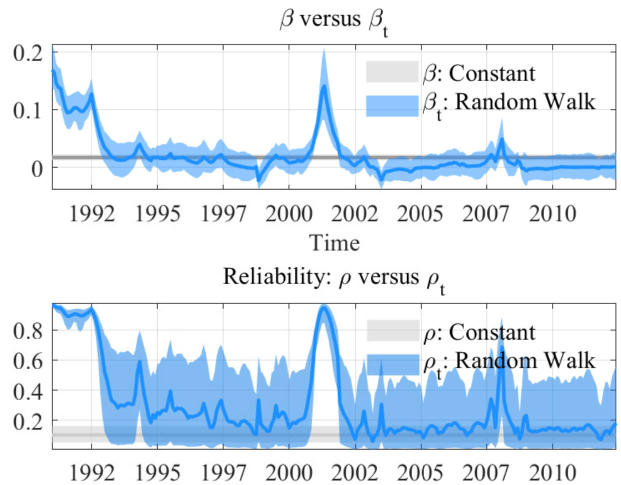


Figure 2: Posterior of β_t and ρ_t (median and 68 percent posterior bands).

The second period our model highlights is in 2001, driven by two intermeeting rate changes in January and April of 2001.[20] The third period of high instrument relevance is the Great Recession around 2008. Our framework thus helps us understand what information is contained in the instruments. We next examine if this time variation in instrument relevance matters for impulse responses.

Figure 3 shows the impulse to a one standard deviation monetary policy shock under the fixed coefficient (gray) and random walk (blue) specifications. We plot the posterior median as well as the 68 percent error bands. For bond yields, IP, and the EBP, the impulse responses are similar. For log CPI, the differences are instead *substantial*. With fixed coefficients, we see a price puzzle appearing, whereas this is not the case at all for the posterior median of the impulse responses when we allow

[20]The rate change around September 11th 2001 is not part of our instrument series as most financial markets were closed until the rate change on September 17th 2001, making it impossible to compute the changes in Fed Funds futures needed to construct the instrument. We thank Eric Swanson for pointing this out to us.

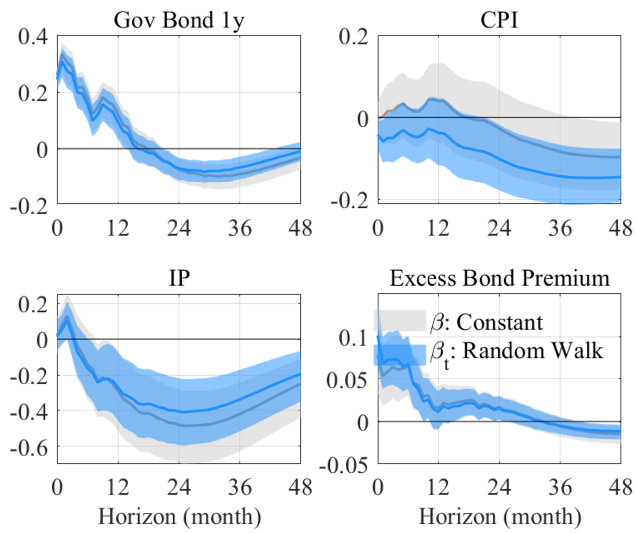


Figure 3: Impulse responses (median and 68 percent posterior bands) to a one standard deviation monetary policy shock, time variation in β.

for time variation in instrument relevance. Furthermore, the response of log CPI is larger in magnitude—after four years, the posterior median of the response is almost 50 percent larger when we allow for time variation in instrument relevance. Our approach discounts periods where the instrument is not informative and can, hence, lead to substantially different impulse responses. As mentioned before, in our example this does not come at a cost in terms of the width of the error bands.

We now contrast these results with the case of stochastic volatility in the noise term. Figure 4 shows that the same volatile periods that were previously identified as high β_t periods are now identified as periods with large noise volatility and basically zero reliability ($\rho_t = 0$). In light of the correlation structure with VAR forecast errors that we analyzed above, it is then not surprising that the resulting impulse responses, as displayed in Figure 5, show no meaningful response in prices or Industrial Production with the posterior median response for prices being positive for the first three years. In light of these findings, we will focus on time variation in β for the rest of the paper.

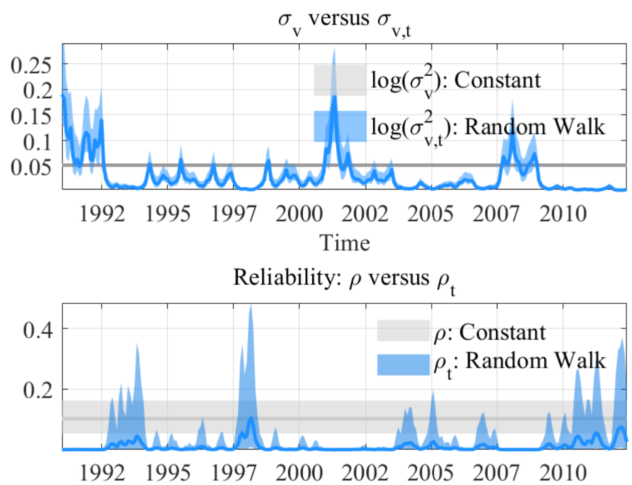


Figure 4: Posterior of $\sigma_{v,t}$ and ρ_t (median and 68 percent posterior bands).

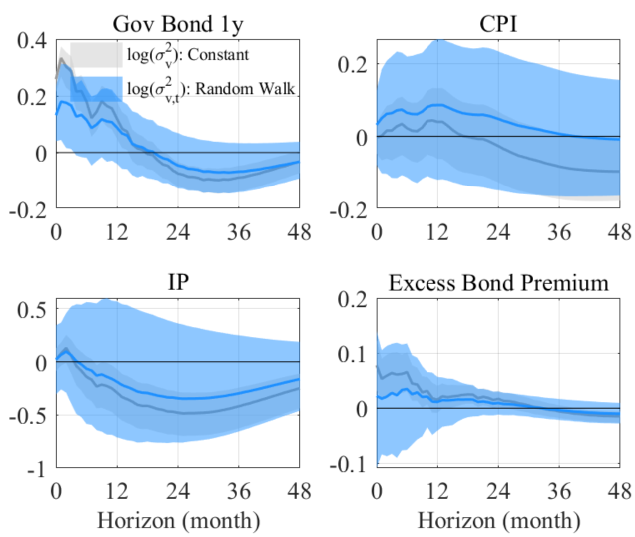


Figure 5: Impulse responses (median and 68 percent posterior bands) to a one standard deviation monetary policy shock, time variation in σ_v.

Modeling changes in β_t means that volatile periods of the instrument are interpreted as informative events; with stochastic volatility in the noise term, they are be interpreted as noise. In most of our applications, it turns out that the instruments are generally not very informative (low

ρ_t) except possibly for clearly delineated short periods of high instrument volatility. Thus, using stochastic volatility in noise implies a prior that, in these specific applications, puts very little faith in the instruments. This stands in contrast to standard priors in the proxy VAR literature (Arias et al. 2021, Caldara & Herbst 2019) that imply that the instruments are indeed useful/reliable. Our preferred assumption of time-varying parameters instead of time variation in the volatility of the noise term σ_v can thus also be seen as a context-specific prior choice that implies at least *some* instrument reliability, in line with the previous literature.

3.2 A Markov-Switching Alternative

Recalling Figure 2, one potential criticism that could be raised is that the estimated path for β_t might be better characterized by a Markov-switching model (Hamilton 1989, Sims & Zha 2006). We think of the random walk as our benchmark exactly because it is flexible enough to approximate many patterns of time variation, including sudden changes as observed in Figure 2. Nonetheless, we next estimate a two-state Markov-switching specification and show that it yields very similar results. The only difference between the Markov-switching specification and the random walk benchmark is the law of motion for β_t as detailed in equations (9) and (11), respectively.

$$\text{Markov Switching in Equation (2): } \beta_t = \beta_{s_t}, Pr(s_t = i | s_{t-1} = j) = p_{ij}. \tag{11}$$

Figure 6 shows the impulse response of log CPI to a one standard deviation monetary policy shock in the two-state Markov-switching model for β_t. We focus here on the response of CPI because that is where the major differences between fixed coefficient and time-varying parameter results occurred in the previous section. That impulse response is very similar to the random walk specification.

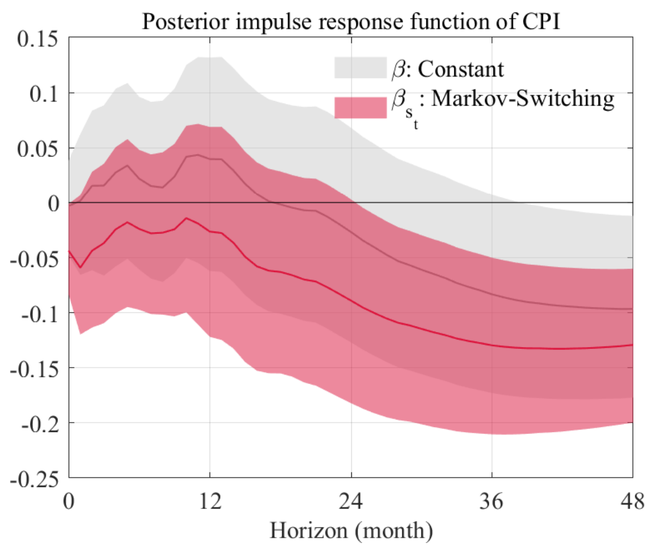


Figure 6: Impulse response of log CPI for Markov-switching specification (median and 68 percent posterior bands).

Figure 7 illustrates instrument relevance for our benchmark random walk specification in blue and the two-state Markov-switching model in red. We can see that both specifications identify largely the same periods of high instrument relevance. The random walk specification is somewhat conservative in that it has fewer spikes, but this does not lead to any meaningful difference in impulse responses, as discussed above. The choice for a specific law of motion for β ultimately comes down to the application in mind as well as preferences. We recommend the random walk as a default choice because of its flexibility.

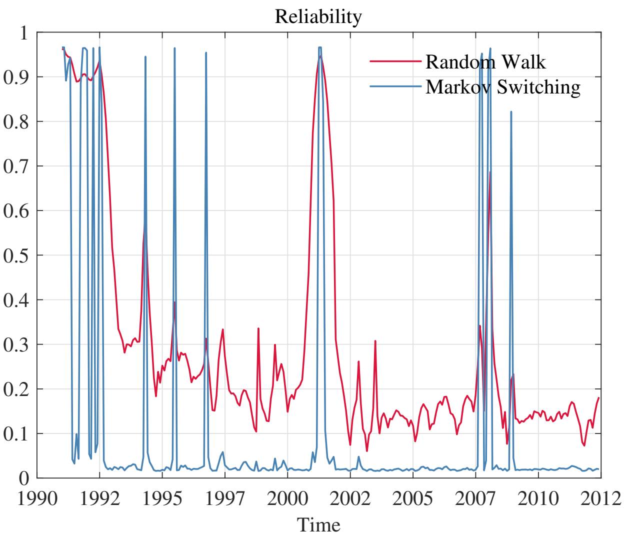


Figure 7: Posterior reliability for Markov-switching and random walk specifications (posterior median).

3.3 Shutting Down Periods Where the Instrument Is Informative/Uninformative

To get a better sense of the role that periods with high instrument relevance play in shaping the posterior distribution of the impulse responses, we now carry out two diametrically opposite thought experiments. First, we compute the posterior probability that $\beta_t = 0$ for each time period t, using the approach in Koop et al. (2010) and our original instrument m_t. We then create two instruments, $\tilde{m}_t$ and $\overline{m}_t$, from our instrument according to the following two rules:

1. $\tilde{m}_t = m_t$ if $Pr(\beta_t = 0) < 0.5$, $\tilde{m}_t = 0$ else

2. $\overline{m}_t = m_t$ if $Pr(\beta_t = 0) \geq 0.5$, $\overline{m}_t = 0$ else

$\tilde{m}_t$ only keeps the original realizations of the instrument that our model deems informative, whereas $\overline{m}_t$ only keeps relatively uninformative realizations, thus exacerbating weak identification problems. The threshold probability of 0.5 only selects the early 1990s and 2001 as informative periods.

Figure 8 shows the results when we use $\tilde{m}_t$ as our instrument. For the sake of comparison, the fixed coefficient VAR in this figure uses our original instrument m_t. We see that our approach still estimates the same periods to have high instrument relevance.[21] The impulse responses (we highlight CPI in this figure but show all responses in the Appendix) are very similar to those in our original setting, making clear that it is indeed *only* those high instrument relevance periods that inform the impulse responses. Naturally, this result depends on the specific application. Had the instrument relevance been reasonably high outside of the spikes in the instrument relevance we document, the procedure in this section would have led to a meaningful loss of information.

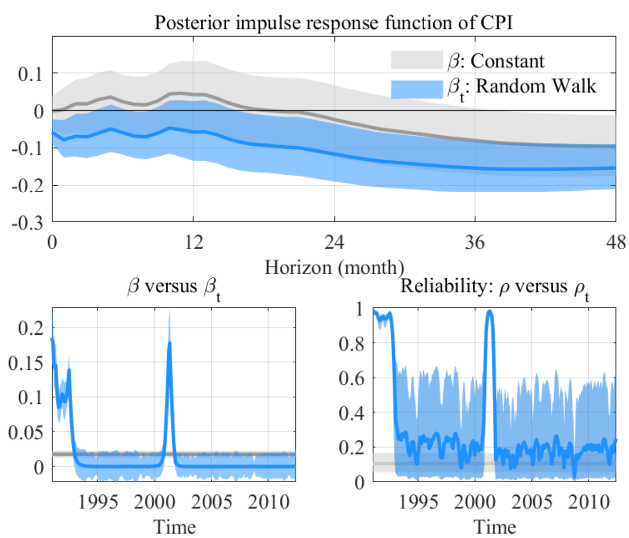


Figure 8: Results for $\tilde{m}_t$ (median and 68 percent posterior bands). The fixed coefficient VAR is based on the original m_t instrument.

Figure 9 shows the corresponding results when we only keep the original instrument if its relevance is low. Zero is now included in the 68 percent posterior bands for all horizons. Posterior instrument relevance

[21]To economize on notation, we also call this parameter β_t, but it is a different object from β_t when we use the instrument m_t.

is low for all periods, implying that identification is weak throughout the sample. This is true even though we keep 90 percent of the observations from the original sample because there is little information contained in those observations. In the Appendix (Figures A-9 and A-10), we show impulse responses obtained with a fixed coefficient VAR and these modified instruments—the resulting impulse responses are basically indistinguishable from the responses obtained with a fixed coefficient VAR and our original instrument. In order to effectively exploit the instrument when it is informative, we need to allow for time variation so that the instrument is not used for identification when it is not informative.

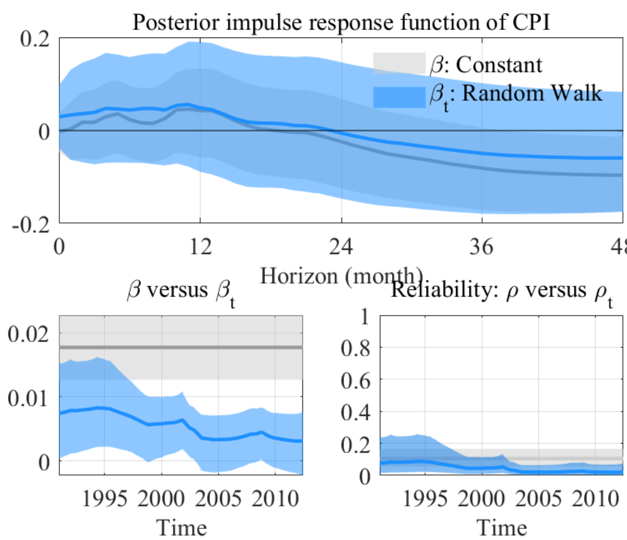


Figure 9: Results for $\overline{m}_t$ (median and 68 percent posterior bands). The fixed coefficient VAR is based on the original m_t instrument.

3.4 Alternative Instruments

Our various observation equations that link m_t and the unobserved monetary shock all imply that m_t is iid, borrowing from Caldara & Herbst (2019). Other papers in the literature (Arias et al. 2021, Plagborg-Møller & Wolf 2021) have posited more flexible relationships where the instrument can be contaminated by past macro variables and/or lags of the instrument. To assess whether this is an issue in our application, we progress in two steps. First, we regress our instrument on two lags of itself and the variables y_t in the VAR. The key results are summarized in Figure 10 and are very similar to our benchmark. The only difference is that the spike in ρ_t and β_t surrounding the Great Recession is less pronounced. The impulse response of CPI is basically unchanged (other impulse responses can be found in Appendix E).

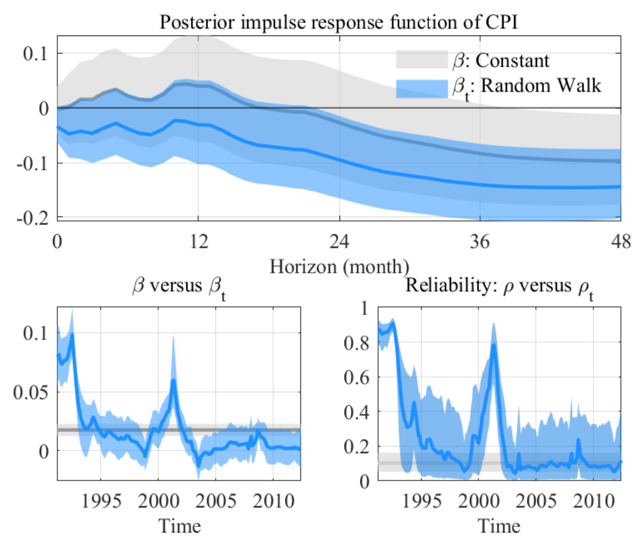


Figure 10: Results for the case of the modified instrument (median and 68 percent posterior bands).

We further use the instrument introduced by Miranda-Agrippino & Ricco (2020). The authors start off with a high-frequency-based instrument like our benchmark choice, but remove any autocorrelation and information available to the FOMC at the time of their meetings (as encoded in the Green Book). The sample for this instrument is January 1991 to December 2009; it is shorter due to the need for Green Book data, which is published with a lag. Figure 11 shows that, the largest spike in β_t by far appears around 2001 for this instrument. We still see a clear tightening of the error bands for instrument reliability in the early 1990s and around the Great Recession as well, but these movements are less pronounced than in our benchmark.

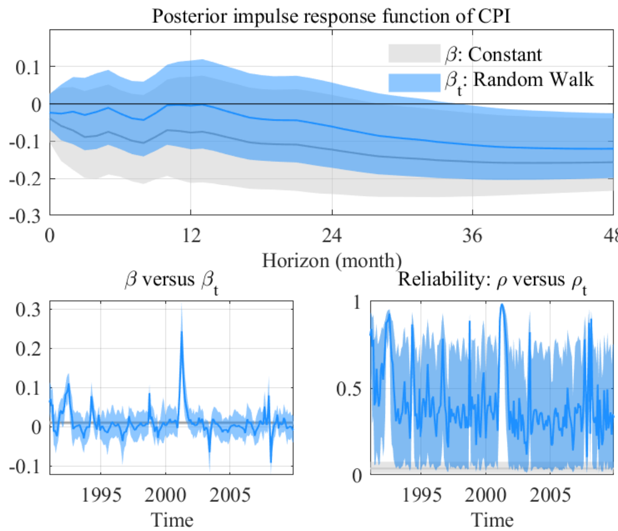


Figure 11: Results for the Miranda-Agrippino & Ricco (2020) instrument (median and 68 percent posterior bands).

Interestingly, the posterior median path of instrument reliability is substantially higher than in our benchmark or in Figure 10. Since the reliability does not fall as much between spikes as in our benchmark, it is not surprising that the difference between the fixed coefficient version of the impulse response of log CPI and its random walk counterpart are very similar—and both are similar to the random walk–based results from our benchmark instrument. While using our approach with the Miranda-Agrippino & Ricco (2020) instrument does not change the conclusions in terms of impulse responses, it adds substantial interpretability. For example, the most informative period (largest β_t value and largest instrument reliability) is around 2001.

As highlighted recently by Bauer & Swanson (2022), instead of controlling for forecasts contained in the Green Book, one can alternatively use private sector forecasts. Figure 12 plots the corresponding results using that instrument instead. Similar to the results based on the Miranda-Agrippino & Ricco (2020) instrument, we find little difference between impulse responses based on a fixed coefficient VAR and the specification with time-varying β.[22] This is reassuring for users of those instruments, but might lead some readers to wonder why they should consider our method in the first place. Our method endogenously cleans instruments, requires substantially less domain-specific knowledge to construct a valid instrument, and data requirements are less strict (we don't require specific variables to clean the original instrument), which leads to our approach being a useful alternative in other applications where cleaned instruments are not available. Importantly, our approach allows users to build a narrative that explains what time periods are important for identification, something that was absent from the previous literature. For the specific application in this paper, the results in this section also serve as a validity check on the popular instruments proposed by Miranda-Agrippino & Ricco (2020), Bauer & Swanson (2022) and highlight why results can differ substantially across various instruments.

[22] In contrast to the results based on Miranda-Agrippino & Ricco (2020) or our benchmark results, reliability is relatively constant over time, but also substantially smaller.

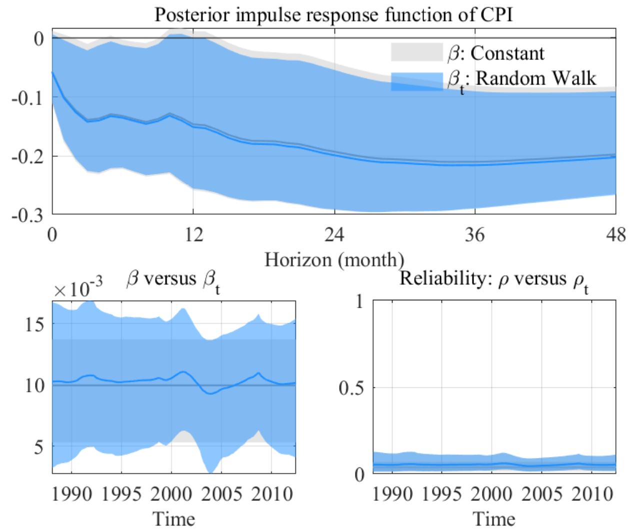


Figure 12: Results for the Bauer & Swanson (2022) instrument (median and 68 percent posterior bands).

4 What Happens When Our Model Is Misspecified?

We next turn to discussing possible sources and consequences of misspecification: What happens if the true data-generating process features changes in volatility in either the noise term v_t or the monetary policy shock, but we estimate a model with time variation in β? We first present some general deliberations for the case of stochastic volatility in monetary policy shocks before studying these issues in further detail using Monte Carlo experiments. One possible source of confusion in the discussion of stochastic volatility in structural shocks is that we can always rewrite the measurement equation linking the instrument and the monetary policy shock as featuring time-varying parameters when the true data-generating process features changes in the volatility of monetary policy shocks. This does *not* mean that the two specifications are equivalent - a homoskedastic VAR specification will be misspecified when the monetary policy shock features stochastic volatility. This leads to different implications for estimated parameter paths of β_t, allowing us to argue that stochastic volatility in monetary policy shocks is not what drives our results. To see this, assume that the true data generating process for the instrument is

$$m_t = \overline{\beta} e_t^{MP} + \sigma_v v_t \tag{12}$$

where the true monetary policy shock e_t^{MP} features changes in volatility σ_t^{MP} and $\overline{\beta}$ is a fixed parameter. We now derive an equivalent representation of the instrument equation that features changes in β_t:

$$m_t = \beta_t \overline{e}_t + \sigma_v v_t \tag{13}$$

where $\overline{e}_t$ is the standardized monetary policy shock that has fixed variance 1 (as in our empirical model assumes). β_t in this case equals $\overline{\beta}\sigma_t^{MP}$.[23] With knowledge of all parameters in the stochastic volatility case (including the entire time path of shock volatility) we can, thus, derive this equivalent representation of the measurement equation. When there are volatility changes in the data-generating process, we now show that our approach will not recover the unit variance shock $\overline{e}_t$. Hence, our approach will not recover the implied $\beta_t = \overline{\beta}\sigma_t^{MP}$. The estimated path of β_t will, instead, be more muted.

To see this, we can rewrite the VAR equation (1) in slightly more compact notation as

$$\mathbf{y}_t = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{\Sigma}\mathbf{e}_t \tag{14}$$

[23]As a side note, in order to get a value of β_t near 0 in this specification (as in our estimated results for US data) we would need the volatility $\overline{\beta}\sigma_t^{MP}$ to go to zero.

where $\mathbf{x}_{t-1}$ collects the relevant lags of $\mathbf{y}_t$ as well as the constant term. Let us assume that the true data-generating process also takes this VAR form with parameter matrices $\mathbf{A}^*$ and $\mathbf{\Sigma}^*$, but with one element of the true structural shocks $\mathbf{e}_t^*$ (namely the true monetary policy shock e_t^{MP}) having non-constant variance σ_t^{MP}. We now study how this affects our estimates of the structural shocks when the estimated model does not feature changes in volatility. Suppose we have at hand parameter values $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{\Sigma}}$ (we can think about a draw from the posterior or a point estimate). Then the implied estimate of the vector of structural shocks $\tilde{\mathbf{e}}_t$ is

$$\tilde{\mathbf{e}}_t = \tilde{\mathbf{\Sigma}}^{-1}([\mathbf{A}^* - \tilde{\mathbf{A}}]\mathbf{x_{t-1}} + \mathbf{\Sigma}^*\mathbf{e}_t^*) \tag{15}$$

It is known that misspecification of the distribution of the structural shocks in linear VARs does not asymptotically bias the estimates of the dynamics of the VAR (Petrova 2022), so asymptotically under standard conditions all elements of $\mathbf{A}^* - \tilde{\mathbf{A}}$ will be zero except possibly for those related to the constant term. In large samples, variation in $\tilde{\mathbf{e}}_t$ is, thus, driven by variation in the true shocks $\mathbf{e}_t^*$, and the estimated shocks $\tilde{\mathbf{e}}_t$ will inherit stochastic volatility from the true shocks $\mathbf{e}_t^*$.

If $\mathbf{\Sigma}^* - \tilde{\mathbf{\Sigma}} = \mathbf{0}$, we recover the true structural shocks with stochastic volatility and as a result in large enough samples our estimates of β_t will converge to a neighborhood of $\overline{\beta}$ because we have the true structural shock as a right hand-side variable in the step of the Gibbs sampler that estimates the path of β_t (this is the aforementioned muted response relative to the implied value of β_t that we derived at the beginning of the section). Stochastic volatility in the true structural shock of interest consequently has different implications for parameter estimates in our model compared to a data-generating process with changes in β_t that do not come from stochastic volatility, even if these assumptions seem similar at first sight.

With estimation error or bias in $\tilde{\Sigma}$ and, due to the sample size, possibly estimation error in $\tilde{\mathrm{A}}$, this argument will not hold exactly, but, as we show in our Monte Carlo study below, holds approximately for a realistic data-generating process and sample size.[24]

We also show by means of these Monte Carlo experiments that our approach performs as well as the standard fixed coefficient approach when it is misspecified and there is either stochastic volatility in the measurement equation that is unrelated to monetary policy or there is stochastic volatility in the monetary policy shock. However, those specifications for the data-generating process lead to estimated paths of β_t that are inconsistent with our findings based on US data, hence providing evidence for our modeling assumptions. While we could, in theory, use marginal likelihoods to compare those different model specifications, we instead choose to use Monte Carlo-based evidence because marginal likelihoods can be substantially influenced by tail behavior of priors even if that tail behavior is inconsequential for most objects of interests, such as posterior error bands for impulse responses. This dependence on priors becomes particularly pronounced when priors are relatively loose, prompting us to instead focus on Monte Carlo experiments.

[24]As far as there is meaningful time variation in the posterior of β_t in this scenario, our estimation will put more weight on periods with high estimated β_t to infer the effects of monetary policy shocks — periods that in this scenario are actually periods with high volatility of the monetary policy shock—, while discounting periods with low estimated β_t — periods with low volatility of the monetary policy shock.

4.1 Monte Carlo Studies

We follow Wolf (2020) and use the Smets & Wouters (2007) model as a laboratory.[25] We use three observables in our VAR: Output, inflation, and nominal interest rates. Since the Smets-Wouters model is a quarterly model, we set the lag length in our VAR to four. We simulate the instrument using equation (2) for various specifications of β_t, the volatility of the monetary shock, and the volatility of the measurement error σ_v.[26]

The first Monte Carlo experiment uses time variation in the measurement equation of the instrument that is along the lines of our estimated models (even though the random walk specification that we use here is still misspecified). The data-generating process features parameter changes in β_t. We choose an extreme scenario where β_t can only take on the values zero and one. The path of β_t in the data-generating process is fixed across all Monte Carlo samples for this first experiment.

We simulate 100 samples of length 250 using the posterior mode as in Wolf (2020). For approximately 10 percent of those periods,[27] we set $\beta_t = 1$ in the data-generating process; otherwise it is zero (hence the instrument is just noise). Figure 13 shows the true impulse response to a one standard deviation monetary shock in black as well as the Monte Carlo average of the 68 percent posterior bands for our approach and the fixed

[25] Wolf (2020) studies an instrumental variables approach in VARs, but in addition to his assumption of fixed coefficients, there are two substantial differences relative to our setup: We use a standard sample size in our simulations, whereas Wolf (2020) studies population properties, and we introduce measurement error in our instrument, which we calibrate to have 25 percent of the variance of the actual monetary shock in our first experiment. Details on the exact calibration of the data-generating processes can be found in Appendix C.

[26] One difference between the equation used to simulate the instrument and equation (2) in our estimated model is that the simulated monetary policy shock does not have unit variance, a point we come back to below.

[27] From periods 1 to 4, 141 to 149, and 231 to 250.

coefficient version. Our results confirm those of the population analysis in Wolf (2020): Even without perfect invertibility, the true responses are well approximated by our VAR. The fixed coefficient version generally has wider error bands, which leads the average posterior bands for output to include 0 for all horizons and a more pronounced probability of a price puzzle for inflation.

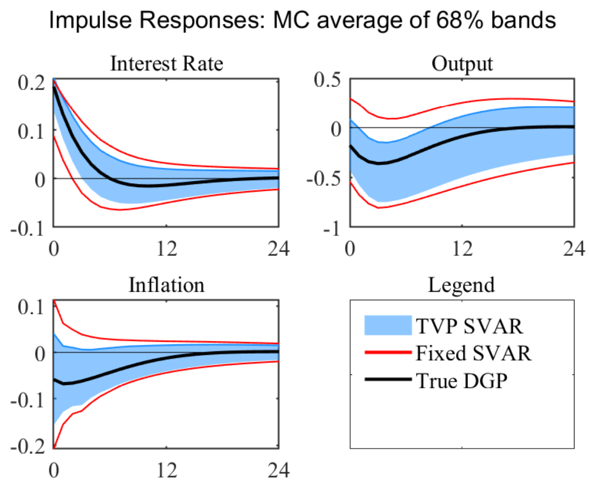


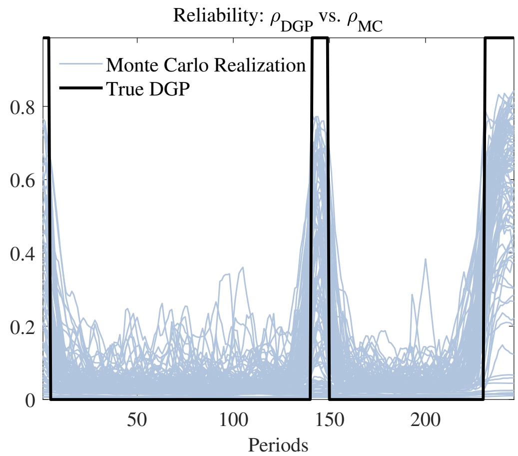
Figure 13: Impulse responses for the data-generating process with time-varying β_t and the Monte Carlo replications, first Monte Carlo experiment.

Figure 14a shows the estimated posterior median of instrument reliability in blue and the true reliability path in black (periods with high values of β, and hence higher instrument volatility, are denoted by gray bars). Our approach captures the changes in instrument reliability well.

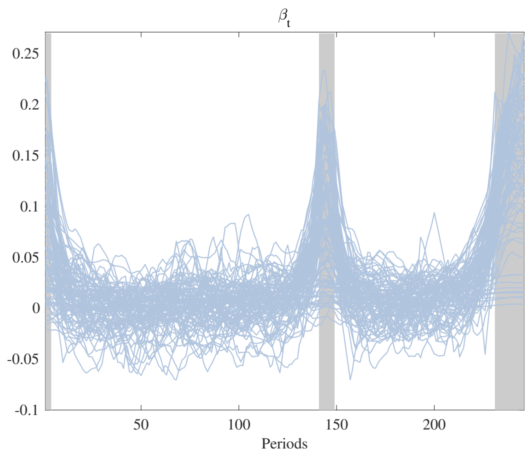
It is instructive to directly study the posterior median paths of β_t. As we discuss in more detail in Appendix C, we must rescale the true β_t values described there by the standard deviation of the monetary shock to make them comparable to our estimation results since our estimated model assumes monetary policy shocks with unit variance.[28] In this

[28] All scale effects in our estimated model are captured by Σ.

Monte Carlo exercise, the properly rescaled true β_t values (which can be directly compared to the estimated values) are 0 and 0.23 (the original values multiplying the monetary policy shock with non-unit variance were 0 and 1).



(a) Reliability (posterior medians).



(b) Posterior medians of β_t.

Figure 14: Reliability and posterior paths of β_t, first Monte Carlo experiment.

The posterior paths shown in Figure 14b closely resemble the true paths and, more importantly, the patterns we find in US data: Close to 0 for most periods, with distinct increases when the true value is non-zero.

Next, we study two more Monte Carlo experiments where our specification of the measurement equation for the instrument is more severely misspecified. We focus here on the posterior impulse responses and the posterior median paths for β_t. First, we simulate the data so that there is stochastic volatility in the measurement error v_t. We choose parameter values to keep the overall volatility of the instrument at each point in time to be the same as in the benchmark case discussed immediately above (see Appendix C for details). Figure 15 shows that our approach performs as well as the fixed coefficient version that is standard in the literature even though both are misspecified in different ways. It is not surprising that both specifications do reasonably well in this specification since β_t is always non-zero in the data-generating process, so the instrument conveys some information about the true monetary policy shock each period, in contrast to the previous experiment.

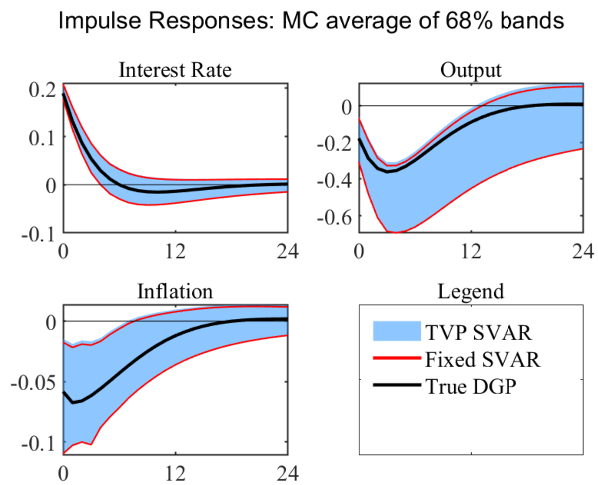


Figure 15: Impulse responses for the data-generating process and the Monte Carlo replications: Stochastic volatility in measurement error.

Turning to the posterior paths of β_t, the properly rescaled true β value in this experiment (which can be directly compared to the estimated values) is $0.5 * 0.23 \approx 0.12$. The posterior paths plotted in Figure 16 are remarkably stable and do generally not hover around zero, but instead are close to the true value of 0.12, a stark contrast to the posterior paths obtained using US data.

Finally, we ask how our approach fares when confronted with data where there is stochastic volatility in the true monetary policy shock.[29] We again keep the paths of the instrument's volatility the same as in our benchmark specification. Figure 17 shows that our approach again performs as well as the fixed coefficient version.[30]

[29] We solve the model linearly and then change the volatility of the monetary policy shock in some periods, along the lines of Justiniano & Primiceri (2008).

[30] Since the volatility of the monetary policy shock changes over time in the data-

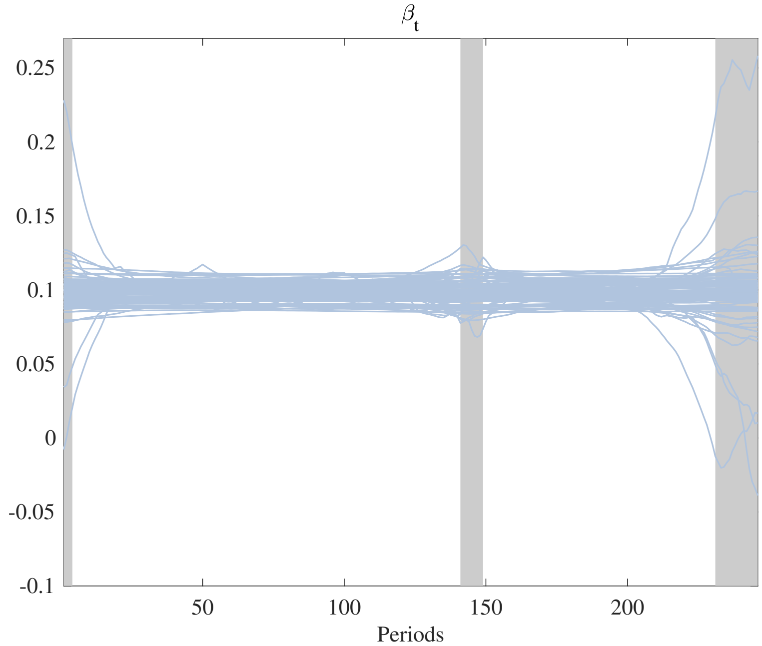


Figure 16: Estimated paths of β_t in the Monte Carlo runs (posterior medians) when the data-generating process features time variation in the volatility of v_t.

We find estimated paths for β_t that do not change much over time and do not decrease toward zero, in contrast to those obtained using US data or in the first experiment (see Figure 18), but in line with our previous discussion. Due to the relative stability, it is then also not surprising that our approach does as well as the fixed coefficient approach in this case. Comparing across experiments, the reason that both algorithms have a harder time identifying significant effects in this experiment is due to our calibration implying that the monetary policy shock is less volatile in most periods compared to the second experiment. This final Monte Carlo experiment involves one subtlety that we alluded to previously and was not present in the earlier experiments: Having knowledge of the true data-generating process, we could rescale the true β_t value (which is con-

generating process, we scale the impulse response for the data-generating process so that the variance of the monetary shock is the simple average of the two possible realizations of the variance we consider, while the estimated impulse responses are still one-standard deviation shocks according to the estimated standard deviation.

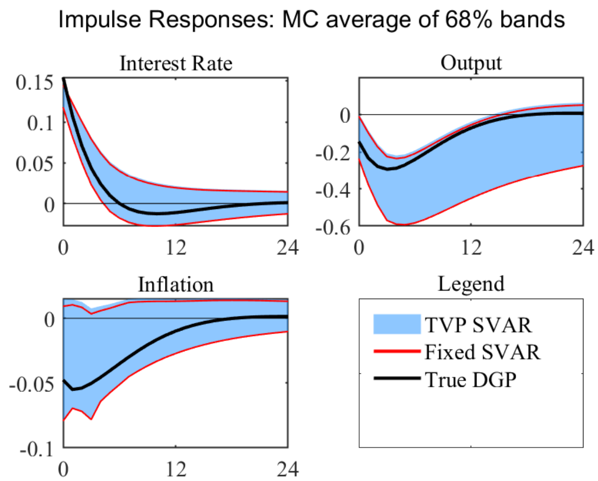


Figure 17: Impulse responses for the data-generating process and the Monte Carlo replications: Stochastic volatility in the monetary policy shock.

stant) by taking into account the changing volatility of the true monetary shock to give β_t values of 0.12 and 0.26 that multiply rescaled versions of monetary policy shock that have unit variance throughout the sample. As we have shown above, such a transformation is always possible in models with stochastic volatility. This does, however, not mean that this last experiment is similar to the first experiment: In the first experiment the VAR in equation (1) is correctly specified, whereas in this last experiment the VAR is misspecified because it assumes constant volatility of the shocks. As discussed before, our estimated monetary policy shock (which in a correctly specified world would have unit variance) inherits changes in volatility via the VAR that is estimated to have fixed coefficients and fixed forecast error variance.[31]

Hence the estimated movements in β_t are more muted than what we would get if the true unit variance (i.e., rescaled) monetary policy shock

[31]To convince readers that the issue of rescaling the monetary policy shock has no impact on our results, we show in Appendix C.3.1 that the estimated impulse responses remain unchanged in a hypothetical scenario where the instrument is directly linked to the period-by-period rescaled (and thus unit variance) monetary policy shock.

from the data-generating process were observable and we would directly estimate the measurement equation for the instrument. Nonetheless, the average level is broadly in line with the rescaled values discussed above.

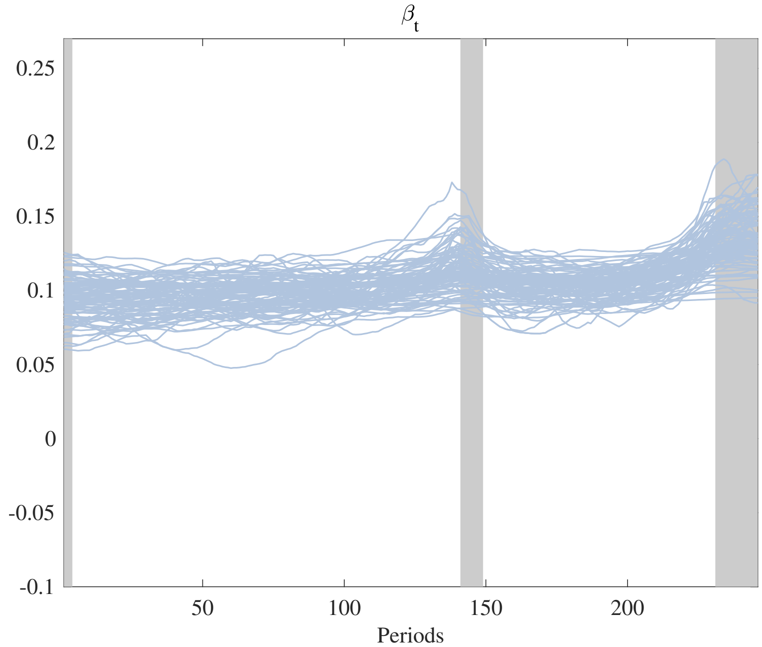


Figure 18: Estimated paths of β_t in the Monte Carlo runs (posterior medians) when the data-generating process features time variation in the volatility of the monetary policy shock.

Summing up, only the first Monte Carlo experiment can qualitatively capture the patterns of the posterior β_t paths that we obtained in our empirical applications. We view this as substantial evidence in favor of our modeling assumptions.

5 Conclusion

In this paper, we study how instrument relevance changes over time in a common application of instrument-based identification in structural VARs. We find substantial time variation in instrument relevance, thus allowing us to isolate periods where instruments are informative, which helps to build a narrative for a given instrument. Furthermore, our approach can substantially alter conclusions by discounting periods where the instrument is not informative, as in the case of the Gertler & Karadi (2015) instrument. As a practical recommendation, we show in our application that "cleaning" an instrument and removing periods that are not informative will generally not help unless a researcher is willing to model time variation in instrument relevance.

While we focus on monetary policy shocks in our application, the estimation approach we develop is general and can be used for any application of external instruments in VARs, such as the effects of government spending shocks, tax shocks, or financial shocks.

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Online Appendix to "Understanding Instruments in Macroeconomics - A Study of High-Frequency Identification"

A Algorithms and Priors

A.1 Time Varying Parameter

The first three steps of the algorithm follows exactly Algorithm 1 of Caldara & Herbst (2019), whose notation we largely borrow. The law of motion of β_t is given by

$$\beta_t = \beta_{t-1} + w_t, w_t \overset{iid}{\sim} N\left(0, \sigma_w^2\right).$$

In addition, we assume following priors:

$$\begin{aligned} p\left(\sigma_w^2\right) &\sim IG(\tau/2, \tau q/2). \\ p(\beta_0) &\sim N(b_0, V_0). \end{aligned}$$

The scale parameter q of the IG prior is crucial for controlling the time variation. We follow the procedure outlined in Amir-Ahmadi et al. (2020) to estimate this parameter.

Our VAR can be stated in companion form as

$$\mathbf{Y}_t = \Phi \mathbf{X}_t + \mathbf{U}_t \tag{A-1}$$

where $\mathbf{Y}_t$ stack current and lagged values of our vector of observables $\mathbf{y}_t$, $\mathbf{X}_t$ contains lags of $\mathbf{Y}_t$ as well as a vector of ones to capture the intercept, and $\mathbf{U}_t \sim_{iid} N(\mathbf{0}, \check{\Sigma})$.

Algorithm 1. For $i = 1, \ldots, N$. At iteration i

1. Draw $\check{\Sigma}, \Phi \mid \mathbf{Y}_{1:T}, \mathbf{M}_{1:T}, \Omega^{i-1}, \beta_{1:T}^{i-1}, \sigma_v^{i-1}, \sigma_w^{i-1}, q^{i-1}$. For $\check{\Sigma}$ we use a mixture proposal distribution (suppressing dependence on parameters for notational convenience):

$$q\left(\check{\Sigma} \mid \check{\Sigma}^{i-1}\right) = \gamma p\left(\check{\Sigma} \mid \mathbf{Y}_{1:T}\right) + (1-\gamma)\mathcal{IW}\left(\check{\Sigma}; \check{\Sigma}^{i-1}, d\right)$$

   where $p\left(\check{\Sigma} \mid \mathbf{Y}_{1:T}\right)$ is the known posterior distribution of $\check{\mathbf{\Sigma}}$ under $\mathbf{Y}_{1:T}$ and $\mathcal{IW}\left(\cdot; \check{\Sigma}^{i-1}, d\right)$ is an Inverse Wishart distribution with scaling matrix $\check{\Sigma}^{i-1}$ and d degrees of freedom. For Φ we use the known distribution $p\left(\Phi \mid \mathbf{Y}_{1:T}, \check{\Sigma}\right)$ as a proposal in an independence MH step:

   - Draw $\check{\Sigma}^*$ according to $q\left(\check{\Sigma} \mid \check{\Sigma}^{i-1}\right)$.
   - Draw Φ^* according to $p\left(\Phi \mid \mathbf{Y}_{1:T}, \check{\Sigma}^*\right)$.
   - With probability α, set $\Phi^i = \Phi^*$ and $\check{\Sigma}^i = \check{\Sigma}^*$, otherwise set $\Phi^i = \Phi^{i-1}$ and $\check{\Sigma}^i = \check{\Sigma}^{i-1}$, defined as

$$\alpha = \min\left\{\frac{p\left(\mathbf{M}_{1:T}, \mathbf{Y}_{1:T}, \Phi^*, \check{\Sigma}^*, \Omega^{i-1}, \beta^{i-1}, \sigma_\nu^{i-1}\right) p\left(\check{\Sigma}^*\right)}{p\left(\mathbf{M}_{1:T}, \mathbf{Y}_{1:T}, \Phi^{i-1}, \check{\Sigma}^{i-1}, \Omega^{i-1}, \beta^{i-1}, \sigma_\nu^{i-1}\right) p\left(\check{\Sigma}^{i-1}\right)} \frac{q\left(\check{\Sigma}^{i-1} \mid \check{\Sigma}^*\right)}{q\left(\check{\Sigma}^* \mid \check{\Sigma}^{i-1}\right)}, 1\right\}$$

2. Draw $\Omega \mid \mathbf{Y}_{1:T}, \mathbf{M}_t, \check{\Sigma}, \Phi, \Omega^{i-1}, \beta_{1:T}^{i-1}, \sigma_v^{i-1}, \sigma_w^{i-1}, q^{i-1}$. Use an Independence Metropolis-Hastings sampler step using the Haar measure on the space of orthogonal matrices:

   - Draw Ω^* using Theorem 9 in Rubio-Ramírez et al. (2010).

- With probability α, set $\Omega^i = \Omega^*$, otherwise $\Omega^i = \Omega^{i-1}$ is defined as

$$\alpha = \min\left\{\frac{p\left(\mathbf{M}_{1:T} \mid \mathbf{Y}_{1:T}, \Phi^i, \check{\Sigma}^i, \boldsymbol{\Omega}^*, \beta^{i-1}, \sigma_\nu^{i-1}\right)}{p\left(\mathbf{M}_{1:T} \mid \mathbf{Y}_{1:T}, \Phi^i, \check{\Sigma}^i, \boldsymbol{\Omega}^{i-1}, \beta^{i-1}, \sigma_\nu^{i-1}\right)}, 1\right\}$$

3. Draw $\sigma_v^2 \mid \mathbf{Y}_{1:T}, \mathbf{M}_{1:T}, \check{\Sigma}, \Phi, \Omega^{i-1}, \beta_{1:T}^{i-1}, \sigma_v^{i-1}, \sigma_w^{i-1}, q^{i-1}$. Sample σ_v^2 from $IG\left(\bar{s}_1/2, \bar{s}_2/2\right)$, the known conditional posterior distribution associated with σ_v^2.

4. Draw $\beta_{1:T} \mid \mathbf{Y}_{1:T}, \mathbf{M}_{1:T}, \check{\Sigma}, \Phi, \Omega^{i-1}, \beta_{1:T}^{i-1}, \sigma_v^{i-1}, \sigma_w^{i-1}, q^{i-1}$. Conditional on all other parameters, the law of motion forms a linear Gaussian state space system. This step can be carried out using the simulation smoother detailed in Carter & Kohn (1994) or Primiceri (2005).

5. Draw $\sigma_w^2 \mid \mathbf{Y}_{1:T}, \mathbf{M}_{1:T}, \check{\Sigma}, \Phi, \Omega^{i-1}, \beta_{1:T}^{i-1}, \sigma_v^{i-1}, \sigma_w^{i-1}, q^{i-1}$. Sample σ_w^2 from $IG\left(\bar{w}_1/2, \bar{w}_2/2\right)$, the known conditional posterior distribution associated with σ_w^2.

6. Draw $q \mid \mathbf{Y}_{1:T}, \mathbf{M}_{1:T}, \check{\Sigma}, \Phi, \Omega^{i-1}, \beta_{1:T}^{i-1}, \sigma_v^{i-1}, \sigma_w^{i-1}, q^{i-1}$. The scale parameter is sampled with a MH step outlined in Amir-Ahmadi et al. (2020).

A.2 Markov switching

In the case of Markov switching in β_t, we assume that β_t follows a two state Markov process with

$$\begin{aligned} \beta_t &= \beta_{s_t} \\ \Pr(s_t &= i|s_{t-1} = j) = p_{ij} \\ i, j &\in \{1, 2\}. \end{aligned}$$

We assume the following priors

$$\begin{aligned}
p(\beta_{s_t=1}) &\sim N(b_1, V_1). \\
p(\beta_{s_t=2}) &\sim N(b_2, V_2). \\
p_{11} &\sim beta\left(a_{11}, b_{11}\right). \\
p_{22} &\sim beta\left(a_{22}, b_{22}\right).
\end{aligned}$$

Algorithm 2. For $i = 1, \ldots, N$. At iteration i. The first 3 steps of the algorithm are the same as Algorithm 1.

4. Draw $\beta_{1:T} \mid \mathbf{Y}_{1:T}, \mathbf{M}_{1:T}, \check{\Sigma}, \Phi, \Omega^{i-1}, \beta_{1:T}^{i-1}, \sigma_v^{i-1}, p_{11}^{i-1}, p_{22}^{i-1}, s_{1:T}^{i-1}$. Sample β_t from $N(\bar{b}_1, \bar{V}_1)$ if $s^{i-1} = 1$ and from $N(\bar{b}_2, \bar{V}_2)$ if $s^{i-1} = 2$. Both are known conditional normal distributions.

5. Draw $p_{11}, p_{22} \mid \mathbf{Y}_{1:T}, \mathbf{M}_{1:T}, \check{\Sigma}, \Phi, \Omega^{i-1}, \beta_{1:T}^{i-1}, \sigma_v^{i-1}, p_{11}^{i-1}, p_{22}^{i-1}, s_{1:T}^{i-1}$. Sample p_{11} from $beta(\bar{a}_{11}, \bar{b}_{11})$ and p_{22} from $beta(\bar{a}_{22}, \bar{b}_{22})$. Both are known conditional beta distributions (see Frühwirth-Schnatter (2006), page 330).

6. Draw $s_{1:T} \mid \mathbf{Y}_{1:T}, \mathbf{M}_{1:T}, \check{\Sigma}, \Phi, \Omega^{i-1}, \beta_{1:T}^{i-1}, \sigma_v^{i-1}, p_{11}^{i-1}, p_{22}^{i-1}, s_{1:T}^{i-1}$. Sample $s_{1:T}$ using the Multi-Move sampler outlined in Frühwirth-Schnatter (2006), algorithm 11.5.

A.3 Stochastic Volatility

We assume that the variance of the measurement error innovation follows a random walk

$$\log\left(\sigma_{v,t}^2\right) = \log\left(\sigma_{v,t-1}^2\right) + w_t, w_t \overset{iid}{\sim} N\left(0, \sigma_u^2\right).$$

In addition, we assume the following priors:

$$
\begin{aligned}
p\left(\sigma_u^2\right) &\sim IG(\tau/2, \tau r/2). \\
p(\log(\sigma_{v,0}^2)) &\sim N(v_0, W_0).
\end{aligned}
$$

Similar to the case of time varying β, the scale parameter r of the IG prior is crucial for controlling the stochastic volatility. We follow the procedure outlined in Amir-Ahmadi et al. (2020) to estimate this parameter.

In practice, we use the same Gibbs steps to draw $\beta_{1:T}$ but set q the hyperparameter controlling the time variation to a very small number, i.e. 10^{-4}. The step 3 of algorithm 1 is then replaced by

3a Draw $\sigma_{v,1:T}^2 \mid \mathbf{Y}_{1:T}, \mathbf{M}_{1:T}, \check{\Sigma}, \Phi, \Omega^{i-1}, \beta_{1:T}^{i-1}, \sigma_{v,1:T}^{i-1}, \sigma_w^{i-1}, q^{i-1}, \sigma_u^{i-1}, r^{i-1}$. The sampler drawing $\log\left(\sigma_{v,t}^2\right)$ is based on Kim et al. (1998) who approximate the distribution of $\log\left(\sigma_{v,t}^2\right)$ by mixtures of normal distributions.

3b Draw $\sigma_u \mid \mathbf{Y}_{1:T}, \mathbf{M}_{1:T}, \check{\Sigma}, \Phi, \Omega^{i-1}, \beta_{1:T}^{i-1}, \sigma_{v,1:T}^{i-1}, \sigma_w^{i-1}, q^{i-1}, \sigma_u^{i-1}, r^{i-1}$. Sample σ_u^{i-1} from $IG\left(\bar{u}_1/2, \bar{u}_2/2\right)$, the known conditional posterior distribution associated with σ_u^2.

3c Draw $r \mid \mathbf{Y}_{1:T}, \mathbf{M}_{1:T}, \check{\Sigma}, \Phi, \Omega^{i-1}, \beta_{1:T}^{i-1}, \sigma_v^{i-1}, \sigma_w^{i-1}, q^{i-1}, \sigma_u^{i-1}, r^{i-1}$. The scale parameter is sampled with a MH step outlined in Amir-Ahmadi et al. (2020).

A.4 More on Priors

We use a benchmark Minnesota prior setting for the VAR with a very loose overall tightness parameter equal to 10.[1] The diagonal elements

[1] For the exact definition of this parameter see Giannone et al. (2015).

of the location matrix of the inverse Wishart prior are fixed to OLS estimates of an AR(1) model based on 12 observations of pre-sample data. We use a fairly uninformative prior for the residual variance $\sigma_v^2 \sim IG(s_1/2, (s_1 s_2^2/2)$. For the estimation of the parameter q, we adopt the half-Cauchy prior with scale parameter θ. The prior hyperparameters are summarized in the following table:

Table A-1: TVP Benchmark Prior Hyperparameters

s_1	s_2	b_0	V_0	τ	θ
2	0.2	0	1	2	0.01

In case of Markov Switching, the Minnesota prior specification remains the same, the other prior hyperparameters are summarized in the following table:

Table A-2: Markov Switching Benchmark Prior Hyperparameters

s_1	s_2	b_1	V_1	b_2	V_2	a_{11}	b_{11}	a_{22}	b_{22}
2	0.2	0	1	0	1	6	1	6	1

In case of stochastic volatility in the measurement error, we use a very similar prior setting as in the case of time varying parameter. In particular, we also estimate the hyperparameter r which controls the degree of stochastic volatility base on Amir-Ahmadi et al. (2020). We assume again a half-Cauchy prior with scale parameter θ. As mentioned in the description of the sampler, we set q the hyperparameter controlling the time variation to 10^{-4}. The other prior hyperparameters are summarized in the following table:

Table A-3: Stochastic Volatility Prior Hyperparameters

v_0	W_0	b_0	V_0	τ	θ
$\log(0.2^2)$	1	0	1	2	0.01

All posterior results except for the Monte Carlo experiments are based on $500,000$ draws from the MCMC sampler.

B Relationship With High-Frequency Identification

In order to analyze the relationship to identification via changes in volatility, we first stack our original VAR and the measurement equation for the instrument m_t (which we assume to be scalar). $\mathbf{A}(L)$ is a polynomial in the lag operator. $\mathbf{e}_{2,t}$ is a vector that collects all structural shocks except for the monetary shock e_t^{MP}. The associated impact effects on $\mathbf{y}_t$ are collected in $\mathbf{\Sigma}^{MP}$ and $\mathbf{\Sigma}^2$.

$$z_t = \begin{bmatrix} m_t \\ \mathbf{y}_t \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{A}(L) \end{bmatrix} \begin{bmatrix} m_{t-1} \\ \mathbf{y}_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} \sigma_v & \beta_t & 0 \\ 0 & \mathbf{\Sigma}^{MP} & \mathbf{\Sigma}^2 \end{bmatrix}}_{\mathbf{\Sigma}_t} \underbrace{\begin{bmatrix} v_t \\ e_t^{MP} \\ \mathbf{e}_{2,t} \end{bmatrix}}_{\mathbf{u}_t}$$

Note that in contrast to standard identification via heteroskedasticity (Rigobon 2003), the volatility of the shocks $E(\mathbf{u}_t\mathbf{u}_t')$ is time-invariant, but the impact matrix $\mathbf{\Sigma}_t$ varies.[2] We show now that if β_t follows a two regime Markov-switching process, we can identify the effects of $\varepsilon_{1,t}$. The Markov-switching structure is only assumed for simplicity to highlight the relationship between our approach and identification via heteroskedasticity. More general laws of motion for β_t would yield the same insights. We call the two possible values for Σ_t Σ_0 and Σ_1. These matrices only differ in their values for β, which is equal to either β_0 or β_1.

[2]We assume throughout, as before, that $E(\mathbf{u}_t\mathbf{u}_t') = I$.

To obtain our result, we start by writing out $\Sigma_1\Sigma_1'$:

$$\begin{bmatrix} \sigma_v & \beta_1 & 0 \\ 0 & \mathbf{\Sigma}^{MP} & \mathbf{\Sigma}^{2} \end{bmatrix} \times \begin{bmatrix} \sigma_v & 0 \\ \beta_1 & \mathbf{\Sigma}^{MP\prime} \\ 0 & \mathbf{\Sigma}^{2\prime} \end{bmatrix} = \begin{bmatrix} \sigma_v^2 + \beta_1^2 & \beta_1\mathbf{\Sigma}^{MP\prime} \\ \beta_1\mathbf{\Sigma}^{MP} & \mathbf{\Sigma}^{MP}\mathbf{\Sigma}^{MP\prime} + \mathbf{\Sigma}^{2}\mathbf{\Sigma}^{2\prime} \end{bmatrix}$$

We also get a similar expression for $\Sigma_0\Sigma_0'$, with β_1 replaced by β_0. The first row of those two quadratic forms allows us to identify $\beta_1^2 - \beta_0^2$ (by taking the difference of the first elements in the first row) and $\frac{\beta_1}{\beta_0}$ (by taking the ratio of any element of $\beta_1\mathbf{\Sigma}^{MP\prime}$ and the corresponding element of $\beta_0\mathbf{\Sigma}^{MP\prime}$). With those two pieces of information, we can uniquely pin down β_1 and β_0 up to one sign normalization. The other elements of the first row of $\Sigma_1\Sigma_1'$ or $\Sigma_0\Sigma_0'$ except for the first element pin down $\mathbf{\Sigma}^{MP}$, which identifies the effects of e_t^{MP}.

C Details on Monte Carlo Exercises

All of our Monte Carlo setups consist of two regimes. Our goal is to match the variance of the instrument for a given regime across specifications. We assume that in the benchmark the monetary policy shocks are $N(0, \sigma_e^2)$ and $\beta = 1$ in one regime and equal 0 in the other. Furthermore, we will assume that in the benchmark the variance of the measurement error v_t is a fixed fraction κ of the variance of the monetary policy shock.[3] Note that in contrast to our estimated model (where e_t^{MP} is assumed to have unit variance and all scaling is captured in the impact matrix Σ),

[3] Compared to the main text, we use non-unit variance shocks in the data-generating process, whereas the shocks entering the estimated model in the main text (monetary shock e_t^{MP} and measurement error u_t^{M}) are unit variance shocks.

the true monetary policy shock does not have unit variance. This affects the scale of the estimated β_t coefficients and needs to be taken into account when comparing to the true values stated here (we give more details when discussing the estimated paths of β_t).

In our Monte Carlo exercise, we simulate 100 samples of length $T = 250$ each. The variables we use in Monte Carlo exercise are the nominal interest rate, output, inflation, and the monetary policy shock from an estimated Smets-Wouters model. The VAR contains simulated nominal interest rate, output and inflation and the lag length is set to 4. In each of the Monte Carlo repetitions (in total 100), posterior results are based on 50,000 MCMC draws. The prior specification is exactly the same as in the empirical estimation.

C.1 Benchmark

The measurement equation and the variance in the two regimes are:

$$m_t = e_t + v_t, Var(m_t) = (1 + \kappa)\sigma_e^2 \quad \text{(A-2)}$$

$$m_t = v_t, Var(m_t) = \kappa\sigma_e^2 \quad \text{(A-3)}$$

We set $\sigma_e^2 = 0.2290^2$ equal to the DGP value and $\kappa = 0.25$. For $T = 5, ..., 140$ and $T = 150, ..., 230$, $\beta = 0$. Otherwise, $\beta = 1$. These values are chosen to be comparable to the Gertler-Karadi instrument.

C.2 Changing Volatility in the measurement error

We now assume that the measurement error v_t has a variance that switches between regimes with values $\sigma_{v,1}^2$ and $\sigma_{v,2}^2$. The measurement equations are given by:

$$m_t = \overline{\beta} e_t + v_t, Var(m_t) = \overline{\beta}^2 \sigma_e^2 + \sigma_{v,1}^2 \tag{A-4}$$

$$m_t = \overline{\beta} e_t + v_t, Var(m_t) = \overline{\beta}^2 \sigma_e^2 + \sigma_{v,2}^2 \tag{A-5}$$

We now need to solve the following two equations:

$$\overline{\beta}^2 \sigma_e^2 + \sigma_{v,1}^2 = (1+\kappa)\sigma_e^2 \tag{A-6}$$

$$\overline{\beta}^2 \sigma_e^2 + \sigma_{v,2} = \kappa \sigma_e^2 \tag{A-7}$$

We actually have three unknowns and two equations here. Since all variances have to be positive, we have additional constraints though. We set $\overline{\beta} = \sqrt{\kappa}$ and $\sigma_{v,2}^2 = 0$. This implies $\sigma_{v,1}^2 = \sigma_e^2$.

We set $\sigma_e^2 = 0.2290^2$ (equal to the DGP value) and $\kappa = 0.25$. For $T = 5, ..., 140$ and $T = 150, ..., 230$, $\sigma_{v,2}^2 = 0$. Otherwise, $\sigma_{v,1}^2 = \sigma_e^2$.

C.3 Changing Volatility in e_t

We now assume that the variance in the monetary policy shocks changes, with variances $\sigma_{e,1}^2$ and $\sigma_{e,2}^2$. We also allow the measurement error variance $\tilde{\sigma}_v^2$ and the coefficient $\tilde{\beta}$ to be different than in the other specifications (they are fixed across regimes though). The equations in this MC are given by

$$m_t = \tilde{\beta} e_t + v_t, Var(m_t) = \tilde{\beta}^2 \sigma_{e,1}^2 + \tilde{\sigma}_v^2 \tag{A-8}$$

$$m_t = \tilde{\beta} e_t + v_t, Var(m_t) = \tilde{\beta}^2 \sigma_{e,2}^2 + \tilde{\sigma}_v^2 \tag{A-9}$$

The equations we need to solve are:

$$\tilde{\beta}^2\sigma_{e,1}^2 + \tilde{\sigma}_v^2 = (1+\kappa)\sigma_e^2 \tag{A-10}$$

$$\tilde{\beta}^2\sigma_{e,2}^2 + \tilde{\sigma}_v^2 = \kappa\sigma_e^2 \tag{A-11}$$

We impose $\tilde{\beta} = 1$ and $\tilde{\sigma}_v^2 = 0$, which implies $\sigma_{e,2}^2 = \kappa\sigma_e^2$ and $\sigma_{e,2}^2 = (1+\kappa)\sigma_e^2$.

We set $\sigma_e^2 = 0.2290^2$ and $\kappa = 0.25$. For $T = 5, ..., 140$ and $T = 150, ..., 230$, $\sigma_{e,2}^2 = \kappa\sigma_e^2$. Otherwise, $\sigma_{e,2}^2 = (1+\kappa)\sigma_e^2$.

C.3.1 Alternative Measurement Equation

Since the previous exercise is somewhat cumbersome to interpret, we also carried out an alternative where the data-generating process for all variables except the instrument is the same as before. For the instrument, we now assume that

$$m_t = \overline{e}_t + v_t, \tag{A-12}$$

where $\overline{e}_t$ is the normalized monetary policy shock that has unit variance each period. We set $v_t \sim N(0, 0.25)$. This exercise has the disadvantage that the path of the instrument's volatility is not the same as in the previous exercise. The advantage is that the instrument equation is independent of changes in the monetary policy shock's volatility. Furthermore, this exercise is certainly not as realistic as the others because the instrument is linked to the normalized true shock. Figure A-1 plots the impulse responses under this alternative specification - results are basically indistinguishable from the original exercise in the main text, as can be seen when comparing Figure A-1 with Figure 17. The posterior median paths of β_t are now flat (Figure A-2).

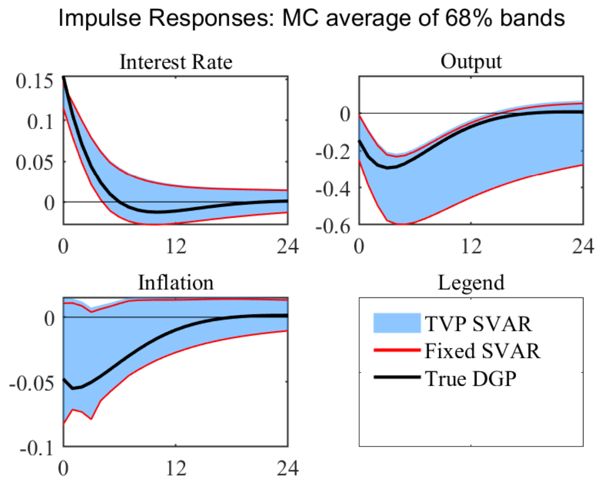

Figure A-1: Impulse responses for the data-generating process and the Monte Carlo replications: Stochastic volatility in the monetary policy shock, alternative instrument.

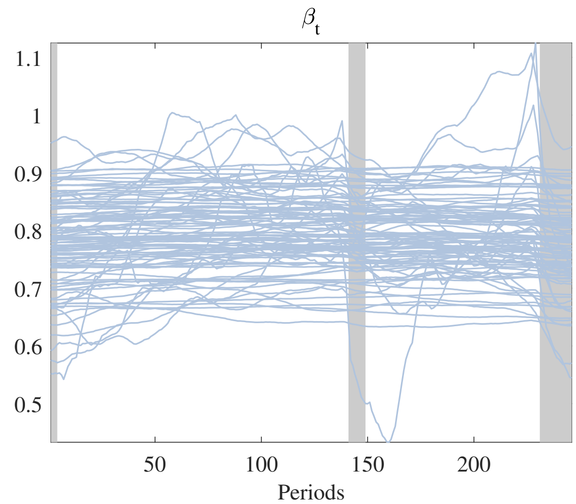

Figure A-2: Estimated paths of β_t in the Monte Carlo runs (posterior medians) when the data-generating process features time variation in the volatility of the monetary policy shock, alternative instrument.

D Data Sources

For the US economy, we follow Gertler & Karadi (2015) and obtained industrial production (INDPRO), consumer price index (CPIAUCSL) and 1-year treasury rate (GS1) from FRED (https://fred.stlouisfed.org/). The data for the excess bond premium is obtained from Board of Governors (https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/files/ebp_csv.csv). The instrument of Gertler & Karadi (2015) is obtained from the replication file of the paper (https://www.openicpsr.org/openicpsr/project/114082/version/V1/view). The instrument of Miranda-Agrippino & Ricco (2022) is obtained from the personal website of Silvia Miranda-Agrippino (http://silviamirandaagrippino.com/s/Instruments_web-x8wr.xlsx). For the UK economy, we use the replication data and instrument of Cesa-Bianchi et al. (2020) from https://github.com/ambropo/MP_HighFrequencyUK/.

E Additional Figures

Here we show the full set of impulse responses for various specifications in the main text.

E.1 Markov Switching

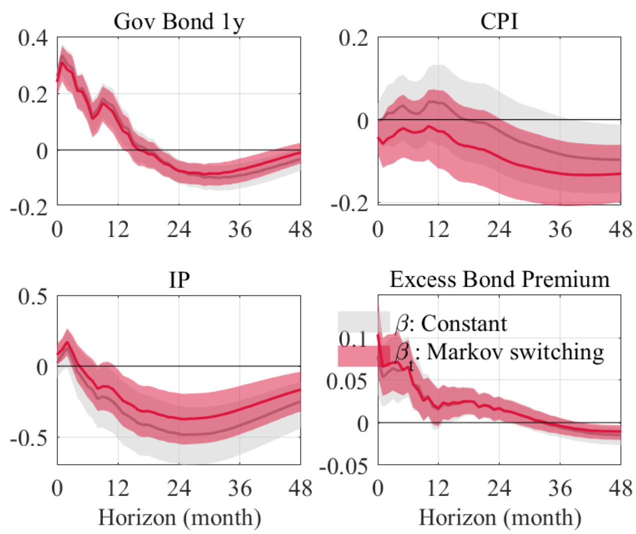

Figure A-3: Impulse responses (median and 68 percent posterior bands) to a one standard deviation monetary policy shock - Markov-switching specification.

E.2 Shutting Down Periods Where the Instrument is Informative/Not Informative

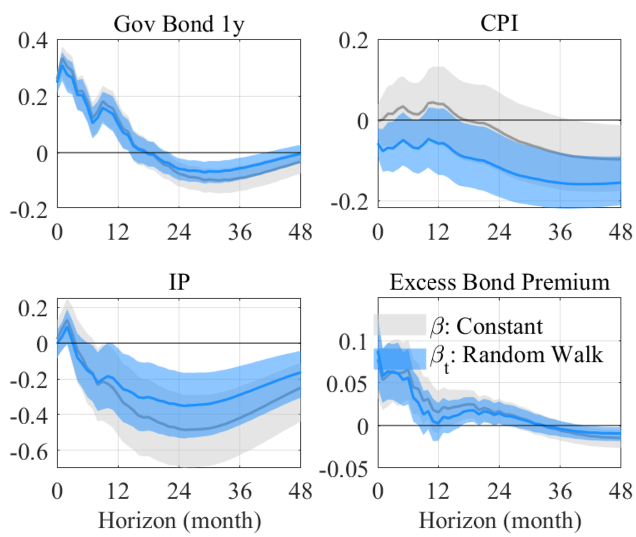

Figure A-4: Impulse responses (median and 68 percent posterior bands) to a one standard deviation monetary policy shock - $\tilde{m}_t$.

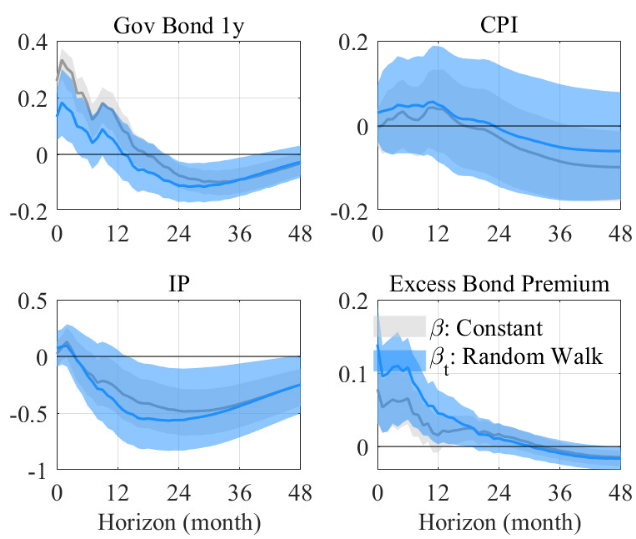

Figure A-5: Impulse responses (median and 68 percent posterior bands) to a one standard deviation monetary policy shock - $\overline{m}_t$.

E.3 Alternative Instruments

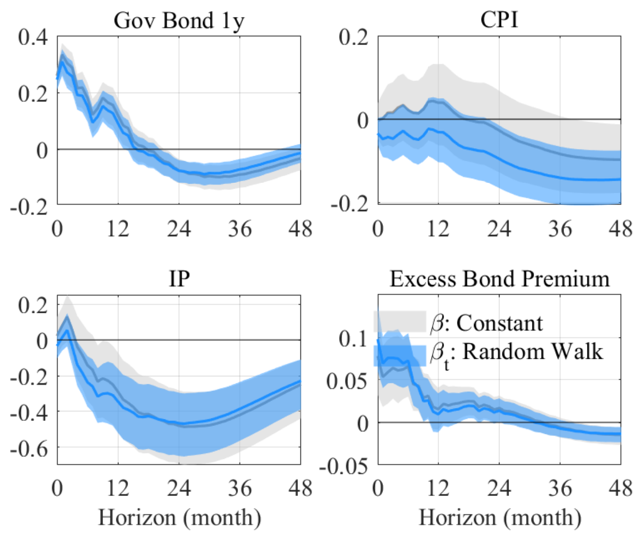

Figure A-6: Impulse responses (median and 68 percent posterior bands) to a one standard deviation monetary policy shock - modified instrument.

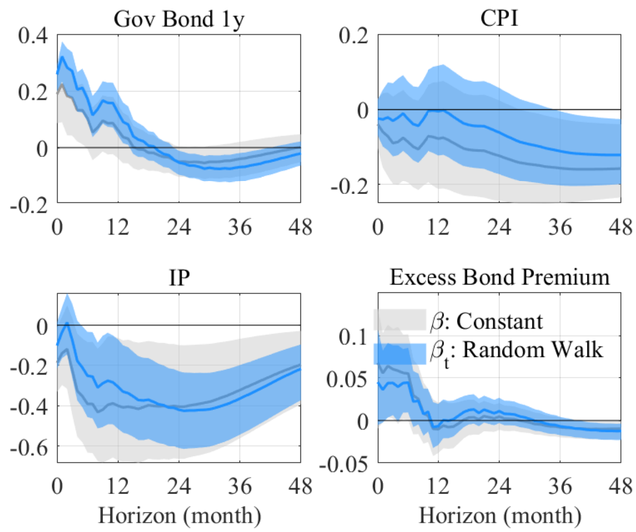

Figure A-7: Impulse responses (median and 68 percent posterior bands) to a one standard deviation monetary policy shock - Miranda-Agrippino & Ricco (2020) instrument.

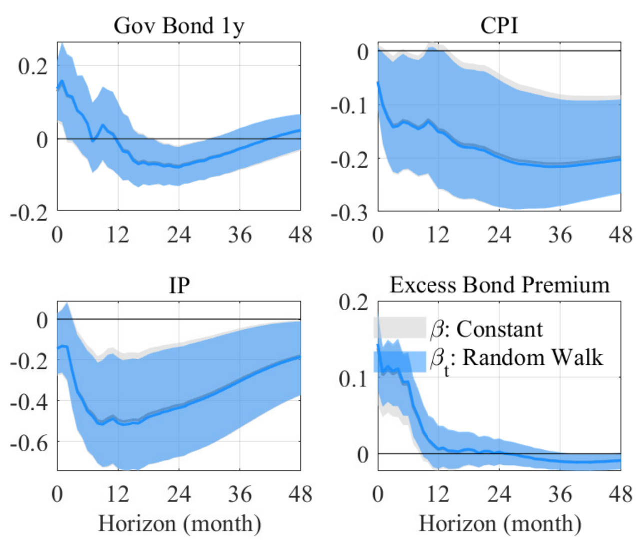

Figure A-8: Impulse responses (median and 68 percent posterior bands) to a one standard deviation monetary policy shock - Bauer & Swanson (2022) instrument.

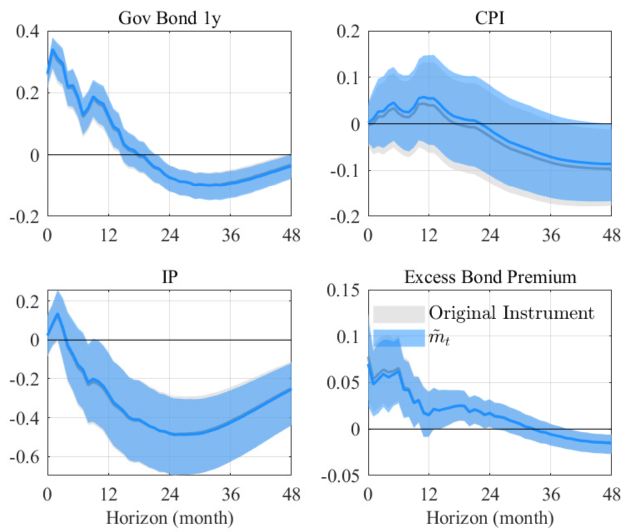

Figure A-9: Impulse responses (median and 68 percent posterior bands) based on fixed coefficient VAR for benchmark instrument and $\tilde{m}_t$

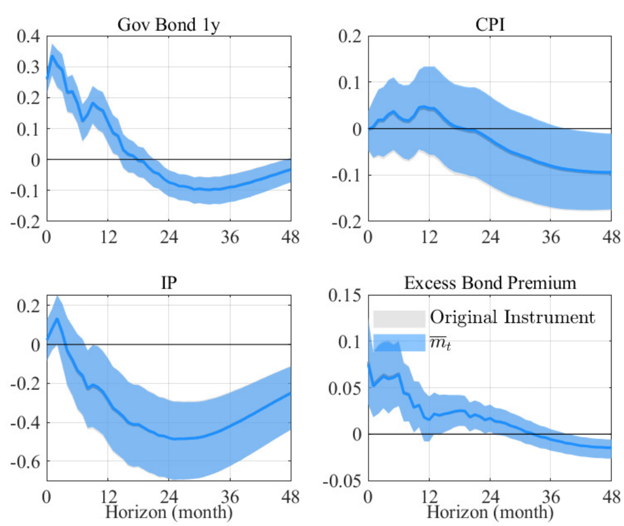

Figure A-10: Impulse responses (median and 68 percent posterior bands) based on fixed coefficient VAR for benchmark instrument and $\overline{m}_t$

F Evidence from the UK

Finally, we present evidence for high-frequency based identification of monetary policy shocks in the United Kingdom. We use both the instrument and the VAR specification (i.e. the choice of variables entering y_t) of Cesa-Bianchi et al. (2020). We focus on the specification with a random walk specification for β_t.

Figure A-11 shows impulse responses for all UK variables in the VAR. In contrast to the US, we find little difference between fixed coefficient-based responses and random walk-based responses. A potential reason can be seen in Figure A-12: The sample for the UK (both for the VAR variables and the instrument) is much shorter, and within that shorter time-span there are more periods where the instrument is informative, making the fixed coefficient estimation generally more informative.

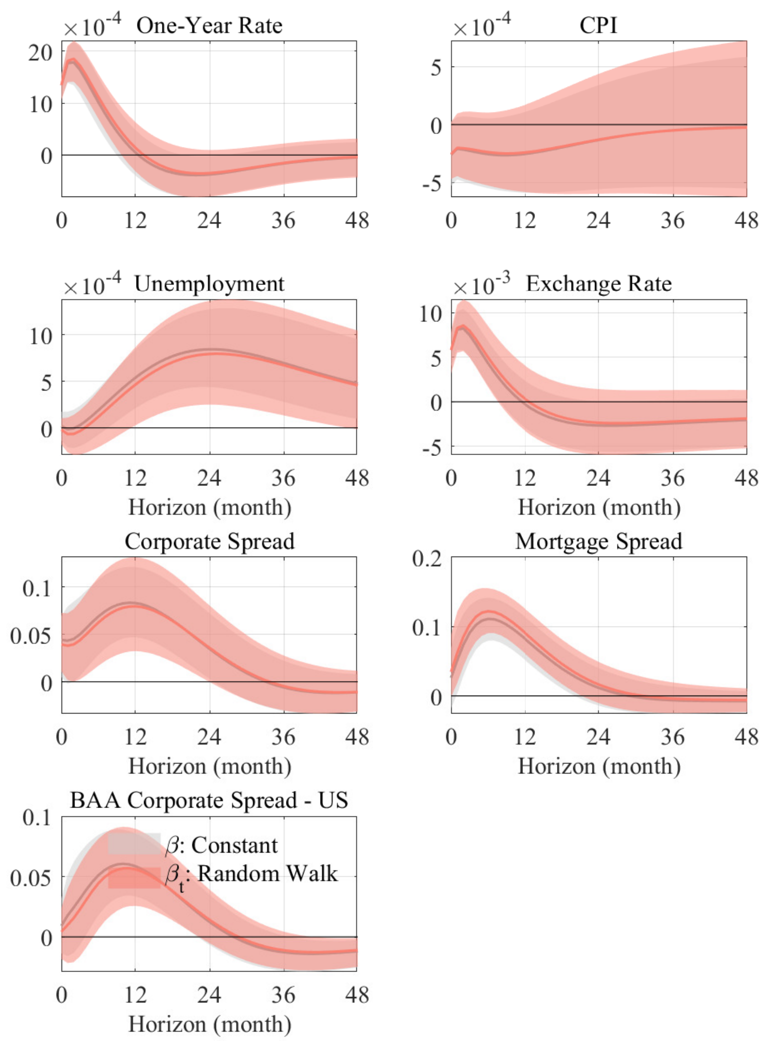

Figure A-11: Impulse responses (median and 68 percent posterior bands) to a one standard deviation monetary policy shock, UK.

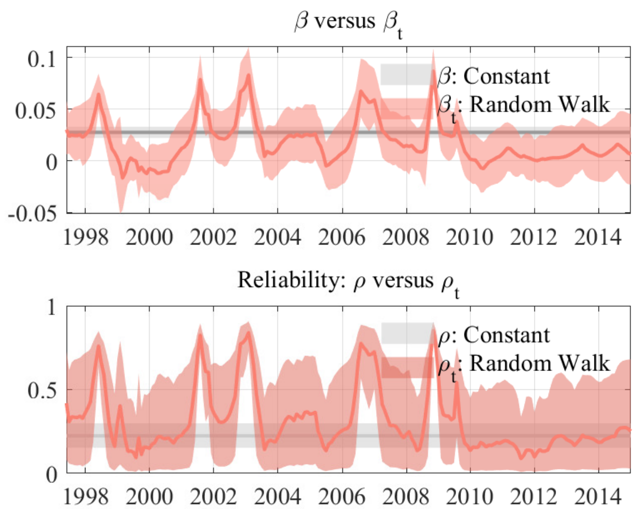

Figure A-12: Posterior of β_t and ρ_t, UK (median and 68 percent posterior bands).