What Do Sectoral Dynamics Tell Us About the Origins of Business Cycles?

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Abstract

We use economic theory to rank the impact of structural shocks across sectors. This ranking helps us to identify the origins of U.S. business cycles. To do this, we introduce a Hierarchical Vector Auto-Regressive model, encompassing aggregate and sectoral variables. We find that shocks whose impact originate in the "demand" side (monetary, household and government consumption) account for 43 percent more of the variance of U.S. GDP growth at business cycle frequencies than identified shocks originating in the "supply" side (technology and energy). Furthermore, corporate financial shocks, which theory suggests propagate to large extent through demand channels, account for an amount of the variance equal to an additional 82 percent of the fraction explained by these supply shocks.

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1 Introduction

What drives business cycles? Macroeconomists have alternatively argued for demand factors such as monetary and fiscal expenditure shocks and supply factors such as technological innovation and the cost of raw materials. So far, comprehensive decompositions of output fluctuations into the contributions of various shocks has only been obtained in tightly specified structural models. Those typically indicate a prominent role for supply shocks.[1]

We propose an approach to identify a variety of demand and supply shocks simultaneously, but within a flexible statistical framework. We identify shocks based on prior knowledge of their impact on different sectors. Thus, for example, an energy cost shock is identified with an aggregate shock that increases energy prices, and has a larger price and output impact on energy intensive sectors. Our analysis suggests a prominent role for shocks that manifest themselves on the demand side.

In order to implement this identification scheme, we introduce a large scale, flexible, and tractable econometric model: the Hierarchical Vector Auto-Regression (Hi-VAR). It allows us to analyze aggregate and sectoral time-series jointly, while allowing for rich internal sectoral dynamics. Aggregate shocks are captured by common factors in the innovations of the various series, and shock identification is obtained by setting priors on factor loadings. The introduction of this particular econometric framework forms a separate methodological contribution of this paper.

In our baseline analysis, we identify six structural shocks: energy cost, technological progress, monetary, corporate finance, government consumption, and household demand. The first five shocks can be easily motivated with reference to an extensive literature. We take the household demand shock to encompass the set of shocks to household credit, wealth, or expectations that have been most heavily emphasized following the 2007-09 recession, and that operate mainly through their effect on household consumption decisions. The identification scheme used for each of the shocks is done by analogy to the energy shock example given above, and mostly relies on input or demand intensity shares that can be read directly from input-output tables. The two exceptions are corporate credit shocks (tied to external financial dependence measures as in Rajan and Zingales (1998)) and monetary shocks (tied to sectoral price stickiness measures by Nakamura and Steinsson (2008)).

We check the reasonableness of our identification scheme by inspecting the resulting impulse response functions for aggregate variables. We find that those largely conform to theoretical priors and findings by studies focusing on single, well identified shocks.

Our results point to a prominent role for fluctuations originating on the demand side of the economy and in access to corporate credit. We find that demand side fluctuations (monetary, household demand, and government consumption) account for a fraction of the variance of GDP growth at business cycle frequencies which is 43 percent larger than the fraction accounted for by identified supply (technology and energy) shocks. Of the three demand shocks, household demand features most prominently, followed by monetary policy and government consumption. Furthermore, corporate financial shocks, which theory implies propagate to a substantial degree through demand channels, account for 82 percent as much as those same supply shocks.

The emphasis on the demand origins of business cycles complements results by Angeletos et al. (2018), who find that most business cycle fluctuations do not seem to be driven by supply shocks such as technological innovations. Bachmann and Zorn (2013) find that demand shocks are the dominant driver of output growth fluctuations in German data. Those findings are in contrast to the results of prominent studies based on tightly specified DSGE models, which tend to emphasize supply factors.

[1] Justiniano et al. (2010) attribute 75% of GDP fluctuations to "neutral" and investment specific TFP shocks.

A standard approach to identifying structural shocks is the use if sign restrictions, as pioneered by Uhlig (2005), Faust (1998), and Canova and Nicolo (2002). The use of sign restrictions on sectoral responses to identify aggregate shocks processes has been analyzed by Amir-Ahmadi and Drautzburg (2017).[2] Relative to this previous work, our main innovation is to develop a method that allows us to investigate the role of several shocks simultaneously in a tractable manner in environments with large amounts of data. This paper thus also falls into a more general trend within macroeconomics of using cross-sectional data to inform on questions of relevance to macroeconomists (Holly and Petrella (2012), Beraja et al. (2016), Sarto (2018), Chen et al. (2018), and Guren et al. (2019), for example). We also add to an existing suite of time series models designed to incorporate large panels, including dynamic factor models (Stock and Watson (2005a)), factor augmented VARs (Bernanke et al. (2005), Boivin et al. (2009)), and global VARs (Chudik and Pesaran (2016), Holly and Petrella (2012)). Lastly, on a more technical note, we add to a literature that relies on Bayesian priors rather than hard identification restrictions (Kociecki (2010) and Baumeister and Hamilton (2015)), where our main contribution is to provide a method to add those restrictions in a maximally tractable manner.

The paper proceeds as follows: Section 2 describes the Hi-VAR model and the identification procedure. Section 3 lays out an analytically tractable multi-sector model with sticky prices and wages to provide the theoretical motivation for the identification assumption. Section 4 presents the results. Section 5 concludes.

2 Hierarchical VAR model: Identification and Estimation

We combine a VAR-type time series model for a vector of aggregate variables Y_t with autoregressive models for vectors of sectoral data X_t^i, where i indicates the sector. Aggregate and sectoral data interact in two ways: (i) via structural shocks that affect both types of data and (ii) via direct feedback from (lagged) aggregate data to sectoral data.[3] We use a Gaussian prior for the effects of the structural shocks on aggregate and sectoral data.[4] This procedure allows us to impose more prior information on the magnitudes of these effects compared to what would be feasible in the standard sign restriction approach.[5] By exploiting our specific model structure, we can efficiently estimate very large scale models. Also, because we directly estimate a structural VAR, our approach can handle set-identified, exactly identified, and over-identified environments - the differences just amount to choosing different priors on the parameters governing the contemporaneous impact of structural shocks.

[2] Also, Schwartzman (2014) and Fulford and Schwartzman (2015) use cross-sectional information to identify shocks. Whereas the first paper uses a structural small open economy model, the second paper leverages the cross-sectional impact of a shock identified from an historical narrative. The case for using information on the relative magnitude of the responses to shocks to help identify shocks has also been made by De Graeve and Karas (2010).

[3] In the appendix we discuss extensions that allow for more flexible feedback.

[4] We can do this because we directly estimate the impact of structural shocks rather than first estimate a reduced-form model and then infer the structural model afterwards, as is common in the VAR literature. In directly estimating a structural representation, we follow in the footsteps of, for example, Baumeister and Hamilton (2015) and Sims and Zha (1998), who directly estimate structural VARs. Baumeister and Hamilton (2018) and Baumeister and Hamilton (2019) are closest to our approach because they also use information on the contemporaneous impact of the structural shocks to inform their priors.

[5] In the standard approach to impose sign restrictions, as outlined in Rubio-Ramirez et al. (2010), inequality restrictions are imposed on impulse responses in conjunction with a uniform (Haar) prior on the rotation matrices that map reduced form parameters to initial impulse responses. In the appendix we show how to incorporate strict inequality restrictions in our framework should a researcher be interested in those.

2.1 Modeling aggregate variables

We model aggregate variables as following a linear vector autoregressive process. A key difference from traditional VARs for aggregate data is that we break the tight link between forecast errors and structural shocks, thus allowing sectoral data to help identify structural shocks.

The aggregate variable vector Y_t (of dimension N by 1) is a function of its past values, structural shocks ε_t, and other shocks w_t:

$$Y_t = \mu + \sum_{l=1}^{L} A_l Y_{t-l} + D\varepsilon_t + w_t \tag{1}$$

where ε_t is of dimension $S \times 1$, and Σ is an $N \times S$ matrix encoding the impact of the Gaussian structural shocks ε on the aggregate variables, and w_t is a $N \times 1$ vector of mean 0, non-structural Gaussian shocks with covariance matrix Ω. We further assume that $\varepsilon \sim N(0, I)$.[6] As will be clear later, we can allow for $S < N$, $S = N$, or $S > N$, whereas standard VAR analyses require $S \leq N$.

For later discussion, it is useful to note that the one-step ahead forecast error for the aggregate level is given by $D\varepsilon_t + w_t$, whereas a standard VAR model for the aggregate variables would assume that any estimate of the structural shock is a linear combination of the aggregate one-step ahead forecast error.[7]

2.2 Modeling idiosyncratic variables

There are observations for I idiosyncratic units (such as industries, regions, or, in our specific application, sectors) with K variables (such as prices and quantities) each. The law of motion for the data from unit i, summarized in the K-dimensional vector X_t^i, is given by:

$$X_t^i = \mu^i + \sum_{l=1}^{L^X} B_l^i X_{t-l}^i + \sum_{l=1}^{L^Y} C_h^i Y_{t-l} + D^i \varepsilon_t + w_t^i \tag{2}$$

where now D^i is a $K \times S$ matrix encoding the impact of shocks ε on the idiosyncratic variables and the mean zero Gaussian vector w_t^i incorporates the impact of idiosyncratic (or non-structural) shocks on individual units. We denote the covariance matrix of w_t^i by Ω^i. We assume that w_t^i is independent across i and independent from w_t.

2.3 Interpreting our model

To gain further intuition, it will be useful to rewrite our model as follows: first, define the vector of all observables

$$Z_t = [Y_t{}' \; X_t^1{}' \; X_t^2{}' \; \ldots \; X_t^I{}']'$$

[6]The distributional assumptions are necessary because we ultimately want to carry out Bayesian inference, for which we need to build a likelihood function.

[7]This is true even if fewer than N shocks are identified, as is common in the literature on sign restrictions in VARs.

We can then recast our model in the following way:

$$Z_t = \mu^Z + \sum_{l=1}^{max(L^X,L^Y,L)} B_l^Z Z_{t-l} + \underbrace{D^Z \varepsilon_t + w_t^Z}_{u_t^Z} \tag{3}$$

where w_t^Z is a vector that stacks the non-structural shocks according to the ordering of observables in Z_t. Our model imposes restrictions on the matrices B_l by assuming that one sector's variables cannot directly respond to any other sector's lagged variables. Note that our one step ahead forecast error u_t^Z is iid and independent of all other right-hand side variables. We now provide a characterization of identification as it pertains to both the parameters of the model as well as the aggregate shocks. Because of the structure of the one-step ahead forecast error, we can identify μ^Z as well as the coefficient matrices B_l^Z. Even single equation-estimation via OLS would yield consistent estimates in our environment. Since we use even more information in our likelihood-based approach, the same identification results carry over. Focusing on u_t^Z, we can see that it follows a factor structure, where the common factors are the iid structural shocks ε_t. Note also that our assumptions on the correlation structure of w_t^Z limits the correlation of those idiosyncratic components across observables (these shocks can be correlated within sector and at the aggregate level, but not across the sets of equations outlined above). In fact, standard results on identification in Factor models apply (Bai and Ng (2008)). While the effects of individual structural shocks are not identified without additional assumption, the overall effect of *all* structural shocks is identified, as we show below and later highlight in a Monte Carlo experiment. To identify ε_t, we need identification restrictions akin to those used in the structural VAR literature. To see this, define

$$u_t = D\varepsilon_t + w_t \tag{4}$$

$$u_t^i = D^i \varepsilon_t + w_t^i \ \forall i \tag{5}$$

For any conformable orthogonal matrix Q, we can construct alternative models that feature the same first and second moments and thus the same Gaussian likelihood:

$$u_t = \underbrace{DQ^{-1}}_{\tilde{D}} \underbrace{Q\varepsilon_t}_{\tilde{\varepsilon}_t} + w_t \tag{6}$$

$$u_t^i = \underbrace{D^i Q^{-1}}_{\tilde{D}^i} \underbrace{Q\varepsilon_t}_{\tilde{\varepsilon}_t} + w_t^i \ \forall i \tag{7}$$

With the sign and magnitude restrictions in this paper, we are not going to pin down a unique value of Q to get exact identification even though the overall impacts $D\varepsilon_t$ and $D^i \varepsilon_t \ \forall i$ are identified. However, even though we are in the realm of set identification, important recent work on the usefulness of restrictions of the kind we use (Amir-Ahmadi and Drautzburg (2017)) shows that they can be very informative. This is especially true for our setting, where we have many sectors on which we impose restrictions. It is important to point out what this identification discussion does *not* say; namely, that the data cannot help tell different identification assumption encoded in different Gaussian priors for D and D^i apart. In fact, these identification assumptions could be assessed via the marginal likelihood. How can we square the previous two statements? First, for every possible combination of D and $D^i \ \forall i$, we can find alternative values that will give the same likelihood. But draws from those parameters from different Gaussian priors over these parameters (which is what we use in this paper) will generally not all imply the same second moments for the variables in our model and thus will not imply the same value of the likelihood function. Priors that not only restrict the sign of a response, but also contain information about the magnitude of a response to shocks (such as our Gaussian priors on D and D^i) might then be more helpful to disentangle different theories or points of view about the impact of different shocks.

We are not the first to impose a factor structure on residuals of time series models. Altonji and Ham (1990),Clark and Shin (1998),Stock and Watson (2005b), and Gorodnichenko (2005) follow the same route to estimate common shocks in time series models with many observables. In particular, Gorodnichenko (2005) offers an interpretation of the w_t shocks as shocks that can arise in equilibrium models due to "expectations errors, measurement errors, heterogeneous information sets (e.g., consumers and the central banker can have different information sets), myopia and other forms of irrational behavior". Gorodnichenko (2005) also describes an equilibrium model with imperfect information that has such a factor structure in residuals. We share with Stock and Watson (2005b) the assumption that non-structural shocks cannot contemporaneously affect variables in other blocks of the model.

What sets our approach apart from the previous literature on structural VARs is that (i) because of our model structure, we can use substantially larger datasets than standard VAR applications can, (ii) for that same reason we can identify several shocks simultaneously, rather than one or two.[8]

Importantly, our approach is computationally very efficient. This is because, as we will show below, it relies solely on standard steps in Gibbs samplers (drawing from Normal and inverse-Wishart priors as described in Koop and Korobilis (2010) as well as using Gibbs sampling for linear and Gaussian state space models as in Carter and Kohn (1994)) that, in our specific case, are especially amenable to parallelization.[9] This implies that our approach can be very efficient even in applications that have a much larger scale than our application in this paper.[10] Finally, how do we interpret the shocks w_t? These are shocks that do not have a contemporaneous effect on sectoral data, while they do affect the aggregate data at time t. Furthermore, these innovations do not have an independent role in determining sectoral data beyond how they influence aggregate data. To safeguard ourselves against a scenario where identified w_t is estimated to be more important for determining aggregate data than it actually is in reality, we suggest adding additional shocks to the vector of structural innovations ε_t on which no identification restrictions are imposed. The additional 'structural' shocks will soak up any explanatory power that would otherwise falsely be attributed to w_t. In our application, we add three of those shocks, but also show in the Appendix results with 10 additional shocks as a robustness check.

[8]By estimating the responses to structural shocks directly, we do not need to post-process reduced-form VAR estimates to obtain the structural representation that allows us to compute the effects of structural shocks. This is useful because the algorithms used to deliver the impulse responses after estimating a reduced-form model can be numerically inefficient because not all proposed candidate parameter vectors of the structural VAR satisfy the identification restrictions Rubio-Ramirez et al. (2010) or because the imposed restrictions are actually overidentifying as in Amir-Ahmadi and Drautzburg (2017).

[9]This parallelization argument does not hold, for example, in large scale VARs. And while certain aspects of Gibbs samplers for factor models might also be amenable to parallelization, these models do not directly emphasize the dynamics of all variables in sector in a transparent fashion.

[10]As a final note on the model, it might be helpful to note that one could interpret the lagged aggregate variables Y_{t-l} as additional, but observable, factors.

2.4 Setting Priors

An important step in our analysis is in the setting of priors. In contrast with traditional approaches, which achieve identification by setting hard constraints on the shock process, we follow Baumeister and Hamilton (2015) in using "soft" prior restrictions for identification. At the same time, by setting Gaussian priors on the direct impact of the structural shocks on variables and inverse Wishart priors on the variances, we can use a Gibbs sampler to estimate a large scale model with several identified shocks very efficiently.

2.4.1 Priors on D and D^i

The most important priors for our identification purposes are the ones we set on the impact matrices, D and D^i. We set priors for both parameters to be Gaussian. To set the average value for element k, s of D^i, we assume that it can decomposed as follows:

$$(E\left[D^i_{k,s}\right])^2 = \gamma^i_k \beta_{k,s} \alpha^i_{k,s} \quad , \tag{8}$$

$$(E\left[D^i_{k}\right])^2 = \sum_{s=1}^{S} (E\left[D^i_{k,s}\right])^2 \quad . \tag{9}$$

where

1. $\alpha^i_{k,s}$: a measure of the *relative* impact of shock s on variable k *for sector i as compared to other sectors*. For example, this variable could encode the fact that a more energy intensive sector ought to be more sensitive to shocks pertaining to the price of energy than a less energy intensive one. This measure, which we derive from cross-sectoral data, is *not* comparable across shocks. To ensure that we are capturing the effect of the shock, we only use identifying information for those sectors that are in either extreme of the distribution of our indicator variables, remaining agnostic on the ones that lie in the middle. The specific indicators that we use to choose $\alpha^i_{k,s}$ are informed by the model presented in Section 3 and will be described in detail in Section 3.7.[11]

2. $\beta_{k,s}$: a measure of the *overall* impact of shock s on variable k *across all sectors*. For example, this variable could encode the notion that aggregate productivity shocks account for a larger fraction of the variance of quantities in all sectors as compared to markup shocks and vice versa for prices. We use an 'ignorance prior' and set this variable to $1/S$, where S is the number of structural shocks.

3. γ^i_k: a measure of the overall sectoral sensitivity to shocks. For example, this variable could encode the notion that consumption of durable goods is overall more sensitive to all shocks than consumption of nondurables. Given $\alpha^i_{k,s}$ and $\beta_{k,s}$, we can back out this variable if we have values for $(D^i_k)^2$. We obtain those by estimating the model in a training sample from our estimation with an agnostic prior. We do not need to impose any identifying restrictions on the structural shocks for that training sample step because we are only interested in estimating how important those shocks are for fluctuations of different variables together. The factor

[11] If there are missing values for the indicator variables for some sectors, we assume that the indicators for those sectors takes on the average value of the relevant indicator.

structure of the shocks allows us to do that even if we cannot disentangle the individual shocks. We also only impose very loose prior on the covariance matrix of the non-structural shocks.

The procedure above allows us to set a magnitude for the prior mean $E\left[D_{k,s}^i\right]$. We use *a priori* information on the sectoral impact of shocks to set the sign.

To set the prior mean for the impact of shocks on aggregate variables, we only use minimal assumptions: a monetary policy shock is more likely to raise the nominal rate than to lower it, a household demand shock tends to increase consumption, a government spending shock tends to increase government spending, a technology shock tends to increase measured TFP, a credit shock shock tends to increase our measure of the spread, and finally an energy shock tends to increase energy price inflation. To be precise, we treat the aggregate variables as another sector. In lieu of sectoral indicators, we just set $\alpha * \beta$ equal to the same value for all shock/variable pairs at the the aggregate level for which we want to assume non-zero prior impulse responses on impact. The specific value is actually numerically irrelevant. The fraction of aggregate fluctuations driven by aggregate shocks is determined by the training sample estimation just as it is for each sector. Those prior assumptions, together with the priors regarding the impact of the shocks on sectoral variables are summarized in table 2 For the impact of those shocks on other variables, we impose an agnostic prior with mean zero and variance 0.25. Given those assumptions, the training sample pins down the prior mean impact of the shocks on the aggregates.

In order to obtain a prior standard deviation for $D_{k,s}^i$, we choose the prior standard deviation to be $k \times abs\left(E\left[D_{k,s}^i\right]\right)$, where we set k to be 0.1.

At this point it is useful to step back. What we have solved for here is the *prior mean* of $D_{k,s}^i$. Let's make this explicit and call the (square root of the) solution to the system of equations above $E\left[D_{k,s}^i\right]$. This means that we do not necessarily force the sign restrictions to hold with certainty. Since the prior on $D_{k,j}^i$ is normal, there will always be some probability that the sign of the posterior mean will be different from that of the prior mean. This also highlights the role of the prior variance.

2.4.2 Setting priors for Ω and Ω^i

To use the Gibbs sampler, we use inverse-Wishart priors for the covariance matrices of the reduced form shocks at the aggregate and sectoral levels. As is well known, this imposes some restrictions on what prior beliefs we can impose on our model. One is that the variances are bounded away from 0 (really not much of a problem in our case), while the main problem is that there is no truly uninformative prior (as we increase the variance, we also have to at some point increase the prior mean since variances are bounded below by 0).
Since the priors for these covariances turn out to be potentially influential in finite samples, we will use the results of the estimation with the agnostic prior to set this prior.

Prior on Ω To set the prior for Ω, we use results from our agnostic prior estimation. We set the prior mean to the estimated posterior mean of Ω and use as degrees of freedom the size of our overall sample.

Prior on Ω^i For Ω^i, we follow the same strategy as for its aggregate counterpart Ω.

2.5 Sampling Strategy

As mentioned before, we exploit the Gibbs sampler throughout by imposing independent Normal-inverse Wishart priors.

2.5.1 Drawing ε_t given all other parameters

We assume Gaussian innovations throughout for tractability. If we use a variant of equations (1) and (2), it is straightforward to see that, conditional on A_l, B_l, C_l, Σ, D, and D^i, ε_t can be drawn via exploiting the Kalman filter (simply put all known quantities on the left-hand-side: all that remain on the right hand side are the ε terms, w^i and w), based on Carter and Kohn (1994). To make this step more numerically efficient, we follow Durbin and Koopman (2012) and collapse the large vector of observables in a vector with the same dimension as the structural shocks. As discussed by Durbin and Koopman (2012), this can be done without loss of information.

2.5.2 Drawing other parameters given ε

Since we condition on ε at this stage, drawing all other parameters amounts to drawing from Gaussian and inverse Wishart posteriors. For the aggregate equations, we use a Minnesota-type prior following Koop and Korobilis (2010). Such a prior is useful because it avoids overfitting. Since there are only two variables per sector, overfitting is less of an issue, so we use priors centered at 0 with a standard deviation of .5 there. One helpful insight here is that conditional on ε, all other blocks can be run in parallel. This means that our approach can be scaled up easily. This is especially useful for extensions where the researcher might want to depart from the Normal-inverse Wishart prior used here.

2.6 Comparison with other approaches

Our model differs from other approaches that try to model large panels of time series by explicitly modeling a distinction between time series at the aggregate and idiosyncratic levels. FAVARs (Bernanke et al. (2005)) do feature this distinction, but do not explicitly model dynamics at the sectoral level. Furthermore, identifying structural shocks in both factor models and FAVARs requires imposing identifying assumptions for both the unobserved factors and the structural shocks, which, in turn, identify the factors. While our model does have a factor structure, we only require identifying assumptions for the structural shocks since all other factors are observable. Another modeling approach that touches on issues similar to ours are Global VARs (Chudik and Pesaran (2016)). Those do not break the tight link between aggregate shocks and one-step ahead forecast errors at the aggregate level and require restrictions on how shocks propagate between idiosyncratic variables. At this point the reader might wonder why breaking the tight link between one-step ahead forecast errors and structural shocks implied by standard VARs is useful. There are two distinct reasons: (i) this allows sectoral data and aggregate data to *jointly* identify structural shocks and (ii) it does not necessarily force structural shocks to explain large fractions of the variances of our observables if the data do not call for structural shocks to be important.[12]

[12] Our model does not preclude structural shocks from being the main drivers of business cycles a priori: the estimated variances of the non-structural shocks could be very small.

In terms of comparison with factor models along the lines of Stock and Watson (2005a), one question that might arise is why those models often select few factors (where the factors are modeled as persistent VAR processes), whereas we will use a larger number of *iid* structural shocks as factors in our empirical application. The answer is that the few 'VAR factors' in the factor model literature could in fact be driven by our larger number of *iid* shocks, but in the factor model literature the assumption is that the number of *iid* shocks driving the 'VAR factors' is the same as the number of 'VAR factors'. Notice that for applications where no structural shocks are identified, assuming as many iid shocks as 'VAR factors' is without loss of generality under Gaussianity and linearity.
Finally, as can be seen from equation 3, our model is a restricted VAR using many variables. As such, there is a natural connection to the literature that uses shrinkage priors for such VARs (Banbura et al. (2010)). Instead of using shrinkage priors (such as the Minnesota prior) in a VAR for all of our variables, we instead impose restrictions implied by the grouping of variables into sectoral and aggregate variables.[13]

2.7 A Monte Carlo Experiment

This section describes the results of a Monte Carlo experiment that is meant to highlight that the overall impact of structural shocks in our environment is identified *independently of identifying restrictions for any specific structural shock*. All variables in this example are stationary, even though this is not necessary for our method. The aggregate level consists of 4 variables, whereas each sector (of which we have 100) consists of 2 variables. There are two structural shocks (elements of ε_t). The additional, non-structural, shocks are correlated within units (sectors or the aggregate level). We assume that most of the variance of the one-step ahead forecast errors is in fact due to these additional shocks - Figure 1 displays the fraction of the one-step ahead forecast error due to structural shocks for the variables at the aggregate and sectoral levels. These numbers are meant to convey that this is in fact a hard inference problem - most of the variation in the simulated data is not due to structural shocks. We simulate 130 datapoints and assume a lag length of 1 in all specifications (which is the correct specification). We use the agnostic prior for this estimation.

The Gaussian priors for all coefficients are centered at 0 with a variance of 10 and are thus loose given the magnitude of the parameters used in the estimation. A loose Wishart prior is used for the covariance of the residual error terms.
Even without *any* specific identifying restrictions on structural shocks, the next two figures show that in finite samples, our approach is able to correctly predict the overall effect of structural shocks. Figure 2 plots the true and estimated (median) effects of the structural shocks on aggregate variables. As outlined in our discussion of identification, these results highlight that while the individual effects of structural shocks can not be estimated without identifying assumption in our framework, the overall effect of all structural disturbances is well identified.

3 A Tractable Multi-Sector Model with Nominal Rigidites

We now lay out a tractable, multi-sector model with nominal rigidities to motivate the shock identification. Nominal rigidities allow for a non-trivial "aggregate demand" channel. Since our main focus is in the cross-sectional differences between industries rather than their individual dynamics, we lay out a static multi-sector economy. This meshes well with our empirical analysis since we only

[13] We do still use a Minnesota-type prior for the aggregate variables in our VAR.

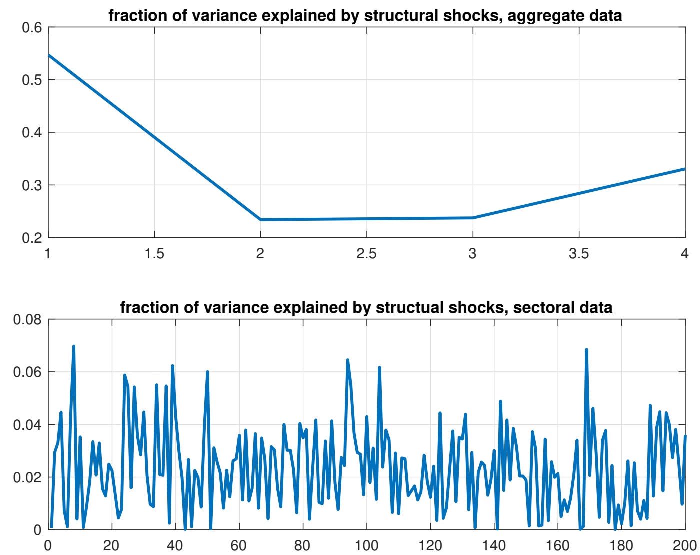


Figure 1: Fraction of variance explained by structural shocks, Monte Carlo experiment, index of variables on the x-axis.

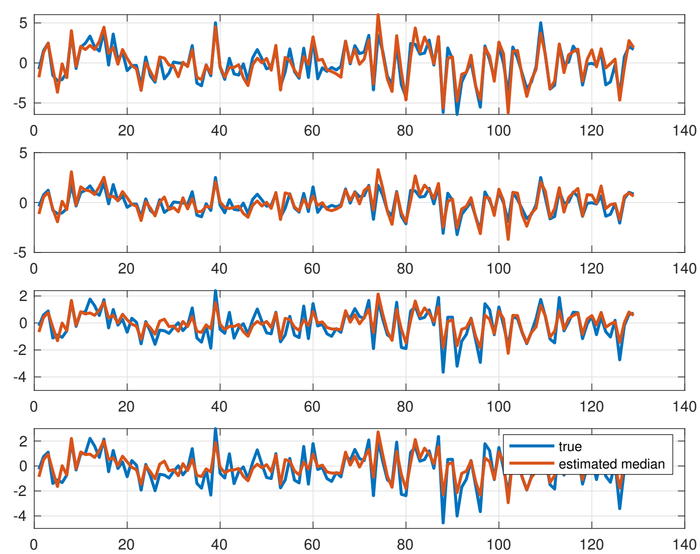


Figure 2: True effect and estimated median effect of the structural shocks on the four aggregate variables, Monte Carlo experiment.

impose identifying restrictions on the impact responses. The model shares many elements with the framework developed in Pasten et al. (2018), while also allowing for nominal wage stickiness and for several aggregate shocks.

3.1 Households

There are I sectors, indexed $i \in \{1, ..., I\}$. There is a representative household with Cobb-Douglas preferences over the various goods, with share-parameter α_i for a good of industry i.

$$U = \prod_i C_i^{\alpha_i},$$

where $\sum_i \alpha_i = 1$. The household chooses its the amount it consumes of good i, C_i, to maximize its utility subject to the budget constraint

$$\sum_i P_i C_i + T = WL + \Pi + \sum_i r_i \bar{K}_i,$$

where T is a lump-sum tax levied by the government to finance its consumption, W is the wage rate, Π are profits rebated from firms, $\bar{K}_i$ is the stock of capital specific to sector i owned by the household, with r_i the corresponding rental rate, and $L < 1$ is employment to be determined in equilibrium. Households supply one unit of labor inelastically, but nominal wages are rigid so that labor is rationed.

Optimal household consumption choice satisfies:

$$P_i C_i = \alpha_i^C P^C C,$$

for $P^C \equiv \prod_i \left(\frac{P_i}{\alpha_i}\right)^{\alpha_i}$ and $C \equiv \prod_i (C_i)^{\alpha_i}$

3.2 Fiscal Authority

The fiscal authority minimizes the cost of consuming an exogenously given aggregate government consumption G,

$$\min \sum_i P_i G_i$$
$$s.t. : \prod_i (G_i)^{\alpha_i^G} = G,$$

where G is exogenously determined and α_i^G are the shares. The optimality condition for the government is:

$$G_i = \alpha_i^G \frac{P_G}{P_i} G$$

where

$$P_G = \prod_i \left(\frac{G_i}{\alpha_i^G}\right)^{\alpha_i^G}.$$

3.3 Firms

Within each sector there is a continuum of varieties of intermediate products indexed $v \in [0,1]$. Those varieties are purchased by final goods producers that bundle them into the I goods according to a CES aggregator:

$$Y_i = \left[\int_0^1 Y_i(v)^{\frac{\theta-1}{\theta}} dv \right]^{\frac{\theta}{\theta-1}}$$

The demand for final good producer in sector i for intermediate input of variety v is

$$Y_i(v) = \left(\frac{P_i(v)}{P_i} \right)^{-\theta} Y_i$$

where

$$P_i = \left[\int P_i(v)^{1-\theta} dv \right]^{\frac{1}{1-\theta}}$$

For each variety, production takes place with a Cobb-Douglas production function:

$$Y_i(v) = e^{\epsilon_i} \prod_j (X_{ji}(v))^{\gamma_{ji}} \times (L_i(v))^{\lambda_i} (K_i(v))^{\chi},$$

where $X_{ji}(v)$ is the quantity of final goods materials produced in sector j used as materials in sector i for variety v, $L_i(v)$ is labor, $K_i(v)$ is sector-specific capital, and ϵ_i is a sector-specific exogenous productivity shock. The share parameter for good j used in sector i is γ_{ji}. We assume that $\sum_j \gamma_{ji} + \lambda_i + \chi = 1$, so that firms in the industry face constant returns to scale.

Producers of varieties are monopolists. Firms differ on the information set available to them regarding prices and the demand for their intermediate input. Letting $\mathbf{s} = \left\{ m^G, m^C, m^Y, \{k_i\}_{i=1}^I, \{\epsilon_i\}_{i=1}^I \right\}$ denote the state of the economy, they take the wage rate, final goods prices, and household demand as given and choose their inputs to maximize expected profits.

$$\begin{aligned} \max_{M_{ji}} E & \left[P_i(v) Y_i(v,\mathbf{s}) - \sum_j P_j(\mathbf{s}) X_{ji}(v,\mathbf{s}) - w(\mathbf{s}) L_i(v,\mathbf{s}) - r_i(\mathbf{s}) K_i(v,\mathbf{s}) | \mathcal{I}_i(v) \right] \\ s.t. : Y_i(v,\mathbf{s}) &= \left(\frac{P_i(v)}{P_i(\mathbf{s})} \right)^{-\theta} Y_i(\mathbf{s}) \\ Y_i(v,\mathbf{s}) &= e^{\epsilon_i} \prod_j (X_{ji}(v,\mathbf{s}))^{\gamma_{ji}} (L_i(v,\mathbf{s}))^{\lambda_i} (K_i(v,\mathbf{s}))^{\chi} \end{aligned}$$

where $\mathcal{I}_i(v)$ is the information set for variety v in sector i. For a fraction ϕ_i of variety producers in sector i ($v \in [0, \phi_i]$) the information set does not includes the realized vector of shocks $\mathbf{s}$. For the remainder, the information set does includes it. Yet, firms commit to producing as much as necessary to satisfy demand at the prices that they choose.

Given cost-minimization, marginal cost is

$$\mathrm{mc}_i(\mathbf{s}) = e^{-\epsilon_i} \prod_j \left(\frac{P_j(\mathbf{s})}{\gamma_{ji}} \right)^{\gamma_{ji}} \left(\frac{w(\mathbf{s})}{\lambda_i} \right)^{\lambda_i} \left(\frac{\mathbf{r}(\mathbf{s})}{\chi} \right)^{\chi}$$

Firms with full information set

$$P_i(v, \mathbf{s}) = \frac{\theta}{\theta - 1} \mathrm{mc}_i(s)$$

Firms without full information set

$$P_i(v) = \frac{\theta}{\theta - 1} E\left[\frac{P_i(\mathbf{s})^\theta Y_i(\mathbf{s})}{E\left[P_i(\mathbf{s})^\theta Y_i(\mathbf{s})\right]} mc_i(\mathbf{s})\right]$$

We thus have that the price index for sector i is

$$P_i(\mathbf{s}) = \left[\phi_i \left(\frac{\theta}{\theta - 1} E\left[\frac{P_i(\mathbf{s})^\theta Y_i(\mathbf{s})}{E\left[P_i(\mathbf{s})^\theta Y_i(\mathbf{s})\right]} mc_i(\mathbf{s})\right]\right)^{1-\theta} + (1 - \phi_i) \left(\frac{\theta}{\theta - 1} \mathrm{mc}_i(\mathbf{s})\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

Given that all firms in a sector have the same marginal cost, we can write the average markup as

$$\mu_i = \frac{P_i(\mathbf{s})}{mc_i(\mathbf{s})} = \left[\phi_i \frac{\theta}{\theta - 1} E\left[\frac{P_i(\mathbf{s})^\theta Y_i(\mathbf{s})}{E\left[P_i(\mathbf{s})^\theta Y_i(\mathbf{s})\right]} mc_i(\mathbf{s})\right]^{1-\theta} \left(\frac{1}{mc_i(\mathbf{s})}\right)^{1-\theta} + (1 - \phi_i) \left(\frac{\theta}{\theta - 1}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

3.4 Market Clearing

Market clearing for each sector i, requires that all output is used either as materials, for household consumption or for government consumption:

$$Y_i = \sum_j X_{ij} + C_i + G_i$$

Also, there is a fixed stock of capital $\bar{K}_i$ for each sector. Market clearing in capital markets thus requires that the demand for capital in sector i equals supply:

$$K_i = \bar{K}_i$$

The resource constraint in the labor market is

$$\sum_i L_i \leq 1$$

With sticky wages the inequality need not hold. We assume that wages are stuck at a level high enough that it doesn't bind. Labor rationing thus implies that

$$L = \sum_i L_i$$

3.5 Shocks

There are two nominal quantities set exogenously: nominal private consumption and nominal government consumption. Specifically, we assume that

$$P^C C = M^C M^Y$$
$$P^G G = M^G M^Y$$

so that nominal private and government consumptions can be affected either by an exogenous component which is specific to each type of final expenditure M^C or M^G, or by a common component M^Y.

We also allow for industry level productivity shocks ϵ_i. We assume that $\epsilon_i = \sum_{r=1}^{R} \lambda_{ir}\epsilon_r + \hat{\epsilon}_i$, where ϵ_r are aggregate shocks, F_i captures the sensitivity of various sectors to that shock, and $\hat{\epsilon}_i$ is a sector-specific shock. In our application, we will allow ϵ_r to incorporate shocks to technology and financial shocks.

3.6 Reduced log-linearized system

After log-linearizing and rearranging, the model can be reduced to:[14]

$$\begin{aligned} p_i &= \frac{1-\phi_i}{1-\chi}\chi \left[-\epsilon_i + \sum_j \gamma_{ji} p_j + \chi(y_i - \bar{k}_i) \right] \\ p_i + y_i &= \sum_j \gamma_{ij} \frac{Y_j}{Y_i} \left(y_j + \frac{1}{1-\phi_j} p_j \right) + \frac{C_i}{Y_i}\left(m^C + m^Y\right) + \frac{G_i}{Y_i}\left(m^G + m^Y\right) \\ c_i + p_i &= m^C + m^Y \end{aligned}$$

where small caps letters denote log deviations from a reference level. The first set of equations are "sectoral supply" equations, relating marginal production cost to prices. The second set of equations are "sectoral demand"'equations, relating nominal expenditures to sectoral prices. The last set of equations link nominal consumption expenditures and exogenous demand shocks.

The system has the form

$$Z = AZ + b = A^N Z + \sum_{n=0}^{N-1} A^n b$$

with Z including prices and quantities in all sectors, b including the direct impact of all exogenous shocks, and A including the indirect impact of shocks through linkages.

As we show in the appendix, the direct impact of shocks is

[14] See the appendix for the detailed derivation.

$$p_i^{\text{Direct}} = \Phi_i \chi \left[\frac{C_i}{Y_i} m^C + \frac{G_i}{Y_i} m^G + m^Y \right] - \Phi_i \left(\epsilon_i + \chi \bar{k}_i \right) \tag{10}$$

$$y_i^{\text{Direct}} = (1 - \Phi_i \chi) \left[\frac{C_i}{Y_i} m^C + \frac{G_i}{Y_i} m^G + m^Y \right] + \Phi_i \left(\epsilon_i + \chi \bar{k}_i \right) \tag{11}$$

$$c_i^{\text{Direct}} = (1 - \Phi_i \chi) \frac{C_i}{Y_i} m^C + (1 - \Phi_i \chi) m^Y - \Phi_i \chi \frac{G_i}{Y_i} m^G + \Phi_i \left(\epsilon_i + \chi \bar{k}_i \right) \tag{12}$$

where

$$\Phi_i \equiv \frac{1 - \phi_i}{\chi(1 - \phi_i) + 1 - \chi}$$

is inversely related to ϕ_i. Indirect effects are

$$p_i^{\text{Indirect}} = \Phi_i \sum_j \left(\chi \frac{f_{ij}}{1 - \phi_j} + b_{ji} \right) p_j + \chi \Phi_i \sum_j f_{ij} y_j \tag{13}$$

$$y_i^{\text{Indirect}} = (1 - \chi \Phi_i) \sum_j f_{ij} y_j + \sum_j \left[\frac{1 - \chi \Phi_i}{1 - \phi_j} f_{ij} - \Phi_i b_{ji} \right] p_j \tag{14}$$

$$c_i^{\text{Indirect}} = -p_i^{\text{Indirect}} \tag{15}$$

where $f_{ij} = \gamma_{ij} \frac{Y_j}{Y_i}$ capture forward linkages and $b_{ji} = \gamma_{ji}$ captures backward linkages.

3.7 Priors on sectoral impact of aggregate shocks

Since we use PCE prices and quantities, we use equations 10 through 15 to put priors on the differential responses of sectoral prices and consumption to shocks (signs for the prior mean in parentheses). We will focus whenever possible on the direct effects of the shocks, as these are likely to be the most salient ones. Also, we will focus on the part of those effects that are most likely to be unique to the shock in part. Thus, for example, whereas the sectoral impact of all shocks is mediated by the degree of price stickiness, indexed by Φ_i, only the shock to consumption demand has its impact depend on the consumption share $\frac{C_i}{Y_i}$. Once other shocks are controlled for, the monetary shock is the only one that has its effects primarily tied to the degree of price stickiness, Φ_i.

For most shocks, a unique source of heterogeneous impact appears most clearly in the direct impact. The exception is the energy shock, the impact of which is felt through its indirect impact. In what follows, we describe the priors chosen to identify each of those shocks following this heuristic.

- Household demand (m^C): $\alpha_{i,k}^s = \frac{C_i}{Y_i}$ (+ for quantities and prices), obtained from Use tables published by the BEA (see equations 10 and 12)

- Government demand: $\alpha_{i,k}^s = \frac{G_i}{Y_i}$ (- for quantities, + for prices), obtained from Use tables published by the BEA (see equations 10 and 12)

- Credit: $\alpha^s_{i,k} = \partial \bar{k}_i / \partial credit$, proxied by dependence on external finance (- for quantities, + for prices) calculated from COMPUSTAT data. (see equations 10 and 12))

- Energy: $\alpha^s_{i,k} = \gamma_{\text{energy},i}$ (- for quantities, + for prices), obtained from Use tables published by the BEA. This follows if we assume that there are forward linkages from the energy sector that are small relative to backward linkages (i.e., energy is an important input in many sectors but is a relatively small consumer of other sectors' output). With $f_{ij} \simeq 0$, the indirect impact of energy shocks on sectoral prices and quantities captured in equations 13 to 15 resembles that of a productivity shock.

- Monetary: $\alpha^s_{i,k} = \phi_i$ for quantities and $1/\phi_i$ for prices, proxied by average price duration, obtained from Nakamura and Steinsson 2008. (see equations 10 and 12))

- Technology: $\gamma^s_{i,k} = \partial \epsilon_i / \partial technology$ proxied by R&D intensity (+ for quantities, - for prices). Here we us the ratio of intermediate goods from tech industries to total intermediate goods following Heckler (2005) (see equations 10 and 12))

Those prior assumptions, together with the priors regarding the impact of the shocks on aggregate variables are summarized in table 2.

4 Data and Results

4.1 Data

We use 8 aggregate US time series (in year-over-year growth rates where applicable): real GDP growth, CPI inflation, the effective Federal Funds rate, growth rate in real government spending, real PCE consumption growth, Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity, Fernald's utility adjusted TFP (Fernald (2014)), and energy inflation based on the relevant producer price index. We use data from the first quarter of 1961 to the last quarter of 2017.

For the sectoral data, we use the growth rate of real activity as measured by the sectoral PCE and sectoral inflation as measured by the associated price. In terms of specification, we use 6 lags throughout, except for the lagged aggregate variables in the sectoral equations, where we only use one lag. We allow for 6 aggregate shocks: monetary policy, government spending, financial, energy, technology, and household demand. We also allow for 3 unidentified sources of sectoral variation, which we include to allow for the possibility that we missed some important aggregate structural shocks. We plot the data we use in the appendix.

Table 1 below summarizes the priors on the different parameters, whereas table 2 summarizes the aggregate variable and sectoral indicators used to pin down the prior means for the impact matrices.

Table 1: Summary of prior distributions

Parameters	Prior Density	Prior Parameters
μ, A_l	Normal	Minnesota prior as in Koop & Korobilis
Ω	Inverse Wishart	mean set via training sample, degrees of freedom set to sample size
D, constrained elements	Normal	mean and standard deviation set by solving system of equations
D, unconstrained elements	Normal	mean 0, standard deviation 0.25
μ^i,B_l^i, C_h^i	Normal	each element has mean 0, standard deviation 0.5
Ω_i	Inverse Wishart	prior mean set via training sample, degrees of freedom set to 15
D^i	Normal	mean and standard deviation set by solving system of equations

Table 2: Summary of prior means

Shock	aggregate impact	Sectoral impact index
Technology	Fernald TFP	high-tech content
Credit	credit spread	financially dependence
Household	household consumption	consumption oriented
Government	government consumption	government oriented
Monetary	fed funds rate	price stickiness
Energy	energy price index	energy intensity

4.2 Impulse Response Functions

We now show the impulse response functions to different shocks. This provides a check on our identification procedure, in that it allows us to evaluate whether responses to identified shocks conform to theory or prior findings based on other identification schemes. To economize on space, we show the 5th percentile, median and 95th percentile of the impulse responses to a one-standard deviation shock for the monetary and household demand shocks in figures 3 and 4. For those shocks we will also highlight how the different components of our identification strategy (on aggregate and sectoral data) interact. For all other shocks, figure 5 shows the median IRFs. The full set of impulse response plots with error bands and the associated figures analyzing the role of our identification assumptions for all other shocks can be found in the appendix. The plots look largely as expected: technology shocks increase consumption and GDP without much of an effect on consumption, while credit shocks depress those. Household demand shocks increase interest rate and GDP at first, the impact on output quickly reverts and the point estimate becomes somewhat negative (Figure 3). Government consumption shocks increase interest rates and inflation, and while they boost consumption at first, they crowd it out later. Energy shocks increase interest rates and inflation and depress GDP and consumption. Monetary policy shocks depress inflation, GDP and consumption (Figure 4).

We also show how incorporating the sectoral data helps with identification. For example, figure 6 shows that, relative to a specification where the shock is identified only from its impact on aggregate consumption, the impulse response functions for the household demand shock becomes much more tightly estimated once we incorporate priors on the sectoral responses. It is those tighter posteriors that make clear the impact of those shocks on inflation and interest rates. Also, figure 7 does the same analysis for monetary policy shocks. Compared to a shock identified solely from its impact

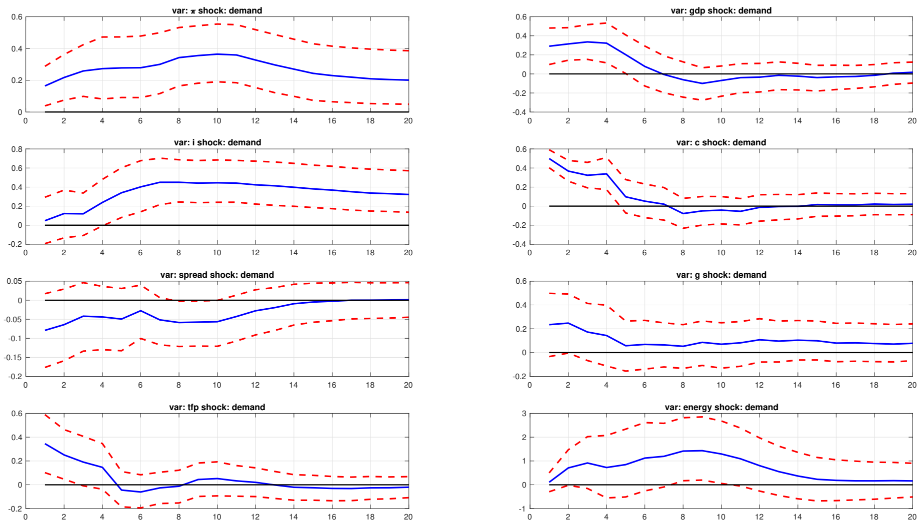


Figure 3: Responses to Household Demand Shock

on the nominal interest rate, the sectoral identification shows the deflationary impact much more clearly.[15]

[15]These figures plot 16th and 84th percentiles.

Figure 4: Responses to Monetary Shock

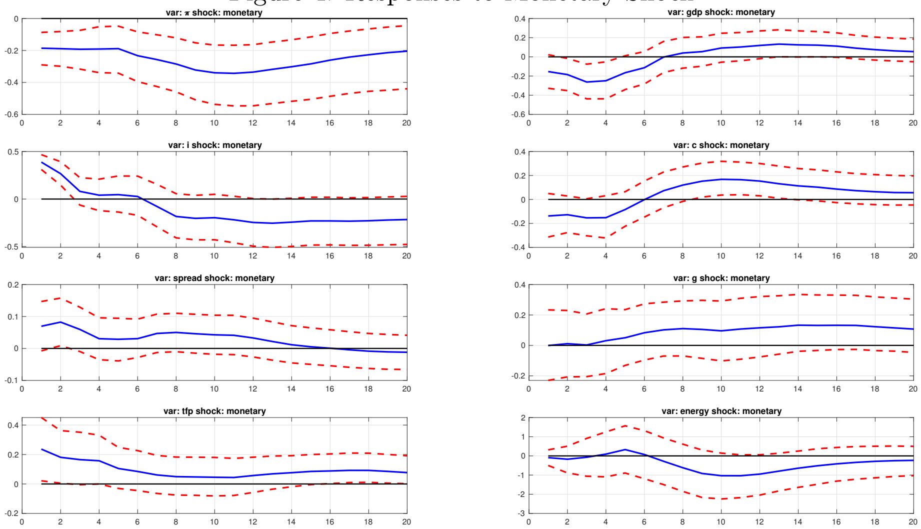


Figure 5: Median Responses to all other Shocks

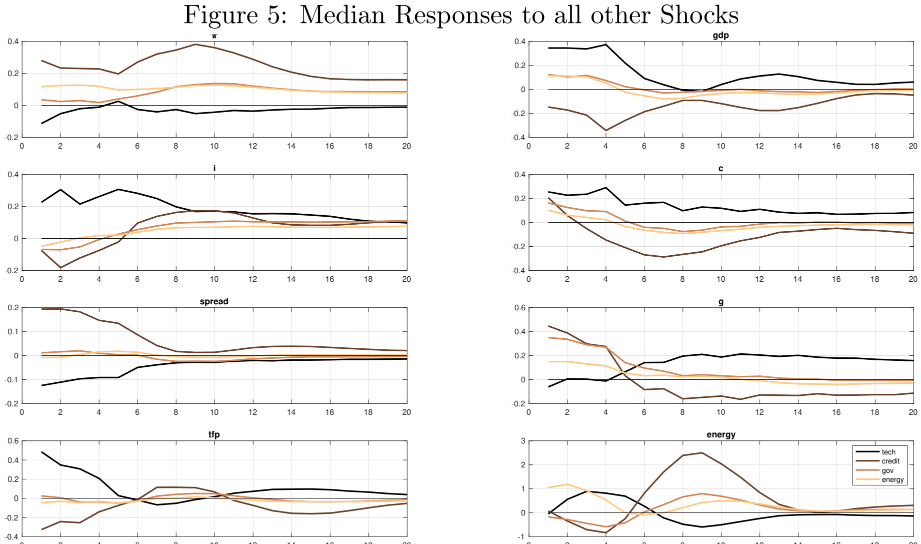


Figure 6: Responses to Household Demand Shocks: Comparison of Identification Schemes

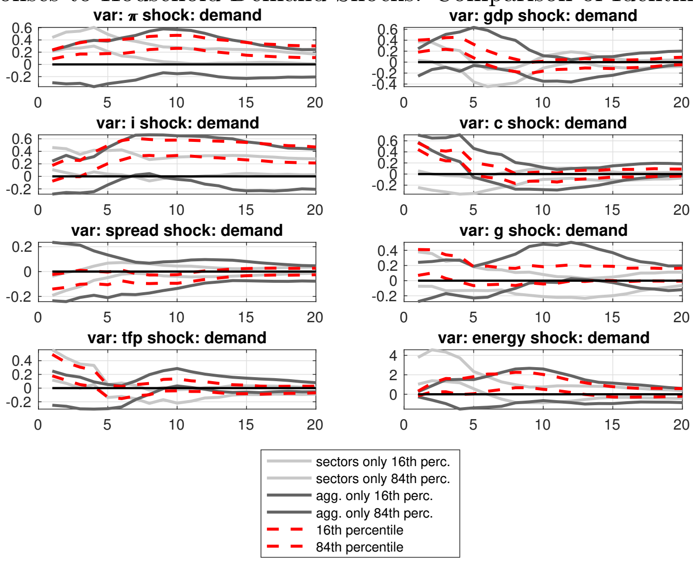
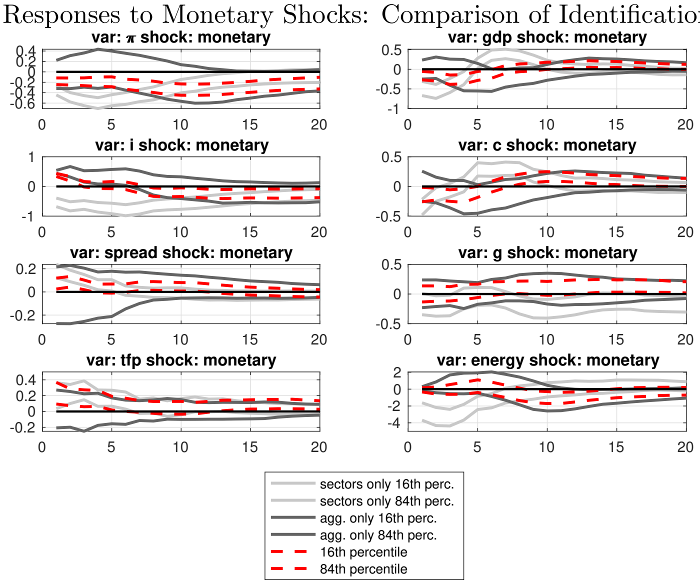


Figure 7: Responses to Monetary Shocks: Comparison of Identification Schemes



4.3 Analysis of Variance

In this section, we show how much our identified structural shocks explain as a fraction of the variance explained by all structural shocks. The results are presented in table 3 below. To obtain the numbers in the table, we decompose for each variable the fraction of the variance of its innovation that is tied to aggregate shocks into different components. The numbers refer to average variances for forecast errors 6 to 32 quarters ahead. The six identified shocks account for more than 80% of overall variance explained by the structural shocks ε, with the one exception being Fernald's TFP series. The table shows that monetary shocks play a prominent role not only in explaining nominal interest rates and inflation (as one would expect), but also GDP, consumption, and energy prices. The other shock with a prominent role is to corporate credit, accounting for a large part of the variance of GDP and consumption. If we count household consumption, government consumption, and monetary policy shocks as "demand" shocks and energy and technology as "supply" shocks, we find that demand shocks account for substantially more of GDP variation at business cycle frequencies than supply shocks.

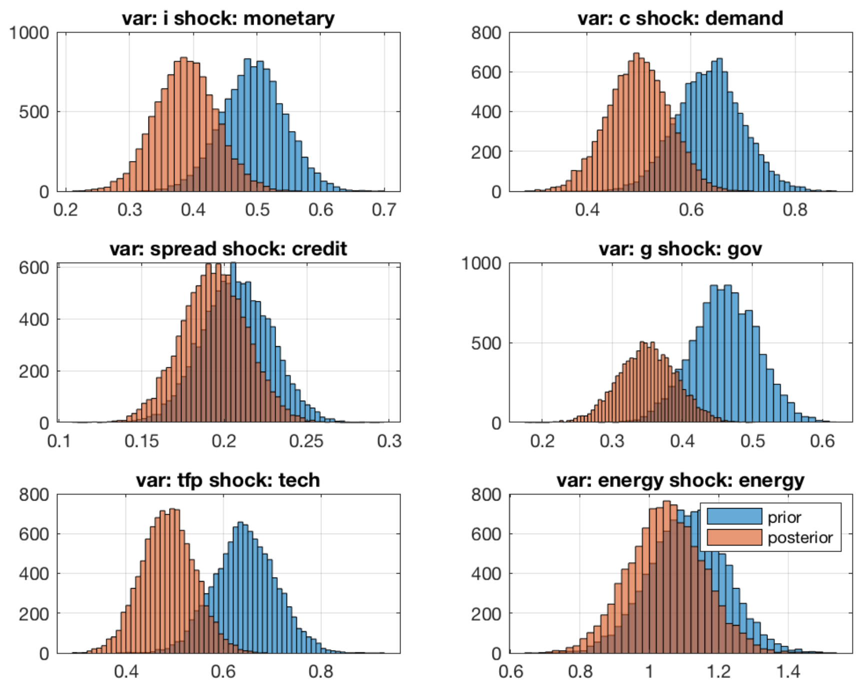
	tech	credit	demand	gov	energy	monetary
π	4.9	21.3	28.6	4.1	4.2	27.1
gdp	21.1	20.8	16.8	3.9	4.1	15.4
i	12.2	10.3	38.4	4.3	2.9	20.6
c	15.9	22.2	28.5	4.3	3.6	14.8
spread	17.1	38.7	14.9	2.6	2.4	15.5
g	18.8	26.7	15.4	9.5	3.8	12.5
tfp	26.4	23.3	16.2	2.1	2.3	14.8
energy	9.6	26.3	17.5	4.6	11.1	16.7

Table 3: Mean of conditional variance decomposition across business cycle frequencies and posterior draws

4.4 The Role of the Prior in Determining Impulse Responses

To get a sense how much information we impose *a priori* about the impulse responses, we next plot histograms of draws from the prior on the impact aggregate impulse responses. Here we focus on those impulse responses for which we imposed informative priors. We then compare those prior histograms with those from the posterior (both are computed using 10,000 draws). We focus here on the impact responses because that is where we use identification restrictions via our choice of priors on D. Figure 8 shows that the data are in fact informative about the impulse responses, but also that the prior certainly is not at odds with the posterior impulse responses.

Figure 8: Prior vs posterior impact IRFs for those IRFs where informative priors are used



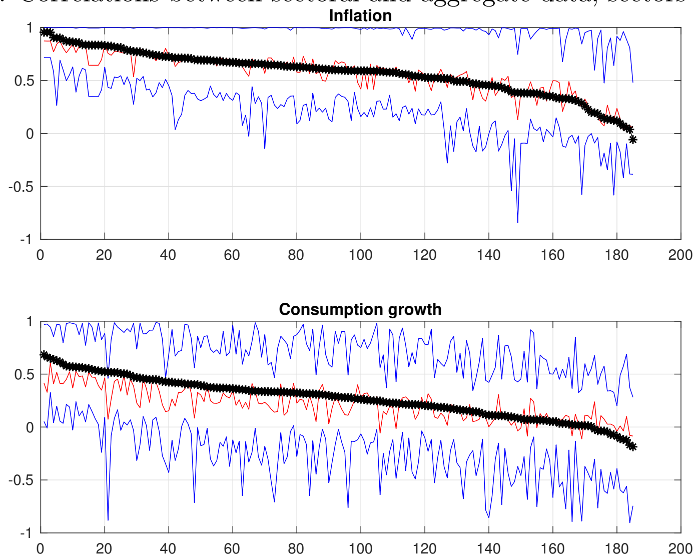
4.5 Model Fit and the Interplay between Sectoral and Aggregate Data

Our model is highly restrictive in that any correlation between sectors as well as between sectors and aggregate variables has to come through either the structural shocks ε_t or lagged aggregate variables. The reader might a priori wonder if this leads to substantial misspecification, which in turn would cast doubt on our identification strategy that is based on sectoral data.

To address this possible concern, we first compute the correlations between aggregate consumption growth and consumption growth at the sectoral level that appear in our dataset as well as the corresponding correlations for aggregate and sectoral inflation. We then draw 1000 parameter values from the posterior, simulate data of the same length as our dataset for each set of parameters (after discarding 1000 burn-in observations) and compute the same correlations for our simulated data. This gives us the posterior distribution of the correlations we are interested in. We are thus carrying out a posterior predictive check as advocated for by Rubin (1984) and further discussed by Gelman et al. (2013) and Geweke (2005), for example.

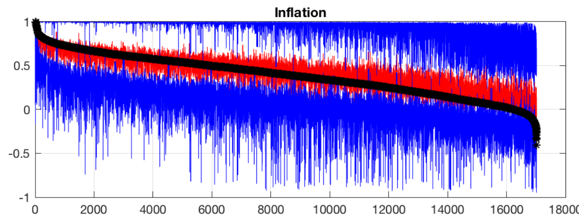
Figure 9 plots the correlations from the data (black) as well as the median (red) and the 5th and 95th percentiles (blue) of the posterior distribution. We sort the correlations from the actual data by size (starting with the largest correlation) to make the figure easier to interpret. We order the sectors the same way for the simulated data. As can be seen from figure 9, our model is able to replicate the correlation patterns between aggregate and sectoral data. An inquisitive reader might

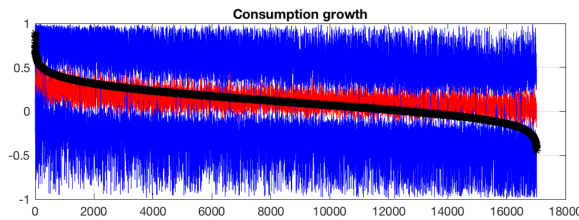
Figure 9: Correlations between sectoral and aggregate data, sectors on x-axis



ask for a more stringent test, namely a check of the correlation of variables *across* sectors rather than between any sector and the corresponding aggregate variable. We show the results for this posterior predictive check in figure 10. The figure looks noisier just because there are many more datapoints (pairwise correlations between the 185 sectors in our sample), but the main pattern remains, our model is able to replicate the broad correlation patterns. Our model misses at the very tail ends of the spectrum of correlations (more so for inflation than for consumption growth), but given that our model is tightly parametrized and parsimonious, we think of these results as very encouraging.

Figure 10: Correlations between sectoral data, sectors on x-axis





5 Conclusion

We lay out a new methodology to identify the effect of aggregate shocks and their role in driving aggregate fluctuations. The hierarchical vector auto-regressive model allows us to isolate innovations to aggregate and sectoral variables. We can then use those innovations to extract common factors that isolate the role of aggregate shocks in driving fluctuations. Those factors are identified through priors on their differential impact on different sectoral prices and quantities combined with minimal identifying assumptions on aggregate variable.

This identification procedure allows us to recover impulse response functions. We find that our estimated impulse responses are consistent with macroeconomic theory and prior studies that focused on one shock at a time. We then use those identified shocks to inquire into the origins of business cycles in the U.S. Identified shocks that originate in household behavior, government expenditure, or monetary policy decisions ("demand" shocks) account for a larger proportion of business cycles than shocks associated with technical progress or energy costs ("supply shocks"). U.S. business cycles thus have their origins more in demand fluctuations rather than supply shocks.

References

Altonji, J. G. and J. C. Ham (1990, January). Variation in Employment Growth in Canada: The Role of External, National, Regional, and Industrial Factors. *Journal of Labor Economics 8*(1), 198–236.

Amir-Ahmadi, P. and T. Drautzburg (2017, May). Identification Through Heterogeneity. Working Papers 17-11, Federal Reserve Bank of Philadelphia.

Angeletos, G.-M., F. Collard, and H. Dellas (2018, July). Business Cycle Anatomy. Nber working papers, National Bureau of Economic Research, Inc.

Bachmann, R. and P. Zorn (2013, April). What Drives Aggregate Investment? Evidence from German Survey Data. NBER Working Papers 18990, National Bureau of Economic Research, Inc.

Bai, J. and S. Ng (2008, June). Large Dimensional Factor Analysis. *Foundations and Trends(R) in Econometrics 3*(2), 89–163.

Banbura, M., D. Giannone, and L. Reichlin (2010). Large Bayesian VARs. *Journal of Applied Econometrics 25*, 71–92.

Baumeister, C. and J. D. Hamilton (2015, September). Sign Restrictions, Structural Vector Autoregressions, and Useful Prior Information. *Econometrica 83*(5), 1963–1999.

Baumeister, C. and J. D. Hamilton (2018). Inference in structural vector autoregressions when the identifying assumptions are not fully believed: Re-evaluating the role of monetary policy in economic fluctuations. *Journal of Monetary Economics 100*(C), 48–65.

Baumeister, C. and J. D. Hamilton (2019). Structural Interpretation of Vector Autoregressions with Incomplete Identification: Revisiting the Role of Oil Supply and Demand Shocks. *American Economic Review forthcoming*.

Beraja, M., E. Hurst, and J. Ospina (2016, February). The Aggregate Implications of Regional Business Cycles. NBER Working Papers 21956, National Bureau of Economic Research, Inc.

Bernanke, B. S., J. Boivin, and P. Eliasz (2005). Measuring the effects of monetary policy: a factor-augmented vector autoregressive (favar) approach. *The Quarterly journal of economics 120*(1), 387–422.

Boivin, J., M. P. Giannoni, and I. Mihov (2009). Sticky prices and monetary policy: Evidence from disaggregated us data. *American economic review 99*(1), 350–84.

Caldara, D. and E. Herbst (2016, May). Monetary Policy, Real Activity, and Credit Spreads : Evidence from Bayesian Proxy SVARs. Finance and Economics Discussion Series 2016-049, Board of Governors of the Federal Reserve System (US).

Canova, F. and G. D. Nicolo (2002, September). Monetary disturbances matter for business fluctuations in the G-7. *Journal of Monetary Economics 49*(6), 1131–1159.

Carter, C. K. and R. Kohn (1994, 09). On Gibbs sampling for state space models. *Biometrika 81*(3), 541–553.

Chen, X., M. Chang, and F. Schorfheide (2018). Heterogeneity and Aggregate Fluctuations. Technical report.

Chudik, A. and M. H. Pesaran (2016, February). Theory And Practice Of Gvar Modelling. *Journal of Economic Surveys 30*(1), 165–197.

Clark, T. E. and K. Shin (1998). The sources of fluctuations within and across countries. Technical report.

De Graeve, F. and A. Karas (2010, July). Identifying VARs through Heterogeneity: An Application to Bank Runs. Working paper series, Sveriges Riksbank (Central Bank of Sweden).

Durbin, J. and S. J. Koopman (2012). *Time Series Analysis by State Space Methods.* Oxford University Press.

Faust, J. (1998). The robustness of identified VAR conclusions about money. *Carnegie-Rochester Conference Series in Public Policy 49*, 207–244.

Fernald, J. (2014). A quarterly, utilization-adjusted series on total factor productivity. Federal Reserve Bank of San Francisco.

Fulford, S. L. and F. Schwartzman (2015). The benefits of commitment to a currency peg: Aggregate lessons from the regional effects of the 1896 u.s. presidential election. Technical report, Working Paper 13-18, Federal Reserve Bank of Richmond.

Gelman, A., J. B. Carlin, H. S. Stern, D. B. Dunson, A. Vehtari, and D. B. Rubin (2013). *Bayesian Data Analysis.* Chapman & Hall.

Geweke, J. (2005). *Contemporary Bayesian Econometrics and Statistics.* Wiley.

Gorodnichenko, Y. (2005). Reduced-rank identification of structural shocks in VARs. Technical report.

Guren, A., A. MacKay, E. Nakamura, and J. Steinsson (2019). What Do We Learn From Cross-Sectional Empirical Estimates in Macroeconomics? Technical report.

Heckler, D. E. (2005). High-technology employment: a naics-based update. *Monthly Lab. Rev. 128*, 57.

Holly, S. and I. Petrella (2012, November). Factor Demand Linkages, Technology Shocks, and the Business Cycle. *The Review of Economics and Statistics 94*(4), 948–963.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2010). Investment shocks and business cycles. *Journal of Monetary Economics 57*(2), 132–145.

Kilian, L. (2009, June). Not All Oil Price Shocks Are Alike: Disentangling Demand and Supply Shocks in the Crude Oil Market. *American Economic Review 99*(3), 1053–1069.

Kociecki, A. (2010). A Prior for Impulse Responses in Bayesian Structural VAR Models. *Journal of Business & Economic Statistics 28*(1), 115–127.

Koop, G. and D. Korobilis (2010, July). Bayesian Multivariate Time Series Methods for Empirical Macroeconomics. *Foundations and Trends(R) in Econometrics 3*(4), 267–358.

Mertens, K. and M. O. Ravn (2013, June). The Dynamic Effects of Personal and Corporate Income Tax Changes in the United States. *American Economic Review 103*(4), 1212–1247.

Nakamura, E. and J. Steinsson (2008). Five facts about prices: A reevaluation of menu cost models. *The Quarterly Journal of Economics 123*(4), 1415–1464.

Pasten, E., R. Schoenle, and M. Weber (2018). The propagation of monetary policy shocks in a heterogeneous production economy. Technical report, National Bureau of Economic Research.

Rajan, R. G. and L. Zingales (1998). Financial dependence and growth. *The American Economic Review 88*(3), 559–586.

Ramey, V. A. (2011). Identifying Government Spending Shocks: It's all in the Timing. *The Quarterly Journal of Economics 126*(1), 1–50.

Romer, C. D. and D. H. Romer (2004, September). A New Measure of Monetary Shocks: Derivation and Implications. *American Economic Review 94*(4), 1055–1084.

Rubin, D. B. (1984, 12). Bayesianly justifiable and relevant frequency calculations for the applied statistician. *The Annals of Statistics 12*(4), 1151–1172.

Rubio-Ramirez, J. F., D. F. Waggoner, and T. Zha (2010). Structural Vector Autoregressions: Theory of Identification and Algorithms for Inference. *Review of Economic Studies 77*(2), 665–696.

Sarto, A. P. (2018). Recovering Macro Elasticities from Regional Data. Technical report.

Schwartzman, F. (2014). Time to produce and emerging market crises. *Journal of Monetary Economics 68*, 37–52.

Sims, C. A. and T. Zha (1998, November). Bayesian Methods for Dynamic Multivariate Models. *International Economic Review 39*(4), 949–968.

Stock, J. H. and M. W. Watson (2005a). Implications of dynamic factor models for var analysis. Technical report, National Bureau of Economic Research.

Stock, J. H. and M. W. Watson (2005b, September). Understanding Changes In International Business Cycle Dynamics. *Journal of the European Economic Association 3*(5), 968–1006.

Uhlig, H. (2005, March). What are the effects of monetary policy on output? Results from an agnostic identification procedure. *Journal of Monetary Economics 52*(2), 381–419.

Zeev, N. B. and E. Pappa (2017, August). Chronicle of a War Foretold: The Macroeconomic Effects of Anticipated Defence Spending Shocks. *Economic Journal 127*(603), 1568–1597.

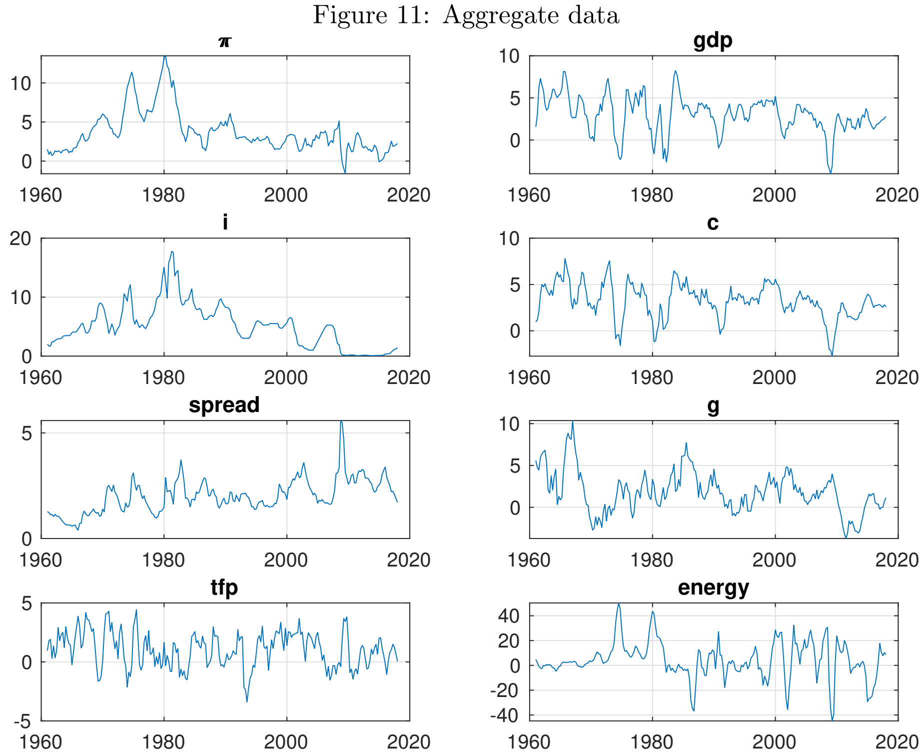
Appendices

In the following appendices we show plots of our data, as well as discuss possible extensions and alternative modes of inference.

A Data Plots

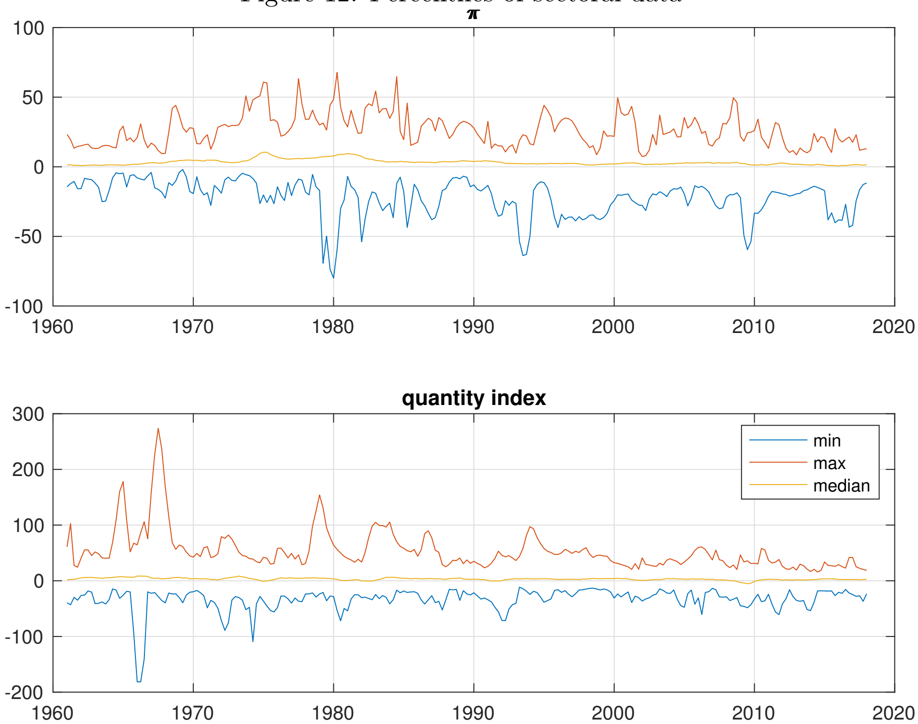
First, we plot the aggregate data we use in our benchmark analysis.

Figure 11: Aggregate data



Next, we look at our sectoral data. We want to highlight the large dispersion across sectors. To do this, we plot the 5th, 50th, and 9th percentile (across sectors) of our sectoral data over time.

Figure 12: Percentiles of sectoral data



B Additional Impulse Responses

B.1 Plots with 5th and 95th Percentile Bands

Figure 13: Responses to Technology Shock

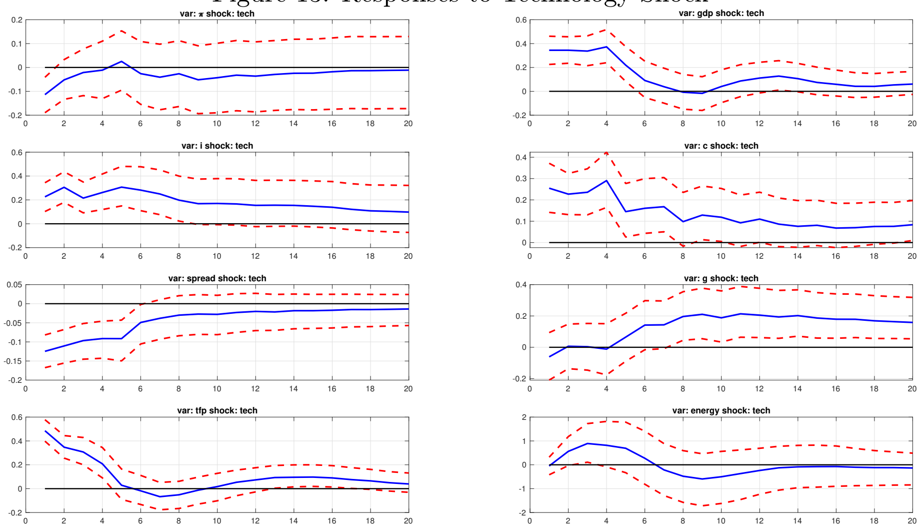


Figure 14: Responses to Corporate credit Shock

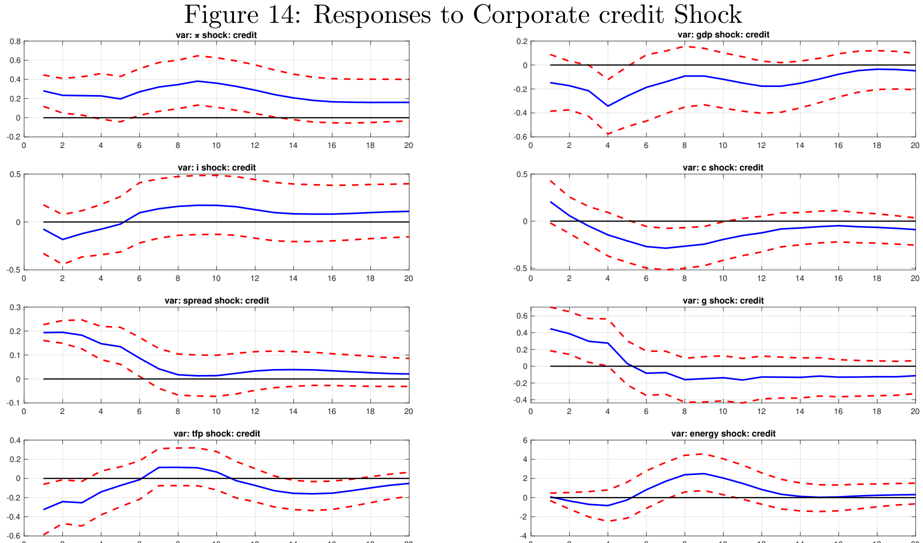


Figure 15: Responses to Government Shock

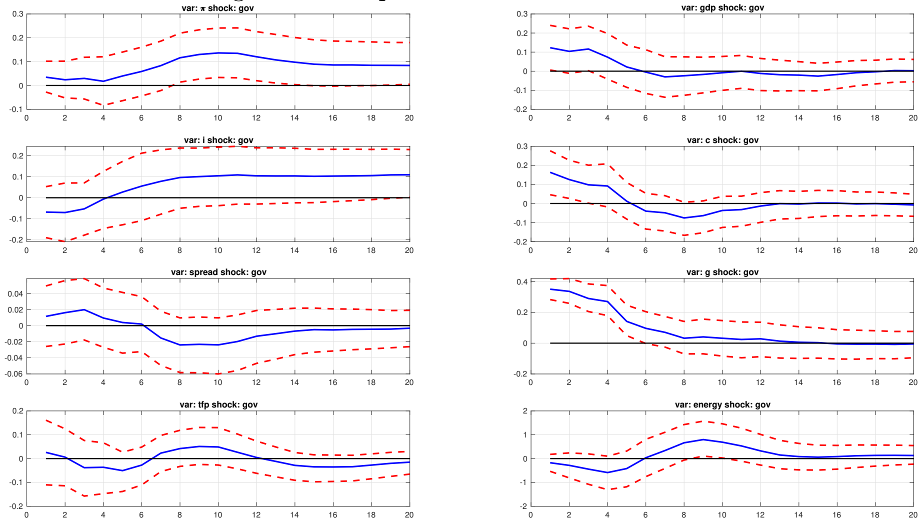
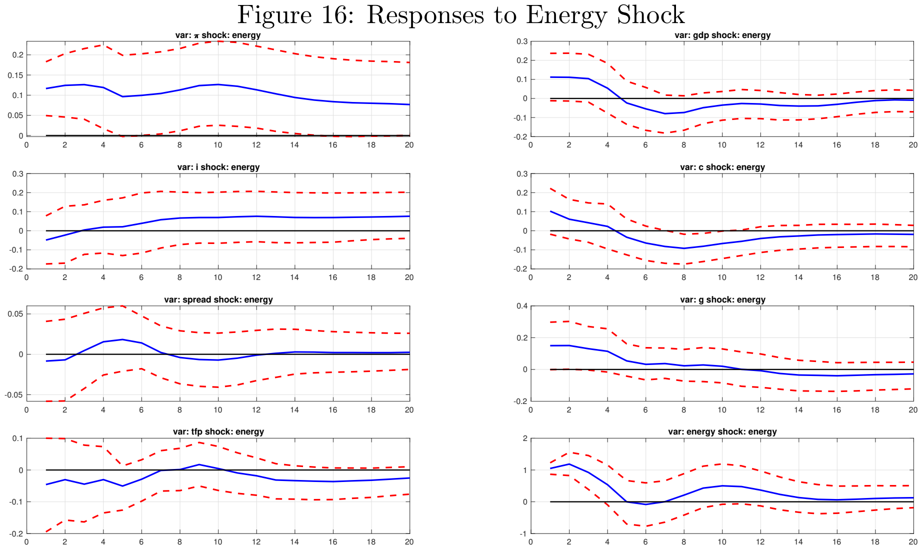


Figure 16: Responses to Energy Shock



B.2 Digging Deeper into Identification Assumptions for other Variables

Figure 17: Responses to Technology Shocks: Comparison of Identification Schemes

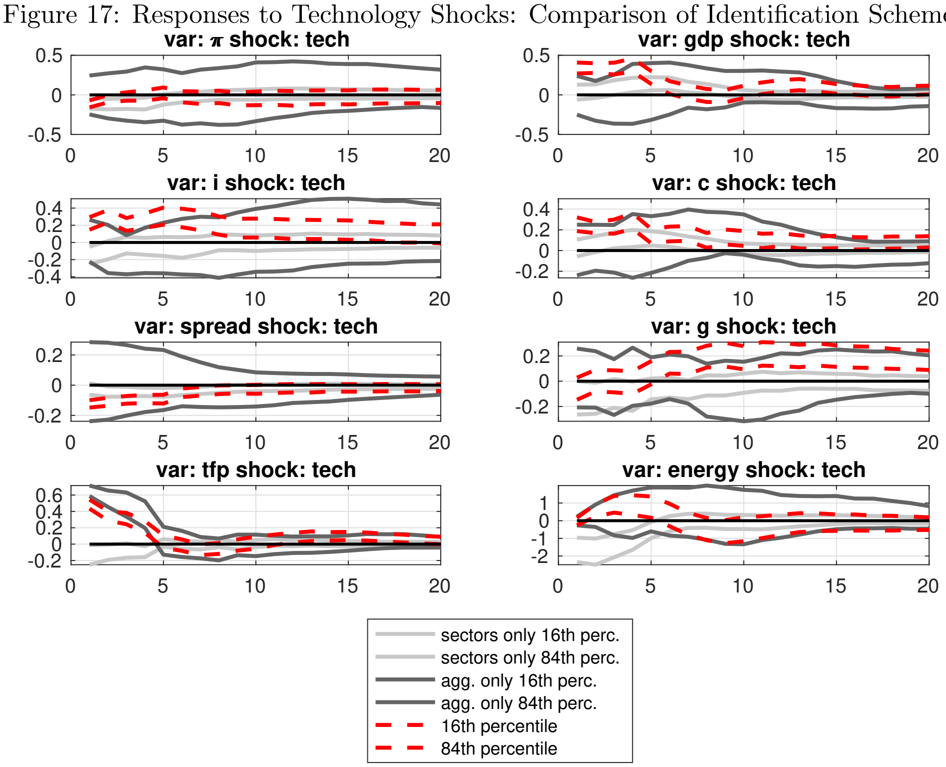


Figure 18: Responses to Credit Shocks: Comparison of Identification Schemes

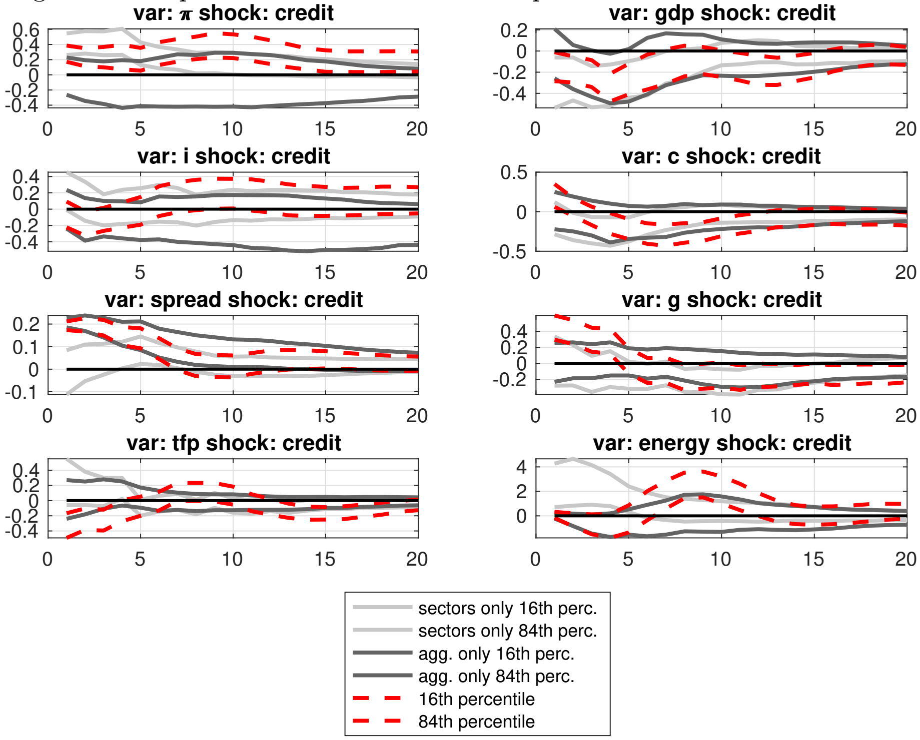


Figure 19: Responses to Government Shocks: Comparison of Identification Schemes

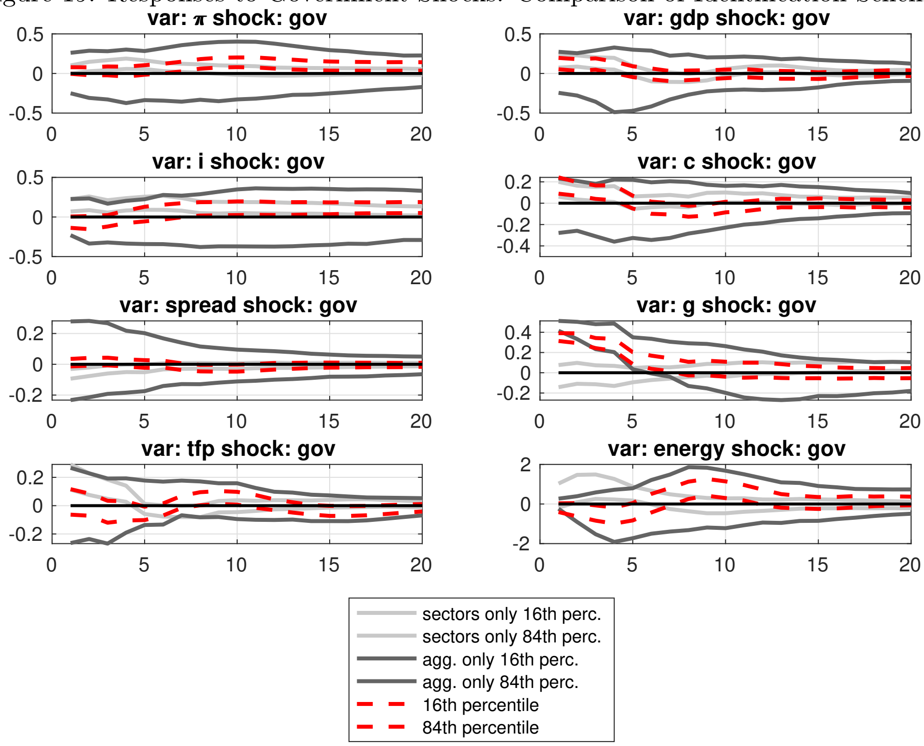
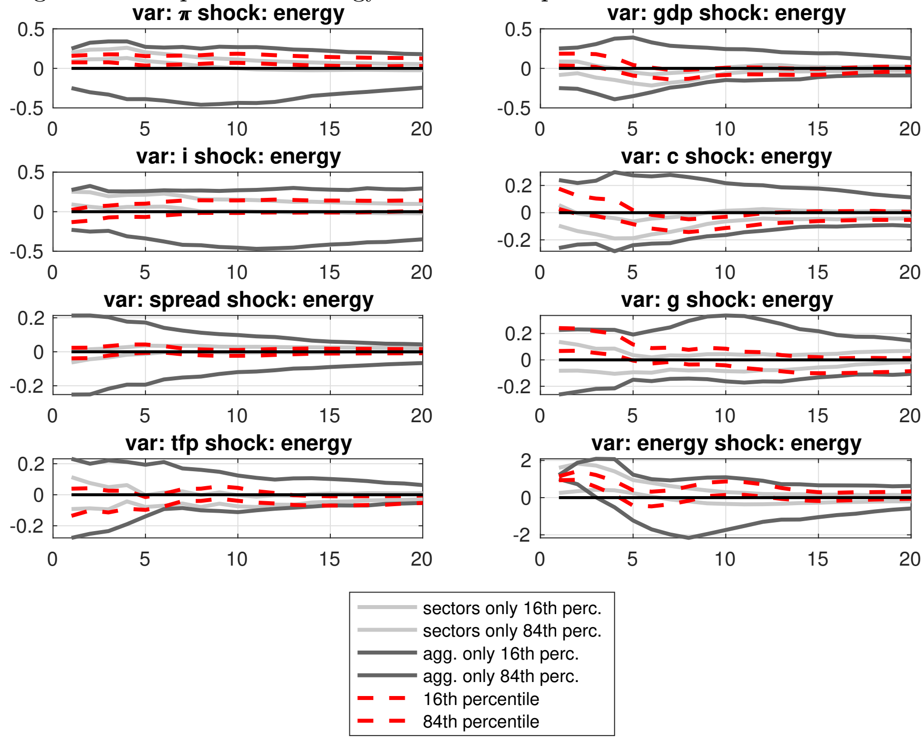


Figure 20: Responses to Energy Shocks: Comparison of Identification Schemes



C Displaying Error Bands for Our Monte Carlo Exercise

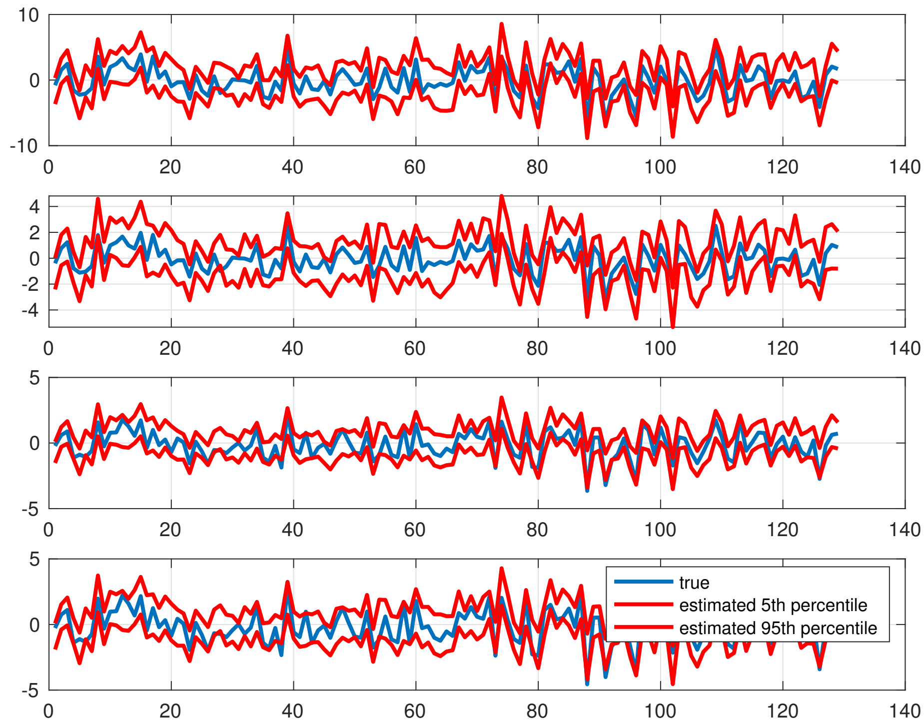


Figure 21: True effect and 90 percent estimated error bands

D A model with multiple hierarchies and feedbacks from idiosyncratic to aggregate variables

This section lays out a more general model (of which the model so far is a special case). Suppose we do not only consider aggregate data and data at (one) sectoral level, but instead consider variables measured at various levels of coarseness or aggregation. Further suppose there are M different levels of aggregation in addition to the aggregate level. Each level has $I_m,\ m = 1, ..., M$ different units (sectors in our application). We assume that $m = 1$ is the coarsest aggregation. For expositional clarity, assume that we use a lag length of 1 throughout. Then the model is given by:

$$Y_t = AY_{t-1} + B\mathcal{F}_y(\{X_{t-1}^{1,i}\}_{i=1}^{I_1}) + \Sigma\varepsilon_t + w_t \tag{16}$$

$$X_t^{m,i} = A^{m,i}X_{t-1}^{m,i} + B^{m,i}Y_{t-1} + C^{m,i}\mathcal{F}_{m,i}(\{X_{t-1}^{m+1,i}\}_{i=1}^{I_{m+1}})) + D^{m,i}\varepsilon_t + w_t^{m,i} \tag{17}$$

$$\mathcal{F}_{M,i} = 0\forall i \tag{18}$$

The superscript m, i denotes the ith unit in the mth level of aggregation. Note that a model of this form is really only necessary if some variables are only available at some levels of aggregation, but not others (otherwise we could solve for the dynamics at higher levels of aggregation by simply adding up the equations at the finest level of aggregation) or if not all units are observable at finer levels of aggregation. The functions $\mathcal{F}$ could be nonlinear (along the lines of Chen et al. (2018)), but we assume they are known *a priori*, possibly up to a finite dimensional parameter vector.[16] These functions allow for feedback from lower levels of aggregation back to higher levels and ultimately the aggregate level.

E Two step inference

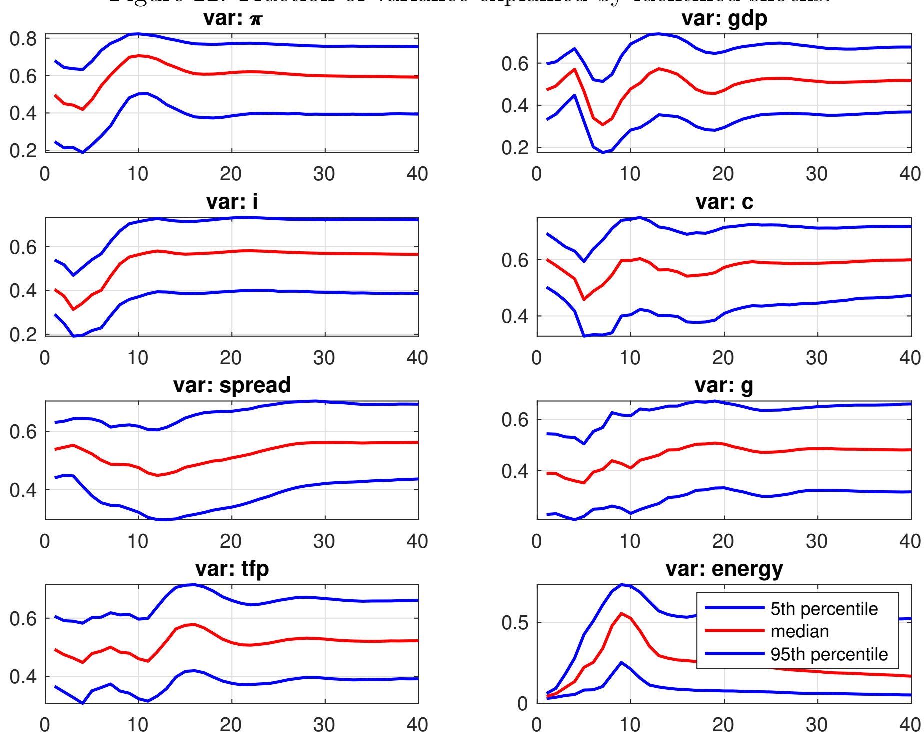
Akin to the estimation of FAVARs (where both one-step fully Bayesian and two step inference are common), two-step inference is possible in this environment. Note that at the aggregate level and for each sector, one can run regressions (of the frequentist or Bayesian persuasion) to identify all parameters *except* that separately identifying the effects of idiosyncratic and structural shocks is not possible. But with estimates of all other parameters in hand, we can back out the sum of the effects of idiosyncratic and structural shocks, which in turn can then be used to back out structural shocks either via the Kalman filter or via principal component analysis (in which case we might have to rotate the estimated structural shocks after estimation so that they satisfy identification restrictions). Note that the structural shocks in our setup really are iid factors. So we could borrow from the large literature on identification of factor models to assess what conditions on the idiosyncratic shocks are necessary to achieve identification (note that this identification would be conditional on estimates of other parameters obtained in the first step). Generally speaking, Bai & Ng (2005) show that we can't have correlations of idiosyncratic shocks across time and variables that is too strong in a sense that they make precise in (for example) assumption E of their Advances in Econometrics summary article.

F The Relative Importance of ε_t and w_t

In this section, we present evidence on how important the identified elements of ε_t are for overall fluctuations. By "identified", we mean those 6 shocks on which we put identification restrictions (remember that we also add 3 additional elements to soak up any misspecification). Figure 23 shows the median as well as the 5th and 95th percentile of the contribution of those shocks to overall fluctuations of the relevant variables at the horizon given on the x-axis. We show below that our results are robust to priors that lead to larger fractions of the overall variance explained by the identified shocks.

[16] In particular, $\mathcal{F}$ could be a weighted average with the weights of the different disaggregated variables being given a priori such as in the GVAR literature, see Chudik and Pesaran (2016).

Figure 22: Fraction of variance explained by identified shocks.



G Robustness

G.1 Using less sectors to identify shocks

In our benchmark analysis, we impose identifying restrictions on all sectors. Here we only use the top and bottom 25 percent of sectors according to a specific indicator to identify the shock associated with the indicator. Those elements of D^i that are not in the top or bottom 25 percent have a prior with mean zero and standard deviation 0.25.

	tech	credit	demand	gov	energy	monetary
π	6.3	10.5	12.5	9.0	5.0	44.8
gdp	17.9	13.3	15.6	7.3	6.5	13.4
i	7.6	9.3	20.8	9.5	4.9	31.8
c	7.9	12.3	42.0	5.9	3.2	11.7
spread	13.5	36.6	11.5	3.4	3.5	21.3
g	11.3	15.7	24.1	12.8	4.0	13.6
tfp	30.9	15.9	12.2	3.1	9.3	11.0
energy	9.6	14.3	15.5	6.8	14.3	25.5

Table 4: Mean of conditional variance decomposition across business cycle frequencies and posterior draws, less sectors used in identification

G.2 Lag Length

In our benchmark analysis we assumed $L = L^X = 6$ lags. Here instead we assume $L = L^X = 4$.

	tech	credit	demand	gov	energy	monetary
π	4.5	28.3	16.1	7.9	9.1	23.5
gdp	11.6	38.6	9.8	8.4	4.5	15.4
i	6.4	14.9	23.4	9.8	4.6	30.8
c	10.5	29.4	22.4	5.2	4.5	18.1
spread	6.3	58.9	7.0	2.2	2.6	16.8
g	6.6	38.1	8.6	13.5	5.3	16.0
tfp	24.7	35.1	7.0	2.1	3.1	12.1
energy	6.8	31.5	10.8	6.3	15.9	16.7

Table 5: Mean of conditional variance decomposition across business cycle frequencies and posterior draws, 4 lags

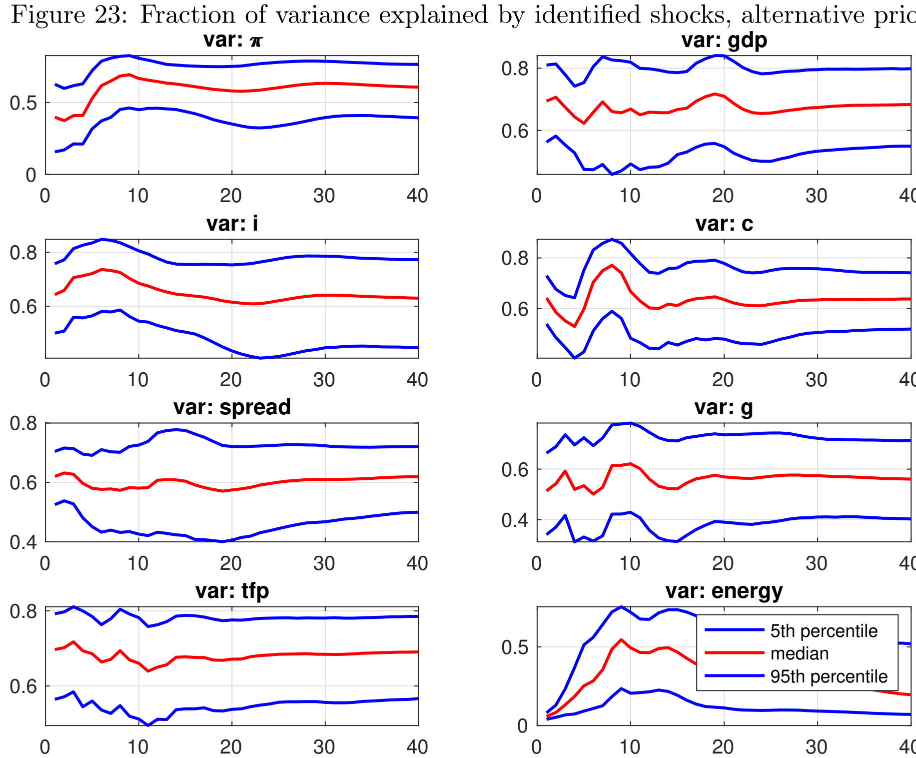
G.3 An Alternative Prior

In this robustness check, we use an alternative prior that encodes the view that aggregate fluctuations should be driven mainly by ε_t. To do so, we simply change the prior mean of Ω, the covariance matrix of w_t, to be 25 percent of the value used in the benchmark. All other prior parameters remain unchanged.

	tech	credit	demand	gov	energy	monetary
π	4.0	9.3	31.7	8.9	2.6	29.2
gdp	3.2	20.2	35.3	13.7	3.6	9.7
i	3.8	6.6	35.3	11.3	2.8	26.8
c	3.5	12.9	40.3	14.8	2.8	13.3
spread	8.1	34.1	13.9	4.3	3.0	28.7
g	4.0	20.7	28.6	15.3	4.0	13.0
tfp	11.2	24.3	27.3	11.1	4.4	14.2
energy	5.6	19.6	22.4	9.1	11.6	20.0

Table 6: Mean of conditional decomposition across business cycle frequencies and posterior draws, , alternative prior

Figure 23: Fraction of variance explained by identified shocks, alternative prior



G.4 More Elements in ε without prior restrictions

In our benchmark model, we add 3 elements to ε_t for which we do not impose any prior information. To check whether the choice of 3 additional shocks is crucial, we now present in table 7 results with 10 additional shocks.

	tech	credit	demand	gov	energy	monetary
π	9.3	19.1	32.8	7.9	3.8	12.4
gdp	13.2	22.1	31.8	5.1	3.2	10.6
i	7.1	16.2	43.4	5.4	2.9	13.1
c	6.8	14.4	53.1	2.9	2.4	9.1
spread	7.8	36.8	28.5	4.5	2.7	11.2
g	4.5	19.9	35.8	12.1	4.5	11.7
tfp	18.7	20.4	19.7	4.9	2.8	13.5
energy	11.1	26.2	22.2	7.4	6.4	13.2

Table 7: Mean of conditional variance decomposition across business cycle frequencies and posterior draws, 10 additional shocks

G.5 Using Instruments for Shocks

Recently, there has been substantial interest in using external information/instruments for structural shocks to help identification of the effects of these shocks (Mertens and Ravn (2013)). Here we borrow ideas from Caldara and Herbst (2016) to incorporate these instruments in our Bayesian framework. To do so, we estimate equations of the following form:

$$m_t^i = m^i + a^i \varepsilon_t^i + u_t^i$$

where m_t^i is an instrument for the ith elements of ε_t. m^i captures any possible differences in means across the instrument and true shock, whereas a^i and the variance of u_t^i (which we assume to be a Gaussian *iid* shock) capture how informative the instrument is by determining the signal to noise ratio in the instrument. Once the parameters are estimated in a separate Gibbs sampling step, we add the instrument equations above to the state space system that is used to generate draws of ε_t. We use four instruments:

1. the government spending news shock from Ramey (2011)
2. the government spending news shock from Zeev and Pappa (2017)
3. the monetary shock from Romer and Romer (2004)
4. the exogenous oil price shock from Kilian (2009)

We truncate our sample to the largest time period so that all shocks are available. Monthly series are averaged to quarterly values. As can be seen from table 8 our results are confirmed. Unfortunately, the instruments themselves do not add substantial information, as the posteriors for a^i broadly centered around 0 and the estimated standard deviation for u_t^i is large for all instruments.

	tech	credit	demand	gov	energy	monetary
π	2.6	13.6	53.1	1.7	2.5	20.0
gdp	9.2	23.5	40.9	1.1	2.4	11.2
i	2.9	12.7	57.9	1.3	2.6	16.0
c	2.6	14.8	60.6	0.9	1.7	10.4
spread	5.2	32.9	39.8	2.8	4.5	10.1
g	3.9	20.2	36.2	6.5	3.1	20.7
tfp	4.1	15.2	54.0	2.5	4.2	13.0
energy	4.2	18.4	36.5	2.4	5.8	26.0

Table 8: Mean of conditional variance decomposition across business cycle frequencies and posterior draws, using instruments

G.6 A Labor Market Shock

In this section, we add one additional element to ε_t relative to our benchmark: a labor market shock, which we identify as a shock that has a larger impact on sectors where compensation to employees as a share of value added is higher.

	tech	credit	demand	gov	energy	monetary	labor
π	6.4	12.7	8.7	9.8	6.7	38.7	5.0
gdp	7.2	12.9	15.2	5.3	12.0	15.0	10.6
i	6.9	8.0	17.1	9.9	8.9	28.2	7.1
c	10.1	12.8	28.5	5.8	7.7	15.6	7.7
spread	9.7	31.2	9.6	4.4	8.1	18.4	8.7
g	8.3	13.1	16.9	10.2	7.9	16.6	9.4
tfp	26.6	18.8	10.7	5.0	8.8	12.8	4.9
energy	6.8	12.8	12.8	6.1	18.6	24.2	5.9

Table 9: Mean of conditional variance decomposition across business cycle frequencies and posterior draws, with labor shock

G.7 A Shorter Sample

To check that our results are not driven by specific events, we re-run our estimation using only data from the first quarter of 1984 to the last quarter of 2007, hence excluding both the Great Inflation and the Great Recession.

	tech	credit	demand	gov	energy	monetary
π	10.0	17.2	50.0	7.1	2.8	10.0
gdp	8.4	15.7	54.0	7.6	2.7	8.9
i	8.8	13.3	59.5	6.7	1.8	7.5
c	6.7	12.6	61.8	7.5	1.9	7.2
spread	9.8	25.1	44.9	6.5	2.8	9.1
g	9.9	18.1	49.3	6.9	2.8	10.2
tfp	9.7	17.4	48.2	6.3	2.7	12.8
energy	9.3	19.0	49.3	6.1	2.9	10.9

Table 10: Mean of conditional variance decomposition across business cycle frequencies and posterior draws, shorter sample

H Model Reduction

Up to a first-order approximation the economy is described by the following system of equations (small letters indicate log deviations from steady-state):

$$
\begin{aligned}
p^C + c &= m^C + m^Y \\
p^G + g &= m^G + m^Y && (19) \\
w &= 0 && (20) \\
g_i - g &= p^G - p_i \ \forall i && (21) \\
c_i - c &= p^C - p_i \ \forall i && (22) \\
y_i &= \epsilon_i + \sum_j \gamma_{ji} x_{ji} + \lambda_i l_i + \chi k_i \ \forall i && (23) \\
w + l_i &= p_i + y_i - \mu_i \ \forall i && (24) \\
p_j + x_{ji} &= p_i + y_i - \mu_i \ \forall i, j && (25) \\
r_i + k_i &= p_i + y_i - \mu_i \ \forall i && (26) \\
k_i &= \bar{k}_i && (27) \\
\mu_i &= -\phi_i \left(\sum_j \gamma_{ji} p_j + \lambda_i w + \chi r_i - \epsilon_i \right) && (28) \\
y_i &= \sum_j \frac{X_{ij}}{Y_i} x_{ij} + \frac{C_j}{Y_i} c_j + \frac{G_j}{Y_i} g_j && (29)
\end{aligned}
$$

The system can be reduced to:

$$
\begin{aligned}
p_i - (1-\chi)\mu_i &= -\epsilon_i + \sum_j \gamma_{ji} p_j + \chi\left(p_i + y_i - \bar{k}_i\right) \\
p_i + y_i &= \sum_j \gamma_{ij} \frac{Y_j}{Y_i}(y_j + p_j - \mu_j) + \frac{C_i}{Y_i}(m^C + m^Y) + \frac{G_i}{Y_i}(m^G + m^Y) \\
\mu_i &= -\frac{\phi_i}{1-\phi_i \chi}\left(-\epsilon_i + \sum_j \gamma_{ji} p_j + \chi\left(p_i + y_i - \bar{k}_i\right)\right)
\end{aligned}
$$

Or, eliminating μ_i,

$$
\begin{aligned}
p_i &= \frac{1-\phi_i}{1-\chi}\left(-\epsilon_i + \sum_j \gamma_{ji} p_j + \chi\left(y_i - \bar{k}_i\right)\right) \\
p_i + y_i &= \sum_j \gamma_{ij} \frac{Y_j}{Y_i}(y_j + \frac{1}{1-\phi_j} p_j) + \frac{C_i}{Y_i}(m^C + m^Y) + \frac{G_i}{Y_i}(m^G + m^Y)
\end{aligned}
$$

The system can be rewritten as

$$
\begin{aligned}
p_i = \frac{1-\phi_i}{1-\chi}\chi &\left[(1-\chi\Phi_i)\left[\sum_j f_{ij}(y_j + \frac{1}{1-\phi_j}p_j) + \frac{C_i}{Y_i}(m^C + m^Y) + \frac{G_i}{Y_i}(m^G + m^Y)\right] + \Phi_i\left(\epsilon_i + \chi\bar{k}_i\right)\right] \\
&- \Phi_i\left(\epsilon_i + \chi\bar{k}_i\right) + \Phi_i \sum_j b_{ji} p_j
\end{aligned}
$$

$$
y_i = (1-\chi\Phi_i)\left[\sum_j f_{ij}(y_j + \frac{1}{1-\phi_j}p_j) + \frac{C_i}{Y_i}(m^C + m^Y) + \frac{G_i}{Y_i}(m^G + m^Y)\right] + \Phi_i\left(\epsilon_i + \chi\bar{k}_i\right) - \Phi_i \sum_j b_{ji} p_j
$$

with $f_{ij} = \gamma_{ij}\frac{Y_j}{Y_i}$ capturing forward links and $b_{ji} = \gamma_{ji}$ capturing backward links
Direct impact of shocks is

$$
\begin{aligned}
p_i^{\text{Direct}} &= \Phi_i \chi \left[\frac{C_i}{Y_i} m^C + \frac{G_i}{Y_i} m^G + m^Y\right] - \Phi_i\left(\epsilon_i + \chi\bar{k}_i\right) \\
y_i^{\text{Direct}} &= (1-\chi\Phi_i)\left[\frac{C_i}{Y_i} m^C + \frac{G_i}{Y_i} m^G + m^Y\right] + \Phi_i\left(\epsilon_i + \chi\bar{k}_i\right)
\end{aligned}
$$

Indirect effects are

$$
\begin{aligned}
p_i^{\text{Indirect}} &= \Phi_i \sum_j \left(\chi \frac{f_{ij}}{1-\phi_j} + b_{ji} \right) p_j + \chi \Phi_i \sum_j f_{ij} y_j \\
y_i^{\text{Indirect}} &= (1 - \chi \Phi_i) \sum_j f_{ij} y_j + \sum_j \left[\frac{1 - \chi \Phi_i}{1 - \phi_j} f_{ij} - \Phi_i b_{ji} \right] p_j
\end{aligned}
$$

Consider special case with $\gamma_{ji} = 0$. Then

$$
\begin{aligned}
(1-\chi) p_i - (1-\phi_i)\, \chi y_i &= -\left(1 - \phi_i\right) \epsilon_i + (1-\phi_i) \chi \bar{k}_i \\
p_i + y_i &= \frac{C_i}{Y_i} m^C + \frac{G_i}{Y_i} m^G + m^Y
\end{aligned}
$$

Solving the system for p_i and y_i yields:

$$
\begin{aligned}
(1-\chi) p_i - (1-\phi_i)\, \chi y_i &= -\left(1 - \phi_i\right) \left(\epsilon_i - \chi \bar{k}_i\right) \\
y_i &= \frac{1-\chi}{1-\chi+(1-\phi_i)\chi} \left[\frac{C_i}{Y_i} m^C + \frac{G_i}{Y_i} m^G + m^Y \right] + \frac{1-\phi_i}{1-\chi+(1-\phi_i)\chi} \left(\epsilon_i - \chi \bar{k}_i\right)
\end{aligned}
$$

,

$$
\begin{aligned}
p_i &= \Phi_i \chi \frac{C_i}{Y_i} m^C + \Phi_i \chi \frac{G_i}{Y_i} m^G + \Phi_i \chi m - \Phi_i \epsilon_i \\
y_i &= (1 - \chi \Phi_i) \frac{C_i}{Y_i} m^C + (1 - \chi \Phi_i) \frac{G_i}{Y_i} m^G + (1 - \chi \Phi_i)\, m + \Phi_i \epsilon_i
\end{aligned}
$$

where $\Phi_i \equiv \frac{1-\phi_i}{\chi(1-\phi_i)+1-\chi}$. Note that Φ is decreasing in ϕ_i for $\chi < 1$.