

Assessing Macroeconomic Tail Risk

Francesca Loria* Christian Matthes† Donghai Zhang‡

October 2020

Abstract

Real GDP and Industrial Production in the US feature substantial tail risk. While this fact is well documented ([Adrian *et al.* \(2019\)](#); [Giglio *et al.* \(2016\)](#)), it is not clear what drives this asymmetry - is there a common propagation mechanism or is it one specific shock that drives this skewness? We provide evidence for the first explanation by showing that the 10th percentile of the GDP and IP distributions responds drastically more to various supply and demand shocks than the median or 90th percentile, and by highlighting that two data-generating processes that feature a common propagation mechanism can match these patterns.

Keywords: Macroeconomic Risk, Shocks, Local Projections

JEL Classification: C21, C53, E17, E37

We thank Marvin Nöller for excellent research assistance on a previous version of the paper. We also want to thank Danilo Cascaldi-Garcia, Dario Caldara, Fabio Canova, Todd Clark, Giovanni Favara, Domenico Giannone, Ricardo Reis, Erick Sager, Rosen Valchev, and Alexander Wolman, as well as seminar/workshop participants at the Chinese University of Hong Kong, the University of Bonn, the International Finance Workshop on Quantile Regressions at the Board of Governors, the 2019 IAAE conference, the 2020 SNDE conference, and the “Modeling The Macroeconomy in Risky Times” Workshop in St. Louis for helpful comments.

Disclaimer: The views presented herein are those of the author and do not necessarily reflect those of the Federal Reserve Board, the Federal Reserve System or their staff.

*Board of Governors of the Federal Reserve System, francesca.loria@frb.gov.

†Indiana University, matthesc@iu.edu.

‡Institute for Macroeconomics and Econometrics — University of Bonn, donghai.zhang@uni-bonn.de.

1 Introduction

Recent contributions in macroeconomics and finance ([Adrian *et al.* \(2019\)](#); [Giglio *et al.* \(2016\)](#)) have highlighted that growth in measures of US output are highly asymmetrical in that they feature substantial *tail risk* - they are left skewed. To illustrate this feature, [Figure 1](#) plots the quantiles of the one-year ahead forecast distribution of growth in Real Gross Domestic Product (GDP) and Industrial Production (IP). We both replicate the well-known results from [Adrian *et al.* \(2019\)](#) based on quarterly GDP as well as the measure that we will rely on in our benchmark analysis, which is based on the monthly GDP model of [Caldara *et al.* \(2020\)](#). All panels convey the same asymmetry - the 10th quantile moves substantially more, especially in a recession (NBER recession are depicted as grey bands in the panels), relative to the median or the 10th percentile.¹

What drives this asymmetry? It is one specific shock that makes GDP and IP have this substantial skewness? Or is it a common propagation mechanism that can make both supply and demand shocks have symmetric effects?

We tackle these questions in two steps. First, we derive a convenient two-step estimation routine that merges ideas from Quantile Regression ([Koenker and Bassett \(1978\)](#)) to determine quantiles of the distribution of interest with Local Projection ([Jordà \(2005\)](#)) to assess the effects of monetary policy shocks, shocks to financial conditions and technology shocks on the distribution of real GDP growth². We devise a bootstrap algorithm to capture all relevant statistical uncertainty in this two-step approach and that might be of independent interest to some readers. We show that responses to *all* shocks considered here show substantial asymmetry - over the first three quarters the 10th percentile moves on average 3 times as much in response to each structural shock as the median.

Second, we use two data-generating processes (DGPs) as laboratories to test our empirical machinery and to further analyze what could drive these results. Both data-generating processes, a "semi-structural" threshold model and the recent model of [Gertler *et al.* \(2019\)](#), give financial conditions a special role in worsening/prolonging downturns. We show that data from both of these DGPs can replicate our main findings. Furthermore, we use these DGPs to highlight specification choices in our framework.

¹Details on the construction can be found in [Section 3.1](#).

²[Giglio *et al.* \(2016\)](#) focus on asymmetries in industrial production growth due to systemic risk. This results is consistent with the results we present. [Figure 1](#) shows that finding asymmetry in IP is robust to using other specifications than those used in [Giglio *et al.* \(2016\)](#).

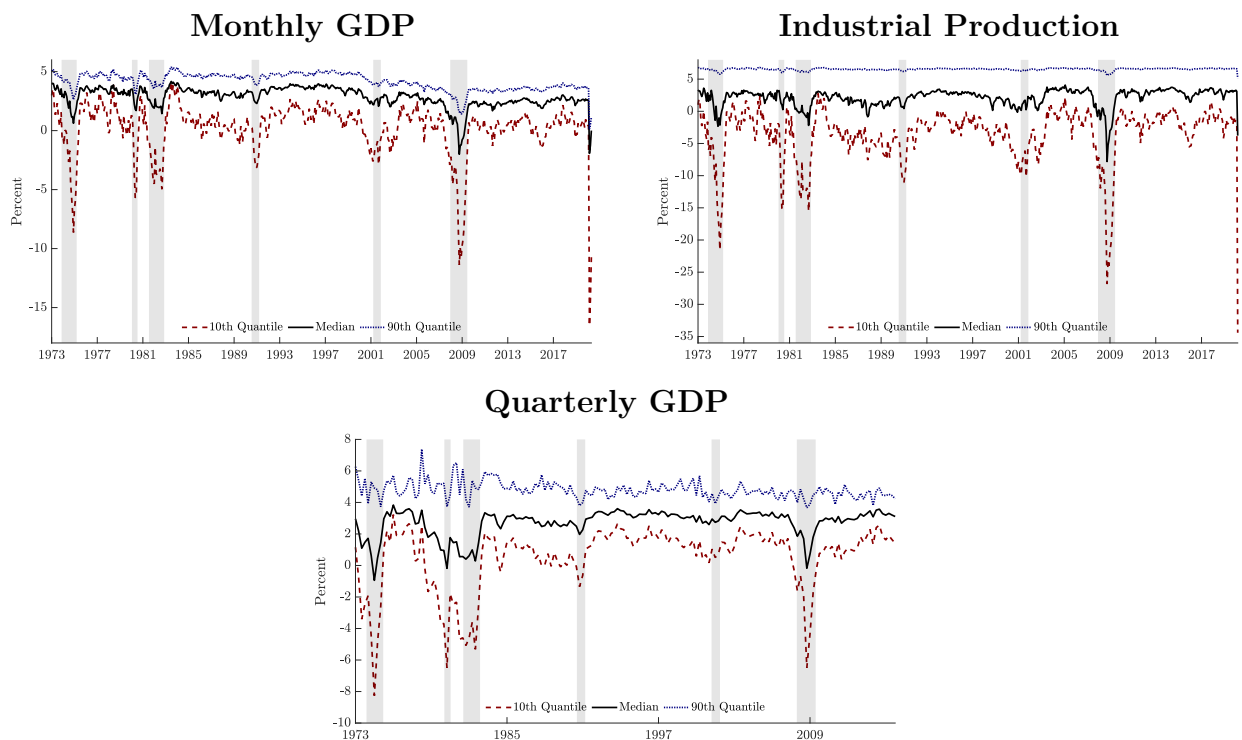


Figure 1: Quantiles of Average Growth over the Next Year of Monthly GDP and Industrial Production (Top Panel) as in Caldara *et al.* (2020) (Left, Top Panel) and of Quarterly GDP as in Adrian *et al.* (2019) (Bottom Panel).

Our results highlight concerns that economic policymakers and market participants have long harbored: They are generally not only worried about what changes to economic conditions will do to the economy *on average*, but also how these changes affect both the probability of *large* harmful events occurring as well as the magnitude of these events.³ In effect, we borrow the concept of *value-at-risk* from the finance literature to operationalize the concept of macroeconomic tail risk. To be more precise, we follow Adrian *et al.* (2019) and study the distribution of macroeconomic risk by estimating a quantile forecast regression of real GDP growth over the next year for various quantiles. We focus on the 10th percentile and, as reference points, the median and 90th percentiles. We interpret this 10th percentile of the predictive distribution of future GDP growth as *macroeconomic tail risk*.

Standard impulse response functions in linear models such as Vector Autoregressions (VARs)

³For research emphasizing that the Federal Reserve is concerned by downside risk, see Kilian and Manganelli (2008). For direct evidence of a policymaker thinking about downside risk (which we will use synonymously with tail risk), see this March 2019 speech by Lael Brainard, member of the Board of Governors of the Federal Reserve System: <https://www.federalreserve.gov/newsevents/speech/brainard20190307a.htm>.

are not built to answer these questions as they track average outcomes. While fully parametric VARs that specify the probability distribution of the one-step ahead forecast error could be used to derive such measures of risk, these VARs put many restrictions on the behavior of forecast densities.⁴ Our goal is to provide a flexible, yet simple framework that can directly tackle these issues. A flexible approach is useful when studying GDP growth because GDP growth is far from Gaussian, as can be seen from papers that interpret the non-normality in GDP growth as coming from fat tails and/or stochastic volatility (Fagiolo *et al.*, 2008, Cúrdia *et al.*, 2014, and Justiniano and Primiceri, 2008).

One key point to emphasize is that our approach is constructed to be as flexible as possible: In the initial quantile regression stage, we model each quantile separately instead of assuming a specific distribution for the forecast distribution of real GDP growth. In the second stage, we use local projections to impose as few restrictions on the data generating process as possible.⁵

2 Some Intuition for Impulse Responses of Quantiles

This section gives three examples where an initial distribution of a variable changes after a shock hits. We show these examples to illustrate how a change in quantiles is linked to changes in the distribution as a whole and how changes in specific moments translate into changes in quantiles.⁶ Our scenario is as follows: After an initial distribution of a scalar variable is hit by a shock, we trace out how this distribution changes on impact and in the period after impact. We consider three experiments:

1. The shock leads to an increase in the variance of our distribution, which is Gaussian.
2. The shock leads to an increase in the mean of our distribution, which is Gaussian.
3. The shock leads to a change in *both* the mean and variance of our distribution, which is Gaussian.

⁴Even non-linear time series models such as VARs with stochastic volatility and time-varying parameters along the lines of Cogley and Sargent (2005) and Primiceri (2005) impose substantial structure. Carriero *et al.* (2020) propose an extension to the standard stochastic volatility structure embedded in these models that helps to capture tail risks.

⁵As shown by Plagborg-Møller and Wolf (2019), local projections and VARs asymptotically estimate the same impulse responses, but are on diametrically opposite ends of the bias-variance trade-off in finite samples.

⁶These examples are not meant to be exhaustive. There can be many other changes in distributions that lead to similar movements in quantiles to those displayed in this section.

Figure 2 plots three panels for each experiment. The first panel in each row shows the initial distribution, the distribution when the shock hits, and the distribution in the period after the shock has materialized. The middle panel in each row shows the evolution of the 10th and 90th percentile for those three periods. The last panel in each row gives the impulse responses for the 10th and 90th percentiles *under the assumption that if the shock that moved the distributions had not materialized, the distribution would have remained at its original position*.⁷ As the impulse response plots the difference between the relevant percentiles and the original values, the impulse response figures only show values for two time periods (the period when the shock hits and the period after). Each row presents the figures for one experiment. Note that the levels of the percentiles are not directly interpretable as IRFs because we do not subtract the baseline value from the quantiles in those figures. As we can see, an increase in the variance of a symmetric distribution makes the quantiles drift apart in a mirror-image fashion, whereas a change in the mean of a symmetric distribution makes the quantiles move in parallel, which in turn makes the impulse responses lie on top of each other. With a non-symmetric distribution (or if a shock makes a distribution non-symmetric) the quantiles can drift apart, but not necessarily in a mirror-image fashion, as highlighted in the third example.

Interpreting changes in multiple quantiles jointly can be challenging because we have to envision how the entire distribution changes. As an example, let us focus on the third experiment. As can be seen from the last panel on the bottom row of Figure 2, the 10th and 90th percentile drift apart because the 90th percentile increases slightly, whereas the 10th percentile decreases substantially. Thus the distribution *spreads out* as a result of the shock—this can also be seen by looking at the leftmost panel of the bottom row, where the yellow distribution is more spread out than the original blue distribution. Let us for a second imagine that this impulse response is the response to a contractionary shock and that quantiles react linearly to those shocks (as will be the case in our local projections). Such shocks would not only move mean and median of the distribution (this can be seen from looking directly at the Gaussian distributions in the bottom left panel, where the mean/median of the Gaussian distribution moves from 5 to 3), but it actually moves the 10th percentile substantially more, thus not only making average outcomes worse, but making outcomes in the left tail much more likely. This is the relevant case in our empirical results.

Another scenario that could occur is that the impulse responses of various quantiles cross.

⁷For our purposes, impulse responses are defined as the difference between two conditional expectations, where one of the expectations conditions on a specific value for one shock in one specific period.

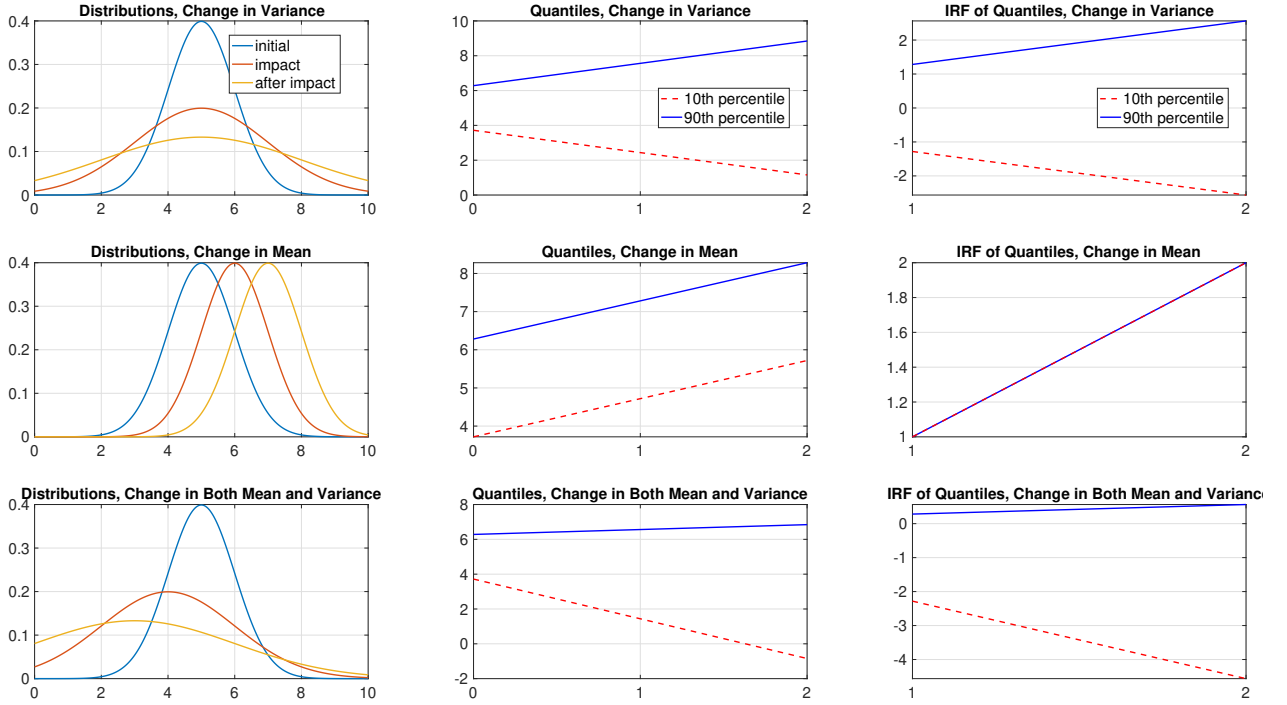


Figure 2: Illustration of Changes in Percentiles.

It is important to emphasize that this does *not* mean that the quantiles themselves cross. In fact, a crossing of impulse responses of various quantiles can just mean that one quantile reverts back to its pre-shock value faster than another. In Appendix B we construct an example along the lines of those presented in Figure 2 where impulse responses cross without the actual quantiles crossing.

Our responses capture the difference between the expected path of the τ th quantile at horizon h after a given shock of specific (one standard deviation) size occurs and the expected path of the τ th quantile conditioning on such a specific shock value. That is, a response equal to 0 at horizon h means that the expected τ th quantile at horizon h is the same independently of whether we condition on a specific shock value in the initial period. In terms of magnitudes, the responses in our application capture the expected change (in percentage points) of the τ th quantile of average future GDP growth to a one-standard-deviation shock.

3 Econometric Methodology

Our approach consists of two steps, which we outline in this section. First, we compute a measure of monthly macroeconomic tail risk following [Adrian *et al.* \(2019\)](#) and [Caldara *et al.*](#)

(2020). Our second step then uses local projections (Jordà, 2005) to assess how structural shocks affect macroeconomic risk and the predictive distribution of average future real GDP growth more generally.

3.1 Conditional Quantiles

Quantile regression models (Koenker and Bassett, 1978) are a flexible tool to quantify the risks surrounding the growth outlook and to study its determinants.⁸ The first step in our analysis is to use this methodology to compute conditional quantiles of future GDP growth as in Caldara *et al.* (2020). Their framework is inspired by Adrian *et al.* (2019), with the important difference of moving to monthly frequency so as to allow a timely assessment of developments in financial markets and the real economy. This is done by estimating GDP growth as well as a financial and a macroeconomic factor at monthly frequency using a Mixed-Frequency Dynamic Factor Model. The data are illustrated in Appendix A. We use this monthly measure of GDP as our benchmark because GDP takes a broader view of real activity compared to Industrial Production and a monthly frequency is helpful to both enlarge our data sample and minimize issues of temporal aggregation when it comes to the identification of shocks. However, our results are very similar if we switch to a quarterly frequency and use standard quarterly data on GDP, as shown in Appendix E and earlier versions of this paper (Loria *et al.*, 2019). Results are also very similar if we use Industrial Production instead of the monthly measure of GDP, as also shown in Appendix E.

Let us denote by $\bar{\Delta}y_{t+1,t+12}$ the average (annualized) real GDP growth over the next 12 months and by FIN_t and MF_t the financial factor and macroeconomic factor, respectively. Formally, the conditional future GDP growth quantiles are estimated from a linear quantile regression⁹ model whose predicted value

$$q_{\tau,t} \equiv \hat{Q}_{\tau}(\bar{\Delta}y_{t+1,t+12}|FIN_t, MF_t) = \hat{\alpha}_{\tau} + \hat{\beta}_{\tau}FIN_t + \hat{\gamma}_{\tau}MF_t, \quad \tau \in (0, 1) \quad (3.1)$$

is a consistent estimator of the quantile function of $\bar{\Delta}y_{t+1,t+12}$ conditional on $\{FIN_t, MF_t\}$.¹⁰

The estimation uses data from January 1973 through May 2019 for the estimation of the

⁸For an introduction to the quantile regression methodology, see Koenker (2005).

⁹A similar approach using factors in quantile regressions can be found in Giglio *et al.* (2016).

¹⁰Formally, the dependency between explanatory variables x_t and a quantile of y_t is measured by $\hat{\beta}_{\tau}$:

$$\hat{\beta}_{\tau} = \underset{\beta_{\tau} \in \mathbb{R}}{\operatorname{argmin}} \sum_{t=1}^T (\tau \cdot \mathbb{1}_{(y_t \geq x_t \beta)} |y_t - x_t \beta| + (1 - \tau) \cdot \mathbb{1}_{(y_t < x_t \beta)} |y_t - x_t \beta|), \quad \tau \in (0, 1)$$

parameters – the last available date for which average GDP growth over the next year is available. In Figure 1 (left panel) we show the time evolution of the future GDP growth distribution by reporting the fitted values for the 10th quantile (left tail), the median and the 90th quantile (right tail)¹¹, which are similar to that obtained by [Adrian *et al.* \(2019\)](#) at quarterly frequency (originally run for the period from 1973:Q1 to 2015:Q4) by conditioning on the National Financial Conditions Index (NFCI) and GDP growth (right panel).

One novel finding in [Caldara *et al.* \(2020\)](#) is that not only deteriorating financial conditions but also decelerating economic activity, once measured by the informationally rich macroeconomic factor instead of GDP growth as in [Adrian *et al.* \(2019\)](#), make the growth outlook more vulnerable. In this sense, we support the view in [Plagborg-Møller *et al.* \(2020\)](#) that financial conditions are not the (only) key determinant of downside risk.

3.2 Impulse Responses

Since the fitted quantiles summarize not only the median but, most importantly, the tails of the future GDP growth distribution, they constitute a measure of macroeconomic (downside and upside) risk. Our interest lies in investigating whether and how these measures of risk respond to aggregate shocks. We estimate responses of different future GDP growth quantiles to a variety of aggregate shocks by applying the local projection method of [Jordà \(2005\)](#). As a baseline, we run the following linear regression:

$$q_{\tau,t+s} = \delta_{\tau}^s + \theta_{\tau}^s \text{ shock}_t + \Psi(L)_{\tau}^s \text{ controls}_t + u_{\tau,t+s}^s, \quad s = \{0, \dots, S\} \quad (3.2)$$

for and where $q_{\tau,t+s}$ is the τ th quantile computed in the previous section, shock_t is the structural shock of interest, and $\Psi(L)_{\tau}^s \text{ controls}_t$ is a lag polynomial of control variables which include the lagged quantiles and model-specific controls that we will discuss in detail later.¹² Note that there are two distinct notions of “horizon” in our application. First, the horizon in the quantile regression h , which we keep fixed at one year and which captures how forward looking our measure of risk is. The second notion of horizon is s in the local projection, which we vary as we trace out how risks respond at different horizons to a shock at time t . The response of quantile q_{τ} at time $t+s$ to a *shock* at time t is then given by θ_{τ}^s .

where $\mathbb{1}_{(\cdot)}$ denotes the indicator function, taking the value one if the condition is satisfied. Note that no distributional assumptions about the error term are required.

¹¹Given parameter estimates and data, predictions can be formed until May 2020, which is what we show in Figure 1 for the [Caldara *et al.* \(2020\)](#) model.

¹²To be specific, we have $\Psi(L)_{\tau}^s = \Psi_{1,\tau}^s L + \Psi_{2,\tau}^s L^2 + \dots + \Psi_{p,\tau}^s L^p$ so that $\Psi(L)_{\tau}^s \text{ controls}_t$ only contains p lags of the control variables.

We construct the impulse-response functions by estimating the sequence of the θ_τ^s 's in a series of univariate regressions for each horizon. Confidence bands are based on the bootstrap procedure described in Appendix C, which controls for serial correlation in the error terms induced by the successive leading of the dependent variable.

At this point it is useful to contrast our approach with another approach that aims to combine quantile regressions with local projections, an approach advocated for by [Linnemann and Winkler \(2016\)](#). We interpret the 10th percentile of average future GDP growth as a measure of downside risk and we then ask how this measure of risk reacts to different shocks. We study a number of shocks and find it useful to use the same quantile (or measure of risk) for all shocks we study in our local projections. [Linnemann and Winkler \(2016\)](#), instead, are interested in one shock only and model the conditional quantiles *conditional on, among other things, a fiscal shock* and thus include the shock directly in the quantile regression. [Linnemann and Winkler \(2016\)](#) cannot distinguish between the two horizons h and s that we emphasized above (given that they ask a different question, they probably would not want to).¹³ An advantage of their approach is that they do not require a separate step to estimate the quantiles. We show later in Section 5 by means of a Monte Carlo exercise that the asymmetry we emphasize is also present when directly estimating the effects of shocks via quantile regressions in a one-step approach, along the lines of [Linnemann and Winkler \(2016\)](#). We highlight in Section 5.1 how our two-step approach yields qualitatively very similar results to this one-step approach. The one-step approach requires more data to obtain reliable estimates as it tries to estimate conditional quantiles that also condition on specific values of the shocks of interest.

4 The Response of Tail Risk to Macroeconomic Shocks

We estimate how quantiles of average GDP growth over the next year respond to various aggregate shocks. Next, we introduce the specifications and data transformations. More details on our data sources are provided in Appendix A. In the baseline specification, we focus on the Great Moderation period and thus start the estimation of the impulse responses in 1986. In Appendix E we show, among other robustness exercises, that our results also hold when starting the estimation in 1973, exploring shocks available starting from that date.

¹³Another approach in empirical macroeconomics that uses quantile regressions is introduced in [Mumtaz and Surico \(2015\)](#), who use quantile autoregressive models to study state dependence in the consumption-interest rate relationship. Recent work that combines quantile regressions with VAR models to estimate impulse responses is presented in [Chavleishvili and Manganelli \(2017\)](#) and [Kim et al. \(2019\)](#).

Monetary Policy Shocks Our benchmark choice of monetary policy shocks are the monetary policy shocks extracted from a version of the [Gertler and Karadi \(2015\)](#) proxy VAR which uses the [Miranda-Agrippino and Ricco \(2020\)](#) shocks (updated up to March 2020) as a proxy for the monetary disturbance in the model.¹⁴ Details on how we extract the monetary policy shocks from the [Gertler and Karadi \(2015\)](#) VAR are provided in Section D. In Appendix E we show that other measures of monetary policy shocks, including the original [Miranda-Agrippino and Ricco \(2020\)](#) shocks as well as the [Romer and Romer \(2004\)](#) shocks, give rise to very similar results.

We estimate the following regression for the January 1986 to December 2019 sample:

$$q_{\tau,t+s} = \delta_{\tau}^s + \theta_{\tau}^s shock_t + \Psi(L)_{\tau}^s \left[q_{\tau,t}, shock_t, \pi_t^{cpi}, \pi_t^{com}, \Delta y_t, i_t, u_t \right]' + u_{\tau,t+s}^s, \quad (4.1)$$

where $shock_t$ denotes the monetary policy shock, π_t^{cpi} and π_t^{comm} are respectively the (month-over-month) headline consumer price and commodity price inflation, Δy_t is the monthly (annualized) GDP growth estimated by [Caldara et al. \(2020\)](#), i_t is the federal funds rate and u_t the unemployment rate. Twelve lags of all controls are included.

Credit Spread Shocks We take the [Gilchrist and Zakrajšek \(2012\)](#) excess bond premium (EBP) updated by [Favara et al. \(2016\)](#) to construct an aggregate credit spread shock. This shock can be identified by setting the controls in the local projection appropriately if one assumes that the bond premium affects interest rates contemporaneously but has no impact on prices and economic activity within a quarter.¹⁵

We estimate the following regression for the sample from January 1986 to March 2020:

$$q_{\tau,t+s} = \delta_{\tau}^s + \theta_{\tau}^s EBP_t + \lambda_{\tau,s} i_t + \Psi(L)_{\tau}^s \left[q_{\tau,t}, EBP_t, \pi_t^{cpi}, \Delta y_t, i_t \right]' + u_{\tau,t+s}^s, \quad (4.2)$$

where we again include twelve lags of all controls.

Technology Shocks We construct technology shocks by taking the growth rate of the [Fernald \(2012\)](#) utilization-adjusted TFP series for the aggregate economy.

¹⁴We use [Miranda-Agrippino and Ricco \(2020\)](#) shocks as instruments to deal with the potential confounding problem in the high-frequency monetary surprises that may arise due to the Fed's private information, see, e.g., [Nakamura and Steinsson \(2018\)](#), [Jarociński and Karadi \(2020\)](#), [Miranda-Agrippino and Ricco \(2020\)](#), and [Zhang \(2019\)](#).

¹⁵For a further discussion of how timing restrictions such as this can be incorporated into local projections, see [Barnichon and Brownlees \(2019\)](#) and [Plagborg-Møller and Wolf \(2019\)](#).

We estimate the following regression for the sample from 1986:Q1 to 2019:Q4:

$$q_{\tau,t+s} = \delta_{\tau}^s + \theta_{\tau}^s \text{shock}_t + \Psi(L)_{\tau}^s \left[q_{\tau,t}, \text{shock}_t, \Delta y_t, \Delta a_t \right]' + u_{\tau,t+s}^s, \quad (4.3)$$

where shock_t denotes the technology shock, Δy_t is (quarter-over-quarter) real GDP growth, Δa_t is the log of productivity (real GDP divided by total hours) in first-differences. The latter is measured as real GDP divided by total hours. Four lags of all controls are included. As the shocks are only available quarterly, the quantiles are estimated using a quarterly model from 1973:Q1 to 2020:Q1 using BEA quarterly real GDP growth and the value of the last month in a given quarter for the financial and the macroeconomic factor.

4.1 Results

Our two-step approach is subject to two key sources of statistical uncertainty: not only are the coefficients in the Local Projections step subject to estimation uncertainty, but the quantiles that are used as data in the local projection step themselves are estimated with uncertainty. We thus design a bootstrap procedure to obtain confidence intervals that is designed to capture the uncertainty of both the quantile regression and the local projection step. The full details are provided in Appendix C.

To highlight both asymmetries and statistical uncertainty, we present our results in two ways in Figure 3, similar to our presentation in Figure 2. In the left panel, we plot the evolution of the quantiles after a *contractionary* specific shock. Starting point for a given quantile at $s = -1$ in the left panel is the average value of a given quantile across the sample considered. These plots are useful because they convey how the overall shape of the distribution changes. To get a better sense of the associated asymmetries, the right panels plot the actual impulse responses of the quantiles (since the statistical uncertainty is the same as in the left panel, we omit it here). To make the responses comparable across shocks, we rescale the responses across quantiles such that the median falls by 25 basis points on impact (this procedure does not distort the sign of the response).

The key takeaway from Figure 3 is that there is a clear asymmetry in the response to all shocks we consider: *The 10th percentile moves more than the median, which in turn moves more than the 90th percentile.* These differences are immense. To give a sense of the quantities, Table 1 shows both the average ratio and the maximum ratio of the impulse responses of the 10th percentile relative to the median for the first three quarters (in other words, we look at the value of the red line in the right panels of Figure 3 divided by the black

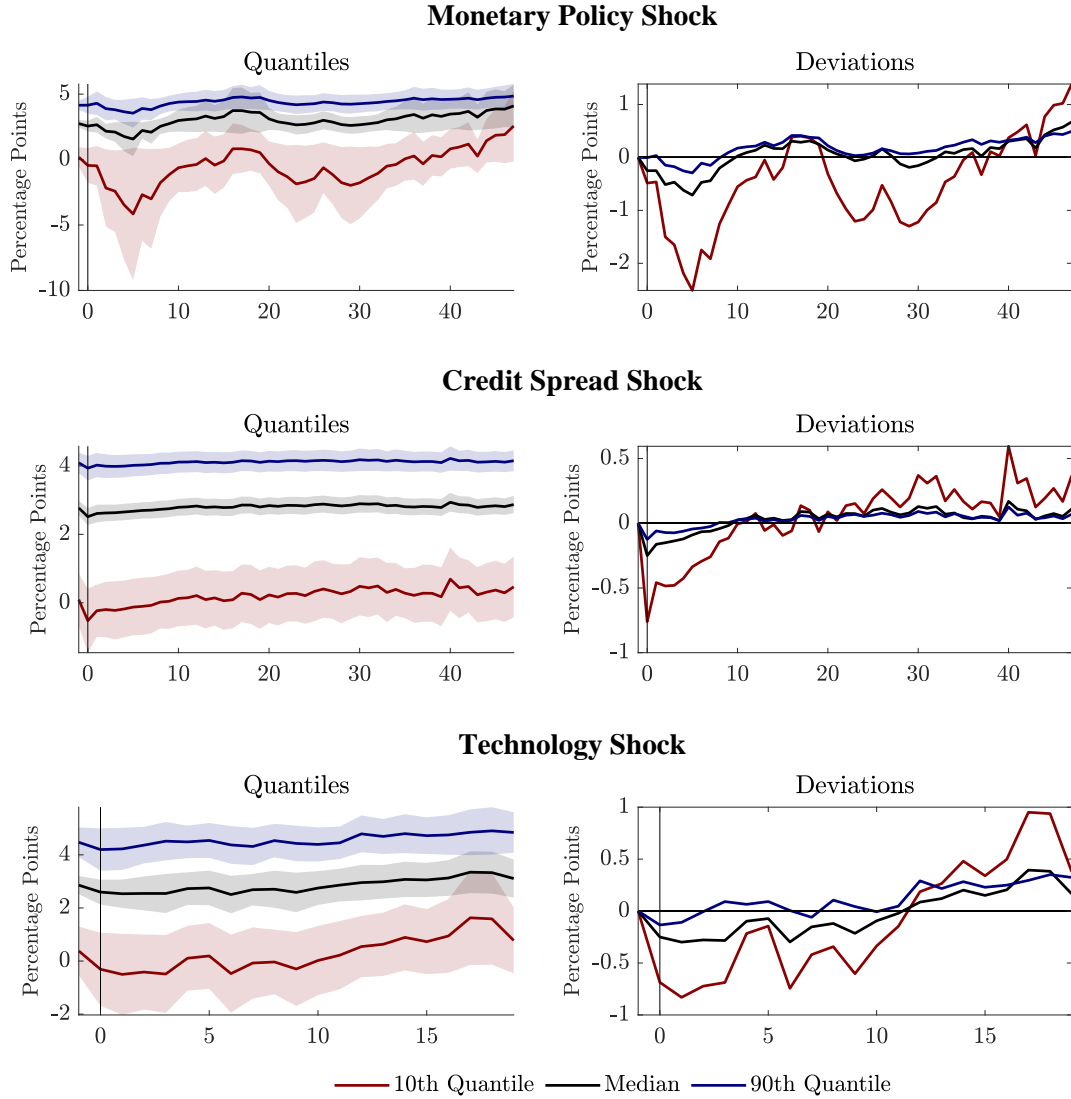


Figure 3: Impulse Responses of Quantiles of Average GDP Growth over the Next Year to *Contractionary* Shocks.

Note: Red is response of the 10th quantile, black is the median response, blue is response of the 90th quantile. Confidence bands correspond to median response, 68% significance level, based on Newey-West and block-of-blocks bootstrapped standard errors. The x-axis gives the horizon of the impulse response, in months for the monetary policy and credit spread shock whereas in quarters for the technology shock. The response on the y-axis is measured in percentage points.

line in the same figure).

The average ratio is around 3 for each of the shock. The maximum ratio across the

Table 1: Ratio of Impulse Responses Over First Three Quarters.

	<i>Monetary Policy Shock</i>	<i>Credit Spread Shock</i>	<i>Technology Shock</i>
Average Ratio	3.5	3.6	2.7
Maximum Ratio	6.2	4.5	2.8

first three quarters can be huge - for example, the response of the 10th percentile for the monetary policy shock can be *six times* as large as the median response. This means that a contractionary shock not only makes average (median) outcomes worse, it moves probability mass to the left tail of the GDP growth distribution even relative to what one would expect from median outcomes. This statement holds for supply shocks (the technology shock we consider here), for demand shocks like our monetary policy shock, as well as for financial shocks. It is thus important to assess how shocks move the entire distribution of future outcomes, not measures of central tendency. This uncertainty is very different from the uncertainty commonly presented in VAR studies where error bands commonly only convey uncertainty due to estimation uncertainty in parameters.¹⁶

A linear model could not generate this type of asymmetry. Hence our results emphasize that nonlinearities are important to understand the business cycle.¹⁷ Furthermore, these nonlinearities cannot be tightly linked to the response to one specific shock, as it is present in all the shocks we study. Our results imply that these asymmetries are feature of the propagation mechanism of shocks, not the original impact. In the next section we describe two data-generating processes that can replicate the asymmetries we have found in the data.

5 Inspecting the Economic Mechanism

In this section, we shed light on the potential economic mechanisms underlying the empirical findings and validate our two-step approach. To do so, we perform Monte Carlo experiments that draw on a “semi-structural” VAR model with switching coefficients and volatilities from [Caldara and Loria \(2020\)](#), as well as the nonlinear DSGE model of [Gertler *et al.* \(2019\)](#) featuring bank panics and financial accelerator mechanisms.

¹⁶This estimation uncertainty is reflected in the error bands in the left panel of Figure 3.

¹⁷As highlighted in [Koenker \(2005\)](#), even linear quantile regressions such as those used here can capture nonlinearities because each quantile is modelled separately.

5.1 A “Semi-Structural” Threshold VAR Model

The Model The VAR model consists of three endogenous variables and three shocks:

$$y_t = \beta_0 + \beta_1(f_t, m_t)f_t + \beta_2(f_t, m_t)m_t + \sigma_y(f_t, m_t)e_t^y \quad (5.1)$$

$$f_t = \alpha_1 f_{t-1} + \alpha_2 m_t + \alpha_3 shock_t + e_t^f \quad (5.2)$$

$$m_t = \gamma_1 m_{t-1} + \gamma_2 f_{t-1} + \gamma_3 shock_t + e_t^m \quad (5.3)$$

where y_t , f_t , m_t respectively denote GDP growth, a financial factor, and a macroeconomic factor and e_t^y , e_t^f and e_t^m are shocks to these variables; $shock_t$ is the structural shock of interest for the impulse response functions. All shocks are drawn from an independent standard normal distribution. β_1 , β_2 , and σ_y are a function of f_t and m_t . In particular, when f_t is larger than a threshold f^* and m_t is smaller than a threshold m^* , the variance σ_y is larger and the effect of financial conditions and macroeconomic activity on future GDP growth is more pronounced. This reflects the idea that during recessions nonlinearities arise which can prolong and exacerbate downturns. The exact parameter values can be found in Appendix G.

This model encodes the view that some recessions (i.e. situations where m_t is low) can be more severe than others - for instance, if the financial variable f_t is not above the threshold the associated recession tends to be milder. A similar idea is pursued in Jordà *et al.* (2020). Even though we focus on one structural shock here for parsimony, our results would be qualitatively the same if instead we had more than one structural shock (i.e. if we replaced $\alpha_3 shock_t$ and $\gamma_3 shock_t$ with $\sum_{i=1}^I \alpha_3^i shock_t^i$ and $\sum_{i=1}^I \gamma_3^i shock_t^i$, respectively, where $shock_t^i$ is the time t realization of the i th structural shock of interest and the number of structural shocks I is larger than 1).

Impulse Responses We simulate the model 1000 times for 413 periods (the number of periods between January 1986 and May 2020) and store $\{y_t, f_t, m_t, shock_t\}$ for each simulation. Exactly as in our empirical analysis, we first construct average future GDP growth over the next year and estimate its quantiles conditional on the financial and the macroeconomic factor. We report representative simulated data and the estimated quantile regression coefficients in Appendix F. In Figure 4 we report the impulse responses of the quantiles to the (contractionary) shock of interest, computed via local projection as in our empirical exercise. We again normalize the responses such that, on impact, the median drops by 25 basis points.

There are two important results that emerge. The first is that in the case of the threshold VAR model (left panel), the shock pushes down the left tail more strongly than other parts of the distribution, as in our empirical findings. This can be explained by the fact that, while the shock enters the model linearly, its relationship to GDP growth is nonlinear to the extent that the shock directly affects the variables that govern the non-linearity in the model (the financial and the macroeconomic factor). In particular, whenever the (contractionary) shock hits, financial conditions tighten and macroeconomic activity weakens. If this movement is strong enough, amplification mechanisms kick in and the economy enters a recession. The second result is that, not surprisingly but reassuringly, no asymmetry is found when we shut down any non-linearities in the DGP (right panel).

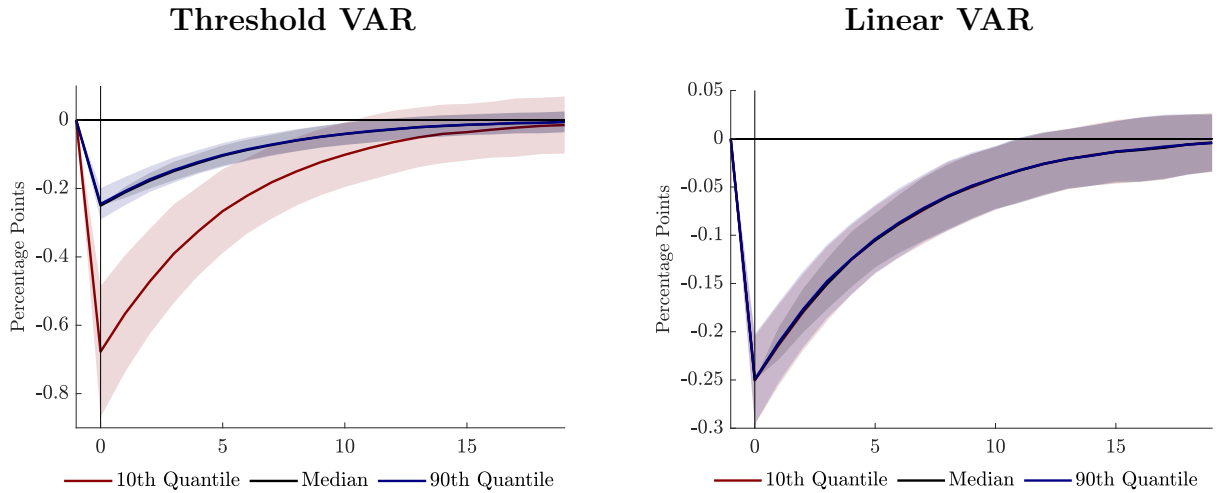


Figure 4: Impulse Responses of Quantiles of Simulated Average Future GDP Growth from Threshold VAR Model.

Note: Straight lines are medians across simulations. Shaded areas are 68% confidence bands.

Alternative Specifications of the DGP In this model, there are multiple channels through which financial and macroeconomic conditions are non-linearly related to GDP growth outcomes. Indeed, there are many possible specifications within this environment that deliver qualitatively the same results (also quantitatively similar to what we find in the data). Here we give a list of data-generating processes that bring about similar findings as in the baseline model:

(QR-I) State-dependent coefficients and constant volatility.

- (QR-II) State-dependence on f_t only, but m_t still affects f_t (i.e., $\alpha_2 \neq 0$). This still holds through even if the coefficient on m_t in the GDP growth equation (β_2) is constant.
- (QR-III) State-dependence on m_t only, but f_t still affects m_t (i.e., $\gamma_2 \neq 0$). However, the asymmetry is starker for m_t and smaller for f_t (since f_t affects m_t with a lag).
- (QR-IV) State-dependence on shock as well as on f_t and/or m_t .
- (QR-V) State-dependence on f_t and/or m_t and shock affects GDP growth independently.¹⁸

In all cases above the shock always has to still affect f_t and m_t for the observational similarity in the IRFs to hold through. In Appendix F we show the results for these cases.

Specification Choices For Our Quantile Regressions Next we turn to a more thorough study of our benchmark specifications. In particular, we ask (i) what would happen if we instead used a one-step approach where we directly include the shock of interest in the quantile regression and (ii) the effects of changing the timing in the quantile regressions.

We investigate the implications of using a one-step approach instead of a two-step approach in estimating the IRFs. To be clear, the one-step approach consists of adding the shock of interest to the conditioning variables in the quantile regression and, at the same time, iteratively leading the dependent variable to estimate the impulse responses of the quantiles to the shock for each horizon s . This approach is formalized in equation (5.4).

$$q_{\tau,t+s} = \hat{\alpha}_{\tau}^{s,B} + \hat{\beta}_{\tau}^{s,B} FIN_{t-1} + \hat{\gamma}_{\tau}^{s,B} MF_{t-1} + \hat{\theta}_{\tau}^{s,B} shock_t \quad (5.4)$$

Note that the timing underlying this specification is the same that we use in our benchmark two-step approach - the quantiles themselves can react to the shocks, but other than that we only control for *lagged* variables. As shown in the left panel of Figure 5, running such a procedure would lead to very similar results as in our benchmark analysis. Then why don't we use this approach? Note that this approach estimates conditional quantiles that condition on a larger set of observables (which now include the shocks). Once we move outside of Monte Carlo experiments, it is hard in a macroeconomic context to compile enough data to estimate these conditional quantiles with enough precision.

¹⁸This case is difficult to justify since GDP growth is a macroeconomic aggregate that is unlikely to be hit by a shock that is specific to GDP growth only and does not affect the other series.

What would happen if we instead control for variables dated t on the right-hand side of our one-step quantile regression? This specification is spelled out in Equation (5.5).

$$q_{\tau,t+s} = \hat{\alpha}_{\tau}^s + \hat{\beta}_{\tau}^s FIN_t + \hat{\gamma}_{\tau}^s MF_t + \hat{\theta}_{\tau}^s shock_t \quad (5.5)$$

The macro and financial factors on the right-hand side can now directly react to the shock, so now $\hat{\theta}_{\tau}^s$ only captures the effect of the shock above and beyond the effects driven by the financial and macro factors. As we can see in the right panel of Figure 5, all nonlinearities are removed. This result seems surprising at first but is actually a result of the fact that, once the contemporaneous financial factor and the macroeconomic factor are accounted for, there is no role left for the shock in explaining the nonlinearity in the data.

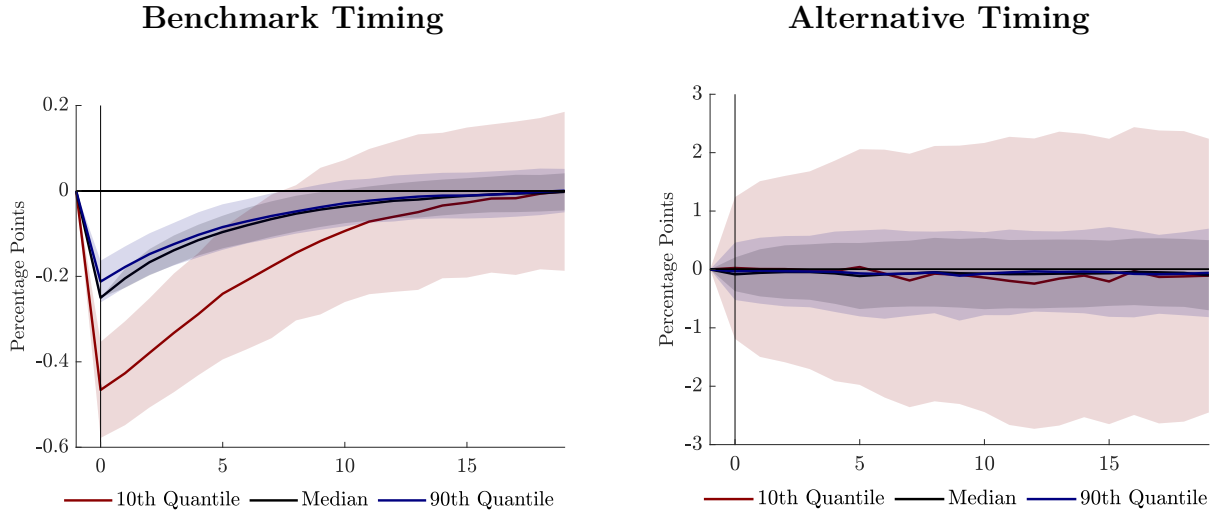


Figure 5: Impulse Responses of Quantiles of Simulated Average Future GDP Growth from Threshold VAR Model. One-Step Approach with Different Timings.

Note: Straight lines are medians across simulations. Shaded areas are 68% confidence bands.

5.2 A Macroeconomic Model with Financial Panics

The Model To assess whether the results we outlined in the threshold model hold in structural models, we now consider the fully micro-founded and nonlinear DSGE model of [Gertler *et al.* \(2019\)](#), which features the possibility of a severe financial crisis. For the sake of conciseness, we briefly discuss the key mechanism that generates the nonlinearity. We

refer interested readers to the original paper for details of the model.¹⁹ The model extends the conventional New Keynesian model with investment by introducing bankers. Bankers are more efficient than households in handling loans. However, bankers are constrained in their ability to raise external funds and are subject to runs. The latter gives rise to multiple equilibria: one with and one without a financial panic.

A financial panic forces the banking system into liquidation, expanding the share of capital held by households. The reallocation of capital holdings from bankers to less efficient households increases the cost of capital, which ultimately disrupts firms' borrowing. Consequently, investment drops substantially more than in the equilibrium without a bank run. A self-fulfilling financial panic equilibrium exists if and only if, in the event of all other depositors' run, an individual household will be better off to follow the "run". When shocks are small, the economy fluctuates around a standard equilibrium. In contrast, a big negative shock pushes the economy into the possibility of a self-fulfilling bank run equilibrium. Combined with a sunspot shock, it triggers a financial panic and a deep recession.

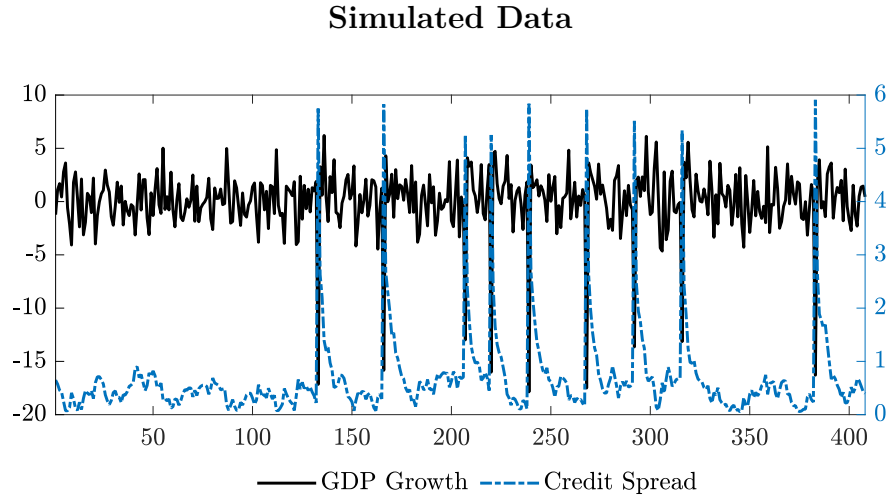
Model Solution The model is highly non-linear, and the non-linear effects of structural shocks to the real economy depend on financial conditions. To allow for these non-linear transition dynamics, the model is solved non-linearly using the collocation method with policy functions solved by time iteration. We follow the original paper in focusing on a capital quality shock as a representative structural shock. However, other shocks would give rise to qualitatively very similar results because the mechanism creating asymmetry is not specifically tied to one structural shock.

Parameterization We simulate the model using the original calibration of the deep parameters and of the capital quality shock process (the only fundamental shock in the model). In order to generate rare financial crisis, we calibrate the process for the sunspot shock such that a bank run equilibrium arises after a big negative shock (above two standard deviations).²⁰

¹⁹The full model consists of 40 equations which we do not include here for the sake of brevity.

²⁰Our calibration intends to make the bank run event (see Figure 2 of the original paper) re-occurring in the simulated sample. In their event study, the authors feed in two consecutive negative capital quality shocks of roughly one standard deviation, together with a sun-spot shock, to generate a bank-run. We deviate slightly from their event study by having sun-spot shocks occurring concurrently with a negative two standard deviations capital quality shocks to ensure the probability of a bank run of 2.5% across simulated samples.

Simulation We simulate this model 1000 times for 413 periods and store the level of GDP, the credit spread, and the capital quality shock. In our analysis, we treat the credit spread as the equivalent of the financial factor though results are robust to the use of alternative measures of financial conditions in the model. The simulated data shown in the top panel of Figure 6 indicate that also in this model there is a non-linear relationship between growth and financial conditions. Indeed, large credit spreads are associated with extremely negative growth realizations.



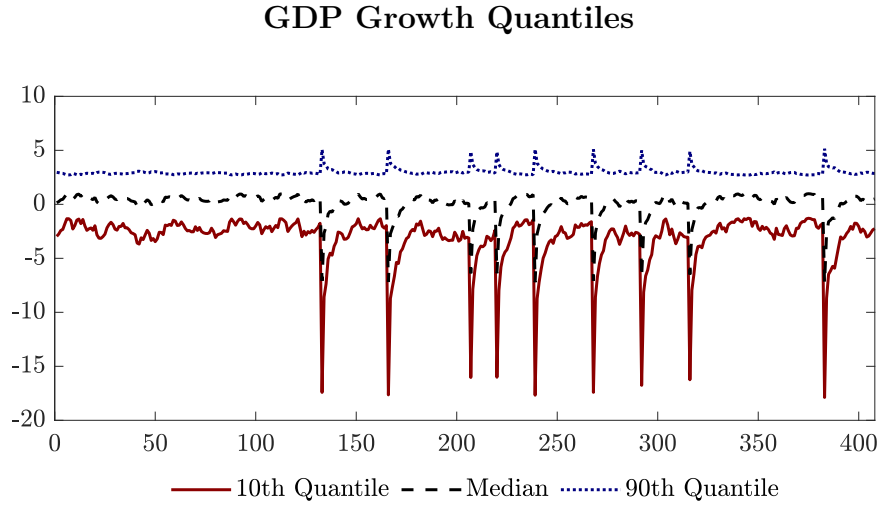
Note: Example from one simulation. GDP Growth (left axis) and Credit Spread (right axis).

Figure 6: Simulated Data from Gertler *et al.* (2019) Model.

Quantile Regression We focus on current (period-over-period) GDP growth as the shocks are transitory and only create a sharp but short-lived recession.²¹ Further, we only consider the credit spread as conditioning variable since the non-linearity in the model is coming from financial conditions. The quantile regression picks up the nonlinear relationship between financial conditions and economic growth, suggesting that financial conditions have a more negative effect on the left tail than on the median.²² Thus, also in this example, the left tail of the distribution falls substantially during times of extreme financial distress, characterizing a vulnerable growth outlook. This is further illustrated in Figure 7 which plots the quantiles of GDP growth associated with the simulated data.

²¹Notice that even though the distribution is for current GDP growth, this exercise is still relevant for the characterization of risks to the growth outlook from a forecasting perspective, as data on credit spreads is available at a higher frequency than GDP growth data within a given period.

²²Averages across simulations of the estimated quantile regression coefficients are shown in Appendix H.



Note: Example from one simulation.

Figure 7: Quantiles from [Gertler *et al.* \(2019\)](#) Model.

Impulse Responses Figure 8 presents the impulse responses of the quantiles to the (contractionary) capital quality shock, computed via local projection as in our empirical exercise. Also in this example, we normalize the responses such that, on impact, the median drops by 25 basis points. We again find that the shock pushes down the left tail more strongly than other parts of the distribution, as in our empirical findings. Indeed, while the shock enters the model linearly, as in the previous exercise, its relationship to GDP growth is nonlinear as it affects the financial variables that transmit and amplify the disturbance nonlinearly.

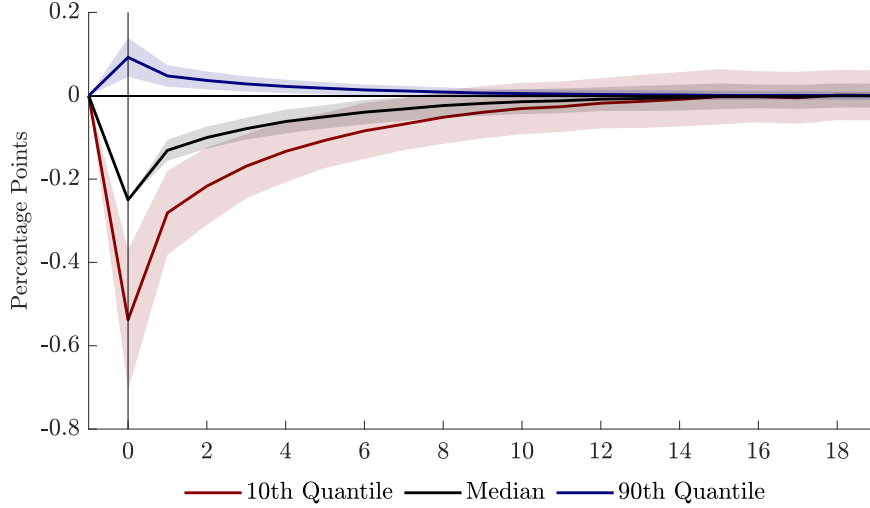


Figure 8: Impulse Responses of Quantiles of Simulated GDP Growth to Capital Quality Shock from [Gertler *et al.* \(2019\)](#) Model.

Note: Straight lines are medians across simulations. Shaded areas are 68% confidence bands.

6 Conclusion

This paper studies tail risk in U.S. aggregate outcomes. In particular, we study how macroeconomic shocks affect tail risk. All shocks considered (monetary policy, credit spread, and productivity shocks) affect tail risk disproportionately more than other quantiles (on average the response of the 10th percentile over the first three quarters after impact is around 3 times larger). Contractionary shocks thus deserve even more attention than what their effect on average outcomes suggests to the extent that they make poor economic conditions more likely. Our results lead us to believe that policymakers should be especially weary of unexpected adverse changes in the economy. The fact that all shocks we study display this tail risk asymmetry makes us suspect that there is a common mechanism behind these asymmetries. Indeed, the two data-generating processes we use as laboratories to test this hypothesis can replicate our findings: In the first experiment, a threshold VAR model, a common mechanism (which could be thought of as a financial accelerator mechanism in a more structural model) amplifies the negative effect on economic growth of a deterioration in financial and macroeconomic conditions. In the second, a macroeconomic model with financial panics, the bank-run equilibrium features nonlinearities that magnify the effects of a financial panic on economic activity.

References

- ADRIAN, T., BOYARCHENKO, N. and GIANNONE, D. (2019). Vulnerable Growth. *American Economic Review*, **109** (4), 1263–1289.
- ANTOLÍN-DÍAZ, J. and RUBIO-RAMÍREZ, J. F. (2018). Narrative Sign Restrictions for SVARs. *American Economic Review*, **108** (10), 2802–29.
- BARNICHON, R. and BROWNLEES, C. (2019). Impulse Response Estimation by Smooth Local Projections. *The Review of Economics and Statistics*, **101** (3), 522–530.
- BERKOWITZ, J., BIEGEAN, I. and KILIAN, L. (1999). *On the Finite-Sample Accuracy of Nonparametric Resampling Algorithms for Economic Time Series*. Finance and Economics Discussion Series 1999-04, Board of Governors of the Federal Reserve System (US).
- CALDARA, D., CASCALDI-GARCIA, D., CUBA BORDA, P. and LORIA, F. (2020). Understanding Growth-at-Risk: A Markov-Switching Approach. *mimeo*.
- and LORIA, F. (2020). Observational Similarity in Quantile Regression Models. *mimeo*.
- CARRIERO, A., CLARK, T. E. and MARCELLINO, M. (2020). *Capturing Macroeconomic Tail Risks with Bayesian Vector Autoregressions*. Working Papers 202002, Federal Reserve Bank of Cleveland.
- CHAVLEISHVILI, S. and MANGANELLI, S. (2017). Quantile Impulse Response Functions.
- COGLEY, T. and SARGENT, T. J. (2005). Drift and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S. *Review of Economic Dynamics*, **8** (2), 262–302.
- CÚRDIA, V., DEL NEGRO, M. and GREENWALD, D. L. (2014). Rare Shocks, Great Recessions. *Journal of Applied Econometrics*, **29** (7), 1031–1052.
- FAGIOLO, G., NAPOLETANO, M. and ROVENTINI, A. (2008). Are Output Growth-Rate Distributions Fat-Tailed? Some Evidence from OECD Countries. *Journal of Applied Econometrics*, **23** (5), 639–669.
- FAVARA, G., GILCHRIST, S., LEWIS, K. F. and ZAKRAJŠEK, E. (2016). Updating the Recession Risk and the Excess Bond Premium. (2016-10-06).

- FERNALD, J. G. (2012). A Quarterly, Utilization-Adjusted Series on Total Factor Productivity. (2012-19).
- GERTLER, M. and KARADI, P. (2015). Monetary Policy Surprises, Credit Costs, and Economic Activity. *American Economic Journal: Macroeconomics*, **7** (1), 44–76.
- , KIYOTAKI, N. and PRESTIPINO, A. (2019). A Macroeconomic Model with Financial Panics. *The Review of Economic Studies*, **87** (1), 240–288.
- GIGLIO, S., KELLY, B. and PRUITT, S. (2016). Systemic risk and the macroeconomy: An empirical evaluation. *Journal of Financial Economics*, **119** (3), 457 – 471.
- GILCHRIST, S. and ZAKRAJŠEK, E. (2012). Credit Spreads and Business Cycle Fluctuations. *American Economic Review*, **102** (4), 1692–1720.
- JAROCIŃSKI, M. and KARADI, P. (2020). Deconstructing Monetary Policy Surprises-the role of Information Shocks. *American Economic Journal: Macroeconomics*, **12** (2), 1–43.
- JORDÀ, O. (2005). Estimation and Inference of Impulse Responses by Local Projections. *American Economic Review*, **95** (1), 161–182.
- JORDÀ, O., SCHULARICK, M. and TAYLOR, A. M. (2020). *Disasters Everywhere: The Costs of Business Cycles Reconsidered*. Working Paper 26962, National Bureau of Economic Research.
- JUSTINIANO, A. and PRIMICERI, G. E. (2008). The Time-Varying Volatility of Macroeconomic Fluctuations. *American Economic Review*, **98** (3), 604–641.
- KILIAN, L. and LÜTKEPOHL, H. (2018). *Structural Vector Autoregressive Analysis*. No. 9781107196575 in Cambridge Books, Cambridge University Press.
- and MANGANELLI, S. (2008). The Central Banker as a Risk Manager: Estimating the Federal Reserve’s Preferences under Greenspan. *Journal of Money, Credit and Banking*, **40** (6), 1103–1129.
- KIM, T.-H., LEE, D. J. and MIZEN, P. (2019). Impulse Response Analysis in Conditional Quantile Models and an Application to Monetary Policy.
- KOENKER, R. (2005). *Quantile Regression*. Econometric Society Monographs, Cambridge University Press.

- and BASSETT, G. (1978). Regression Quantiles. *Econometrica*, **46** (1), 33–50.
- LINNEMANN, L. and WINKLER, R. (2016). Estimating Nonlinear Effects of Fiscal Policy Using Quantile Regression Methods. *Oxford Economic Papers*, **68** (4), 1120–1145.
- LORIA, F., MATTHES, C. and ZHANG, D. (2019). *Assessing Macroeconomic Tail Risk*. Working Paper 19-10, Federal Reserve Bank of Richmond.
- MERTENS, K. and RAVN, M. O. (2013). The Dynamic Effects of Personal and Corporate Income Tax Changes in the United States. *American Economic Review*, **103** (4), 1212–1247.
- MIRANDA-AGRIPPINO, S. and RICCO, G. (2020). The Transmission of Monetary Policy Shocks. *American Economic Journal: Macroeconomics*, **forthcoming**.
- MUMTAZ, H. and SURICO, P. (2015). The Transmission Mechanism In Good And Bad Times. *International Economic Review*, **56**, 1237–1260.
- NAKAMURA, E. and STEINSSON, J. (2018). High-frequency Identification of Monetary Non-neutrality: the Information Effect. *The Quarterly Journal of Economics*, **133** (3), 1283–1330.
- NEWKEY, W. K. and WEST, K. D. (1987). A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica*, **55** (3), 703–708.
- PLAGBORG-MØLLER, M., REICHLIN, L., RICCO, G. and HASENZAGL, T. (2020). *When is Growth at Risk?* Tech. rep., Brooking Papers on Economic Activity.
- and WOLF, C. (2019). Local Projections and VARs Estimate the Same Impulse Responses.
- PRIMICERI, G. E. (2005). Time Varying Structural Vector Autoregressions and Monetary Policy. *Review of Economic Studies*, **72** (3), 821–852.
- RAMEY, V. (2016). Macroeconomic Shocks and Their Propagation. *Handbook of Macroeconomics*, **2**, 71 – 162.
- ROMER, C. D. and ROMER, D. H. (2004). A New Measure of Monetary Shocks: Derivation and Implications. *American Economic Review*, **94** (4), 1055–1084.

- UHLIG, H. (2005). What Are the Effects of Monetary Policy on Output? Results from an Agnostic Identification Procedure. *Journal of Monetary Economics*, **52** (2), 381–419.
- ZHANG, D. (2019). Term Structure, Forecast Revision and the Signaling Channel of Monetary Policy. *mimeo*.

A Data

This section gives a brief overview of the data we use throughout this paper.

A.1 Growth-at-Risk Data

The data for the quantile regression com from [Caldara *et al.* \(2020\)](#). The dependent variable is monthly GDP growth, shown in Figure [A-1](#) whereas the conditioning variables are a financial and a macroeconomic factor, displayed in Figure [A-2](#). These three variables are estimated from a Dynamic Factor Model (DFM). The reader is referred to the original paper for the details on the DFM model and its estimation procedure.

The following set of data inform the two factors used as conditioning variables in the quantile regression:

(I) Financial factor

- Volatility index of the S&P 100 (VXO)
- Excess bond premium
- 3-month LIBOR rate minus 3-month Treasury bill
- 3-month financial commercial paper rate minus 3-month Treasury bill.

(II) Macroeconomic factor

- Industrial Production
- Retail Sales
- New Export Orders Component of Purchasing Managers' Index (PMI)
- Initial Unemployment Claims
- GDP

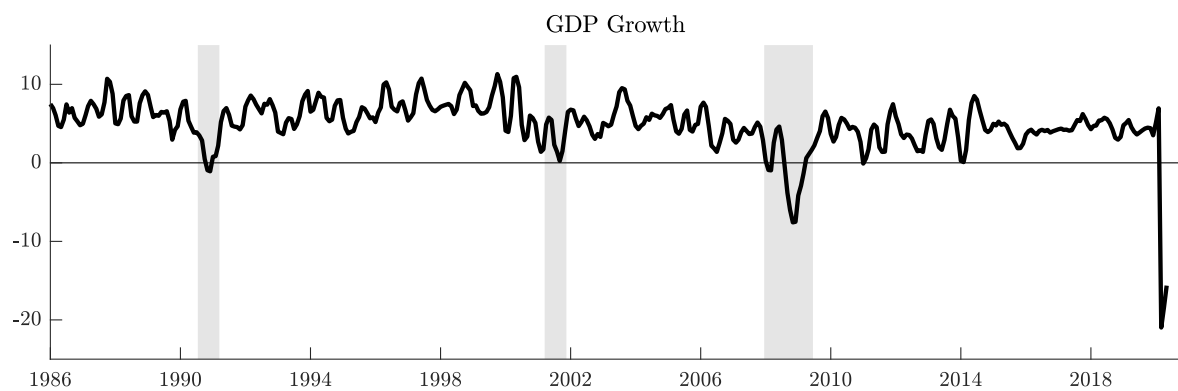


Figure A-1: Monthly GDP Growth.

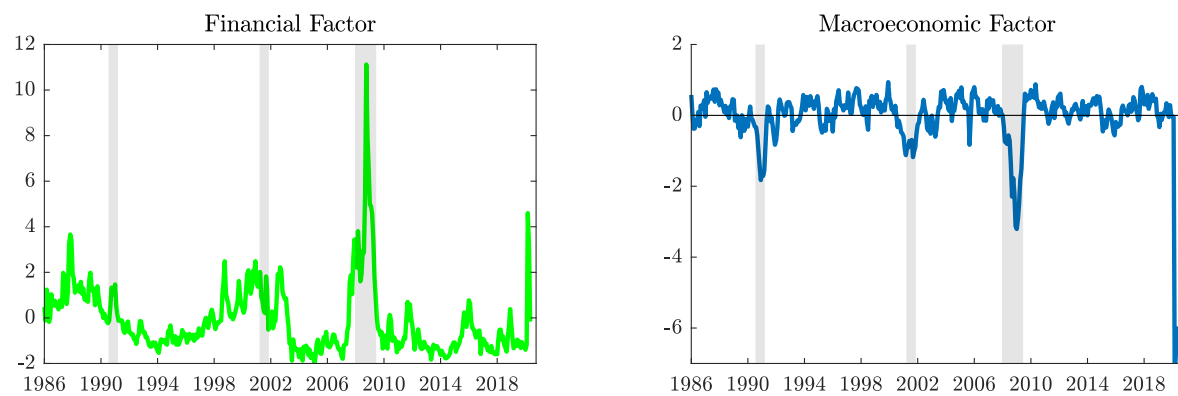


Figure A-2: Conditioning Variables in the Quantile Regressions.

A.2 Local Projection Variables

Most control variables used in the local projection stage are available in FRED. The source data for the control variables are:

- Consumer Price Index for all Urban Consumers: All Items. FRED Mnemonic: CPIAUCSL.
- CRB Commodity Index. Haver Mnemonic: PZALL@USECON.
- Unemployment Rate. Percent, Seasonally Adjusted. FRED Mnemonic: A191RL1Q225SBEA.
- Federal funds rate. FRED Mnemonic: FEDFUNDS.
- Excess bond premium. Source: Favara *et al.* (2016)²³.
- Real GDP. Percent Change from Preceding Period, Seasonally Adjusted Annual Rate. FRED Mnemonic: A191RL1Q225SBEA.
- Chicago Fed National Financial Conditions Index. FRED Mnemonic: NFCI.
- Total hours worked given by the hours of wage and salary workers on non-farm payrolls. FRED Mnemonic: TOTLQ. The latter is used to compute productivity (real GDP divided by total hours).
- Industrial production. FRED Mnemonic: INDPRO.
- One-year government bond rate. FRED Mnemonic: GS1.

As to the shocks used in the robustness exercises, the Romer and Romer (2004) monetary shock is provided by Ramey (2016). We aggregate the monthly shock series to quarterly frequency by taking the quarterly average. We take the narrative monetary policy shock provided by Antolín-Díaz and Rubio-Ramírez (2018), again aggregated to quarterly frequency by calculating the quarterly average.

²³The series can be downloaded at <https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/updating-the-recession-risk-and-the-excess-bond-premium-20161006.html>.

B Some More Intuition

In this section we show an example where the impulse responses of the quantiles cross. By construction, however, the quantiles themselves cannot cross because we directly model changes in the entire distribution. The example is similar to the first example from the main text, but we change the sequence of variances for the Gaussian distribution to achieve the crossing of the impulse responses of the 10th and 90th percentiles.

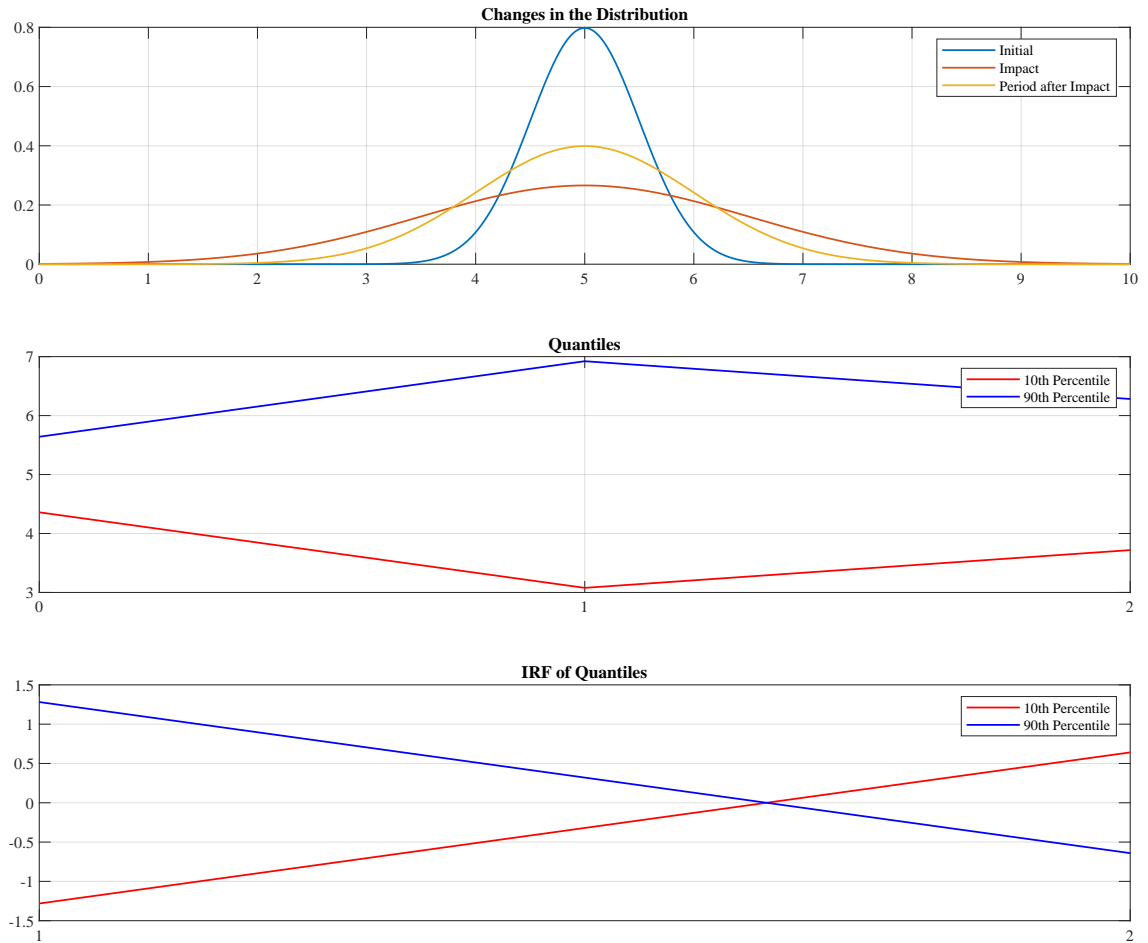


Figure B-1: Example Where Impulse Responses Cross But Quantiles Don't.

C Bootstrap Procedure

This section gives a brief overview of the bootstrap procedure used to obtain the confidence bands. The procedure is designed to capture the uncertainty involved both with the quantile regression and with the local projection step of our estimation approach.

Quantile Regression Step The first step of the bootstrap procedure involves quantifying the uncertainty around the quantile regression estimates. To do so, we revert to “blocks-of-blocks” bootstrap. While more details on this methodology can be found in [Kilian and Lütkepohl \(2018\)](#) (see Chapter 12 therein), we here provide a brief summary of the bootstrap procedure. “Blocks-of-blocks” bootstrap is used in cases where a researcher is interested in computing confidence intervals around nonsymmetric statistics of the underlying data (e.g., autocorrelations or estimators of autoregressive slope coefficients in a time-series context). This is relevant in our case since not only the quantile regression slopes are non-linear functions of the data but also, we are de facto running a h -step predictive regression of inflation on its (past) determinants. The “blocks-of-blocks” bootstrap procedure preserves the (time-series) dependency in the data, which in most cases would get destroyed by a naive bootstrap.

More specifically, the “blocks-of-blocks” bootstrap procedure relies on first dividing the dependent variable y and the regressors X into consecutive blocks of all possible m -tuples. For a total number of 100 bootstrap replications, blocks of data are randomly drawn to form a new sample of the same size as the original data. Importantly, the blocks are resampled in the same order for both the dependent variable y and the regressors X , a key step which preserves the time-dependency in the data. In our particular application, we run the quantile and store the estimates of the local projection coefficient $\hat{\theta}_\tau^s$, capturing the effect of the shock on the quantile τ at horizon s , for each bootstrap replication. The procedure is asymptotically valid for stationary processes if the block size l increases at a suitable rate as $T \rightarrow \infty$. Following [Berkowitz *et al.* \(1999\)](#) we set $m = \sqrt[3]{T}$, where T is the sample size. Finally, this bootstrap procedure preserves the quantile regression feature of being agnostic about the underlying distribution of the error terms, as this is not a residual-based approach.

Local Projection Step At each horizon s , 100 bootstrap replications of the local projection estimates are obtained by drawing the impulse response coefficients from their asymptotic distribution. This distribution is known and given by $\theta_\tau^s \sim \mathcal{N}(\hat{\theta}_\tau^s, \hat{\Sigma}_u^s)$, where $\hat{\theta}_\tau^s$ is the estimated coefficient and $\hat{\Sigma}_u^s$ the variance-covariance matrix of the local projection residuals

u_{t+s}^s , estimated by [Newey and West \(1987\)](#) with lag order $s - 1$ due to the serial correlation in the error term induced by the successive leading of the dependent variable in the s -step ahead direct forecasting regression.

Combining the Uncertainty We merge the two distributions of the impulse response coefficient $\hat{\theta}_\tau^s$ into one distribution. 68 percent confidence intervals are constructed by looking at the 16th and 84th percentile of that distribution. These intervals are then centered around the point estimate $\hat{\theta}_\tau^s$ obtained with the original sample. We then add the point estimate $\hat{\theta}_\tau^s$ and the confidence bands to the average over the estimated sample of the quantile $q_{\tau,t+s}$ in order to visualize the uncertainty around the entire distribution.

D [Gertler and Karadi \(2015\)](#) Monetary Policy Shocks

We construct the monetary policy shocks implied by the proxy VAR used in [Gertler and Karadi \(2015\)](#) using the following procedure.

First, we update the data used in the [Gertler and Karadi \(2015\)](#) baseline VAR. They use monthly data from 1979M7 to 2012M6. We update all time-series to 2019M12. The VAR includes (the log of) industrial production, (the log of) the consumer price index, the one-year government bond rate, and the excess bond premium. As instrument for the monetary policy shock in the VAR we use our updated [Miranda-Agrippino and Ricco \(2020\)](#) shocks series.

Then, we estimate the proxy VAR and compute the implied structural monetary policy shocks, see the appendix of [Mertens and Ravn \(2013\)](#) for details. The identification of monetary policy shocks is achieved by relying on the correlation between the reduced form residuals of the one-year government bond rate and the instrument and that the later is orthogonal to other structural shocks.

E Robustness

Horizon - Average GDP Growth over the Next Six Months We explore the sensitivity of our results to the choice of a shorter horizon for the growth outlook. In particular, we replicate our results for the choice of average GDP growth over the next six months as a dependent variable in the quantile regression. Figure E-1.

Industrial Production Instead of Monthly GDP In Figure E-2 we explore the sensitivity of our baseline results for the monthly specifications to the choice of the dependent variable. In particular, we replace monthly GDP growth by the (month-over-month) growth in industrial production. As an additional sensitivity, we also report in that Figure the responses using the pure [Miranda-Agrippino and Ricco \(2020\)](#) monetary policy shocks (MAR) on top of the [Gertler and Karadi \(2015\)](#) (GK) shocks shown in the baseline results.

Starting the Quantile Regression in 1986 The quantile regression starts in 1986 as the local projection. See Figure E-3.

Pre-Great-Recession Sample Local projection stops in 2007 September/Q3. To allow for enough observations the local projection for the technology shock starts in 1973 and for all shocks we use lags for half a year. See Figure E-4.

Quarterly Specification of [Adrian et al. \(2019\)](#) Specification uses the National Financial Conditions Index and annualized (quarter-over-quarter) GDP growth. Quantile regression starts in 1973:Q1 and ends in 2019:Q1 (last available data point for average GDP growth over the next year). Local projections start as in baseline. See Figure E-5. Monthly shocks are averaged.

Quarterly Specification of [Adrian et al. \(2019\)](#) with Shocks Available from 1973 This specification is the same as the previous one, but uses shocks available from 1973 so that also the local projection starts in 1973. For this exercise, we explore two types of monetary policy shocks. First, we use the [Romer and Romer \(2004\)](#) (RR henceforth) narrative-based monetary shocks provided by [Ramey \(2016\)](#). They regress the federal funds target rate on Greenbook forecasts at each FOMC meeting date and use the residuals as the monetary policy shock. The sample period runs from 1973Q1 to 2007Q4. As a second measure, we use the monetary policy shocks identified by [Antolín-Díaz and Rubio-Ramírez \(2018\)](#) (AR henceforth) who add narrative sign restrictions to the VAR model in [Uhlig \(2005\)](#). Here the sample period is 1973Q1 to 2007Q3.

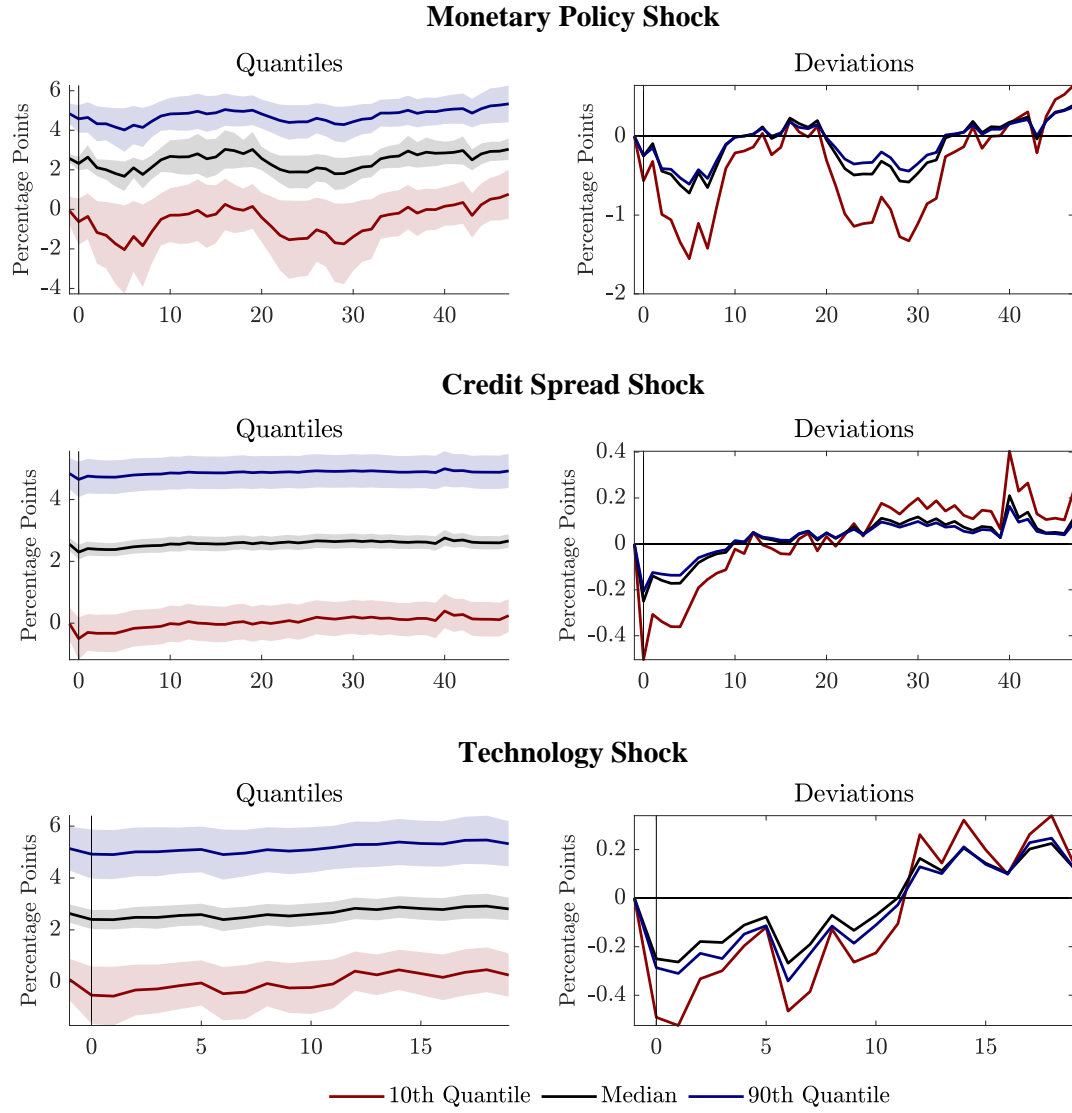


Figure E-1: Impulse Responses of Quantiles of Average GDP Growth over the Next Six Months to *Contractionary* Shocks.

Note: Red (dashed) is response of the 10th quantile, black (solid) is the median response, blue (dotted) is response of the 90th quantile. Confidence bands correspond to median response, 68% significance level, based on Newey-West and block-of-blocks bootstrapped standard errors. The x-axis gives the horizon of the impulse response, in months for the monetary policy and credit spread shock whereas in quarters for the technology shock. The response on the y-axis is measured in percentage points.

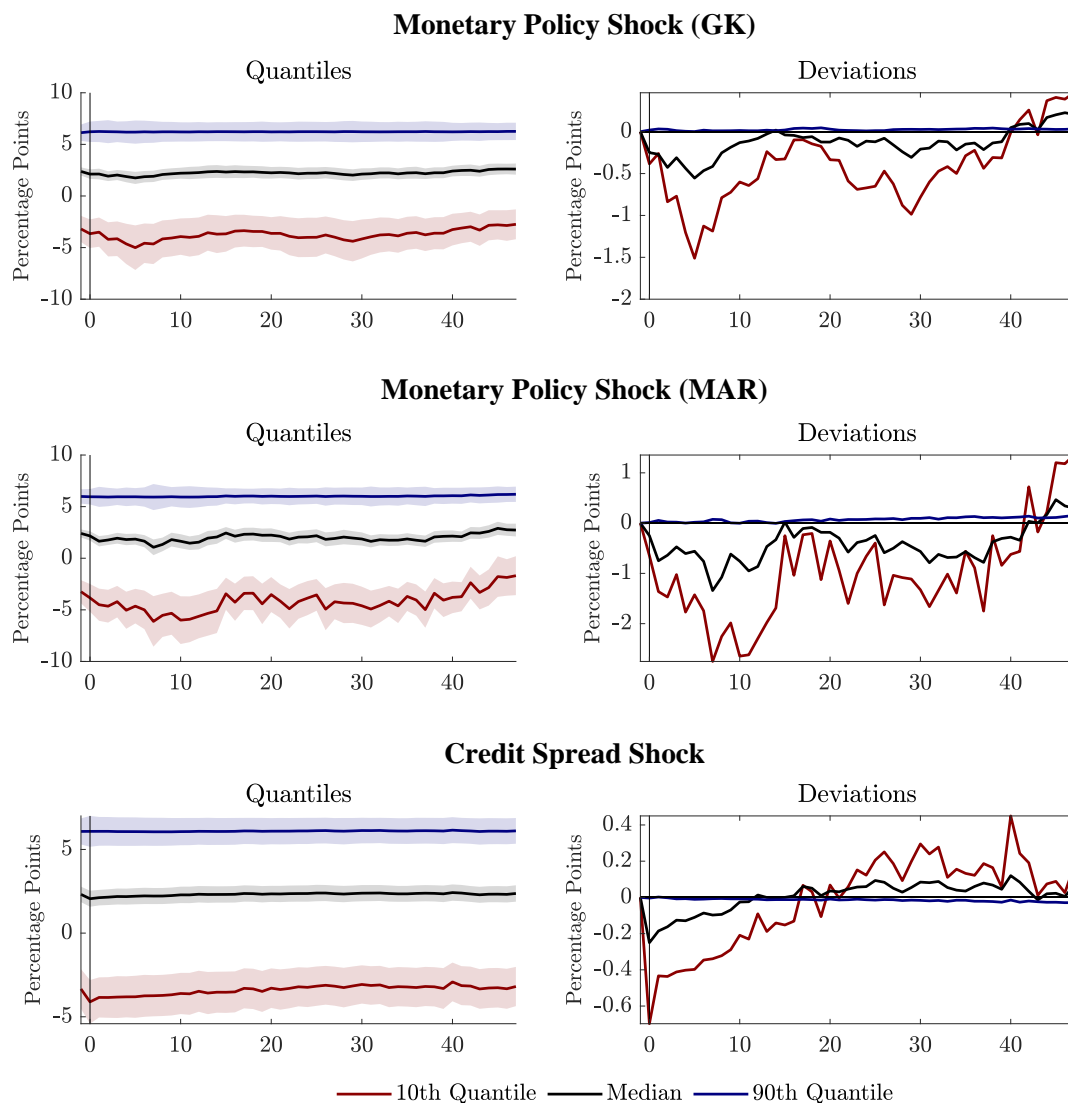


Figure E-2: Impulse Responses of Quantiles of Average Industrial Production Growth over the Next Year to *Contractionary* Shocks. Specification using Industrial Production Instead of Monthly GDP as Dependent Variable.

Note: Red (dashed) is response of the 10th quantile, black (solid) is the median response, blue (dotted) is response of the 90th quantile. Confidence bands correspond to median response, 68% significance level, based on Newey-West and block-of-blocks bootstrapped standard errors. The x-axis gives the horizon of the impulse response, in months. The response on the y-axis is measured in percentage points.

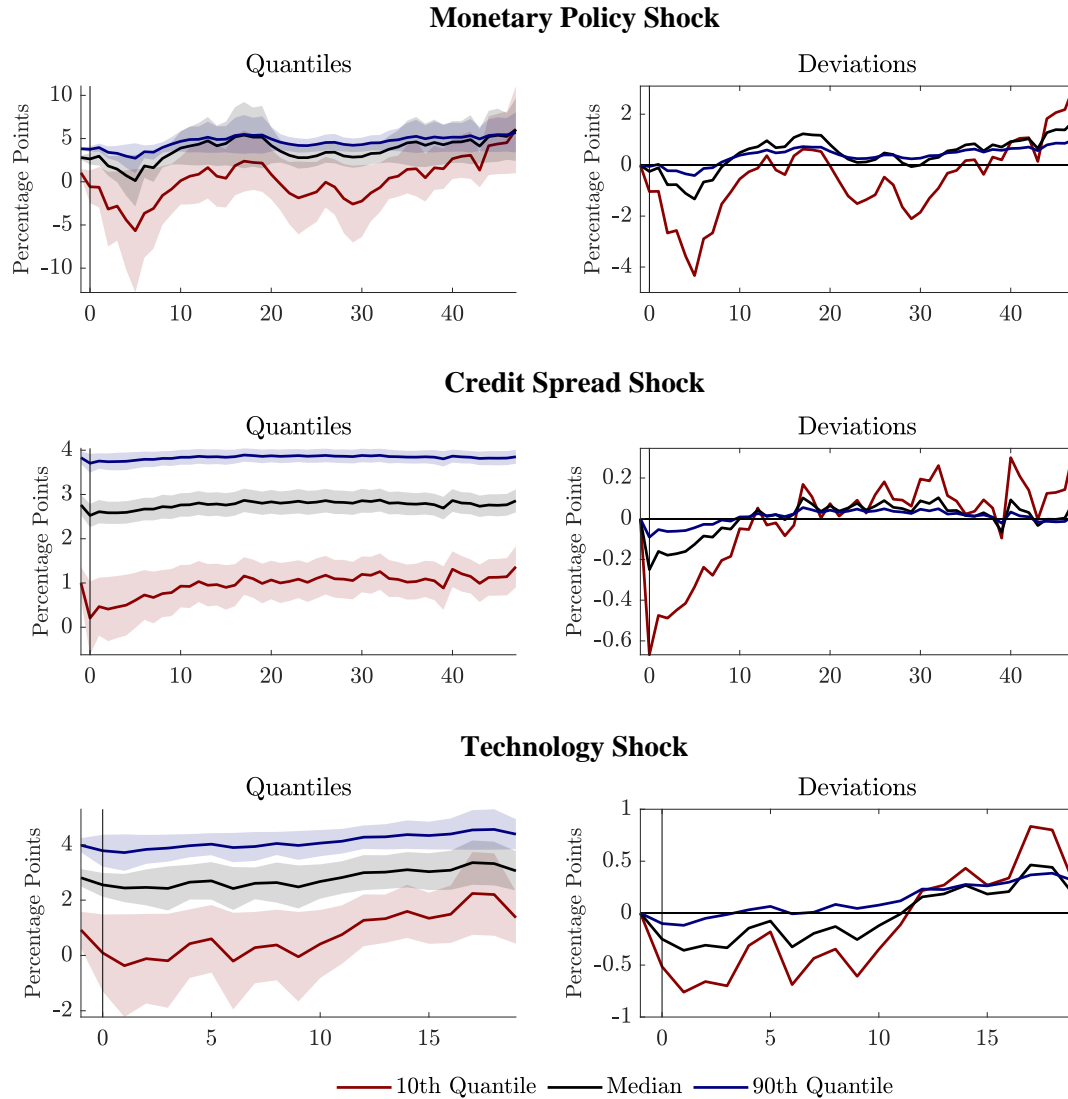


Figure E-3: Impulse Responses of Quantiles of Average GDP Growth over the Next Year to *Contractionary* Shocks. Sample Starting in 1986.

Note: Red (dashed) is response of the 10th quantile, black (solid) is the median response, blue (dotted) is response of the 90th quantile. Confidence bands correspond to median response, 68% significance level, based on Newey-West and block-of-blocks bootstrapped standard errors. The x-axis gives the horizon of the impulse response, in months for the monetary policy and credit spread shock whereas in quarters for the technology shock. The response on the y-axis is measured in percentage points.

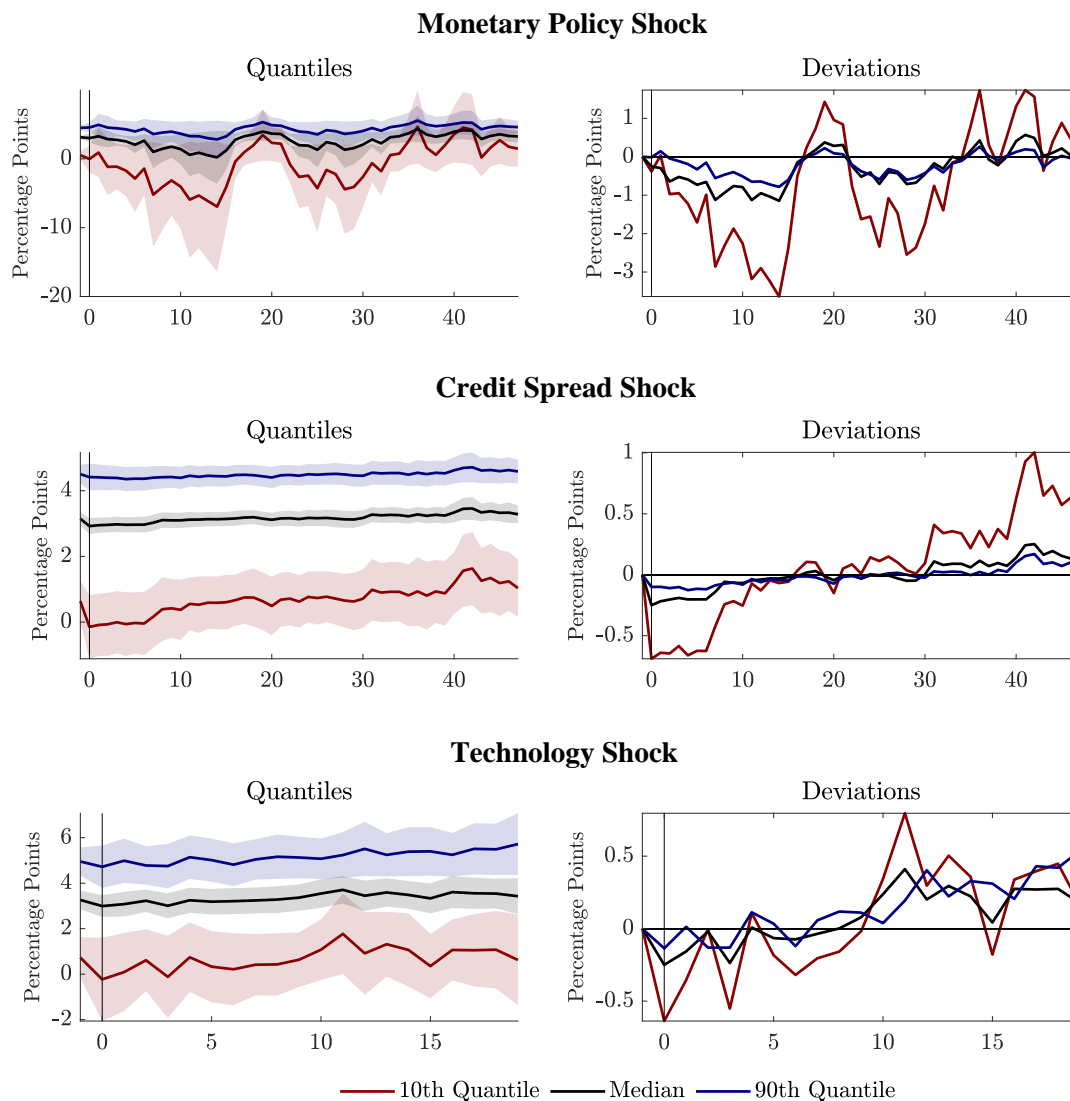


Figure E-4: Impulse Responses of Quantiles of Average GDP Growth over the Next Year to *Contractionary* Shocks. Sample Stopping Before Great Recession.

Note: Red (dashed) is response of the 10th quantile, black (solid) is the median response, blue (dotted) is response of the 90th quantile. Confidence bands correspond to median response, 68% significance level, based on Newey-West and block-of-blocks bootstrapped standard errors. The x-axis gives the horizon of the impulse response, in months for the monetary policy and credit spread shocks whereas in quarters for the technology shock. The response on the y-axis is measured in percentage points.

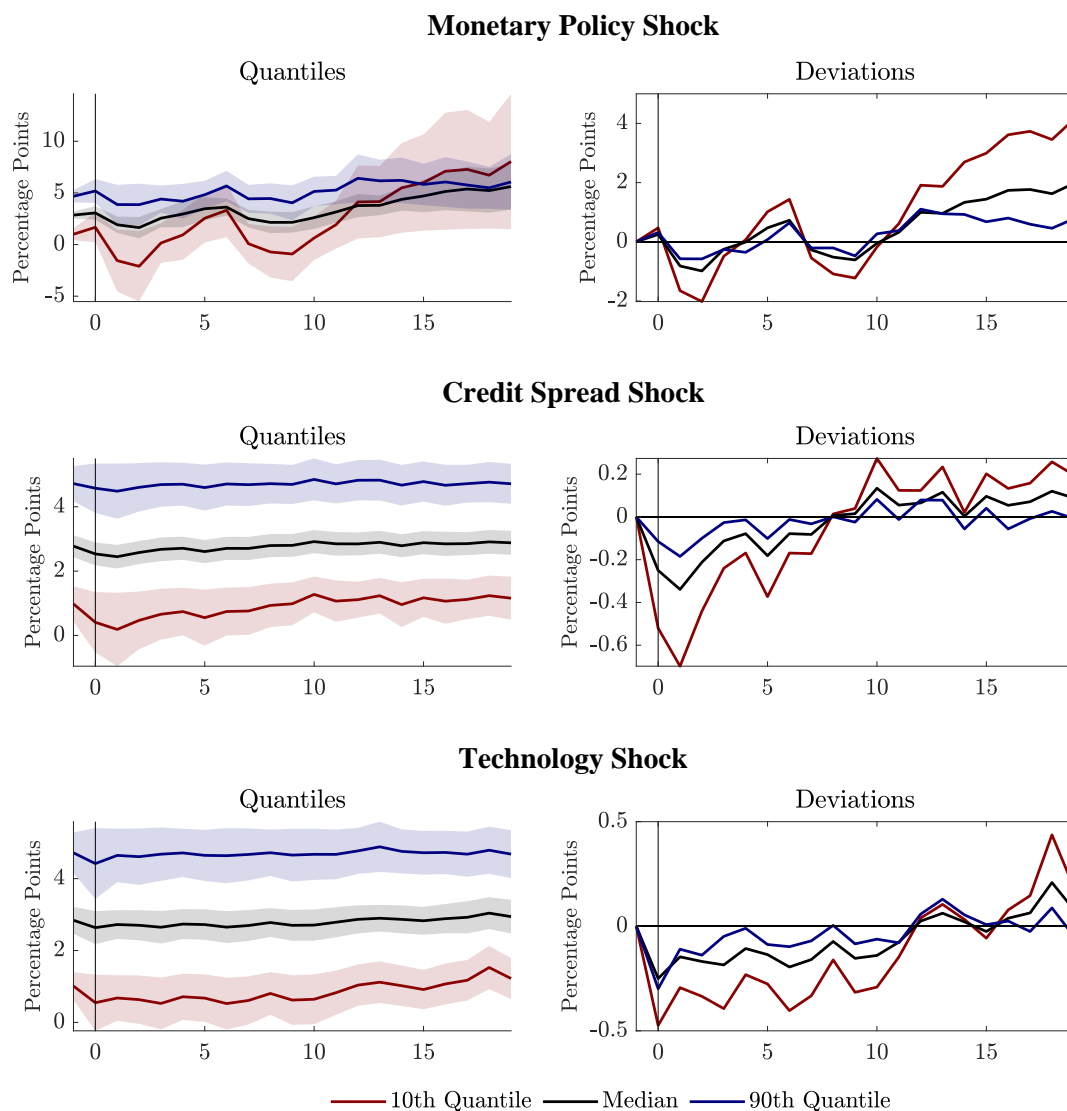


Figure E-5: Impulse Responses of Quantiles of Average GDP Growth over the Next Year to *Contractionary* Shocks. Quarterly Specification with NFCI and GDP Growth as Conditioning Variables.

Note: Red (dashed) is response of the 10th quantile, black (solid) is the median response, blue (dotted) is response of the 90th quantile. Confidence bands correspond to median response, 68% significance level, based on Newey-West and block-of-blocks bootstrapped standard errors. The x-axis gives the horizon of the impulse response, in months for the monetary policy and credit spread shocks whereas in quarters for the technology shock. The response on the y-axis is measured in percentage points.

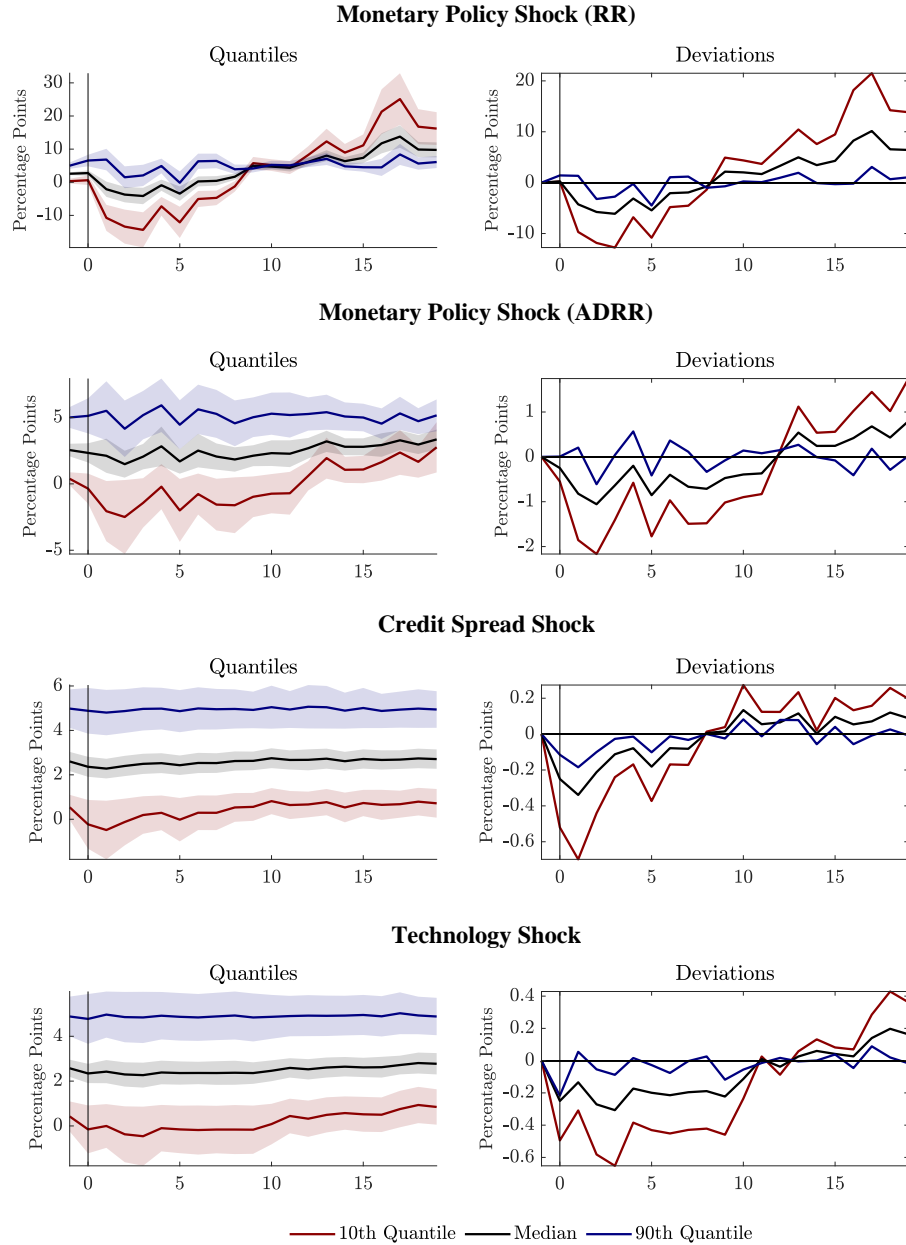


Figure E-6: Impulse Responses of Quantiles of Average GDP Growth over the Next Year to *Contractionary* Shocks. Quarterly Specification with NFCI and GDP Growth as Conditioning Variables and with Shocks Starting in 1973:Q1.

Note: Red (dashed) is response of the 10th quantile, black (solid) is the median response, blue (dotted) is response of the 90th quantile. Confidence bands correspond to median response, 68% significance level, based on Newey-West and block-of-blocks bootstrapped standard errors. The x-axis gives the horizon of the impulse response, in months for the monetary policy and credit spread shocks whereas in quarters for the technology shock. The response on the y-axis is measured in percentage points.

F More Results on the Threshold VAR Model

F.1 Baseline Threshold VAR Specification

Table F-1: Quantile Regression Coefficients - Threshold VAR Model.

Variable	10th Quantile	Median	90th Quantile
Intercept	-5.34	0.99	3.87
Financial Factor	-0.42	-0.19	-0.19
Macroeconomic Factor	1.12	0.37	0.36

* Note: Average Across Simulations.

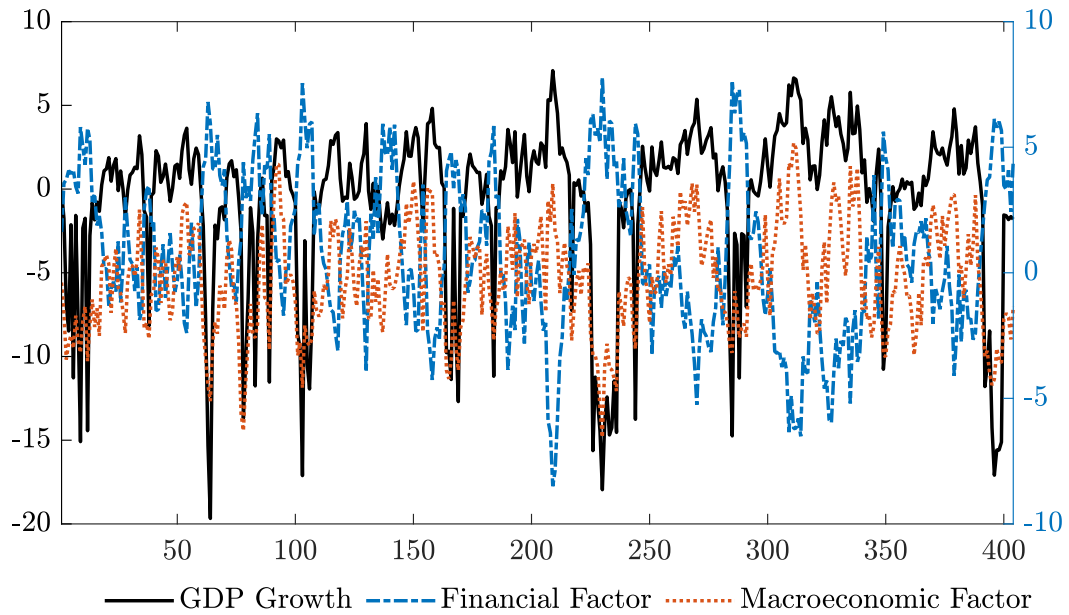
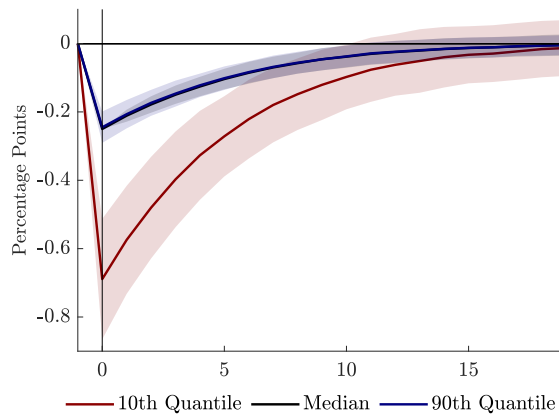


Figure F-1: Simulated Data from Threshold VAR Model.

Note: Example from one simulation. GDP Growth (left axis), Financial & Macroeconomic Factor (right).

F.2 Results For Alternative Specifications of Threshold VAR Model

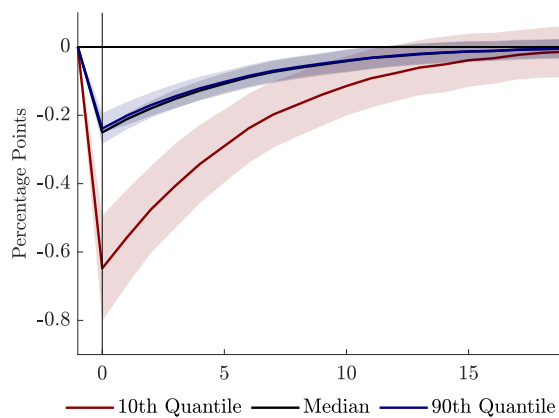
F.2.1 QR-I: State-Dependent Coefficients and Constant Volatility



Note: Straight lines are medians across simulations. Shaded areas are 68% confidence bands.

Figure F-2: Results from Threshold VAR Model (QR-I).

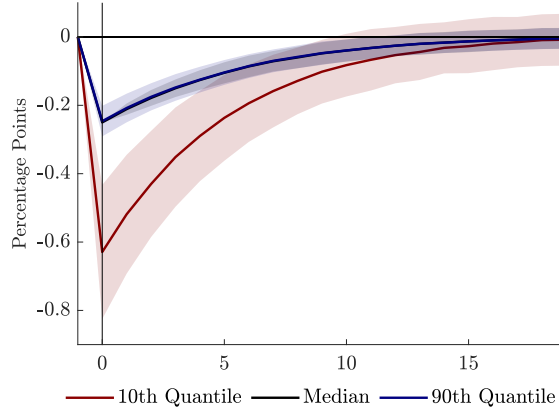
F.2.2 QR-II: State-Dependence on f_t only, But m_t Still Affects f_t



Note: Straight lines are medians across simulations. Shaded areas are 68% confidence bands.

Figure F-3: Results from Threshold VAR Model (QR-II).

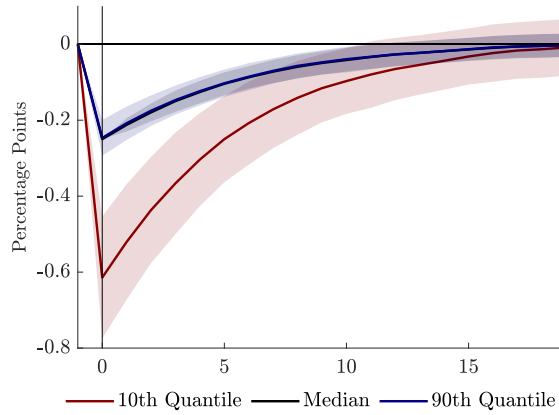
F.2.3 QR-III: State-Dependence on m_t only, But f_t Still Affects m_t



Note: Straight lines are medians across simulations. Shaded areas are 68% confidence bands.

Figure F-4: Results from Threshold VAR Model (QR-III).

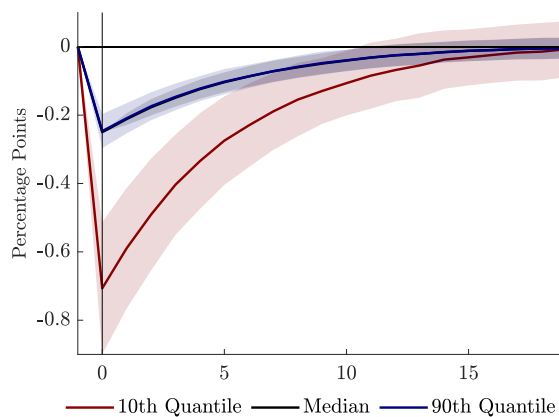
F.2.4 QR-IV State-Dependence on Shock as well as on f_t and/or m_t



Note: Straight lines are medians across simulations. Shaded areas are 68% confidence bands.

Figure F-5: Results from Threshold VAR Model (QR-IV).

F.2.5 QR-V State-Dependence on f_t and/or m_t and Shock Affects GDP Growth Independently



Note: Straight lines are medians across simulations. Shaded areas are 68% confidence bands.

Figure F-6: Results from Threshold VAR Model (QR-V).

G Details on Threshold VAR Model

Parameterization The parameters $\beta_1(f_t, m_t)$, $\beta_2(f_t, m_t)$ and $\sigma_y(f_t, m_t)$ take on the values:

$$\beta_1(f_t, m_t) = \begin{cases} -1.5, & \text{if } f_t > f^* \text{ \& } m_t < m^* \\ -0.5, & \text{normal state} \end{cases}, \quad \beta_2(f_t, m_t) = \begin{cases} 1.5, & \text{if } f_t > f^* \text{ \& } m_t < m^* \\ 0.5, & \text{normal state} \end{cases},$$

$$\sigma_y(f_t, m_t) = \begin{cases} 2, & \text{if } f_t > f^* \text{ \& } m_t < m^* \\ 0.1, & \text{normal state} \end{cases}$$

where f^* and m^* are chosen such that, on average, the constraint binds in 10% of the sample (on average, across simulations), which is about percentage of NBER-dated recessions over the January 1978 to May 2020 sample. Negative values for $\beta_1(f_t, m_t)$ mean that an increase in f_t is associated with a tightening in financial conditions, which depresses growth. Conversely, positive values for $\beta_2(f_t, m_t)$ mean that an increase in m_t is associated with an improvement in macroeconomic activity, which fosters growth. In the symmetric model $\beta_1(f_t, m_t)$, $\beta_2(f_t, m_t)$ and $\sigma_y(f_t, m_t)$ take on the values of the normal state.

We assume $\beta_0 = 1$, $\alpha_1 = \gamma_1 = 0.8$ for equal persistence of f_t and m_t , and $\alpha_2 = \gamma_2 = -0.2$ as the financial and macroeconomic factor are negatively related, $\alpha_3 = 1$ and $\gamma_3 = -1$ as a positive (contractionary) shock increases the financial factor (tightens financial conditions) and decreases the macroeconomic factor (weakens economic activity).

H Details on [Gertler *et al.* \(2019\)](#) Model

Table H-1: Quantile Regression Coefficients - [Gertler *et al.* \(2019\)](#).

Variable	10th Quantile	Median	90th Quantile
Intercept	-0.94	0.91	2.55
Credit Spread	-2.64	-1.32	0.45

* Note: Average Across Simulations.