The Influence of Fiscal and Monetary Policies on the Shape of the Yield Curve

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Abstract

We study how fiscal and monetary policy shape the nominal yield curve and associated term premia. Government spending affects the long end of the curve, while tax changes and monetary policy influence the short end on impact. Within spending categories, only government consumption shifts the short end, but these effects dissipate within a year. While monetary policy and government consumption operate primarily through expected short rates, other fiscal interventions affect yields mainly by altering term premia.

JEL Classification: E50, E62, G10

Key Words: Yield Curve, Fiscal Policy, Monetary Policy, Functional Time Series

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1 Introduction

How do policy actions influence the yield curve? We study this question using a unified and flexible statistical framework, combined with standard identification strategies for fiscal and monetary policy shocks. Understanding these effects matters for several reasons.

First, the yield curve plays a central role in the economy. Many interest rates that shape macroeconomic outcomes—such as mortgage rates—track it closely. Second, recent theoretical work identifies conditions under which fiscal and monetary policy can have similar effects on the broader economy (Correia *et al.*, 2008; Wolf, 2021). We complement this literature by showing that, historically, fiscal policy—especially government spending—has affected the yield curve in ways that differ from monetary policy. Third, we examine how fiscal and monetary decisions influence government borrowing costs. While governments finance a large share of spending through debt issuance, surprisingly little work has studied how fiscal actions shape the yield curve for nominal government liabilities. Existing research focuses primarily on the macroeconomic effects of tax and spending changes (e.g., Romer and Romer, 2010; Mertens and Ravn, 2013; Blanchard and Perotti, 2002; Auerbach and Gorodnichenko, 2012; Ramey, 2011; Ramey and Zubairy, 2018), not on how those decisions affect the government's own financing terms.[1]

Our paper fills this gap by documenting how fiscal and monetary policy move the nominal yield curve and, in turn, affect government borrowing costs. These effects are critical for setting fiscal policy. In equilibrium models of optimal fiscal policy, governments must consider how their decisions—and possibly those of the central bank—shift the yield curve (see, e.g., Lucas and Stokey, 1983; Barro, 1979; Buera and Nicolini, 2004; Angeletos, 2002).[2] We estimate the effects of both types of policy within a single empirical framework, avoiding the comparability issues that arise when monetary and fiscal effects are studied separately (e.g., using results from Piazzesi, 2005; Ireland, 2015).

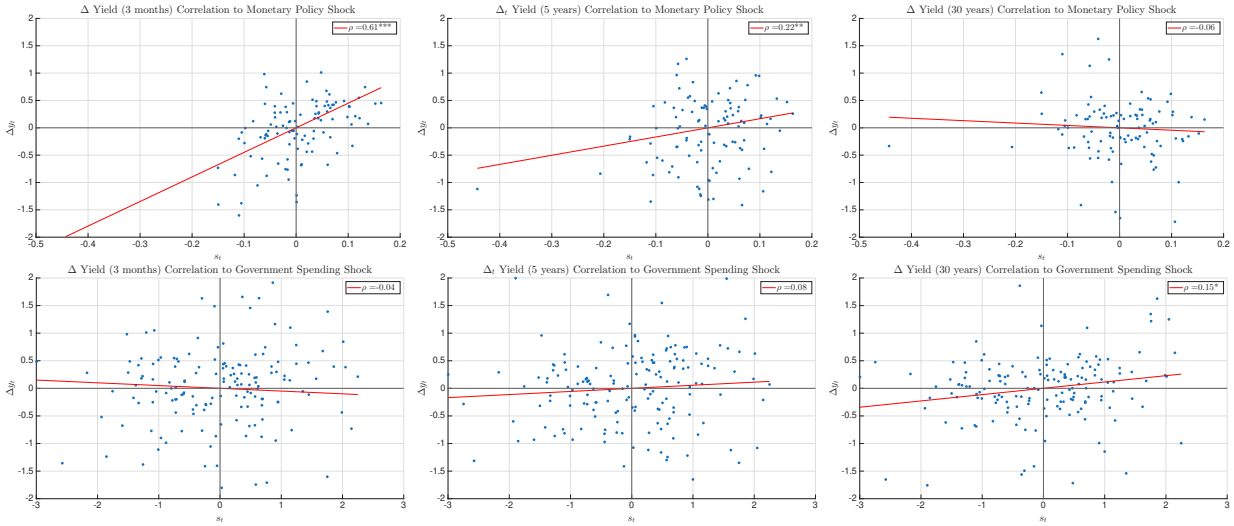
Beyond these theoretical motivations, there are empirical reasons to expect fiscal policy to influence the yield curve. Ang and Piazzesi (2003) and Evans and Marshall (2007), for example, show that macroeconomic factors are key drivers of the nominal yield curve. Moreover, a growing literature finds that fiscal policies can exert powerful effects on economic

[1] We focus on nominal U.S. government debt. The market for inflation-indexed bonds (TIPS) is smaller, less liquid, and covers a shorter sample. Moreover, the nominal yield curve is already a central object of study in economics and finance.

[2] Even without assuming rational expectations, understanding how fiscal actions affect prices remains important for fiscal policy design; see Karantounias (2020).

activity. Whether and how those policies translate into changes in borrowing costs is the central question we address.

Figure 1: Correlation Between Policy Shocks and Changes in the Yield Curve



Note: Scatter-plots showing changes in yields Δy_t and instruments for policy shocks: monetary policy (Aruoba and Drechsel, 2022) and government spending shock (Auerbach and Gorodnichenko, 2012). We report the estimated correlation in the legend of each panel, where the significance levels are denoted by $^{*}: 10\%, ^{**}: 5\%, ^{***}: 1\%$. Information on data sources can be found in Appendix B.

Figure 1 provides a first look at how government decisions move the yield curve. It plots quarterly changes in yields at different maturities against instruments for monetary policy shocks (Aruoba and Drechsel, 2022) and government spending shocks (Auerbach and Gorodnichenko, 2012). Both sets of shocks are significantly correlated with changes in yields, but they affect different segments of the curve: monetary policy moves short maturities, while government spending shifts longer maturities.

These scatterplots are only suggestive. First, we do not directly observe structural shocks but instead rely on noisy instruments, and the figures do not correct for this measurement error. Second, they do not account for how the yield curve moves as a whole. Our empirical framework addresses both issues and extends the analysis to include tax changes and disaggregated components of government spending. Still, the broad pattern—different policy shocks affecting different parts of the yield curve—remains one of our key findings.

To study the yield curve without imposing strong functional form assumptions, we adopt recent tools from the theory of functional time series (Chang *et al.*, 2016). We treat the yield curve at each point in time as a realization of a random function. With minimal assumptions, any such function can be expressed as a sum of basis functions weighted by time-varying coefficients. A key insight from the functional data literature is that a small

number of basis functions often suffice to approximate the curve well. Our analysis focuses on these time-varying weights, which summarize the dynamics of the yield curve. We show that this representation admits a natural state-space formulation.

This framework allows us to use the full yield curve—rather than just a few points or linear combinations of yields—to analyze how it evolves over time. In contrast to standard principal components approaches, which reduce dimensionality by analyzing fixed-weight combinations of yields, our method captures changes in the curve as a function. For further discussion and comparisons of the functional and traditional approaches, see Chang, Durlauf, Lee and Park (2023a) and Bjørnland, Chang and Cross (2023).

Beyond tracking the evolution of the yield curve, our goal is to identify the causal effects of government actions. To do so, we use standard instruments for fiscal and monetary shocks. Specifically, we employ exogenous variation in total government spending, defense spending, government consumption, and investment from Auerbach and Gorodnichenko (2012); tax shocks identified by Romer and Romer (2010); and monetary policy shocks constructed by Aruoba and Drechsel (2022). We estimate how these instruments relate to the dynamics of the yield curve—captured by the weights on the basis functions—which allows us to compute impulse responses of the entire curve to each type of policy shock.

Our paper builds on but is distinct from several related contributions. Berndt *et al.* (2012), for example, examine how defense spending shocks influence government financing decisions, such as net surpluses and portfolio returns. Our focus, instead, is on how different types of fiscal policy shift nominal borrowing costs across the maturity spectrum. Plosser (1987) studies how fiscal forecast errors propagate to yields, while we estimate causal effects using external instruments. Dai and Philippon (2005) analyze the yield-curve impact of fiscal policy in a no-arbitrage framework with Blanchard and Perotti (2002)-style identification. We adopt a more flexible statistical approach that incorporates both fiscal and monetary policy shocks and emphasizes causal identification via instruments, arguably the most common approach in empirical macroeconomics nowadays.

The closest applied paper using a functional approach is Inoue and Rossi (2021), who apply a Nelson-Siegel specification within a VAR to estimate how unconventional monetary policy affects the level, slope, and curvature of the yield curve. Chang *et al.* (2021) uses functional data methods to study how cross-sectional micro distributions respond to macroeconomic shocks and relate to aggregate dynamics.

In the next section, we draw on economic theory to further motivate our study and to provide potential mechanisms for how policy affects the yield curve. Section 3 outlines our

empirical methodology with an emphasis on accessibility for applied researchers. We then present our main results.

2 Two Concepts from Economic Theory

We begin by illustrating one link between fiscal and monetary policy changes and the nominal yield curve via the government budget constraint, and then introduce a yield decomposition that defines a term premium.

Following Berndt *et al.* (2012), we analyze the government budget constraint but focus on the *nominal* version to match our interest in nominal yields. Specifically,

$$B_{t+1} = R^b_{t+1}\Big(B_t - S_t\Big), \tag{1}$$

where B_t is the nominal value of outstanding debt at the start of period t, S_t is the nominal primary surplus, and R^b_{t+1} is the nominal gross return on the government's portfolio between t and $t+1$.[3]

By log-linearizing, we can approximate this constraint (see also Berndt *et al.*, 2012) as

$$ns_t - b_t = E_t\left[\sum_{j=1}^{\infty} \rho^j \left(r^b_{t+j} - \Delta ns_{t+j}\right)\right], \tag{2}$$

where ns_t denotes the weighted log nominal primary surplus ratio (think of it as a measure of nominal surpluses), $b_t = \log B_t$, $r^b_t = \log R^b_t$, and $\rho \in (0,1)$. Intuitively, changes in $ns_t - b_t$ must show up in either future returns on the government portfolio (r^b_{t+j}) or future surpluses.

Berndt *et al.* (2012) focus on how defense spending affects this decomposition. Our emphasis is broader: we study how changes in different components of government spending, tax rates, and monetary policy alter the government's borrowing costs. These costs, captured by r^b_t, are encoded in the nominal yield curve (see Hall and Sargent, 2011). We therefore examine the effects of policy actions across the *entire* yield curve.

We next use the expectations hypothesis to define a time-t term premium $r_t(\tau)$ for a bond of maturity τ. Let $y_t(\tau)$ be the yield of that maturity, and let τ_0 denote a shorter reference

[3] Hall and Sargent (2011) propose methods for computing theory-consistent measures of R^b_{t+1}. See also Hilscher *et al.* (2014), Giannitsarou and Scott (2008), and Chung and Leeper (2007).

maturity. The expectations hypothesis implies

$$y_t(\tau) = \underbrace{\frac{1}{n}\sum_{i=0}^{n-1} E_t\left[y_{t+i}(\tau_0)\right]}_{\text{expectation}} + \underbrace{r_t(\tau)}_{\text{term premium}}, \tag{3}$$

where n is the number of periods between τ_0 and τ. Our modeling framework will produce estimates of the expectation term, and we define the term premium $r_t(\tau)$ as the difference between the observed yield $y_t(\tau)$ and this expectation component.

3 A Hitchhiker's Guide to Functional Time Series Methods

We now provide a high-level explanation of the functional time series methodology used throughout this paper.[4] This approach is particularly effective when large amounts of data describe economic variables that are theoretically linked via a functional relationship, such as nominal yields along the yield curve. By treating these data as realizations of a random function, we can leverage their inherent structure more efficiently than traditional methods.

In Appendix G, we provide evidence that such a functional time series approach delivers competitive and often superior forecasts for nominal yields relative to standard VAR and principal components approaches.

We assume that observations of the nominal yield curve at time t can be represented by a function

$$y_t : I \to \mathbb{R},$$

where I is an interval of maturities (from three months to 30 years in our case). For a security maturing in $t+\tau$, the yield at time t is $y_t(\tau)$, with $\tau \in I$. We treat the entire function $y_t(\tau)$ as a random variable in a functional space, acknowledging that it varies stochastically across time. For brevity, we often write y_t instead of $y_t(\tau)$.

The data for the yield curve we use here (Gürkaynak, Sack and Wright, 2007) allows us to obtain a yield for all values of τ between the aforementioned bounds of three months to thirty years. In Appendix B we discuss their approach to computing yield curves and how it relates to our functional principal components approach. We describe in Appendix D how

[4] See the Appendix or Chang *et al.* (2022, 2023b) for further details.

we, in practice, use a fine grid of values to represent the interval I and their corresponding images (yields) for each quarter t.

So far we have not restricted the yield curve in any way - the function $y_t(\tau)$ can take on arbitrary value for each maturity τ at any point in time t. We next describe the mild restriction we impose on the function $y_t(\tau)$ before turning to a description of a finite-dimensional approximation of this function, which we can then exploit in our empirical analysis.

3.1 Restrictions on the Yield Curve

In order to econometrically exploit the fact that all yields are linked via the yield curve, we will put one mild restriction on the yield curve. We only study yield curves that are in the space $H = \mathcal{L}^2(I)$, the space of square integrable functions.[5] While this space of functions is very general (it includes functions that are not continuous, for example), it still imposes a surprising amount of regularity. In particular, we can now define a scalar product and a norm in H: For f and g in the space H we obtain

$$\langle f, g\rangle = \int_I f(x)g(x)dx \quad \text{and} \quad \|f\| = \sqrt{\langle f, f\rangle}. \tag{4}$$

In addition to the inner product and the norm, we also can define a tensor product[6]

$$(f \otimes g)v = \langle v, g\rangle f \tag{5}$$

for all v in H. In Appendix E we show how to use these constructs (scalar and tensor products) to define the expectation function and the covariance operator of random functions in H.

Using results from functional analysis[7] we find that the space H is a *separable Hilbert space*. These are spaces that admit a scalar product, such as the one defined above, and have *countable* bases. This means that every yield curve in H can be expressed as the linear

[5] The space of all (real) functions f_t defined over I such that $\int_I |f(x)|^2 dx < \infty$.

[6] If $H \equiv \mathbb{R}^n$, we have $f \otimes g = fg'$, *i.e.*, $f \otimes g$ reduces to the outer product, in contrast to the inner product $\langle f, g\rangle = f'g$, where f' and g' are the transposes of f and g. Note that $(f \otimes g)v = (fg')v = (v'g)f$ for all $v \in \mathbb{R}^n$ in this case.

[7] See for example Folland (1999).

combination of countable many functions $\{v_i\}_{i=1,2,3,\ldots}$:[8]

$$y_t = \sum_{i=1}^{\infty} \alpha_{it} v_i. \tag{6}$$

Since the functions $\{v_i\}$ are independent of t, once they are determined, the yield curve y_t is fully characterized by the sequence of real numbers $(\alpha_{1t}, \alpha_{2t}, \ldots)$. In other words, the yield curve can be analyzed through a sequence of real numbers, and every sequence of real numbers can be traced back to a yield curve by combining the basis functions $\{v_1, v_2, \ldots\}$ with the sequence $(\alpha_{1t}, \alpha_{2t}, \ldots)$.

This approach is different from models of the yield curve that start with focusing on the level, slope, and curvature of the yield curve (Diebold and Rudebusch, 2012): We are not imposing a particular set of functions to describe the yield curve - instead, we choose basis functions that jointly describe most of the fluctuations in the yield curve.

3.1.1 A Finite Dimensional Representation of the Yield Curve

The dimension of a space is given by the number of elements in its basis. By this logic, the space H is infinite dimensional as the basis $\{v_i\}_{i=1,2,3,\ldots}$ that we used in (6) has infinitely many elements.

The next step in our approach is to define a finite-dimensional subspace of H. We do this by considering only functions resulting from a linear combination of the first m elements of the basis $\{v_1, v_2, \ldots, v_m\}$, these functions define the finite-dimensional space H_m (a subspace of H).

The function y_t is not an element of H_m given that we need more than just the first m elements of the basis to represent it as we can see in (6). However, we can consider the projection of y_t on H_m given by

$$\tilde{y}_t = \sum_{i=1}^{m} \alpha_{it} v_i. \tag{7}$$

This gives us an equation akin to an observation equation in a state space model

$$y_t = \sum_{i=1}^{m} \alpha_{it} v_i + w_t, \tag{8}$$

[8] Note that we have omitted the argument τ of the function, but v_i (and y_t) still refers to a *function*.

where $w_t = y_t - \tilde{y}_t$ is the approximation error we make by restricting ourselves to H_m. Under suitable conditions, this approximation error becomes asymptotically negligible. In what follows, we assume that $\{v_i\}$ is an orthonormal basis, i.e. $\|v_i\| = 1$ for all i and $\langle v_i, v_j \rangle = 0$ for all $i \neq j$. Under this assumption, we have

$$\alpha_{it} = \langle v_i, y_t \rangle$$

for all i and t.

Let us now introduce a mapping from H_m to $\mathbb{R}^m$

$$H_m \ni \tilde{y}_t \mapsto \alpha_t = \begin{pmatrix} \alpha_{1t} \\ \alpha_{2t} \\ \vdots \\ \alpha_{mt} \end{pmatrix} \in \mathbb{R}^m.$$

This mapping is one-to-one correspondence between H_m and $\mathbb{R}^m$. Therefore, with the basis $\{v_1, v_2, \ldots, v_m\}$ and $\alpha_t = (\alpha_{1t}, \alpha_{2t}, \ldots, \alpha_{mt})'$ we can recover $\tilde{y}_t$ through (7). The mapping is an isometry between H_m and $\mathbb{R}^m$, which preserves the norm.[9] As a result, we can study a vector autoregression (VAR) for α_t by least squares, rather than having to work directly in a functional space.

Using functional principal components, whose properties we discuss in Appendix C, we determine a basis of functions $\{v_i\}_{i=1,2,3,\ldots}$ such that its first m elements generate an approximation of y_t. Note that we can thus effectively choose a very efficient set of basis functions for our purposes rather than restrict ourselves to an a priori chosen basis function such as monomials $\{1, \tau, \tau^2, \ldots\}$. Comparing our approach to a standard principal components approach, it might be useful to note that the basis function here serve the same role as the weights on the principal components in standard finite dimensional principal component analysis. Intuitively, the functional approach, because it uses infinite dimensional data, replaces the sums present in standard finite dimensional principal component analysis with integrals, but the underlying ideas remain the same.

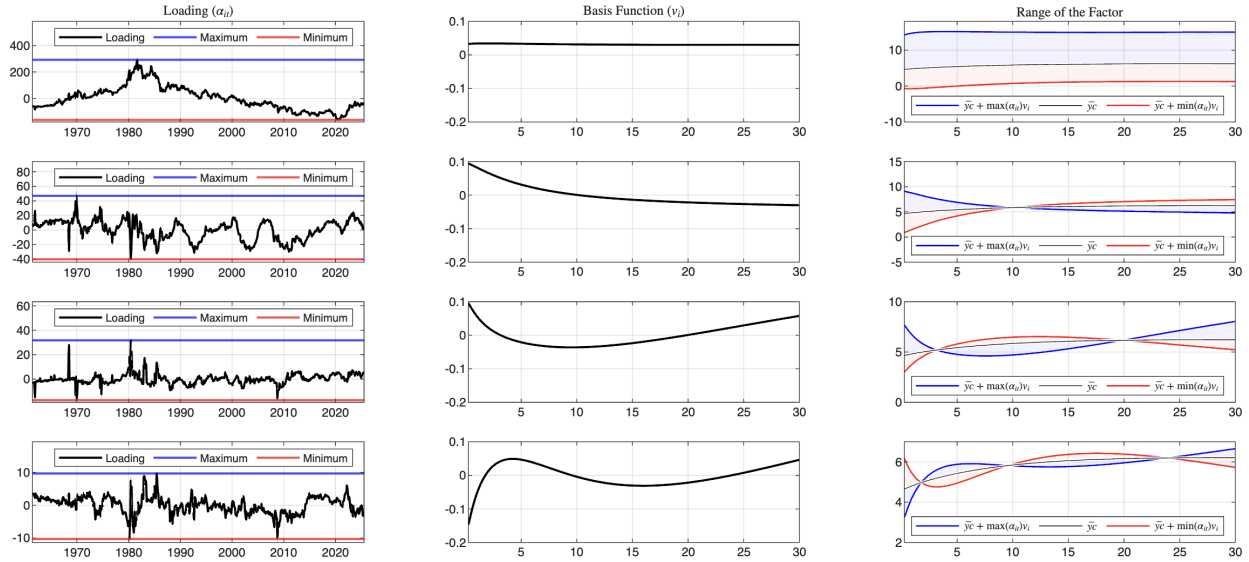
[9] We can show that

$$\|\tilde{y}_t\|^2 = \sum_{i=1}^{m} \langle v_i, y_t \rangle^2 = \sum_{i=1}^{m} \alpha_{it}^2 = \|\alpha_t\|^2,$$

where we use the same notation $\|\cdot\|$ to denote the norm of a function in H_m and the norm of a vector in $\mathbb{R}^m$.

This principal components analysis also delivers a time series for the vector of weights $\alpha_t = \begin{pmatrix} \alpha_{1t} & \alpha_{2t} & \dots & \alpha_{mt} \end{pmatrix}'$. Figure 2 displays the α_{it} values (left column), the v_i's (center

Figure 2: Description of the Functional Principal Components



Note: The first column shows the time series of weights (α_{it}) for each component (one in each row). The shape of the component is described in the second column. The last column shows the range of effects that each component has on the yield curve using the sample mean yield curve (black line) as a benchmark. The blue (red) lines in the top/middle/bottom panel signify the yield curves obtained with positive (negative) realizations of the weights (left panel) and the associated basis function (center panel).

column), and the range of yield curves generated by time series fluctuations in the α vector (right column). This range is derived from the yield curve data described in Section 4. Each plot (from top to bottom) shows how variations in a single element of α_{it} influence the sample mean yield curve (in black). Note how the first three basis functions resemble the level, slope, and curvature factors common in the the yield curve literature (Diebold and Rudebusch, 2012). However, we did not impose these shapes ex ante. The vector α_t is not directly interpretable as yields; only in conjunction with the basis functions $\{v_1, v_2, \dots, v_m\}$ does it reconstruct the yield curve. Nonetheless, α_t serves as the *state* in our state-space representation of the yield curve.[10]

A crucial feature of this framework is that for each maturity τ, the yield $y_t(\tau)$ is a *linear* function of α_t. This linearity simplifies the construction of impulse responses, given the linear

[10]Here, the analogy to state-space models is slightly loose because we first estimate α_t via principal components and then specify its law of motion. In contrast, standard state-space applications often use a filtering algorithm (e.g., the Kalman Filter) that leverages the assumed law of motion to estimate the states. Our approach instead resembles the two-step estimation in linear factor models (e.g., Stock and Watson, 2016), yet the resulting yield-curve model is still in state-space form.

dynamics we impose on α_t (discussed in the next section).

3.2 The Dynamics of α_t and the Identification of Impulse Responses

We specify a VAR law of motion for α_t and the policy-shock instrument z_t.[11] In particular, we use a VAR(p) model and later discuss how we select p jointly with the number of factors m:

$$\gamma_t = c + \sum_{l=1}^{p} A_l \gamma_{t-l} + u_t, \tag{9}$$

where $\gamma_t \equiv [\, z_t \;\; \alpha_t' \,]'$. This equation can be seen as the state equation for a state-space model for the yield curve at each time t.

We identify the shock of interest by assuming a linear relationship between the forecast error u_t and the vector of structural shocks of interest e_t as

$$u_t = \Omega e_t, \tag{10}$$

and assume that Ω is computed via the lower triangular Cholesky decomposition of the covariance matrix of u_t so that $E(e_t e_t') = I$,[12] as proposed by Plagborg-Møller and Wolf (2021). The policy shock of interest is related to the first element of e_t, as we discuss below. This approach has a number of advantages, even beyond its simplicity. First, it automatically corrects for possible autocorrelation of the instrument and dependence of the instrument on past yield curve movements (which are generally thought to encode macroeconomic outcomes). To see this, it is useful to write the first equation of the set of Equations (9), using Equation (10):

$$z_t = \sum_{l=1}^{p} A_l^{1,1} z_{t-l} + \sum_{l=1}^{p} \sum_{j=1}^{m} A_l^{1,j+1} \alpha_{j,t-l} + \Omega_{1,1} e_t^1, \tag{11}$$

where $A_l^{i,j}$ is the element of the matrix A_l in row i and column j. Following Plagborg-Møller and Wolf (2021), it is worthwhile to point out that this identification approach will correctly identify normalized impulse responses even if the yield curve itself does not contain enough information to identify the shock of interest ε_t (i.e., non-invertibility) and if there is

[11] We estimate a separate VAR for each instrument because the sample sizes differ across instruments.
[12] I denotes the identity matrix.

measurement error w_t present in e_t^1, so that $e_t^1 = \theta\varepsilon_t + w_t$, where $\theta \neq 0$ is a parameter that influences the strength of identification and w_t is an i.i.d. measurement error. This comes at a cost, as we can only identify normalized impulse responses if there is non-invertibility. Throughout this paper, we plot impulse responses that increase the first element of e_t by one unit (which is equal to a one standard deviation change in the first element of the one-step ahead forecast error u_t). This has the advantage of giving us some sense of magnitude of the effects if the shock is indeed invertible.

For statistical inference, we use a bootstrap procedure that is detailed in Appendix F. [13]

4 Yield Curve Data and Instruments

Our nominal yield-curve data come from Gürkaynak, Sack and Wright (2007), available on the Board of Governors' website.[14] The sample begins in June 1961. We use quarterly data, taking the yield curve observed on the last day of each quarter so that any shocks within the quarter can affect that quarter's yield curve. For each policy shock, we use the longest intersection between the yield-curve sample and the relevant shock series. We opt for quarterly data because many of our fiscal shocks are available only at that frequency.

As instruments for government spending shocks, we rely on the VAR-identified shocks of Auerbach and Gorodnichenko (2012).[15] Using VAR-identified shocks as instruments is common in applied work (e.g., Känzig, 2021). For tax shocks, we use the exogenous series documented by Romer and Romer (2010). We take our monetary policy shocks from Aruoba and Drechsel (2022), who employ machine learning and natural language processing to augment FOMC staff forecasts with information from internal FOMC documents. The deviation of actual FOMC decisions from these enriched forecasts serves as the monetary policy shock measure. A key advantage of this series is its longer sample coverage compared to popular high-frequency instruments based on interest-rate futures (Kuttner, 2001; Gertler and Karadi, 2015). In Appendix A.1, we show that our findings are robust to using the shock measure in Miranda-Agrippino and Ricco (2021) instead.

[13] This bootstrap procedure is valid, as shown by Chang, Park and Pyun (2023b).

[14] https://www.federalreserve.gov/data/nominal-yield-curve.htm

[15] We deviate slightly from Auerbach and Gorodnichenko (2012) by controlling for forecasts of overall government spending in *all* VAR specifications to ensure our identified shocks are truly unforecastable. The results are very similar if we follow Auerbach and Gorodnichenko (2012) exactly.

5 Response of the Yield Curve to Policy Shocks

We begin by plotting impulse responses for the entire yield curve, illustrating how yields at different maturities change h periods after a shock. Here, h is the *horizon* of the response, distinct from the *maturity* τ on the x-axis (in years). To estimate our model, we must choose the number of factors m and the lag length p. We select these jointly by minimizing average one-step-ahead forecast errors for yields with maturities between three months and 30 years. As a benchmark, we pick m and p from a VAR containing only α_t, allowing us to use one specification for all results. This procedure yields $m = 4$ and $p = 1$. These four factors explain 99.87% of the yield-curve variance. In Appendix A.3, we confirm that our findings are robust to choosing m and p separately for each shock, allowing for differences in observables and sample periods.

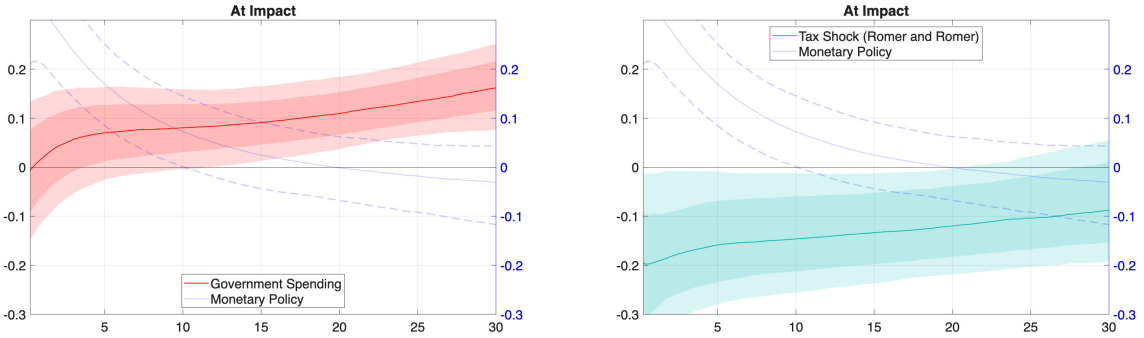
We first examine the impact response ($h = 0$) of various fiscal and monetary policy shocks. Our yield data represent quarter-end yields, so these impact responses capture changes within the same quarter as the shock. In all figures, the yield curve's response to monetary policy appears in light blue for ease of comparison.

Figure 3 (left panel) shows the response to a total government spending shock, identified using the Auerbach and Gorodnichenko (2012) instrument. In contrast to the short-end response triggered by monetary policy —significant out to maturities of about 10 years— government spending does not significantly affect yields under 12 years, after which the response becomes significant at longer maturities. The right panel shows the response to a tax shock, identified via the exogenous tax changes in Romer and Romer (2010). Interestingly, its impact response resembles the monetary policy response but in the opposite direction, and remains significant up to maturities of 15 years, suggesting a more persistent yield-curve impact than monetary policy.

We next investigate whether different components of government spending have the same effect on the yield curve. This question is motivated by Boehm (2020), who shows that the fiscal multiplier can vary significantly between government investment and consumption. To measure shocks to defense spending, government consumption, and government investment, we use instruments constructed as in Auerbach and Gorodnichenko (2012).

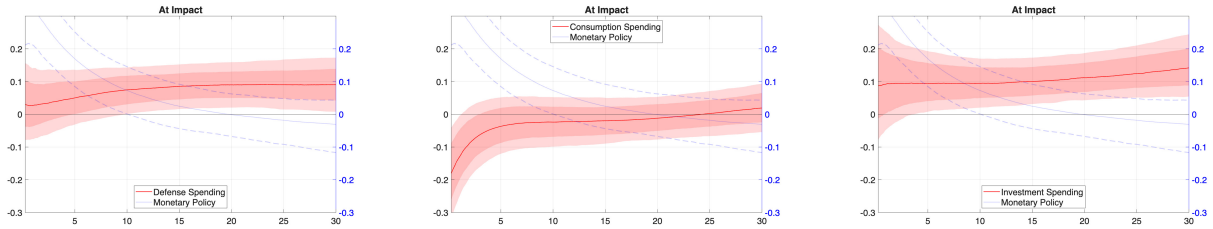
Figure 4 (left panel) shows that defense spending shocks generate a response similar to

Figure 3: Response at Impact of the Yield Curve to a Government Spending Shock (Auerbach and Gorodnichenko, 2012) and Exogenous Tax Change (Romer and Romer, 2010)



Note: For reference the response to monetary policy (Aruoba and Drechsel, 2022) and its 90% confidence band is represented in blue. The lighter (darker) shade signifies 90 % (68%) confidence bands. 90% and 68% confidence bands estimated using bootstrap methods (see Appendix F).

Figure 4: Response at Impact of the Yield Curve to a Government Defense Spending Shock (left), Government Consumption Spending Shock (center), and, Government Investment Spending Shock (right) (Auerbach and Gorodnichenko, 2012)



Note: For reference the response to monetary policy (Aruoba and Drechsel, 2022) and its 90% confidence band is represented in blue. The lighter (darker) shade signifies 90 % (68%) confidence bands. 90% and 68% confidence bands estimated using bootstrap methods.

overall government spending, significantly affecting only the long end of the yield curve.[16] The same pattern emerges for government investment, as shown in the right panel. Government consumption, however, moves only the very short end of the yield curve, and its effect is even more concentrated at short maturities than that of monetary policy.

We now consider responses for up to five years after the shock. To visualize these, we select certain maturities and plot their impulse responses over time. Figures 5–7 report these results for overall government spending, tax shocks, and monetary policy shocks, respectively.

For government spending shocks, the immediate effect grows larger at longer maturities, aligning with our previous analysis. After the shock, responses at all maturities quickly return to zero, except for the 30-year maturity, which displays slightly more persistence. Beyond that, the overall response shapes remain broadly similar across horizons. Tax shocks

[16] One might wonder why we do not use the defense news shock of Ramey (2011). Our sample starts after the Korean War, and as Ramey (2016) notes, that instrument has limited relevance in any sample beginning after the Korean War.

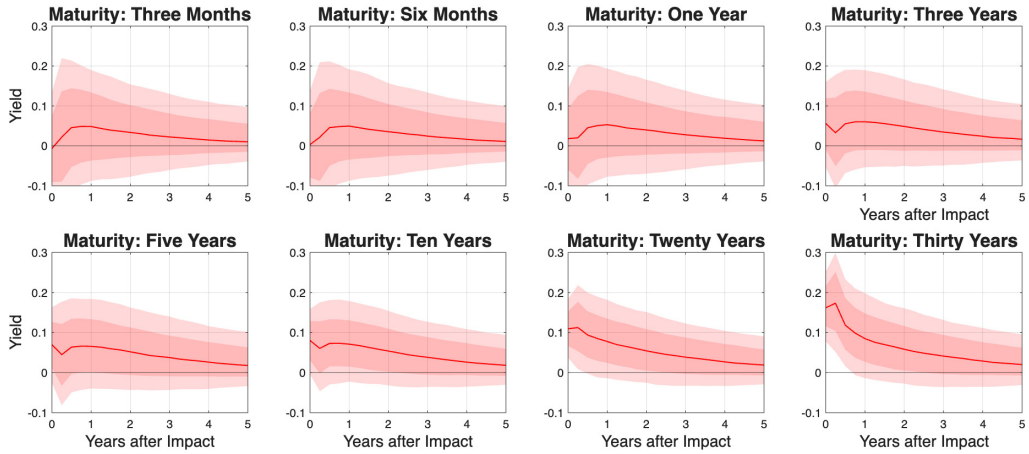


Figure 5: Response of yields of specific maturities (given in the title of each subplot) to a government spending shock. The x-axis denotes periods after the shock occurred h. The lighter (darker) shade signifies 90 % (68%) confidence bands. 90% and 68% confidence bands estimated using bootstrap methods.

cause similar changes at all maturities, and these also revert to zero soon after the initial negative impact on yields. In contrast, monetary policy shocks reveal notable differences across maturities. At 20 years and longer, we observe a brief decline, possibly reflecting how higher current short-term rates can dampen inflation expectations, allowing the central bank to reduce short-term nominal rates in the future.

Finally, we decompose the impulse responses into average expected short rates up to each maturity and the term premium. Recall the decomposition in Equation (3), and note how the yield for a specific maturity τ depends on α_t:

$$y_t(\tau) = \sum_{i=1}^{m} \alpha_{it}\, v_i(\tau) + w_t(\tau),$$

where we now highlight τ. The expected future yield at horizon h is

$$E_t\Big[y_{t+h}(\tau)\Big] = \sum_{i=1}^{m} E_t[\alpha_{i,t+h}]\, v_i(\tau),$$

and we can compute $E_t[\alpha_{i,t+h}]$ using the VAR in Equation (9). This lets us obtain impulse responses for the expectations component of $y_t(\tau)$, and from those we derive the term premium's response as the difference between the total yield response and the response of the expectations component.

Figure 8 reports the decomposition at impact ($h = 0$) for all maturities from three months

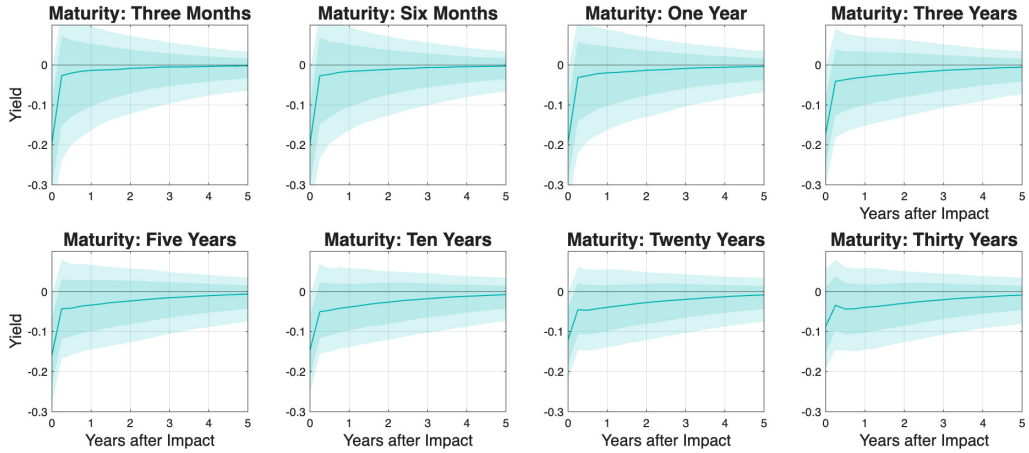


Figure 6: Response of yields of specific maturities (given in the title of each subplot) to a tax shock. The x-axis denotes periods after the shock occurred h. The lighter (darker) shade signifies 90 % (68%) confidence bands. 90% and 68% confidence bands estimated using bootstrap methods.

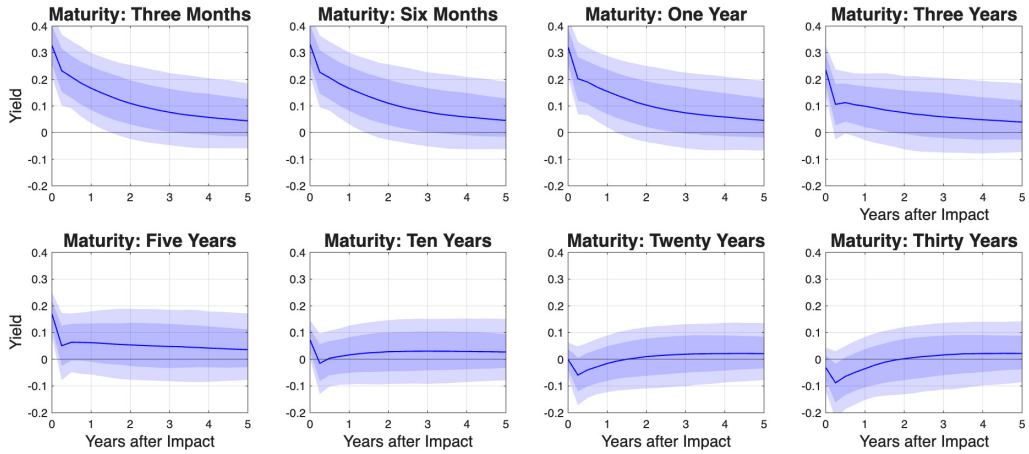


Figure 7: Response of yields of specific maturities (given in the title of each subplot) to a monetary policy shock. The x-axis denotes periods after the shock occurred h. The lighter (darker) shade signifies 90 % (68%) confidence bands. 90% and 68% confidence bands estimated using bootstrap methods.

to 30 years, comparing government spending, tax, and monetary policy shocks. Government spending affects average expected short rates only for long maturities, but it also moves the term premium significantly at the very low and very high ends of the yield curve. Tax shocks influence the term premium only for short maturities. Monetary policy responses are driven mainly by shifts in expected current and future short rates, except at the longest maturities. Comparing the left and right panels of Figure 8 reveals that fiscal policy primarily affects yields through term premia, while monetary policy, unsurprisingly, operates via changes in expected short rates. This result suggests that monetary policymakers, insofar as they adjust short-term rates, do not substantially respond to these fiscal shocks over the time horizons we consider.

Finally, Figure 9 shows the same decomposition for various components of government spending. Interestingly, government consumption shocks again stand out as inducing a similar response qualitatively to monetary policy shocks - both move short rate expectations of short horizons. Investment and defense spending shocks show patterns that are qualitatively similar to those of overall government spending shocks.

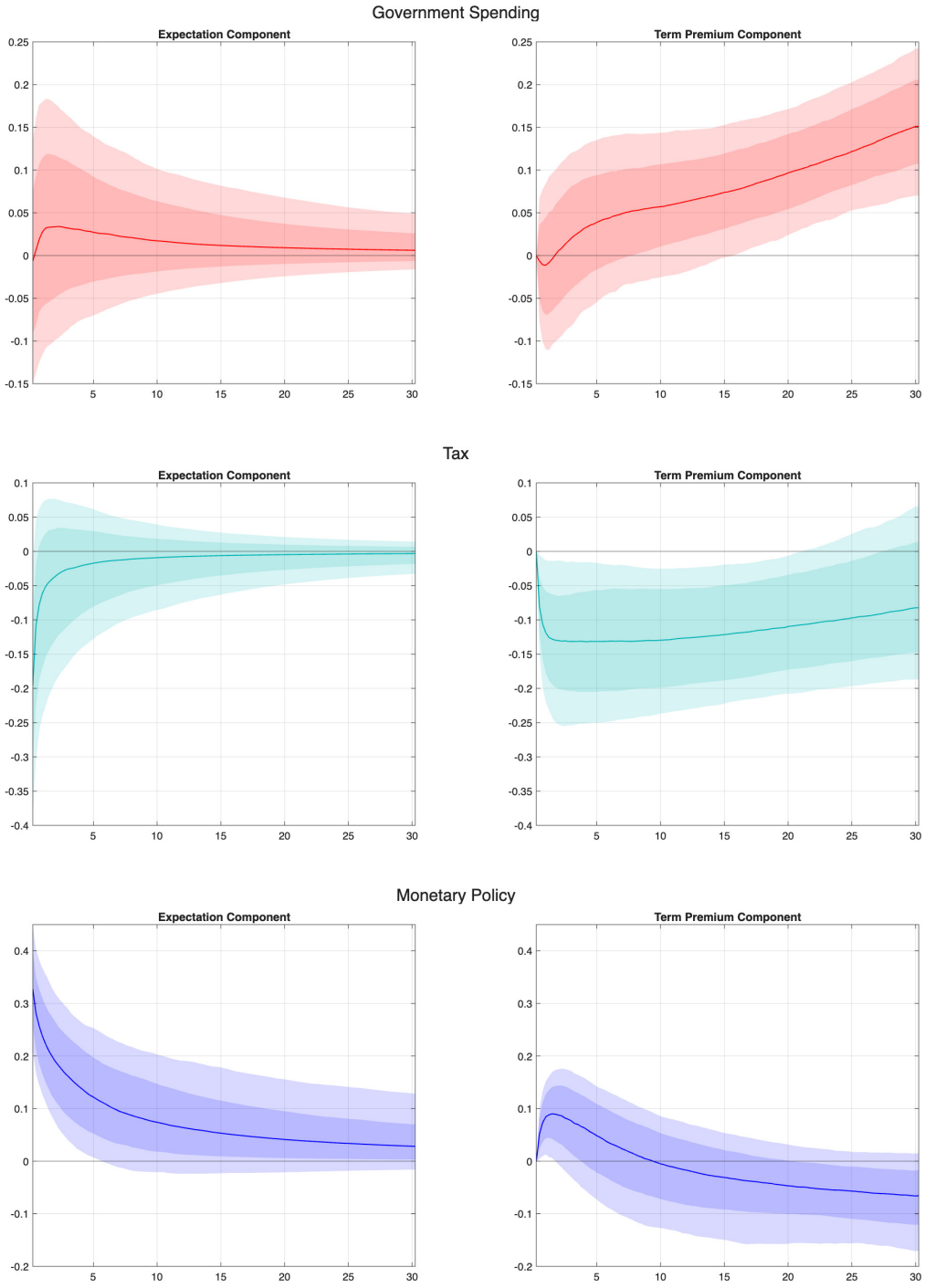


Figure 8: Decomposition of the responses on impact for the average short-term rate expectation (left) and the term-premium (right) for total government spending (top), the tax shock (middle), and the monetary policy shock (bottom).

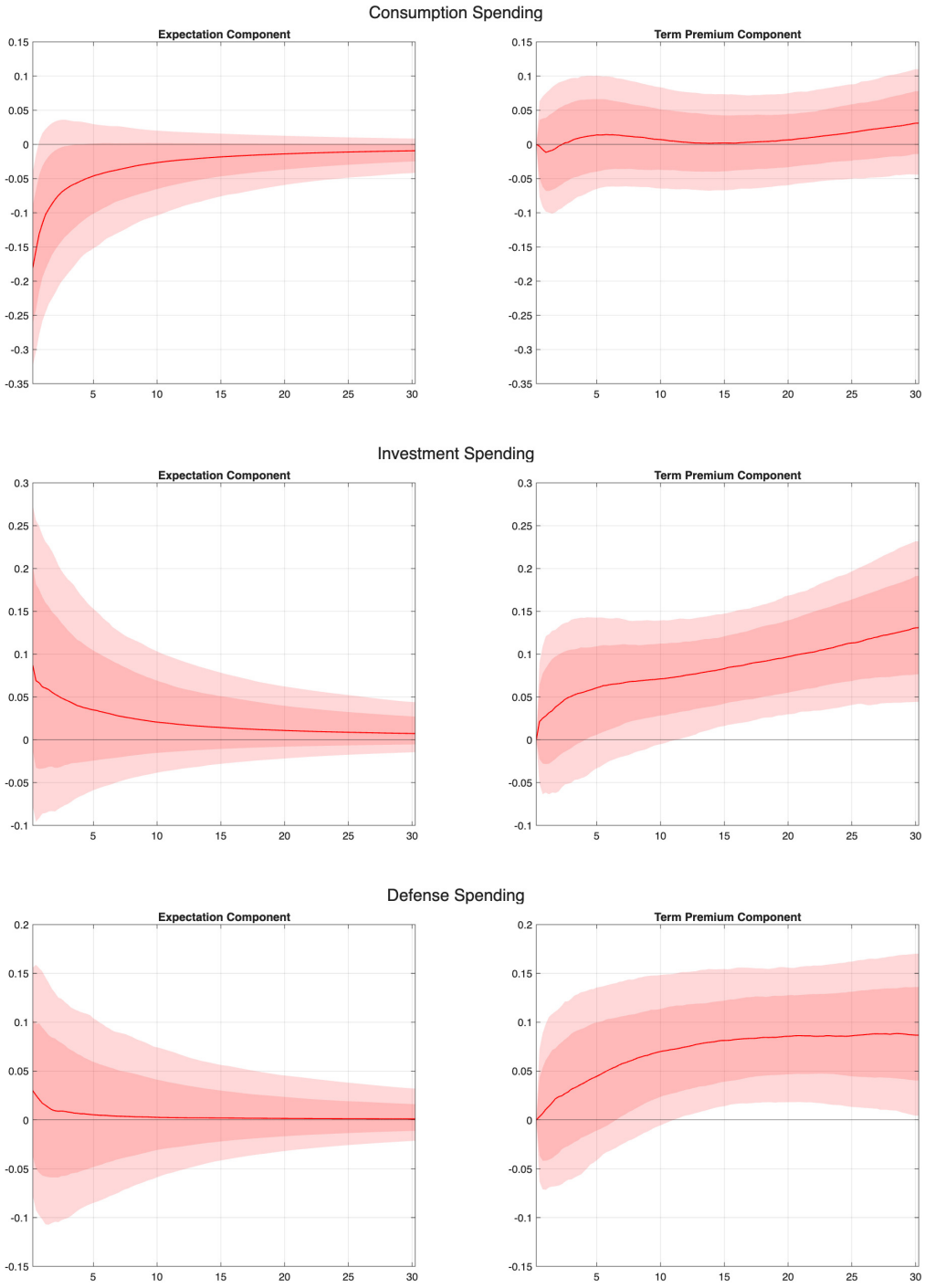


Figure 9: Decomposition of the responses on impact for the average short-term rate expectation (left) and the term-premium (right) for total government spending consumption (top),investment (middle) and defense (bottom).

6 Conclusion

We study the effects of monetary and fiscal policies on the yield curve and find that they have qualitatively very different consequences for the yield curve. These findings are useful for both policymakers, who often view the yield curve as a major aspect of policy transmission, in addition to directly encoding a government's borrowing costs. Furthermore, our results can be useful as calibration targets for macroeconomists who want to develop quantitative equilibrium models that take the yield curve seriously.

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Online Appendix for "The Influence of Fiscal and Monetary Policies on the Shape of the Yield Curve"

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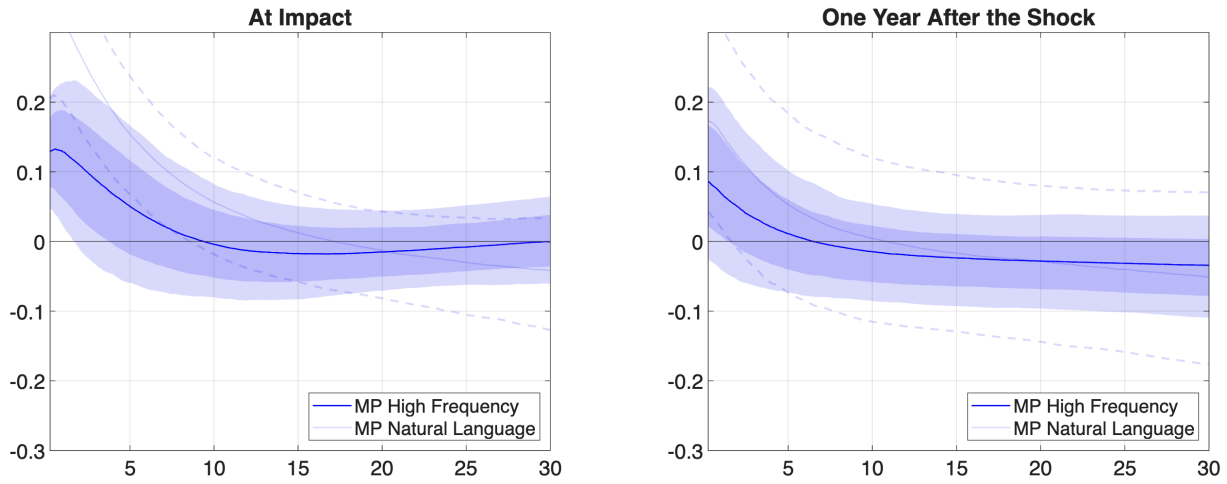
A Additional Figures

A.1 Response of the Yield Curve to Monetary Policy Shocks Identified Using High Frequency Variation

When comparing the responses of the yield curve to monetary policy shocks, two different studies provide valuable insights. Aruoba and Drechsel (2022) and Miranda-Agrippino and Ricco (2021) utilize different shocks to analyze the transmission of monetary policy to the yield curve. The shock identified by Aruoba and Drechsel (2022) focuses on a natural language analysis, while the shock from Miranda-Agrippino and Ricco (2021) represents a different perspective, based on high frequency estimation.

Figure A-1: Response at Impact of the Yield Curve to a Monetary Policy Shock (Miranda-Agrippino and Ricco, 2021)

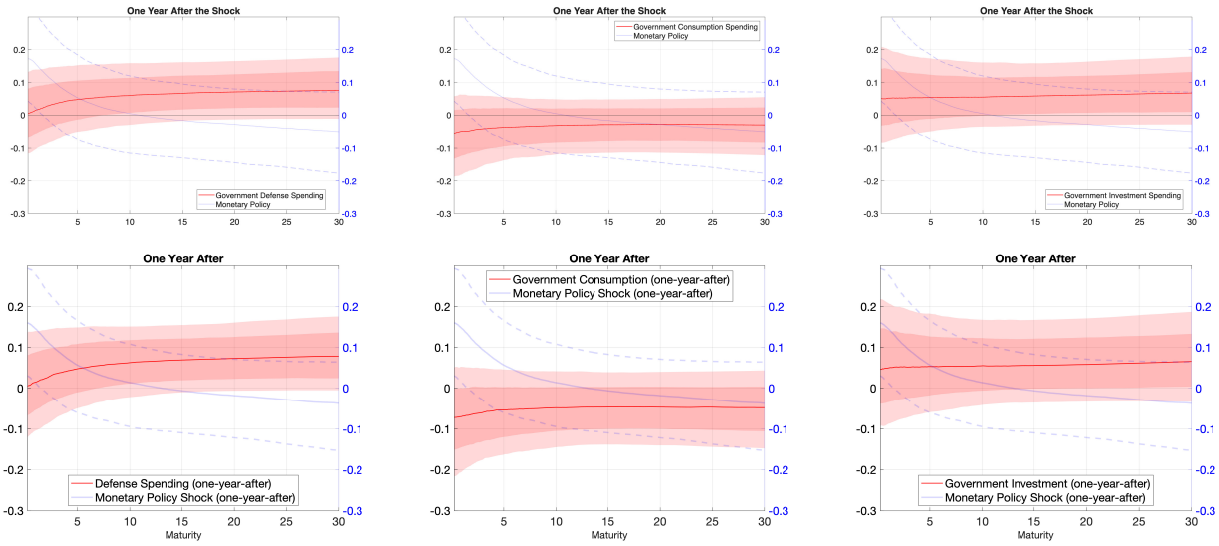


Note: For reference the response to monetary policy (Aruoba and Drechsel, 2022) and its 90% confidence band is represented in light blue. The lighter (darker) blue shade signifies the 90% (68%) confidence band for the response to high frequency monetary policy shock (Miranda-Agrippino and Ricco, 2021). The confidence bands are estimated using bootstrap methods.

A.2 Additional Results

The following figure shows the response of the yield curve to shocks to components of government spending one year after the shock. These plots confirm that the effects of fiscal policy become very weak or even negligible after just one year.

Figure A-2: Response after one year of the yield curve to a Government Defense Spending Shock (left), Government Consumption Spending Shock (center), and, Government Investment Spending Shock (right) (Auerbach and Gorodnichenko, 2012)



Note: For reference the response to monetary policy (Aruoba and Drechsel, 2022) and its 90% confidence band is represented in light blue. The lighter (darker) blue shade signifies the 90% (68%) confidence band for the response to different government spending shocks(Auerbach and Gorodnichenko, 2012). The confidence bands are estimated using bootstrap methods.

A.3 Comparison of different choices of m and p

In this section, we compare impulse responses (on impact) when choosing m and p optimally for each shock separately rather than using the benchmark values $m = 4$ and $p = 1$ that were obtained using the entire sample of the yield curve data without using a specific shock measure in the VAR.

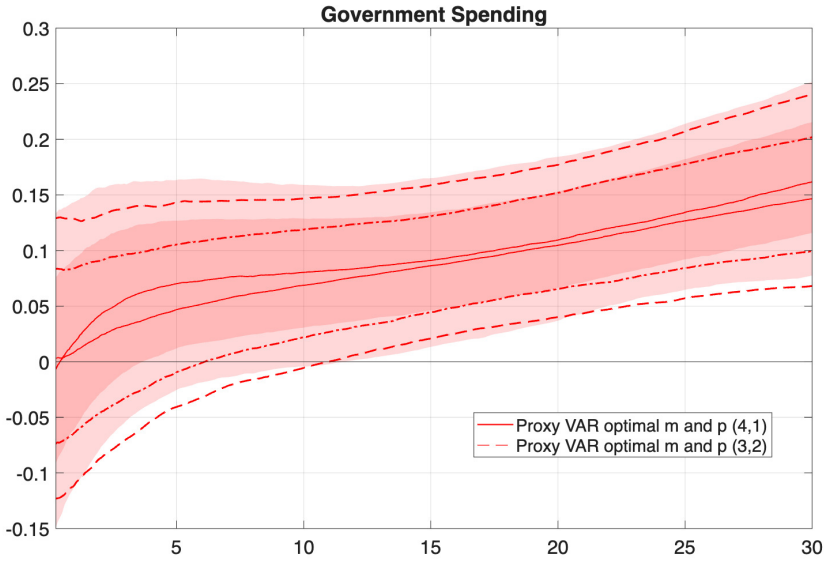


Figure A-3: Response (at Impact) of the Yield Curve to a Government Spending Shock. Comparison different general and specific values of m and p.

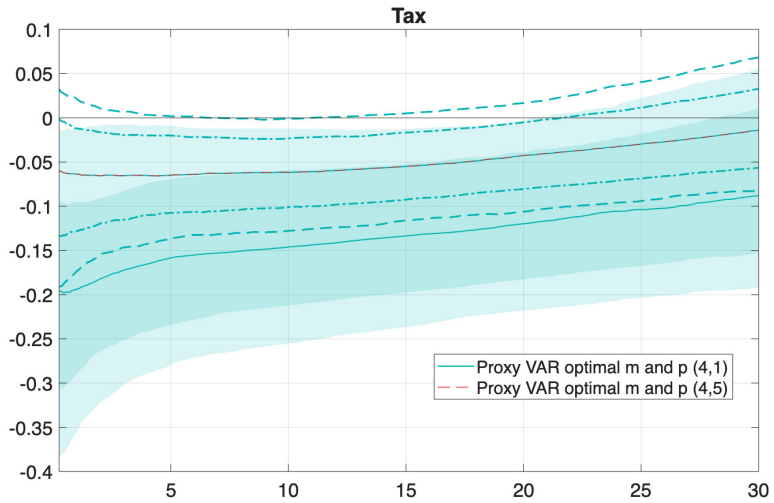


Figure A-4: Response (at Impact) of the Yield Curve to a Tax Change Shock. Comparison different general and specific values of m and p.

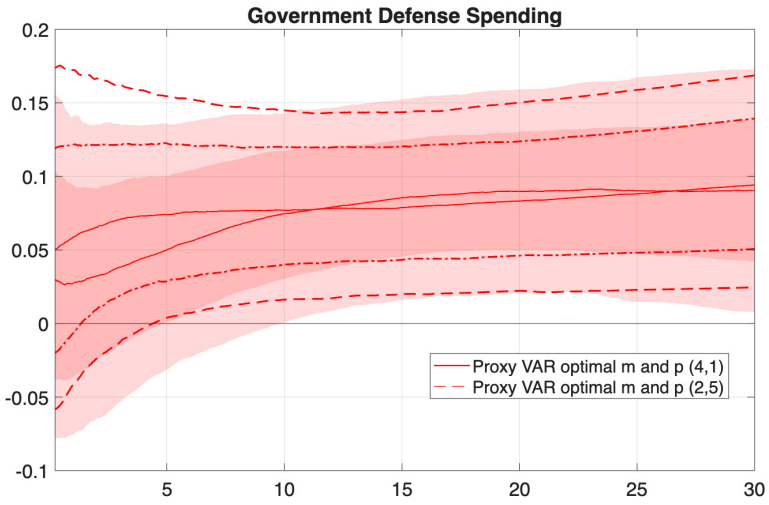


Figure A-5: Response (at Impact) of the Yield Curve to a Government Defense Spending Shock. Comparison different general and specific values of m and p.

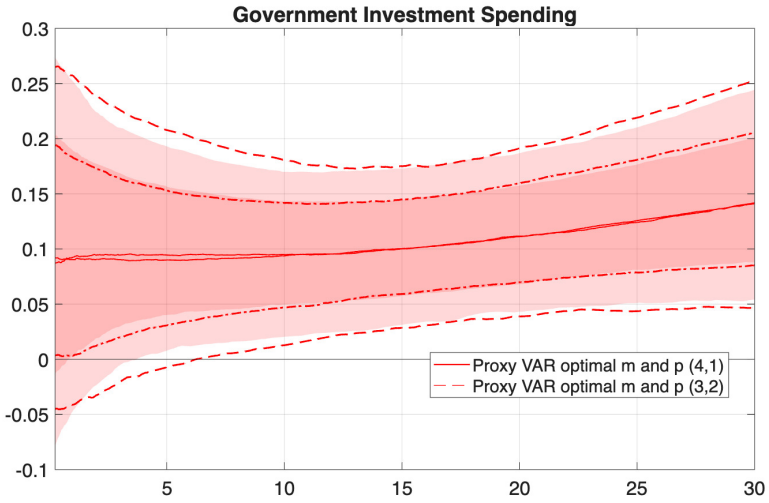


Figure A-6: Response (at Impact) of the Yield Curve to a Government Investment Spending Shock. Comparison different general and specific values of m and p.

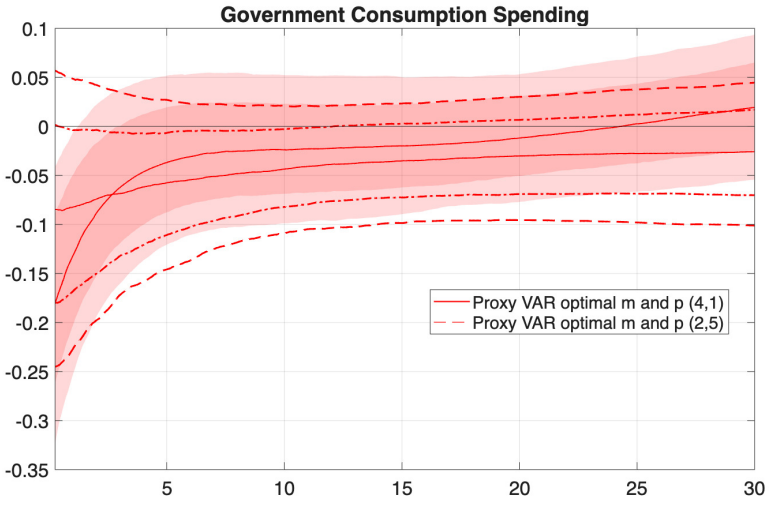


Figure A-7: Response (at Impact) of the Yield Curve to a Government Consumption Shock. Comparison different general and specific values of m and p.

B Data

In this section, we provide a comprehensive overview of the various data sources utilized in this paper.

B.1 Yield Curve Data

The yield curve data we use as a starting point are taken from the Federal Reserve Board[1], based on the model by Gürkaynak, Sack and Wright (2007). From this source, we obtain daily estimates of six parameters: $\beta_{0t}, \beta_{1t}, \beta_{2t}, \beta_{3t}, \tau_{1t}$, and τ_{2t}. These parameters are used to construct the following yield curve function:

$$y_t(M) = \beta_{0t} + \beta_{1t}\left(\frac{1-e^{-\frac{M}{\tau_{1t}}}}{\frac{M}{\tau_{1t}}}\right) + \beta_{2t}\left(\frac{1-e^{-\frac{M}{\tau_{1t}}}}{\frac{M}{\tau_{1t}}} - e^{-\frac{M}{\tau_{1t}}}\right) + \beta_{3t}\left(\frac{1-e^{-\frac{M}{\tau_{2t}}}}{\frac{M}{\tau_{2t}}} - e^{-\frac{M}{\tau_{2t}}}\right)$$

Conditional on τ_1 and τ_2, these specifications represent a linear combination of three basis functions. However, since these parameters vary over time—unlike our approach in the main text—the basis functions are not time-invariant. This means that our approach still involves a substantial dimensionality reduction relative to the original data. The daily sample of the yield curve starts on June 14, 1961, and is updated weekly.

B.2 External Shocks

Throughout the paper, we use several external shocks borrowed from the literature. We consider three main groups of shocks: monetary policy, government spending, and tax changes. In the following sections, we describe the data sources used to construct each of these external shocks.

B.2.1 Monetary Policy

The primary references for measuring changes in monetary policy are Aruoba and Drechsel (2022) and Miranda-Agrippino and Ricco (2021).

Aruoba and Drechsel (2022) This paper applies natural language processing techniques to analyze documents prepared by economists for FOMC meetings, aiming to capture the

[1] https://www.federalreserve.gov/data/nominal-yield-curve.htm

information available to the committee at the time of policy decisions. Using machine learning methods, the authors predict changes in the target interest rate based on this information and identify monetary policy shocks as the residuals. Their identified shock, covering the period from 1982Q3 to 2008Q4, is available at: http://econweb.umd.edu/ drechsel/files/Aruoba_Drechsel_Data.xlsx.

Miranda-Agrippino and Ricco (2021) This study constructs a high-frequency instrument for monetary policy shocks that accounts for informational rigidities. The series of shocks is available at: http://silviamirandaagrippino.com/s/Instruments_web-x8wr.xlsx. The sample period for this shock spans from 1991Q1 to 2009Q4.

B.2.2 Government Spending

Auerbach and Gorodnichenko (2012) We use the linear (non-regime-switching) version of the VAR model described in Auerbach and Gorodnichenko (2012) to obtain government spending shocks. The identification strategy (variable ordering), decomposition of the government spending variable, and treatment of predictable components of fiscal shocks remain consistent with the original study.

The variables included in the model are as follows: government spending, representing the log of real government (federal, state, and local) purchases (consumption and investment); government revenue, representing the log of real government receipts from direct and indirect taxes net of transfers to businesses and individuals; and output, representing the log of real gross domestic product (GDP) in chained 2000 dollars.

To account for anticipated shocks, we incorporate quarterly forecasts of fiscal and aggregate variables (government purchases, output, and taxes) from the University of Michigan's Research Seminar in Quantitative Economics (RSQE) macroeconometric model, the Survey of Professional Forecasters (SPF), and the forecasts prepared by the Federal Reserve Board (FRB) staff for FOMC meetings. These forecasts are included in the SVAR to eliminate the effects of "innovations" in fiscal variables that were predicted by professional forecasters.

The resulting shocks are obtained using the inverse of the Cholesky decomposition of the estimated covariance matrix and the estimated residuals. The sample period for these shocks spans from 1967Q3 to 2008Q4. Replication files are available at: http://doi.org/10.3886/E114783V1.

B.2.3 Tax Changes

We estimate the response of the yield curve to changes in tax legislation as described in Romer and Romer (2010).

Tax Changes The authors use narrative records, such as presidential speeches and congressional reports, to identify the size, timing, and principal motivation for all major postwar tax policy actions. Their analysis distinguishes between legislated changes motivated by prospective economic conditions and those driven by more exogenous factors. The sample period for this shock spans from 1961Q3 to 2007Q4. The shock series is available at: https://www.aeaweb.org/aer/data/june2010/20080421_app.zip.

C Functional Principal Components

The eigenvalues of a compact operator are defined as the non-zero scalars λ for which there exists a non-zero function v in the underlying vector space such that the operator S applied to v is a scalar multiple of v. More formally, for a compact operator S defined on a Banach space or a Hilbert space, the eigenvalue-eigenvector equation is given by:

$$Sv = \lambda v$$

Here, v is the eigenfunction associated with the eigenvalue λ, and λ is a scalar. Compact operators are typically defined on infinite-dimensional spaces, and their eigenvalues may include accumulation points or have a discrete or continuous spectrum depending on the properties of the operator and the underlying space. The eigenvalues of a compact operator provide important information about its spectral properties and behavior. In the case of a bounded compact operator on a Hilbert spaces, the eigenvalues are a sequence of numbers with 0 as only accumulation point.

For a time series of functions $y_1, y_2, y_3, \ldots, y_T$ with sample mean $\bar{y}$ we define the operator

$$S = T^{-1} \sum_{i=1}^{T} (y_i - \bar{y}) \otimes (y_i - \bar{y})$$

the operator S is the sample covariance operator of y_t.

Let $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots$ the eigenvalues of S we call the eigenfunctions $v_1, v_2, v_3, \ldots$ the **functional principal components of** y_t.

There are multiple norms we can use for S. We use the trace norm $\|S\|$ to measure the variability of y_t. The trace norm equals the summation of all eigenvalues of the compact operator

$$\|S\| = \lambda_1 + \lambda_2 + \lambda_3 + \cdots$$

A screeplot measures the amount of the total variability explained by a subset of principal components:

$$s(m) = \frac{\lambda_1 + \lambda_2 + \cdots + \lambda_m}{\lambda_1 + \lambda_2 + \lambda_3 + \cdots}$$

In the Appendix D we show how to computationally obtain the functional principal components of a functional time series.

D How to Model the Yield Curve Computationally?

In the main text, we use the interval $I = [0.25, 30]$ representing maturities from three months to 30 years. We use a grid of 1024 equidistant points, from $x_1 = 0.25$ to $x_{1024} = 30$.

Computationally, the sample of the yield curve is a matrix Y of dimensions $T \times 1024$, where each row contains the values y_{it} for $i = 1, \ldots, 1024$ and $t = 1, 2, 3, \ldots, T$, representing the yield in period t and maturity x_t.

The main operation using the data are described as follows:

1. Sample mean yield curve: $\bar{y} = (\bar{y}_{\cdot 1}, \bar{y}_{\cdot 2}, \ldots, \bar{y}_{\cdot 1024})$. This represents the mean yield curve in the sample.

2. The scalar product between two functions, is the inner product of their vector representation.

3. The tensor product is the outter product of their vector representation.

4. The sample variance matrix S with dimensions 1024×1024, calculated as:

$$S = (Y - \bar{y})'(Y - \bar{y})$$

5. The estimated functional principal components are obtained from the eigenvalue decomposition of the matrix S. The principal components are given by the eigenvectors of S, and the portion of the variance explained by each component is given by the eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots$

E Random Functions

In this section, we describe some of the main concepts of random functions. Similar to random variables and random vectors, we define a random function on a probability space $(\Omega, \mathcal{A}, P)$, which consists of a sample space, an event space, and a probability measure, respectively.

Expected function Given a random function f and an arbitrary element v of H, the scalar product $\langle f, v\rangle$ becomes a scalar random variable. This random variable has an expected value denoted as $\mathbb{E}\langle f, v\rangle$. The mapping

$$v \mapsto \mathbb{E}\langle f, v\rangle$$

is proven to be a linear functional from H to $\mathbb{R}$. By Riesz' representation theorem, there exists a non-random element in H referred to as "$\mathbb{E}f$", such that

$$v \mapsto \mathbb{E}\langle f, v\rangle = \langle \mathbb{E}f, v\rangle$$

In other words, we use the representation of a linear functional as the scalar product with a fixed element of H to characterize the expected function of the random function f.

Covariance operator If f and g are random functions taking values in H, then their covariance operator $\mathbb{E}(f \otimes g)$ is generally defined as a linear operator satisfying

$$\Big\langle u, [\mathbb{E}(f \otimes g)]v \Big\rangle = \mathbb{E}\langle u, f\rangle\langle v, g\rangle$$

for all u and v in H.

The combination of these two concepts allows us to define, for example, a functional white noise (ε_t) as follows: We set $\mathbb{E}\varepsilon_t = 0$ for all $t \geq 1$, and (ε_t) to be serially uncorrelated with $\mathbb{E}(\varepsilon_t \otimes \varepsilon_t) = \Sigma$ for all $t \geq 1$.

F Bootstrapping

We use the bootstrap to determine confidence intervals for the statistics we estimate regarding the yield curve and its reactions to external shocks.

In our investigation of the yield curve's response to an external shock, we consider a sample of the yield curve that corresponds to the sample of the external shock. The majority of the external shocks examined in this study occur at a quarterly frequency, so we utilize the most recent daily observation of the yield curve from the corresponding quarter. This ensures that we have two identical samples, in terms of size and frequency, for both the yield curve and the external shock.

For simplicity, the bootstrap method discussed here is illustrated using a VAR of order $p = 1$. However, this approach can be readily extended to cases with multiple lags, as suggested in the main text.

In the following, we outline the procedure for generating copies of functional time series and the external shock, each of size n:

1. Obtain the residuals from estimating model (9): $\hat{u}_t = \gamma_t - \hat{A}\gamma_{t-1}$.

2. Randomly select, with replacement, a sample of n residuals from the set $\{\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_n\}$, and center (demean) it to obtain a new set of residuals (u_t^*).

3. Generate a new time series γ_t^* using the equation $\gamma_t^* = \hat{A}\gamma_{t-1}^* + u_t^*$ with the (u_t^*) generated in the previous step and the initial value $\gamma_0^* = \gamma_0$. Note that $\hat{A}$ is the same as in the first step.

With the new copy of the time series $\{\gamma_t^*\}_{t=0,1,2,\ldots,n}$, estimate model (9) and obtain $\hat{u}_t^* = \gamma_t^* - \hat{A}^*\gamma_{t-1}^*$. With the new estimation, obtain impulse response functions of γ^* as discussed in the main text. Then, use the basis functions $v_1, v_2, v_3, \ldots, v_m$ to recover a yield curve from the components of the response that belong to the α vector. Repeat the steps 2-3 a large number B of times (e.g., $B = 1000$). Calculate the desired confidence bands as the quantiles of the B saved estimates.

G Models of Yield Curve Dynamics

This appendix evaluates how our functional time-series (FTS) specification compares with established approaches to modeling yield-curve dynamics. Because the FTS framework is

used to trace responses to external monetary—and especially fiscal—shocks, it is important to verify that it also delivers competitive predictive accuracy. We assess out-of-sample performance using mean squared forecast errors (MSFE) over multiple maturities and forecast horizons.

For two models—the FTS specification and an alternative—fix a maturity $\mathfrak{m}$ and forecast horizon h. For each forecast origin $t-h$, let $\hat{y}^{(i)}_{t|t-h}(\mathfrak{m})$ denote the h-step-ahead forecast for period t from model $i \in \{1,2\}$. The corresponding forecast error is

$$FE^{(i)}_t(\mathfrak{m},h) \;=\; y_t(\mathfrak{m}) - \hat{y}^{(i)}_{t|t-h}(\mathfrak{m})\,.$$

Forecasts are produced with a rolling window of $W = 120$ months. To ensure comparability, we align the set of forecast origins across models (using the maximal lag order when relevant), so that both generate the same number $N(h)$ of h-step errors. For each $(\mathfrak{m},h)$, the MSFE of model i is

$$\mathrm{MSFE}^{(i)}(\mathfrak{m},h) \;=\; \frac{1}{N(h)}\sum_{t}\Big[FE^{(i)}_t(\mathfrak{m},h)\Big]^2,$$

and we report

$$\Delta\mathrm{MSFE}(\mathfrak{m},h) \equiv \mathrm{MSFE}^{(\mathrm{Alt})}(\mathfrak{m},h) - \mathrm{MSFE}^{(\mathrm{FTS})}(\mathfrak{m},h),$$

where positive values favor the FTS model. To assess statistical significance, we use Diebold-Mariano tests (Diebold and Mariano, 2002) with HAC standard errors suitable for overlapping h-step forecasts.

We employ the zero-coupon Treasury yield data of Gürkaynak *et al.* (2007), which provide a long monthly sample (over 700 observations). With a rolling 120-month scheme, this yields several hundred forecast origins for each horizon.

June 1961 $\rightarrow$ oooooooooooooo$\underbrace{\text{oooooo}}_{\#\text{ lags}}\underbrace{\text{ooooooooo}\cdots\text{ooooooooooo}}_{120\text{ months}}$ooooooooooooo $\leftarrow$ June 2025

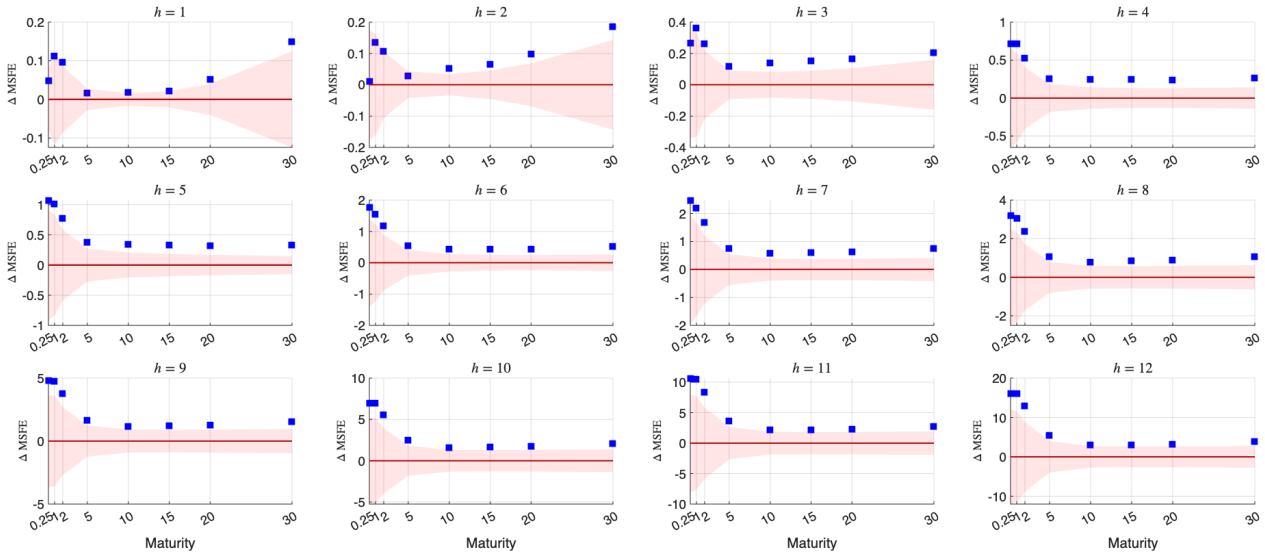
G.1 A Vector Autoregression with Multiple Maturities

A straightforward benchmark is to stack a set of maturities into a vector. Let k maturities $\mathfrak{m}_1 < \mathfrak{m}_2 < \cdots < \mathfrak{m}_k$ and define

$$Y_t = \Big(y_t(\mathfrak{m}_1),\, y_t(\mathfrak{m}_2),\, \ldots,\, y_t(\mathfrak{m}_k)\Big)'.$$

The main limitation of this approach is that it only models the chosen maturities, whereas the FTS specification naturally delivers forecasts for the entire curve. Nonetheless, we evaluate out-of-sample accuracy of the FTS and the VAR on equal footing, selecting the number of lags and components to minimize MSFE for each specification.

Figure A-8: Mean Square Forecast Error Comparison



Note: Each panel shows the difference in MSFE between the FTS model and the VAR (blue squares). We consider eight maturities: three months, one, two, five, ten, fifteen, twenty, and thirty years. The shaded area represents the one–standard deviation non-rejection region based on the Diebold and Mariano (2002) statistic. Positive values indicate larger MSFE for the VAR relative to the FTS model.

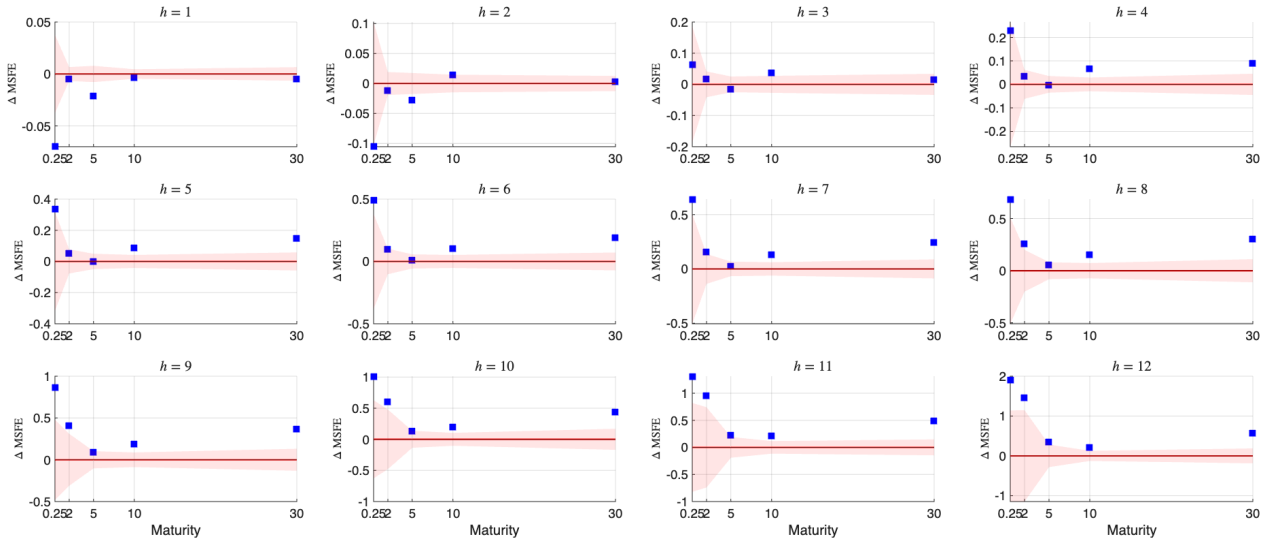
Figure A-8 reports the MSFE difference between the FTS and VAR across eight maturities and forecast horizons h (quarters ahead). A positive value implies an advantage for our FTS approach. The FTS gains are modest at short horizons ($h = 1$–3) but widen beyond $h = 5$, becoming both larger and statistically significant at several maturities. At long horizons ($h = 9$–12), the FTS framework consistently outperforms the VAR, underscoring its efficiency when forecasting further into the future.

Figure A-9 considers a smaller set of maturities. Results mirror the earlier figure: little difference at short horizons (note that the y-axis is different for shorter horizons), but consistent and statistically significant gains for the FTS model at longer horizons.

Figure A-10 restricts attention further, to four maturities. Again, the FTS model delivers smaller forecast errors at medium- and long-term horizons, with some improvements statistically significant, showing robustness to narrower maturity choices.

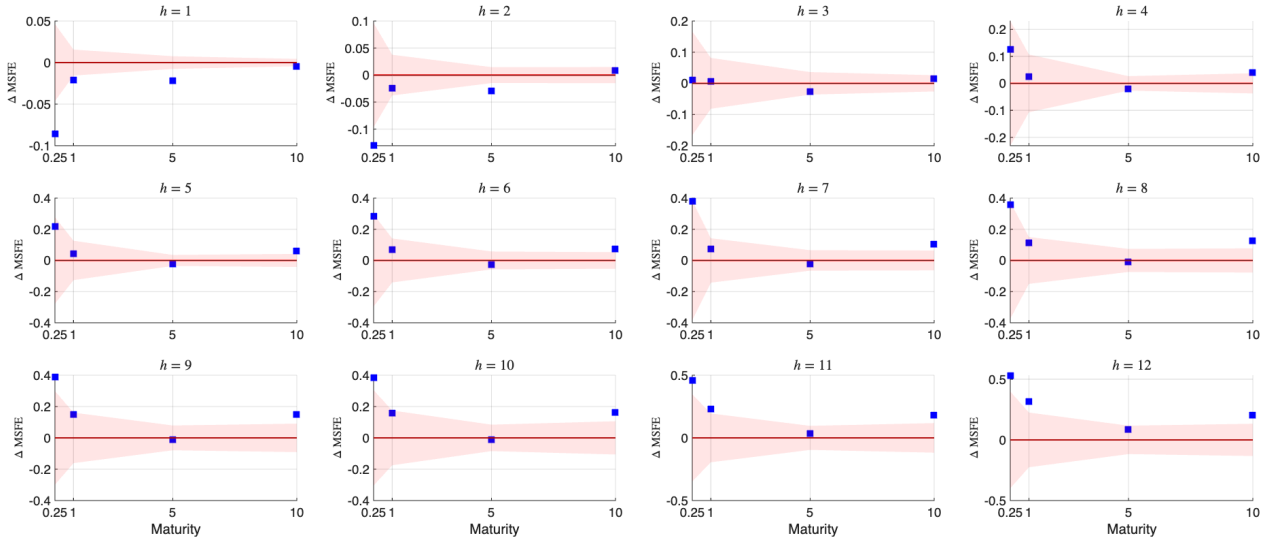
Figures A-11 and A-12 extend the robustness analysis. With just four maturities or with a richer nine-maturity grid, the findings remain consistent: modest differences at short

Figure A-9: Mean Square Forecast Error Difference: FTS *versus* VAR



Note: As above, with five maturities: three months, two, five, ten, and thirty years.

Figure A-10: Mean Square Forecast Error Difference: FTS *versus* VAR

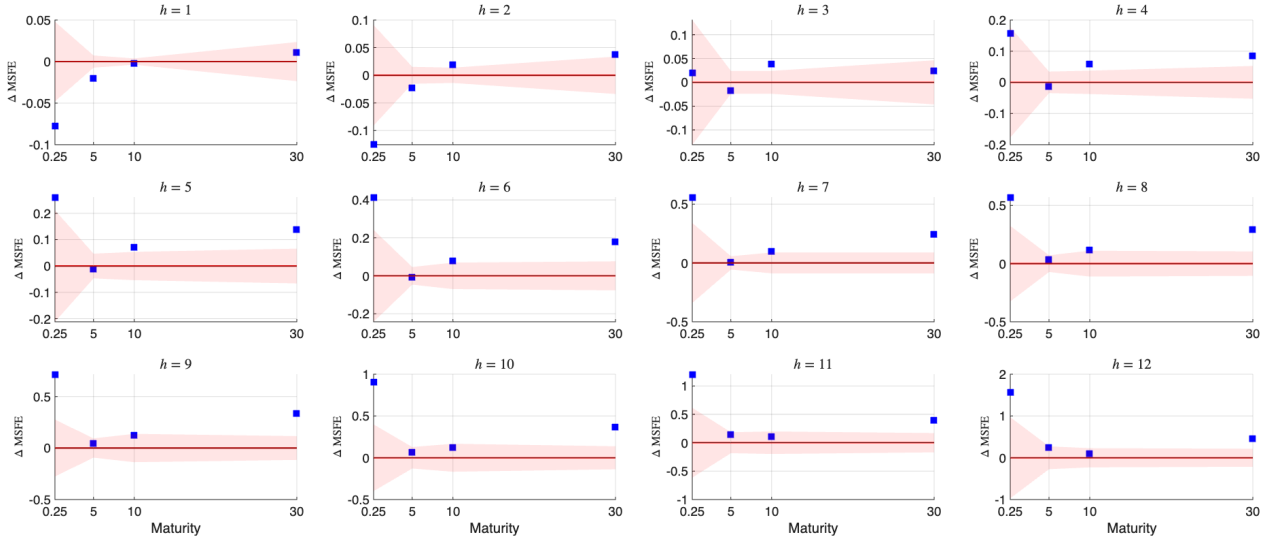


Note: Four maturities: three months, one, five, and ten years.

horizons, but clear FTS gains at medium and long horizons, with significance in many cases. The broader grid in Figure A-12 makes the advantage of the functional approach especially visible, as it scales more effectively with the dimensionality of the maturity space.

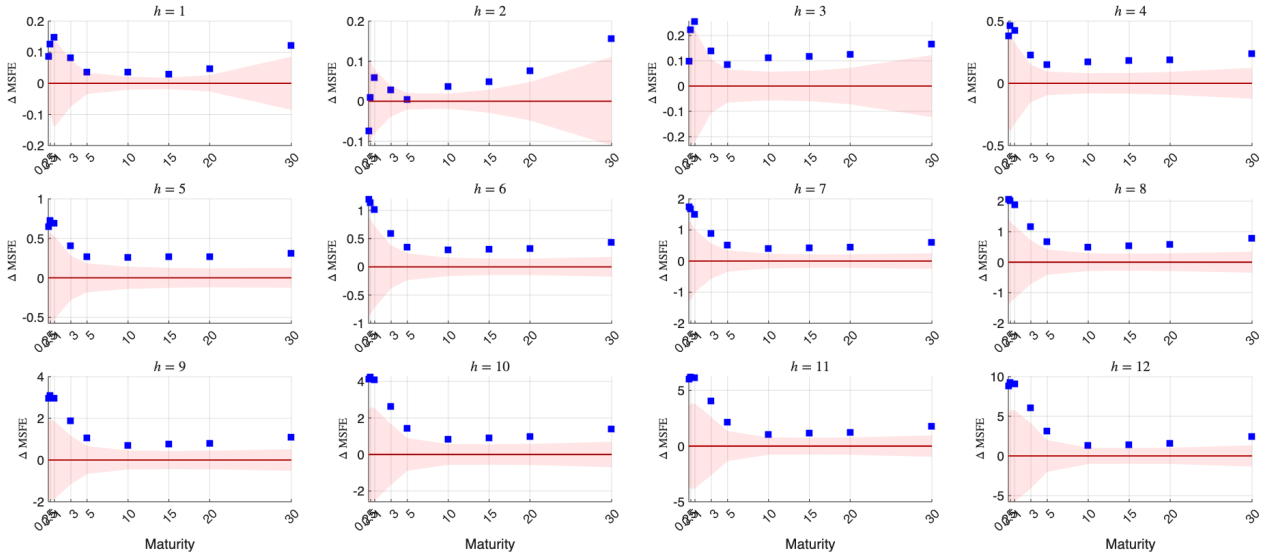
Summary. Across Figures A-8 to A-12, the FTS model systematically outperforms the VAR approach, except for very short horizons, where the difference in MSFEs is, however,

Figure A-11: Mean Square Forecast Error Difference: FTS *versus* VAR



Note: Four maturities: three months, five, ten, and thirty years.

Figure A-12: Mean Square Forecast Error Difference: FTS *versus* VAR



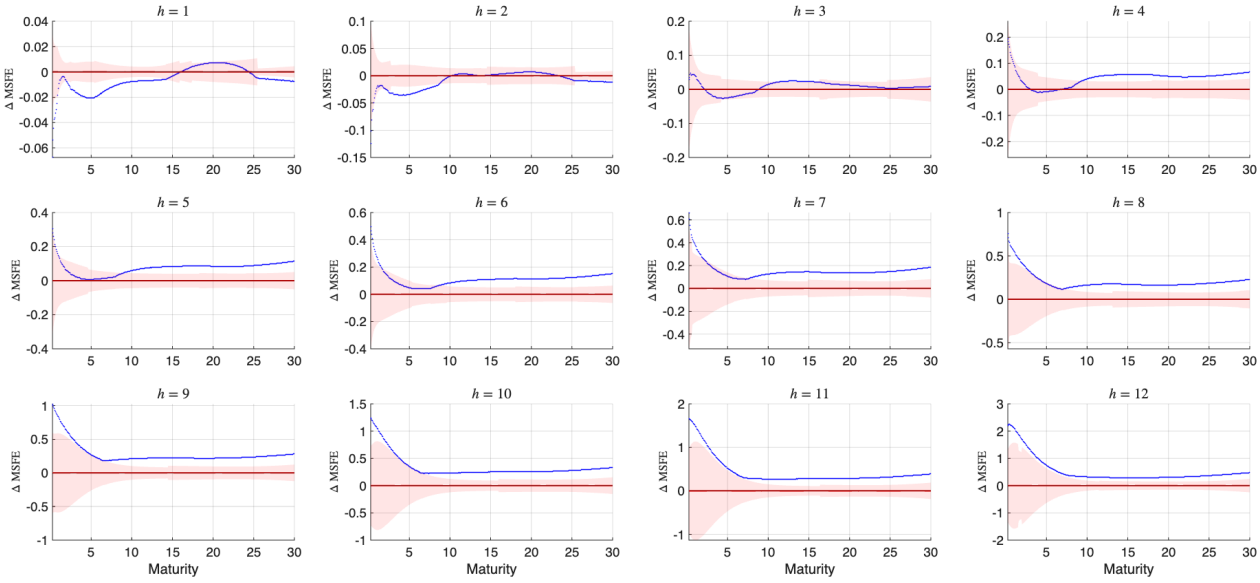
Note: Nine maturities: three and six months, one, three, five, ten, fifteen, twenty, and thirty years.

not large. The superiority is not driven by the particular choice of maturities but reflects the inherent advantage of modeling the entire yield curve as a functional object. Gains are modest at short horizons but become substantial and statistically significant at medium and long horizons, confirming the efficiency of the functional specification.

G.2 Principal Components Vector

A second benchmark is principal component analysis (PCA), which summarizes a set of yields with a small number of common factors. If implemented on a dense grid of maturities (e.g., 1024 points between 0.25 and 30 years), PCA and the FTS approach would coincide. In practice, however, PCA is typically applied to coarser sets of yields, often on a monthly grid. To provide a fair comparison, we construct a PCA-based benchmark using yields at monthly maturities from 3 to 360 months.

Figure A-13: Mean Square Forecast Error Difference: FTS *versus* PCA



Note: Each panel shows the difference in MSFE between the FTS model and the PCA approach (blue line). We consider monthly maturities from 3 months to 30 years. The shaded area represents the one–standard deviation non-rejection region based on the Diebold and Mariano (2002) statistic. Positive values indicate larger MSFE for PCA relative to FTS.

Figure A-13 reports the MSFE differences between the FTS model and this PCA benchmark. At short horizons, both approaches perform similarly, with differences generally inside the confidence bands. At longer horizons, however, the FTS model consistently delivers smaller forecast errors, with improvements that grow in size and significance across several maturities. While PCA is a useful dimension-reduction tool, these results show that the

functional framework better captures the smooth dynamics of the yield curve, yielding more accurate forecasts at medium and long horizons.

These results confirm that the FTS specification provides forecasts that are, at the very least, competitive with these competitors and often superior, especially once we move past very forecast horizons. Its strength lies in treating the yield curve as a continuous functional object, rather than a finite set of yields or factors, allowing it to exploit more of the information embedded in the term structure.