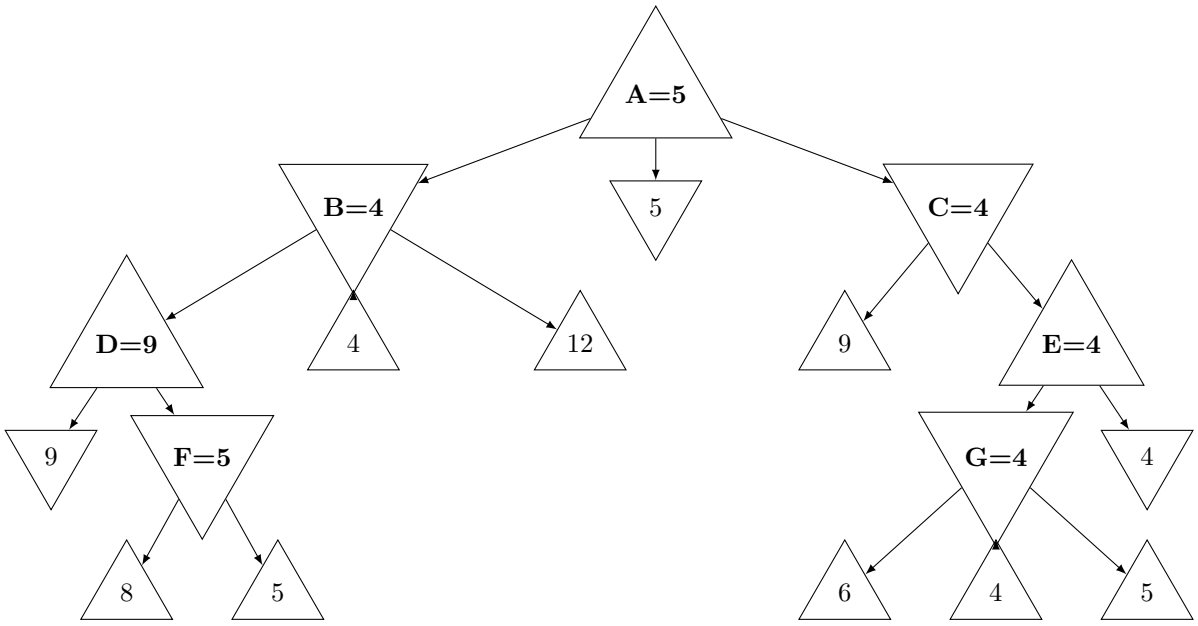
Introduction to AI Homework 5

Chetankumar Mistry

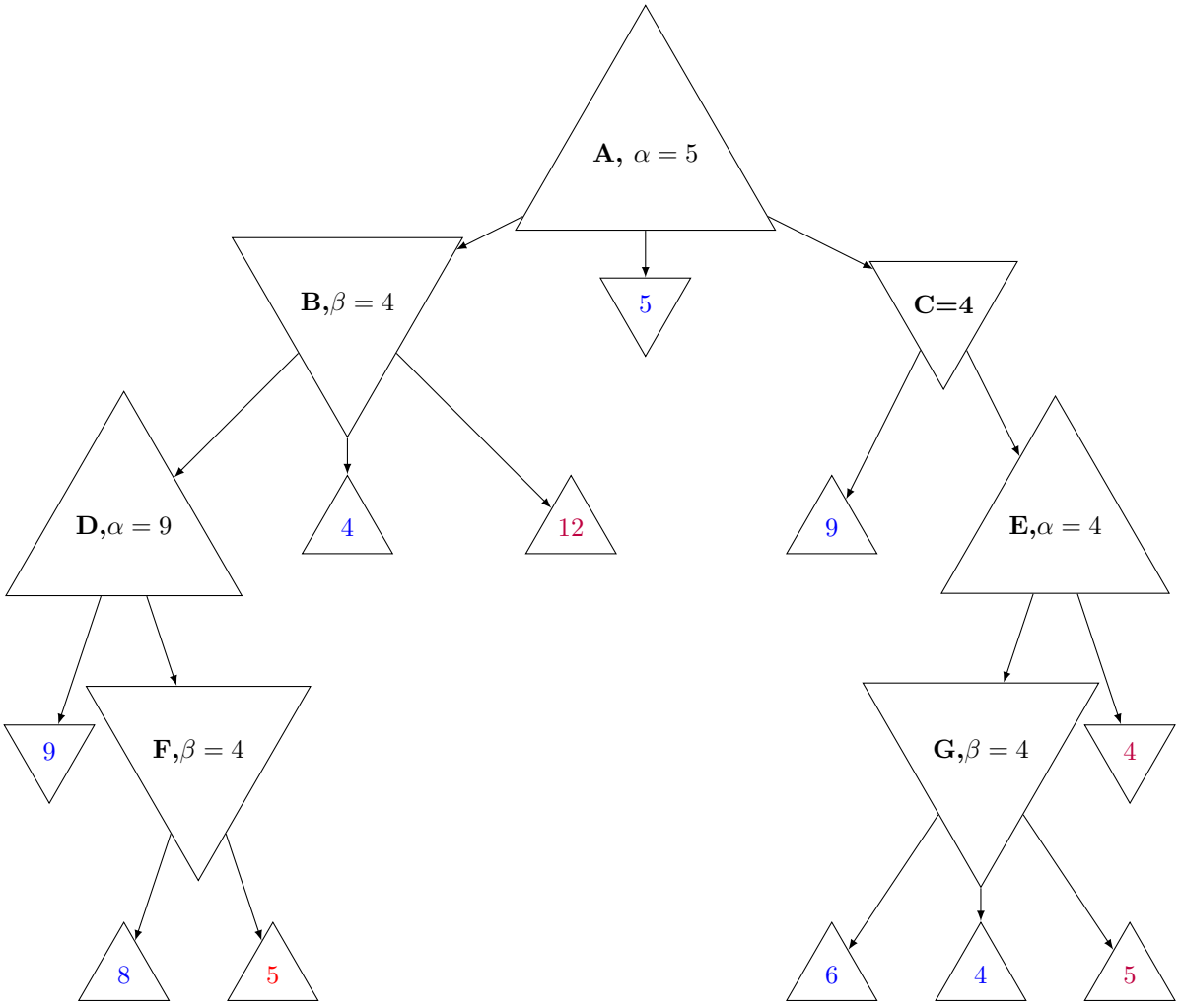
October 2018

1 Question 1: Game Tree Search



Node Scores:

- A=5, Path: A → 5
- B=4, Path: A → B → 4
- C=4, Path: A → C → E → 4 or A → C → E → G → 4
- D=9, Path: A → B → D → 9
- E=4, Path: A → C → E → 4 or A → C → E → G → 4
- F=5, Path: A → B → D → F → 5


A, $\alpha = 5$
B,$\beta = 4$
5
C=4
D,$\alpha = 9$
4
12
9
E,$\alpha = 4$
9
F,$\beta = 4$
G,$\beta = 4$
4
8
5
6
4
5

- G=4, Path: A $\rightarrow$ C $\rightarrow$ E $\rightarrow$ G $\rightarrow$ 4

Nodes in red are pruned:

- F $\rightarrow$ 5

3) If alpha-beta pruning is applied to a standard minimax game tree then the outputs of both the standard minimax algorithm and one with alpha-beta pruning will be the same. This is because of the way that alpha-beta pruning is implemented; it will only prune a node and its sub-trees if they are proven to be sub-optimal.

1.1 Probability

2 Dice rolled to give a score of 7:

- 1:6
- 2:5
- 3:4
- 4:3
- 5:2
- 6:1

This is 6 possible combinations of 2 die, and each pair has a $\frac{1}{6} * \frac{1}{6} = \frac{1}{36}$ chance of happening.
This is equal to:

$$6 * \frac{1}{36} = \frac{6}{36} = \frac{1}{6} \tag{1}$$

3 Dice rolled to give a score of 7:

- 5:1:1
- 4:2:1
- 3:3:1
- 2:4:1
- 1:5:1
- 1:4:2
- 1:3:3

- 1:2:4
- 1:1:5
- 2:1:4
- 3:1:3
- 4:1:2
- 3:2:2
- 2:3:2
- 2:2:3

This is 15 possible combinations of 3 die, and each triplet has a $\frac{1}{6} * \frac{1}{6} * \frac{1}{6} = \frac{1}{216}$ chance of happening.
This is equal to:

$$15 * \frac{1}{216} = \frac{15}{216} = \frac{5}{72} \tag{2}$$

2) John and Robert play a game with 52 cards, the first to pick up the Heart Ace wins.
Given that John picks up 2 cards shows to Robert that neither of them are the Heart Ace, we can model the remaining pile as a uniform distribution where we have a $\frac{1}{50}$ chance of picking the Heart Ace.
Robert also picks up 2 cards from this remaining pile:

$$2 * \frac{1}{50} = \frac{2}{50} = \frac{1}{25} \tag{3}$$

Therefore John has a $\frac{1}{25}$ chance of winning the game in this round.

3) 1 out of 10000 clovers has 4 leaflets. How many clovers does one need to pick to in order to find a 4 leaf-clover with at least 0.9 probability?

$$\begin{aligned} P(4Leaf) &= \frac{1}{100000} \\ 1 - P(4Leaf) &= \frac{9999}{10000} \end{aligned} \tag{4}$$

We can calculate the number of clovers needed to pick so that the probability of NOT seeing one is < 0.1.

$$
\begin{aligned}
(\frac{9999}{100000})^x &= 0.1 \\
x \log \frac{9999}{10000} &= \log 0.1 \\
x &= \frac{\log 0.1}{\log \frac{9999}{10000}} \\
x &= 23024.699 \\
&= 23025
\end{aligned} \tag{5}
$$

Therefore 23025 clovers needed to be picked to ensure that there is a greater than 0.9 probability of finding a 4-leafed clover.

4) There are three cards in an opaque box. The first card is painted black on both sides, the second card is painted red on both sides, and the third card has one side black and one side red. One picks a card randomly and observes one side of this card is black (without seeing the other side). What is the probability that the other side is red?

Let $C_1 = bb, C_2 = br, C_3 = rr$

$$
\begin{aligned}
P(C = C_2|b) &= \frac{P(b|C = C_2) * P(C = C_2)}{P(b)} \\
&= \frac{\frac{1}{2} * \frac{1}{3}}{\frac{1}{2}} \\
&= \frac{1}{3}
\end{aligned} \tag{6}
$$

5) Suppose that the probability of the word lottery appearing in a spam email is 42%, and the probability of it appearing in a non-spam email is 5%. If every email received has equal probability of being spam or not spam, what is the probability that an email is spam given it contains the word lottery?

$$
\begin{aligned}
P(W|S) &= 0.42 \\
P(W|\neg S) &= 0.05 \\
P(S) = P(\neg S) &= 0.5 \\
P(S|W) = \frac{P(W|S) * P(S)}{P(W|S) * P(S) + P(W|\neg S) * P(\neg S)} \\
&= \frac{0.42 * 0.5}{0.42 * 0.5 + 0.05 * 0.5} \\
&= \frac{42}{47} \\
&= 0.894
\end{aligned} \tag{7}
$$