Calibrating the SIDM Gravothermal Catastrophe with N-body Simulations

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Introduction

The nature of dark matter is one of the most persistent mysteries in our Universe. One dark matter candidate, self-interacting dark matter (SIDM), predicts that dark matter halos experience gravothermal catastrophe, a process where the halo's inner region rapidly increases in density and decreases in size. Models for the collapse process require calibration to N-body simulations, but the meaning of that calibration is unclear, and there are many differing values of the calibration parameter across the literature.

In this work we use the N-body code `Arepo` to study this calibration, and to extend the useful range of core-collapse models. **We show that this calibration changes as a function of halo parameters and SIDM cross-section, but can be well described as a function of just one variable.** With this calibrated model, dark matter researchers can study the nature of SIDM without running costly numerical simulations.

The Gravothermal Catastrophe

SIDM predicts that dark matter halos (the spherical overdensities of dark matter that house galaxies) eventually undergo the gravothermal catastrophe. This means that the centers of these halos will eventually grow increasingly small and dense, at a rapidly accelerating rate.

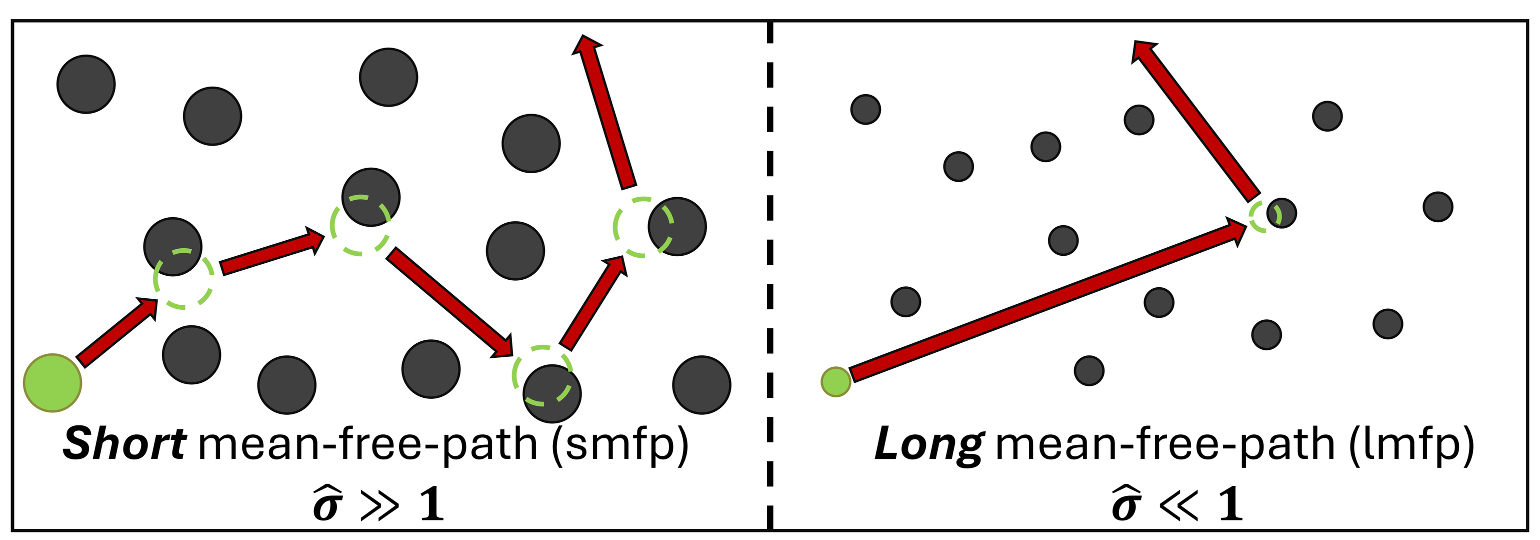

Figure 1. Two regimes for SIDM: smfp (dominated by self-scattering) and lmfp (dominated by gravitational interaction).

Numerical gravothermal fluid models have been successful in predicting the evolution of halos in the lmfp regime [1], but those models face two major challenges:

1. These models **require calibration** to N-body simulations to get an accurate heat transfer rate
2. These models **break down** as $\hat{\sigma}$ approaches 1, limiting their application

To resolve these issues, we derive a model for an effective calibration parameter $\beta_{\rm eff}$, based on the expected variation in heat transfer between the smfp and lmfp regimes:

$$\beta_{\rm eff}(\hat{\sigma}) = \frac{1}{0.20\hat{\sigma}}\left((0.20\beta\hat{\sigma})^{-\alpha} + \left(\frac{0.63}{\hat{\sigma}}\right)^{-\alpha}\right)^{-1/\alpha}.$$

This model is a function of $\hat{\sigma}$, with two fit parameters β and α.

How does an N-body simulation work?

Given two particles with masses m_1 and m_2 at a distance r from each-other, it is straightforward to calculate their mutual force of gravity as:

$$F = G\frac{m_1 m_2}{r^2}$$

but for large scale simulations doing those calculations for each particle pair is inefficient and costly. Most N-body simulations, including ours, take numerical shortcuts [2]. These shortcuts include **large simulation particles** that represent many "real" particles, and **approximations to the gravitational force** between distant particles.

The general steps in an SIDM N-body simulation are:

1. **Calculate forces:** The distance between particles is measured, and the gravitational acceleration on each particle is calculated.
2. **SIDM collisions:** Determine which dark matter particles will scatter off of each-other, and compute the velocity changes from those scatterings.
3. **Update positions:** With the acceleration of gravity and the SIDM interaction, the simulation "steps" forwards a fixed time interval. The new position of each particle after this time is calculated.
4. **Repeat:** These steps are repeated, until the simulation is run as long as desired.

N-body Simulations

To test our model for $\beta_{\rm eff}$ and determine values for the fit parameters β and α, we run 96 simulations across a variety of halo and SIDM parameters. In our simulations we vary the halo mass, halo concentration, and the strength of the SIDM scattering. Our predicted model (Equation 1) only depends on $\hat{\sigma}$, which is a specific combination of those three parameters.

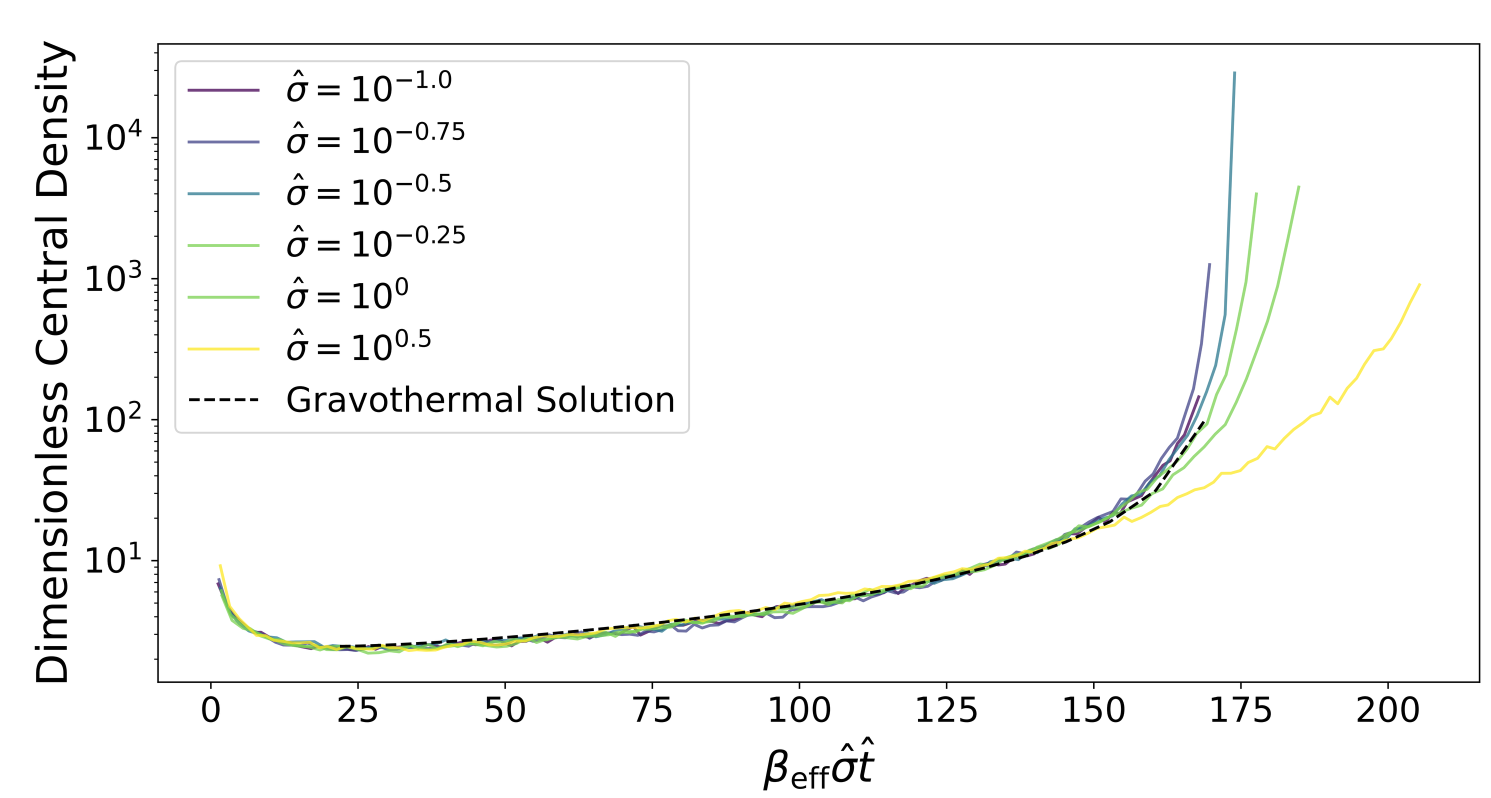

Figure 2. The central density evolution of core collapse simulations as a function of time. Each color of line corresponds to a different value of $\hat{\sigma}$, where smaller values are deeper in the long mean-free-path regime. We find the best fit $\beta_{\rm eff}$ for each simulation by varying it until the gravothermal fluid prediction (black) matches the simulation data.

Results

We find that our model accurately describes the evolution of halos with $\hat{\sigma} < 1$ (Figure 3). Our best fit parameters are $\beta = 0.954$ and $\alpha = 0.901$. These fit parameters, and the success of our model, are not dependent on halo parameters or SIDM cross-section. The success of this model shows that regardless of the specific mass, concentration, and cross-section of a halo, its evolution can be predicted by just $\hat{\sigma}$.

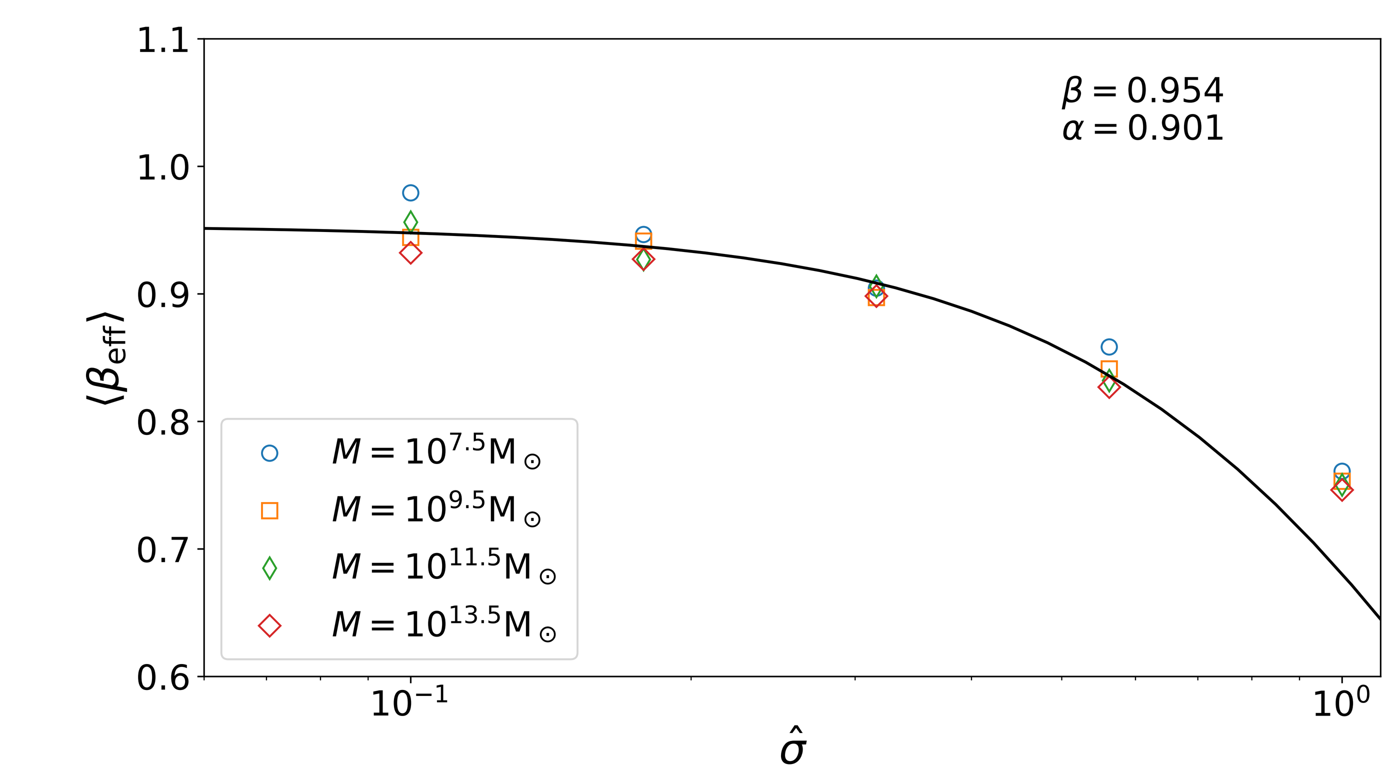

Figure 3. The effective calibration parameter $\beta_{\rm eff}$ as a function of $\hat{\sigma}$. Different halo masses are shown by marker, and each $\beta_{\rm eff}$ value is averaged over halo concentrations.

Implications

Figure 4. Multiply imaged galaxies (JWST, 2024).

Our $\beta_{\rm eff}$ model allows dark matter researchers to model the evolution of SIDM halos accurately, without running costly N-body simulations. We have also expanded the effective range of these gravothermal models by calibrating to the transition regime where $\hat{\sigma}$ approaches 1. While the variation in $\beta_{\rm eff}$ is small, there are applications that call for this level of precision.

One such application for precise collapse timescales is substructure lensing, where the gravitational lensing caused by dark subhalos within a galaxy can be used to infer the properties of dark matter. If subhalos of the lensing galaxy undergo gravothermal core-collapse, the properties of the lensed image can change drastically.

References

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