Learning Decision-Making Functions Given Cardinal and Ordinal Consensus Data

Anonymous submission

"I shall be telling this with a sigh
Somewhere ages and ages hence:
Two roads diverged in a wood, and I—
I took the one less traveled by,
And that has made all the difference."

-Robert Frost, *The Road Not Taken*

Abstract

Decision-making and reaching consensus are an integral part of everyday life, and studying how individuals reach these decisions is an important problem in psychology, economics, and social choice theory. Our work develops methods and theory for learning the nature of decisions reached upon by individual decision makers or groups of individuals using data.

We consider two tasks, where we have access to data on: 1) Cardinal utilities for d individuals with cardinal consensus values that the group or decision maker arrives at, 2) Cardinal utilities for d individuals for pairs of actions, with ordinal information about the consensus, i.e., which action is better according to the consensus. Under some axioms of social choice theory, the set of possible decision functions reduces to the set of weighted power means, $M(\mathbf{u}, \mathbf{w}, p) = \left(\sum_{i=1}^{d} w_i u_i^p\right)^{1/p}$, where $\{u_i\}_{i=1}^{d}$ indicates the d utilities, $\mathbf{w} \in \Delta_{d-1}$ denotes the weights assigned to the d individuals, and $p \in \mathbb{R}$ (Cousins 2023). For instance, $p = 1$ corresponds to a weighted utilitarian function, and $p = -\infty$ is the egalitarian welfare function.

Our goal is to learn $\mathbf{w} \in \Delta_{d-1}$ and $p \in \mathbb{R}$ for the two tasks given data. The first task is analogous to regression, and we show that owing to the monotonicity in $\mathbf{w}$ and p (Qi et al. 2000), simple gradient descent methods are viable options for learning. For the second task, we wish to learn $\mathbf{w}, p$ such that, given pairs of actions $\mathbf{u}, \mathbf{v} \in \mathbb{R}_+^d$, the preference is given as $C\left((\mathbf{u}, \mathbf{v}), \mathbf{w}, p\right) = \mathrm{sign}\left(\ln\left(M(\mathbf{u}, \mathbf{w}, p)\right) - \ln\left(M(\mathbf{v}, \mathbf{w}, p)\right)\right)$. This is analogous to classification; however, convexity of the loss function in $\mathbf{w}$ and p is not guaranteed.

We analyze two related cases. We first assume all the weights are the same, $\mathbf{w} = \mathbf{1}_d/d$, and learn p, calling this the unweighted case. We then remove this assumption on $\mathbf{w}$ and learn it as well, calling this the weighted case. We prove that both cases are PAC-learnable given positive $\mathbf{u}, \mathbf{v}$ by giving an $\mathcal{O}(d)$ bound on the VC dimension for the weighted case, and an $\mathcal{O}(d^2)$ bound for the weighted case. Moreover, we show the hinge loss to be Lipschitz continuous, finding the Lipschitz constant for both cases. We then make use of Lipschitz continuity to design practical algorithms (Malherbe and Vayatis 2017) and get theoretical bounds for some cases of noisy data (Natarajan et al. 2013).

References

Cousins, C. 2023. Revisiting Fair-PAC Learning and the Axioms of Cardinal Welfare. In *International Conference on Artificial Intelligence and Statistics*, 6422–6442. PMLR.

Malherbe, C.; and Vayatis, N. 2017. Global optimization of Lipschitz functions. In *International Conference on Machine Learning*, 2314–2323. PMLR.

Natarajan, N.; Dhillon, I. S.; Ravikumar, P. K.; and Tewari, A. 2013. Learning with noisy labels. *Advances in neural information processing systems*, 26.

Qi, F.; Mei, J.-Q.; Xia, D.-F.; and Xu, S.-L. 2000. New proofs of weighted power mean inequalities and monotonicity for generalized weighted mean values. *Math. Inequal. Appl*, 3(3): 377–383.