Benson Earth Sciences Reinforced Concrete Design

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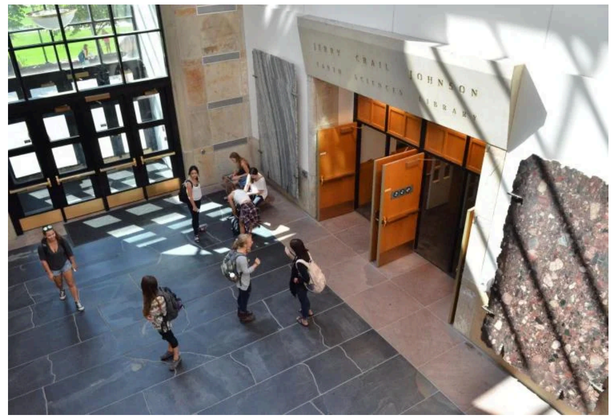
Section 1: Building Description



Address

2200 Colorado Ave, Boulder, CO 80309

Architect of Record

Anderson Mason Dale Architects

Engineer of Record

Martin/Martin Inc.

Description of Building

Benson Earth Sciences is the home of the University of Colorado Boulder's Department of Geological Sciences and a notable feature of the campus as a whole. This four-story building (excluding a mechanical room on the fifth floor) houses a mixture of large lecture halls, small classrooms, offices, and laboratories, in addition to a large multi-story entrance room. Built in 1997, Benson Earth Sciences is a reinforced concrete structure built with a joist system for floors, and a column and beam system comprising the frame of the building. Overall, Benson is 84,000 square feet with an 11,000 square foot library, a 173 seat auditorium, and seven classrooms. Some rooms span multiple stories while others span only one, making the design somewhat varied and complicated compared to a traditional office building. This also means that there is no single "typical" structural bay. The building is a reinforced concrete structure consisting of a concrete column and beam frame, with joists supporting the floor system. The exterior facade is stone masonry, while most of the interior walls are concrete. Overall, Benson is a cherished building which is beloved by students and faculty.

Section 2: Plan, Profile, and 3-D View of Typical Structural Bay of Building

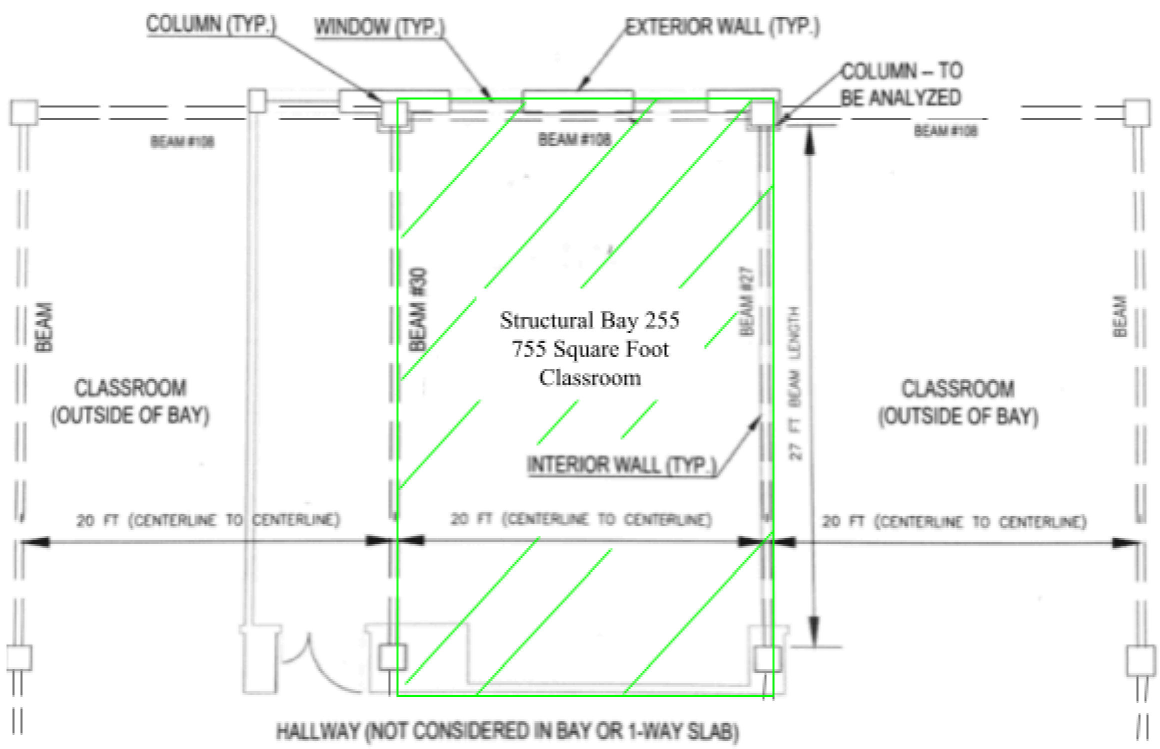


Figure 1. Benson Earth Sciences Bay 255 Plan View

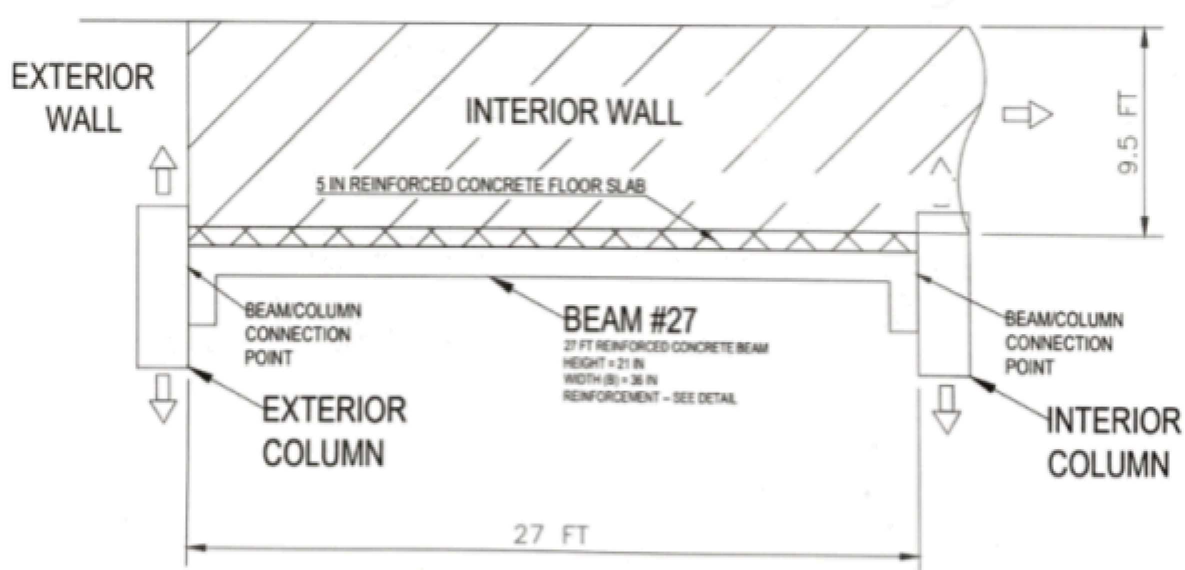


Figure 2. Benson Earth Sciences Bay 255 Profile View

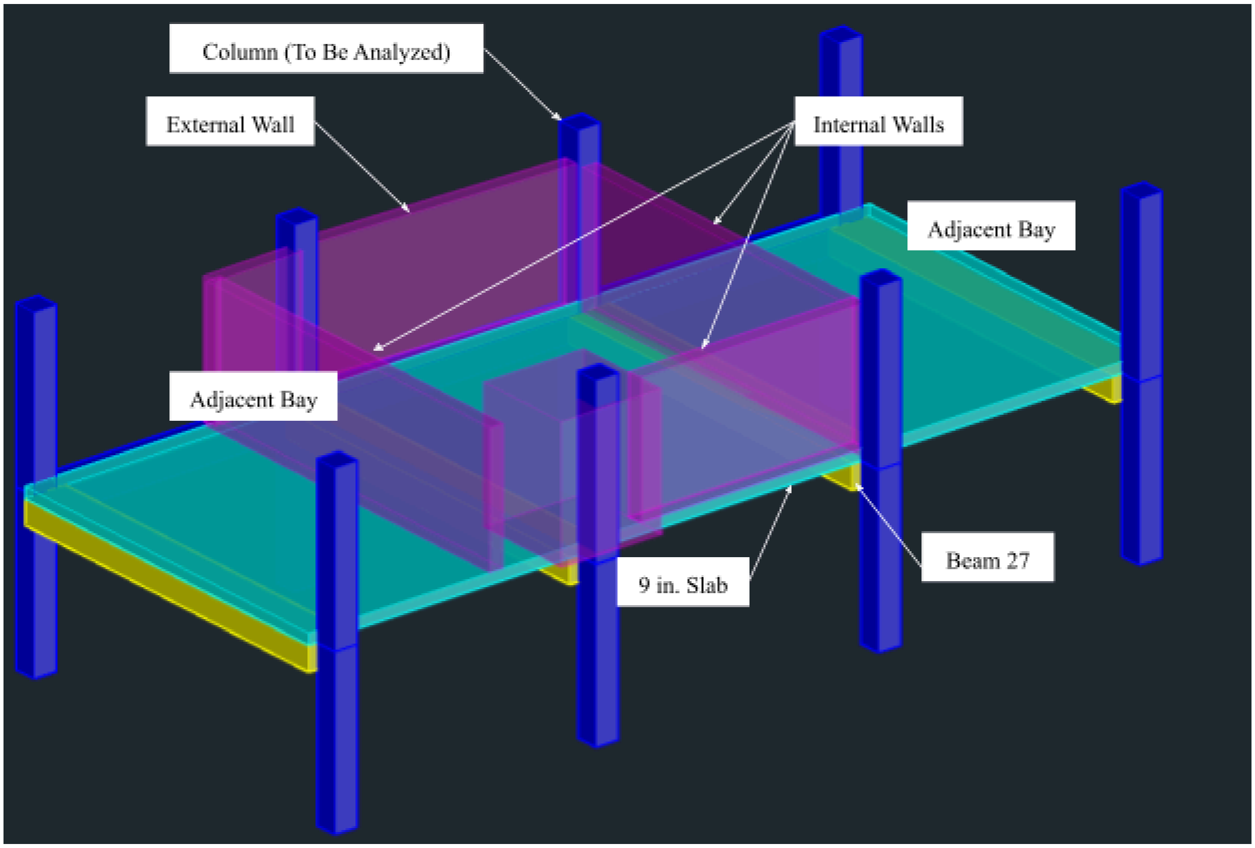


Figure 3. 3D Drawing of Bay and Surrounding Structural Elements

Section 3: Preliminary Estimation of Relevant Loads on Members

Horizontal Element 1. One-Way Slab

For this structural design Bay 255 is considered to comprise part of classroom 255, and is framed to the North by beam 108, the West by beam 30, the East by beam 27, and the South by the hallway, which is a different slab. All of Bay 255 is designed as a 9 in. one-way reinforced concrete slab.

Bay 255 and the surrounding two bays are classrooms. According to ASCE 7-16 the estimated live load in a classroom is 40 psf. The slab dead load is assumed to be self weight plus the dead load of other static elements that are not structural elements, and calculated as if the slab is pure concrete rather than a composite of concrete and reinforcement. The dead load of other non-structural elements is assumed to be 30 psf. Calculations of slab dead load are shown below.

$$DL_{Slab} = \left(\frac{9\,in}{12\,\frac{in}{ft}}\right)\left(150\,\frac{lbs}{ft^3}\right) + (30\,psf) = 142.5\,\frac{lbs}{ft^2}$$

The LRFD load combinations are shown below. In this case, combination two yields the highest factored load and is therefore the basis for design. We are designing for post-construction conditions, so there is no snow load or roof live load on this interior slab. Seismic Loads are not a concern in this area.

LC	LRFD	ASD
1	$1.4D$	D
2	$1.2D+1.6L+0.5(L_r \text{ or } S \text{ or } R)$	$D+L$
3	$1.2D+1.6(L_r \text{ or } S \text{ or } R)+(L \text{ or } 0.5W)$	$D+(L_r \text{ or } S \text{ or } R)$
4	$1.2D+W+L+0.5(L_r \text{ or } S \text{ or } R)$	$D+0.75L+0.75(L_r \text{ or } S \text{ or } R)$
5	$0.9D+W$	$D+0.6W$
6	$1.2D+E_v+E_h+L+0.2S$	$D+0.75L+0.75(0.6W)+0.75(L_r \text{ or } S \text{ or } R)$
7	$0.9D-E_v+E_h$	$0.6D+0.6W$
8		$D+0.7E_v+0.7E_h$
9		$D+0.525E_v+0.525E_h+0.75L+0.75S$
10		$0.6D-0.7E_v+0.7E_h$

Figure 4. LRFD Load Combinations

$$W_{u,\,slab} = (1.2)\left(142.5\,\frac{lbs}{ft^2}\right) + (1.6)\left(40\,\frac{lbs}{ft^2}\right) = 235\,\frac{lbs}{ft^2}$$

Horizontal Element 2. Beam 27

The tributary area of beam 27 is 20 ft, and there are two identical bays to the West and East. The beam is attached to columns on either end. Beam 27 is 27 ft. long and supports a 9.5 ft tall, 6 in. wide precast concrete interior wall. Calculations of Dead and Live Load are shown Below. The weight of the exterior wall in the North of the Bay is carried by beam 108, and the interior walls in the Southern interior wall are assumed to be carried by separate structural members in the hallway.

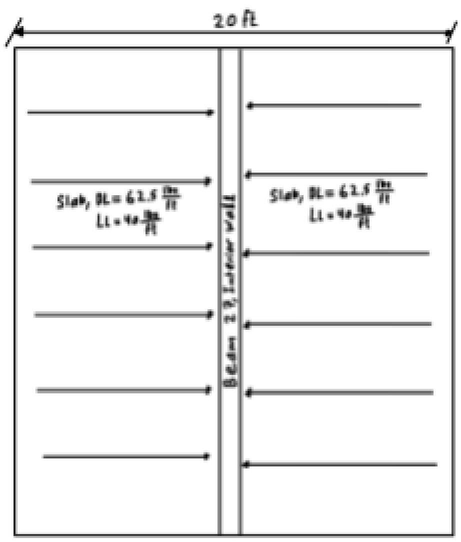


Figure 5. Sketch of Beam Loads

Beam 27 dimensions: $h = 21\ in.\ b = 36\ in.$

$DL_{Beam} = Self\ Weight + Wall\ Weight + Slab\ Weight$

$Self\ Weight = (\frac{21\ in.}{12\ \frac{in.}{ft}})(\frac{36\ in.}{12\ \frac{in.}{ft}})(150\ \frac{lbs}{ft^3})$

$Self\ Weight = 787.5\ \frac{lbs}{ft}$

$Wall\ Weight = (9.5\ ft)(\frac{6\ in.}{12\ \frac{in.}{ft}})(150\ \frac{lbs}{ft^3})$

$Wall\ Weight = 712.5\ \frac{lbs}{ft}$

$Slab\ Weight = (142.5\frac{lbs}{ft^2})(20\ ft) = 2,850\ \frac{lbs}{ft}$

$DL = 787.5\ \frac{lbs}{ft} + 712.5\ \frac{lbs}{ft} + 2850\ \frac{lbs}{ft}$

$DL = 4,350\ \frac{lbs}{ft}$

The live load on the beam is more straightforward, as the only live loads on the beam come from the live loads on the slabs it supports.

$$LL_{Beam} = (40\frac{lbs}{ft^2})(20\ ft) = 800\ \frac{lbs}{ft}$$

Again, there are no roof live loads, snow loads, or seismic loads to consider in this case. Referring to the LRFD load combinations (Figure 2.), the controlling load combination is combination 2.

$$W_{u,\ Beam} = (1.2)(4,350\ \frac{lbs}{ft}) + (1.6)(800\ \frac{lbs}{ft}) = 6,500\ \frac{lbs}{ft} = 6.5\ \frac{Kips}{ft}$$

Vertical Element 1. Column (See Plan View Above)

This column extends from the ground floor to the fourth floor of the building, and in order to accurately design a sufficient column, the total load supported by the column must be considered, not just the load from floor two. In this design, the column is assumed to be homogeneous between all floors (I.E. the dimensions of the column do not change). Therefore the dimensions of the column will be based on the load at the base of the column on the first floor (assuming the basement segment of the column is different).

Floors two, three, and four all have classrooms in the bays around the column, so the live load for all floors supported by the column is assumed to be 40 psf. It is also worth noting that there is a smaller floor five, but this floor is not directly supported by the column of interest and therefore its weight was neglected in this design. Due to the similar layout of floors three and four, the dead and live load on the column from floor two will be calculated and then multiplied by three to account for the dead and live load from the additional floors, the load from the roof and the self weight of the column will be calculated separately.

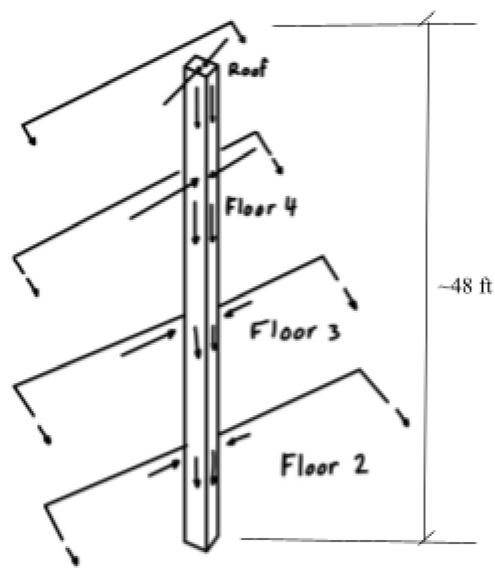


Figure 6. Rough Sketch of Floors Supported by Column

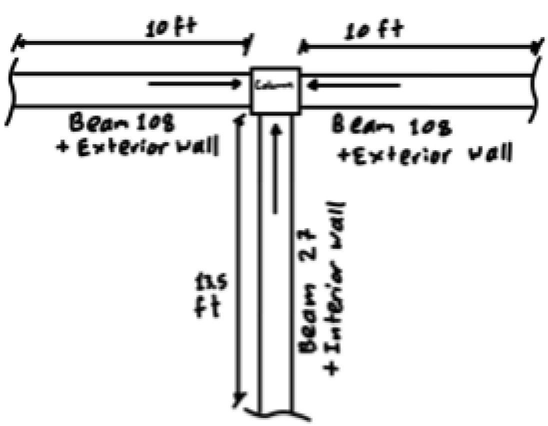


Figure 7. Sketch of Load Path Into Column on Floor 2

Column
is designed with dimensions h = 20 in. b=20 in.

$$DL_{Column} = (3)(DL_{Floor\ 2}) + SW + DL_{Roof}$$

$$SW = (\frac{20\ in.}{12\ \frac{in.}{ft}})(\frac{20\ in.}{12\ \frac{in.}{ft}})(48\ ft)(150\ \frac{lbs}{ft^3})$$

$$SW = 20,000\ lbs$$

$$DL_{Floor\ 2} = (20\ ft)(DL_{Beam\ 108}) + (13.5\ ft)(DL_{Beam\ 27})$$

Beam 108 has dimensions h = 21 in. b = 28 in. Also, the exterior wall above beam 108 is a 14 in. thick stone masonry wall, which is assumed to have a unit weight of 120 $\frac{lbs}{ft^3}$.

$$DL_{Floor\ 2} = (20\ ft)[(\frac{21\ in.}{\frac{12\ in.}{ft}})(\frac{28\ in.}{\frac{12\ in.}{ft}})(150\frac{lbs}{ft^3}) + (9.5\ ft)(\frac{14\ in.}{\frac{12\ in.}{ft}})(120\frac{lbs}{ft^3})]$$
$$+ (13.5\ ft)(4,350\ \frac{lbs}{ft}) = 97,575\ lbs$$

Structural Information for the roof was not available in the plans, so the roof load on the column was estimated to be equal to the load from one of the floors. This is a conservative estimate as typically a roof pan, slab, and shingles are significantly lighter than a floor slab and floor system. In fact, a typical roof dead load is estimated to be only 60 psf, which would result in a very low total DL of 16,200 lbs. However, we do not believe that this load adequately accounts for the weight so the conservative estimate of the roof weighing the same as the floor below was used.

$DL_{Roof} = 100,000\ lbs$ (Conservative Estimate)

$$DL_{Column} = (3)(97,575\ lbs) + (20,000\ lbs) + (100,00\ lbs) = 412,725\ lbs = 412.725\ kips$$

The occupancy live load for the column is considerably more straightforward, as the roof produces no occupancy live load.

$$LL_{Column} = (3)(LL_{Floor\ 2}) = (3)(13.5\ ft)(800\ \frac{lbs}{ft}) = 32,400\ lbs = 32.4\ kips$$

A snow load and roof live load also need to be calculated for the roof. For this region of Colorado is 40 $\frac{lbs}{ft^2}$. The roof live load is 20 $\frac{lbs}{ft^2}$.

$$SL_{Column} = Tributary\ Snow\ Load\ on\ Roof = (13.5\ ft)(20\ ft)(40\ \frac{lbs}{ft^2}) = 10,800\ lbs = 10.8\ Kips$$
$$L_{R,\ column} = Tributary\ L_R\ on\ Roof = (13.5\ ft)(20\ ft)(20\ \frac{lbs}{ft^2}) = 5,400\ lbs = 5.4\ Kips$$

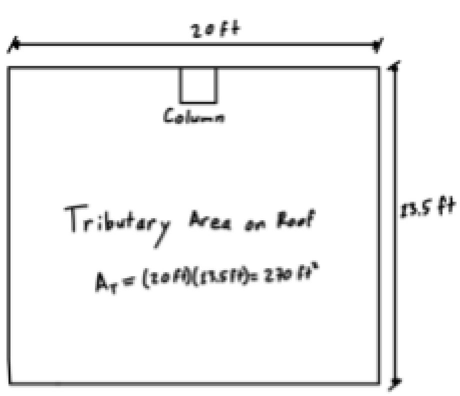


Figure 8. Hand Sketch of Tributary Area on Roof

To compute the total factored load on the column, the LRFD Load Combination Table is again referenced. In this case, load combination one produces the most load.

$$P_{U,\ Column} = (1.4)(412.725\ Kips) = 577.8\ Kips$$

Section 4: SAP Model To Determine Shear and Moment in Beam and Column

Beam 27.

First, the moment on the beam was determined using SAP 2000. This model was simple as only a two-dimensional structural model had to be used because the beam was fixed to columns at either end (this was later cross-checked with a 3-D model which produced similar loads). In order to get an accurate result, the dimensions of the beam and columns were set as well as default reinforcement (which should not have significant impact on the shear and moment applied to the beam). Furthermore, the connections were determined to be fixed-end, making accurate hand calculations difficult.

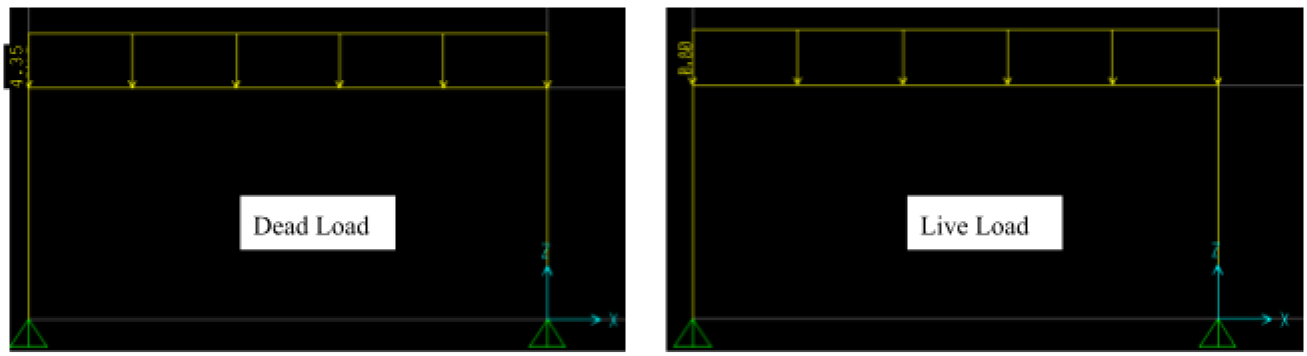


Figure 9. Frame and Loads in SAP 2000

After running the analysis in SAP 2000, the following results were obtained:

$|Shear\ at\ Either\ End\ of\ the\ Beam|$ = 87.82 Kips

$|Shear\ in\ the\ Middle\ of\ the\ Beam|$ = 0 Kips

$Moment\ at\ Either\ End\ of\ the\ Beam\ =\ -171.31\ Kips$

$Moment\ in\ the\ Middle\ of\ the\ Beam\ =\ 421.48\ Kips$

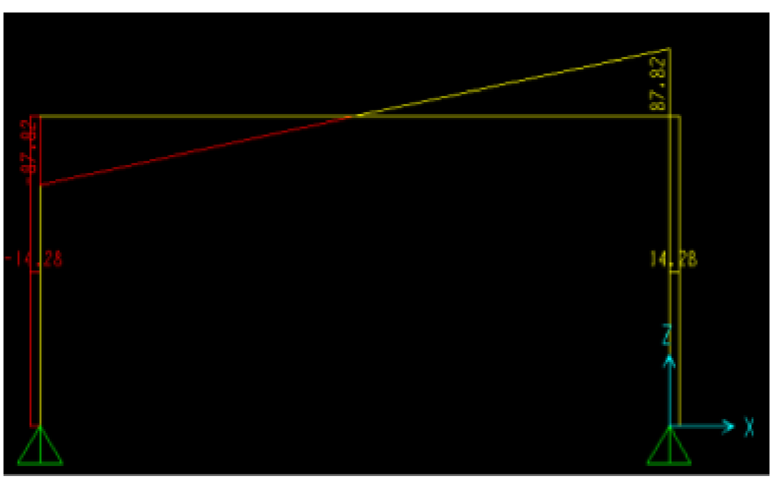


Figure 10. Shear Diagram for Beam

In order to confidently use the results obtained from the SAP 2000 analysis, we briefly assessed whether SAP's calculations made sense with manual calculations. The shear result appears to be accurate as the shear at either end of the beam should be close to the below calculations.

$$V_{Beam} = \frac{(Distributed\ Load)(Length)}{2}$$

$$V_{Beam} = \frac{(6.5\frac{Kips}{ft})(27\ ft)}{2} = 87.75\ Kips$$

The bending diagram produced by SAP 2000 appears to be mostly accurate, however the distribution of moment along the beam should be parabolic, not linear. To check the accuracy of SAP's results a brief hand calculation for the moment at the midspan of a fixed-fixed beam is shown below.

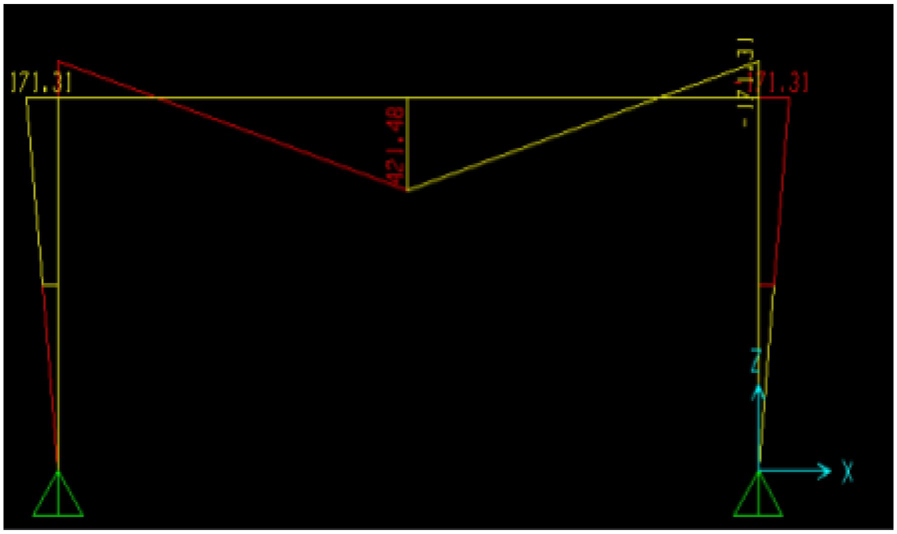


Figure 11. Bending Moment Diagram for Beam

$$Moment_{Midspan} = \frac{w_u L^2}{12} = \frac{(6.5\,\frac{Kips}{ft})(27\,ft)^2}{12} = 394\,Kips - ft$$

This result is similar to that obtained by SAP 2000, SAP's result is likely more accurate as it accounts for the geometry of the elements.

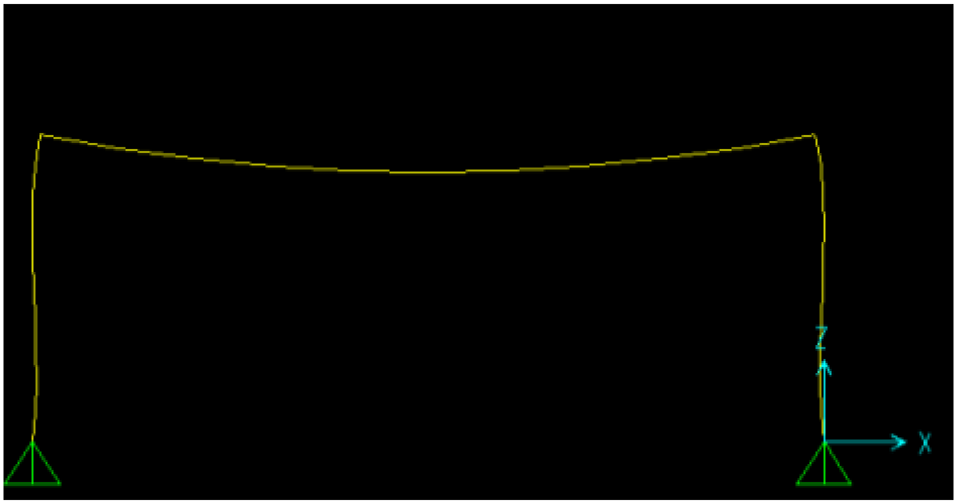


Figure 12. Deflection

Column

In order to accurately determine the shear and moment acting on the column, a 3-Dimensional SAP 2000 model is required. A model of the relevant structural elements of floor two was created in SAP, and the loads from the column self weight and the floors above were added as a point load to the column directly. The applied dead-load point load is equal to the total dead load minus the load of floor 2, the snow load was applied to the column as a point load, and the roof live load was also applied directly to the column as a point load. While this is not a perfect representation of the forces acting on the column, it is sufficient for design.

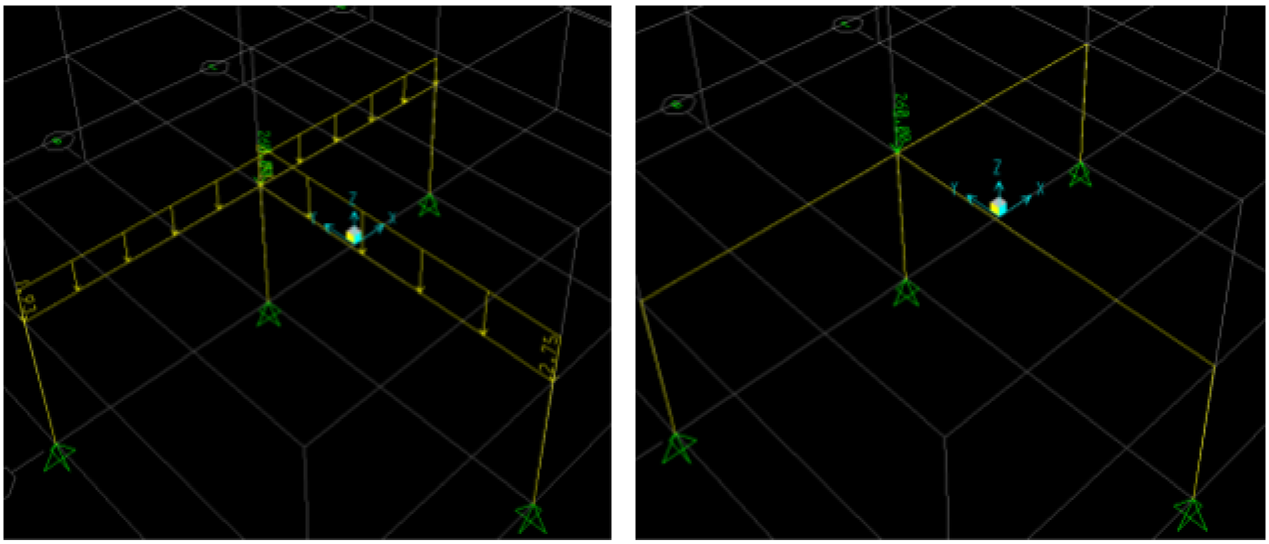


Figure 13. Dead Loads on Bay 255

Figure 11 shows the assigned dead loads for the 3-D model of the bay (and outside-of-bay beam). Snow, live, and roof live loads were also added to the SAP 2000 model. Different elements use different load combinations, and the governing load combination was determined by SAP 2000. For the column, the governing load combination was combination 1.

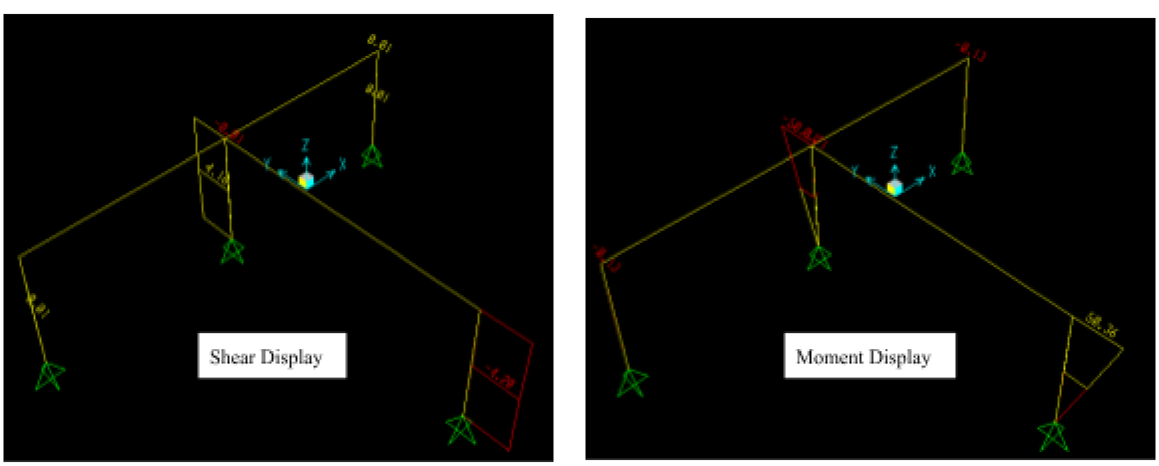


Figure 13. Shear and Moment Displays From SAP 2000

The results from SAP 2000 are shown below:

$$Maximum\ Moment = -50.01\ Kips - ft = -600\ Kips - in$$
$$Distributed\ Shear = 4.18\ Kips$$
$$Axial\ Force = 466.1\ Kips$$

The beam moment and shear was also checked with the 3-D diagram and is similar to the results obtained with the 2-D analysis.

At first glance, the shear and moment in the column appear to be extremely low. However, considering the frequent column bracing and the lack of transverse loads, these results are reasonable. The axial force in the column will dictate the design as according to SAP 2000, the axial force in the column is 466.1 Kips. According to manual calculations (in section 3 of this report), $P_U = 578\ Kips$. While SAP 2000 is likely more accurate than manual calculations, the column will be designed to withstand $P_U = 578\ Kips$ as this is a more conservative design. This answer likely differs from SAP 2000, because the load combination in the analysis software was set to maximize moment rather than axial load.

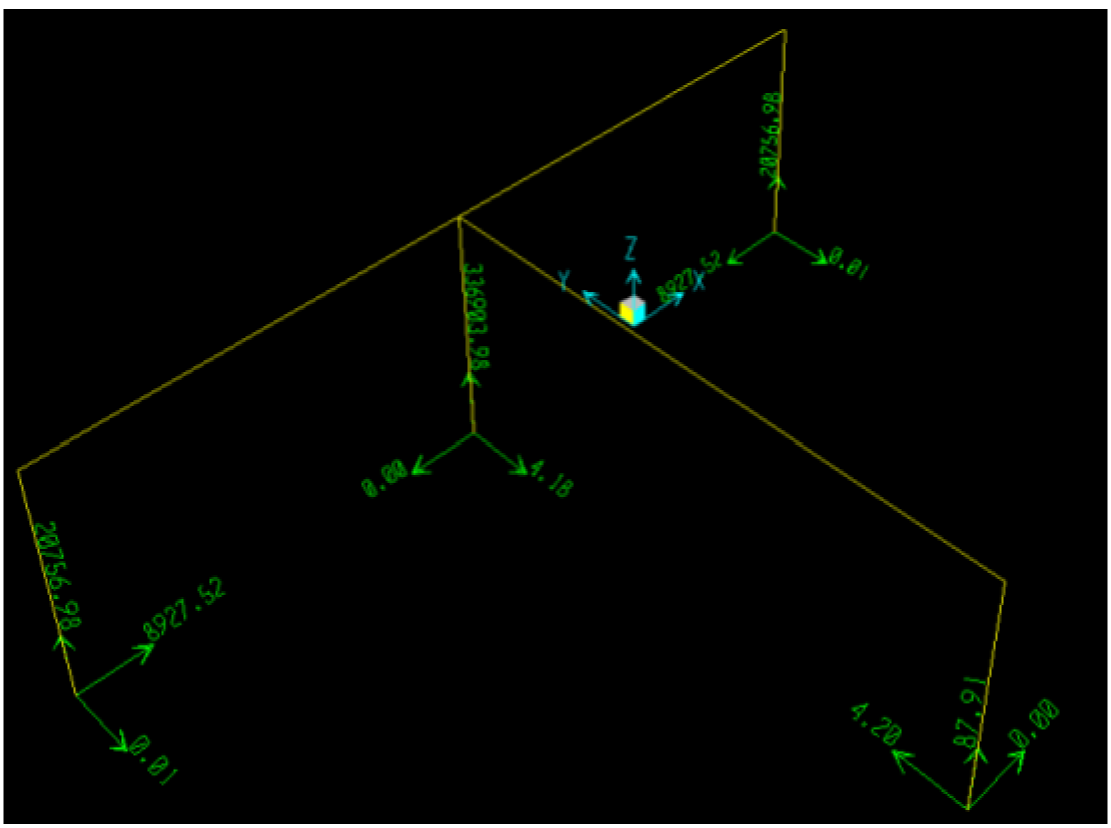


Figure 14. Reactions (Partial Axial Forces)

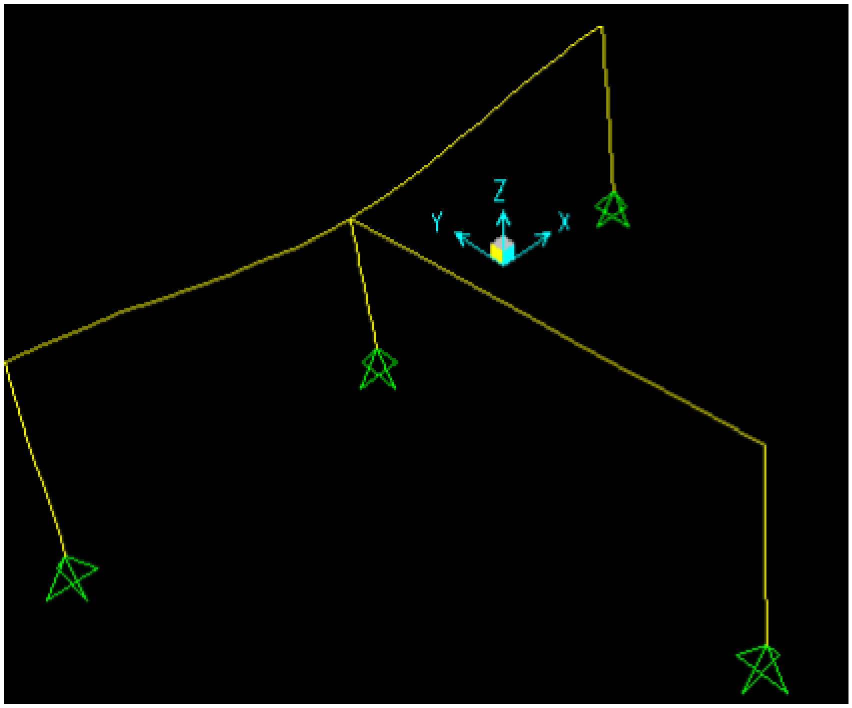


Figure 15. Deformed Structure

Section 5: Design of Structural Elements

Note – the concrete to be used in our design has f'_c=4,000 PSI, and $f_y = 60,000$ PSI.

Reinforced Concrete Slab (Horizontal Element 1)

First, the minimum slab thickness was found using the ACI Code. For this slab, both ends are continuous as the slab section extends into the adjacent classroom bays.

$$h_{min} = \frac{L}{28} = \frac{(20\,ft)(12\,\frac{in.}{ft})}{28} = 8.57\,in$$

This means that a slab thickness of 9 in. should be sufficient for design. The next step in designing the slab was to treat the slab as 27 1-foot thick beams and perform calculations for one "beam" before multiplying for the rest of the slab. The ACI coefficient for moments at the supports and at midspan were found from ACI Table 6.5.2.

Moment	Location	Condition	M_u
Positive	End span	Discontinuous end integral with support	$w_u\ell_n^2/14$
		Discontinuous end unrestrained	$w_u\ell_n^2/11$
	Interior spans	All	$w_u\ell_n^2/16$
Negative[1]	Interior face of exterior support	Member built integrally with supporting spandrel beam	$w_u\ell_n^2/24$
		Member built integrally with supporting column	$w_u\ell_n^2/16$
	Exterior face of first interior support	Two spans	$w_u\ell_n^2/9$
		More than two spans	$w_u\ell_n^2/10$
	Face of other supports	All	$w_u\ell_n^2/11$
	Face of all supports satisfying (a) or (b)	(a) slabs with spans not exceeding 10 ft (b) beams where ratio of sum of column stiffnesses to beam stiffness exceeds 8 at each end of span	$w_u\ell_n^2/12$

[1]To calculate negative moments, ℓ_n shall be the average of the adjacent clear span lengths.

Figure 16. ACI Table 6.5.2

This slab is an interior slab with more than two spans, so the maximum moment for design is $\frac{(w_u)(L^2)}{16}$ at midspan, and $\frac{(w_u)(L^2)}{11}$ near the supports where the moment is negative.

$$W_u = (235\,\frac{lbs}{ft^2})(1\,ft) = 235\,\frac{lbs}{ft}$$

$$M_{u,\,midspan} = \frac{(235\,\frac{lbs}{ft})(20\,ft)^2}{(16)} = 5.875\,kips - ft$$

$$M_{u,\,supports} = \frac{(235\,\frac{lbs}{ft})(20\,ft)^2}{(11)} = 8.55\ kips - ft$$

$$Depth\ to\ Reinforcement\ = d = h - 1\,in. = 8\,in.$$

Next, the flexural resistance factor is found in order to find ρ.

$$R_{Midspan} = \frac{M_U}{(\phi)(b)(d)^2} = \frac{(5.875\ kips-ft)(12\,\frac{in.}{ft})}{(.9)(12\,in.)(8\,in)^2} = 102\ \frac{lbs}{in^2}$$

$$R_{Supports} = \frac{M_U}{(\phi)(b)(d)^2} = \frac{(8.55\ kips-ft)(12\,\frac{in.}{ft})}{(.9)(12\,in.)(8\,in)^2} = 148\ \frac{lbs}{in^2}$$

TABLE A.5a
Flexural resistance factor: $R = \rho f_y\left(1 - 0.588\frac{\rho f_y}{f'_c}\right) = \frac{M_n}{bd^2} = \frac{M_u}{\phi bd^2}$, **psi**

	f_y = 40,000 psi				f_y = 60,000 psi				f_y = 80,000 psi			
	f'_c, psi				f'_c, psi				f'_c, psi			
ρ	3000	4000	5000	6000	3000	4000	5000	6000	3000	4000	5000	6000
0.0005	20	20	20	20	30	30	30	30	40	40	40	40
0.0010	40	40	40	40	59	59	60	60	79	79	79	79
0.0015	59	59	60	60	88	89	89	89	117	118	118	119
0.0020	79	79	79	79	117	118	118	119	155	156	157	157
0.0025	98	99	99	99	146	147	147	148	192	194	195	196
0.0030	117	118	118	119	174	175	176	177	229	232	233	234
0.0035	136	137	138	138	201	204	205	206	265	268	271	272

Figure 17. Flexural Resistance Factor (R) Table

Using the R Table (conservative estimate) to find p

ρ $= .002$ at midspan
ρ $= .0026$ at supports

This is below the minimum ρ $= .0033$, but as long as A_S is above the minimum this is acceptable for design.

$$A_{S,\,Midspan} = \rho bd = (.002)(12\,in.)(8\,in.) = .192\,in^2$$

$$A_{S,\,Supports} = \rho bd = (.0026)(12\,in.)(8\,in.) = .2496\,in^2$$

Table 7.6.1.1—$A_{s,min}$ for nonprestressed one-way slabs

Reinforcement type	f_y, psi	$A_{s,min}$	
Deformed bars	< 60,000	$0.0020A_g$	
Deformed bars or welded wire reinforcement	≥ 60,000	Greater of:	$\frac{0.0018 \times 60{,}000}{f_y}A_g$
			$0.0014A_g$

Figure 18. ACI Table 7.6.1.1

Next, the minimum A_s was checked according to ACI table 7.6.1.1

$$A_{S,\,min} = (.0018)(b)(h) = (.0018)(12\ in.)(9\ in.) = .1944\ in^2$$
$$A_{s,\,min} > A_{s,\,calculated,\,midspan}$$
$$A_{s,\,min} < A_{s,\,calculated,\,supports}$$

The minimum area of steel, $A_{s,\,min}$, will be used at the midspan, while the calculated area of steel will be used for design of reinforcement near the supports. Before finalizing a reinforcement design, the slab was checked to make sure it could withstand the shear forces acting on it.

$$V_U = \frac{(W_U)(L)}{2} = \frac{(235\ \frac{lbs}{ft})(20\ ft)}{2} = 2.35\ Kips$$

$$\phi V_C = \phi 2\lambda\sqrt{f'_c}bd = (.75)(2)(1)(\sqrt{4,000\frac{lbs}{in^2}})(12\ in)(8\ in) = 9.107\ Kips$$

$\phi V_C >> V_U$ so there is no shear reinforcement needed in the slab.

Finally, a final design of reinforcement was selected for the slab. For this slab, the moment is positive near the midspan and negative near the slab-beam interface. This means that the required reinforcement found from the maximum moments will be near the bottom (8 inches from the top) in the middle of the beam, and near the top (8 inches from the bottom) near the supports. To determine where the change in reinforcement occurs, SAP 2000 was once again used. This time to model an imaginary one-foot-wide beam spanning 20 feet. It was determined that the vast majority of the slab is in positive bending aside from at the very ends. For a conservative design, the middle 12 feet of the beam should be reinforced for positive bending, while the four feet near either support should be reinforced for negative bending. In addition, a significant overlap was added. See the Rebar Schedule for detailed information.

TABLE A.3[5]
Areas of bars in slabs, in²/ft

Spacing, in.	Inch-Pound: / SI:	Bar No. 3 / 10	4 / 13	5 / 16	6 / 19	7 / 22	8 / 25	9 / 29	10 / 32	11 / 36
3		0.44	0.78	1.23	1.77	2.40	3.14	4.00	5.06	6.25
3½		0.38	0.67	1.05	1.51	2.06	2.69	3.43	4.34	5.36
4		0.33	0.59	0.92	1.32	1.80	2.36	3.00	3.80	4.68
4½		0.29	0.52	0.82	1.18	1.60	2.09	2.67	3.37	4.17
5		0.26	0.47	0.74	1.06	1.44	1.88	2.40	3.04	3.75
5½		0.24	0.43	0.67	0.96	1.31	1.71	2.18	2.76	3.41
6		0.22	0.39	0.61	0.88	1.20	1.57	2.00	2.53	3.12
6½		0.20	0.36	0.57	0.82	1.11	1.45	1.85	2.34	2.89
7		0.19	0.34	0.53	0.76	1.03	1.35	1.71	2.17	2.68
7½		0.18	0.31	0.49	0.71	0.96	1.26	1.60	2.02	2.50
8		0.17	0.29	0.46	0.66	0.90	1.18	1.50	1.89	2.34
9		0.15	0.26	0.41	0.59	0.80	1.05	1.33	1.69	2.08
10		0.13	0.24	0.37	0.53	0.72	0.94	1.20	1.52	1.87
12		0.11	0.20	0.31	0.44	0.60	0.78	1.00	1.27	1.56

Figure 19. Areas of Bars in Slabs Table

Longitudinal Reinforcement was selected from table A.3. Near the supports, the reinforcement should consist of #4 bars at 9 inch spacing, while the reinforcement at midspan is #4 bars at 12 inch spacing.

To finish the calculations, ρ still needs to be compared to $\rho_{.005}$ to ensure that the assumption of $\phi = .9$ was accurate. Only the ρ near the supports (where A_s is highest) will be compared to $\rho_{.005}$.

$$\rho = \frac{A_s}{bd} = \frac{.26\ in^2}{(12\ in)(8\ in)} = .0027 < \rho_{.005} = .0181$$

The assumption of tension controlled was accurate and $\phi = .9$ so no change to the design is necessary.

Beam 27 (Longitudinal Element 2)

First, the depth to the steel reinforcement was assumed to be $h - 1.5$, $d = 19.5\ in$.

The factored moment, M_u, at the middle of the beam and at the ends of the beam was used to calculate the required area of longitudinal steel reinforcement.

Using the known dimensions and material properties of the beam, we set up the following equations:

$M_u = \Phi M_n = A_s f_y (d - a/2)$.

$a = (A_s f_y)/(0.85 * f'_c * b)$

All values in the previous two equations are known except for the area of longitudinal shear reinforcement, A_s. The factored moment, M_u, is obtained from the SAP 2000 analysis. The following A_s values were obtained using Excel:

Middle of the beam: $A_s = 5.2\ in^2$

Ends of the beam: $A_s = 2.1\ in^2$

Afterwards, the reinforcement ratios were checked to make sure that a phi factor of 0.9 is appropriate. For the reinforcement at the midspan of the beam, the actual reinforcement ratio fell within acceptable limits. For the reinforcement at the ends of the beams, the actual reinforcement ratio was less than the minimum reinforcement ratio. Because of this, the required area of steel reinforcement was increased to meet the minimum reinforcement ratio requirements.

Ends of the beam: $A_s = 2.3166$

TABLE A.2 [4]

Areas of groups of standard bars, in²

Bar No. Inch-Pound	SI	Number of Bars 1	2	3	4	5	6	7	8	9	10	11	12
4	13	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40
5	16	0.31	0.62	0.93	1.24	1.55	1.86	2.17	2.48	2.79	3.10	3.41	3.72
6	19	0.44	0.88	1.32	1.76	2.20	2.64	3.08	3.52	3.96	4.40	4.84	5.28
7	22	0.60	1.20	1.80	2.40	3.00	3.60	4.20	4.80	5.40	6.00	6.60	7.20
8	25	0.79	1.58	2.37	3.16	3.95	4.74	5.53	6.32	7.11	7.90	8.69	9.48
9	29	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00
10	32	1.27	2.54	3.81	5.08	6.35	7.62	8.89	10.16	11.43	12.70	13.97	15.24
11	36	1.56	3.12	4.68	6.24	7.80	9.36	10.92	12.48	14.04	15.60	17.16	18.72
14	43	2.25	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50	24.75	27.00
18	57	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00	44.00	48.00

Figure 20. Area of Bars in Beams Table

Longitudinal Reinforcement from table A.2

Middle of the beam: 7 #8 bars
Ends of the beam: 3 #8 bars

From the shear diagram, we find that longitudinal reinforcement must be used in the bottom of the beam from $x = 3.93$ ft to $x = 23.07$ ft. Longitudinal reinforcement must be used in the top of the beam everywhere else.

Reinforcement in top of beam: $0\,ft < x < 3.93\,ft,\ 23.07\,ft < x < 27\,ft$
Reinforcement in bottom of beam: $3.93\,ft < x < 23.07\,ft$

Next, the transverse reinforcement must be found using the maximum factored shear load, $V_{u,max}$. $V_{u,max}$ is determined using the following equation:

$$V_{u,max} = V_u - (2V_u)/L * d$$

We know that no transverse reinforcement is required in the part of the beam where the shear is less than $\frac{1}{2}\Phi V_c$. Using the shear diagram produced by SAP 2000, and a phi factor of 0.75 for shear, we determined that no shear reinforcement is necessary from $x = 8.38$ ft to $x = 18.62$ ft.

No shear reinforcement: $8.38\,ft < x < 18.62\,ft$

Using the known dimensions and material properties of the beam, we set up the following equations:

$$V_{u, max} = \Phi V_n = \Phi(V_c + V_s)$$
$$V_c = 2 * \sqrt{f'_c} * b_w * d$$
$$V_s = A_v * F_y * d/s$$

The transverse reinforcement used is #4 bars arranged such that there are 2 legs (stirrups). $A_v = 2 * 0.2 = 0.4$. All values of the previous two equations are known except for the spacing, s. The following spacing was obtained using Excel:

$$s = 32\ in$$

Clearly, this satisfies the minimum spacing requirement, but the maximum spacing requirement still needs to be checked. The maximum spacing that governs the transverse reinforcement is $d/2 = 9.75\ in$. To make sure the concrete does not fail in shear, the maximum spacing will be used in place of the calculated spacing.

$$s_{max} = 9.75\ in$$
$$V_S = 48\ Kips$$

Checks to Ensure Appropriate Design:

$$V_S \leq 8\sqrt{f'_c}bd = 355.2\ Kips$$
$$V_S \leq 4\sqrt{f'_c}bd = 178\ Kips$$

Column (Vertical Element 1)

Similar to the horizontal structural elements, the column must be designed to withstand the calculated load including axial force, moment, and shear. Transverse loads were not considered in this design, so the moment and shear forces on the column are extremely low, and the primary basis for design is axial force.

$P_U = 577.8\ Kips$

$M_U = 600\ Kips - in = 50\ Kips - ft$

$e = \frac{M_U}{P_U} = \frac{50\ Kips - ft}{577.8\ Kips} = .08\ ft$

This column is extremely compression controlled, so distributed reinforcement will be used. $\phi = .65$ was assumed.

$K_U = \frac{P_U}{\phi f'_c A_g}$

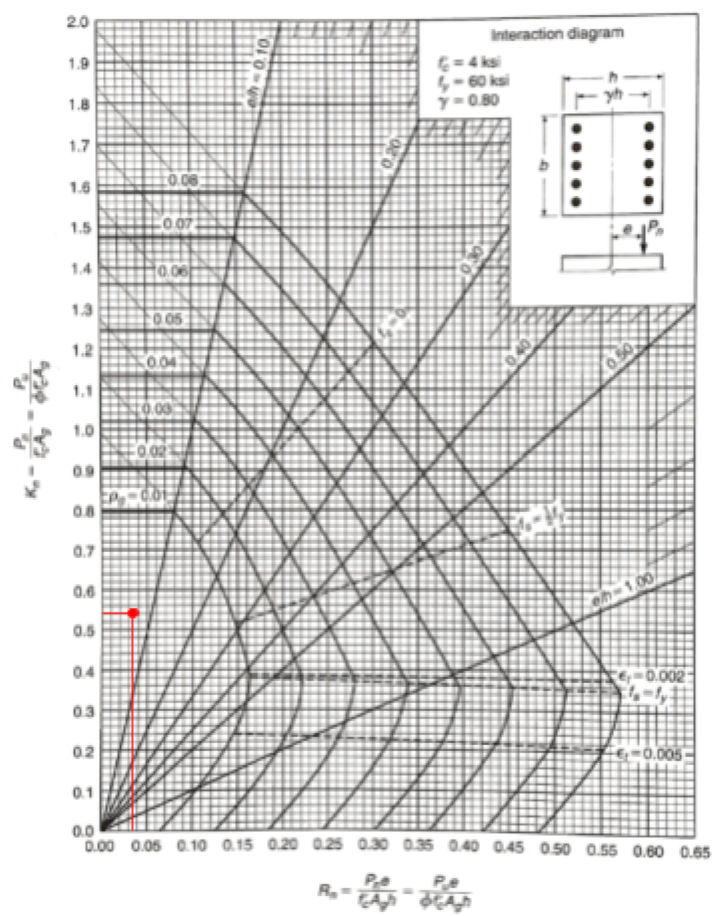
$K_U = \frac{577.8\ Kips}{(.65)(4\ \frac{Kips}{in^2})(20\ in)(20\ in)} = .556$

$R_U = \frac{M_U}{\phi f'_c A_g h}$

$R_U = \frac{600\ Kips - in}{(.65)(4\ \frac{Kips}{in^2})(20\ in)(20\ in)(20\ in)} = .028$

$\gamma = \frac{20\ in - (2)(d)}{(20\ in)} = .8$

$d = 2\ in,\ \gamma = .8$



From this design-aid interaction diagram, the gross reinforcement ratio was found. It is worth noting that the point on this diagram indicates only one (modified) load combination from the SAP model (load combination 1). However, running the structural model for other load combinations lowers the axial load (sometimes significantly), but never increases the moment. So the gross reinforcement ratio remains in the $\rho_g = .01$ section of the graph because the moment is not high enough to push it past the $\rho_g = .01$ line on the chart.

$A_s = (A_g)(\rho_g) = (400\ in^2)(.01) = 4\ in^2$

From this A_S, a preliminary reinforcement design is found. Again referencing Table A2 above to find the areas of rebar, 4 #9 $Bars$ is sufficient for design. However, this result must be compared to the actual interaction diagram.

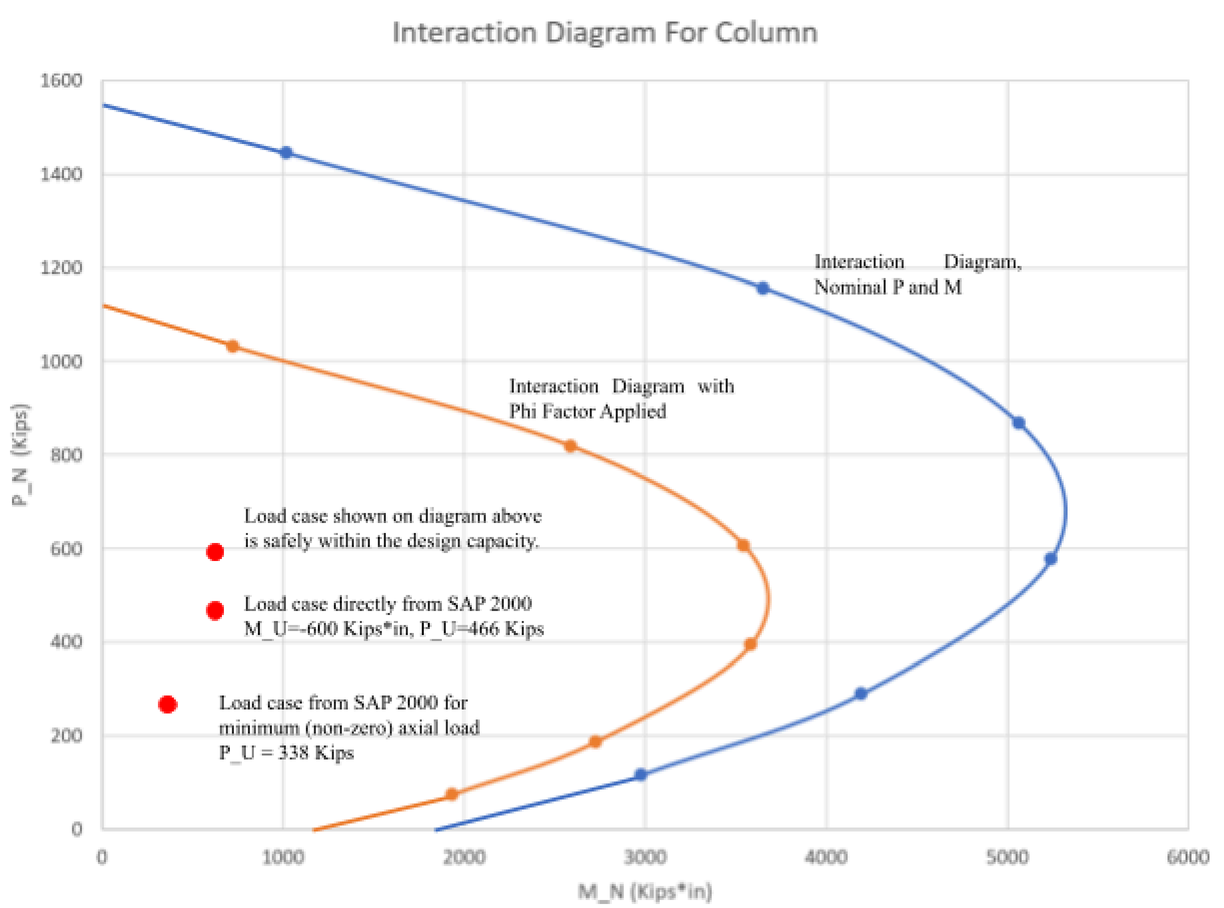


Figure 22. Interaction Diagram for Selected Column Design

The Excel tables used to calculate the interaction diagram are shown in Appendix A. This diagram shows that the selected column longitudinal reinforcement design is sufficient.

The shear force acting on the column was extremely small (4.2 Kips maximum). However, to be thorough with the design of the column, shear capacity was checked.

$$V_U = 4.2\ Kips$$

$$\phi V_C = \phi 2\lambda\sqrt{f'_c}bd = (.75)(2)(1)(\sqrt{4,000\frac{lbs}{in^2}})(20\ in)(18\ in) = 34.15\ Kips$$

$\phi V_C >> V_U$ so there is no shear reinforcement needed in the slab. However, in order to design an adequate column, ties must still be included. ACI specifies that the maximum distance between ties is the minimum of 16 longitudinal bar diameters, 48 tie diameters, or the smaller exterior column dimension. In this case, the maximum distance between ties is $S = (16)(1.128\, in.) = (48)(.375\, in.) = 18\, inches$. The ties for this column are #3 bars spaced at 18 inches.

Section 6: Rebar Schedule

Reinforced Concrete Slab (Horizontal Element 1)

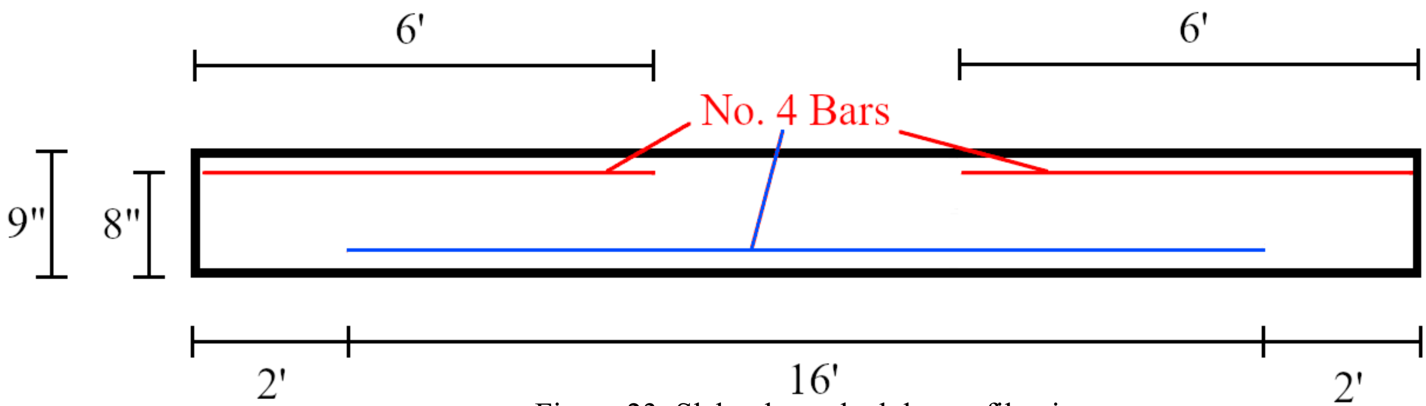


Figure 23: Slab rebar schedule, profile view

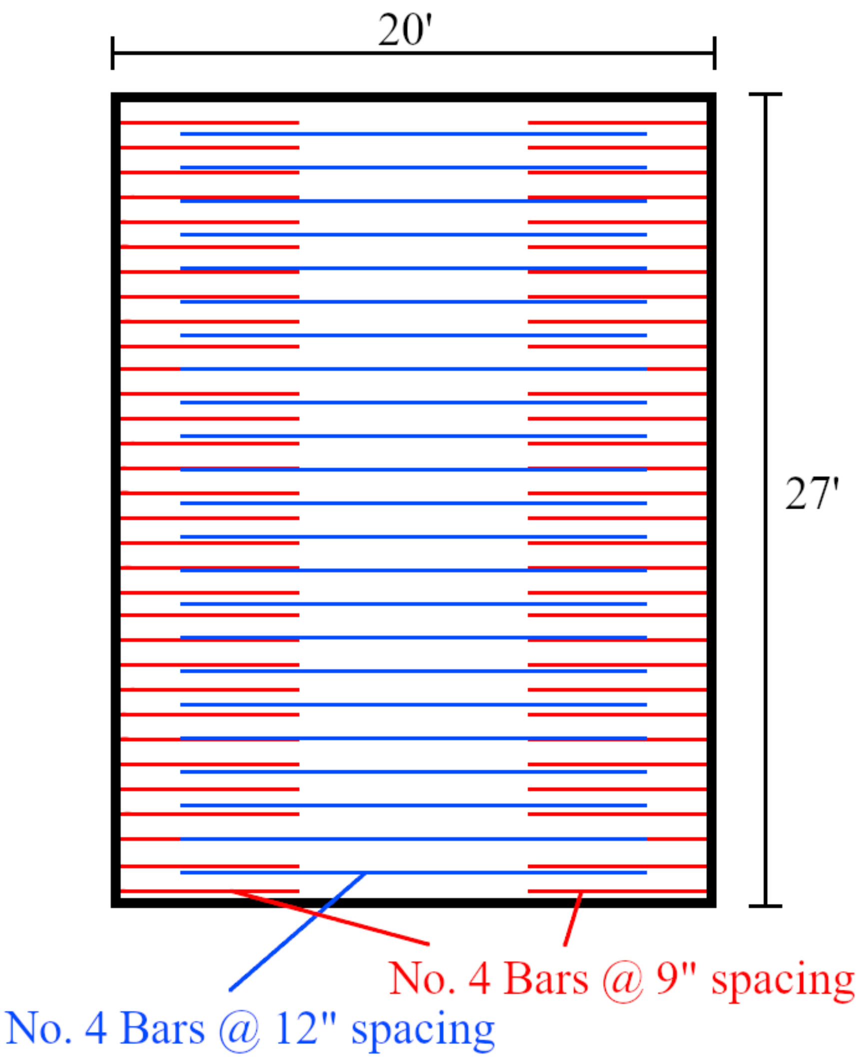


Figure 24: Slab rebar schedule, plan view

Beam 27 (Longitudinal Element 2)

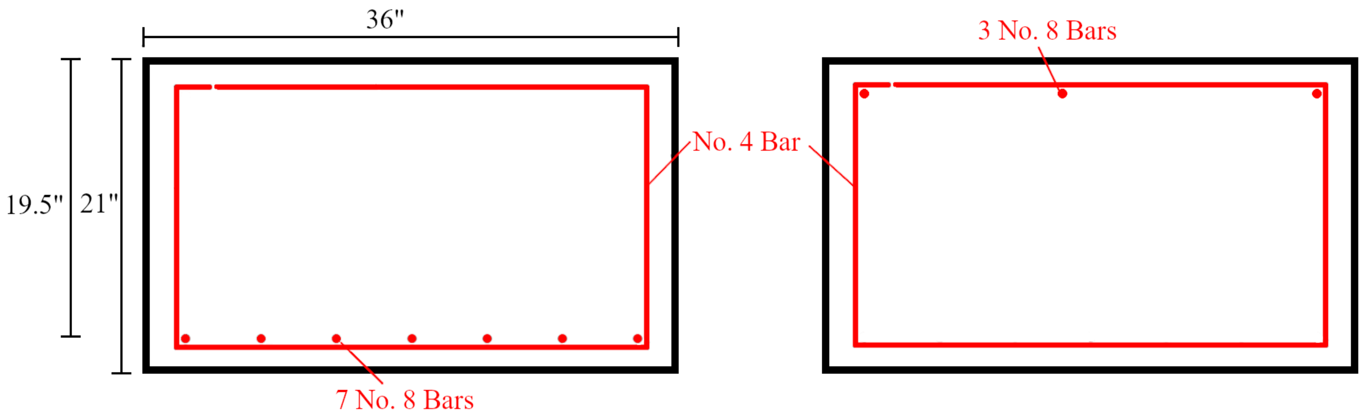


Figure 25: Beam rebar schedule, cross section

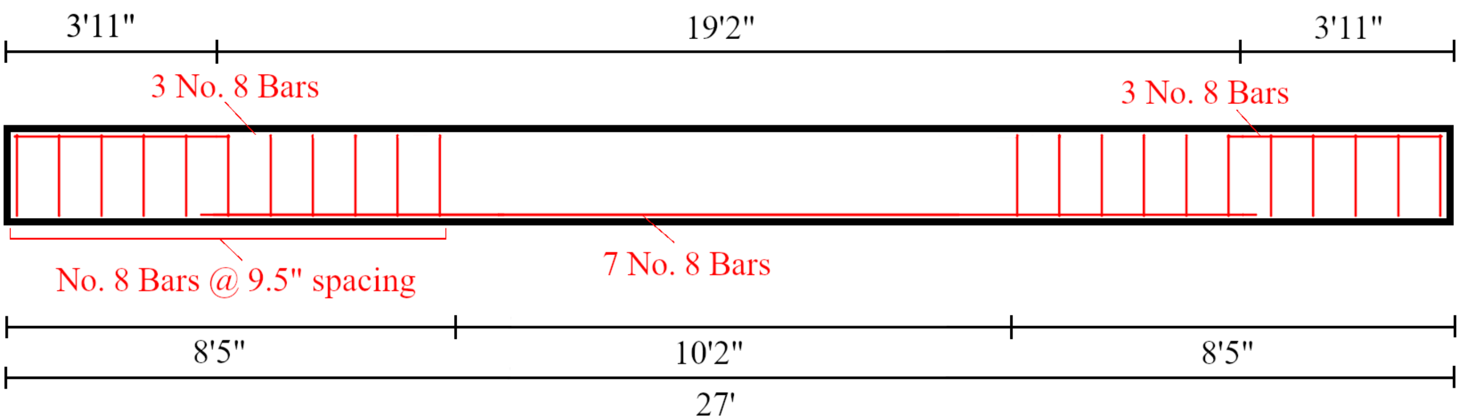


Figure 26: Bar rebar schedule, profile view, overlap top and bottom longitudinal reinforcement 1 foot

Column (Vertical Element 1)

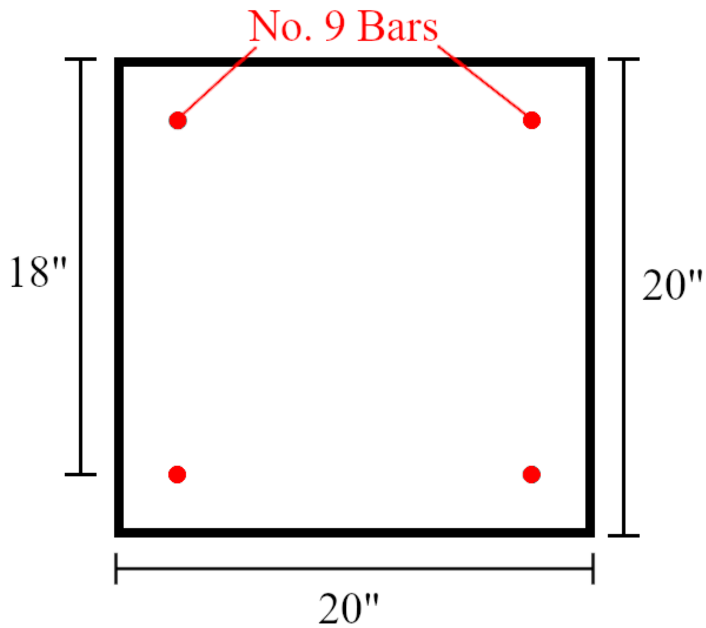


Figure 27: Column rebar schedule, cross section

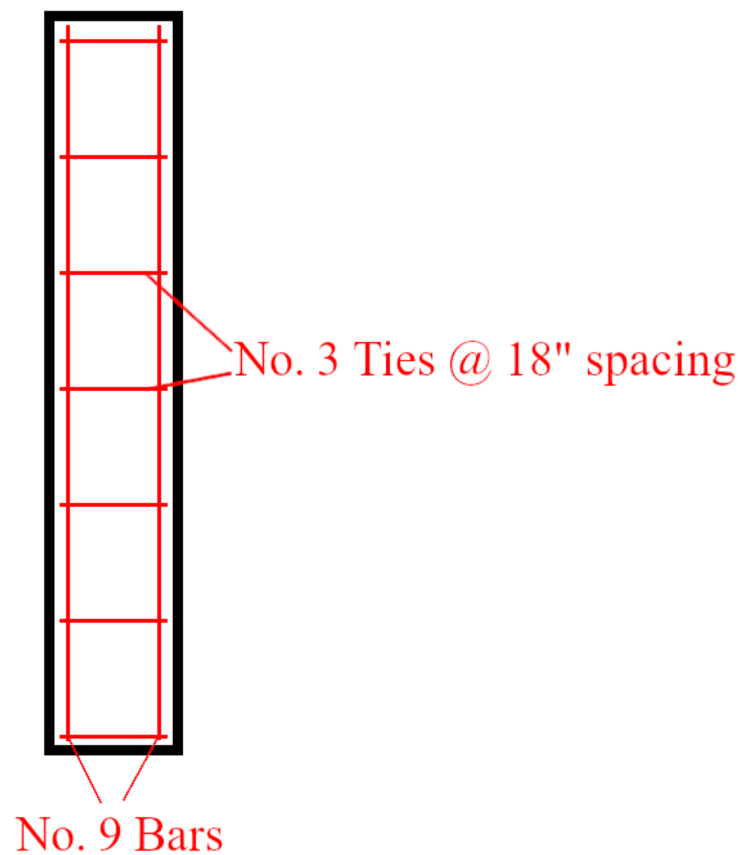


Figure 28: Column rebar schedule, profile view

Section 7: Comments on Design

In our redesign of the structural elements of Bay 255, the dimensions of the beam and column were not changed from the original structural plans, instead the reinforcement was changed to account for our analysis of loads on the structure. However, in the design of both the beam and column, relatively little reinforcement was used. The column required only four #9 longitudinal bars for longitudinal reinforcement, and required the minimum transverse reinforcement. The beam required relatively low reinforcement particularly near the beam-column interface (3 #8 bars), and used the maximum spacing for shear reinforcement. Due to the low requirements for reinforcement, a more economical and energy-efficient design would likely use structural elements with smaller dimensions and higher reinforcement ratios. In fact, when transverse loads are not considered, it might be possible to consolidate the columns and have longer clear spans for the beams. This would save money and be a more efficient design.

In the original design, the structural engineers accounted for transverse loads, which somewhat impacted the design of structural members. In the original design of the column, the reinforcement consists of only four #8 bars for longitudinal reinforcement and #3 bars spaced every 16 inches for transverse reinforcement. This means that the original design also had fairly low reinforcement requirements even when accounting for additional transverse loads. In the original design, the column was only 16 X 16 inches on the second floor. Our design was actually more conservative (and less efficient) than the original design. It is likely that our low reinforcement ratio requirements (especially in the column) were a result of not accounting for the increased shear and moment forces from lateral loadings. Lateral loads could include wind and temperature loads, as well as interaction between structural elements of the building.

In the original beam design, Beam #27 had the same dimensions as our design but was reinforced longitudinally with five #8 bars in the bottom and three #7 bars in the top. From the plans, it appears that Martin and Martin accounted for compression reinforcement, which was not included in our design. The stirrups in the original design were #4 bars at nine inch spacing. Overall our beam design was very similar to the original design. It is likely that we overestimated the moment on the beam and underestimated the shear when comparing our design to the original beam.

There were several inefficiencies in our design, some were unique to just our redesign while some were present in the original design as well. The slab was the most inefficient aspect of the design. While it is sufficient to withstand the required loads, it is much thicker than a typical floor slab (4-6 inches). A 9 inch slab on the second floor of the building would be difficult to pour and extremely heavy. In the original design, Martin and Martin used a joist system for the floor, which was likely more efficient. Due to our lack of experience with concrete joists, we used the slab. Furthermore, an ideal beam design would have a height equal to two-to-three times the length of the width. In our design, the width of the beam was 36 inches, while the height was only 21 inches, which is not a good design from a structural standpoint. However, this design was used in the original building likely to conserve vertical space.

Overall, our design works and is fairly straightforward and reasonable. Aside from the slab, no excessively large structural members were required and the amount of reinforcement was well within reasonable limits. If a more in-depth analysis was used to account for transverse loads the required structural design would change significantly, but for the scope of this project, the selected structural member designs were adequate.

Appendix A

Excel Tables For Beam Design (Bending Moment)

Mu (k*ft)	As	Fy (psi)	Fs (psi)	b (in)	d (in)	a/2 (in)	Mn (lb*in)	Mn (k*ft)	ρ0.005	ρactual	ρmin	phi	phi*Mn
421.48	5.2	60000	4000	36	19.5	1.2745098	5686352.94	473.862745	0.0181	0.00740741	0.0033	0.9	426.476471
-171.31	2.3166	60000	4000	36	19.5	0.56779412	2631500.89	219.291741	0.0181	0.0033	0.0033	0.9	197.362567

Excel Tables For Beam Design (Shear)

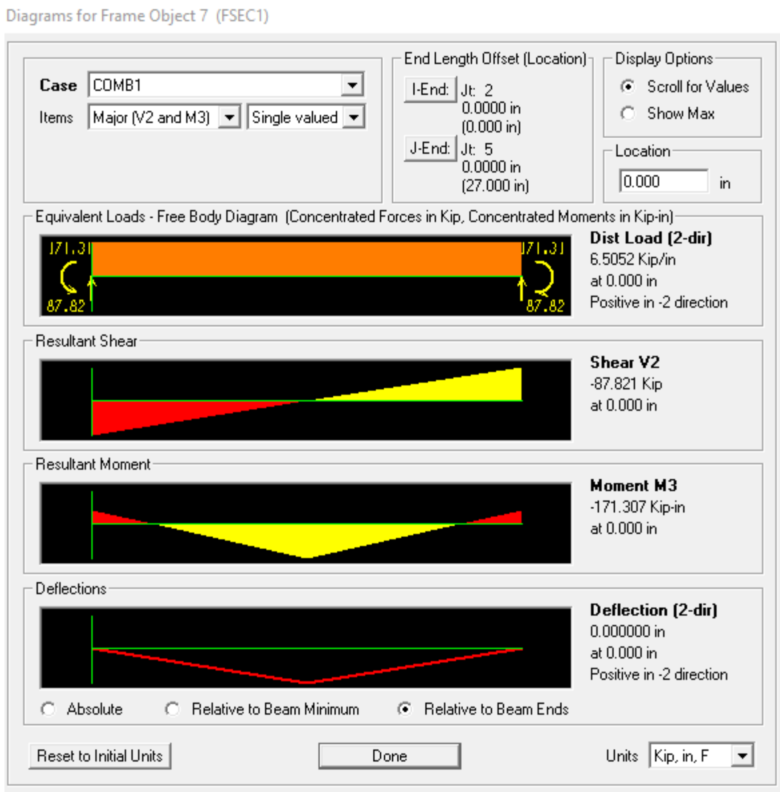
Vu (kips)	Vu, max (kips)	Av	Fy (psi)	Fs (psi)	b (in)	d (in)	s (in)	smax (in)	Vs (lbs)	Vc (kips)	1/2 phi Vc	Vn (kips)	phi	phi Vs	phi Vc	phi*Vn
87.82	77.25	0.4	60000	4000	36	19.5	32	9.75	48	88.7967567	33.2987838	136.796757	0.75	36	66.5975675	102.597568

Excel Tables For Calculation of Interaction Diagram

As (One Side) (in^2)	Fy (KSI)	d (In)	h (in)	d' (In)	Beta	Es (KSI)	Epsilon (u)	f'c (KSI)	b (In)
2	60	18	20	2	0.85	29000	0.003	4	20
f's	fs	cb	ab	Cconc	Pb	Epsilon (y)	Mb	eb	Strain y
70.66666667	60	10.65306	9.055102	615.7469	615.7469	483.3333	5289.644	8.590613	0.002069

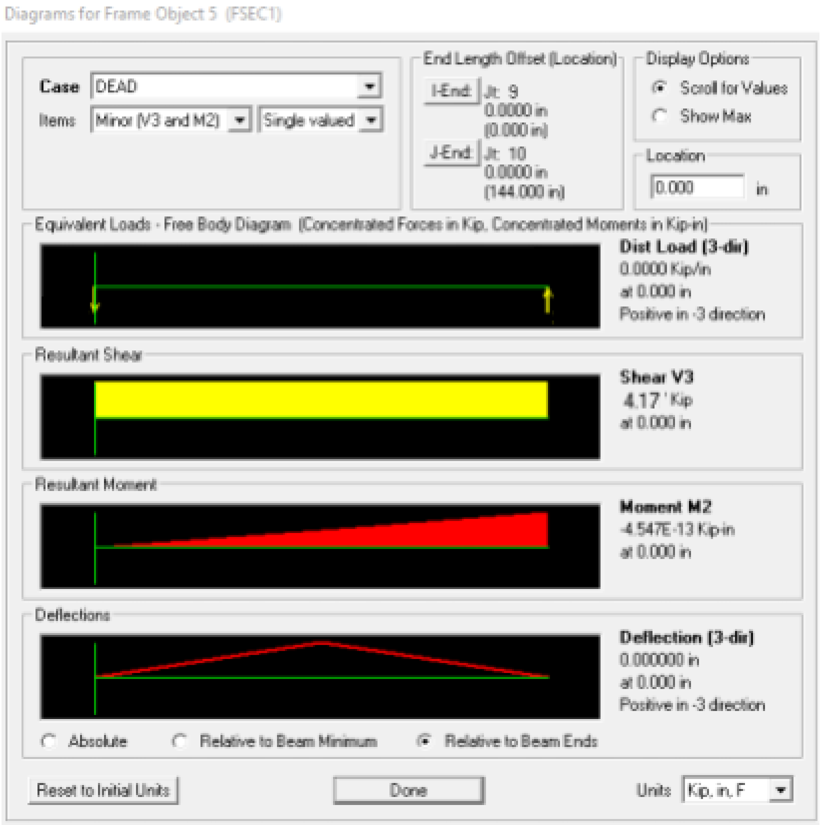
C	a	Strain s'	Cconc	fs'	P_N	M_N	es'	Phi	Phi*M_N	Phi*P_N
2	1.7	0	115.6	0	115.6	2977.74	0	0.65	1935.531	75.14
5	4.25	0.0018	289	52.2	289	4195.875	0.0018	0.65	2727.319	187.85
10	8.5	0.0024	578	69.6	578	5243.5	0.0024	0.683333	3583.058	394.9667
15	12.75	0.0026	867	75.4	867	5062.875	0.0026	0.7	3544.013	606.9
20	17	0.0027	1156	78.3	1156	3654	0.0027	0.708333	2588.25	818.8333
25	21.25	0.00276	1445	80.04	1445	1016.875	0.00276	0.713333	725.3708	1030.767

Results Table From 2-D Beam Analysis SAP 2000



Note that due to the way the model was built the results in Kips-in above are actually in Kips-ft.

Results Table From 3-D Column Analysis SAP 2000



Note – the maximum moment is not displayed in the results table above, as it is displaying results at 0.00 in which is not the location of maximum moment.

References

"New Cu-Boulder Geology Building to Be Dedicated on Oct. 30." *CU Boulder Today*, 20 July 2016, www.colorado.edu/today/1997/10/20/new-cu-boulder-geology-buIlding-be-dedicated-oct-30.