Let $w^\star$ be the ground truth score vector.

Let $P^\star$ be the ideal matrix where $P^\star_{ij} = \frac{w^\star_i}{w^\star_i + w^\star_j}$

We construct a synthetic data matrix P which approximates $P^\star$, so $P_{ij} \approx \frac{w^\star_i}{w^\star_i + w^\star_j}$

Our goal is to construct a vector w that approximates the true vector $w^\star$. To do this, we first apply a link function $\psi()$ to P, such that $\hat{P} = \psi(P)$ and $\hat{P}_{ij} \approx \log(w^\star_i) - \log(w^\star_j)$

Let Y represent the link-transformed ideal $P^\star$ matrix: $Y = \psi(P^\star)$, so $Y_{ij} = \log(w^\star_i) - \log(w^\star_j)$.

Then, the optimal estimated vector w is that which minimizes the difference between Y and $\hat{P}$.

Thus, we define our optimization problem as follows: $\min ||Y - \hat{P}||_1$

We can further establish the constraints of this problem. First, Let $Z = Y - \hat{P}$ and let $x = \log(w)$

$$\min ||Z||_1$$

$$\min \left[\sum_{i=1}^{n} \sum_{j=1}^{n} Z_{ij} \right]$$

$$-Z_{ij} \leq Y_{ij} - \hat{P}_{ij} \leq Z_{ij}$$

$$-Z_{ij} \leq [\log(w_i) - \log(w_j)] - \hat{P}_{ij} \leq Z_{ij}$$

$$-Z_{ij} \leq x_i - x_j - \hat{P}_{ij} \leq Z_{ij}$$

Now, we can define our problem as $\min ||Z||_1$, with the following constraints:

$$x_i - x_j - \hat{P}_{ij} + Z_{ij} \leq Z_{ij}$$

$$x_i - x_j - \hat{P}_{ij} - Z_{ij} \geq -Z_{ij}$$