R4Stats

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Contents

```
require(UsingR)
```

```
## Loading required package: UsingR

## Loading required package: MASS

## Loading required package: HistData

## Loading required package: Hmisc

## Loading required package: lattice

## Loading required package: survival

## Loading required package: Formula

## Loading required package: ggplot2

##
## Attaching package: 'Hmisc'

## The following objects are masked from 'package:base':
##
##     format.pval, units

##
## Attaching package: 'UsingR'

## The following object is masked from 'package:survival':
##
##     cancer
```

About

This is a *sample* book written in **Markdown**. You can use anything that Pandoc's Markdown supports; for example, a math equation $a^2 + b^2 = c^2$.

Usage

Each **bookdown** chapter is an .Rmd file, and each .Rmd file can contain one (and only one) chapter. A chapter *must* start with a first-level heading: `# A good chapter`, and can contain one (and only one) first-level heading.

Use second-level and higher headings within chapters like: `## A short section` or `### An even shorter section`.

The `index.Rmd` file is required, and is also your first book chapter. It will be the homepage when you render the book.

Render book

You can render the HTML version of this example book without changing anything:

1. Find the **Build** pane in the RStudio IDE, and

2. Click on **Build Book**, then select your output format, or select "All formats" if you'd like to use multiple formats from the same book source files.

Or build the book from the R console:

```
bookdown::render_book()
```

To render this example to PDF as a `bookdown::pdf_book`, you'll need to install XeLaTeX. You are recommended to install TinyTeX (which includes XeLaTeX): https://yihui.org/tinytex/.

Preview book

As you work, you may start a local server to live preview this HTML book. This preview will update as you edit the book when you save individual .Rmd files. You can start the server in a work session by using the RStudio add-in "Preview book", or from the R console:

```
bookdown::serve_book()
```

Chapter 1

Getting Started with R

1.1 Problems

```
1+2*(3+4)
```

```
## [1] 15
```

```
(4^3)+(3^(2+1))
```

```
## [1] 91
```

```
sqrt(4+3)*(2+1)
```

```
## [1] 7.937254
```

```
(1+(2*3^4))/(5/6)-7
```

```
## [1] 188.6
```

```
(0.25 - 0.2) / (0.2 * (1 - 0.2)/100)^(1/2)
```

```
## [1] 1.25
```

```r
x <- 2
y <- 3
z <- 4
w <- 5

x*y*z*w
```

```
## [1] 120
```

```r
rivers
```

```
##   [1]  735  320  325  392  524  450 1459  135  465  600  330  336  280  315  870
##  [16]  906  202  329  290 1000  600  505 1450  840 1243  890  350  407  286  280
##  [31]  525  720  390  250  327  230  265  850  210  630  260  230  360  730  600
##  [46]  306  390  420  291  710  340  217  281  352  259  250  470  680  570  350
##  [61]  300  560  900  625  332 2348 1171 3710 2315 2533  780  280  410  460  260
##  [76]  255  431  350  760  618  338  981 1306  500  696  605  250  411 1054  735
##  [91]  233  435  490  310  460  383  375 1270  545  445 1885  380  300  380  377
## [106]  425  276  210  800  420  350  360  538 1100 1205  314  237  610  360  540
## [121] 1038  424  310  300  444  301  268  620  215  652  900  525  246  360  529
## [136]  500  720  270  430  671 1770
```

```r
Orange
```

```
##    Tree  age circumference
## 1     1  118            30
## 2     1  484            58
## 3     1  664            87
## 4     1 1004           115
## 5     1 1231           120
## 6     1 1372           142
## 7     1 1582           145
## 8     2  118            33
## 9     2  484            69
## 10    2  664           111
## 11    2 1004           156
## 12    2 1231           172
## 13    2 1372           203
## 14    2 1582           203
## 15    3  118            30
## 16    3  484            51
## 17    3  664            75
## 18    3 1004           108
```

```
## 19    3 1231            115
## 20    3 1372            139
## 21    3 1582            140
## 22    4  118             32
## 23    4  484             62
## 24    4  664            112
## 25    4 1004            167
## 26    4 1231            179
## 27    4 1372            209
## 28    4 1582            214
## 29    5  118             30
## 30    5  484             49
## 31    5  664             81
## 32    5 1004            125
## 33    5 1231            142
## 34    5 1372            174
## 35    5 1582            177
```

```
mean(Orange$age)
```

```
## [1] 922.1429
```

```
max(Orange$circumference)
```

```
## [1] 214
```

Chapter 2

Univariate data

Levels of measurement

The view in most textbooks is from Stanley Smith Stevens (1964)

Definition 2.1. Nominal
Such data is qualitative or descriptive, but not numeric. An example might be the name of a person or the town they are from, or the number on a bib a runner wears in a race.

Definition 2.2. Ordinal
Ordinal data is data with some order, so that we can sort the data from largest to smallest. An example might be the place a runner takes in a race.

Definition 2.3. Interval
Interval data is ordinal data where the difference between two values has some interpretation. The clock time a person finishes might be an example. If we know runner a finishes at noon and runner B at 1PM then we know that runner B took longer. Since we haven't specified when they started, we don't know what percent longer though.

Definition 2.4. Ratio
Ration data has a meaningful 0. If we record not the time of finishing, but the time since starting, then 0 has a meaning and we can take a ration of the total time for runner A and B to compare the two.

However, working with data on a computer is different, requiring different categories…

Definition 2.5. Factor
When we look at many variables, some may simply record categories used to group the data. In R we will use *factors* to store these variables. An example might be the browser a user has used to view a website, as gleaned from a log.

Definition 2.6. character
Some categorical data are factors, but others are really just identifiers, and are not used for grouping. An example might be a user's IP address. Difference can be thought of as distinguishing between *categorizing* a case or *characterizing* a case. While both factor and categorical data are *nominal*, we keep the distinction as we will interact with the data differently.

Definition 2.7. discrete
Discrete data comes from measurements where there are essentially only distinct and separate possible values that can be counted. For example, the number of visits a person makes to a website will always be integer data, as will other counting data.

Definition 2.8. continuous
Data which could conceivably come from a continuum of variables. The recording of time in milliseconds of a visit to a website might be such data. A useful distinction is that for discrete data we expect that cases will share values, whereas for continuous data this will be impossible, or at least very unlikely. We can also turn continuous data into discrete data by truncating (record the minute instead of the millisecond) or by binning.Rather than draw distinctions between ordinal, interval, and ratio, it is more important for statistical theory - in finding a model for the recorded data - to know if the data is discrete or continuous.

Definition 2.9. time and date
Though we just saw that time and date can be considered continuous or discrete, for computers there are often separate ways to handle date and time data. Issues that complicate matters are leap days and time zones, but also scale (some people want millisecond data)

Definition 2.10. hierarchical
while much data is several measurements for several cases and fits nicely onto a rectangular spreadsheet, data on networks does not fit this

2.1 Data Vectors

Suppose the number of whale beachings in Texas during the 1990s was

`74 122 235 111 292 111 211 133 156 79`

We can combine these into a data set through

```r
whale <- c(74, 122, 235, 111, 292, 111, 211, 133, 156, 79)
```

The `whale` object is a *data vector*.

the size of the data set is retreived with the `length` function

```
length(whale)
```

```
## [1] 10
```

```
sum(whale)
```

```
## [1] 1524
```

Average can be found with combining the two...

```
sum(whale)/length(whale)
```

```
## [1] 152.4
```

or

```
mean(whale)
```

```
## [1] 152.4
```

2.1.1 Vectorization

The arithmetic operations and the mathematical functions are vectorized, in that they will be called for each element in a data vector.

```
whale - mean(whale)
```

```
##  [1] -78.4 -30.4  82.6 -41.4 139.6 -41.4  58.6 -19.4   3.6 -73.4
```

```
whale^2 / length(whale)
```

```
##  [1]  547.6 1488.4 5522.5 1232.1 8526.4 1232.1 4452.1 1768.9 2433.6  624.1
```

```
sqrt(whale)
```

```
##  [1]  8.602325 11.045361 15.329710 10.535654 17.088007 10.535654 14.525839
##  [8] 11.532563 12.489996  8.888194
```

2.1.2 Missing values

```
hipcost <- c(10500, 45000, 74100, NA, 83500, 86000, 38200, NA, 44300, 12500, 55700, 43
```

NA is interpreted as a missing value, but which may have meaning, so it is not 0

```
sum(hipcost)
```

```
## [1] NA
```

- leads to NA
- solution is to use na.rm

```
sum(hipcost, na.rm = TRUE)
```

```
## [1] 627600
```

```
mean(hipcost, na.rm = TRUE)
```

```
## [1] 52300
```

- multivariate datasets have more options related to NA values

2.1.3 Attributes: names

```
head(precip)
```

```
##      Mobile      Juneau     Phoenix Little Rock Los Angeles  Sacramento
##        67.0        54.7         7.0        48.5        14.0        17.2
```

```
head(sort(precip, decreasing=TRUE))
```

```
##       Mobile        Miami     San Juan  New Orleans       Juneau Jacksonville
##         67.0         59.8         59.2         56.8         54.7         54.5
```

```
head(names(precip))
```

```
## [1] "Mobile"      "Juneau"      "Phoenix"     "Little Rock" "Los Angeles"
## [6] "Sacramento"
```

2.1.4 Structured Data

```
1:length(whale)
```

```
##  [1]  1  2  3  4  5  6  7  8  9 10
```

```
0:length(whale)-1
```

```
##  [1] -1  0  1  2  3  4  5  6  7  8  9
```

```
0:(length(whale)-1)
```

```
##  [1] 0 1 2 3 4 5 6 7 8 9
```

…

2.2 Numeric summaries

2.2.1 Center

- most common - mean, median, mode

```
wts <- kid.weights
```

```
sort(wts$weight, decreasing = TRUE)
```

```
##   [1] 150 150 144 131 125 108 105 100  98  94  93  90  89  87  86  85  85  80
##  [19]  80  80  78  76  74  72  70  70  69  69  65  65  65  64  61  60  60  60
##  [37]  59  58  55  55  55  54  53  52  52  52  52  52  50  50  50  50  50  50
##  [55]  49  48  47  47  47  47  46  46  45  45  45  45  45  45  45  45  44  43
##  [73]  43  43  42  42  42  42  42  41  41  41  40  40  40  40  40  40  40  40
##  [91]  40  40  40  38  38  38  38  38  38  37  37  36  36  35  35  35  35  35
## [109]  35  34  34  34  34  34  33  33  32  32  32  32  32  32  32  32  32  32
## [127]  31  31  31  30  30  30  30  30  30  30  30  30  30  30  30  30  30  29
## [145]  29  29  29  29  28  28  28  28  28  28  28  28  27  27  27  27  27  27
## [163]  26  26  26  26  26  26  26  26  25  25  25  25  25  25  24  24  24  23
## [181]  23  23  23  23  23  22  22  22  22  22  22  21  21  21  20  20  20  20
## [199]  20  20  19  19  19  19  19  19  18  18  18  18  18  18  18  17  17  17
## [217]  17  17  16  16  16  16  16  15  15  15  14  14  14  14  14  14  14  14
## [235]  14  13  13  13  13  13  13  13  13  12  12  12  11  11  11  10
```

- sample mean known as `x bar` or $\bar{x}$

```
mean(wts$weight)
```

```
## [1] 38.384
```

2.2.2 Spread

- variability of the data

2.2.2.1 `sample variance`

$$s^2 = \frac{1}{n-1}\sum_i (x_i - \bar{x})^2$$

```
var(wts$weight)
```

```
## [1] 615.3781
```

2.2.2.2 Sample `standard deviation`

$$\sqrt{s^2} = \sqrt{\frac{1}{n-1}\sum_i (x_i - \bar{x})^2}$$

```
sd(hipcost, na.rm=TRUE)
```

```
## [1] 24848.85
```

2.2.2.3 `z-score`

- deviations, $d_i = x_i - \bar{x}$, express data relative to its centre, rather than absolute
$$z - score = \frac{x_i - \bar{x}}{s}$$
gives the size of the data point in terms of its relative position to centre on a scale of standard deviations, so z-score of 3 means the data point is 3sd larger than mean

2.2.2.4 defining function

```r
z_score <- function(x)(x-mean(x))/sd(x)
```

```r
z_score(wts$weight)
```

```
##   [1] -0.01547962  1.95978406  0.46825843  2.40321060  0.34732391 -0.33797165
##   [7] -0.57984067  0.26670091  4.25753976 -0.57984067  0.06514339 -0.94264420
##  [13] -0.74108668 -1.14420172 -0.78139819 -0.37828315  1.67760353 -0.74108668
##  [19] -0.41859465 -0.57984067 -0.13641413 -0.33797165  0.54888143  1.35511150
##  [25] -0.41859465 -0.98295570 -0.82170969 -1.10389021 -0.29766014 -0.41859465
##  [31] -0.29766014 -0.98295570 -0.01547962 -0.94264420 -0.45890616  0.87137346
##  [37]  0.06514339  0.06514339 -0.70077518  0.34732391  0.30701241 -0.33797165
##  [43]  1.23417699  1.51635752 -0.90233270 -1.10389021 -0.01547962 -0.53952916
##  [49] -0.37828315 -0.70077518 -0.09610262 -0.41859465  0.10545489 -1.02326721
##  [55] -0.70077518 -0.21703714  0.06514339 -0.25734864 -0.45890616 -0.74108668
##  [61] -0.33797165  0.26670091 -0.17672563 -0.78139819  0.10545489  0.06514339
##  [67] -0.78139819 -0.98295570  0.18607790  2.08071857  0.26670091 -0.37828315
##  [73]  2.68539112  2.24196458 -0.66046367  1.23417699 -0.86202119 -0.45890616
##  [79] -1.06357871  0.06514339 -0.53952916 -1.02326721 -0.74108668 -0.90233270
##  [85]  0.26670091 -0.45890616 -0.74108668  0.62950444  0.87137346 -0.33797165
##  [91] -0.82170969  0.66981594 -0.98295570 -0.17672563  0.26670091  0.38763542
##  [97] -0.33797165 -0.53952916 -0.01547962 -1.02326721 -0.66046367 -0.78139819
## [103]  1.43573451 -0.33797165  2.04040706 -0.13641413 -0.21703714  1.59698053
## [109]  0.18607790  0.06514339 -0.25734864 -0.29766014 -0.94264420 -0.78139819
## [115]  0.66981594  0.06514339 -0.05579112 -0.17672563  0.26670091  0.22638940
## [121]  0.46825843  0.46825843 -0.41859465 -0.25734864 -0.66046367  0.54888143
## [127]  0.06514339  0.14576640 -0.66046367 -0.62015217 -0.90233270 -0.86202119
## [133] -0.33797165  0.06514339 -0.25734864 -0.49921766 -0.33797165  0.91168496
## [139]  1.07293098  0.54888143 -0.62015217 -0.37828315 -0.82170969 -0.25734864
## [145]  0.46825843 -1.02326721 -0.09610262  0.34732391  1.67760353 -0.62015217
## [151]  1.03261948  0.26670091 -0.53952916  1.07293098 -0.49921766 -0.01547962
## [157] -0.74108668  4.49940878  0.14576640  0.58919294 -0.13641413 -0.90233270
## [163]  4.49940878 -0.53952916 -0.33797165  0.54888143 -0.45890616 -0.45890616
## [169] -0.66046367 -0.33797165 -0.13641413  0.14576640 -1.06357871  0.34732391
## [175] -0.98295570 -0.25734864 -0.53952916  2.20165308 -0.33797165  0.83106196
## [181]  2.48383360 -0.17672563  0.54888143  0.18607790 -0.25734864 -1.02326721
## [187] -0.82170969 -0.01547962 -0.86202119  0.30701241 -0.37828315  0.46825843
## [193] -0.98295570 -1.02326721 -0.49921766 -1.10389021 -0.25734864  1.27448850
## [199] -0.90233270  0.46825843 -0.49921766 -0.62015217 -0.05579112  0.87137346
## [205]  0.26670091 -0.49921766  3.73349022 -0.66046367 -1.06357871  1.27448850
## [211]  1.91947255 -0.13641413 -0.98295570  1.87916105  0.79075045 -0.41859465
## [217] -0.86202119 -0.17672563 -0.78139819 -1.02326721 -0.82170969 -0.62015217
## [223] -0.86202119 -0.33797165  0.10545489 -0.33797165 -1.02326721  3.49162119
## [229] -0.25734864 -0.82170969 -0.13641413 -0.25734864  2.80632563  0.06514339
## [235]  0.14576640 -0.41859465 -0.49921766 -0.49921766 -0.62015217  0.14576640
```

```
## [241] -0.41859465  0.66981594  1.07293098  1.67760353  0.42794692 -0.82170969
## [247] -0.98295570 -0.49921766  1.87916105 -0.98295570
```

Example

Prof scales on z-scores and those who have z-score value of greater than 1.28, get an A

```
x <- c(54, 50, 79, 79, 51, 69, 55, 62, 100, 80)
z <- (x-mean(x))/sd(x)
x[z >= 1.28]
```

```
## [1] 100
```

what score is just good enough for an A?

```
mean(x) + 1.28 * sd(x)
```

```
## [1] 88.91046
```

- formula reverses the z-score formula is read as the score which is 1.28 SD above the mean
- z-score allow datasets with different scales to be compared

2.2.3 Shape (distribution)

- normal

2.3 categorical data

```
x <- babies$smoke
x <- factor(x, labels=c("never", "now", "until current", "once, quit", "unknown" ))
table(x)
```

```
## x
##         never           now until current    once, quit       unknown
##           544           484            95           103            10
```

```
out <- table(x)
prop <- 100 * out / sum(out)
round(prop, digits = 2)
```

```
## x
##          never            now until current    once, quit        unknown
##          44.01          39.16          7.69          8.33           0.81
```

Chapter 3

Bivariate Data

```
fat
```

```
##      case body.fat body.fat.siri density age weight height  BMI ffweight neck
## 1       1     12.6          12.3  1.0708  23 154.25  67.75 23.7    134.9 36.2
## 2       2      6.9           6.1  1.0853  22 173.25  72.25 23.4    161.3 38.5
## 3       3     24.6          25.3  1.0414  22 154.00  66.25 24.7    116.0 34.0
## 4       4     10.9          10.4  1.0751  26 184.75  72.25 24.9    164.7 37.4
## 5       5     27.8          28.7  1.0340  24 184.25  71.25 25.6    133.1 34.4
## 6       6     20.6          20.9  1.0502  24 210.25  74.75 26.5    167.0 39.0
## 7       7     19.0          19.2  1.0549  26 181.00  69.75 26.2    146.6 36.4
## 8       8     12.8          12.4  1.0704  25 176.00  72.50 23.6    153.6 37.8
## 9       9      5.1           4.1  1.0900  25 191.00  74.00 24.6    181.3 38.1
## 10     10     12.0          11.7  1.0722  23 198.25  73.50 25.8    174.4 42.1
## 11     11      7.5           7.1  1.0830  26 186.25  74.50 23.6    172.3 38.5
## 12     12      8.5           7.8  1.0812  27 216.00  76.00 26.3    197.7 39.4
## 13     13     20.5          20.8  1.0513  32 180.50  69.50 26.3    143.5 38.4
## 14     14     20.8          21.2  1.0505  30 205.25  71.25 28.5    162.5 39.4
## 15     15     21.7          22.1  1.0484  35 187.75  69.50 27.4    147.0 40.5
## 16     16     20.5          20.9  1.0512  35 162.75  66.00 26.3    129.3 36.4
## 17     17     28.1          29.0  1.0333  34 195.75  71.00 27.3    140.8 38.9
## 18     18     22.4          22.9  1.0468  32 209.25  71.00 29.2    162.5 42.1
## 19     19     16.1          16.0  1.0622  28 183.75  67.75 28.2    154.3 38.0
## 20     20     16.5          16.5  1.0610  33 211.75  73.50 27.6    176.8 40.0
## 21     21     19.0          19.1  1.0551  28 179.00  68.00 27.3    145.1 39.1
## 22     22     15.3          15.2  1.0640  28 200.50  69.75 29.1    169.8 41.3
## 23     23     15.7          15.6  1.0631  31 140.25  68.25 21.2    118.2 33.9
## 24     24     17.6          17.7  1.0584  32 148.75  70.00 21.4    122.6 35.5
## 25     25     14.2          14.0  1.0668  28 151.25  67.75 23.2    129.8 34.5
```

```
## 26     26       4.6              3.7  1.0911  27 159.25  71.50 21.9     151.9 35.7
## 27     27       8.5              7.9  1.0811  34 131.50  67.50 20.3     120.3 36.2
## 28     28      22.4             22.9  1.0468  31 148.00  67.50 22.9     114.9 38.8
## 29     29       4.7              3.7  1.0910  27 133.25  64.75 22.4     127.0 36.4
## 30     30       9.4              8.8  1.0790  29 160.75  69.00 23.8     145.7 36.7
## 31     31      12.3             11.9  1.0716  32 182.00  73.75 23.6     159.7 38.7
## 32     32       6.5              5.7  1.0862  29 160.25  71.25 22.2     149.8 37.3
## 33     33      13.4             11.8  1.0719  27 168.00  71.25 23.3     142.5 38.1
## 34     34      20.9             21.3  1.0502  41 218.50  71.00 30.5     172.7 39.8
## 35     35      31.1             32.3  1.0263  41 247.25  73.50 32.2     170.4 42.1
## 36     36      38.2             40.1  1.0101  49 191.75  65.00 32.0     118.4 38.4
## 37     37      23.6             24.2  1.0438  40 202.25  70.00 29.1     154.5 38.5
## 38     38      27.5             28.4  1.0346  50 196.75  68.25 29.7     142.6 42.1
## 39     39      33.8             35.2  1.0202  46 363.15  72.25 48.9     240.5 51.2
## 40     40      31.3             32.6  1.0258  50 203.00  67.00 31.8     139.4 40.2
## 41     41      33.1             34.5  1.0217  45 262.75  68.75 39.1     175.8 43.2
## 42     42      31.7             32.9  1.0250  44 205.00  29.50 29.9     140.1 36.6
## 43     43      30.4             31.6  1.0279  48 217.00  70.00 31.2     151.1 37.3
## 44     44      30.8             32.0  1.0269  41 212.00  71.50 29.2     146.7 41.5
## 45     45       8.4              7.7  1.0814  39 125.25  68.00 19.1     114.7 31.5
## 46     46      14.1             13.9  1.0670  43 164.25  73.25 21.3     141.1 35.7
## 47     47      11.2             10.8  1.0742  40 133.50  67.50 20.6     118.5 33.6
## 48     48       6.4              5.6  1.0665  39 148.50  71.25 20.6     139.0 34.6
## 49     49      13.4             13.6  1.0678  45 135.75  68.50 20.4     117.6 32.8
## 50     50       5.0              4.0  1.0903  47 127.50  66.75 20.2     121.2 34.0
## 51     51      10.7             10.2  1.0756  47 158.25  72.25 21.3     141.4 34.9
## 52     52       7.4              6.6  1.0840  40 139.25  69.00 20.6     129.0 34.3
## 53     53       8.7              8.0  1.0807  51 137.25  67.75 21.1     125.3 36.5
## 54     54       7.1              6.3  1.0848  49 152.75  73.50 19.9     142.0 35.1
## 55     55       4.9              3.9  1.0906  42 136.25  67.50 21.1     129.6 37.8
## 56     56      22.2             22.6  1.0473  54 198.00  72.00 26.9     154.1 39.9
## 57     57      20.1             20.4  1.0524  58 181.50  68.00 27.6     145.1 39.1
## 58     58      27.1             28.0  1.0356  62 201.25  69.50 29.3     146.7 40.5
## 59     59      30.4             31.5  1.0280  54 202.50  70.75 28.4     141.0 40.5
## 60     60      24.0             24.6  1.0430  61 179.75  65.75 29.2     136.7 38.4
## 61     61      25.4             26.1  1.0396  62 216.00  73.25 28.2     161.2 41.4
## 62     62      28.8             29.8  1.0317  56 178.75  68.50 26.8     127.4 35.6
## 63     63      29.6             30.7  1.0298  54 193.25  70.25 27.6     136.1 38.0
## 64     64      25.1             25.8  1.0403  61 178.00  67.00 27.9     133.3 37.4
## 65     65      31.0             32.3  1.0264  57 205.50  70.00 29.5     141.7 40.1
## 66     66      28.9             30.0  1.0313  55 183.50  67.50 28.3     130.4 40.9
## 67     67      21.1             21.5  1.0499  54 151.50  70.75 21.3     119.6 35.6
## 68     68      14.0             13.8  1.0673  55 154.75  71.50 21.3     133.1 36.9
## 69     69       7.1              6.3  1.0847  54 155.25  69.25 22.8     144.2 37.5
## 70     70      13.2             12.9  1.0693  55 156.75  71.50 21.6     136.1 36.3
## 71     71      23.7             24.3  1.0439  62 167.50  71.50 23.1     127.8 35.5
```

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## 72    72       9.4            8.8  1.0788  55 146.75  68.75 21.9    132.9 38.7
## 73    73       9.1            8.5  1.0796  56 160.75  73.75 20.8    146.1 36.4
## 74    74      13.7           13.5  1.0680  55 125.00  64.00 21.5    107.9 33.2
## 75    75      12.0           11.8  1.0720  61 143.00  65.75 23.3    125.9 36.5
## 76    76      18.3           18.5  1.0666  61 148.25  67.50 22.9    121.1 36.0
## 77    77       9.2            8.8  1.0790  57 162.50  69.50 23.7    147.5 38.7
## 78    78      21.7           22.2  1.0483  69 177.75  68.50 26.7    139.1 38.7
## 79    79      21.1           21.5  1.0498  81 161.25  70.25 23.0    127.2 37.8
## 80    80      18.6           18.8  1.0560  66 171.25  69.25 25.1    139.5 37.4
## 81    81      30.2           31.4  1.0283  67 163.75  67.75 25.1    114.3 38.4
## 82    82      26.0           26.8  1.0382  64 150.25  67.25 23.4    111.2 38.1
## 83    83      18.2           18.4  1.0568  64 190.25  72.75 25.3    155.6 39.3
## 84    84      26.2           27.0  1.0377  70 170.75  70.00 24.5    126.0 38.7
## 85    85      26.1           27.0  1.0378  72 168.00  69.25 24.7    124.1 38.5
## 86    86      25.8           26.6  1.0386  67 167.00  67.50 26.0    123.9 36.5
## 87    87      15.0           14.9  1.0648  72 157.75  67.25 24.6    134.1 37.7
## 88    88      22.6           23.1  1.0462  64 160.00  65.75 26.0    123.8 36.5
## 89    89       8.8            8.3  1.0800  46 176.75  72.50 23.7    161.1 38.0
## 90    90      14.3           14.1  1.0666  48 176.00  73.00 23.3    150.9 36.7
## 91    91      20.2           20.5  1.0520  46 177.00  70.00 25.4    141.3 37.2
## 92    92      18.1           18.2  1.0573  44 179.75  69.50 26.2    147.3 39.2
## 93    93       9.2            8.5  1.0795  47 165.25  70.50 23.4    150.1 37.5
## 94    94      24.2           24.9  1.0424  46 192.50  71.75 26.3    145.9 38.0
## 95    95       9.6            9.0  1.0785  47 184.25  74.50 23.4    166.6 37.3
## 96    96      17.3           17.4  1.0991  53 224.50  77.75 26.1    185.7 41.1
## 97    97      10.1            9.6  1.0770  38 188.75  73.25 24.8    169.6 37.5
## 98    98      11.1           11.3  1.0730  50 162.50  66.50 25.9    143.5 38.7
## 99    99      17.7           17.8  1.0582  46 156.50  68.25 23.7    128.8 35.9
## 100  100      21.7           22.2  1.0484  47 197.00  72.00 26.7    154.2 40.0
## 101  101      20.8           21.2  1.0506  49 198.50  73.50 25.9    157.2 40.1
## 102  102      20.1           20.4  1.0524  48 173.75  72.00 23.6    138.9 37.0
## 103  103      19.8           20.1  1.0530  41 172.75  71.25 24.0    138.6 36.3
## 104  104      21.9           22.3  1.0480  49 196.75  73.75 25.5    153.7 40.7
## 105  105      24.7           25.4  1.0412  43 177.00  69.25 26.0    133.2 39.6
## 106  106      17.8           18.0  1.0578  43 165.50  68.50 24.8    136.0 31.1
## 107  107      19.1           19.3  1.0547  43 200.25  73.50 26.0    162.0 38.6
## 108  108      18.2           18.3  1.0569  52 203.25  74.25 26.0    166.3 42.0
## 109  109      17.2           17.3  1.0593  43 194.00  75.50 24.0    160.6 38.5
## 110  110      21.0           21.4  1.0500  40 168.50  69.25 24.7    133.1 34.2
## 111  111      19.5           19.7  1.0538  43 170.75  68.50 25.6    137.5 37.2
## 112  112      27.1           28.0  1.0355  43 183.25  70.00 26.3    133.5 37.1
## 113  113      21.6           22.1  1.0486  47 178.25  70.00 25.6    139.7 40.2
## 114  114      20.9           21.3  1.0503  42 163.00  70.25 23.3    128.9 35.3
## 115  115      25.9           26.7  1.0384  48 175.25  71.75 24.0    129.9 38.0
## 116  116      16.7           16.7  1.0607  40 158.00  69.25 23.4    131.7 36.3
## 117  117      19.8           20.1  1.0529  48 177.25  72.75 23.6    142.1 36.8
```

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## 118  118      14.1              13.9  1.0671  51 179.00  72.00 24.3     153.8 41.0
## 119  119      25.1              25.8  1.0404  40 191.00  74.00 24.6     143.1 38.3
## 120  120      17.9              18.1  1.0575  44 187.50  72.25 25.3     153.8 38.0
## 121  121      27.0              27.9  1.0358  52 206.50  74.50 26.2     150.7 40.8
## 122  122      24.6              25.3  1.0414  44 185.25  71.50 25.5     139.6 39.5
## 123  123      14.8              14.7  1.0652  40 160.25  68.75 23.9     136.5 36.9
## 124  124      16.0              16.0  1.0623  47 151.50  66.75 23.9     127.3 36.9
## 125  125      14.0              13.8  1.0674  50 161.00  66.50 25.6     138.5 37.7
## 126  126      17.4              17.5  1.0587  46 167.00  67.00 26.2     137.9 36.6
## 127  127      26.4              27.2  1.0373  42 177.50  68.75 26.4     130.7 38.9
## 128  128      17.4              17.4  1.0590  43 152.25  67.75 23.4     125.8 37.5
## 129  129      20.4              20.8  1.0515  40 192.25  73.25 25.2     153.0 39.8
## 130  130      15.0              14.9  1.0648  42 165.25  69.75 23.9     140.5 38.3
## 131  131      18.0              18.1  1.0575  49 171.75  71.50 23.7     140.9 35.5
## 132  132      22.2              22.7  1.0472  40 171.25  70.50 24.3     133.3 36.3
## 133  133      23.1              23.6  1.0452  47 197.00  73.25 25.8     151.2 37.8
## 134  134      25.3              26.1  1.0398  50 157.00  66.75 24.8     117.2 37.8
## 135  135      23.8              24.4  1.0435  41 168.25  69.50 24.5     128.3 36.5
## 136  136      26.3              27.1  1.0374  44 186.00  69.75 26.8     137.1 37.8
## 137  137      21.4              21.8  1.0491  39 166.75  70.75 23.5     131.0 37.0
## 138  138      28.4              29.4  1.0325  43 187.75  74.00 24.1     134.4 37.7
## 139  139      21.8              22.4  1.0481  40 168.25  71.25 23.3     131.6 34.3
## 140  140      20.1              20.4  1.0522  49 212.75  75.00 26.6     169.9 40.8
## 141  141      24.3              24.9  1.0422  40 176.75  71.00 24.6     133.8 37.4
## 142  142      18.1              18.3  1.0571  40 173.25  69.50 25.3     141.8 36.5
## 143  143      22.7              23.3  1.0459  52 167.00  67.75 25.6     129.0 37.5
## 144  144       9.9               9.4  1.0775  23 159.75  72.25 21.6     143.9 35.5
## 145  145      10.8              10.3  1.0754  23 188.15  77.50 22.1     168.4 38.0
## 146  146      14.4              14.2  1.0664  24 156.00  70.75 21.9     133.6 35.7
## 147  147      19.0              19.2  1.0550  24 208.50  72.75 27.7     168.9 39.2
## 148  148      28.6              29.6  1.0322  25 206.50  69.75 29.8     147.5 40.9
## 149  149       6.1               5.3  1.0873  25 143.75  72.50 19.3     135.0 35.2
## 150  150      24.5              25.2  1.0416  26 223.00  70.25 31.8     168.3 40.6
## 151  151       9.9               9.4  1.0776  26 152.25  69.00 22.5     137.2 35.4
## 152  152      19.1              19.6  1.0542  26 241.75  74.50 30.7     195.1 41.8
## 153  153      10.6              10.1  1.0758  27 146.00  72.25 19.7     130.5 34.1
## 154  154      16.5              16.5  1.0610  27 156.75  67.25 24.4     130.9 37.9
## 155  155      20.5              21.0  1.0510  27 200.25  73.50 26.1     159.3 38.2
## 156  156      17.2              17.3  1.0594  28 171.50  75.25 21.6     142.0 35.6
## 157  157      30.1              31.2  1.0287  28 205.75  69.00 30.4     143.9 38.5
## 158  158      10.5              10.0  1.0761  28 182.50  72.25 24.6     163.4 37.0
## 159  159      12.8              12.5  1.0704  30 136.50  68.75 20.3     119.1 35.9
## 160  160      22.0              22.5  1.0477  31 177.25  71.50 24.4     138.3 36.2
## 161  161       9.9               9.4  1.0775  31 151.25  72.25 20.4     136.2 35.0
## 162  162      14.8              14.6  1.0653  33 196.00  73.00 25.9     167.0 38.5
## 163  163      13.3              13.0  1.0690  33 184.25  68.75 24.4     159.8 40.7
```

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## 164  164      15.2             15.1  1.0644  34 140.00  70.50 19.8     118.8 36.0
## 165  165      26.5             27.3  1.0370  34 218.75  72.00 29.7     160.8 39.5
## 166  166      19.0             19.2  1.0549  35 217.00  73.75 28.1     175.8 40.5
## 167  167      21.4             21.8  1.0492  35 166.25  68.00 25.3     130.7 38.5
## 168  168      20.0             20.3  1.0525  35 224.75  72.25 30.3     179.7 43.9
## 169  169      34.7             34.3  1.0180  35 228.25  69.50 33.3     149.3 40.4
## 170  170      16.5             16.5  1.0610  35 172.75  69.50 25.2     144.2 37.6
## 171  171       4.1              3.0  1.0926  35 152.25  67.75 23.4     146.1 37.0
## 172  172       1.9              0.7  1.0983  35 125.75  65.50 20.6     123.4 34.0
## 173  173      20.2             20.5  1.0521  35 177.25  71.00 24.8     141.7 38.4
## 174  174      16.8             16.9  1.0603  36 176.25  71.50 24.3     146.6 38.7
## 175  175      24.6             25.3  1.0414  36 226.75  71.75 31.0     170.9 41.5
## 176  176      10.4              9.9  1.0763  37 145.25  69.25 21.3     130.2 36.0
## 177  177      13.4             13.1  1.0689  37 151.00  67.00 23.7     130.8 35.3
## 178  178      28.8             29.9  1.0316  37 241.25  71.50 33.2     171.7 42.1
## 179  179      22.0             22.5  1.0477  38 187.25  69.25 27.5     146.1 38.0
## 180  180      16.8             16.9  1.0603  39 234.75  74.50 29.8     195.3 42.8
## 181  181      25.8             26.6  1.0387  39 219.25  74.25 28.0     162.7 40.0
## 182  182       0.0              0.0  1.1089  40 118.50  68.00 18.1     118.5 33.8
## 183  183      11.9             11.5  1.0725  40 145.75  67.25 22.7     128.4 35.5
## 184  184      12.4             12.1  1.0713  40 159.25  69.75 23.0     139.5 35.3
## 185  185      17.4             17.5  1.0587  40 170.50  74.25 21.8     140.8 37.7
## 186  186       9.2              8.6  1.0794  40 167.50  71.50 23.1     152.1 39.4
## 187  187      23.0             23.6  1.0453  41 232.75  74.25 29.7     179.2 41.9
## 188  188      20.1             20.4  1.0524  41 210.50  72.00 28.6     168.3 38.5
## 189  189      20.2             20.5  1.0520  41 202.25  72.50 27.0     161.4 40.8
## 190  190      23.8             24.4  1.0434  41 185.00  68.25 28.0     141.0 38.0
## 191  191      11.8             11.4  1.0728  41 153.00  69.25 22.5     135.0 36.4
## 192  192      36.5             38.1  1.0140  42 244.25  76.00 29.8     155.2 41.8
## 193  193      16.0             15.9  1.0624  42 193.50  70.50 27.4     162.6 40.7
## 194  194      24.0             24.7  1.0429  42 224.75  74.75 28.3     170.8 38.5
## 195  195      22.3             22.8  1.0470  42 162.75  72.75 21.6     126.5 35.4
## 196  196      24.8             25.5  1.0411  42 180.00  68.25 27.2     135.4 38.5
## 197  197      21.5             22.0  1.0488  42 156.25  69.00 23.1     122.6 35.5
## 198  198      17.6             17.7  1.0583  42 168.00  71.50 23.1     138.4 36.5
## 199  199       7.3              6.6  1.0841  42 167.25  72.75 22.3     155.1 37.6
## 200  200      22.6             23.6  1.0462  43 170.75  67.50 26.4     132.1 37.4
## 201  201      12.5             12.2  1.0709  43 178.25  70.25 25.4     155.9 37.8
## 202  202      21.7             22.1  1.0484  43 150.00  69.25 22.0     117.5 35.2
## 203  203      27.7             28.7  1.0340  43 200.50  71.50 27.6     144.9 37.9
## 204  204       6.8              6.0  1.0854  44 184.00  74.00 23.7     171.4 37.9
## 205  205      33.4             34.8  1.0209  44 223.00  69.75 32.3     148.5 40.9
## 206  206      16.6             16.6  1.0610  44 208.75  73.00 27.6     174.2 41.9
## 207  207      31.7             32.9  1.0250  44 166.00  65.50 27.2     113.5 39.1
## 208  208      31.5             32.8  1.0254  47 195.00  72.50 26.1     133.6 40.2
## 209  209      10.1              9.6  1.0771  47 160.50  70.25 22.9     144.3 36.0
```

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## 210  210     11.3            10.8  1.0742  47 159.75  70.75 22.5     141.8 34.5
## 211  211      7.8             7.1  1.0829  49 140.50  68.00 21.4     129.5 35.8
## 212  212     26.4            27.2  1.0373  49 216.25  74.50 27.4     159.3 40.2
## 213  213     19.3            19.5  1.0543  49 168.25  71.75 23.0     135.9 38.3
## 214  214     18.5            18.7  1.0561  50 194.75  70.75 27.4     158.7 39.0
## 215  215     19.3            19.5  1.0543  50 172.75  73.00 22.8     139.4 37.4
## 216  216     45.1            47.5  0.9950  51 219.00  64.00 37.6     120.2 41.2
## 217  217     13.8            13.6  1.0678  51 149.25  69.75 21.6     128.7 34.8
## 218  218      8.2             7.5  1.0819  51 154.50  70.00 22.2     141.9 36.9
## 219  219     23.9            24.5  1.0433  52 199.25  71.75 27.2     151.7 39.4
## 220  220     15.1            15.0  1.0646  53 154.50  69.25 22.7     131.2 37.6
## 221  221     12.7            12.4  1.0706  54 153.25  70.50 24.5     151.3 38.5
## 222  222     25.3            26.0  1.0399  54 230.00  72.25 31.0     171.9 42.5
## 223  223     11.9            11.5  1.0726  54 161.75  67.50 25.0     142.6 37.4
## 224  224      6.1             5.2  1.0874  55 142.25  67.25 22.2     133.6 35.2
## 225  225     11.3            10.9  1.0740  55 179.75  68.75 26.8     159.5 41.1
## 226  226     12.8            12.5  1.0703  55 126.50  66.75 20.0     110.3 33.4
## 227  227     14.9            14.8  1.0650  55 169.50  68.25 25.6     144.2 37.2
## 228  228     24.5            25.2  1.0418  55 198.50  74.25 25.3     149.9 38.3
## 229  229     15.0            14.9  1.0647  56 174.50  69.50 25.4     148.3 38.1
## 230  230     16.9            17.0  1.0601  56 167.75  68.50 25.2     139.4 37.4
## 231  231     11.1            10.6  1.0745  57 147.75  65.75 24.1     131.4 35.2
## 232  232     16.1            16.1  1.0620  57 182.25  71.75 24.9     152.9 39.4
## 233  233     15.5            15.4  1.0636  58 175.50  71.50 24.2     148.4 38.0
## 234  234     25.9            26.7  1.0384  58 161.75  67.25 25.2     119.9 35.1
## 235  235     25.5            25.8  1.0403  60 157.75  67.50 24.1     117.5 40.4
## 236  236     18.4            18.6  1.0563  62 168.75  67.50 26.1     137.6 38.3
## 237  237     24.0            24.8  1.0424  62 191.50  72.25 25.8     145.2 40.6
## 238  238     26.4            27.3  1.0372  63 219.15  69.50 31.9     161.2 40.2
## 239  239     12.7            12.4  1.0705  64 155.25  69.50 22.6     135.5 37.9
## 240  240     28.8            29.9  1.0316  65 189.75  65.75 30.9     135.1 40.8
## 241  241     17.0            17.0  1.0599  65 127.50  65.75 20.8     105.9 34.7
## 242  242     33.6            35.0  1.0207  65 224.50  68.25 33.9     149.2 38.8
## 243  243     29.3            30.4  1.0304  66 234.25  72.00 31.8     165.6 41.4
## 244  244     31.4            32.6  1.0256  67 227.75  72.75 30.3     156.3 41.3
## 245  245     28.1            29.0  1.0334  67 199.50  68.50 29.9     143.6 40.7
## 246  246     15.3            15.2  1.0641  68 155.50  69.25 22.8     131.8 36.3
## 247  247     29.1            30.2  1.0308  69 215.50  70.50 30.5     152.7 40.8
## 248  248     11.5            11.0  1.0736  70 134.25  67.00 21.1     118.9 34.9
## 249  249     32.3            33.6  1.0236  72 201.00  69.75 29.1     136.1 40.9
## 250  250     28.3            29.3  1.0328  72 186.75  66.00 30.2     133.9 38.9
## 251  251     25.3            26.0  1.0399  72 190.75  70.50 27.0     142.6 38.9
## 252  252     30.7            31.9  1.0271  74 207.50  70.00 29.8     143.7 40.8
##      chest abdomen   hip thigh knee ankle bicep forearm wrist
## 1     93.1    85.2  94.5  59.0 37.3  21.9  32.0    27.4  17.1
## 2     93.6    83.0  98.7  58.7 37.3  23.4  30.5    28.9  18.2
```

```
## 3    95.8    87.9  99.2  59.6 38.9  24.0  28.8    25.2  16.6
## 4   101.8    86.4 101.2  60.1 37.3  22.8  32.4    29.4  18.2
## 5    97.3   100.0 101.9  63.2 42.2  24.0  32.2    27.7  17.7
## 6   104.5    94.4 107.8  66.0 42.0  25.6  35.7    30.6  18.8
## 7   105.1    90.7 100.3  58.4 38.3  22.9  31.9    27.8  17.7
## 8    99.6    88.5  97.1  60.0 39.4  23.2  30.5    29.0  18.8
## 9   100.9    82.5  99.9  62.9 38.3  23.8  35.9    31.1  18.2
## 10   99.6    88.6 104.1  63.1 41.7  25.0  35.6    30.0  19.2
## 11  101.5    83.6  98.2  59.7 39.7  25.2  32.8    29.4  18.5
## 12  103.6    90.9 107.7  66.2 39.2  25.9  37.2    30.2  19.0
## 13  102.0    91.6 103.9  63.4 38.3  21.5  32.5    28.6  17.7
## 14  104.1   101.8 108.6  66.0 41.5  23.7  36.9    31.6  18.8
## 15  101.3    96.4 100.1  69.0 39.0  23.1  36.1    30.5  18.2
## 16   99.1    92.8  99.2  63.1 38.7  21.7  31.1    26.4  16.9
## 17  101.9    96.4 105.2  64.8 40.8  23.1  36.2    30.8  17.3
## 18  107.6    97.5 107.0  66.9 40.0  24.4  38.2    31.6  19.3
## 19  106.8    89.6 102.4  64.2 38.7  22.9  37.2    30.5  18.5
## 20  106.2   100.5 109.0  65.8 40.6  24.0  37.1    30.1  18.2
## 21  103.3    95.9 104.9  63.5 38.0  22.1  32.5    30.3  18.4
## 22  111.4    98.8 104.8  63.4 40.6  24.6  33.0    32.8  19.9
## 23   86.0    76.4  94.6  57.4 35.3  22.2  27.9    25.9  16.7
## 24   86.7    80.0  93.4  54.9 36.2  22.1  29.8    26.7  17.1
## 25   90.2    76.3  95.8  58.4 35.5  22.9  31.1    28.0  17.6
## 26   89.6    79.7  96.5  55.0 36.7  22.5  29.9    28.2  17.7
## 27   88.6    74.6  85.3  51.7 34.7  21.4  28.7    27.0  16.5
## 28   97.4    88.7  94.7  57.5 36.0  21.0  29.2    26.6  17.0
## 29   93.5    73.9  88.5  50.1 34.5  21.3  30.5    27.9  17.2
## 30   97.4    83.5  98.7  58.9 35.3  22.6  30.1    26.7  17.6
## 31  100.5    88.7  99.8  57.5 38.7  33.9  32.5    27.7  18.4
## 32   93.5    84.5 100.6  58.5 38.8  21.5  30.1    26.4  17.9
## 33   93.0    79.1  94.5  57.3 36.2  24.5  29.0    30.0  18.8
## 34  111.7   100.5 108.3  67.1 44.2  25.2  37.5    31.5  18.7
## 35  117.0   115.6 116.1  71.2 43.3  26.3  37.3    31.7  19.7
## 36  118.5   113.1 113.8  61.9 38.3  21.9  32.0    29.8  17.0
## 37  106.5   100.9 106.2  63.5 39.9  22.6  35.1    30.6  19.0
## 38  105.6    98.8 104.8  66.0 41.5  24.7  33.2    30.5  19.4
## 39  136.2   148.1 147.7  87.3 49.1  29.6  45.0    29.0  21.4
## 40  114.8   108.1 102.5  61.3 41.1  24.7  34.1    31.0  18.3
## 41  128.3   126.2 125.6  72.5 39.6  26.6  36.4    32.7  21.4
## 42  106.0   104.3 115.5  70.6 42.5  23.7  33.6    28.7  17.4
## 43  113.3   111.2 114.1  67.7 40.9  25.0  36.7    29.8  18.4
## 44  106.6   104.3 106.0  65.0 40.2  23.0  35.8    31.5  18.8
## 45   85.1    76.0  88.2  50.0 34.7  21.0  26.1    23.1  16.1
## 46   96.6    81.5  97.2  58.4 38.2  23.4  29.7    27.4  18.3
## 47   88.2    73.7  88.5  53.3 34.5  22.5  27.9    26.2  17.3
## 48   89.8    79.5  92.7  52.7 37.5  21.9  28.8    26.8  17.9
```

```
## 49   92.3    83.4  90.4  52.0 35.8  20.6  28.8    25.5  16.3
## 50   83.4    70.4  87.2  50.6 34.4  21.9  26.8    25.8  16.8
## 51   90.2    86.7  98.3  52.6 37.2  22.4  26.0    25.8  17.3
## 52   89.2    77.9  91.0  51.4 34.9  21.0  26.7    26.1  17.2
## 53   89.7    82.0  89.1  49.3 33.7  21.4  29.6    26.0  16.9
## 54   93.3    79.6  91.6  52.6 37.6  22.6  38.5    27.4  18.5
## 55   87.6    77.6  88.6  51.9 34.9  22.5  27.7    27.5  18.5
## 56  107.6   100.0  99.6  57.2 38.0  22.0  35.9    30.2  18.9
## 57  100.0    99.8 102.5  62.1 39.6  22.5  33.1    28.3  18.5
## 58  111.5   104.2 105.8  61.8 39.8  22.7  37.7    30.9  19.2
## 59  115.4   105.3  97.0  59.1 38.0  22.5  31.6    28.8  18.2
## 60  104.8    98.3  99.6  60.6 37.7  22.9  34.5    29.6  18.5
## 61  112.3   104.8 103.1  61.6 40.9  23.1  36.2    31.8  20.2
## 62  102.9    94.7 100.8  60.9 38.0  22.1  32.5    29.8  18.3
## 63  107.6   102.4  99.4  61.0 39.4  23.6  32.7    29.9  19.1
## 64  105.3    99.7  99.7  60.8 40.1  22.7  33.6    29.0  18.8
## 65  105.3   105.5 108.3  65.0 41.2  24.7  35.3    31.1  18.4
## 66  103.0   100.3 104.2  64.8 40.2  22.7  34.8    30.1  18.7
## 67   90.0    83.9  93.9  55.0 36.1  21.7  29.6    27.4  17.4
## 68   95.4    86.6  91.8  54.3 35.4  21.5  32.8    27.4  18.7
## 69   89.3    78.4  96.1  56.0 37.4  22.4  32.6    28.1  18.1
## 70   94.4    84.6  94.3  51.2 37.4  21.6  27.3    27.1  17.3
## 71   97.6    91.5  98.5  56.6 38.6  22.4  31.5    27.3  18.6
## 72   88.5    82.8  95.5  58.9 37.6  21.6  30.3    27.3  18.3
## 73   93.6    82.9  96.3  52.9 37.5  23.1  29.7    27.3  18.2
## 74   87.7    76.0  88.6  50.9 35.4  19.1  29.3    25.7  16.9
## 75   93.4    83.3  93.0  55.5 35.2  20.9  29.4    27.0  16.8
## 76   91.6    81.8  94.8  54.5 37.0  21.4  29.3    27.0  18.3
## 77   91.6    78.8  94.3  56.7 39.7  24.2  30.2    29.2  18.1
## 78  102.0    95.0  98.3  55.0 38.3  21.8  30.8    25.7  18.8
## 79   96.4    95.4  99.3  53.5 37.5  21.5  31.4    26.8  18.3
## 80  102.7    98.6 100.2  56.5 39.3  22.7  30.3    28.7  19.0
## 81   97.7    95.8  97.1  54.8 38.2  23.7  29.4    27.2  19.0
## 82   97.1    89.0  96.9  54.8 38.0  22.0  29.9    25.2  17.7
## 83  103.1    97.8  99.6  58.9 39.0  23.0  34.3    29.6  19.0
## 84  101.8    94.9  95.0  56.0 36.5  24.1  31.2    27.3  19.2
## 85  101.4    99.8  96.2  56.3 36.6  22.0  29.7    26.3  18.0
## 86   98.9    89.7  96.2  54.7 37.8  33.7  32.4    27.7  18.2
## 87   97.5    88.1  96.9  57.2 37.7  21.8  32.6    28.0  18.8
## 88  104.3    90.9  93.8  57.8 39.5  23.3  29.2    28.4  18.1
## 89   97.3    86.0  99.3  61.0 38.4  23.8  30.2    29.3  18.8
## 90   96.7    86.5  98.3  60.4 39.9  24.4  28.8    29.6  18.7
## 91   99.7    95.6 102.2  58.3 38.2  22.5  29.1    27.7  17.7
## 92  101.9    93.2 100.6  58.9 39.7  23.1  31.4    28.4  18.8
## 93   97.2    83.1  95.4  56.9 38.3  22.1  30.1    28.2  18.4
## 94  106.6    97.5 100.6  58.9 40.5  24.5  33.3    29.6  19.1
```

```
## 95   99.6    88.8 101.4  57.4 39.6  24.6  30.3    27.9  17.8
## 96  113.2    99.2 107.5  61.7 42.3  23.2  32.9    30.8  20.4
## 97   99.1    91.6 102.4  60.6 39.4  22.9  31.6    30.1  18.5
## 98   99.4    86.7  96.2  62.1 39.3  23.3  30.6    27.8  18.2
## 99   95.1    88.2  92.8  54.7 37.3  21.9  31.6    27.5  18.2
## 100 107.5    94.0 103.7  62.7 39.0  22.3  35.3    30.9  18.3
## 101 106.5    95.0 101.7  59.0 39.4  22.3  32.2    31.0  18.6
## 102  99.1    92.0  98.3  59.3 38.4  22.4  27.9    26.2  17.0
## 103  96.7    89.2  98.3  60.0 38.4  23.2  31.0    29.2  18.4
## 104 103.5    95.5 101.6  59.1 39.8  25.4  31.0    30.3  19.7
## 105 104.0    98.6  99.5  59.5 36.1  22.0  30.1    27.2  17.7
## 106  93.1    87.3  96.6  54.7 39.0  24.8  31.0    29.4  18.8
## 107 105.2   102.8 103.6  61.2 39.3  23.5  30.5    28.5  18.1
## 108 110.0   101.6 100.7  55.8 38.7  23.4  35.1    29.6  19.1
## 109 110.1    88.7 102.1  57.5 40.0  24.8  35.1    30.7  19.2
## 110  97.8    92.3 100.6  57.5 36.8  22.8  32.1    26.0  17.3
## 111  96.3    90.6  99.3  61.9 38.0  22.3  33.3    28.2  18.1
## 112 108.0   105.0 103.0  63.7 40.0  23.6  33.5    27.8  17.4
## 113  99.7    95.0  98.6  62.3 38.1  23.9  35.3    31.1  19.8
## 114  93.5    89.6  99.8  61.5 37.8  21.9  30.7    27.6  17.4
## 115 100.7    92.4  97.5  59.3 38.1  21.8  31.8    27.3  17.5
## 116  97.0    86.6  92.6  55.9 36.3  22.1  29.8    26.3  17.3
## 117  96.0    90.0  99.7  58.8 38.4  22.8  29.9    28.0  18.1
## 118  99.2    90.0  96.4  56.8 38.8  23.3  33.4    29.8  19.5
## 119  95.4    92.4 104.3  64.6 41.1  24.8  33.6    29.5  18.5
## 120 101.8    87.5 101.0  58.5 39.2  24.5  32.1    28.6  18.0
## 121 104.3    99.2 104.1  58.5 39.3  24.6  33.9    31.2  19.5
## 122  99.2    98.1 101.4  57.1 40.5  23.2  33.0    29.6  18.4
## 123  99.3    83.3  97.5  60.5 38.7  22.6  34.4    28.0  17.6
## 124  94.0    86.1  95.2  58.1 36.5  22.1  30.6    27.5  17.6
## 125  98.9    84.1  94.0  58.5 36.6  23.5  34.4    29.2  18.0
## 126 101.0    89.9 100.0  60.7 36.0  21.9  35.6    30.2  17.6
## 127  98.7    92.1  98.5  60.7 36.8  22.2  33.8    30.3  17.2
## 128  95.9    78.0  93.2  53.5 35.8  20.8  33.9    28.2  17.4
## 129 103.9    93.5  99.5  61.7 39.0  21.8  33.3    29.6  18.1
## 130  96.2    87.0  97.8  57.4 36.9  22.2  31.6    27.8  17.7
## 131  97.8    90.1  95.8  57.0 38.7  23.2  27.5    26.5  17.6
## 132  94.6    90.3  99.1  60.3 38.5  23.0  31.2    28.4  17.1
## 133 103.6    99.8 103.2  61.2 38.1  22.6  33.5    28.6  17.9
## 134 100.4    89.4  92.3  56.1 35.6  20.5  33.6    29.3  17.3
## 135  98.4    87.2  98.4  56.0 36.9  23.0  34.0    29.8  18.1
## 136 104.6   101.1 102.1  58.9 37.9  22.7  30.9    28.8  17.6
## 137  92.9    86.1  95.6  58.8 36.1  22.4  32.7    28.3  17.1
## 138  97.8    98.6 100.6  63.6 39.2  23.8  34.3    28.4  17.7
## 139  98.3    88.5  98.3  58.1 38.4  22.5  31.7    27.4  17.6
## 140 104.7   106.6 107.7  66.5 42.5  24.5  35.5    29.8  18.7
```

```
## 141  98.6    93.1 101.6  59.1 39.6  21.6  30.8    27.9  16.6
## 142  99.5    93.0  99.3  60.4 38.2  22.0  32.0    28.5  17.8
## 143 102.7    91.0  98.9  57.1 36.7  22.3  31.6    27.5  17.9
## 144  92.1    77.1  93.9  56.1 36.1  22.7  30.5    27.2  18.2
## 145  96.6    85.3 102.5  59.1 37.6  23.2  31.8    29.7  18.3
## 146  92.7    81.9  95.3  56.4 36.5  22.0  33.5    28.3  17.3
## 147 102.0    99.1 110.1  71.2 43.5  25.2  36.1    30.3  18.7
## 148 110.9   100.5 106.2  68.4 40.8  24.6  33.3    29.7  18.4
## 149  92.3    76.5  92.1  51.9 35.7  22.0  25.8    25.2  16.9
## 150 114.1   106.8 113.9  67.6 42.7  24.7  36.0    30.4  18.4
## 151  92.9    77.6  93.5  56.9 35.9  20.4  31.6    29.0  17.8
## 152 108.3   102.9 114.4  72.9 43.5  25.1  38.5    33.8  19.6
## 153  88.5    72.8  91.1  53.6 36.8  23.8  27.8    26.3  17.4
## 154  94.0    88.2  95.2  56.8 37.4  22.8  30.6    28.3  17.9
## 155 101.1   100.1 105.0  62.1 40.0  24.9  33.7    29.2  19.4
## 156  92.1    83.5  98.3  57.3 37.8  21.7  32.2    27.7  17.7
## 157 105.6   105.0 106.4  68.6 40.0  25.2  35.2    30.7  19.1
## 158  98.5    90.8 102.5  60.8 38.5  25.0  31.6    28.0  18.6
## 159  88.7    76.6  89.8  50.1 34.8  21.8  27.0    34.9  16.9
## 160 101.1    92.4  99.3  59.4 39.0  24.6  30.1    28.2  18.2
## 161  94.0    81.2  91.5  52.5 36.6  21.0  27.0    26.3  16.5
## 162 103.8    95.6 105.1  61.4 40.6  25.0  31.3    29.2  19.1
## 163  98.9    92.1 103.5  64.0 37.3  23.5  33.5    30.6  19.7
## 164  89.2    83.4  89.6  52.4 35.6  20.4  28.3    26.2  16.5
## 165 111.4   106.0 108.8  63.8 42.0  23.4  34.0    31.2  18.5
## 166 107.5    95.1 104.5  64.8 41.3  25.6  36.4    33.7  19.4
## 167  99.1    90.4  95.6  55.5 34.2  21.9  30.2    28.7  17.7
## 168 108.2   100.4 106.8  63.3 41.7  24.6  37.2    33.1  19.8
## 169 114.9   115.9 111.9  74.4 40.6  24.0  36.1    31.8  18.8
## 170  99.1    90.8  98.1  60.1 39.1  23.4  32.5    29.8  17.4
## 171  92.2    81.9  92.8  54.7 36.2  22.1  30.4    27.4  17.7
## 172  90.8    75.0  89.2  50.0 34.8  22.0  24.8    25.9  16.9
## 173 100.5    90.3  98.7  57.8 37.3  22.4  31.0    28.7  17.7
## 174  98.2    90.3  99.9  59.2 37.7  21.5  32.4    28.4  17.8
## 175 115.3   108.8 114.4  69.2 42.4  24.0  35.4    21.0  20.1
## 176  96.8    79.4  89.2  50.3 34.8  22.2  31.0    26.9  16.9
## 177  92.6    83.2  96.4  60.0 38.1  22.0  31.5    26.6  16.7
## 178 119.2   110.3 113.9  69.8 42.6  24.8  34.4    29.5  18.4
## 179 102.7    92.7 101.9  64.7 39.5  24.7  34.8    30.3  18.1
## 180 109.5   104.5 109.9  69.5 43.1  25.8  39.1    32.5  19.9
## 181 108.5   104.6 109.8  68.1 42.8  24.1  35.6    29.0  19.0
## 182  79.3    69.4  85.0  47.2 33.5  20.2  27.7    24.6  16.5
## 183  95.5    83.6  91.6  54.1 36.2  21.8  31.4    28.3  17.2
## 184  92.3    86.8  96.1  58.0 39.4  22.7  30.0    26.4  17.4
## 185  98.9    90.4  95.5  55.4 38.9  22.4  30.5    28.9  17.7
## 186  89.5    83.7  98.1  57.3 39.7  22.6  32.9    29.3  18.2
```

```
## 187 117.5   109.3 108.8  67.7 41.3  24.7  37.2    31.8  20.0
## 188 107.4    98.9 104.1  63.5 39.8  23.5  36.4    30.4  19.1
## 189 109.2    98.0 101.8  62.8 41.3  24.8  36.6    32.4  18.8
## 190 103.4   101.2 103.1  61.5 40.4  22.9  33.4    29.2  18.5
## 191  91.4    80.6  92.3  54.3 36.3  21.8  29.6    27.3  17.9
## 192 115.2   113.7 112.4  68.5 45.0  25.5  37.1    31.2  19.9
## 193 104.9    94.1 102.7  60.6 38.6  24.7  34.0    30.1  18.7
## 194 106.7   105.7 111.8  65.3 43.3  26.0  33.7    29.9  18.5
## 195  92.2    85.6  96.5  60.2 38.9  22.4  31.7    27.1  17.1
## 196 101.6    96.6 100.6  61.1 38.4  24.1  32.9    29.8  18.8
## 197  97.8    86.0  96.2  57.7 38.6  24.0  31.2    27.3  17.4
## 198  92.0    89.7 101.0  62.3 38.0  22.3  30.8    27.8  16.9
## 199  94.0    78.0  99.0  57.5 40.0  22.5  30.6    30.0  18.5
## 200 103.7    89.7  94.2  58.5 39.0  24.1  33.8    28.8  18.8
## 201 102.7    89.2  99.2  60.2 39.2  23.8  31.7    28.4  18.6
## 202  91.1    85.7  96.9  55.5 35.7  22.0  29.4    26.6  17.4
## 203 107.2   103.1 105.5  68.8 38.3  23.7  32.1    28.9  18.7
## 204 100.8    89.1 102.6  60.6 39.0  24.0  32.9    29.2  18.4
## 205 121.6   113.9 107.1  63.5 40.3  21.8  34.8    30.7  17.4
## 206 105.6    96.3 102.0  63.3 39.8  24.1  37.3    23.1  19.4
## 207 100.6    93.9 100.1  58.9 37.6  21.4  33.1    29.5  17.3
## 208 102.7   101.3 101.7  60.7 39.4  23.3  36.7    31.6  18.4
## 209  99.8    83.9  91.8  53.0 36.2  22.5  31.4    27.5  17.7
## 210  92.9    84.4  94.0  56.0 38.2  22.6  29.0    26.2  17.6
## 211  91.2    79.4  89.0  51.1 35.0  21.7  30.9    28.8  17.4
## 212 115.6   104.0 109.0  63.7 40.3  23.2  36.8    31.0  18.9
## 213  98.3    89.7  99.1  56.3 38.8  23.0  29.5    27.9  18.6
## 214 103.7    97.6 104.2  60.0 40.9  25.5  32.7    30.0  19.0
## 215  98.7    87.6  96.1  57.1 38.1  21.8  28.6    26.7  18.0
## 216 119.8   122.1 112.8  62.5 36.9  23.6  34.7    29.1  18.4
## 217  92.8    81.1  96.3  53.8 36.5  21.5  31.3    26.3  17.8
## 218  93.3    81.5  94.4  54.7 39.0  22.6  27.5    25.9  18.6
## 219 106.8   100.0 105.0  63.9 39.2  22.9  35.7    30.4  19.2
## 220  93.9    88.7  94.5  53.7 36.2  22.0  28.5    25.7  17.1
## 221  99.0    91.8  96.2  57.7 38.1  23.9  31.4    29.9  18.9
## 222 119.9   110.4 105.5  64.2 42.7  27.0  38.4    32.0  19.6
## 223  94.2    87.6  95.6  59.7 40.2  23.4  27.9    27.0  17.8
## 224  92.7    82.8  91.9  54.4 35.2  22.5  29.4    26.8  17.0
## 225 106.9    95.3  98.2  57.4 37.1  21.8  34.1    31.1  19.2
## 226  88.8    78.2  87.5  50.8 33.0  19.7  25.3    22.0  15.8
## 227 101.7    91.1  97.1  56.6 38.5  22.6  33.4    29.3  18.8
## 228 105.3    96.7 106.6  64.0 42.6  23.4  33.2    30.0  18.4
## 229 104.0    89.4  98.4  58.4 37.4  22.5  34.6    30.1  18.8
## 230  98.6    93.0  97.0  55.4 38.8  23.2  32.4    29.7  19.0
## 231  99.6    86.4  90.1  53.0 35.0  21.3  31.7    27.3  16.9
## 232 103.4    96.7 100.7  59.3 38.6  22.8  31.8    29.1  19.0
```

```
## 233 100.2    88.1  97.8  57.1 38.9  23.6  30.9    29.6  18.0
## 234  94.9    94.9 100.2  56.8 35.9  21.0  27.8    26.1  17.6
## 235  97.2    93.3  94.0  54.3 35.7  21.0  31.3    28.7  18.3
## 236 104.7    95.6  93.7  54.4 37.1  22.7  30.3    26.3  18.3
## 237 104.0    98.2 101.1  59.3 40.3  23.0  32.6    28.5  19.0
## 238 117.6   113.8 111.8  63.4 41.1  22.3  35.1    29.6  18.5
## 239  95.8    82.8  94.5  61.2 39.1  22.3  29.8    28.9  18.3
## 240 106.4   100.5 100.5  59.2 38.1  24.0  35.9    30.5  19.1
## 241  93.0    79.7  87.6  50.7 33.4  20.1  28.5    24.8  16.5
## 242 119.6   118.0 114.3  61.3 42.1  23.4  34.9    30.1  19.4
## 243 119.7   109.0 109.1  63.7 42.4  24.6  35.6    30.7  19.5
## 244 115.8   113.4 109.8  65.6 46.0  25.4  35.3    29.8  19.5
## 245 118.3   106.1 101.6  58.2 38.8  24.1  32.1    29.3  18.5
## 246  97.4    84.3  94.4  54.3 37.5  22.6  29.2    27.3  18.5
## 247 113.7   107.6 110.0  63.3 44.0  22.6  37.5    32.6  18.8
## 248  89.2    83.6  88.8  49.6 34.8  21.5  25.6    25.7  18.5
## 249 108.5   105.0 104.5  59.6 40.8  23.2  35.2    28.6  20.1
## 250 111.1   111.5 101.7  60.3 37.3  21.5  31.3    27.2  18.0
## 251 108.3   101.3  97.8  56.0 41.6  22.7  30.5    29.4  19.8
## 252 112.4   108.5 107.1  59.3 42.2  24.6  33.7    30.0  20.9
```

```
names(fat)
```

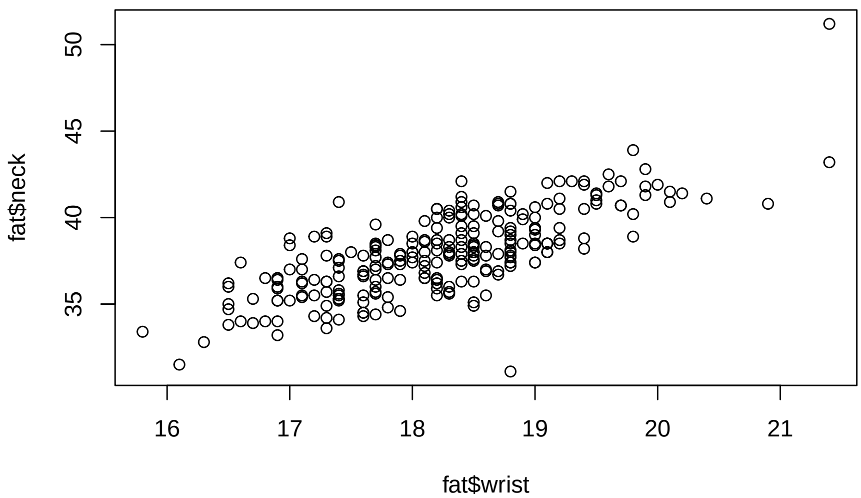
```
##  [1] "case"          "body.fat"      "body.fat.siri" "density"
##  [5] "age"           "weight"        "height"        "BMI"
##  [9] "ffweight"      "neck"          "chest"         "abdomen"
## [13] "hip"           "thigh"         "knee"          "ankle"
## [17] "bicep"         "forearm"       "wrist"
```

3.1 Correlation

p.106

- `correlation` is a numeric summary of how closely related are the measures of two numeric variables when they are in a linear relationship.
- perfect correlation would mean data on a straight line;
- no correlation means values are scattered

```
plot(fat$wrist, fat$neck)
```

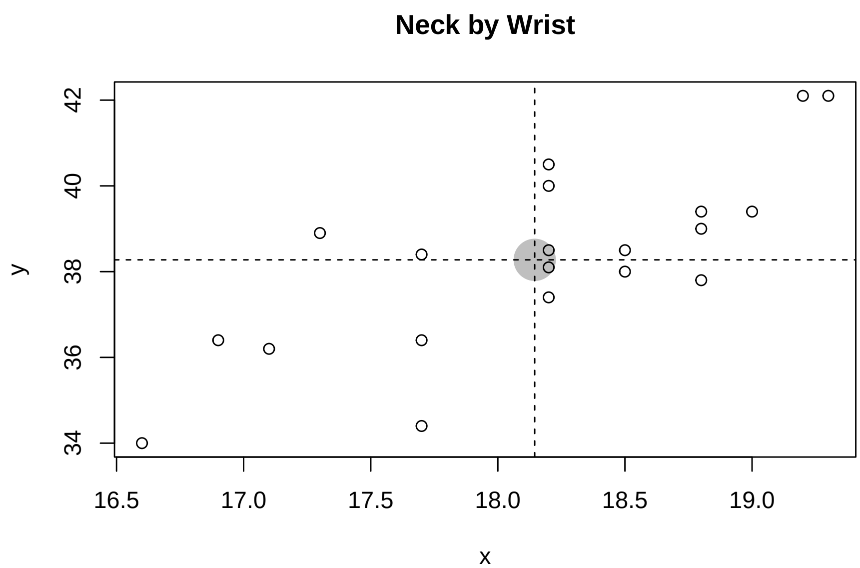


or

```
plot(neck ~ wrist, data = fat)
```

To investigate `correlation`, shift centre of scatter plot to the mean of `x` variable and the mean of `y` variable and add `abline` and `point` functions - plot below separates the data into 4 quadrants as determined by points that are above and below the mean of all `x` values and those above and below the mean of all `y` values

```
x <- fat$wrist[1:20]; y <- fat$neck[1:20]                    # data
plot(x, y, main = "Neck by Wrist")
abline(v = mean(x), lty=2)                                   # dashed vertical line
abline(h = mean(y), lty=2)                                   # dashed horizontal line
points(mean(x), mean(y), pch=16, cex=4, col=rgb(0,0,0, .25))
```



- correlated data shows in opposite regions (bottom left and top right)

3.1.1 covariance

$$cov(x, y) = \frac{1}{n-1} \sum (x_i - \bar{x})(y_i - \bar{y})$$

3.1.2 `Pearson correlation coefficient`

- uses `z-score` instead of deviations…

$$cor(x, y) = \frac{1}{n-1} \sum \frac{(x_i - \bar{x})}{s_x} \frac{(y_i - \bar{y})}{s_y} = cov(x, y)/(s_x s_y)$$

- formula is symmetric, so `x` and `y` can have order switched
- using `z-score` removes the effect of spread and centre by standardizing

3.1.2.1 `cov` and `cor` functions

```
cor(fat$wrist, fat$neck)    # strongly correlated
```

```
## [1] 0.7448264
```

```
cor(fat$wrist, fat$height)  # mildly correlated
```

```
## [1] 0.3220653
```

```
cor(fat$age, fat$ankle)      # basically uncorrelated
```

```
## [1] -0.1050581
```

3.1.3 Spearman correlation coefficient

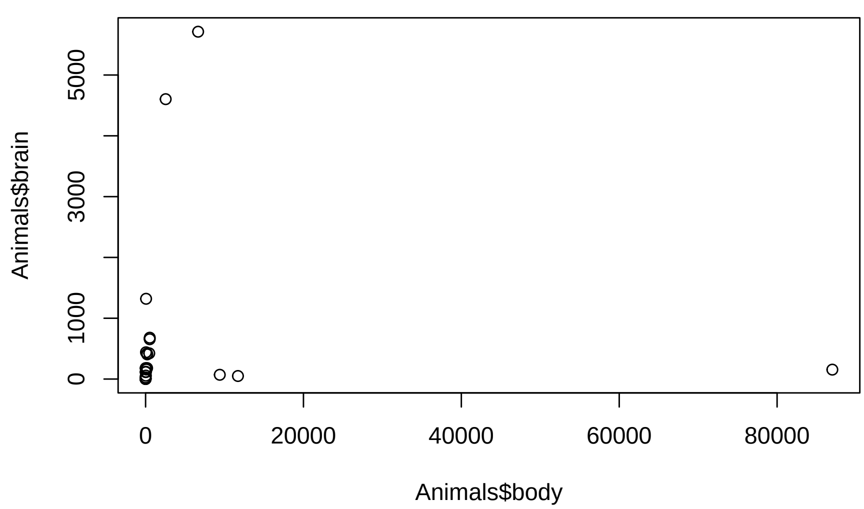
- `Pearson` measures the strength of linear ralationship

```
cor(Animals$body, Animals$brain)
```

```
## [1] -0.005341163
```

- very low correlation...

```
plot(Animals$body, Animals$brain)
```



```
body <- Animals$body; brain <- Animals$brain
cross_prods <- (body - mean(body)) * (brain - mean(brain))
Animals[cross_prods < 0, ]
```

```
##                   body  brain
## Dipliodocus      11700   50.0
## Asian elephant    2547 4603.0
## Horse              521  655.0
## Giraffe            529  680.0
## Human               62 1320.0
## Triceratops       9400   70.0
## Brachiosaurus    87000  154.5
```

- brachiosaurus is a huge outlier…
- so, rank the data

```
cor(rank(body), rank(brain))
```

```
## [1] 0.7162994
```

or

```
cor(body, brain, method="spearman")
```

```
## [1] 0.7162994
```

3.1.3.1 correlation with replication

```
cor(ToothGrowth$dose, ToothGrowth$len)
```

```
## [1] 0.8026913
```

- positive correlation
- for each dosage there are several experimental units -split data into 3 groups and then compute the correlation for these dosage values and the group averages

```
l <- split(ToothGrowth$len, ToothGrowth$dose)
group_means <- c(mean(l[[1]]), mean(l[[2]]), mean(l[[3]]))
cor(c(0.5, 1, 2), group_means)
```

```
## [1] 0.9574428
```

- 0.95 for aggregated data is higher than 0.8 for individual data. In general, correlations formed from averages are typically closer to 1 or -1.

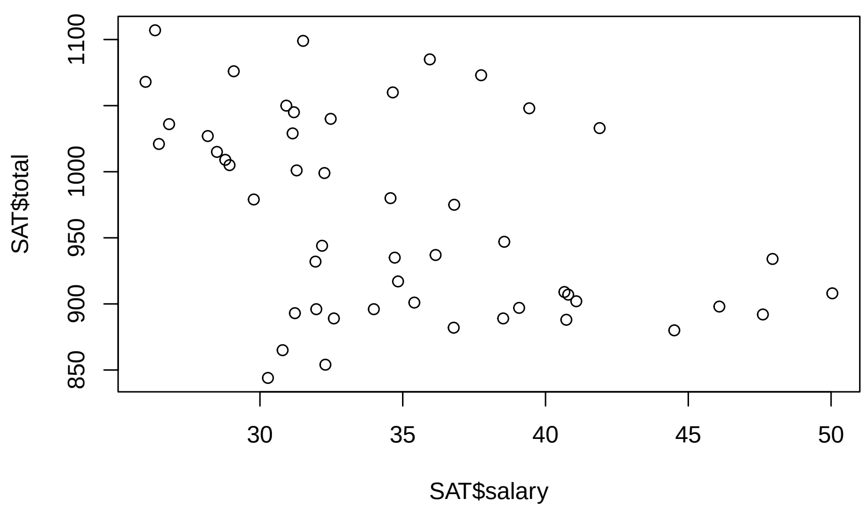
3.1.3.2 correlation v causation

- what is the relationship between average teacher pay and SAT scores?

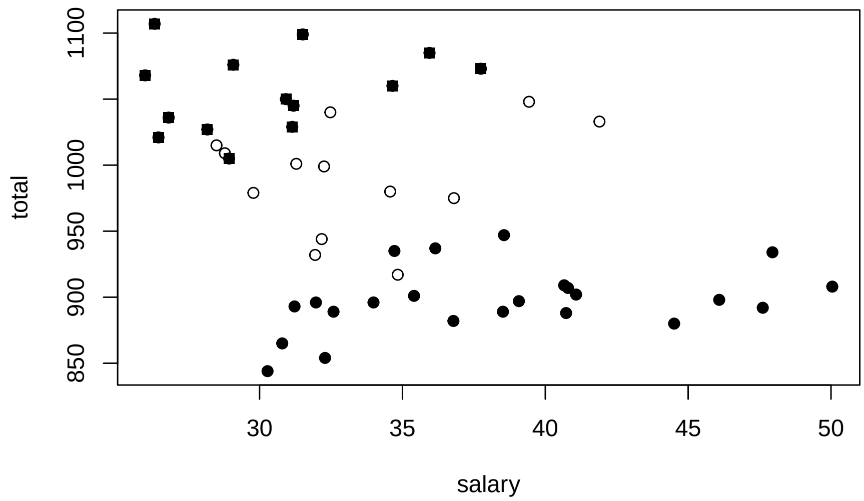
```
cor(SAT$salary, SAT$total)
```

```
## [1] -0.4398834
```

```
plot(SAT$salary, SAT$total)
```




```
plot(total~salary, SAT)
points(total~salary, SAT, subset = perc < 10, pch=15)  # square
points(total ~ salary, SAT, subset = perc > 40, pch=16) #solid
```



- shows correlation for each subgroup is positive

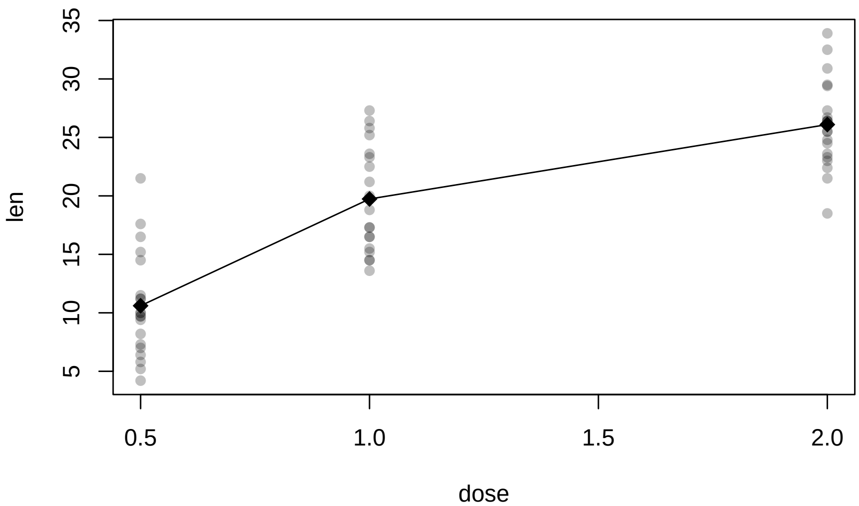
```
total <- SAT$total; salary <- SAT$salary; perc <- SAT$perc
less_10 <- perc < 10
more_40 <- perc > 40
between <- !less_10 & !more_40
c(less = cor(total[less_10], salary[less_10]),
  between = cor(total[between], salary[between]),
  more = cor(total[more_40], salary[more_40]))
```

```
##      less   between      more
## 0.2588199 0.2224926 0.3673461
```

- all correlations are positive for subgroups, yet overall correlation is negative, called `Simpson's paradox` where some trend that exists for subgroups disappears when data are aggregated

3.2 Trends

```
plot(len ~ dose, data=ToothGrowth, pch=16, col=rgb(0,0,0, .25))
points(c(0.5, 1, 2), group_means, cex=1.5, pch=18)
lines(c(0.5, 1, 2), group_means)
```



- summarize a relationship between two numeric variables
- model for a linear trend can be specified as follows: > The mean response value depends linearly on the predictor value.

$$\mu_{y|x} = \beta_0 + \beta_1 x$$

where $\mu_{y|x}$ means the mean of the response variable for a specified value of the predictor x

- for individual data points this becomes

  $$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

  where ϵ_i are the error terms
- we make assumptions about ϵ_i, that on average, the values of ϵ_i are 0

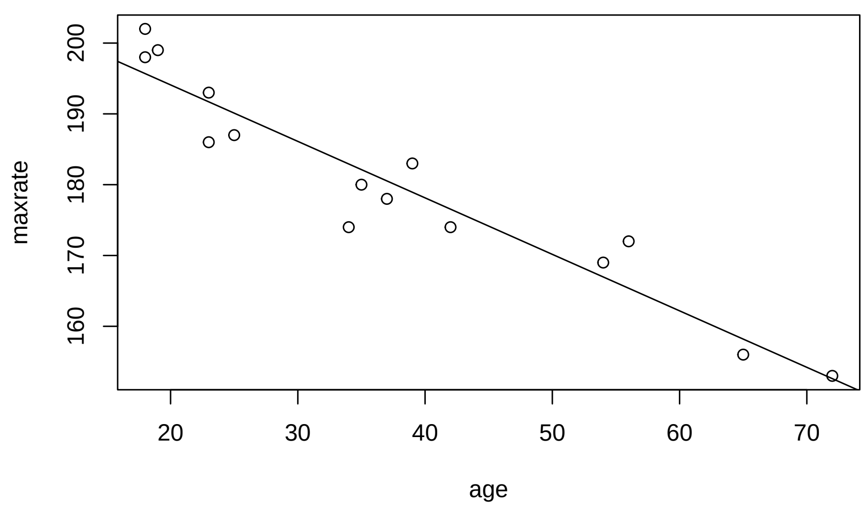
3.2.1 `lm` function

```
res <- lm(maxrate ~ age, data=heartrate)
res
```

```
##
## Call:
## lm(formula = maxrate ~ age, data = heartrate)
##
## Coefficients:
## (Intercept)          age
##    210.0485      -0.7977
```

Visualizing the regression line

```
plot(maxrate ~ age, data=heartrate)
abline(res)
```



3.3 Bivariate Categorical Data

p132

- usually presented in form of a `two-way contingency table`
- count occurrences of each possible pair of levels and place frequencies on a grid
- then compare rows and columns

3.3.1 Summarized data

- create tables in R with `rbind`or `cbind`

```
rbind(c(56,8), c(2,16)) # bind as rows
```

```
##      [,1] [,2]
## [1,]   56    8
## [2,]    2   16
```

```
cbind(c(56,2), c(8,16)) # bind as columns
```

```
##      [,1] [,2]
## [1,]   56    8
## [2,]    2   16
```

- can also use `matrix`

```
seatbelts <- matrix(c(56, 2, 8, 16), nrow=2)
seatbelts
```

```
##      [,1] [,2]
## [1,]   56    8
## [2,]    2   16
```

```
rownames(seatbelts) <- c("buckled", "unbuckled")
colnames(seatbelts) <- c("buckled", "unbuckled")
seatbelts
```

```
##           buckled unbuckled
## buckled        56         8
## unbuckled       2        16
```

3.3.2 Unsummarized data

p134

```
headtail(grades, k=3)
```

```
##      prev grade
## 1     B+    B+
## 2     A-    A-
## 3     B+    A-
##    ...
## 119   F     D
## 120   A     A-
## 121   A     A
## 122   B     B
```

```
table(grades)
```

```
##       grade
## prev    A   A-  B+  B   B-  C+  C   D   F
##    A   15   3   1   4   0   0   3   2   0
##    A-   3   1   1   0   0   0   0   0   0
##    B+   0   2   2   1   2   0   0   1   1
##    B    0   1   1   4   3   1   3   0   2
##    B-   0   1   0   2   0   0   1   0   0
##    C+   1   1   0   0   0   0   1   0   0
##    C    1   0   0   1   1   3   5   9   7
##    D    0   0   0   1   0   0   4   3   1
##    F    1   0   0   1   1   1   3   4  11
```

3.4 Marginal distributions of two-way tables

p135

- distribution of each variable is called `marginal distribution`
- marginal distributions can be found by summing down the rows or columns with `margin.table`

```
dimnames(seatbelts) <- list(parent=c("Buckled", "unbuckled"),
                            child=c("buckled", "unbuckled"))
seatbelts
```

```
##            child
## parent      buckled unbuckled
##   Buckled        56         8
##   unbuckled       2        16
```

```
margin.table(seatbelts, margin=1)  # 1 is for rows
```

```
## parent
##   Buckled unbuckled
##        64        18
```

```
margin.table(seatbelts, margin=2)  # 2 is for columns
```

```
## child
##   buckled unbuckled
##        58        24
```

`addmargins` will return marginal distributions by extending the table

```
addmargins(seatbelts)
```

```
##             child
## parent        buckled unbuckled Sum
##   Buckled          56         8  64
##   unbuckled         2        16  18
##   Sum              58        24  82
```

```
tbl <- with(grades, table(prev, grade))
margin.table(tbl, 1)
```

```
## prev
##  A    A-   B+   B    B-   C+   C    D    F
##   28    5    9   15    4    3   27    9   22
```

```
margin.table(tbl, 2)
```

```
## grade
##  A    A-   B+   B    B-   C+   C    D    F
##   21    9    5   14    7    5   20   19   22
```

3.4.1 Conditional distributions of two-way tables

- comparing rows of a two-way table
  - does a previous grade influence a grade

```
prop.table(tbl, margin=1) * 100
```

```
##       grade
## prev          A         A-        B+         B        B-        C+         C
##    A   53.571429 10.714286  3.571429 14.285714  0.000000  0.000000 10.714286
##    A-  60.000000 20.000000 20.000000  0.000000  0.000000  0.000000  0.000000
##    B+   0.000000 22.222222 22.222222 11.111111 22.222222  0.000000  0.000000
##    B    0.000000  6.666667  6.666667 26.666667 20.000000  6.666667 20.000000
##    B-   0.000000 25.000000  0.000000 50.000000  0.000000  0.000000 25.000000
##    C+  33.333333 33.333333  0.000000  0.000000  0.000000  0.000000 33.333333
##    C    3.703704  0.000000  0.000000  3.703704  3.703704 11.111111 18.518519
##    D    0.000000  0.000000  0.000000 11.111111  0.000000  0.000000 44.444444
##    F    4.545455  0.000000  0.000000  4.545455  4.545455  4.545455 13.636364
##       grade
## prev          D         F
##    A    7.142857  0.000000
##    A-   0.000000  0.000000
##    B+  11.111111 11.111111
##    B    0.000000 13.333333
##    B-   0.000000  0.000000
##    C+   0.000000  0.000000
##    C   33.333333 25.925926
##    D   33.333333 11.111111
##    F   18.181818 50.000000
```

- comparing rows shows that previous grade has a strong influence on the current grade

3.4.2 `xtabs` function

- althernative to `table`, where the structure of the table is specified with a model formula

```
headtail(Fingerprints)
```

```
##    Whorls Loops count
## 1       0     0    78
## 2       1     0   106
## 3       2     0   130
##   ...
## 33      2     5    NA
## 34      3     5    NA
## 35      4     5    NA
## 36      5     5    NA
```

```
xtabs(count ~ Whorls + Loops, Fingerprints)
```

```
##       Loops
## Whorls   0   1   2   3   4   5
##      0  78 144 204 211 179  45
##      1 106 153 126  80  32   0
##      2 130  92  55  15   0   0
##      3 125  38   7   0   0   0
##      4 104  26   0   0   0   0
##      5  50   0   0   0   0   0
```

3.4.3 `ftable`

- flattens contingency tables

```
tbl <- xtabs( ~ Origin + Type + Passengers, Cars93)
ftable(tbl, row.vars=2, col.vars=c(1,3))
```

```
##         Origin     USA                non-USA
##         Passengers   2  4  5  6  7  8       2  4  5  6  7  8
## Type
## Compact              0  0  5  2  0  0       0  1  8  0  0  0
## Large                0  0  0 11  0  0       0  0  0  0  0  0
## Midsize              0  0  6  4  0  0       0  2  9  1  0  0
## Small                0  2  5  0  0  0       0  6  8  0  0  0
## Sporty               1  7  0  0  0  0       1  5  0  0  0  0
## Van                  0  0  0  0  4  1       0  0  0  0  4  0
```

3.5 Graphical summaries of two-way contingency tables

p.140

- bar plots as with numerical data

3.5.1 Mosaic plots

- suitable for viewing relationships between two or more categorical variables

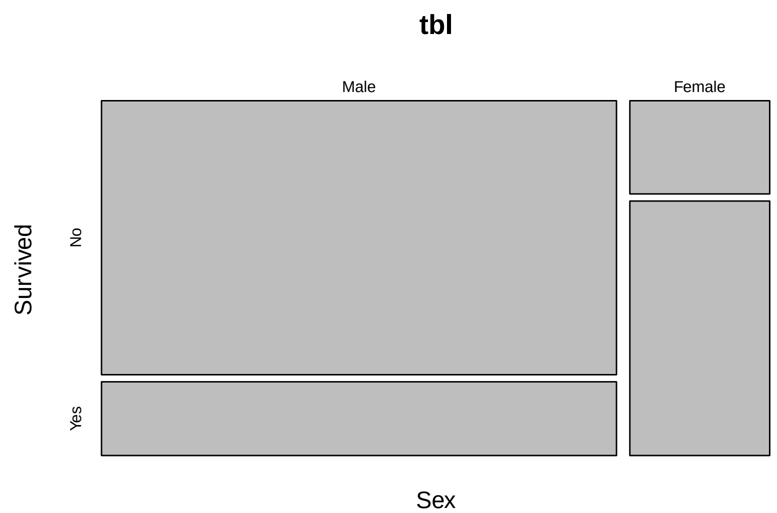
```
titanic <- as.data.frame(Titanic)
xtabs(Freq ~ Survived + Class, data=titanic, subset=Sex=="Female")
```

```
##         Class
## Survived 1st 2nd 3rd Crew
##      No    4  13 106    3
##      Yes 141  93  90   20
```

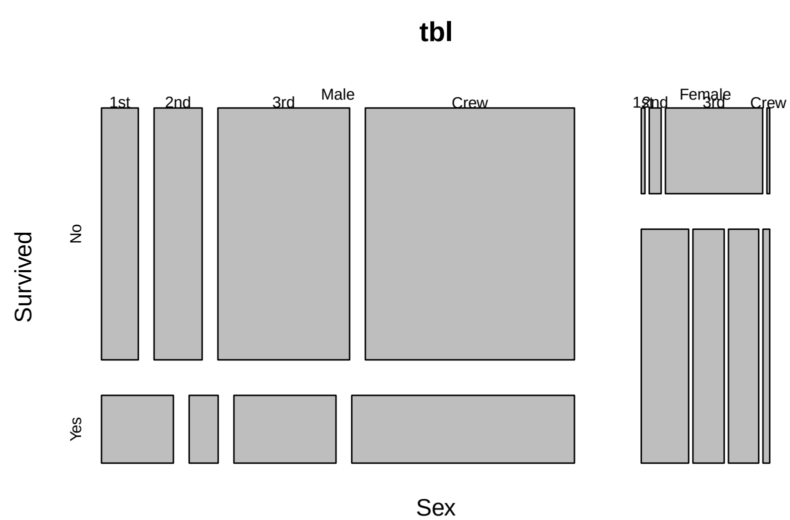
- need to convert `Titanic` to a dataframe as it was a contingency table and `xtabs` displays tabular data

```
tbl <- xtabs(Freq ~ Sex, titanic)
mosaicplot(tbl)
```


```
tbl <- xtabs(Freq ~ Sex + Survived, titanic)
mosaicplot(tbl)
```



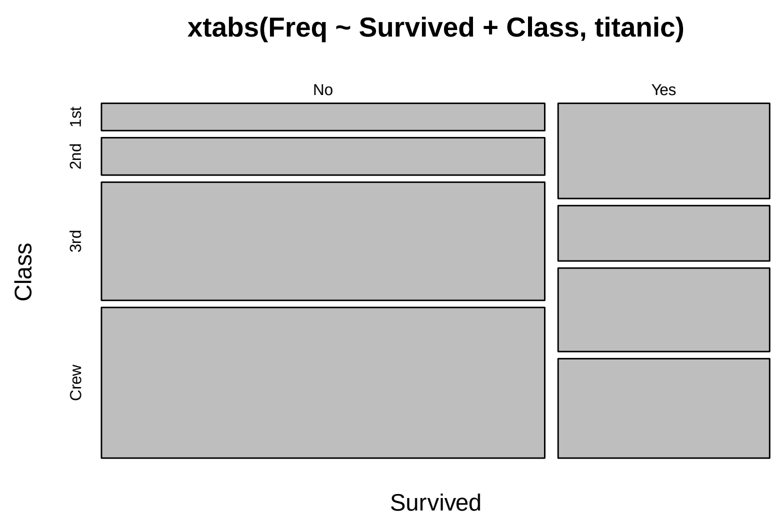

```r
tbl <- xtabs(Freq ~ Sex + Survived + Class, titanic)
mosaicplot(tbl)
```



3.6 Measures of association for categorical data

p.143

```
mosaicplot(xtabs(Freq ~ Survived + Class, titanic))
```



Segmentation of the `survived` variable by `class` is different showing they are "correlated" - the value of one depends on the other. - Pearson's correlation is for numerical data but these are categorical -the variables `class` and `survived` are not numeric but they are naturally ordered - to make ordered factors out of the data.

```
survived <- rep(titanic$Survived, titanic$Freq)
survivied <- ordered(survived)
class <- rep(titanic$Class, titanic$Freq)
class <- ordered(class)
head(class)
```

```
## [1] 3rd 3rd 3rd 3rd 3rd 3rd
## Levels: 1st < 2nd < 3rd < Crew
```

- That the levels are ordered is indicated with `<` above
- now we can coerce them into numeric data with `as.numeric` and calculate `correlation`

```
cor(as.numeric(survived), as.numeric(class), method="kendall")
```

```
## [1] -0.224474
```

- negative correlation is due to the ordering of class with `1st` being a 1 and `crew` being a 4

3.6.1 Kendall τ

Chapter 4

Footnotes and citations

4.1 Footnotes

Footnotes are put inside the square brackets after a caret `^[]`. Like this one [1].

4.2 Citations

Reference items in your bibliography file(s) using `@key`.

For example, we are using the **bookdown** package [**?**] (check out the last code chunk in index.Rmd to see how this citation key was added) in this sample book, which was built on top of R Markdown and **knitr** [Xie, 2015] (this citation was added manually in an external file book.bib). Note that the `.bib` files need to be listed in the index.Rmd with the YAML `bibliography` key.

The RStudio Visual Markdown Editor can also make it easier to insert citations: https://rstudio.github.io/visual-markdown-editing/#/citations

[1] This is a footnote.

Chapter 5

Blocks

5.1 Equations

Here is an equation.

$$f(k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{5.1}$$

You may refer to using `\@ref(eq:binom)`, like see Equation (5.1).

5.2 Theorems and proofs

Labeled theorems can be referenced in text using `\@ref(thm:tri)`, for example, check out this smart theorem 5.1.

Theorem 5.1. *For a right triangle, if c denotes the* length *of the hypotenuse and a and b denote the lengths of the* ***other*** *two sides, we have*

$$a^2 + b^2 = c^2$$

Read more here https://bookdown.org/yihui/bookdown/markdown-extensions-by-bookdown.html.

5.3 Callout blocks

The R Markdown Cookbook provides more help on how to use custom blocks to design your own callouts: https://bookdown.org/yihui/rmarkdown-cookbook/custom-blocks.html

Chapter 6

Sharing your book

6.1 Publishing

HTML books can be published online, see: https://bookdown.org/yihui/bookdown/publishing.html

6.2 404 pages

By default, users will be directed to a 404 page if they try to access a webpage that cannot be found. If you'd like to customize your 404 page instead of using the default, you may add either a `_404.Rmd` or `_404.md` file to your project root and use code and/or Markdown syntax.

6.3 Metadata for sharing

Bookdown HTML books will provide HTML metadata for social sharing on platforms like Twitter, Facebook, and LinkedIn, using information you provide in the `index.Rmd` YAML. To setup, set the `url` for your book and the path to your `cover-image` file. Your book's `title` and `description` are also used.

This `gitbook` uses the same social sharing data across all chapters in your book- all links shared will look the same.

Specify your book's source repository on GitHub using the `edit` key under the configuration options in the `_output.yml` file, which allows users to suggest an edit by linking to a chapter's source file.

Read more about the features of this output format here:

https://pkgs.rstudio.com/bookdown/reference/gitbook.html

Or use:

```
?bookdown::gitbook
```

Bibliography

Yihui Xie. *Dynamic Documents with R and knitr*. Chapman and Hall/CRC, Boca Raton, Florida, 2nd edition, 2015. URL http://yihui.org/knitr/. ISBN 978-1498716963.