Algebraic Graphs with Class

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Abstract

[8]

1 Introduction

- graph representations (in maths, in Haskell/other programming languages)
  - "standard" tuple $G = (V, E), E \subseteq V \times V$
  - containers [6], fgl [4] (state of the art Haskell libraries)
- wellformedness ($\rightarrow$ implicit/explicit vertex set (?), partial functions)
- introduce datatype (?)
- sound and complete representation for graphs
- overview

2 Type class/locale

2.1 The Core

- graph construction primitives
- abstract from datatype $\rightarrow$ type class
- fundamental functions (vertex, edge, etc.)

2.2 Algebraic Structure

- axioms (for digraphs)
- subgraph - partial ordering
- axioms for undirected graphs, other classes
- instances in Haskell (directed, undirected, etc. via Eq instance)

2.3 Graph Transformation Library

- graph families (path, circuit, etc.)
- transpose
- functor
- monad
- removing edges

2.4 Verification

- equational reasoning
- reasoning in Isabelle/HOL → locale
  - "advanced" graph concepts (walks, reachability, SCCs, etc.)
    * more experimentation needed for actual feasibility (additional axioms required for e.g. $Vertex\ u \neq \epsilon$?)
    * first impression: similar as with other representations; abstraction to more advanced concepts quickly hides representation
  - "polymorphic" lemmas
  - compare to Kruskal AFP entry [5] (possibility to get algorithms for any graph representation which can instantiate locale)
  - instantiation with pair_digraph [10] - soundness and completeness proof

3 Deep Embedding

- compact representation
- compare to different concrete/executable representations wrt. efficiency in time and space
  - sparse graphs (edge list)
  - adjacency map
- performance (haskell-perf)
- reasoning in Isabelle/HOL → induction is great
- algorithms directly on deep embedding? → future work/open question (Haskell implementation converts to adjacency map for algorithms)

3.1 Quotient Type

- deep embedding which satisfies axioms
- additional lemmas
- minimization [7]
  - interesting for quotient type, disconnect from Noschinski/any other representation for equality, same applies in Haskell for Eq instance)
  - interesting for algorithms (assuming we can exploit compact representation)

4 Conclusion

- summarize findings
- restate open questions, future work
- labeled graphs (outlook)
- related work: other algebraic approaches (semiring on matrices [3], relational algebra [2])
- usage examples [9, 1]

References

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