Chapter 27. Computing Plane Sundials

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27.1 Introduction

Many ancient sundials are found on the facades of old churches and other old houses. Beautiful examples are found in [3, 4]. Silent and often hardly noticed, they tell of a past age when man measured time using sundials. Even clocks and watches were set by the sun until the end of the 19th century. Today, sundials reappear in gardens or on the facades of houses mainly as a decorative element. Often, these sundials are not very accurate. From time to time, a look at one's watch confirms not insignificant differences. On the other hand, there are sundials whose accuracy surprise us.

There are many different types of sundials. Virtually anything casting a shadow can be made into a sundial. The aim of this chapter is to convey the mathematics which is necessary to design accurate plane sundials. These are sundials where the shadow cast by the tip of a pointer onto a plane surface marked with hour lines (the *dial*) indicates the time. The pointer is called the *gnomon*. Such sundials can not only show *local real* time but also the *mean* time, which we use in daily life, and even other time marks.

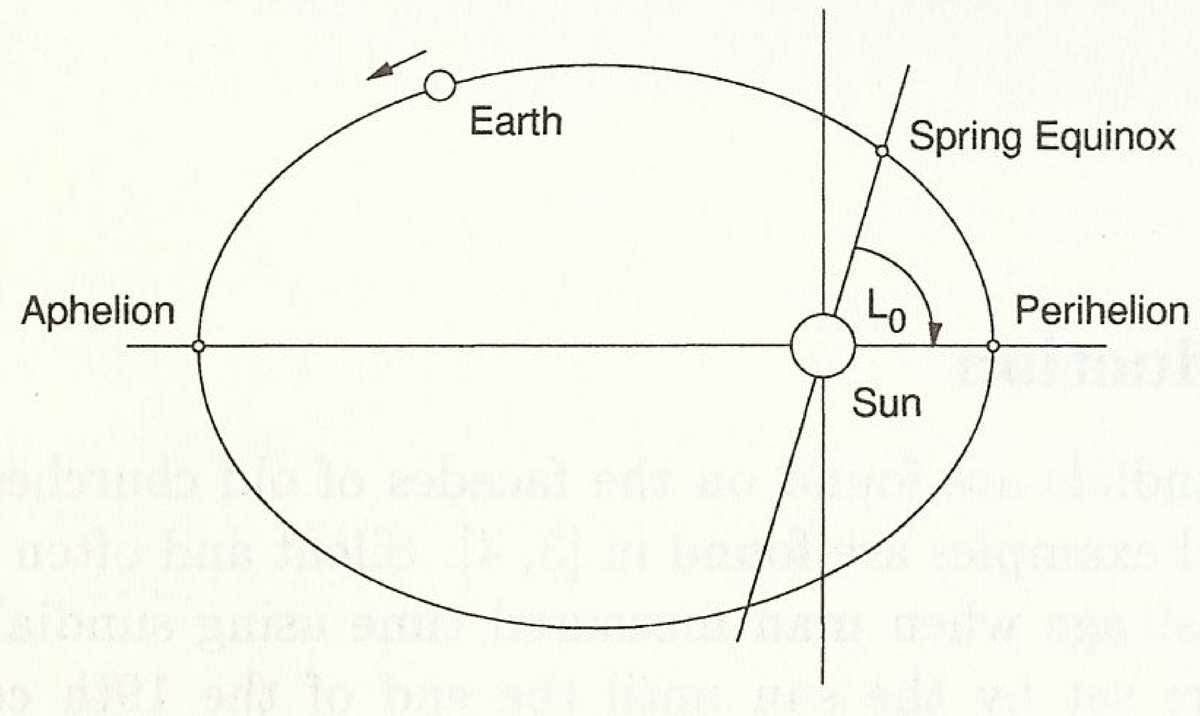
It is important to note, that to be accurate, a sundial must be specially designed for the spot it is to be used in and must also be pointed in the right direction. The algorithms written in MATLAB allow the reader to perform these calculations for his own sundial.

The chapter is based on [1] and its outline is as follows. First, the necessary astronomical fundamentals are introduced and some useful coordinate systems are defined for later use. The gnomonic projection conveys the basic understanding for all further calculations. In Section 27.3, various time marks are introduced for horizontal sundials. The restriction to horizontal dials is removed in Section 27.4. The Chapter concludes with a real example. To complete the picture we mention the related article [2].

27.2 Astronomical Fundamentals

We all know the astronomical fundamentals which are necessary to design a sundial. The Earth orbits on an elliptic path around the sun once a year (revolution) satisfying Kepler's three laws. On this path, the Earth rotates steadily around its axis (rotation), which is tilted with respect to its orbital plane around the sun. The orbital plane is called *ecliptic*. The change of day and night is due to the rotation, while the revolution gives us the seasons. Prominent points of the day are sunrise and sunset as well as the culmination, i.e., the time when the sun is in its highest position (noon).

FIGURE 27.1. *Earth's Elliptic Path*



The sun is located at one focus of the Earth's elliptic path, see Figure 27.1. As the Earth travels away from the sun it slows down until it reaches the aphelion (farthest vertex). At this point the Earth is moving the slowest and is positioned at the summer solstice. The gravitational pull of the sun begins to pull the Earth back toward the sun and increases its speed. It continues to accelerate until it reaches the opposite vertex, the perihelion. At the spring equinox, the day and night become equal in length.

The astronomical constants we need, are the numerical eccentricity of the Earth's elliptic path. It has the value $e = 0.01672$. Further, we need the angle $\epsilon = 23.44°$ between ecliptic and the equatorial plane and finally the ecliptic longitude of the perihelion $L_0 = -77.11°$ counted counterclockwise starting from the spring equinox. Actually these constants are not exactly constant. For instance, the perihelion rotates around the sun once in 21′000 years. This is neglected in what follows.

27.2.1 Coordinate Systems

Various coordinate systems from the astronomers are used. All have a base of orthogonal vectors $(\mathbf{x}_a, \mathbf{y}_a, \mathbf{z}_a)$. If $\mathbf{s}$ is a vector in space then $\mathbf{s}_a$ denotes its vector representation regarding to the base $(\mathbf{x}_a, \mathbf{y}_a, \mathbf{z}_a)$.

The *horizontal coordinate system* is shown in Figure 27.2. The observer is located in the origin of a horizontal plane. The point located on the vertical above the observer is the *zenith*, the opposite on the sphere is the *nadir*. The elevation h gives the angle of a star above the horizon. Negative values refer to positions below the horizon. The azimuth a gives the angle between the south point S and the foot point of a vertical circle going through the star and the zenith.

FIGURE 27.2. *Horizontal System*

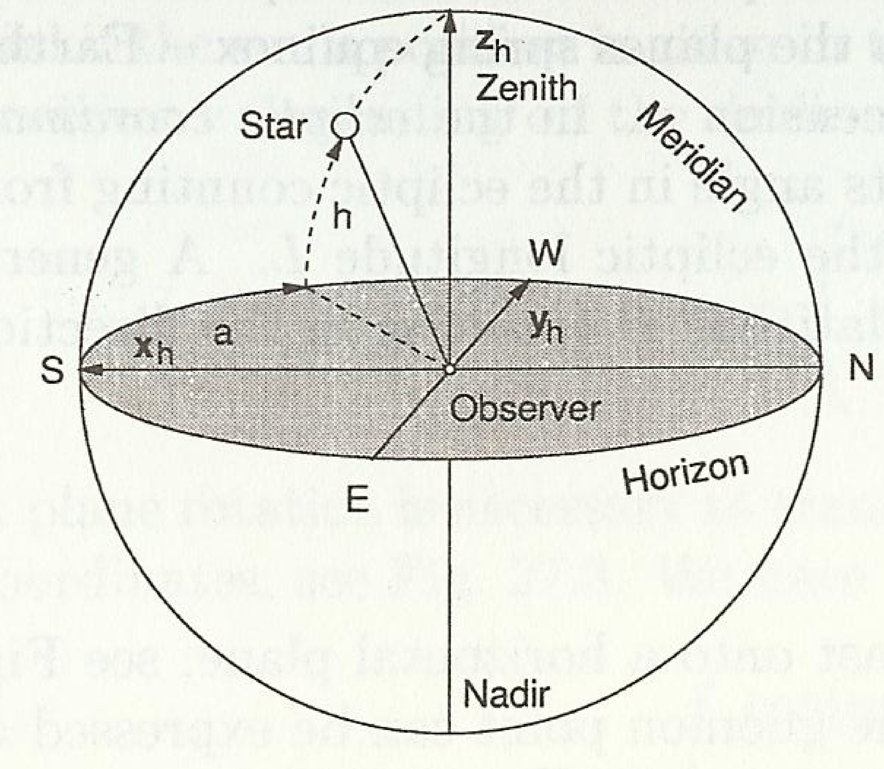
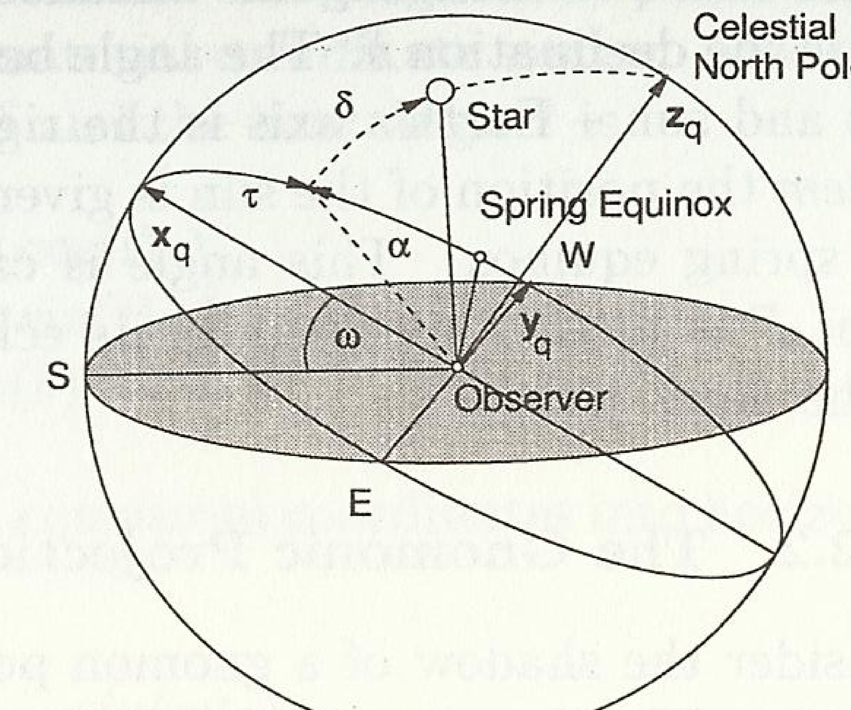


FIGURE 27.3. *Equatorial System*

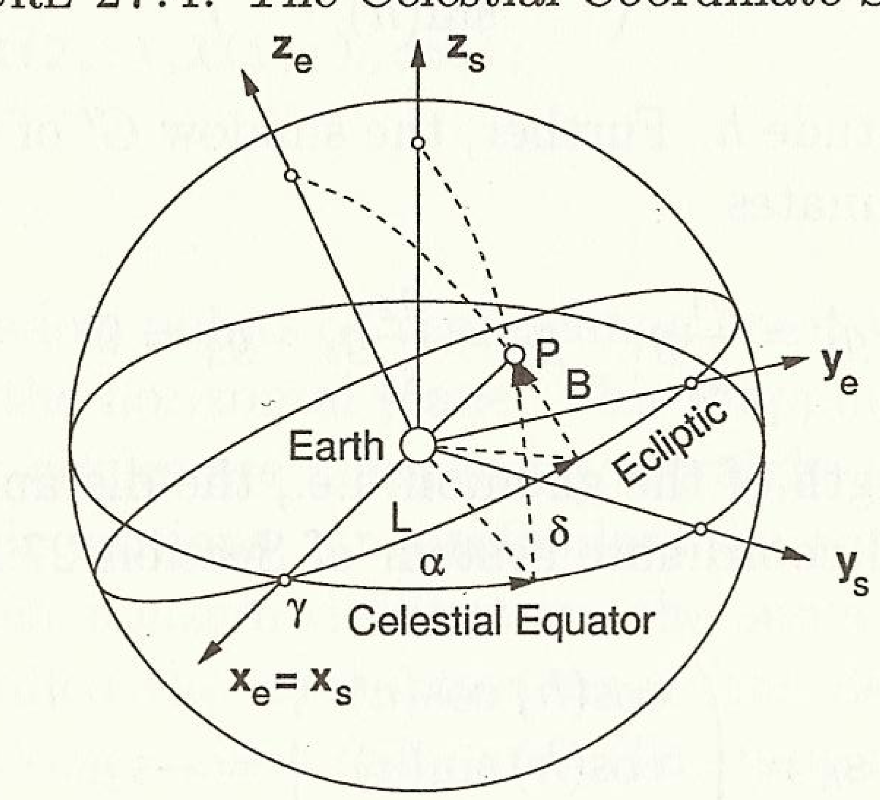


The horizontal coordinate system has the drawback that it refers to the zenith, which is not a fixed point in the cosmos. For astronomical observations it is more convenient to use the *local equatorial system*, which uses the north pole and the local equator as reference, see Figure 27.3. The hour angle τ of a star gives the angle between the meridian and the star measured on the equator in the direction of the daily motion (the meridian is the great circle from the north to the south pole). The *declination* δ of a star is its angular distance from the equator. It is positive if the star lies to the north of the equator, negative otherwise. Instead of the hour angle τ, the *right ascension* α, measured on the equator starting from the spring equinox in the positive direction, is often used.

Two additional celestial coordinate systems are convenient to describe the Earth's yearly motion around the sun. Both systems are geocentric, fade out the Earth's rotation and serve to specify the sun's position relative to the Earth. In contrast to the coordinate systems above they are independent of a specific location on the Earth.

The equinoxes are the points where the ecliptic intersects the celestial equator. The spring equinox is denoted by γ in Figure 27.4. On one hand we have the *system of the celestial equator* in which the sun's position is given by the two

FIGURE 27.4. *The Celestial Coordinate Systems*

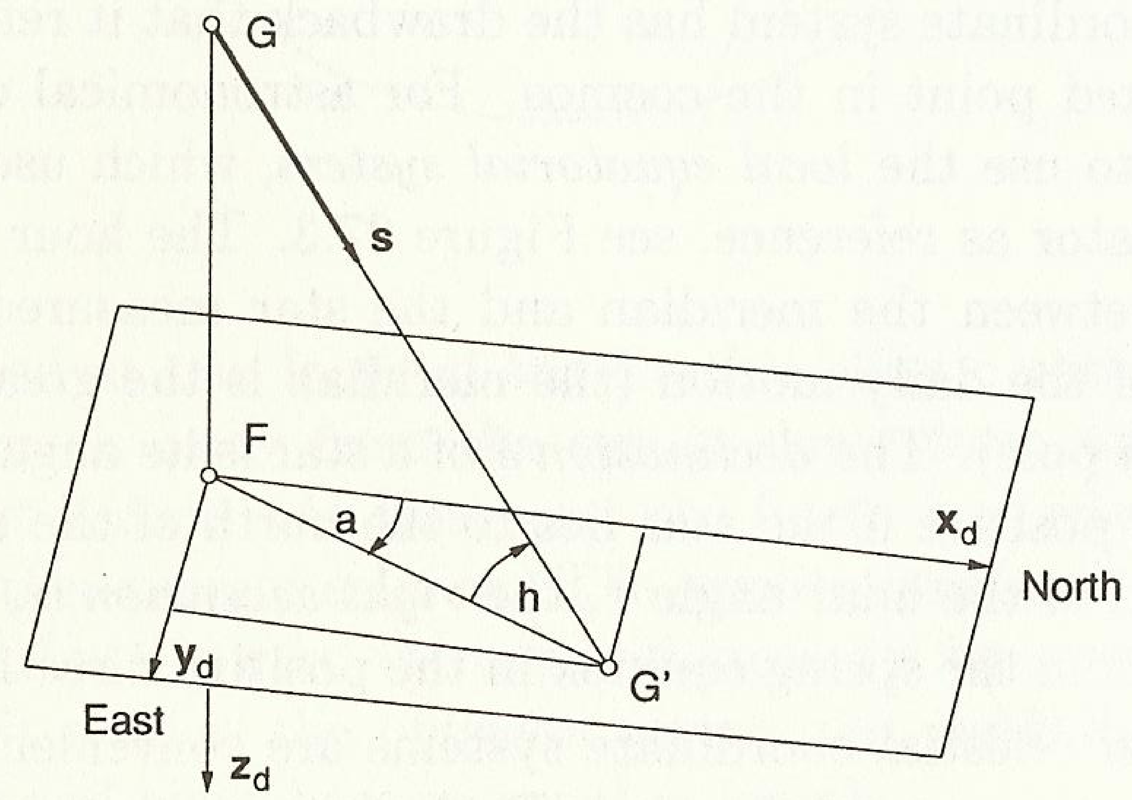


angles δ and α. The angular distance from the plane of celestial equator to the sun is the declination δ. The angle between the planes spring equinox – Earth's axis and sun – Earth's axis is the right ascension α. In the *ecliptic coordinate system* the position of the sun is given by its angle in the ecliptic counting from the spring equinox. This angle is called the ecliptic longitude L. A general point P is further specified by its ecliptic latitude B (positive in the direction of the north pole).

27.2.2 The Gnomonic Projection

Consider the shadow of a gnomon point cast onto a horizontal plane, see Figure 27.5. The direction of the shadow of the gnomon point can be expressed as

FIGURE 27.5. *Coordinate System of Dial*



a vector s in the following coordinate system. Its origin is at the foot F of the perpendicular from the gnomon point G onto the plane. The x_d-axis points to north, the y_d-axis to east and the z_d-axis in direction of the perpendicular. In this coordinate system, vector s is given as

$$\mathbf{s}_d = \begin{pmatrix} \cos(h)\cos(a) \\ \cos(h)\sin(a) \\ \sin(h) \end{pmatrix},$$

with azimuth a and altitude h. Further, the shadow G' of the gnomon point on the plane has the coordinates

$$g_1' = \frac{s_1}{s_3}g, \quad g_2' = \frac{s_2}{s_3}g, \quad g_3' = 0,$$

where g denotes the length of the gnomon, i.e., the distance GF.

Using the horizontal coordinate system of Section 27.2.1, the vector s can be expressed as

$$\mathbf{s}_h = \begin{pmatrix} \cos(h)\cos(a) \\ \cos(h)\sin(a) \\ \sin(h) \end{pmatrix} = -\mathbf{s}_d.$$

Consider now the task to determine the shadow of the gnomon point on the dial if the sun's position is given in the equatorial coordinate system. The sun's position v depending on the declination δ and the hour angle t is

$$\mathbf{v}_q = \begin{pmatrix} \cos(\delta)\cos(t) \\ \cos(\delta)\sin(t) \\ \sin(\delta) \end{pmatrix}.$$

A plane rotation is necessary to transform equatorial coordinates into horizontal coordinates, see Fig. 27.3. We have

$$\mathbf{v}_h = \begin{pmatrix} \cos(w) & 0 & -\sin(w) \\ 0 & 1 & 0 \\ \sin(w) & 0 & \cos(w) \end{pmatrix} \mathbf{v}_q,$$

where $w = 90° - \phi$ and ϕ is the observer's latitude. Thus, given the hour angle t and declination δ of the sun, the local latitude ϕ and the gnomon length g, we can calculate the shadow as given in the MATLAB procedure of Algorithm 27.1.

ALGORITHM 27.1. *Function* project

```
function project(t,d,phi,g,cmd)

X = [cos(d).*cos(t);                        % rays in equatorial sys.
     cos(d).*sin(t);
     sin(d).*ones(size(t))];
w = pi/2 - phi;
R = [cos(w)  0  -sin(w);
      0      1    0;
     sin(w)  0   cos(w)];
X = R*X;                                    % equatorial -> horizontal
ix = (X(3,:) > 0);                          % only rays from above
if any(ix),
   X = g*X(1:2,ix)./(ones(2,1)*X(3,ix));
                                            % shadow points
   plot(X(2,:),X(1,:),cmd);
end
```

The calculation of shadow points G' represents a point-wise mapping of the upper hemisphere onto the horizontal plane. This mapping is called the *gnomonical projection.* Great circles are mapped into straight lines, while other circles are mapped into conic sections. In particular, the equator is mapped into a straight line. For each nonzero declination, the sun's daily arcs are mapped into conic sections called the *declination lines* (the declination of the sun is practically constant along a given daily arc). The type of conic section depends on the latitude ϕ and the declination δ.

27.3 Time Marks

The previous section presented the fundamentals that allow one to determine the shadow of a gnomon point on a horizontal plane, knowing the declination δ and the hour angle t. Now various interesting time marks are presented. The hours are counted from 0 to 24 starting from midnight. Note that the hours on ancient sundials might be counted starting from the solar culmination, i.e., at noon.

27.3.1 Local Real Time

The time which is determined directly by the motion of the sun is called *local real time* or local apparent time. When the sun is in its highest point of the day, it is noon in local real time. But no two points share the same time unless they lie on the same meridian. This time is thus localized to a particular meridian. In addition, the hours of local real time are not equal in length during the year. Because of the Earth's elliptical orbit around the sun and the inclination of the Earth's axis to the ecliptic, the sun's apparent motion across the sky is not uniform throughout the year. A sundial showing local real time may run about 16 minutes fast or slow when compared with a watch, see Sec. 27.3.2.

The hour lines for local real time are especially simple. The hour angle t_r is independent of the declination and is given by

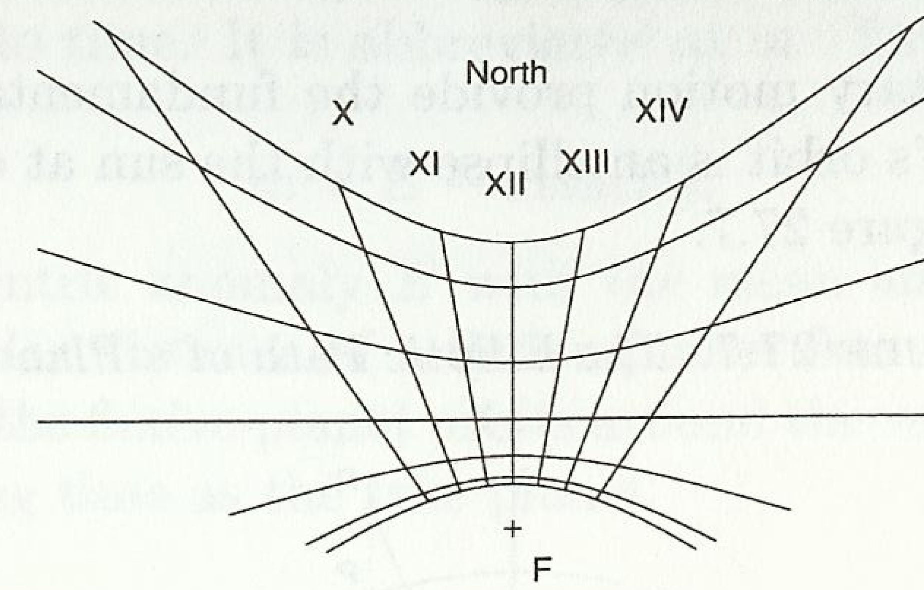
$$t_r = 15^\circ(k - 12).$$

The hour lines are gnomonic projections of great circles and therefore are straight lines on the dial as shown in Figure 27.6. In addition, the dial in Figure 27.6 shows seven declination lines which mark the beginning of a new sign of zodiac. This is a division of the ecliptic into twelve equal sections of 30° according to an old Babylonic tradition. Table 27.1 gives the declination δ and ecliptic longitude L for the beginning of the twelve signs of zodiac. These values are easily derived using the relation between the ecliptic coordinate system and the system of the celestial equator, see Sec. 27.2.1.

TABLE 27.1. *The Signs of the Zodiac*

L	sign	δ	L	sign	δ
0°	Aries	0.0°	180°	Libra	0.0°
30°	Taurus	11.47°	210°	Scorpio	−11.47°
60°	Gemini	20.15°	240°	Sagittarius	−20.15°
90°	Cancer	23.44°	270°	Capricorn	−23.44°
120°	Leo	20.15°	300°	Aquarius	−20.15°
150°	Virgo	11.47°	330°	Pisces	−11.47°

FIGURE 27.6. *Horizontal Sundial for Real Time and Latitude +47°*



The program excerpt below uses the MATLAB procedure `project` to plot a dial for local real time given the latitude ϕ.

```
>> g = 1;  eps = 23.44;  p = pi/180;
>> clg;  axis([-5 5 -2 5]);  hold on;
>> phi = 47*p;                          % latitude
>> k = [7:17];
>> t = 15*p*(k-12);
>> d = p*[23.44 20.15 11.47 0 -11.47 -20.15 -23.44];
>> for i=1:length(d),                   % declination lines
>>    project(t,d(i),phi,g,'-');
>> end
>> d = eps*p*[-1 1];
>> for k=8:16,                          % hour lines
>>    t = 15*p*(k-12);
>>    project(t,d,phi,g,'-');
>> end
```

27.3.2 Mean Time

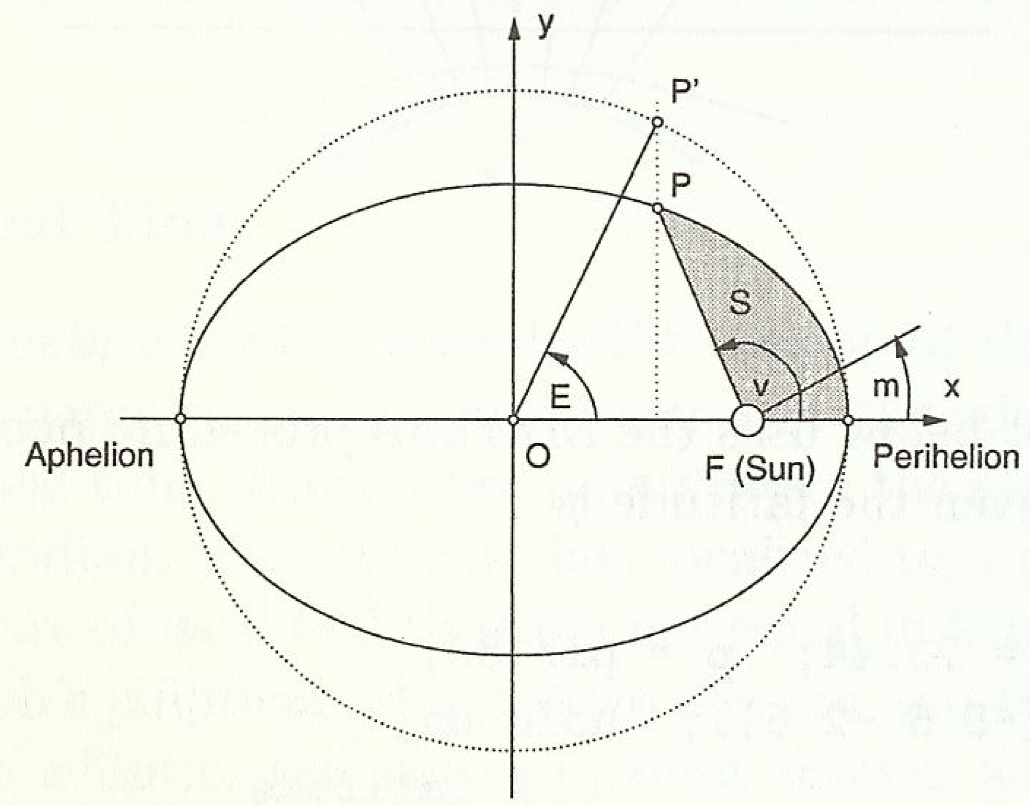
The time we use in daily life is a *mean time*, where all hours have the same length regardless of the season. This mean time allows the rational use of clocks. The local mean time is defined by the motion of an imaginary sun progressing at a constant speed, which is just the average speed with which the real sun moves in the ecliptic, around the celestial equator instead of around the ecliptic. It leaves the spring equinox γ at the same instant as the true sun and meets it again after the lapse of a tropical year.

In order to indicate the mean time on a sundial a correction has to be applied to the real time. This correction, defined as the difference between local real time and local mean time, is called the *equation of time*. Before going into designing hour lines for the mean time the equation of time is introduced in the following two sections.

Kepler's Equation

Kepler's laws of planetary motion provide the fundamentals to introduce the mean time. The Earth's orbit is an ellipse with the sun at one focus, according to the first law, see Figure 27.7.

FIGURE 27.7. *The Elliptic Path of a Planet*



A point P on the ellipse has the coordinates

$$\mathbf{x} = (a\cos(E), \quad b\sin(E)),$$

where a and b are the larger and smaller half axis of the ellipse respectively. The angle E is called the *eccentric anomaly.* The distance of the focus F to the zero point O is ae, where e is the *numerical eccentricity.* Parameter e determines the shape of the ellipse. The Earth's orbit has nowadays the numerical eccentricity $e = 0.01672$. Further, the angle v is called the *true anomaly.* It is the polar angle for a coordinate system with F as origin. Given the coordinates of the points F and P we can calculate the length r of the vector FP. It is

$$r = a(1 - e\cos(E)), \qquad r\cos(v) = a(\cos(E) - e).$$

The first equation is obtained using the fact that the ellipse is an affine mapping of the dotted circle, and the latter is a simple comparison of distances on the x-axis. A tedious manipulation of these two equations yields a relation between v and E:

$$\tan(E/2) = \tan(v/2)\tan(\arccos(e)/2). \tag{27.1}$$

The motion of a planet is such that the quantities v and E change irregularly. However, the radius vector FP sweeps out equal areas in equal intervals of time, according to Kepler's second law. Therefore, the hatched area S in Fig. 27.7 increases regularly. With the help of an affine mapping one finds

$$S = \frac{ab}{2}(E - e\sin(E)),$$

where E is given in radians. The quantity in parenthesis is called *mean anomaly* and is proportional to time. It is abbreviated as m. This leads us to *Kepler's equation*

$$m = E - e\sin(E), \tag{27.2}$$

which links the eccentric anomaly E with the mean anomaly m. The mean anomaly is the angular distance of a fictive planet from its perihelion as seen from the sun, where the fictive planet orbits around the sun with constant speed and the same orbiting time as the true planet.

The Equation of Time

Due to the Earth's varying speed in its orbit around the sun, which is described by Kepler's second law and the tilt of the Earth's axis relative to the ecliptic, the sun's apparent motion across the sky is not uniform throughout the year. Therefore, a correction has to be applied to the real time in order to indicate the mean time. This correction is called the equation of time.

The contribution z_k to the equation of time due to the Earth's varying speed is the difference between true and mean anomaly, i.e.,

$$z_k = m - v.$$

This quantity is best computed with respect to the ecliptic longitude L of the true sun. Remember that L is counted from spring equinox, see Sec. 27.2.1. On the other hand, the true anomaly v is counted from the Perihelion, which currently has the longitude $L_0 = -77.11°$, see Fig. 27.1. Therefore, the true anomaly v is given as

$$v = L - L_0.$$

Then, the eccentric anomaly E and the mean anomaly m are calculated using equations (27.1) and (27.2).

The second contribution to the equation of time is caused by the tilt of the Earth's axis with respect to the ecliptic. This tilt has the angle $\epsilon = 23.44°$ and points always in the same direction. If the Earth's axis was perpendicular to the ecliptic, this correction would not be necessary. Therefore, the fictive mean sun, which defines the mean time, moves around the celestial equator. Consequently, the contribution z_t to the equation of time is the difference between longitude L and right ascension α of the position of the true sun, i.e.,

$$z_t = L - \alpha.$$

The right ascension α is determined from L using the relation

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\epsilon) & -\sin(\epsilon) \\ 0 & \sin(\epsilon) & \cos(\epsilon) \end{pmatrix} \begin{pmatrix} \cos(L) \\ \sin(L) \\ 0 \end{pmatrix} = \begin{pmatrix} \cos(\delta)\cos(\alpha) \\ \cos(\delta)\sin(\alpha) \\ \sin(\delta) \end{pmatrix}$$

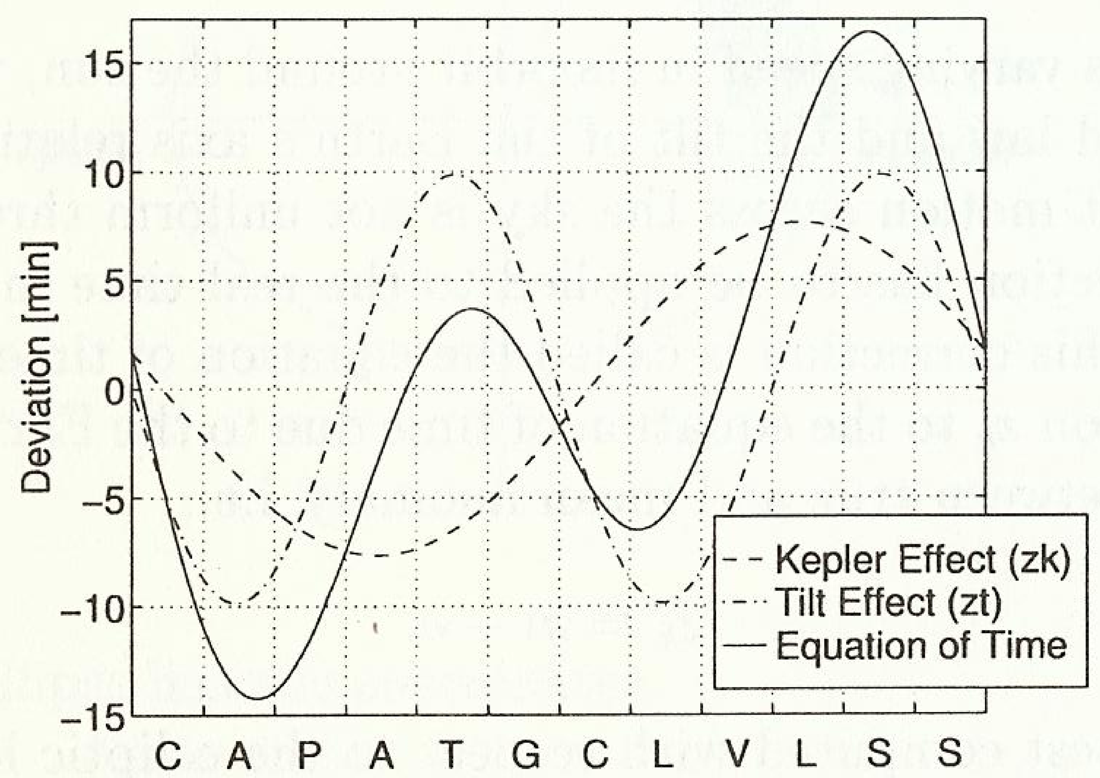
for the true sun's position in the two celestial coordinate systems.

The equation of time z_g is the sum of z_k and z_t. As α makes a jump at $L=180°$ and E makes a jump at the aphelion ($v=180°$), the continuity of z_g must be forced by setting $z_g = \arctan(\tan(z_g))$. Thus, the observed value for the equation of time is

$$z_g = \arctan(\tan(z_k + z_t)).$$

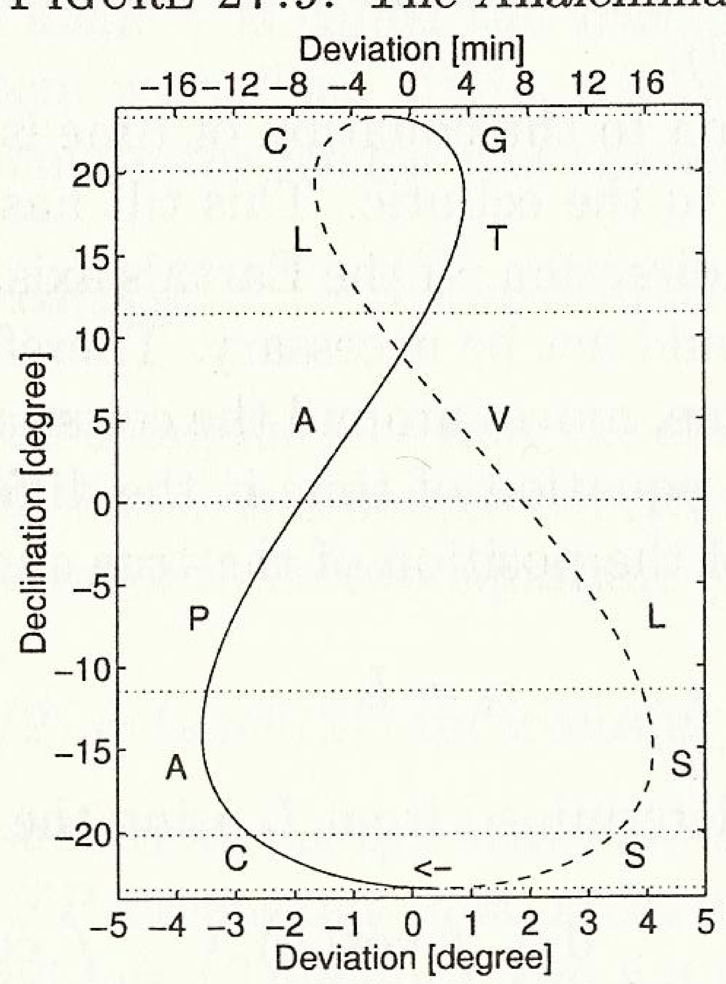
Figure 27.8 shows the equation of time z_g as well as the two fractions z_k and z_t plotted against the signs of the zodiac (z_g depending on the day of the year varies slightly from year to year because of adjustments due to the leap years). While z_k is zero at the aphelion and perihelion of the Earth's orbit, z_t is in step

FIGURE 27.8. *The Equation of Time as a function of the zodiac*



with the seasons, being zero at the equinoxes and the solstices (June 21 and December 21). Figure 27.9 shows the declination of the sun depending on the equation of time.

FIGURE 27.9. *The Analemma*



The graph looks like a figure-8 and is called the *analemma*. If one was to photograph the sun's position at the same time each day throughout the year, the result would be an analemma. The solid part of the curve belongs to the time from December 21 to June 21, while the dashed part belongs to the other half of the year. The dotted declination lines mark the beginning of a new sign of zodiac.

Local Mean Time

The time defined by the hour angle of the fictive mean sun introduced in the previous section, is called the *local mean time*. That is, the local mean time T_m is the local real time T_r subtracted by the equation of time:

$$T_m = T_r - z_g.$$

Consequently the hour angle t_m of the kth hour local mean time is

$$t_m = 15°(k - 12) + z_g.$$

By marking each hour of mean time by an analemma, the mean time can be read directly from the dial without the need to apply an additional correction. The following program excerpt first computes the declination of the sun and the equation of time during the course of a year. With this information the analemma for the kth hour is easily plotted.

```
>> L0 = -77.11;  e = 0.01672;  eps = 23.44;  p  = pi/180;
>> L  = 3*p*[-30:90];                     % Sun's True Longitude
>> v  = L - L0*p;                         % True Anomaly
>> c  = sqrt((1-e)/(1+e));                % c=tan(arccos(e)/2)
>> E  = 2*atan(c*tan(v/2));               % Eccentric Anomaly
>> zk = E-e*sin(E)-v;
>> x  = [cos(L);                          % Sun's Coordinates in System
>>       sin(L)*cos(eps*p);               % of Celestial Equator
>>       sin(L)*sin(eps*p)];
>> r  = sqrt(x(1,:).^2+x(2,:).^2);
>> al = atan2(x(2,:),x(1,:));             % Right Ascension
>> d  = atan2(x(3,:),r);                  % Declination
>> zt = L-al;
>> zg = atan(tan(zk+zt));                 % Equation of Time
>> for k = 9:15,                          % Individual Hour Lines
>>    t  = 15*p*(k-12)+zg;
>>    project(t,d,phi,g,'-y');
>> end;
```

Zone Time

The local times discussed so far are astronomically correct for one meridian. A sundial located farther west is behind a local one. But a system of timekeeping, which is so narrowly localized, is impractical in daily life. Therefore, *time zones* or standard times were introduced at the end of the 19th century. The Earth is divided into 24 time zones, each approximately 15° of longitude apart beginning at the prime meridian. Time increases one hour for each zone east of the prime meridian where time is called Greenwich Mean Time (GMT) and decreases for each zone west of GMT.

The zone time differs from the local mean time by the difference in longitude between the local meridian λ and the meridian of the time zone λ_0. Thus, the hour angle for the kth hour zone time is

$$t_z = 15^\circ(k-12) + z_g + (\lambda_0 - \lambda).$$

27.3.3 Babylonic and Italic Hours

The people of Babylon, to whom we owe the division of the day into 24 hours, used to count the hours starting from sunrise. In contrast, the Italians of the Middle Ages started a new day at sunset. Both of these systems of timekeeping have been in use for a very long time. Countless old European dials are to be found even now, which refer to Babylonic or Italic hours. Let us study dials with these hour marks, although they are not used today anymore.

Let

$$\mathbf{u}_q = \begin{pmatrix} \cos(\delta)\cos(\tau) \\ \cos(\delta)\sin(\tau) \\ \sin(\delta) \end{pmatrix}$$

be the position of the sun at sunset. Vector $\mathbf{u}$ is in the horizontal plane. Thus

$$\mathbf{u}_h = \begin{pmatrix} \cos(w) & 0 & -\sin(w) \\ 0 & 1 & 0 \\ \sin(w) & 0 & \cos(w) \end{pmatrix} \mathbf{u}_q,$$

with $w = 90^\circ - \phi$ and latitude ϕ, must have a zero z-component. This leads to the equation

$$\cos(\tau) = -\tan(\delta)\tan(\phi)$$

for the hour angle at sunset τ. The negative solution, i.e., $t = -\tau$ is the hour angle for the sunrise. Note that τ also is half the length of the light day. The hour angles for the bth Babylonic and ith Italic hours are

$$t_b = 15^\circ b - \tau \qquad \text{and} \qquad t_i = 15^\circ i + \tau$$

respectively. The hour angle t depends not only on b (or i respectively) but also on the declination of the sun. Remember the gnomonic projection: the Babylonic as well as the Italic hour lines are projections of great circles and therefore are straight lines. Further, the intersection of the bth Babylonic and the ith Italic hour line lies on the hour line $t_r = (b+i)/2$ real time.

For latitudes $\phi > |90^\circ - \epsilon|$, Babylonic and Italic hours are only defined for declinations for which there is neither polar day nor polar night. That means, they are only defined for $\delta \in [-\delta_0, \delta_0]$ with

$$\delta_0 = \begin{cases} \epsilon & : \ |\phi| + \epsilon \le 90^\circ \\ 90^\circ - \phi & : \ \text{otherwise.} \end{cases}$$

The program excerpt below plots Babylonic hour lines. Three points are calculated per hour line which must lie on a straight line.

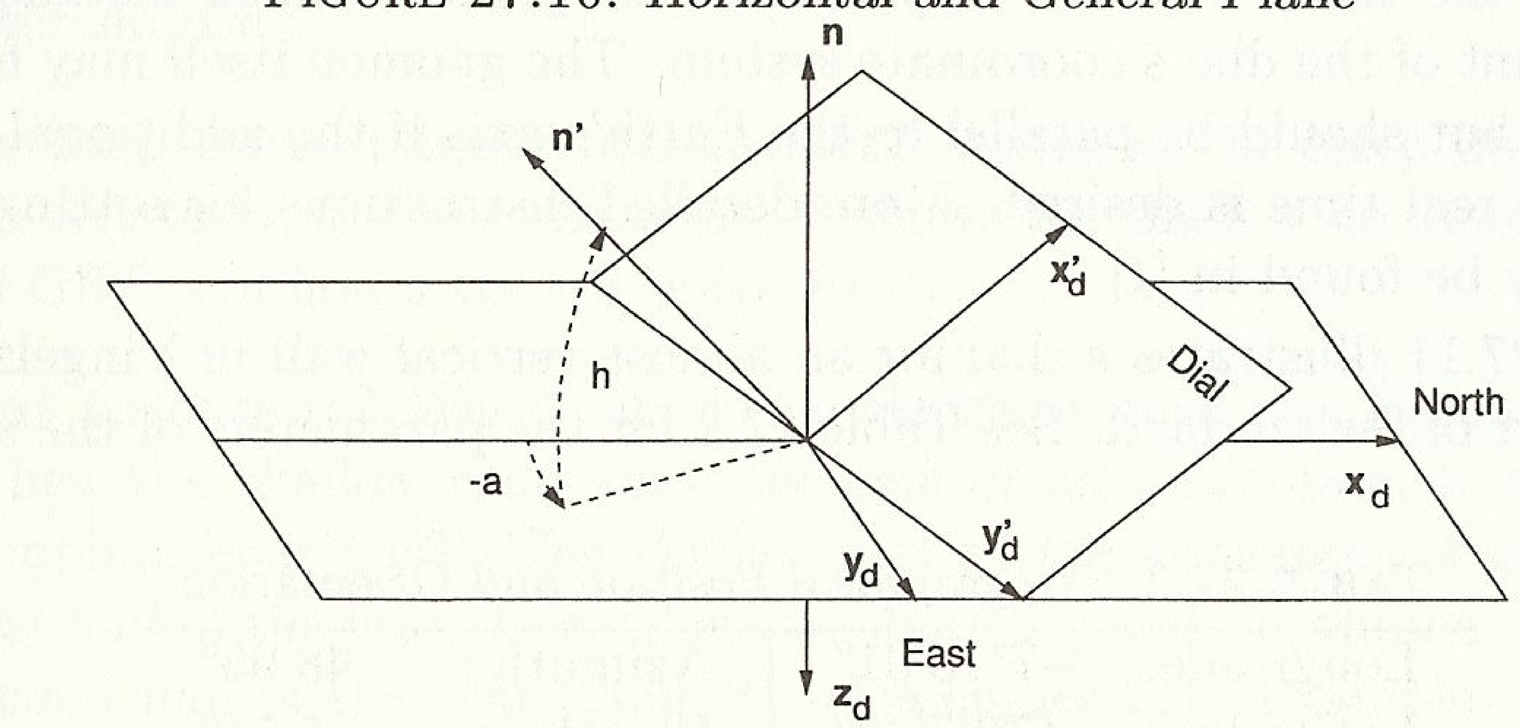
```
>> % Babylonic hours
>> if (abs(phi)+eps*p > pi/2),   % declination restricted above
>>     d0 = pi/2-phi;            %  polar circle
>> else
>>     d0 = eps*p;
>> end
>> s0 = acos(-tan(d0)*tan(phi));
>> k3 = fix(s0/7.5/p);           % number of hours for longest day
>> d = d0*[-1:1];
>> s = acos(-tan(d)*tan(phi));
>> for k=0:k3,
>>     t = 15*k*p-s;
>>     project(t,d,phi,g,'-.r');
>> end
```

A dial with Babylonic and Italic hour lines is given in Figure 27.11.

27.4 Sundials on General Planes

So far, we discussed sundials on horizontal planes. It is not difficult however to generalize them to sundials on planes with any orientation. The orientation of a general plane is defined for instance by the azimuth a and elevation h of the perpendicular $\mathbf{n}$ onto the plane, see Figure 27.10.

FIGURE 27.10. *Horizontal and General Plane*



There are two different approaches to calculate the dial for this general plane. On the one hand, two rotations transform the coordinate system of the horizontal dial ($\mathbf{x}_d$, $\mathbf{y}_d$, $\mathbf{z}_d$) into the coordinate system of the dial in the general plane ($\mathbf{x}_d'$, $\mathbf{y}_d'$, $\mathbf{z}_d'$). On the other hand, we can determine a position on the Earth, for which the direction of the azimuth is parallel to the perpendicular of the desired plane. Then we can calculate a horizontal sundial for this new position. This approach requires some spherical trigonometry and modifications in several places of the existing programs would be necessary [1]. Let us take the first approach.

A sun-ray $\mathbf{v}_d$ in the local coordinate system of the horizontal dial is transformed to a sun-ray $\mathbf{v}'_d$ in the coordinate system of the general plane by the two rotations. In detail, vector $\mathbf{v}'_d$ is determined by

$$\mathbf{v}'_d = \begin{pmatrix} \cos(b) & 0 & -\sin(b) \\ 0 & 1 & 0 \\ \sin(b) & 0 & \cos(b) \end{pmatrix} \begin{pmatrix} \cos(a) & \sin(a) & 0 \\ -\sin(a) & \cos(a) & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{v}_d,$$

where a is the azimuth measured clockwise from the south point and $b = 90° - h$ with elevation h. The corresponding modification of the procedure `project` is straightforward and is left to the reader.

27.5 A Concluding Example

The code fragments given in this Chapter are easily combined into a MATLAB program, which calls the procedure `project` for plotting the dial. An appropriately magnified output can serve as a draft for a skilled painter to make an accurate sundial. As input the program requires the precise geographic position and orientation of the planed sundial as well as its desired size, what determines the length of the gnomon. The longitude of the time zone's meridian λ_0 is further needed if the dial shall also show zone time. The geographical parameters have to be determined with care to guarantee the accuracy. A final difficulty is the correct placement of the gnomon. Only the tip of the gnomon is used for reading the time. It must be placed in the predetermined distance above the zero point of the dial's coordinate system. The gnomon itself may have any orientation but should be parallel to the Earth's axis if the additional reading of the local real time is desired. More detailed instructions for setting up the sundial may be found in [1].

Figure 27.11 illustrates a dial for an almost vertical wall in Vingelz, in the western part of Switzerland. See Table 27.2 for the parameters of the sundial.

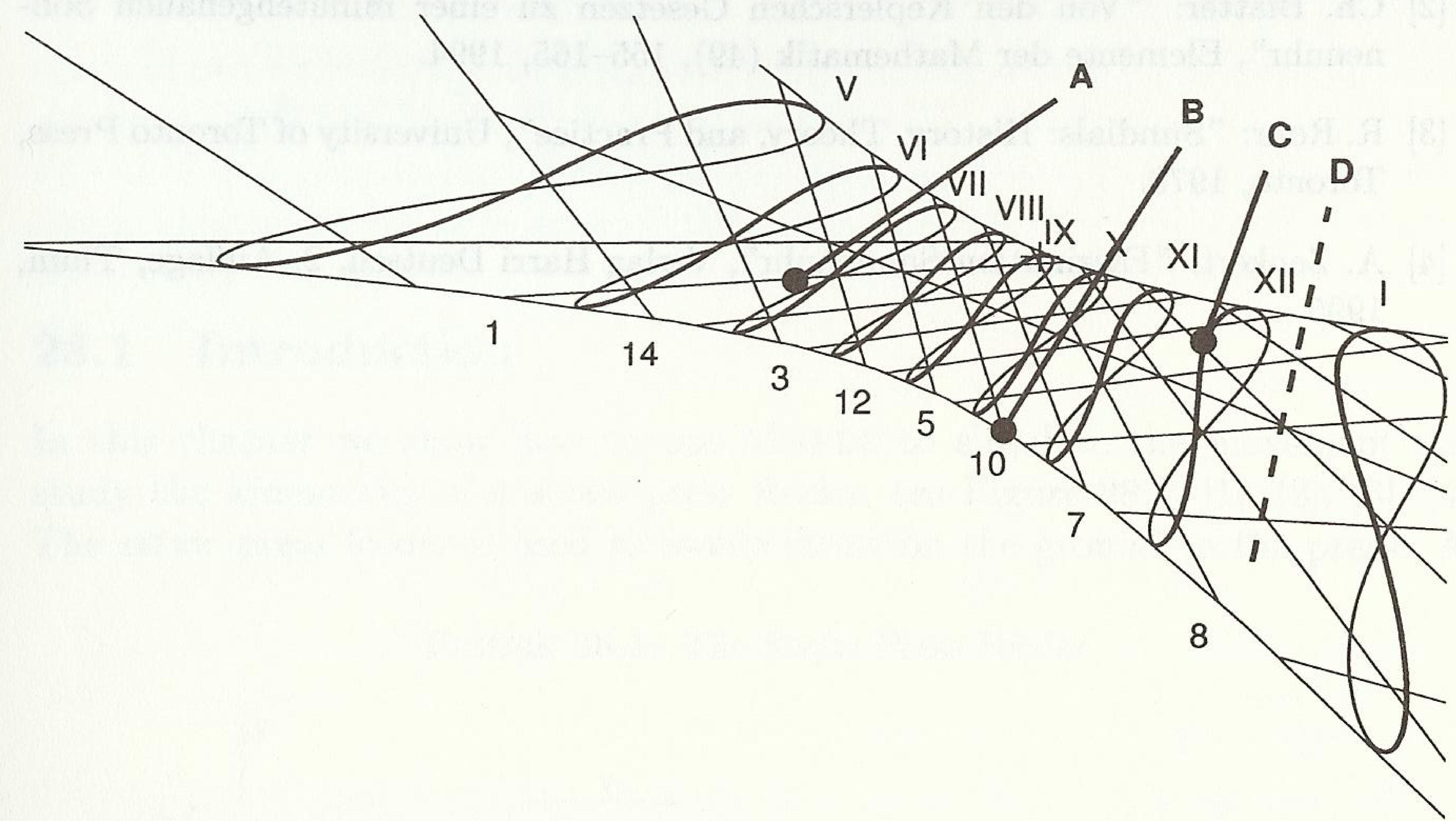
TABLE 27.2. *Geographical Position and Orientation*

Longitude:	$-7°13'01''$	Azimuth :	$-48.95°$
Latitude :	$47°07'48''$	Elevation:	$5.71°$

The dial indicates Central European Time (CET) as well as Babylonic and Italic hours. Even local real time can be read, as the gnomon is parallel to Earth's axis. The net of straight lines behind the analemmas shows the Babylonic and Italic hours. Only every second line is marked with an Arabic number. The lines 1, 3, 5, 7 indicate how many hours have past since sunrise (Babylonic hours) while the lines 8, 10, 12, 14 show how many hours are left until sunset (Italic hours).

Watch at the center of the filled circle to read the time. It represents the shadow of the tip of the gnomon.

FIGURE 27.11. *Sundial at Vingelz, Switzerland*



A: How many hours past since sunrise? Watch at the slightly inclined lines from lower left to upper right. Example A shows the shadow either on April 20 at 6 h 40 CET or on 23. August 6 h 45 CET. It is one hour after sunrise.

B: How long does it last until sunset? Watch at the steep lines from upper left to lower right. Example B shows the shadow on June 21 at 10 h 20 CET: ten hours are left until sunset.

C: What time is it? Watch at the analemmas with the Roman numbers. When the shadow casts onto the line of an analemma, it is exactly a complete hour CET. The thicker part of the analemma is good for the first half of the year (December 21 - June 21) and the thinner part for the second half of the year (June 21 - December 21). Example C shows the shadow on February 19 at 12 h CET.

D: Local real time. The intersection between Babylonic and Italic hour lines can be used to determine the local real time. Just watch at the shadow of the gnomon for this. Example D shows the shadow at noon 12 h local real time. The sun is at its highest point of the day for Vingelz.

References

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Chapter 28. Agriculture Kinematics

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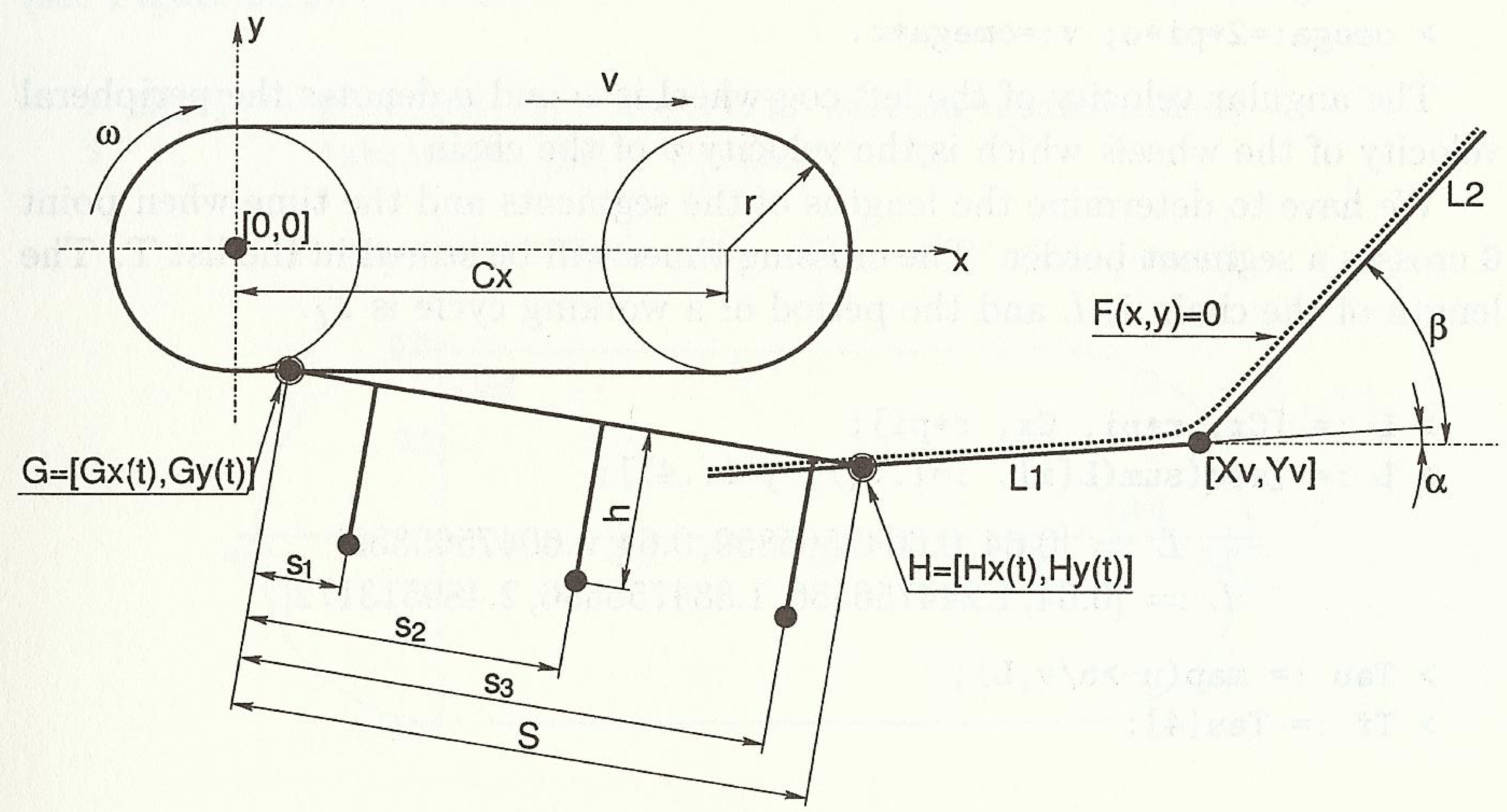
28.1 Introduction

In this chapter we show how to use MAPLE to simulate the movement and study the kinematics of a straw press feeder, see Figure 28.1, [1], [2], [3], [4]. The straw press feeder is used to sweep straw on the ground to the press. At

FIGURE 28.1. *The Straw Press Feeder*



the bar $\overline{GH}$ (of length S) there are three brooms or scrapers attached: b_1, b_2 and b_3 at the positions s_1, s_2 and s_3. Each scraper has the length h.

A chain moves around two cog-wheels with constant velocity v. The two wheels have the radius r and are C_x apart. One end of the supporting bar is connected to the chain (point G), the other end (point H) glides on two rail bars which are connected at the point $[X_v, Y_v]$. One rail bar L_1, is inclined with the angle α the second L_2, with β.

We would like to compute as a function of time the position, the velocity and the acceleration of the ends of the scrapers. The results will be plotted and also a simulation of the movements of the machine will be presented.