

VARIATION IN THE EQUATION OF TIME

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I have set myself the somewhat daunting task of designing a heliochronometer that is accurate to within 30 seconds for the next 250 years. It must operate anywhere in the world where the sun normally shines. The aim is to do better than both my father's Longine watch and Pilkington & Gibbs' engineering marvels. The Longine was within 30 seconds for 60 years. Pilkington & Gibbs is still doing well after nearly a century. One of the first steps in the design task was a rigorous study of the differences or errors between Civil Time and Sun Time. I use the word 'difference' for those items that are natural and therefore cannot be errors. The word 'error' is used when our carelessness, inattention or inability is to blame.

DIFFERENCE/ERROR CLASSES

Four classes of difference or error may be distinguished:

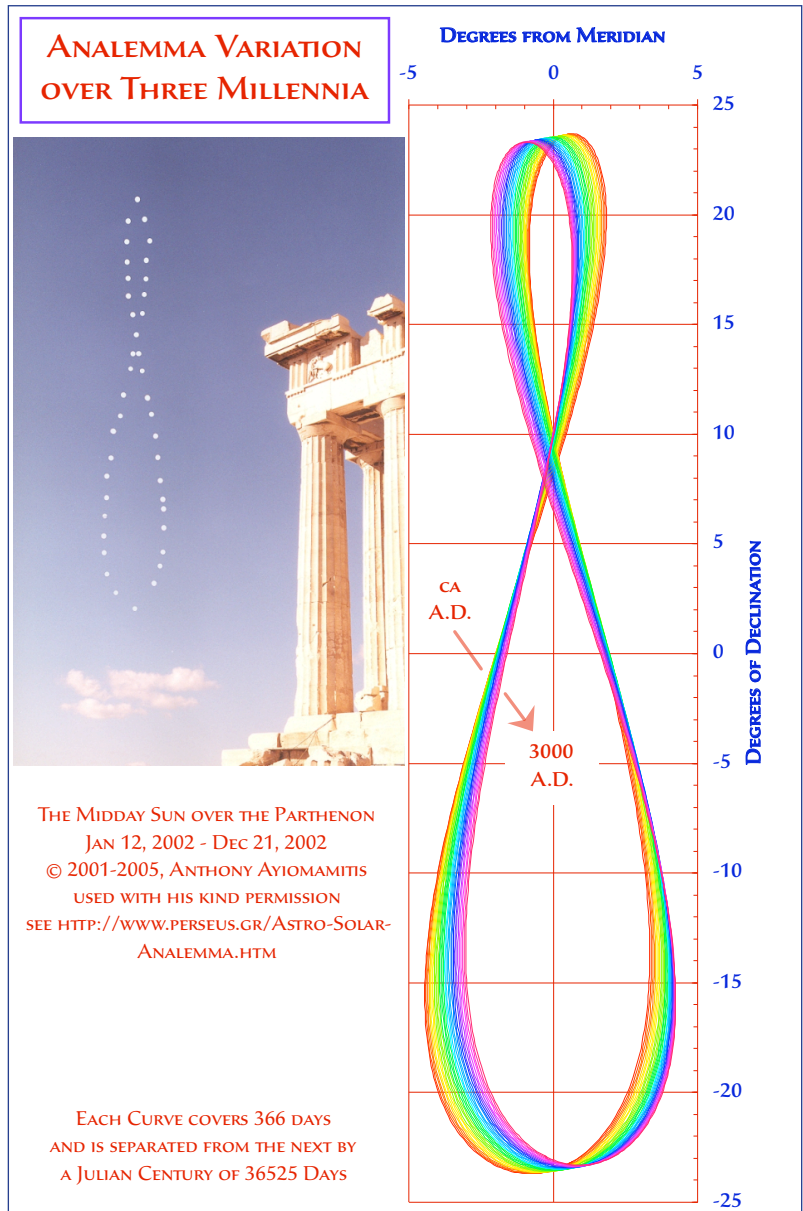
- **Astronomical Differences:**
Orbital Eccentricity, Rotational Obliquity, Luni-Solar Precession, Planetary Precession, Solar Aberration, Nutation Effects.
- **Sun Finding Differences:**
Locational Parallax, Atmospheric Refraction, Penumbra Effects.
- **Calendar Alignment Errors:**
Leap Year, Leap Second, Summer Time
- **Human Errors:**
Longitude set-up, Gnomon set-up, Plate Direction & Angle, Engraving

This article covers only those items marked in italics which are almost entirely responsible for the Equation of Time.

EQUATION OF TIME COMPONENTS

Human kind has settled on the straightforward concept that our time should be based on the perceived daily spin of the earth and on its annual orbit around the sun. Furthermore noon should occur on average when the sun is overhead and the equinoxes on average occurring on the same date. Thus, our civil time is based on the concept of a **Mean Sun**, which *by definition* rotates at a **Uniform rate** around the **Equatorial plane**. Sundials however read the **Real Sun**, which *in fact* appears to rotate at a **Non-uniform rate** around the **Ecliptic plane**, which has an obliquity of 23.4° to the equatorial plane. The difference between the uniform and non-uniform rotational rates gives rise to the so-called Eccentricity effect. The angle between the ecliptic and equatorial planes gives rise to the so-called Obliquity effect.

The Eccentricity effect is an offshoot of the operation of Johannes Kepler laws proposed in 1609. In his 1st law, he states that planets move in ellipses with the Sun at one focus; in his 2nd law, that a line joining the earth to the sun sweeps out equal areas in equal times. The laws imply



that the earth is moving fastest when it is closest to the sun at perihelion (around 2nd January) and slowest at aphelion. Compared with a uniformly moving mean ecliptic sun, the real sun races ahead after perihelion. But by aphelion, the mean sun has caught up as the real sun slows down. In the second half of the year, the mean sun is ahead of the real sun, which only catches up again at perihelion. This is shown in the left of Fig ii on the next page.

The eccentricity of our orbit around the sun is only 0.0167 – the orbit is very nearly circular. However, it does give rise to a difference that is nearly – but not quite – a sine curve with an annual period, of magnitude $7 \frac{1}{2}$ minutes and phased with perihelion/aphelion.

The Obliquity effect can be viewed as the result of projecting the path of the real sun (in the ecliptic) onto the plane of the mean sun (in the equator). The right half of Fig ii overleaf, which for simplicity shows a 'mean ecliptic sun' rather

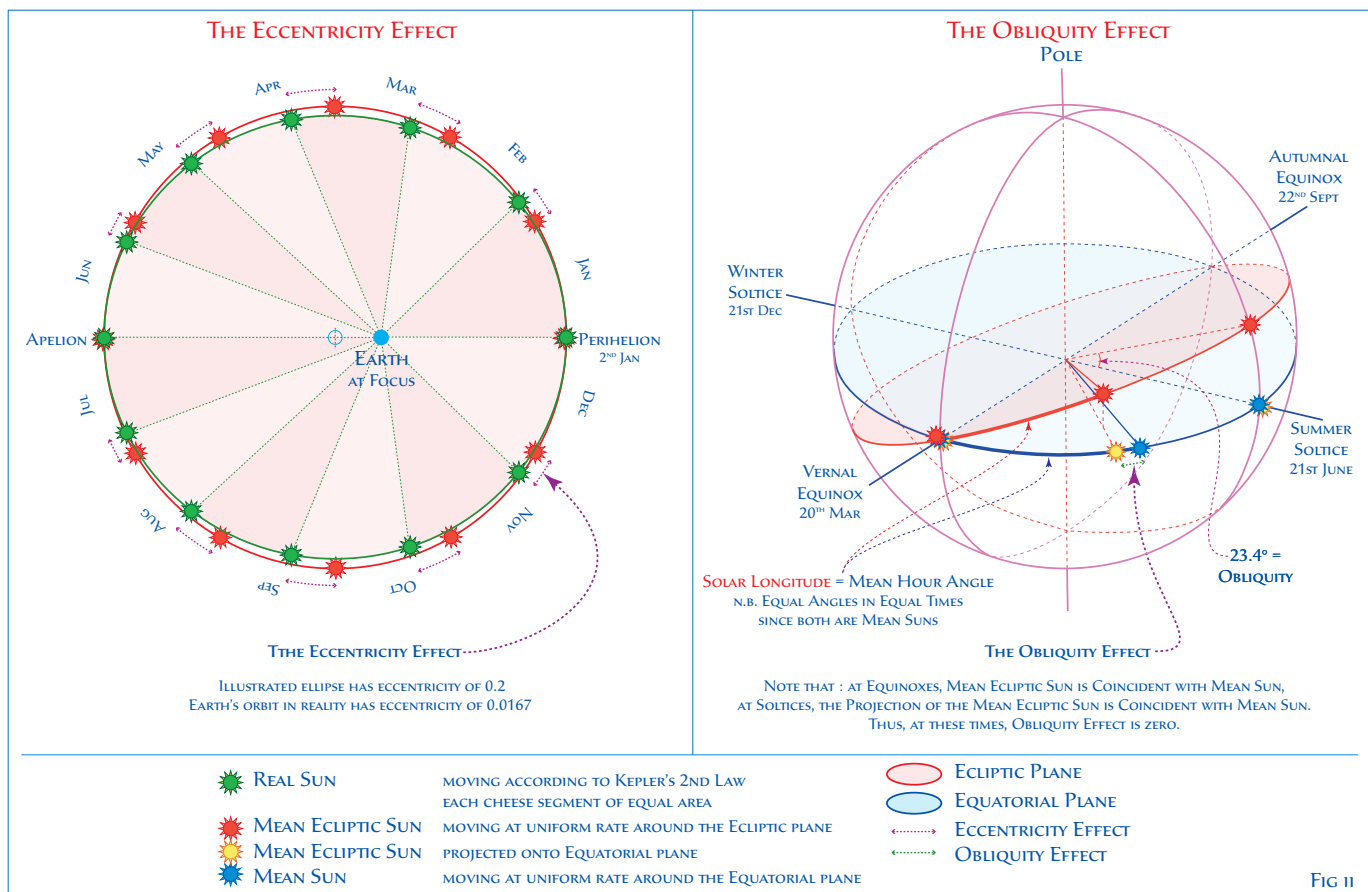


FIG 11

than the real sun, illustrates how this manifests itself. At both the equinoxes and the solstices, there is no difference, but at all other times, there is a difference that turns out to be nearly – but not quite – a sine curve of biannual period, of magnitude 10 minutes and phased with the equinoxes.

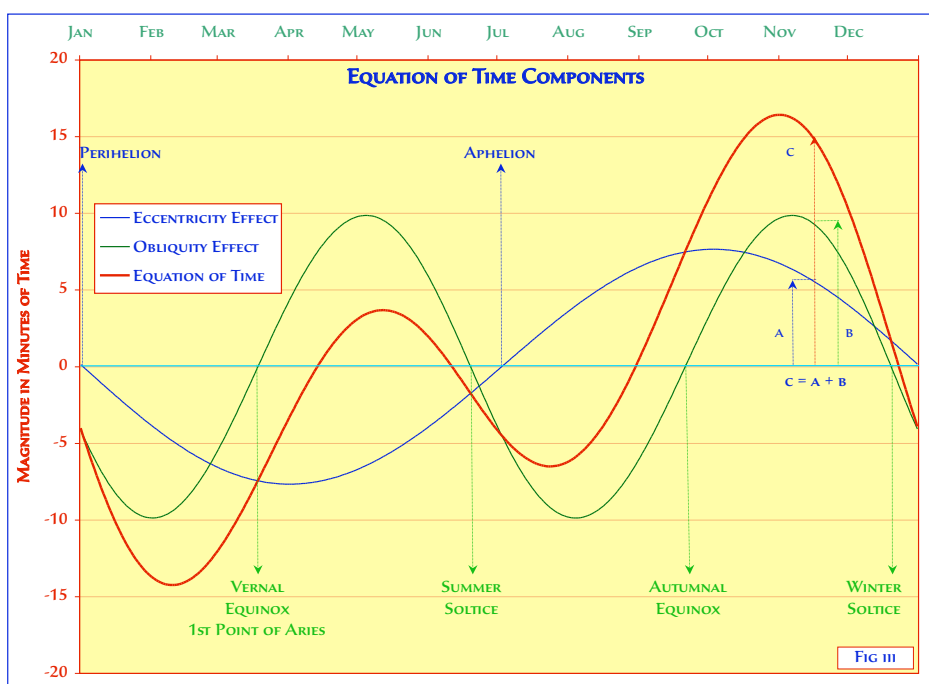
The sum of the eccentricity and obliquity effects, together with a number of relatively minor other effects, add up to give the familiar shape of the Equation of Time (Fig iii) and its alternative guise as the Analemma (Fig i).

A table showing values of the Equation of Time was first published by Thomas Tompion in 1683, using data probably supplied by the Astronomer Royal, John Flamsteed. Since then, numerous methods have been proposed to calculate the values. See references for a few examples.

EQUATION OF TIME CHANGES

Referring to Fig iii, there are four values that dictate the shape of the Equation during a given year.

- The value of the eccentricity of the earth's orbit;
- The date/time of Perihelion;
- The value of the earth's obliquity;
- The date/time of the Vernal Equinox.



All of these are changing slowly with time as a result the gravitational pull of the Sun, the Moon and other planets, each of which moves in a plane that is *not* the same as the equator. In the case of the Sun and Moon (and marginally Jupiter), the pull acts unevenly due to the equatorial bulge of the Earth's shape. The varying pulls cause a general short-term periodic wobble in the earth's axis called Nutation, which has only a minor effect on the Equation of Time. It also causes a number of longer-term larger-scale effects called Luni-Solar Precession (see A below) & Planetary Precession (see B, C & D below).

A) against the backdrop of the stars, the whole axis of the earth gyrates like an out-of-balance spinning top in a 25,770 year cycle. This is a major astronomical effect: e.g. North no longer points at the Pole star; the pyramids are no longer aligned with the correct stars; the astrologer's 1st Point of Aries - the Vernal Equinox - is now in the constellation of Pisces).

On the one hand, it has only a very minor effect *long term* on the Equation of Time, because we have chosen the length of our civil year - which averages at 365.242500 days - to match the mean equinox-to-equinox 'tropical' year of 365.242191 days. This ensures that the Equinoxes do on average stay at the same time of the year : see blue dotted line in Fig iv.

On the other hand, however, the saw-tooth shape of the curve which echoes our leap years does cause significant *short-term* variation - see blue solid line. On each successive year, the time of the equinox is shifted by 1/4 day, until it is pulled back "into line" by the leap year cycles. If a given leap-year is taken as a base-line and considering mid December when the rate of change of the Equation is greatest at 30 seconds/day, the Equation will vary with 7.5 secs in the 1st year following, by 15 secs in the 2nd, by 22.5 secs in the 3rd. It will then 'self-correct' in the following leap year.

B) the ellipse of the earth's orbit around the sun is itself turning against the background of the stars in a 21,000 year cycle. This is a significant effect in relation to the Equation of Time, since it is shifts the time of perihelion in relation to that of the vernal equinox - see red lines in Fig iv.

C) the obliquity of the Earth ranges between 22.1° & 24.5° in a 41,000 year cycle, mainly under the influence of Jupiter - see the green line in Fig v.

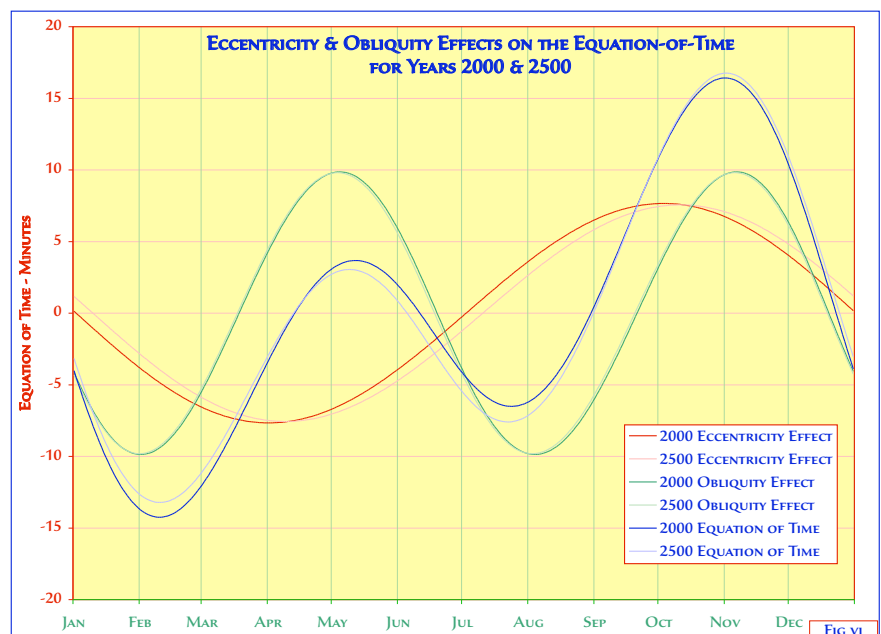
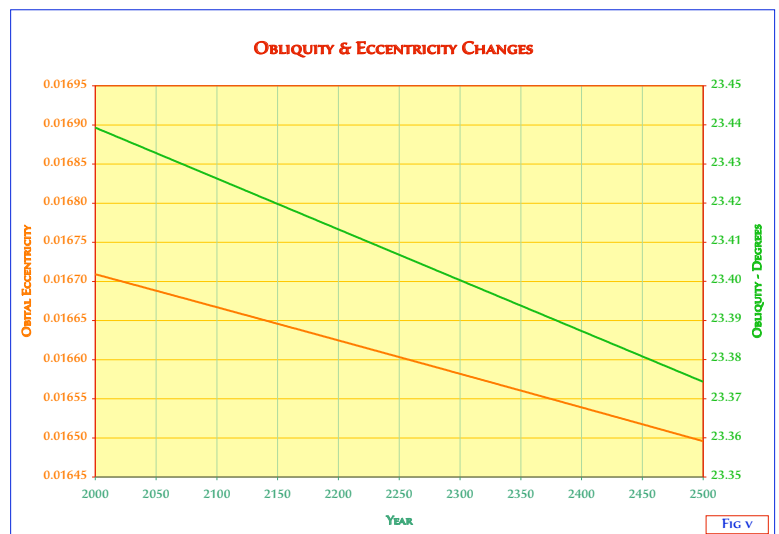
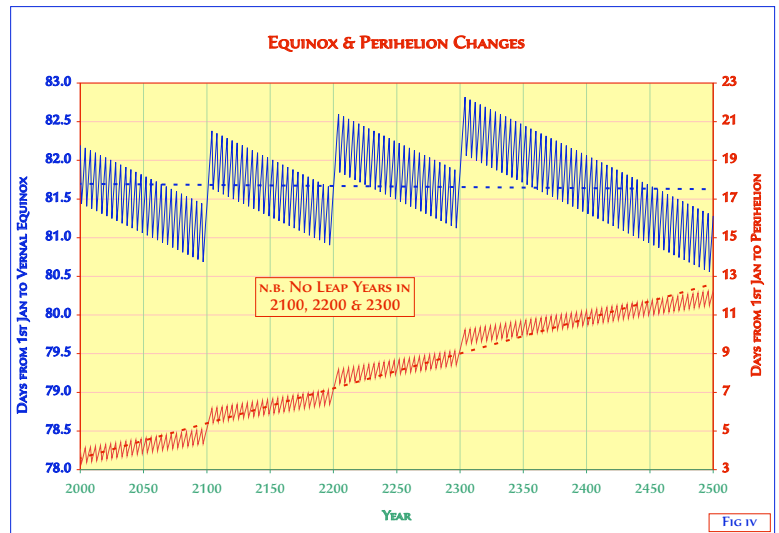
D) the eccentricity of the Earth's orbit changes in a complicated manner in cycles of 96,000 and 413,000 years. - see the orange line in Fig v.

ADDING THE CHANGES

Fig vi shows these four items together over a 500 year period. One sees:

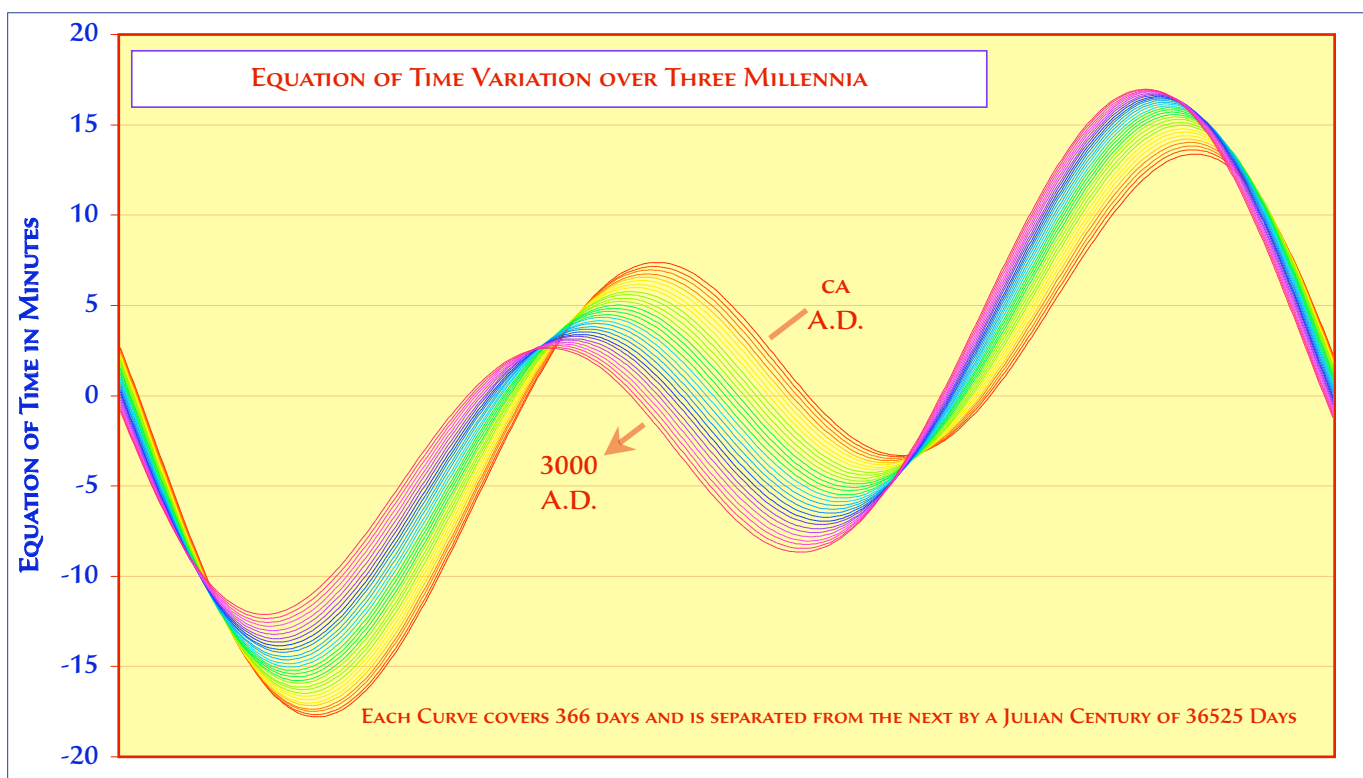
- almost no change in the obliquity effect (the green lines),
- a slight reduction in the magnitude of the eccentricity effect (the height of the red lines)
- a significant shift in the position of the eccentricity effect, as perihelion gets nearer to the vernal equinox (the position of the red lines).

The blue lines shows the result of adding the red and green to give the Equation of Time - providing significant change.



IMPORTANT OBSERVATIONS ARE:

- variability is mainly controlled by the period between perihelion and the equinox;
- in the long term, since our calendar fixes the average date of the equinox, the change in date of perihelion, as a result of planetary precession, is the prime variable.



- in the short term, this period is mostly changed by the vagaries of our leap year cycles;

IS THE VARIATION IMPORTANT?

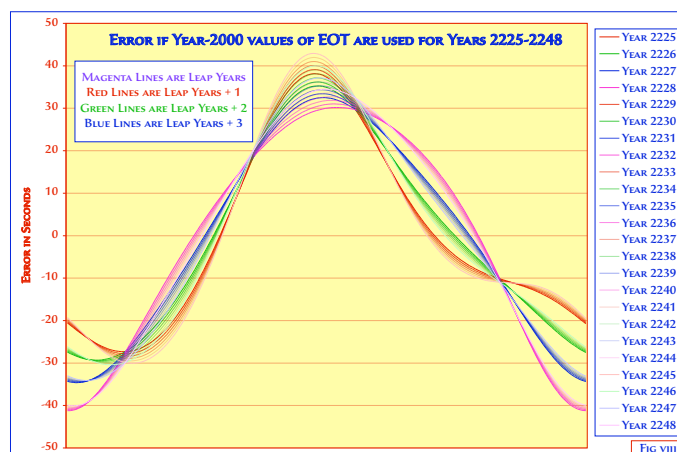
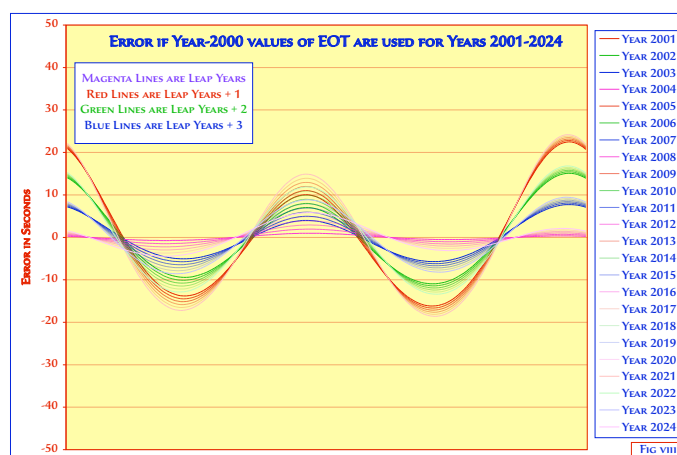
Fig vii shows the considerable shift that the Equation makes over a number of centuries. From which, it is concluded that:

- old dials have old tables which become progressively more out-of-date;
- new dials, which incorporate a table or curve, should ensure that the Equation used is appropriate for the expected lifetime of the dial and duly dated;
- designs for heliochronometers must seek the means to overcome the variability or accept that accuracy will degrade. Fig viii and ix show the error introduced if the Equation of Time for the Year 2000 is used in successive years. These show bands reflecting the various leap and non-leap years cycles and the progressive change within each band. Note that errors of 40 seconds are reached within 250 years.

Subsequent articles will trace strategies to eliminate the effect of leap year cycles and how to combat the longer-term perihelion drift. They will also cover the other sources of error or difference.

REFERENCES

- 1) *Admiralty Manual of Navigation*, Volume III Chap IX, HM Stationary Office, London (1955).
- 2) Peter Duffet-Smith, *Practical Astronomy with your Calculator*, Cambridge Univ. Press, Cambridge (1988).
- 3) Jean Meeus, *Astronomical Algorithms*, Willman-Bell, Richmond (1998).
- 4) For the Equation of Time with ultimate precision, see NASA/JPL's: <http://ssd.jpl.nasa.gov/cgi-bin/eph>
- 5) For an animation of the E-o-T build-up and changes, see <http://precise/directions.co.uk>



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