Investigating the Effects of Parameters in a Steady-State Latitude-Dependent Energy Balance Model

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GenAI Statement: Generative AI tools were not used in this coursework.

Introduction

Energy Balance Models (EBMs) are climate models used to describe the temperature of the Earth. These models balance the heat energy absorbed and emitted from the Earth to calculate temperature over time and space. This study investigates a steady-state latitude-dependent EBM, where Temperature ($T(y,t)$) is constant for all time t at a given latitude y.

The study of energy balance models provides conceptual tools for understanding climate changes [Lohmann, 2020].

Analysing the effects of model parameters allows for a greater understanding of what affects ice line positioning. This, in combination with real-world context, allows for the optimisation of EBMs to fit a system with specific characteristics.

Methodology

The study examines the following EBM:

$$E_{in} - E_{Out} + E_{Transport} = 0$$
$$Qs(y)[1 - a(y)] - (A + BT) + k(\overline{T} - T) = 0$$

Where default values / formulae are the following:

$$Q = 342Wm^{-2}, A = 202Wm^{-2},$$

$$B = 1.9Wm^{-2}\,^{\circ}C^{-1},\ T_c = -10^{\circ}C,\ k = 1.6B$$

$$s(y) = 1 - S_2P_2,\ S_2 = 0.482,\ P_2 = \frac{3y^2 - 1}{2},$$

$$a(y) = \begin{cases} a_i = 0.62 & y > y_s \\ a_w = 0.32 & y < y_s \\ \frac{a_i + a_w}{2} & y = y_s \end{cases}$$

Ice lines are reflected about the equator, meaning the conditions on $a(y)$ are flipped in the Southern hemisphere (negative latitude).

This study evaluates the effects of the default formulae and key variable values on the overall EBM, and hence their effect on ice line position y_s.

The default EBM ice lines are found at ±0.256 and ±0.939, which are hemispherically symmetric.

Analysis

1 The Effect of Parameters on the Ice Age State

A planet is in an 'ice age' when its surface area is completely glaciated, meaning its ice line is at latitude 0.

This section discusses the effect of model constants on the ice age state, and on the climate more generally. Here, climate relates to ice line position (location of ice climates and water climates). Global mean temperature is also a climate-related factor which will be discussed where appropriate.

How the Earth can Enter and Leave Ice-Age State using the Default Parameters

Fundamental to the ice-age state is understanding how the planet enters and leaves it. Specifically, when starting a trajectory at the stable mixed steady state, what decrease in solar input fully glaciates the Earth, and then what increase in solar input will take the Earth out of this ice age state.

This trajectory can be visualised in a $Q - \overline{T}$ bifurcation diagram, Figure 1.1:

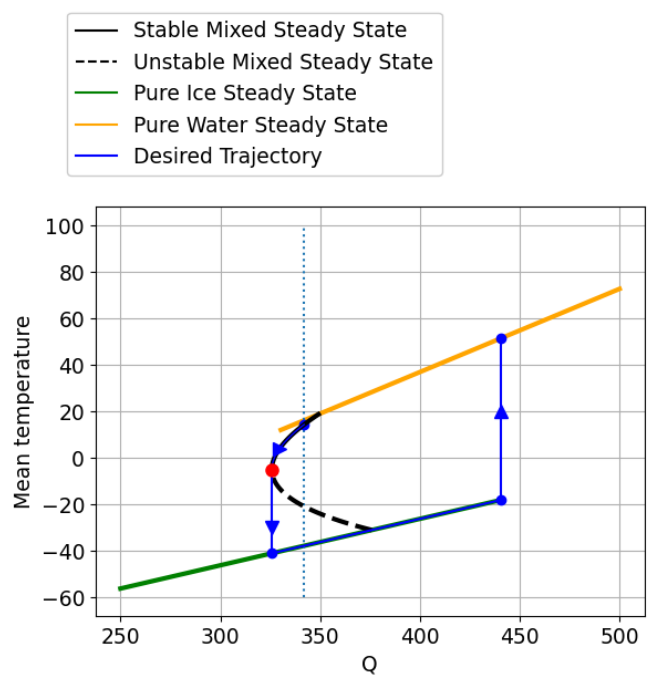

Figure 1.1: $Q - \overline{T}$ bifurcation diagram with trajectory depicting how to enter and leave ice-age state from mixed steady state

The blue trajectory highlighted in Figure 1.1 depicts the scenario described above. It commences at the stable steady state where $Q = Q_0 = 342Wm^{-2}$, the default value. By decreasing Q, the trajectory remains at the stable mixed steady state, until reaching a tipping point (Red), from which the trajectory falls from mixed steady state to ice age. This occurs at $Q \approx 325.8Wm^{-2}$. Hence, Q must be decreased by $\approx 16.2Wm^{-2}$ from default value $Q = 342Wm^{-2}$ to place Earth in a fully glaciated state.

Once in glaciated state, Q is increased until it exceeds the maximum Q of the ice age, from which the state turns to pure water. The maximum ice age solar input is $440.7Wm^{-2}$, hence the solar input is increased by $114.9Wm^{-2}$ for the ice to retreat from the equator.

The Effect of the transport coefficient k

Various authors suggest different values of the transport coefficient k; the knowledge of its effects allows for optimal selection based on the Earth's characteristics.

The effect of k on the ice line is depicted in Figure 1.2

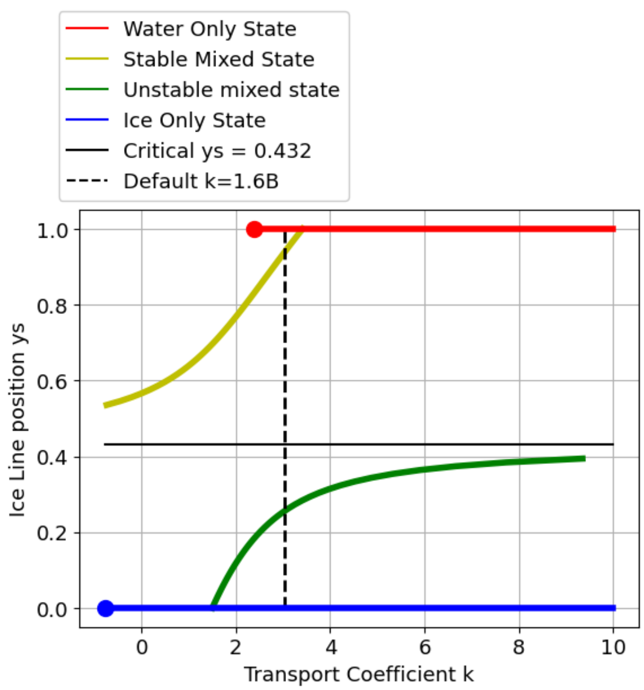

Figure 1.2: Bifurcation diagram of ice line position y_s with varying values of k

Figure 1.2 highlights that there is only a small region of k values for which both stable and unstable mixed state ice lines exist. For large k, only the unstable ice line exists, and for small k, only the stable ice line exists. As $k \to \infty$ or $k \to 0, y_s \to 0.432$, termed the critical y_s. This is approached from above for small k, and from below for large k.

This change in steady state existence can be further demonstrated in $Q - \bar{T}$ bifurcation diagrams:

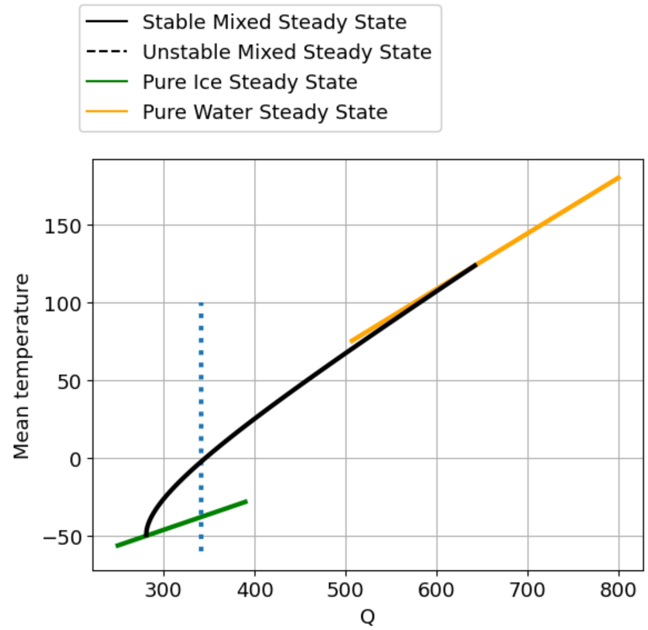

Figure 1.3: Comparing Solar constant Q with mean temperature, for $k = 0.05$

Figure 1.3 shows that using a low value of k means that the system cannot move from pure ice state to pure water state without having a stable mixed state in between.

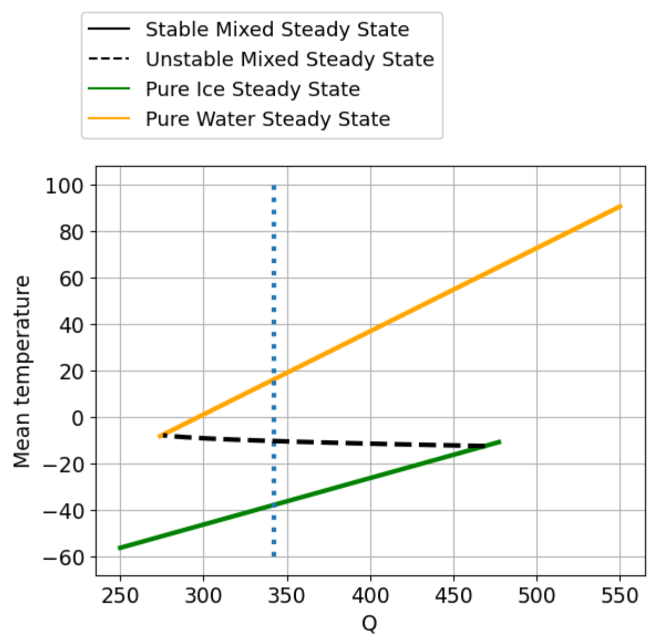

Figure 1.4: Comparing Solar constant Q with mean temperature, for $k = 50$

Figure 1.4 shows that the system cannot exhibit a mixed steady state; it will only ever be in pure ice or pure water states by varying solar input.
The results for the large and small values of k are expected based on physical principles for stability. Note that $T \sim y$ since there are no oscillations, so $\frac{dQ}{dT} \sim \frac{dQ}{dy_s}$ at steady state, which provides stability conditions. As a function of steady state position:

$$Q(y_s) = \frac{(T_c + \frac{A}{B})(B+k)}{s(y_s)(1 - \frac{a_i + a_w}{2}) + k/B(1 - \overline{a}(y_s)}$$

Taking $k << 1$, it can be shown that:

$$\frac{dQ}{dy_s} = \frac{-B(T_c + \frac{A}{B})}{1 - \frac{a_i + a_w}{2}} \frac{1}{s(y_s)^2}(-0.241(6y_s)) > 0 \text{ (Stable)}$$

Taking k large, it can also be shown that:

$$\frac{dQ}{dy_s} = B(a_w - a_i)\frac{s(y_s)}{(1 - \overline{a}(y_s))^2} < 0 \text{ (Unstable)}$$

Note: $Q_{k\to\infty}(y_s) \approx Q_{k\to-\infty}(y_s)$ and $\frac{dQ}{dy_s} \to 0$, which identifies the asymptote in Figure 1.2.

The Effect of Critical Temperature T_c

Ice lines y_s occur at the critical temperature T_c, which implies that changing T_c will change ice line position. This means with prior knowledge of a system's characteristics, T_c can be selected to obey the criteria.

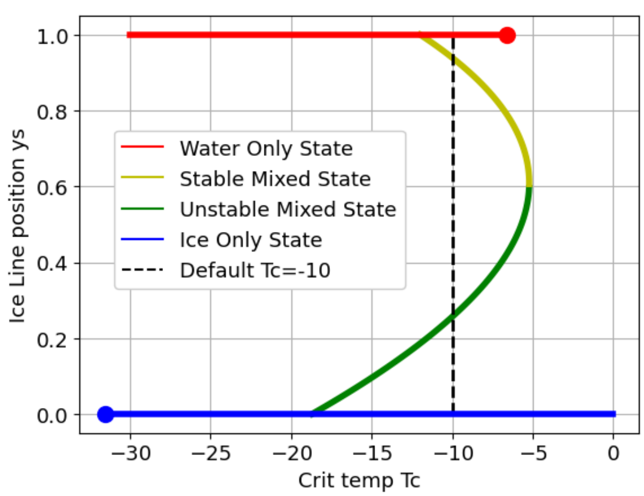

Figure 1.5: Bifurcation diagram of ice line position y_s with varying values of T_c

Figure 1.5 highlights the different climates attainable by varying T_c. For example, $T_c > -5.2°C$ places all latitudes in an ice age, and choosing $-12°C < T_c < -5.2°C$ will return two ice lines, one stable and one unstable, at different latitudes.

The mean temperatures of the extreme outcomes can be demonstrated in terms of varying solar constant Q.

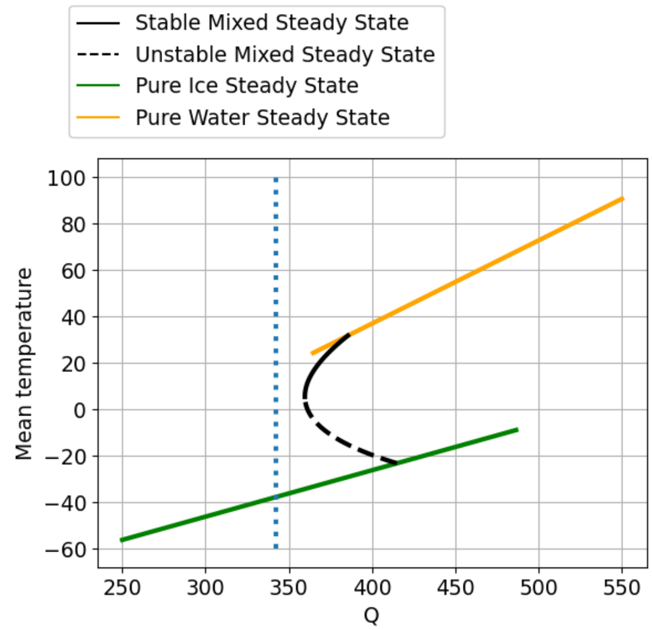

Figure 1.6: Comparing solar constant Q with mean temperature for $T_c = 0°C$

Figure 1.6 demonstrates a system with $T_C > -5.2°C$. It depicts a right transformation of Figure 1.1, meaning increasing T_c increases the necessary Q for the same solutions. With the default $Q = 342Wm^{-2}$, only the pure-ice state can be found for this T_c.

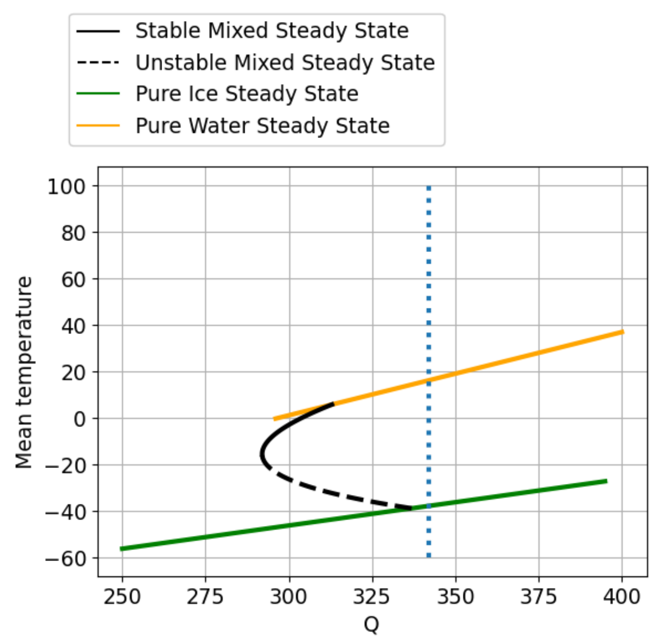

Figure 1.7: Comparing solar constant Q with mean temperature for $T_c = -20°C$

Figure 1.7 demonstrates a system with $T_C < -18.7°C$. It depicts a left transformation of Figure 1.1, meaning decreasing T_c decreases the necessary Q for the same solutions. With the default $Q = 342Wm^{-2}$, only the pure-ice and pure water states can be found for this T_c.

Observations show that oceans freeze all year-round at $-13°C$ ($T = -13°C$ is pure ice state), and that land freezes in winter at $0°C$ ($T = 0°C$ is a stable mixed steady state). These characteristics are shown in the default model (Figure 1.1), and hence no update to the model is required to account for these findings.

The Effects of Constants A and B

$(A+BT)$ is the outgoing longwave radiation emitted from the Earth. Various authors suggest different values of A and B, which typically correspond to varying surface material and levels of cloud cover. Two specific papers use $A = 202Wm^{-2}, B = 1.45Wm^{-2°}C^{-1}$ (Budyko 1969) and $A = 212Wm^{-2}, B = 1.6Wm^{-2°}C^{-1}$ (Cess 1976). Comparing these models to the default yields Figure 1.8:

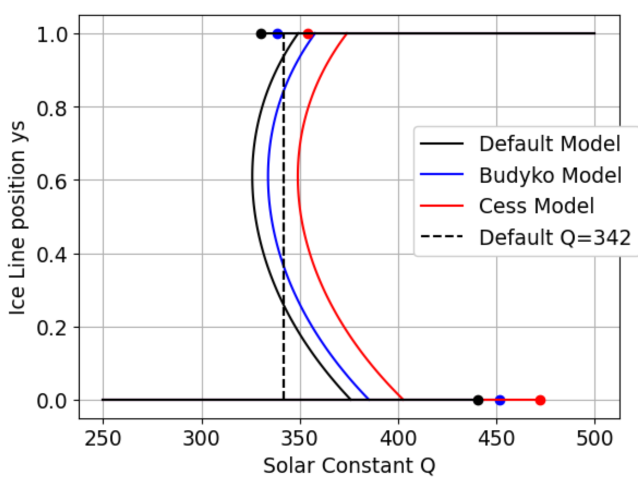

Figure 1.8: Comparing Ice Line solutions for the Cess 1976 model, Budyko 1969 model, and default model.

Figure 1.8 shows that in both the Budyko and Cess model, Solar Constant Q must be increased to achieve ice lines at the same latitudes as default. Using $Q = 342Wm^{-2}$, the Cess model only has the pure-ice steady state solution. For pure water and mixed solutions to be viable, Q must be increased.

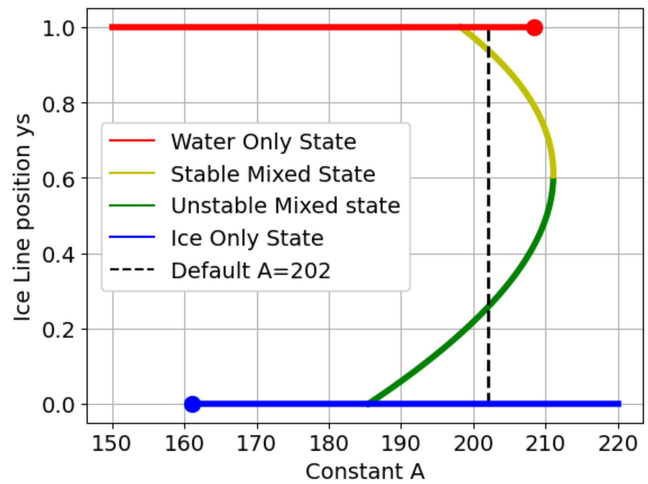

Figure 1.9: Bifurcation diagram to show how A affects ice line position

When B is fixed, Figure 1.9 shows that mixed state solutions exist between $199 < A < 211$. It also shows that increasing A beyond this region yields an ice-only state (as shown for the Cess model in Figure 1.8).

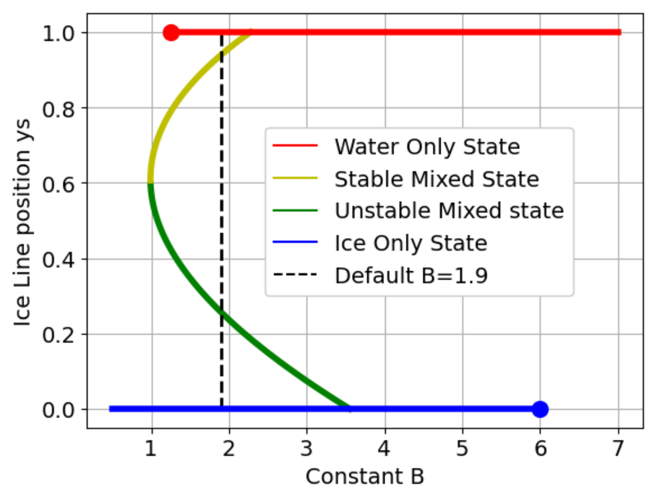

Figure 1.10: How Ice Line position is affected by varying B

Figure 1.10 uses the default A and $k = 1.6B$ to demonstrate the effect of changing B on the default model. It shows that B has two mixed state solutions between $1 < B < 2.2$; decreasing B below this yields only a pure-ice state.

The Sensitivity of Constants on Modelling

By defining sensitivity as the size of the normalised region for which two mixed steady states exist, the model is proportionally most sensitive to changes in A. This is because A is two magnitudes larger than the other parameters, meaning single-digit differences are more granular. These sensitivities are compared in Table 1:

Constant	Sensitivity: $= \frac{\lvert MaxVal - MinVal \rvert}{DefaultVal}$
A	$\frac{211-199}{202} = 0.06$
B	$\frac{2.2-1}{1.9} = 0.63$
T_c	$\frac{-5.2--12}{10} = 0.68$

Table 1: The sensitivity of solutions by varying constants discussed in Section 1

2 The Search for Other Steady State Solutions

2.1 The Possibility of Asymmetric Steady States about the Equator

The Earth exhibits hemispherical differences in multiple ways, ranging from differing ocean proportions between the Northern and Southern hemispheres (61% to 81% respectively [Webb, 2023]) to different average temperatures (Northern hemisphere warmer than Southern [Feulner, 2013]).
Sources suggest these differences are not significant enough to impact albedo symmetry on Earth [Kang,2021]. However, it is relevant to assess the model's capability for modelling asymmetrically, even if not currently relevant for Earth.

Asymmetric Steady States by Changing $s(y)$ and/or $a(y)$
The simplest way of generating asymmetric results is to make $s(y)$ or $a(y)$ asymmetric about $y = 0$. However, to be sensible, functions and results must obey the following conditions (s_{North} denotes the $s(y)$ function in the Northern hemisphere etc.):

1. $s_{North}(0) = s_{South}(0)$
2. $Q_{North}(0) = Q_{South}(0)$
3. $T_{North}(0) = T_{South}(0)$
4. $a_{North}(0) = a_{South}(0)$
5. $a_{North}(1) = a_{South}(-1)$ for ice age consistency

Changing a continuous and piecewise continuous $s(y)$ lead to difficulties meeting conditions 1), 2) and 3) simultaneously, where results seem to tend towards symmetric steady states.
Changing the discrete albedo function leads to difficulties obeying conditions 4) and 5).
This study focuses on generating asymmetric steady states which abide by all conditions listed above. For cases where some conditions are not necessary, asymmetric steady states can be found. For example, if it is not crucial for a pure ice steady state to exist, then $s(y)$ and $a(y)$ can be chosen to abide by the other conditions.

Considering Axial Tilt (obliquity)
The Earth experiences axial tilt (obliquity), where its axis of rotation is not vertical.
During a year, from the point of view of the Sun, the Earth's obliquity ranges from 0° to approximately 23.5° in Summer and Winter [Hays, 2000].

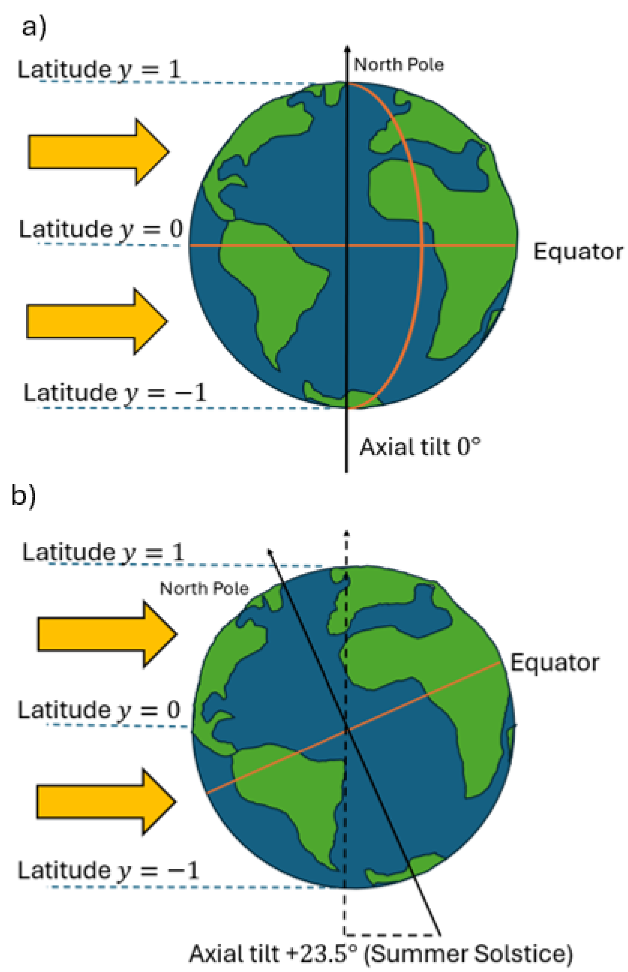

Figure 2.1: a) Example of Earth with zero tilt (equinox). b) Example of Earth in Northern hemisphere Summer.

[Nadeau, 2019] presents a paper investigating the possibility of stable asymmetric ice lines on Pluto by modelling obliquity. The governing equation used is as follows:

$$R\frac{\partial T}{\partial t} = Q(s(y,\beta)(1-a(y))) - (A + BT(y,t))$$
$$+K(\overline{T} - T(y,t))$$

Where T represents temperature, R represents the heat capacity of the planet's surface layer and β represents the angle of obliquity. The chosen $s(y)$ is

$$s(y,\beta) \approx 1 - \frac{5}{8}p_2(cos(\beta))p_2(y) - \frac{9}{64}p_4(cos(\beta))p_4(y)$$
$$-\frac{65}{1024}p_6(cos(\beta))p_6(y)$$

where p_i is a Legendre polynomial of order i, and β represents the obliquity of the planet. The albedo function remains the same.

The basis for finding asymmetric steady states in this paper relies on solving for Temperature in terms of time, which is not applicable to a steady-state EBM. Hence, it remains to determine whether their method yields asymmetric solutions when $\frac{\partial T}{\partial t} = 0$. Using the Nadeau $s(y)$ function in the default model, only symmetric solutions are generated.
Overall, the study finds that on Earth, using the steady-state latitude-dependent EBM can't yield asymmetric steady states when strictly abiding by sensibility conditions. Converting the model to a latitude-dependent EBM with variable time (as suggested by Nadeau) can result in stable asymmetric steady states, given suitable obliquity and rotational speed parameters. However, when using Earth's parameters, the steady states generated by this model are symmetric. Therefore, although Earth does not have asymmetric ice lines, it is possible to modify the model such that asymmetric ice lines can be generated.

2.2 The Effect of a Continuous Albedo Function

The discontinuous default albedo function finds critical y_s values (ice lines), which correspond to steady state latitude y values. These values occur at the point of the discontinuous step, and hence are depicted by straight lines in $Q - y_s$ and $Temperature - latitude$ plots. The physical interpretation of these ice lines is that the Earth is in ice-state on the polar side of the ice line and is in water state on the equatorial side of the ice line.
Employing a continuous albedo function (ramp function) means that critical y_s values which cause a step no longer exist. Instead, the climate of the Earth is modelled by a continuous smoothed curve. This means instead of separating the water and ice states with an ice line, they are separated by a band, for which both water and ice can exist. This is defined as an ice band in this study. To be comparable with the discontinuous default, the albedo function must be chosen to map as closely to the step as possible.
First, note that no oscillations are expected in the discontinuous case, and hence T maps to y in a 1-to-1 relationship. This means that albedo can be considered as a function of T, rather than as a function of y. This means that for the discontinuous case:

$$a(T) = \begin{cases} a_w & T > T_c \\ a_i & T < T_c \\ \frac{a_i + a_w}{2} & T = T_c \end{cases}$$

where $T(y_s) = T_c$.
Then a continuous albedo which could model this is:

$$a(T) = \frac{a_i + a_w}{2} - \frac{a_i - a_w}{2} tanh(X(T - T_c))$$

where X dictates the gradient of the ramp function. A smaller value of X yields a more smoothed function, and hence lessens the gradient of the ramp.

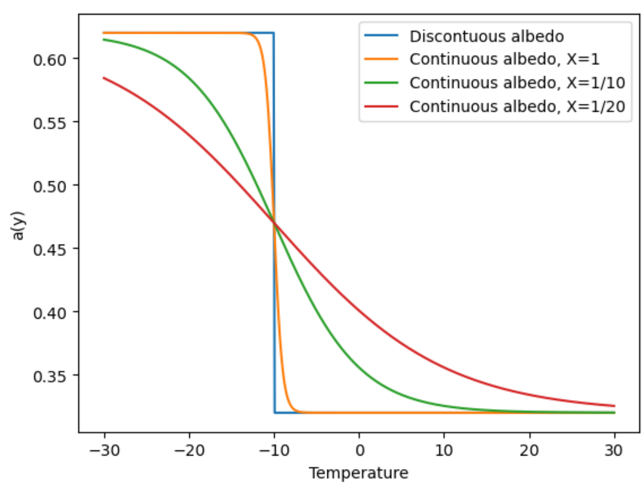

Figure 2.2: Plot demonstrating the fit of a continuous albedo plot with varying X values against the default discontinuous albedo

Figure 2.2 shows that the larger the value of X, the steeper the ramp and the smaller the ice band. As X increases, the ice band tends towards the discontinuous ice line y_s.
It remains to determine the steady state solutions for this continuous $a(T)$, so these can be compared to the default steady states. X is chosen to be $\frac{1}{14}$ because this enabled the best fit for both steady state solutions. The results are shown in Figure 2.3:

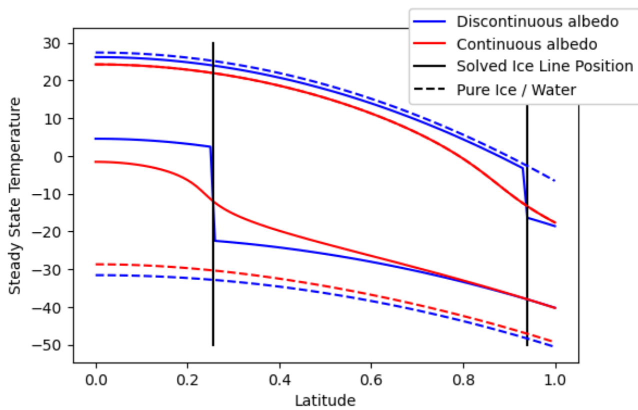

Figure 2.3: Plot comparing steady state temperature solutions over latitude for continuous and discontinuous albedo functions

Figure 2.3 was generated by solving the EBM directly for different temperatures at the equator and poles, returning the different solution paths. These temperatures were chosen such that the continuous solutions follow the default trajectories. It is noted that a pure water steady state cannot be found, meaning there is always an ice band.
Overall, the implementation of a continuous albedo function reduces the extremity of temperature steady state solutions when compared to the discontinuous albedo, especially in medium latitudes. The ramp function smooths the step generated by the discontinuous function. Furthermore, the use of a continuous function means that instead of ice lines, ice bands are found, which seem more realistic.

3 Incorporating Real Diffusion

Including Real Diffusive Heat Transport
Incorporating real diffusive heat transport, $\nabla^2 T$, improves model realism. By modelling in spherical coordinates, ρ and ϕ are scaled out from the spherical coordinate formula, since only latitude varies. This returns:

$$\nabla^2 T = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial T}{\partial\theta})$$
$$0 \le \theta \le \pi$$

A sensible transformation is to set φ as the angle from the equator, separating the Northern and Southern hemispheres with positive and negative φ.

$$\varphi = \theta - \frac{\pi}{2} \iff \theta = \varphi + \frac{\pi}{2}, \varphi \in [-\frac{\pi}{2}, \frac{\pi}{2}]$$

Applying this transformation to the diffusive heat equation yields:

$$\begin{aligned}\nabla^2 T &= \frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}(\cos\varphi\frac{\partial T}{\partial\varphi}) \\ &= \frac{1}{\cos\varphi}(\cos\varphi\frac{\partial^2 T}{\partial\varphi^2} - \sin\varphi\frac{\partial T}{\partial\varphi})\end{aligned} \tag{1}$$

Using that $y = sin(\varphi)$, an expression for $\frac{\partial^2 T}{\partial\varphi^2}$ can be derived:

$$\begin{aligned}\frac{\partial T}{\partial\varphi} &= \frac{\partial T}{\partial y}\frac{\partial y}{\partial\varphi} = \frac{\partial T}{\partial y}\cos\varphi \\ \frac{\partial^2 T}{\partial\varphi^2} &= \frac{\partial}{\partial\varphi}(\frac{\partial T}{\partial y}\cos\varphi) \\ &= \frac{\partial}{\partial\varphi}(\frac{\partial T}{\partial y})\cos\varphi - \frac{\partial T}{\partial y}\sin\varphi \\ &= \frac{\partial^2 T}{\partial y^2}\cos^2\varphi - \frac{\partial T}{\partial y}\sin\varphi\end{aligned}$$

This can substituted into (1) to obtain the following:

$$\begin{aligned}\nabla^2 T &= \frac{1}{\cos\varphi}(\cos\varphi(\frac{\partial^2 T}{\partial y^2}\cos^2\varphi - \frac{\partial T}{\partial y}\sin\varphi) \\ &\quad - \sin\varphi\cos\varphi\frac{\partial T}{\partial y}) \\ &= \frac{\partial^2 T}{\partial y^2}\cos^2\varphi - 2\frac{\partial T}{\partial y}\sin\varphi \\ &= \frac{\partial^2 T}{\partial y^2}(1-\sin^2\varphi) - 2\sin\varphi\frac{\partial T}{\partial y} \\ &= \frac{\partial^2 T}{\partial y^2}(1-y^2) - 2y\frac{\partial T}{\partial y} \\ &= \frac{\partial}{\partial y}(\frac{\partial T}{\partial y}(1-y^2))\end{aligned}$$

Recompiling the EBM with $E_{Transport} = \kappa\frac{d}{dy}[\frac{dT}{dy}(1-y^2)$ yields the following:
C (1): For $y \in [0,1]$

$$Qs(y)[1-a(y)] - [A+BT] + \kappa\frac{d}{dy}[\frac{dT}{dy}(1-y^2)] = 0$$

Where $E_{In} - E_{Out} + E_{Transport} = 0$ is the new EBM.
C (2): For $y = 0$ or $y = 1$

$$[\frac{dT}{dy}(1-y^2)] = 0$$

Intuitively, a latitude-dependent energy transport coefficient implies a latitude dependent $\overline{T}$, which implies that the surface albedo is likely not a disjoint function. Assuming that the desired equatorial albedo is a_w and polar albedo is a_i, an appropriate monotonic continuous equivalent is:

$$a(y) = \frac{a_i + a_w}{2} - \frac{a_i - a_w}{2} tanh(M(y - y_s))$$

for a constant $M > 0$ (M=40 used). Since $a(y \to \infty) = a_i$ and $a(y \to -\infty) = a_w$, M is chosen to appropriately ramp between the two values in the region [0,1].

Results of Including Diffusive Heat Transport

The new EBM is a second order boundary value problem with Neumann boundary conditions. C (2) can be substituted into C (1) to obtain a numerical solution of the BVP. The boundary conditions can be transformed to Dirichlet conditions:

$$T(y = 0 \text{ or } 1) = \frac{Qs(y)(1 - a(y)) - A}{B}, \text{ (constant)}$$

Using these conditions, numerical solution (Figure 3.1) is found:

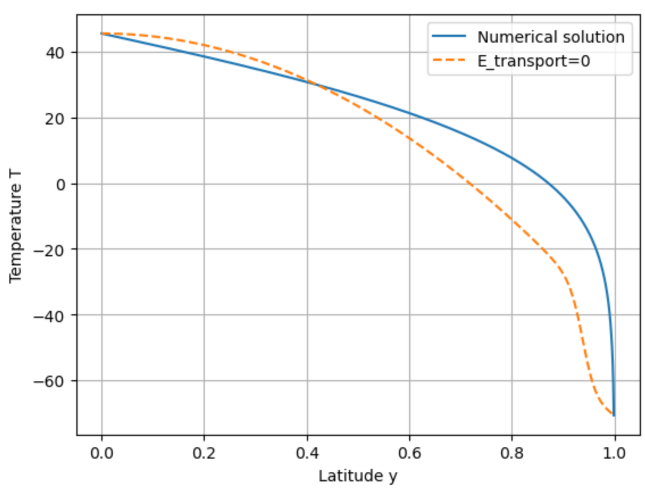

Figure 3.1: Numerical solution for the EBM with real diffusion

Figure 3.1 highlights that the numerical solution of the new EBM increases the temperatures at greater latitudes, with a steep temperature decline until the minimum.

Analytically, a boundary layer exists near to $y = 1$, because as $y \to 1, (1 - y^2) \to 0$. The first interpretation of this is setting $(1-y^2)\frac{d^2T}{dy^2} = 0$, which would give the inner layer. However, $\frac{d^2T}{dy^2}$ could scale inversely with $1 - y^2$, leaving $(1-y^2)\frac{d^2T}{dy^2} = O(1)$. This would give the outer layer, hence returning a boundary layer close to 1. The difficulty implementing this is that y is not always small, so the boundary layer solution is only solvable when $y \approx 1$. This means the EBM cannot be easily rescaled to the form $\epsilon\frac{d^2T}{dy^2} + K_1\frac{dT}{dy} + K_2T + K_3 = 0$, since $\epsilon = 1 - y^2$ is a function of y.

Conclusion

There exist many ways to alter the default EBM in order to specify it for a system with known characteristics.

The effects of parameters were quantified by varying a given parameter and fixing all others to default.

Increasing k past ≈ 3.5 leaves only the unstable mixed state, and decreasing k past ≈ 1.6 leaves only the stable mixed state. This means for large k, no mixed state can exist, and for small k, a mixed state must exist between pure-ice and pure-water states.

Bifurcations of T_c, A and B, with respect to ice line position highlighted finite regions for which stable mixed steady states could be found. The steady state solutions, and hence climate, are most sensitive to a proportional change in A, since its larger base magnitude means changes are proportionally smaller.

Asymmetric steady states were shown to be possible if considering the EBM without being at steady state in time. This can be done by modelling obliquity and solving for $T(t)$ [Nadeau, 2019]. In steady-state time, only symmetric steady states were found when considering all conditions for sensibility.

A continuous albedo function could be considered more realistic; instead of ice lines, it generates ice bands. With sufficient tailoring, the continuous albedo function can output similar solutions to the discontinuous case by 'ramping' the discontinuous step, highlighted in Figure 2.3.

Including diffusive heat transport in the $E_{Transport}$ term further improves the realism of the model, reducing temperature difference in median latitudes.

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