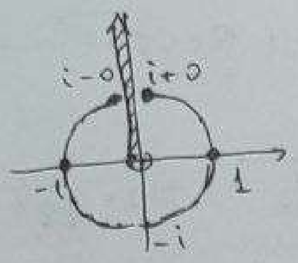
① a) $\varphi(z) = \sqrt[3]{z}, \quad z \subset [0; i\infty], \quad \varphi(-1) = e^{\frac{i\pi}{3}}$



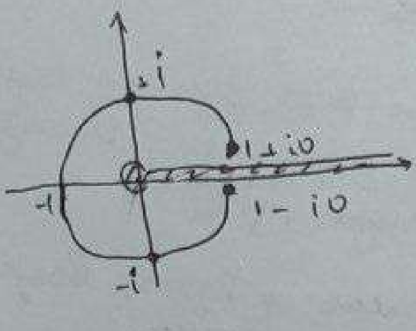
$$\varphi(1) = e^{i\frac{\pi}{3}} \sqrt[3]{\left|\frac{1}{-1}\right|}\, e^{\frac{i\pi}{3}} = e^{i\frac{2\pi}{3}}$$

$$\varphi(i+0) = e^{i\frac{\pi}{3}} \sqrt[3]{\left|\frac{1}{i}\right|}\, e^{i\frac{\pi}{2}} = e^{i\frac{5\pi}{6}}$$

$$\varphi(i-0) = e^{i\frac{\pi}{3}} \sqrt[3]{\left|\frac{1}{i}\right|}\, e^{-i\frac{\pi}{2}\cdot\frac{1}{3}} = e^{i\frac{\pi}{6}}$$

δ) $\varphi(z) = \ln z \quad , z \in [0; +\infty], \quad \varphi(1-i0) = 0$

$$\varphi(z) = \ln\left|\frac{g(z)}{g(z_0)}\right| + i \Delta \underset{z_0 \to z}{\arg} + \varphi(z_0)$$

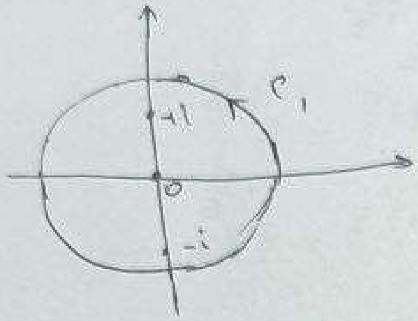


$$\varphi(1+i0) = \ln\frac{1}{1} + i(-2\pi) + 0 = -i2\pi$$

$$\varphi(i) = \ln\frac{1}{1} + i\left(-\frac{3\pi}{2}\right) + 0 = -i\frac{3\pi}{2}$$

$$\varphi(-i) = \ln\frac{1}{1} + i\left(-\frac{\pi}{2}\right) + 0 = -i\frac{\pi}{2}$$

(3) $f(z) = \sqrt{1+z^2} = \sqrt{(1+iz)(1-iz)}$

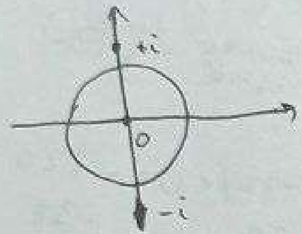


$$I_1 = \oint_{|z|>1} \sqrt{1+z^2}\,dz = -2\pi i \cdot \operatorname{Res}_{z=\infty} \sqrt{1+z^2} = -2\pi i \cdot c_{-1}$$

$$\sqrt{1+z^2} = z\sqrt{1+\frac{1}{z^2}} = z\left(1+\frac{1}{2z^2}+\dots\right) \Rightarrow$$

$$\Rightarrow \operatorname{Res}_{z=\infty} = c_{-1} = \frac{1}{2}$$

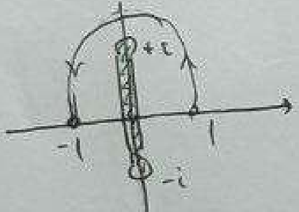
$$\Rightarrow I_1 = 2\pi i \cdot \frac{1}{2} = \pi i$$



$$I_2 = \oint_{|z|<1} \sqrt{1+z^2}\,dz = 0$$

При контуре, вн. к-ого лежит одна точка нужно знать зн. многозн. функций вида $\sqrt{g(z)}$.

Т.к. нам нужен контур, к-ый можно взять вокруг точки i:



$\cancel{f(z)=}\ f(1)=\sqrt{2}$

$\cancel{\Rightarrow f(z)} = \sqrt{2} \cdot 2$

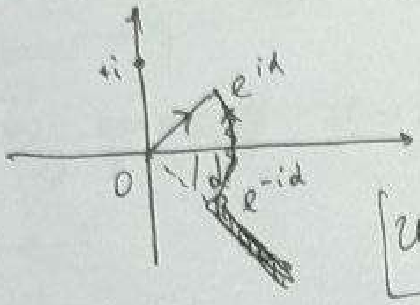
$f(1) = \sqrt{2}$, $f(-1) = \sqrt{2}\sqrt{\frac{2}{2}}\, e^{i\left(\frac{3\pi}{2}+\frac{\pi}{2}\right)} = -\sqrt{2}$

$\Rightarrow$ следовательно, нам нужен ~~контур~~ а) разрез

② $\varphi_1(z) = \sqrt{z - e^{-i\alpha}}$, $\varphi_2(z) = \ln(z - e^{-i\alpha})$, $\alpha \in (0, \frac{\pi}{2})$

$\varphi_1(0) = ie^{-i\frac{\alpha}{2}}$, $\varphi_2(0) = -i\pi - i\alpha$

a)

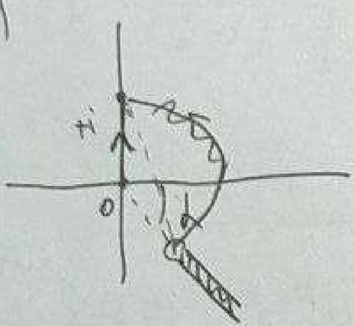


a.1) $\varphi_1(e^{i\alpha}) = ie^{-i\frac{\alpha}{2}} \sqrt{\left|\frac{e^{i\alpha}-e^{-i\alpha}}{1}\right|}\, e^{-i\frac{1}{2}(\frac{\pi}{2}-\alpha)} =$

$\boxed{\text{Из. рис. } \Delta \arg = \frac{\pi}{2} - \alpha} = \sqrt{2\sin\alpha}\cdot ie^{-\frac{i\pi}{4}} =$

$= \sqrt{\sin\alpha} + i\sqrt{\sin\alpha}$

a.2)



Из. рис. $\Delta \arg = \frac{\pi}{4} - \frac{\alpha}{2}$

$\cancel{\varphi_2(e^{i\alpha})}\ \varphi_2(i) = ie^{-i\frac{\alpha}{2}} \sqrt{\left|\frac{i-e^{-i\alpha}}{1}\right|}\, e^{-i\frac{1}{2}(\frac{\pi}{4}-\frac{\alpha}{2})} =$

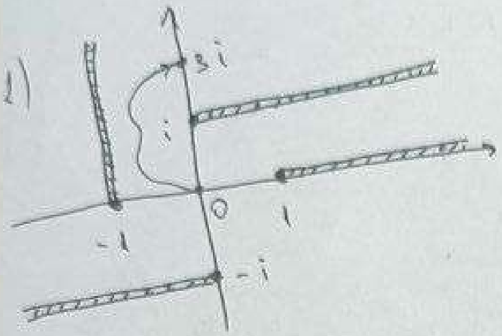
$= e^{i\frac{\pi}{2} - i\frac{\alpha}{2} + \frac{i\alpha}{4} - \frac{i\pi}{8}} \sqrt{2\sin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)} =$

$= \sqrt{2\sin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)}\, e^{i\frac{3\pi}{8} - i\frac{\alpha}{4}}$

④ $\varphi(z) = \sqrt[3]{1+z^2}$, $\varphi(0) = 1$

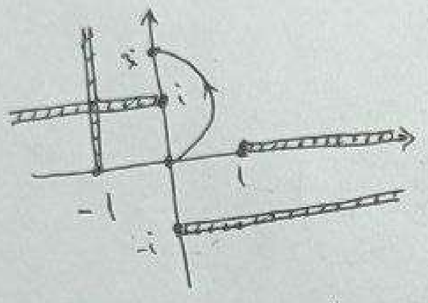
$\varphi(3i) = 1 \cdot \sqrt[3]{\left|\frac{1-8}{1}\right|}\, e^{\frac{i}{3}(-\pi+0)} = 2e^{-i\frac{\pi}{3}}$

1)

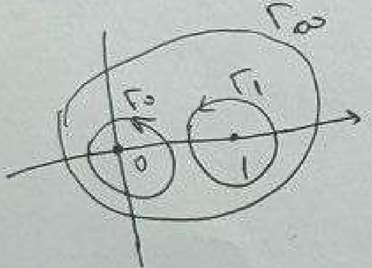


2)

$\varphi(3i) = 1 \cdot \sqrt[3]{\left|\frac{-8}{1}\right|}\, e^{-\frac{i}{3}(\pi+0)} = 2e^{i\frac{\pi}{3}}$



⑥ $\varphi(z) = z^a (z-1)^b$



Для $N(1,1)$:

$\Gamma_0: \Delta\arg z + \Delta\arg(z-1) = 2\pi + 0 = 2\pi$

$\Gamma_1: \Delta\arg z + \Delta\arg(z-1) = 0 + 2\pi = 2\pi$

$\Gamma_\infty: \Delta\arg z + \Delta\arg(z-1) = 4\pi$

$\Rightarrow N(1,1) = 0$

Для $N(1, \frac{1}{2})$:

$\Gamma_0: \Delta\arg z + \frac{1}{2}\Delta\arg(z-1) = 2\pi$

$\Gamma_1: \Delta\arg z + \frac{1}{2}\arg(z-1) = \pi$

$\Gamma_\infty: \Delta\arg z + \frac{1}{2}\arg(z-1) = \frac{3\pi}{2}$

$\Rightarrow N(1, \frac{1}{2}) = 2$

Для $N(\frac{1}{2}, \frac{1}{3})$:

$\Gamma_0: \frac{1}{2}\Delta\arg z + \frac{1}{3}\Delta\arg(z-1) = \pi$

$\Gamma_1: \frac{1}{2}\Delta\arg z + \frac{1}{3}\Delta\arg(z-1) = \frac{2\pi}{3}$

$\Gamma_\infty: \frac{1}{2}\Delta\arg z + \frac{1}{3}\Delta\arg(z-1) = \frac{5\pi}{3}$

$\Rightarrow N(\frac{1}{2}, \frac{1}{3}) = 3$

Для $N(\frac{2}{3}, \frac{1}{3})$:

$\Gamma_0: \frac{2}{3}\Delta\arg z + \frac{1}{2}\Delta\arg(z-1) = \frac{4\pi}{3}$

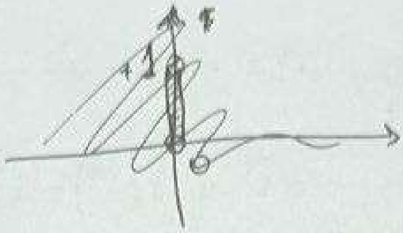
$\Gamma_1: \frac{2}{3}\Delta\arg z + \frac{1}{2}\Delta\arg(z-1) = \frac{2\pi}{3}$

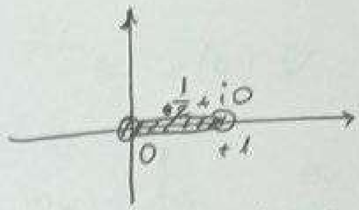
$\Gamma_\infty: \frac{2}{3}\Delta\arg z + \frac{1}{2}\Delta\arg(z+1) = 2\pi$

$\Rightarrow N(\frac{2}{3}, \frac{1}{3}) = 2$

Problem 4.4.

$\varphi(z) = z^{\mu}(1-z)^{1-\mu}$



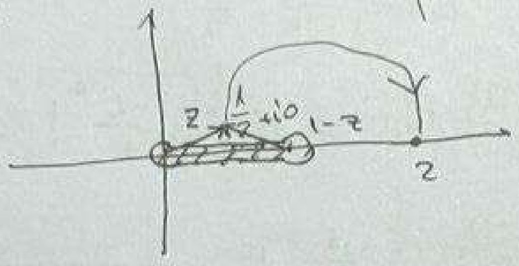


$$\varphi(z) = \frac{\varphi(z)}{\varphi(z_0)}\varphi(z_0) = \varphi_0 \left|\frac{z}{z_0}\right|^{\mu} \left|\frac{1-z}{1-z_0}\right|^{1-\mu} e^{i\mu \Delta \arg z_{z_0 \to z}} e^{i(1-\mu)\Delta \arg(1-z)_{z_0\to z}}$$

$\varphi(\frac{1}{2}+i0) = \frac{1}{2}$

1) $\varphi(2) - ?$

$$\varphi(2) = \frac{1}{2}\left|\frac{2}{1/2}\right|^{\mu}\left|\frac{1-2}{1-\frac{1}{2}}\right|^{1-\mu} e^{i\mu \cdot 0} e^{-i(1-\mu)\pi}$$



$$= \frac{1}{2} 4^{\mu} \cdot 2^{1-\mu} e^{-i\pi + i\mu\pi} =$$

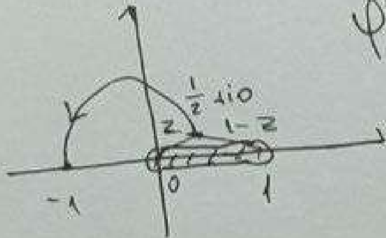
$$= 4^{\mu} 2^{-\mu} (-1) e^{i\mu\pi} =$$

$$= -2^{2\mu} \cdot 2^{-\mu} e^{i\mu\pi} =$$

$$= -2^{\mu} e^{i\mu\pi}$$

2) $\varphi(-1)$

$$\varphi(-1) = \frac{1}{2}\left|\frac{-1}{1/2}\right|^{\mu}\left|\frac{1+1}{1-\frac{1}{2}}\right|^{1-\mu} e^{i\mu\pi} e^{i(1-\mu)\cdot 0} =$$

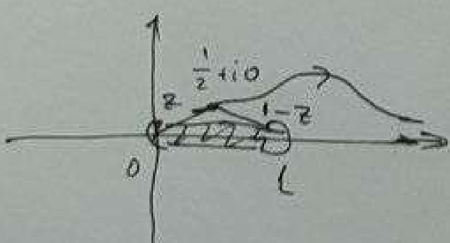


$$= \frac{1}{2} 2^{\mu} \cdot 4^{1-\mu} e^{i\mu\pi} =$$

$$= 2^{\mu-1} 2^{2-2\mu} e^{i\mu\pi} = 2^{1-\mu} e^{i\mu\pi}$$

3) $\lim\limits_{z\to\infty} \frac{\varphi(z)}{z}$; $\varphi(z) = \frac{1}{2} 2^{\mu} x^{\mu} \cdot 2^{1-\mu}(1-x)^{1-\mu} e^{i\mu 0} e^{-i(1-\mu)\pi}$

$$= x^{\mu} x^{1-\mu}\left(\frac{1}{x}-1\right)^{1-\mu} e^{i(\mu-1)\pi}$$



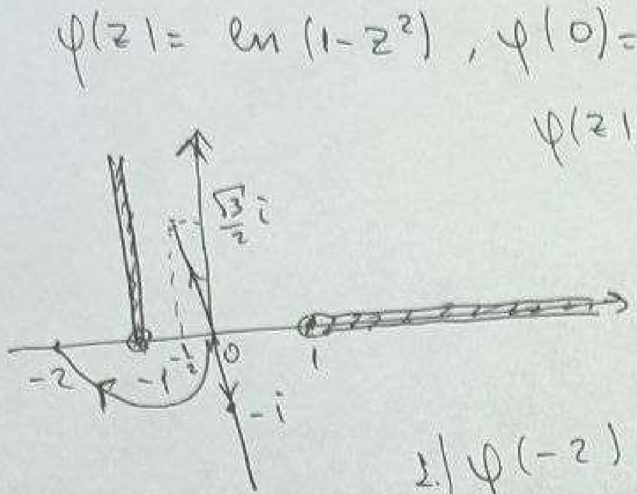
$$\lim_{z\to\infty}\frac{\varphi(z)}{z} = (-1)^{1-\mu} e^{i(\mu-1)\pi} =$$

$$= e^{i\pi(1-\mu)} e^{i\pi(\mu-1)} = 1$$

Problem 4.5.

$$\varphi(z) = \ln(1-z^2), \quad \varphi(0) = -2\pi i$$

$$\varphi(z) = \ln[(1-z)(1+z)]$$



$$1)\ \varphi(-2) = -2\pi i + \ln\frac{3}{1} + i(0-\pi) =$$
$$= \ln 3 - 3\pi i$$

$$2)\ \varphi(-i) = -2\pi i + \ln\frac{2}{1} + i\left(-\frac{\pi}{4} + \frac{\pi}{4}\right) = \ln 2 - 2\pi i$$

$$3)\ \varphi\left(\frac{-1+\sqrt{3}i}{2}\right) = -2\pi i + \ln\sqrt{3} + i\left(\frac{\pi}{3} + \frac{\pi}{6}\right) =$$
$$= \frac{1}{2}\ln 3 - \frac{11\pi}{6}i$$