Задача 2.

$$\vec{H} = \nabla\times\vec{A} = \frac{e}{c}\,\nabla\times\frac{\vec{v}}{\left(R-\frac{\bar{v}\bar{R}}{c}\right)} =$$

$$= \frac{e\gamma^2}{cR^2}\left[\frac{R}{\gamma}\,\nabla\times\vec{v} - \nabla\left(R-\frac{\bar{v}\bar{R}}{c}\right)\times v\right] = \quad \text{из лекции}$$

$$= \frac{e\gamma^2}{cR^2}\left[\frac{R}{\gamma}\,\nabla t'\times\dot{\vec{v}} - \left[\bar{n}\gamma\left(1-\frac{v^2}{c^2}\right)-\frac{\bar{v}}{c}+\gamma\,\frac{\bar{n}\,(\bar{R}\cdot\bar{v})}{c^2}\right]\times\vec{v}\right] =$$

$$= \cancel{n\times\frac{e\gamma^3}{cR}\left[\frac{\vec{R}}{c}\times\left[\left(n-\frac{\bar{v}}{c}\right)\cdot\dot{\vec{v}}\right]\right]+\frac{1}{R}}$$

$$= \vec{n}\times\frac{e\gamma^3}{c^2R}\left\{\left[\vec{n}\times\left[\left(\bar{n}-\frac{\bar{v}}{c}\right)\times\dot{\vec{v}}\right]\right]\right\} + n\times\frac{e\gamma^3}{R^2}\left(\vec{n}-\frac{\bar{v}}{c}\right)\left(1-\frac{v^2}{c^2}\right) =$$

$$= \vec{n}\times\left(\frac{e\gamma^3}{R}\left[\frac{1}{c^2}\vec{n}\times\left[\left(\bar{n}-\frac{\bar{v}}{c}\right)\times\dot{\vec{v}}\right]+\frac{1}{R}\left(\vec{n}-\frac{\bar{v}}{c}\right)\left(1-\frac{\bar{v}^2}{c^2}\right)\right]\right) =$$

$$\vec{n}\times\vec{E}$$

Задача 3

$$\vec{S} = \frac{c}{4\pi}\,\vec{E}\times\vec{H} = \frac{c}{4\pi}\,\vec{E}\times[\vec{n}\times\vec{E}] =$$

$$= \frac{c}{4\pi}\left[\vec{n}\,|\vec{E}|^2 - \underbrace{(\vec{n}\vec{E})}_{0}\vec{E}\right] = \frac{c}{4\pi}\,|\vec{E}|^2\,\vec{n}$$

$$\begin{cases} R\to\infty \\ v\ll c\end{cases} \Rightarrow \bar{E} = \frac{e}{c^2R}\,n\times[n\times\dot{\vec{v}}]$$

$$|S| = \frac{c}{4\pi}\cdot\frac{e^2}{c^4\cdot R^2}\,\left|n\times[n\times\dot{\vec{v}}]\right|^2 =$$

$$= \frac{e^2}{4\pi c^3R^2}\,\left|n\times[n\times\dot{\vec{v}}]\right|^2$$

Задача 4

$$v(t) = \begin{cases} -v_0 + \frac{4v_0}{T}t, & t\in[0;\frac{T}{2}] \\ v_0 - \frac{4v_0}{T}(t-\frac{T}{2}), & t\in[\frac{T}{2};T] \end{cases}$$

При $v \ll c$ и $R\to\infty$

$$S = \frac{c}{4\pi}\frac{e^2}{c^3R^2}\sin^2\theta|\dot{\vec{v}}|^2$$

найдем $\dot{\vec{v}} = \frac{4v_0}{T}$

$$\langle P\rangle_T = \frac{e^2}{c^3 4\pi}\int_0^{2\pi}\int_0^{\pi}\sin^3\theta\, d\theta\, d\varphi\cdot\left(\frac{4v_0}{T}\right)^2 =$$

$$= \frac{2}{3}\frac{e^2}{c^3}\left(\frac{4v_0}{T}\right)^2$$ - (совпадает с формулой Лармора)

Задача 5.

т.к. $\vec{v} = \text{const}$: $\overline{R}_{t'} - \frac{\vec{v}}{c}R_{t'} = \overline{R}_{t'} - \vec{v}(t-t')$

$$\vec{s} = \vec{R} - \vec{R}_0(t) = \vec{R} - \vec{R}_0(t') - \vec{v}(t-t') =$$

$$= R(\vec{n} - \frac{\vec{v}}{c})$$

$$\vec{E} = \frac{e\gamma^3}{R^2}\left(1-\frac{v^2}{c^2}\right)\left(\vec{n}-\frac{\vec{v}}{c}\right)$$

$$= \frac{1}{\vec{n}\vec{s}} =$$

$$R_{t'} - \frac{1}{c}R_{t'}\vec{v} = R_{t'}\sqrt{1-\frac{v^2}{c^2}\sin^2\theta}$$

$$\Rightarrow E = \frac{e\vec{s}}{s^3}\frac{1-v^2/c^2}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}$$

Задача 6.

$$\vec{A} = \frac{\varphi}{c}\vec{v}$$

$$\cancel{\nabla\times\vec{A} = \nabla\times\left(\frac{\varphi}{c}\vec{v}\right) = \frac{1}{c}(\nabla\varphi)\times\vec{v} + \frac{\varphi}{c}(\nabla\times\vec{v})}$$

$$\cancel{\nabla\times\vec{v} = -[\dot{\vec{v}}\,\mathrm{grad}\,t'] = -\frac{\gamma}{c}[\vec{n}\,\dot{\vec{v}}]}$$

$$\nabla\vec{A} = \frac{1}{c}\vec{v}(\nabla\varphi) + \frac{\varphi}{c}(\nabla\cdot\vec{v})$$

$$\nabla\varphi = \frac{e\gamma^2}{R^2}\left[-\vec{n}\gamma\left(1-\frac{v^2}{c^2}\right) + \frac{\vec{v}}{c} - \gamma\frac{\vec{n}(\vec{R}\cdot\dot{\vec{v}})}{c^2}\right]$$

1. $$\nabla\varphi\frac{\vec{v}}{c} = -\frac{e\gamma^3}{cR^2}(\vec{n}\vec{v}) + \frac{e\gamma^3 v^2}{c^3R^2}(\vec{n}\vec{v}) + \frac{e\gamma^2}{c^2R^2}\vec{v}^2 - \frac{e\gamma^3}{c^3R^2}(\vec{n}\vec{v})(\vec{R}\dot{\vec{v}})$$

2. $$\frac{1}{c}\frac{\partial\varphi}{\partial t} = \frac{e}{c}\frac{\partial}{\partial t}\frac{1}{R-\frac{\vec{R}\vec{v}}{c}} = -\frac{e}{c}\frac{1}{\left(R-\frac{\vec{v}\vec{R}}{c}\right)^2}\frac{\partial}{\partial t}\left(R-\frac{\vec{R}\vec{v}}{c}\right) =$$

$$= \frac{e\gamma^3}{cR^2}\left(\vec{n}\vec{v} - \frac{e\gamma^3 v^2}{c^2R^2} + \frac{e\gamma^3}{c^2R^2}(\vec{R}\dot{\vec{v}})\right)$$

$$\nabla\cdot\vec{v} = \dot{\vec{v}}\,\nabla t' = -\frac{\vec{n}\dot{\vec{v}}}{c}\gamma$$

3. $$\frac{\varphi}{c}(\nabla\vec{v}) = -\frac{\vec{n}\dot{\vec{v}}}{c^2R}\gamma^2$$

Сложив 1, 2 и 3, получаем:

$$\nabla\vec{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t} = \nabla\varphi\frac{\vec{v}}{c} + \frac{\varphi}{c}(\nabla\vec{v}) + \frac{1}{c}\frac{\partial\varphi}{\partial t} =$$

$= \ldots = 0$, что и требовалось показать

Гуаев. М.А. БФЗ-22-КТ-1 . Задача 1.

$dx = \cos\varphi\, d\rho - \sin\varphi\, \rho\, d\varphi$

$dy = \sin\varphi\, d\rho + \cos\varphi\, \rho\, d\varphi$

$dz = dz$

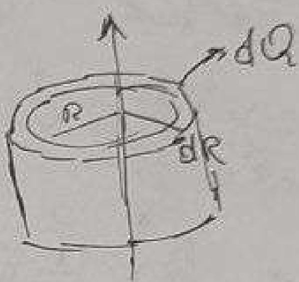
$\Rightarrow ds^2 = \cos^2\varphi\, d\rho^2 + \sin^2\varphi\, d\varphi^2 - 2\sin\cos\varphi\, d\rho d\varphi +$

$+ \sin^2\varphi\, d\rho^2 + \cos^2\varphi\, d\varphi^2 + 2\sin\varphi\cos\varphi\, d\rho d\varphi +$

$+ dz^2 \Rightarrow ds^2 = d\rho^2 + \rho^2 d\varphi^2 + dz^2$

$H_\rho = 1,\ H_\varphi = \rho,\ H_z = 1$

Т. Гаусса:



$E_r 2\pi r \cdot h = 4\pi Q$

$E_{r+dr} \cdot 2\pi (r+dr) \cdot h = 4\pi Q + 4\pi dQ$

$\Rightarrow d(E_r r) \cdot 2\pi h = 4\pi dQ$

$dQ = 2\pi r dr \cdot h \cdot \rho$

$\Rightarrow d(E_r r) 2\pi \cancel{h} = 4\pi\ 2\pi r\, dr \cdot \cancel{h}\rho$

$\Rightarrow \frac{1}{r}\frac{d(Er)}{dr} = 4\pi\rho = \nabla\vec{E}$; $\vec{E} = -\operatorname{grad}\varphi = -\frac{d\varphi}{dr}$

$\Rightarrow \operatorname{div}\operatorname{grad}\varphi = \frac{1}{r}\frac{d}{dr}\left(r\frac{d\varphi}{dr}\right) \Rightarrow$

$\Rightarrow \Delta\varphi = \frac{1}{r}\frac{d}{dr}\left(r\frac{d\varphi}{dr}\right)$