Assignment 2

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Download all python codes from

```
https://github.com/cmapsi/AI1103−Probability−and
    −random−variables/tree/main/Assignment−2/
    codes
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and latex-tikz codes from

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https://github.com/cmapsi/AI1103−Probability−and
    −random−variables/blob/main/Assignment−2/
    main.tex
```

1 Problem

Two dice, one blue and one grey, are thrown at the same time. Write down all the possible outcomes.What is the probability that the sum of the two numbers appearing on the top of the dice is
(i) 8?
(ii) 13?
(iii) less than or equal to 12?

2 solution

Let $X_i \in \{1, 2, 3, 4, 5, 6\}$, $i = 1, 2$ be the random variables representing the outcomes of each die. The probability mass function is given below.

$$p_{X_i}(n) = \Pr(X_i = n) = \begin{cases} \frac{1}{6} & 1 \le n \le 6 \\ 0 & otherwise \end{cases} \tag{2.0.1}$$

Desired outcomes

$$X = X_1 + X_2 = n \tag{2.0.2}$$

$$p_X(n) = \Pr(X_1 + X_2 = n) = \Pr(X_1 = n - X_2) \tag{2.0.3}$$

$$= \sum_k \Pr(X_1 = n - k|X_2 = k) p_{X_2}(k) \tag{2.0.4}$$

Since X_1 and X_2 are independents events, we can deduce

$$\Pr(X_1 = n - k|X_2 = k) = \Pr(X_1 = n - k) = p_{X_1}(n - k) \tag{2.0.5}$$

From (2.0.4) and (2.0.5)

$$p_X(n) = \sum_k p_{X_1}(n - k) p_{X_2} = p_{X_1}(n) * p_{X_2}(n) \tag{2.0.6}$$

From (2.0.1) and (2.0.6)

$$p_X(n) = \frac{1}{6} \sum_{k=1}^{6} p_{X_1}(n - k) \tag{2.0.7}$$

$$p_{X_1}(k) = 0, \text{ if } k \notin \{1, 2, 3, 4, 5, 6\} \tag{2.0.8}$$

$$p_X(n) = \begin{cases} 0 & n < 1 \\ \frac{1}{6} \sum_{k=1}^{n-1} p_{X_1}(k) & 1 \le n - 1 \le 6 \\ \frac{1}{6} \sum_{k=n-6}^{6} p_{X_1}(k) & 1 < n - 6 \le 6 \\ 0 & n > 12 \end{cases} \tag{2.0.9}$$

Since,

$$p_{X_1}(k) = 0, \text{ if } k \notin \{1, 2, 3, 4, 5, 6\} \tag{2.0.10}$$

We can reduce it to

$$p_X(n) = \begin{cases} 0 & n < 1 \\ \frac{n-1}{36} & 2 \le n \le 7 \\ \frac{13-n}{36} & 7 < n \le 12 \\ 0 & n > 12 \end{cases} \tag{2.0.11}$$

Using (2.0.11) we get the following answers (i) $\frac{5}{36} = 0.138889$
(ii) 0
(iii) Since the probability of getting sum of greater than 12 is 0, the probability of getting a sum to be less than or equal to 12 should be 1.

The graph in given below