

# Challenging Problem-1

Chirag Mehta - AI20BTECH11006

Download all latex-tikz codes from

[https://github.com/cmapsi/AI1103-Probability-and-random-variables/blob/main/Challenging\\_1/main.tex](https://github.com/cmapsi/AI1103-Probability-and-random-variables/blob/main/Challenging_1/main.tex)

## 1 QUESTION: CHALLENGING PROBLEM 1

Let  $X$  be a random variable such that  $E(X) = E(X^2) = 1$ . Then  $E(X^{100}) = ?$

- (A) 0
- (B) 1
- (C)  $2^{100}$
- (D)  $2^{100} + 1$

## 2 SOLUTION

$$\sigma_X^2 = E[X^2] - (E[X])^2 = 0 \quad (2.0.1)$$

$X$  is almost surely constant random variable, it follows degenerate distribution, in mathematical terms

$$\Pr(X \neq E[X]) < \delta, \forall \delta > 0 \quad (2.0.2)$$

$$E[X^{100}] = \sum_x x^{100} p_X(x) \quad (2.0.3)$$

$$1 \leq E[X^{100}] \leq 1 + \sum_x x^{100} \delta \quad (2.0.4)$$

$$\forall \epsilon > 0 \exists \delta > 0 \ni |E[X^{100}] - 1| < \epsilon \quad (2.0.5)$$

$$\text{Choose } \delta < \frac{\epsilon}{\sum_x x^{100}}$$

$$\therefore E[X^{100}] = 1 \quad (2.0.6)$$