Challenging Problem-1

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Download all latex-tikz codes from

https://github.com/cmapsi/AI1103−Probability−and−random−variables/blob/main/Challenging_1/main.tex

1 Question: Challenging Problem 1

Let X be a random variable such that $E(X) = E(X^2) = 1$. Then $E(X^{100}) =?$

(A) 0
(B) 1
(C) 2^{100}
(D) $2^{100} + 1$

2 Solution

$$\sigma_X^2 = E\left[X^2\right] - (E\left[X\right])^2 = 0 \tag{2.0.1}$$

X is almost surely constant random variable, it follows degenrate distribution, in mathematical terms

$$\Pr\left(X \neq E\left[X\right]\right) < \delta, \forall \delta > 0 \tag{2.0.2}$$

$$\begin{aligned} E\left[X^{100}\right] &= \sum_x x^{100} p_X(x) \\ &= 1^{100} \Pr\left(X = 1\right) + \sum_{x \neq 1} x^{100} \Pr\left(X = x\right) \end{aligned} \tag{2.0.3}$$

We know that $0 \leq \Pr\left(X \neq 1\right) < \delta$, $\Pr\left(X = 1\right) = 1 - \delta$

$$0 \leq \left|E\left[X^{100}\right] - 1\right| \leq \sum_x x^{100} \delta \tag{2.0.4}$$

$$\forall \epsilon > 0 \exists \delta > 0 \ni \left|E\left[X^{100}\right] - 1\right| < \epsilon \tag{2.0.5}$$

$$\text{Choose } \delta < \frac{\epsilon}{\sum_x x^{100}}$$

$$\therefore E\left[X^{100}\right] = 1 \tag{2.0.6}$$