Assignment 8

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Download all python codes from

https://github.com/cmapsi/AI1103−Probability−and−random−variables/tree/main/Assignment−8/codes

and latex-tikz codes from

https://github.com/cmapsi/AI1103−Probability−and−random−variables/blob/main/Assignment−8/main.tex

1 Problem

(CSIR UGC NET EXAM (Dec 2015), Q.106) Let $X_1, X_2, ..., X_n$ be independent and identically distributed, each having a uniform distribution on $(0, 1)$. Let $S_n = \sum_{i=1}^{n} X_i$ for $n \geq 1$. Then, which of the following statements are true?

$(A)\frac{S_n}{nlog(n)} \to 0$ as $n \to \infty$ with probability 1.

$(B)\Pr\left(\left(S_n > \frac{2n}{3}\right) \text{occurs for infinitely many n}\right) = 1$

$(C)\frac{S_n}{log(n)} \to 0$ as $n \to \infty$ with probability 1.

$(D)\Pr\left(\left(S_n > \frac{n}{3}\right) \text{occurs for infinitely many n}\right) = 1$

2 Solution

Given

$$S_n = \sum_{i=1}^{n} X_i, n \geq 1 \tag{2.0.1}$$

Dividing by n on both sides, defining μ_n as follows

$$\frac{S_n}{n} = \frac{1}{n}\sum_{i=1}^{n} X_i = \mu_n \tag{2.0.2}$$

Let X be an independent continuous variable identical to $X_1, X_2, ..., X_n$
It can be said that $X_1, X_2, ..., X_n$ are the trials of X, and μ_n represents arthimetic mean of outcomes.
By definition

$$E[X] = \lim_{n\to\infty} \frac{\sum_{i=1}^{n} X_i}{n} = \lim_{n\to\infty} \frac{S_n}{n} \tag{2.0.3}$$

$$\lim_{n\to\infty} \frac{S_n}{n} = \frac{1}{2} \tag{2.0.4}$$

$$\therefore \lim_{n\to\infty} \frac{S_n}{nlog(n)} = 0 \tag{2.0.5}$$

It is easy to observe from (2.0.4) that option C is false.
Using weak law

$$\lim_{n\to\infty} \Pr(|\mu_n - E[x]| > \epsilon) = 0, \forall \epsilon > 0 \tag{2.0.6}$$

$$\therefore \Pr\left(\left(S_n > \frac{n}{3}\right) \text{occurs for infinitely many n}\right) = 1 \tag{2.0.7}$$

It can be easily implied from (2.0.6) that option B is false.

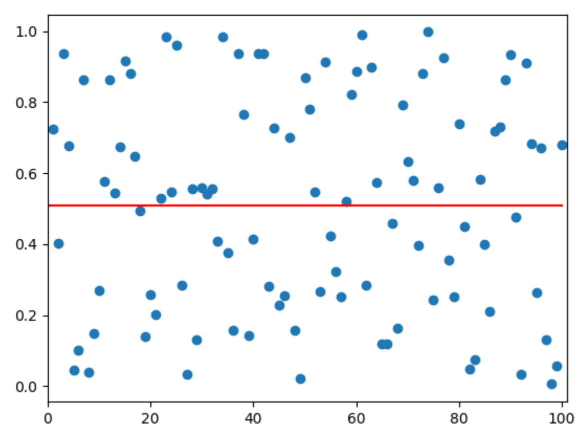

Fig. 0: Mean simulation with n=100