Assignment 5

Chirag Mehta - AI20BTECH11006

Download all python codes from

https://github.com/cmapsi/AI1103−Probability−and−random−variables/tree/main/Assignment−5/codes

and latex-tikz codes from

https://github.com/cmapsi/AI1103−Probability−and−random−variables/blob/main/Assignment−5/main.tex

1 Problem

(GATE-XE-A 2017 Q.170) Two dice are thrown simultaneously. The probability that the product of the numbers appearing on the top faces of the dice is a perfect square is
(A) $\frac{1}{9}$ (B)$\frac{2}{9}$ (C)$\frac{1}{9}$ (D)$\frac{4}{9}$

2 solution

Let $X_i \in \{1,2,3,4,5,6\},\ i = 1,2$ be the random variables representing the outcomes of each die. The probability mass function is given below.

$$p_{X_i}(n) = \Pr(X_i = n) = \begin{cases} \frac{1}{6} & 1 \le n \le 6 \\ 0 & otherwise \end{cases} \tag{2.0.1}$$

$$X = X_1 \times X_2 = n \tag{2.0.2}$$

Defining set $S = \{1,4,9,16,25,36\}$
Let there be k ordered pair of factors for each n. $a_i \times b_i = n,\ a_i, b_i \in S,\ \forall i \in \{1,2,..,k\}$ We have the following expression for probability

$$p_X(n) = \sum_k p_{X_1}(a_k) \times p_{X_2}(b_k) = \frac{k}{36} \tag{2.0.3}$$

Let $g(n)$ be defined as follows

$$g(n) = \sum_{j=1}^{6} \left\lfloor \left\lfloor \frac{n}{j} \right\rfloor - \frac{n}{j} \right\rfloor + 1 \tag{2.0.4}$$

$$\tag{2.0.5}$$

$g(n)$ returns the number of factors of n, all being in range 1 to 6

$$\therefore p_X(n) = \frac{g(n)}{36} \tag{2.0.6}$$

$$\Pr(X \in S) = \sum_{k \in S} p_X(k) \tag{2.0.7}$$

Using (2.0.3) and (2.0.7)

$$\Pr(X \in S) = \frac{2}{9} \tag{2.0.8}$$

The graph for theoretical result vs simulation is given below