

Assignment 5

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Download all python codes from

<https://github.com/cmapsi/AI1103-Probability-and-random-variables/tree/main/Assignment-5/codes>

and latex-tikz codes from

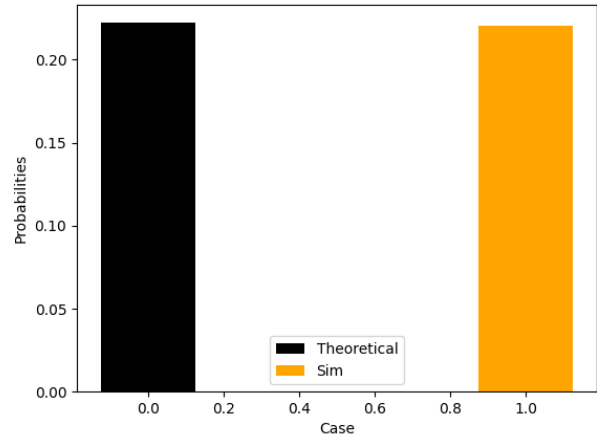
<https://github.com/cmapsi/AI1103-Probability-and-random-variables/blob/main/Assignment-5/main.tex>

$$\Pr(X \in S) = \sum_{k \in S} p_X(k) \quad (2.0.6)$$

Using (2.0.3) and (2.0.6)

$$\Pr(X \in S) = \frac{2}{9} \quad (2.0.7)$$

The graph for theoretical result vs simulation is given below



1 PROBLEM

(GATE-XE-A 2017 Q.170) Two dice are thrown simultaneously. The probability that the product of the numbers appearing on the top faces of the dice is a perfect square is

- (A) $\frac{1}{9}$ (B) $\frac{2}{9}$ (C) $\frac{1}{9}$ (D) $\frac{4}{9}$

2 SOLUTION

Let $X_i \in \{1, 2, 3, 4, 5, 6\}$, $i = 1, 2$ be the random variables representing the outcomes of each die. The probability mass function is given below.

$$p_{X_i}(n) = \Pr(X_i = n) = \begin{cases} \frac{1}{6} & 1 \leq n \leq 6 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.1)$$

$$X = X_1 \times X_2 = n \quad (2.0.2)$$

Defining set $S = \{1, 4, 9, 16, 25, 36\}$

Let there be k ordered pair of factors for each n . $a_i \times b_i = n$, $a_i, b_i \in S$, $\forall i \in \{1, 2, \dots, k\}$ We have the following expression for probability

$$p_X(n) = \sum_k p_{X_1}(a_k) \times p_{X_2}(b_k) = \frac{k}{36} \quad (2.0.3)$$

Let $g(x)$ be defined as follows

$$g(n) = \sum_{j=1}^6 \left[\left\lfloor \frac{n}{j} \right\rfloor - \frac{n}{j} \right] + 1 \quad (2.0.4)$$

$$\therefore p_X(n) = \frac{g(n)}{36} \quad (2.0.5)$$