

Challenging Problem-1

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Download all latex-tikz codes from

https://github.com/cmapsi/AI1103-Probability-and-random-variables/blob/main/Challenging_1/main.tex

1 QUESTION: CHALLENGING PROBLEM 1

Let X be a random variable such that $E(X) = E(X^2) = 1$. Then $E(X^{100}) = ?$

- (A) 0
- (B) 1
- (C) 2^{100}
- (D) $2^{100} + 1$

2 SOLUTION

$$\sigma_X^2 = E[X^2] - (E[X])^2 = 0 \quad (2.0.1)$$

X is almost surely constant random variable, it follows degenerate distribution, in mathematical terms

$$\Pr(X \neq E[X]) < \delta, \forall \delta > 0 \quad (2.0.2)$$

$$\begin{aligned} E[X^{100}] &= \sum_x x^{100} p_X(x) \\ &= 1^{100} p_X(1) + \sum_{x \neq 1} x^{100} p_X(x) \end{aligned} \quad (2.0.3)$$

We know that $0 \leq \Pr(X \neq 1) < \delta$, $p_X(1) = 1 - \delta$

$$0 \leq \left| E[X^{100}] - 1 \right| \leq \sum_x x^{100} \delta \quad (2.0.4)$$

$$\forall \epsilon > 0 \exists \delta > 0 \ni \left| E[X^{100}] - 1 \right| < \epsilon \quad (2.0.5)$$

$$\text{Choose } \delta < \frac{\epsilon}{\sum_x x^{100}}$$

$$\therefore E[X^{100}] = 1 \quad (2.0.6)$$