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Ejercicios del 11/05/2020.

Ejercicio 1 Determinar $f'(0.3)$, $f(x) = \cos x$, $h=0.15$

a) Nivel 6

$$h_1 = 0.15 \quad h_2 = \frac{0.15 - 0.075}{2} = 0.075 \quad h_3 = \frac{0.075}{2} = 0.0375$$

$$h_4 = \frac{0.0375}{2} = 0.01875 \quad h_5 = \frac{0.01875}{2} = 0.009375 \times 10^{-3} \quad h_6 = 4.6875 \times 10^{-3}$$

$$f'(x_0) = \frac{f(x_0 + 2h) + 8f(x_0 + h) - 8f(x_0 - h) + f(x_0 - 2h)}{12h}$$

Diferencias centrales

$$\star h_1 = 0.15$$

$$f'(0.3) = \frac{-f(0.3 + 2(0.15)) + 8f(0.3 + 0.15) - 8f(0.3 - 0.15) + f(0.3 - 2(0.15))}{12(0.1)}$$
$$= -0.295515233$$

$$\star h_2 = 0.075$$

$$f'(0.3) = \frac{-f(0.3 + 2(0.075)) + 8f(0.3 + 0.075) - 8f(0.3 - 0.075) + f(0.3 - 2(0.075))}{12(0.075)}$$
$$= -0.295519895$$

$$* h_3 = 0.0375$$

$$f'(0.3) = \frac{-f(0.3+2(0.0375)) + 8f(0.3+0.0375) - 8f(0.3-0.0375) + f(0.3-2(0.0375))}{12(0.0375)}$$
$$= -0.295520187$$

$$* h_4 = 0.01875$$

$$f'(0.3) = \frac{-f(0.3+2(0.01875)) + 8f(0.3+0.01875) - 8f(0.3-0.01875) + f(0.3-2(0.01875))}{12(0.01875)}$$
$$= -0.295520205$$

$$* h_5 = 0.09375$$

$$f'(0.3) = \frac{-f(0.3+2(0.09375)) + 8f(0.3+0.09375) - 8f(0.3-0.09375) + f(0.3-2(0.09375))}{12(0.09375)}$$
$$= -0.295520207$$

$$* h_6 = 4.6875 \times 10^{-3}$$

$$f'(0.3) = \frac{-f(0.3+2(4.6875 \times 10^{-3})) + 8f(0.3+4.6875 \times 10^{-3}) + 8f(0.3-4.6875 \times 10^{-3}) + f(0.3-2(4.6875 \times 10^{-3}))}{12(0.09375)}$$
$$= -0.295520207$$

$$D(k+1, i) = \frac{4}{3} D(k, i+1) - D(k-1, i)$$

Nivel 2

$$k=1$$

$$D(2, i) = \frac{4}{3} D(1, i+1) - \frac{1}{3} D(1, i)$$

$$D(2, 1) = \frac{4}{3} (-0.295519895) - \frac{1}{3} (-0.2955233) \\ = -0.295521449$$

$$D(2, 2) = \frac{4}{3} (-0.295520187) - \frac{1}{3} (-0.295519895) \\ = -0.295520285$$

$$D(2, 3) = \frac{4}{3} (-0.295520205) - \frac{1}{3} (-0.295520187) \\ = -0.295520212$$

$$D(2, 4) = \frac{4}{3} (-0.295520207) - \frac{1}{3} (-0.295520205) \\ = -0.295520207$$

$$D(2, 5) = \frac{4}{3} (-0.295520207) - \frac{1}{3} (-0.295520207) \\ = -0.295520207$$

Para nivel 3

$$D(x_i, i) = \frac{4^k D(x_{i+1}, i+1) - D(x_{i-1}, i)}{4^{k-1}}$$

$$k+1=3 \Rightarrow k=2$$

$$D(3, 1) = \frac{4^2 D(2, 2) - D(2, 1)}{4^2 - 1}$$

$$= \frac{16 D(2, 2) - 1}{15} D(2, 1)$$

$$D(3, 1) = \frac{16 (-0.29552085)}{15} - \frac{1}{15} (-0.295521449)$$
$$= -0.295520207$$

$$D(3, 2) = \frac{16 (-0.295520212)}{15} - \frac{1}{15} (-0.29552085)$$
$$= -0.295520207$$

$$D(3, 3) = \frac{16 (-0.295520207)}{15} - \frac{1}{15} (0.295520212)$$
$$= -0.295520207$$

$$D(3, 4) = \frac{16 (-0.295520207)}{15} - \frac{1}{15} (-0.295520207)$$
$$= -0.295520207$$

Para nivel 4

$$D(4,i) = \frac{64}{63} D(3,i+1) - \frac{1}{63} D(3,i)$$

$$D(4,1) = \frac{64}{63} (-0.295520207) - \frac{1}{63} (-0.295520207)$$
$$= -0.295520207$$

$$D(4,1) = \frac{64}{63} (-0.295520207) - \frac{1}{63} (-0.295520207)$$
$$= -0.295520207$$

$$D(4,3) = \frac{64}{63} (-0.295520207) - \frac{1}{63} (-0.295520207)$$

Para el nivel 5

$$D(5,i) = \frac{256}{255} D(4,i+1) - \frac{1}{255} (4,i)$$

$$D(5,1) = \frac{256}{255} (-0.295520207) - \frac{1}{255} (-0.295520207)$$
$$= -0.295520207$$

$$D(5,1) = \frac{256}{255} (-0.295520207) - \frac{1}{255} (-0.295520207)$$
$$= -0.295520207$$

Para el nivel 6

$$D(k+1, i) = \frac{4^k D(k, i) - D(k+1, i)}{4^k - 1}$$

$$k+1=6$$

$$k=5 \quad D(6, i) = \frac{4^5 D(5, i) - D(6, i)}{4^5 - 1}$$

$$= \frac{1024 D(5, i+1) - 1 D(5, i)}{1023}$$

$$D(6, 1) = \frac{1024 (-0.295520207)}{1023} - \frac{1}{1023} (-0.295520207)$$
$$= \underline{-0.295520207}$$

b) Nivel 7

$$h_7 = 2.34375 \times 10^{-3}$$

$$f'(0.3) = \frac{-f(0.3 + 2(2.34375 \times 10^{-3})) + 8f(0.3 + 2.34375 \times 10^{-3}) - 8f(0.3 - 2(2.34375 \times 10^{-3})) + f(0.3 - 2(2.34375 \times 10^{-3}))}{12(2.34375 \times 10^{-3})}$$

- Restante del nivel 2

$$D(2,6) = \frac{4}{3} (-0.295520207) - \frac{1}{3} (-0.295520207)$$
$$= -0.295520207$$

= Restante del nivel 3

$$D(3,5) = \frac{16}{15} (-0.295520207) - \frac{1}{15} (-0.295520207)$$
$$= -0.295520207$$

- Restante del nivel 4

$$D(4,4) = \frac{64}{63} (-0.295520207) - \frac{1}{63} (-0.295520207)$$
$$= -0.295520207$$

- Restante del nivel 5

$$D(5,3) = \frac{256}{255} (-0.295520207) - \frac{1}{255} (-0.295520207)$$
$$= -0.295520207$$

- Restante del nivel 6

$$D(6,2) = \frac{1024}{1023} (-0.295520207) - \frac{1}{1023} (-0.295520207)$$
$$= -0.295520207$$

* Para nivel 7

$$D(k+1, i) = 4^k D(k+1, i+1) - D(k+1, i)$$

$$k+1=7$$

$$k=6$$

$$\begin{aligned} D(7, 1) &= \frac{4^6 D(6, i+1)}{4^6 - 1} - \frac{1}{4^6 - 1} D(6, i+1) \\ &= \frac{4,096 D(6, i+1)}{4095} - \frac{1}{4095} D(6, i) \end{aligned}$$

$$\begin{aligned} D(7, 1) &= \frac{4,096(-0.295520207)}{4,095} - \frac{1}{4,095} (-0.295520207) \\ &= \underline{-0.295520207} \end{aligned}$$

Ejercicio 2

Determinar la integral

$$\int_3^5 \frac{dx}{1+x^5}$$

a) Simple

$$\int_a^b f_1(x) dx = (b-a) \left[\frac{f(a) + f(b)}{2} \right]$$

$$\int_3^5 \frac{dx}{1+x^5} = (3-5) \left[\frac{f(3) + f(5)}{2} \right] = \underline{4.418258288 \times 10^{-3}}$$

b) compuesta

* 6 intervalos

$$f(x) = \frac{1}{1+x^2}$$

$$h = \frac{b-a}{n} = \frac{5-3}{6} = \frac{1}{3}$$

i	x _i	f(x _i)
0	a = x ₀	3
1	x ₁ = x ₀ + h	x ₁ = 3 + 1/3 = 10/3
2	x ₂ = x ₁ + h	x ₂ = 10/3 + 1/3 = 11/3
3	x ₃ = x ₂ + h	x ₃ = 11/3 + 1/3 = 4
4	x ₄ = x ₃ + h	x ₄ = 4 + 1/3 = 13/3
5	x ₅ = x ₄ + h	x ₅ = 13/3 + 1/3 = 14/3
6	x ₆ = x ₅ + h	x ₆ = 14/3 + 1/3 = 5

$$\sum_{i=1}^{n-1} f(x_i) h$$

$$\sum_{i=1}^{n-1} = 0.006011943$$

$$\int_a^b f(x) dx = (b-a) \left[f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right]$$

$$\int_3^5 \frac{dx}{1+x^2} = (5-3) \left[0.0046983606556 + 2(0.006011943) + 0.00319897633 \right]$$

$$= 0.016442144371$$

* 8 intervalos

$n=8$

$$h = \frac{b-a}{n} = \frac{5-3}{8} = \frac{2}{8} = \frac{1}{4}$$

i	x_i	$f(x_i)$
0	$x_0=3$	0.00409836656
1	$x_1=3.25$	0.002750344465
2	$x_2=3.5$	0.001900350377
3	$x_3=3.75$	0.001346661424
4	$x_4=4$	0.000975669756
5	$x_5=4.25$	0.0006720679635
6	$x_6=4.50$	0.000416291289
7	$x_7=4.75$	0.000213382783
8	$x_8=5$	0.000131897633

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0.008648658

$$\int_3^5 \frac{dx}{1+x^5} = (\bar{x}-3) \left[0.00409836656 + 2(0.008648658) + 0.00031897633 \right]$$
$$= 0.021715574$$