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Ejercicios del 30 de marzo

Ejercicio 1

$f(x) = \cos x \qquad x = \{-\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi\}$

$P(\pi/4) = ?$

$E_r = ?$

$E_T = ?$

x	x_0	x_1	x_2	x_3	x_4
y	$f(x_0)$	$f(x_1)$	$f(x_2)$	$f(x_3)$	$f(x_4)$

x	$-\pi$	$-\pi/2$	0	$\pi/2$	π
y	$\cos(-\pi)$	$\cos(-\pi/2)$	$\cos(0)$	$\cos(\pi/2)$	$\cos\pi$

x	$-\pi$	$-\pi/2$	0	$\pi/2$	π
y	-1	0	1	0	-1

$$P_4(x) = f(x_0)\frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}$$

$$+ \cancel{f(x_1)\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}} \to 0$$

$$+ f(x_2)\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}$$

$$+ \cancel{f(x_3)\frac{(x-x_0)(x-x_2)(x-x_1)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}} \to 0 \quad +$$

$$f(x_4)\ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_2)(x_4-x_3)(x_4-x_1)}$$

$$P_4(x) = -1\,\frac{(x-(-\pi/2))(x-0)(x-(\pi/2))(x-\pi)}{(-\pi-(-\pi/2))(-\pi-0)(-\pi-\pi/2)(-\pi-\pi)}$$
$$+\frac{(x-(-\pi))(x-(-\pi/2))(x-\pi/2)(x-\pi)}{(0-(\pi))(0-(-\pi/2))(0-\pi/2)(0-\pi)}$$
$$-1\,\frac{(x-(-\pi))(x-(-\pi/2))(x-0)(x-\pi/2)}{(\pi-(-\pi))(\pi-(-\pi/2))(\pi-0)(\pi-\pi/2)}$$

$$P_4(x) = \frac{-(x+\pi/2)(x)(x-\pi/2)(x-\pi)}{(-\pi/2\,\pi/2)(-\pi)(-3\pi/2)(-2\pi)}$$
$$+\frac{(x+\pi)(x+\pi/2)(x-\pi/2)(x-\pi)}{(-\pi)(\pi/2)(-\pi/2)(-\pi)}$$
$$-\frac{(x+\pi)(x+\pi/2)(x)(x-\pi/2)}{(2\pi)(3\pi/2)(\pi)(\pi/2)}$$

$$P_4(x) = \frac{-(x^2-\pi^2/4)(x^2-x\pi)}{-3/2\,\pi^4}+\frac{(x^2-\pi^2)(x^2-\pi^2/4)}{\pi^4/4}$$
$$-\frac{(x^2+\pi x)(x^2-\pi^2/4)}{\frac{3}{2}\pi^4}$$

$$P_4(x) = \frac{-(x^4-\pi x^3-\pi^2/4\,x^2+\pi^3/4\,x)}{-3/2\,\pi^4}+\frac{(x^4-\pi^2/4x^2-\pi^2x^2+\pi^4/4)}{\pi^4/4}$$
$$-\frac{(x^4-\pi^4/4x^2+\pi x^3-\pi^5/4\,x)}{\frac{3}{2}\pi^4}$$

$$\frac{x^4 - \pi x^3 - \pi^2/4\, x^2 + \pi^3/4\, x}{3/2\ \pi^4} + \frac{x^4 - 5/4\ \pi^2 x^2 + \pi^4/4}{\pi^4/4}$$

$$- \frac{x^4 - \pi^4/4\, x^2 + \pi x^3 - \pi^5/4\, x}{3/2\ \pi^4}$$

$$= \frac{2x^4}{3\pi^4} - \frac{2x^3}{3\pi^3} - \frac{1}{6}\frac{x^2}{\pi^2} + \frac{1}{6}\frac{x}{\pi} + \frac{4x^4}{\pi^4} - \frac{5}{4}\frac{x^2}{\pi^2} + 1$$

$$-\left(\frac{2}{3\pi^4}x^4 - \frac{1}{6}x^2 + \frac{2}{3\pi^3}x^3 - \frac{1}{6\pi}\right)$$

$$= \frac{14\,\cancel{x^4}}{\cancel{3\pi^4}} - \frac{2x^3}{\cancel{3\pi^3}} - \frac{31\,\cancel{x^2}}{\cancel{6\pi^2}} + \frac{1}{6\pi}x + 1 - \cancel{\frac{2}{3\pi^4}x^4} + \cancel{\frac{1}{6}x^2} - \cancel{\frac{2}{3\pi^3}x^3} + \frac{1}{6\pi}$$

$$p(x) = \frac{4}{\pi^4}x^4 - \frac{4}{3\pi^3}x^3 + \left(\frac{1}{6} - \frac{31}{6\pi^2}\right)x^2 + \frac{1}{6\pi}x + \frac{1}{6\pi}$$

$$p(\pi/4) = \frac{4}{\pi^4}(\pi/4)^4 - \frac{4}{3\pi^3}(\pi/4)^3 + \left(\frac{1}{6} - \frac{31}{6\pi^2}\right)(\pi/4)^2 + \frac{1}{6\pi}(\pi/4) + \frac{1}{6\pi}$$

$$= \frac{4}{\cancel{\pi^4}}\left(\frac{\cancel{\pi^4}}{256}\right) - \frac{4}{3\cancel{\pi^3}}\left(\frac{\cancel{\pi^3}}{64}\right) + \left(\frac{1}{6} - \frac{31}{6\pi^2}\right)\left(\frac{\pi^2}{16}\right) + \frac{1}{24} + \frac{1}{6\pi}$$

$$= \frac{1}{64} - \frac{1}{48} + \left(\frac{6\pi^2 - 186}{36\cancel{\pi^2}}\right)\frac{\cancel{\pi^2}}{16} + \frac{1}{24} + \frac{1}{6\pi}$$

$$= -\frac{1}{192} + \frac{6\pi^2 - 186}{576} + \frac{6\pi + 24}{144}$$

$$= 0.072249739$$

Valor real

$V_v = \cos(\pi/4)$

$$E_r = \frac{\cos(\pi/4) - 0.07224939}{\cos(\pi/4)} =$$

$E_r = 0.89782343 9$

Error teórico

$n+1 = 4+1 = 5$

$f(x) = \cos(x)$

$f'(x) = -\operatorname{sen}(x)$

$f''(x) = -\cos(x)$

$f'''(x) = \operatorname{sen}(x)$

$f^{IV}(x) = \cos(x)$

$f^{V}(x) = -\operatorname{sen}(x)$

$f^{VI}(x) = -\cos(x)$

$$E = \frac{-\operatorname{sen}(\pi)}{5!}(\pi/4 - (-\pi))(\pi/4 - (-\pi/2))(\pi/4 - 0)(\pi/4 - \pi/4)(\pi/4 - \pi)$$

$= \frac{\pm 0}{6!}$

$= 1.917495 \times 10^{-24}$

Ejercicio 2

$f(x) = e^x$

	x_0	x_1	x_2	x_3	x_4
x	-2	-1	0	1	2
$f(x)$	e^{-2}	e^{-1}	e^{0}	e	e^{2}

$P(-0.5)?$

$E_r = ?$

$E_T = ?$

$$P_4(x) = f(x_0)\frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}$$
$$+ f(x_1)\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}$$
$$+ f(x_2)\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}$$
$$+ f(x_3)\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}$$
$$+ f(x_4)\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)}$$

$$P_4(x) = e^{-2}\frac{(x-(-1))(x-0)(x-1)(x-2)}{(-2-(-1))(-2-0)(-2-1)(-2-2)} + e^{-1}\frac{(x-(-2))(x-0)(x-1)(x-2)}{(-1-(-2))(-1-0)(-1-1)(-1-2)}$$
$$+ e^{0}\frac{(x-(-2))(x-(-1))(x-1)(x-2)}{(0-(-2))(0-(-1))(0-1)(0-2)} + e\frac{(x-(-2))(x-(-1))(x-0)(x-2)}{(1-(-2))(1-(-1))(1-0)(1-2)}$$
$$+ e^{2}\frac{(x-(-2))(x-(-1))(x-0)(x-1)}{(2-(-2))(2-(-1))(2-0)(2-1)}$$

$$P_4(x) = e^{-2}\frac{(x+1)(x)(x-1)(x-2)}{(-1)(-2)(-3)(-4)} + e^{-1}\frac{(x+2)(x)(x-1)(x-2)}{(1)(-1)(-2)(-3)}$$

$$+ e^{0}\frac{(x+2)(x+1)(x-1)(x-2)}{(-2)(-1)(-1)(-2)} + e\frac{(x+2)(x+1)(x)(x-2)}{(3)(2)(1)(-1)}$$

$$+ e^{2}\frac{(x+2)(x+1)(x)(x-1)}{(4)(3)(2)(1)}$$

$$P_4(x) = e^{-2}\frac{(x^2-1)(x^2-2x)}{24} + e^{-1}\frac{(x^2-4)(x^2-x)}{-6}$$

$$+ \frac{(x^2-4)(x^2-1)}{4} + e\frac{(x^2-4)(x^2+x)}{-6}$$

$$+ e^{2}\frac{(x^2+2x)(x^2-1)}{24}$$

$$P_4(x) = e^{-2}\frac{(x^4-2x^3-x^2+2x)}{24} + e^{-1}\frac{(x^4-x^3-4x^2+4x)}{-6}$$

$$+ \frac{x^4-x^2-4x^2+4}{4} + e\frac{(x^4+x^3-4x^2-4x)}{-6}$$

$$+ e^{2}\frac{(x^4-x^2+2x^3-2x)}{24}$$

$$= \cancel{5.638970135\times10^{-3}x^4} - \cancel{0.01127794x^3} - \cancel{5.63897035\times10^{-3}x^2} +$$

$$\cancel{0.01127794x} - \cancel{0.06131324x^4} + \cancel{0.06131324x^3} + \cancel{0.24525296x^2}$$

$$- \cancel{0.24525296x} + \cancel{0.25x^4} - \cancel{1.25x^2} + 1 - \cancel{0.453046971x^4}$$

$$- \cancel{0.453046971x^3} + \cancel{1.812187886x^2} + \cancel{1.812187886x}$$

$$+ \cancel{0.307877337x^4} - \cancel{0.307877337x^2} + \cancel{0.615754674x^3}$$

$$- \cancel{0.615754674x}$$

$$= 0.049156097x^4 + 0.212743003x^3 + 0.493899687x^2 + 0.96245815x$$

$$+1$$

Error = [illegible] 2126 41

$n+1 = 4+1 = 5$

$f(x) = e^x$

$f'(x) = e^x$

$f''(x) = e^x$

$f'''(x) = e^x$

$f^{iv}(x) = e^x$

$f^{v}(x) = e^x$

$$\frac{f^{(n+1)}(\xi_x)}{(n+1)!}(x-x_1)(x-x_2)(x-x_3)\cdots(x-x_n)$$

$$\frac{e^{(2)}}{5!}(-0.5-(-2))(-0.5-(-1))(-0.5-0)(-0.5-1)(-0.5-2)$$

$$= \frac{e^{(2)}}{5!}(1.5)(0.5)(-0.5)(-1.5)(-2.5) = -0.086590501$$

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Ejercicio 3

$f(2) = 0.5103757 \rightarrow f(x_0)$

$f(2.2) = 0.5207843 \rightarrow f(x_1)$

$f(2.4) = 0.5104147 \rightarrow f(x_2)$

$f(2.6) = 0.4813306 \rightarrow f(x_3)$ aproximar $f(2.5)$

$f(2.8) = 0.4359160 \rightarrow f(x_4)$

a) Grado uno (2 puntos)

$$P_1(x) = f(x_0)\frac{x-x_1}{x_0-x_1} + f(x_1)\frac{x-x_0}{x_1-x_0}$$

$$= (0.51037357)\frac{(x-2.2)}{2-2.2} + 0.5207843\frac{(x-2)}{2.2-2}$$

$$= (0.51037357)\frac{(x-2.2)}{-0.2} + 0.520743\frac{(x-2)}{0.2}$$

$$= -2.5518685(x-2.2) + 2.6039215(x-2)$$

$$= -2.5518685x + 5.6141107 + 2.603715x - 5.20743$$

$$= 0.0518465x + 0.4066807$$

$P_1(x) = 0.518465x + 0.4066807$

$P_1(2.5) = 0.51865(2.5) + 0.4066807 = 1.7033057$

b) Grado dos (3 puntos)

$$P_2(x) = f(x_0)\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + f(x_1)\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + f(x_2)\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$=0.5103757\frac{(x-2.2)(x-2.4)}{(2-2.2)(2-2.4)}+0.5207843\frac{(x-2)(x-2.4)}{(2.2-2)(2.2-2.4)}$$

$$+0.5104147\frac{(x-2)(x-2.2)}{(2.4-2)(2.4-2.2)}$$

$$=0.5103757\frac{(x^2-2.4x-2.2x+5.28)}{0.08}+0.5207843\frac{(x^2-2.4x-2x+4.8)}{-0.04}$$

$$+0.5104147\frac{(x^2-2.2x-2x+4.4)}{0.08}$$

$$=6.3796625(x^2-4.6x+5.28)-13.0196075(x^2-4.4x+4.8)$$
$$+6.38018375(x^2-4.2x+4.4)$$

$$=\cancel{6.37966225x^2}-\cancel{29.34644475x}+33.6844618-\cancel{13.0196075x^2}$$
$$+\cancel{57.286273x}-62.494116+\cancel{6.38018375x^2}-\cancel{26.79677175x}$$
$$+28.0728085$$

$$P_2(x)=-0.2597615x^2+1.14305375x-0.7366895$$

$$P_2(2.5)=-0.2597615(2.5)^2+1.14305375(2.5)-0.7366895$$

$$=0.4974355$$

c) Grado 3 (cuatro puntos)

$$P_3(x) = f(x_0)\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + f(x_1)\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$+ f(x_2)\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + f(x_3)\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_2)(x_3-x_1)}$$

$$P_3(x) = 0.5103757\frac{(x-2.2)(x-2.4)(x-2.6)}{(2-2.2)(2-2.4)(2-2.6)} + 0.5207843\frac{(x-2)(x-2.4)(x-2.6)}{(2.2-2)(2.2-2.4)(2.2-2.6)}$$

$$+0.5104147\frac{(x-2)(x-2.2)(x-2.6)}{(2.4-2)(2.4-2.2)(2.4-2.6)} + 0.4813306\frac{(x-2)(x-2.2)(x-2.4)}{(2.6-2)(2.6-2.2)(2.6-2.4)}$$

$$P_3(x) = 0.5103757\frac{(x^3-7.2x^2+17.24x-13.728)}{-0.048} + 0.5207843\frac{(x^3-7x^2+16.24x-12.48)}{0.016}$$

$$+0.5104147\frac{(x^3-6.8x^2+15.32x-11.44)}{-0.016} + 0.4813306\frac{(x^3-6.6x^2+14.48x-10.56)}{0.048}$$

$$= -10.63282708(x^3-7.2x^2+17.24x-13.728) + 32.549018.75(x^3-7x^2+16.24x-12.48)$$

$$-31.90091875(x^3-6.8x^2+15.32x-11.44) + 30.0831625(x^3-6.6x^2+14.48x-10.56)$$

$$= -10.63282708x^3 + 76.556355x^2 - 183.3099389x + 145.9674562$$

$$+ 32.54901875x^3 - 227.8431313x^2 + 528.5960645x - 406.211754$$

$$-31.9009875x^3 + 216.9267153x^2 - 488.7231291x + 364.9472975$$

$$+30.0831625x^3 - 198.5488725x^2 + 435.604193x - 317.678196$$

$$P_4(x) = 20.0983667x^3 - 132.908933x^2 + 292.1671158x - 212.9752701$$

$$P(2.5) = 20.0983667(2.5)^3 - 132.908933(2.5)^2 + 292.1671158(2.5) - 212.9752701$$

$$P(2.5) = 0.79866783 7$$

Ejercicio 4

$P(x)$ para $f(x) = \frac{1}{x}$ evaluar

	x_0	x_1	x_2
x	1	2	3
$f(x)$	1	$\frac{1}{2}$	$\frac{1}{3}$

$$P(x) = f(x_0)\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + f(x_1)\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + f(x_2)\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$= \frac{(x-2)(x-3)}{(1-2)(1-3)} + \frac{1}{2}\frac{(x-1)(x-3)}{(2-1)(2-3)} + \frac{1}{3}\frac{(x-1)(x-2)}{(3-1)(3-2)}$$

$$= \frac{x^2-5x+6}{2} - \frac{1}{2}(x^2-4x+3) + \frac{1}{3}\frac{(x^2-3x+2)}{2}$$

$$\cancel{0.5x^2} - \cancel{2.5x} + \cancel{3} - \cancel{0.5x^2} + \cancel{2x} - \cancel{3/2} + \frac{1}{6}x^2 - \cancel{\frac{1}{2}x} + \frac{1}{3}$$

$$P(0.7) = \frac{1}{6}x^2 - x + \frac{11}{6} = 1.215$$

$$V_v = \frac{1}{0.7} = \frac{10}{7}$$

$$E_r\% = \left|\frac{10/7 - 1.215}{10/7}\right| \times 100 = 14.95\%$$

Error teórico

$n+1 = 2+1 = 3$

$f(x) = \frac{1}{x}$

$f'(x) = -1/x^2$

$f''(x) = 2/x^3$

$f'''(x) = -6/x^4$

$f''''(x) = 24/x^5$

$E = \frac{-6/3^4}{3!}(0.7-1)(0.7-2)(0.7-3)$

$E = 0.0110740740^{-3}$

Ejercicios del 03/abril

Ejercicio 1

Determinar polinomio de Newton

$f(x) = e^{x^2}$

$x = \{0, 2, 3, 4, 6\}$ ($x_0\ x_1\ x_2\ x_3\ x_4$) interpolar $= x = 1.7$

$P(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)(x-x_2)$
$b_4(x-x_0)(x-x_1)(x-x_2)(x-x_3)$

* $(x-0)(x-2) = x^2-2x$
* $(x-0)(x-2)(x-3) = (x^2-2x)(x-3)$
$= x^3-3x^2-2x^2+6x$
$= x^3-5x^2+6x$

* $(x-0)(x-2)(x-3)(x-4) = (x^3-5x^2+6x)(x-4)$
$= x^4-4x^3-5x^3+20x^2+6x^2-24x$
$= x^4-9x^3+26x^2-24x$

$f(x) = e^{x^2}$

x	x_0	x_1	x_2	x_3	x_4
	0	2	3	4	6
$f(x)$	1	e^4	e^9	e^{16}	e^{36}

$b_0 = F[x_0] = f(0) = 1$

$b_1 = F[x_0, x_1] = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \dfrac{e^4 - 1}{2} = 26.79907502$

$b_2 = F[x_0, x_1, x_2] = \dfrac{F[x_1, x_2] - \overbrace{F[x_1 - x_0]}^{b_1}}{x_2 - x_0}$

$= \dfrac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - b_1}{x_2 - x_0} = \dfrac{\dfrac{e^9 - e^4}{3-2} - b_1}{3}$

$= 2673.895568$

$b_3 = F[x_0, x_1, x_2, x_3] = \dfrac{F[x_3, x_2, x_1] - \overbrace{F[x_0, x_1, x_2]}^{b_2}}{x_3 - x_0}$

$= \dfrac{\dfrac{F[x_3, x_2] - F[x_2, x_1]}{x_3 - x_1} - b_2}{4} =$

$= \dfrac{\dfrac{\dfrac{e^{16} - e^9}{4-3} - \dfrac{e^9 - e^4}{3-2}}{4-2} - b_2}{4} = 1108676.395$

$$P(x) \; b_4 = f[x_0, x_1, x_2, x_3, x_4] = \frac{f[x_4, x_3, x_2, x_1] - \overbrace{f[x_3, x_2, x_1, x_0]}^{b_3}}{x_4 - x_0}$$

$$f[x_4, x_3, x_2, x_1] = \frac{f[x_4, x_3, x_2] - f[x_3, x_2, x_1]}{x_4 - x_1}$$

$$= \frac{\dfrac{f[x_4, x_3] - f[x_3, x_2]}{x_4 - x_2} - \dfrac{f[x_3, x_2] - f[x_2, x_1]}{x_3 - x_1}}{x_4 - x_1}$$

$$= \frac{\dfrac{\dfrac{e^{36} - e^{16}}{2} - \dfrac{e^{16} - e^{4}}{1}}{3} - 887800.7437}{4} = 7.1853858317 \times 10^{14}$$

$$= \frac{7.185385867 \times 10^{14} - 44349\,79.475}{4} = 1.796346456 \times 10^{14}$$

$$P(x) = 1 + 26.7907502x + 2673.89568(x^2 - 2x) + 1108076.395(x^3 - 5x^2 + 6x) + 1.796346456 \times 10^{14}(x^4 - 9x^3 + 26x^2 - 24x)$$

$$= 1 + \cancel{26.7907502x} + \cancel{2673.89568x^2} - \cancel{5{,}347.7936x} + \cancel{1108076.395x^3} - \cancel{5{,}540381.975x^2} + \cancel{6{,}648458.37x} + 1.7963464 5 \times 10^{14} x^4 - \cancel{1.61671181 \times 10^{15}} x^3 + \cancel{4.670500786 \times 10^{15}} x^2 - \cancel{4.311231494 \times 10^{15}} x$$

$$= 1 - 4.311231487 \times 10^{15} x + 4.670500731 \times 10^{15} x^2 - 1.616711809 \times 10^{15} x^3 + 1.79634645 \times 10^{14} x^4$$

$$P(1.7) = 1 - 4.311231487 \times 10^{15}(1.7) + 4.670500731 \times 10^{15}(1.7)^2 - 1.616711809 \times 10^{15}(1.7)^3 + 1.74634645 \times 10^{14}(1.7)^4 = -1.103375081 \times 10^{15}$$

Valor real:

$f(1.7) = e^{1.7^2} = 17.9933096$

$$E\% = \left| \frac{e^{1.7^2} - f(1.1033750\text{81}\times 10^{5})}{e^{1.7^2}} \right| \times 100$$

$= 6.1321408\text{04}\times 10^{13}\ \%$

Ejercicio 2

Determinar $f(x) = \frac{1}{x}$ para $x = \{3, 5, 7\}$

* Grado 1

x	x_0	x_1	x_2
	3	5	7
f(x)	1/3	1/5	1/7

$b_0 = f(x_0) = \frac{1}{3}$

$b_1 = F[x_1, x_0] = \frac{1/5 - 1/3}{5-3} = -\frac{1}{15}$

$P_1(x) = \frac{1}{3} - \frac{1}{15}(x-3)$

$= \frac{1}{3} - \frac{1}{15}x + \frac{1}{5} = -\frac{1}{5}x + \frac{8}{15}$

Grado 2

$$P(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$b_0 = f(x_0) = 1/3$$

$$b_1 = f[x_1, x_0] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{1/5 - 1/3}{5-3} = -\frac{1}{15}$$

$$b_2 = f[x_2, x_1, x_0] = \frac{f[x_2, x_1] - \overbrace{f[x_1, x_0]}^{b_1}}{x_2 - x_0} = \frac{\frac{1/7 - 1/5}{2} - \left(-\frac{1}{15}\right)}{4} = \frac{1}{105}$$

$$(x-3)(x-5) = x^2 - 5x - 3x + 15$$
$$= x^2 - 8x + 15$$

$$P(x) = \frac{1}{3} - \frac{1}{15}(x-3) + \frac{1}{105}(x^2 - 8x + 15)$$

$$= \cancel{\frac{1}{3}} - \cancel{\frac{x}{15}} + \cancel{\frac{1}{5}} + \frac{x^2}{105} - \cancel{\frac{8}{105}x} + \cancel{\frac{1}{7}}$$

$$= \frac{71}{105} - \frac{11}{7}x + \frac{x^2}{105}$$

Ejercicios del 14/04/2020

Ejercicio 1

Interpolación de Newton $x = -1$

	x_0	x_1	x_2	
x	2	0	-2	x
y	15	-1	-17	$\rightarrow f(x)$

x	b_0	b_1	b_2
2	+15		
0	-1	8	
-1	-17	8	0

$$P(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$
$$= 15 + 8(x-2) + 0(x-2)(x-0)$$
$$= 15 + 8x - 16$$
$$= \underline{8x - 1}$$

$$P(-1) = 8(-1) - 1 = -9$$

Ejercicio 2

x	0	1	2	3
y	-1	6	31	18

$$P(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$
$$b_3(x-0)(x - x_1)(x - x_2)$$

	b_0	b_1	b_2	b_3
0	-1			
1	6	7		
2	31	25	9	
3	18	-13	-19	$-28/3$

$$(x-0)(x-1) = x^2 - x$$
$$(x-0)(x-2)(x-31) = (x-31)(x^2-x)$$
$$= x^3 - x^2 - 31x^2 + 31x$$
$$= x^3 - 33x^2 + 31x$$

$$P(x) = -1 + 7x + 9(x^2 - x) - \frac{28}{3}(x^3 - 33x^2 + 31x)$$

$$P(x) = -1 + \cancel{7x} + 9x^2 \cancel{-9x} - \frac{28}{3}x^3 + 308x^2 - \frac{868}{3}x$$

$$P(x) = -1 - \frac{874}{3}x - \frac{1}{3}x^2 - \frac{28}{3}x^3$$

Ejercicios

$x = 2$, $x = 6$

x	1	3	5	9
y	2	4	3	8

x	b_0	b_1	b_2	b_3
1	2			
3	4	1		
5	3	$-1/2$	$-3/8$	
9	8	$5/4$	$7/24$	$1/12$

$$= \frac{\frac{5}{4} - \left(-\frac{1}{2}\right)}{9-3} = \frac{7}{24}$$

$$= \frac{7/24 - (-3/8)}{9-1} = \frac{1}{12}$$

$$P(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)(x-x_2)$$

$$(x-1)(x-3) = x^2 - 3x - x + 3$$
$$= x^2 - 4x + 3$$

$$(x-1)(x-3)(x-5) = (x-5)(x^2-4x+3)$$
$$= x^3 - \cancel{4x^2} + 3x - \cancel{5x^2} + 20x - 15$$
$$= x^3 - 9x^2 + 23x - 15$$

$$P(x) = 2 + x - 1 - 3/8(x^2 - 4x + 3) + \frac{1}{12}(x^3 - 9x^2 + 23x - 15)$$
$$=$$
$$\cancel{1} + \cancel{x} - \frac{3}{8}\cancel{x^2} + \frac{3}{2}\cancel{x} - \cancel{\frac{9}{8}} + \frac{1}{12}x^3 - \frac{3}{\cancel{4}}\cancel{x^2} + \cancel{\frac{23x}{12}} - \cancel{\frac{15}{4}}$$

$$P(x) = -\frac{11}{8} + \frac{53}{12}x - \frac{9}{8}x^2 + \frac{1}{12}x^3$$

Ejercicios del 20/04/2020

Se considera la funcion $f(x) = \ln x$

a) Calcula el polinomio de Newton

$x = \{1, 2, 3, 4\}$ $\qquad x = 0.73$

	1	2	3	4
$f(x) =$	$\ln(1)$	$\ln 2$	$\ln 3$	$\ln 4$

x	b_0	b_1	b_2	b_3
1	0			
2	$\ln 2$	0.6931 4718		
3	$\ln 3$	0.405465108	-0.14384 1036	
4	$\ln 4$	0.28768 2072	-0.05889 518	0.02831 6506

$p(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)(x - x_2)$

$(x-1)(x-2) = x^2 - 2x - x + 2$

$= x^2 - 3x + 2$

$(x-1)(x-2)(x-3) = (x^2 - 3x + 2)(x - 3)$

$= x^3 - 3x^2 - 3x^2 + 9x + 2x - 6$

$= x^3 - 6x^2 + 11x - 6$

$P(x) = 0 + 0.69314718(x-1) - 0.143841036(x^2-3x+2) + 0.028316506$
$*(x^3-6x^2+11x-6)$

$P(x) = \cancel{0.69314718x} - \cancel{0.69314718} - \cancel{0.143841036x^2} + \cancel{0.431523108x} - \cancel{0.287682072}$
$+ 0.028316506x^3 - \cancel{0.169899036x^2} + \cancel{0.311481566x} - \cancel{0.169899036}$

$P(x) = 1.436151854x - 0.575364144 - 0.313740072x^2 + 0.028316506x^3$

$P(0.73) = 1.436151854(0.73) - 1.130728288 - 0.313740072(0.73)^2$
$+ 0.028316506(0.73^3)$

$P(0.73) = 0.238513916$

$\boxed{\text{Error porcentual}}$

$$E\% = \left|\frac{V_v - V_c}{V_v}\right| \times 100 = \left|\frac{\ln(0.73) - (-0.238513916)}{\ln(0.73)}\right| \times 100$$

$$= 24.2117021 3\ \%$$

Error teorico

$f(x) = \ln x$
$f'(x) = 1/x$
$f''(x) = -1/x^2$
$f'''(x) = 2/x^3$
$f''''(x) = -6/x^4$

$n+1 = 3+1 = 4$

$$E = \frac{-6/(4)^4}{4!}(0.73-1)(0.73-2)(0.73-3)(0.73-4)$$

$$E = 2.485656657 \times 10^{-3}$$

b) Polinomio de Hermite para $x = \{1, 2\}$

$f(x) = \ln x$

x	b_0	b_1	b_2	b_3
1	0			
1	0	1		
2	ln2	ln2	-0.3068528191	
2	ln2	1/2	-0.1931147886	0.1137056388

$b_1 \quad \frac{1/1}{1} = 1 \qquad \frac{\ln 2 - 0}{2-1} = \ln 2$

$\frac{\frac{1}{2}}{1} = \frac{1}{2} \qquad \frac{1/2 - \ln 2}{2-1} = -0.1931147186$

$b_2 \quad \frac{\ln 2 - 1}{2-1} = -0.3068528191$

$b_3 = \frac{0.1931147786 - (-0.3068528191)}{12-1} = 0.1137056388$

$p(x) = b_1(x-x_0) + b_2(x-x_0)^2 + b_3(x-x_0)^2(x-x_1)$

$= 0 + (x-1) - 0.3068528191(x-1)^2 + 0.1137056388(x-1)^2(x-2)$

$p(x) = x - 1 - 0.306852191(x^2-2x+1) + 0.113705638(x^2-2x+1)(x-2)$

Ejercicios del 27/04/20

Ejercicio 1

Grado uno

x	6	7	8	10	12
y	3	4	7	9	5

$[6,7], [7,8], [8,10], [10,12]$

$$s(x) = \begin{cases} a_0 x + b_0 & x \in [6,7] \\ a_1 x + b_1 & x \in [7,8] \\ a_2 x + b_2 & x \in [8,10] \\ a_3 x + b_3 & x \in [10,12] \end{cases}$$

$s(6) = 3$;
$s(7) = 4$;
$s(8) = 7$;
$s(10) = 9$;
$s(12) = 15$

$s(6) = 3 \rightarrow 6a_0 + b_0 = 3$

$s(7) = 4 \rightarrow 7a_0 + b_0 = 4$

$s(7) = 4 \rightarrow 7a_1 + b_1 = 4$

$s(8) = 7 \rightarrow 8a_1 + b_1 = 7$

$s(8) = 7 \rightarrow 8a_2 + b_2 = 7$

$s(10) = 9 \rightarrow 10a_2 + b_2 = 9$

$s(10) = 9 \rightarrow 10a_3 + b_3 = 9$

$s(12) = 15 — 12a_3 + b_3 = 15$

$$\begin{cases} 6a_0 + b_0 = 3 \\ 7a_0 + b_0 = 4 \quad (-1) \end{cases} \rightarrow \begin{array}{r} 6a_0 + \cancel{b_0} = 3 \\ -7a_0 - \cancel{b_0} = -4 \\ \hline -a_0 = -1 \end{array}$$

$$\boxed{a_0 = 1}$$

$G(1) = b_0 = 3$

$b_0 = 3$

$$\begin{cases} 7a_1 + b_1 = 4 \\ 8a_1 + b_1 = 7 \quad (-1) \end{cases}$$

$$\begin{array}{r} 7a_1 + b_1 = 4 \\ -8a_1 - b_1 = -7 \\ \hline -a_1 = -3 \\ a_1 = 3 \end{array}$$

$8(3) + b_1 = 7$

$24 + b_1 = 7$

$b_1 = 7 - 24$

$b_1 = -17$

$$\begin{cases} 8a_2 + b_2 = 7 \\ 10a_2 + b_2 = 9 \quad (-1) \end{cases}$$

$$\begin{array}{r} 8a_2 + b_2 = 7 \\ -10a_2 - b_2 = -9 \\ \hline -2a_2 = -2 \\ a_2 = 1 \end{array}$$

$10(1) + b_2 = 9$

$10 + b_2 = 9$

$b_2 = -1$

$$\begin{cases} 10a_3 + b_3 = 9 \quad (-1) \\ 12a_3 + b_3 = 15 \end{cases}$$

$$\begin{array}{r} -10a_3 - \cancel{b_3} = -9 \\ 12a_3 + \cancel{b_3} = 15 \\ \hline 2a_3 = 6 \\ a_3 = 3 \end{array}$$

$10(3) + b_3 = 9$

$30 + b_3 = 9$

$b_3 = -21$

$$s(x) = \begin{cases} x - 3 & x \in [6,7] \\ 3x - 17 & x \in [7,8] \\ x - 1 & x \in [8,10] \\ 3x - 21 & x \in [10,12] \end{cases}$$

Grado dos

$[6,7]$, $[7,8]$, $[8,10]$, $[10,12]$

$s(6)=3$
$s(7)=4$
$s(8)=7$
$s(10)=9$
$s(12)=15$

$$s(x)=\begin{cases} a_0x^2+b_0x+c_0 & x\in[6,7] \\ a_1x^2+b_1x+c_1 & x\in[7,8] \\ a_2x^2+b_2x+c_2 & x\in[8,10] \\ a_3x^2+b_3x+c_3 & x\in[10,12] \end{cases}$$

$s(6)=3$ — $36a_0+6b_0+c_0=3$
$s(7)=4$ — $49a_0+7b_0+c_0=4$
$s(7)=4$ — $49a_1+7b_1+c_1=4$
$s(8)=7$ — $64a_1+8b_1+c_1=7$
$s(8)=7$ — $64a_2+8b_2+c_2=7$
$s(10)=9$ — $100a_2+10b_2+c_2=9$
$s(10)=9$ — $100a_3+10b_3+c_3=9$
$s(12)=15$ — $144a_3+12b_3+c_3=15$

$$s'(x)=\begin{cases} 2a_0+b_0 & x\in[6,7] \\ 2a_1+b_1 & x\in[7,8] \\ 2a_2+b_2 & x\in[8,10] \\ 2a_3+b_3 & x\in[10,12] \end{cases}$$

Puntos 7,8,10 pueden tener discontinuidad por tanto

$2(7)a_0+b_0=2(7)a_1+b_1$

$14a_0+b_0=14a_1+b_1$ /

$a_0=0$

$$2(8)a_1 + b_1 = 2(8)a_2 + b_2 \qquad 2(10)a_2 + b_2 = 2(10)a_3 + b_3$$

$$16a_1 + b_1 = 16a_2 + b_2 \qquad 20a_2 + b_2 = 20a_3 + b_3$$

$$6b_0 + c_0 = 3 \quad (1)$$

$$7b_0 + c_0 = 4 \quad (2)$$

$$49a_1 + 7b_1 + c_1 = 4 \quad (3)$$

$$64a_1 + 8b_1 + c_1 = 7 \quad (4)$$

$$64a_2 + 8b_2 + c_2 = 7 \quad (5)$$

$$100a_2 + 10b_2 + c_2 = 9 \quad (6)$$

$$100a_3 + 10b_3 + c_3 = 9 \quad (7)$$

$$144a_3 + 12b_3 + c_3 = 15 \quad (8)$$

$$b_0 = 14a_1 + b_1 \quad (9)$$

$$16a_1 + b_1 = 16a_2 + b_2 \quad (10)$$

$$20a_2 + b_2 = 20a_3 + b_3 \quad (11)$$

$$\begin{array}{c|ccccccccccc|c|c|}
 & b_0 & c_0 & a_1 & b_1 & c_1 & a_2 & b_2 & c_2 & a_3 & b_3 & c_3 & & \\
\hline
b_0 & 6 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 3 \\
c_0 & 7 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 4 \\
a_1 & 0 & 0 & 49 & 7 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & & 4 \\
b_1 & 0 & 0 & 64 & 8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & & 7 \\
c_1 & 0 & 0 & 0 & 0 & 0 & 64 & 8 & 1 & 0 & 0 & 0 & \times & 7 \\
a_2 & 0 & 0 & 0 & 0 & 0 & 100 & 10 & 1 & 0 & 0 & 0 & & 9 \\
b_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & 10 & 1 & & 9 \\
c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 144 & 12 & 1 & & 15 \\
a_3 & 1 & 0 & -14 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 \\
b_3 & 0 & 0 & 16 & 1 & 0 & -16 & -1 & 0 & 0 & 0 & 0 & & 0 \\
c_3 & 0 & 0 & 0 & 0 & 0 & 20 & 1 & 0 & 20 & -1 & 0 & & 0
\end{array}$$

$a_0 = 0$

$b_0 = 1$

$c_0 = -3$

$a_1 = 2$

$b_1 = -27$

$c_1 = 95$

$a_2 = -2$

$b_2 = 37$

$c_2 = -161$

$a_3 = 3$

$b_3 = -63$

$c_3 = 339$

$$s(x) = \begin{cases} x-3 & \\ 2x^2-27x+95 & x\in[6,7] \\ -2x^2+37x-161 & x\in[7,8] \\ 3x^2-63x+339 & x\in[10,12] \end{cases}$$

Grado 3

$[6,7], [7,8], [8,10], [10,12]$

$$s(x) = \begin{cases} a_0x^3+b_0x^2+c_0x+d_0 & x\in[6,7] \\ a_1x^3+b_1x^2+c_1x+d_1 & x\in[6,8] \\ a_2x^3+b_2x^2+c_2x+d_2 & x\in[8,10] \\ a_3x^3+b_3x^2+c_3x+d_3 & x\in[10,12] \end{cases}$$

$s(6)=3$
$s(7)=4$
$s[8]=7$
$s[10]=0$
$s(12)=15$

$s(6)=3 \rightarrow 216a_0+36b_0+6c_0+d_0=3$ (1)
$s(7)=4 - 343a_0+49b_0+7c_0+d_0=4$ (2)
$s(7)=4 - 343a_1+49b_1+7c_1+d_1=4$ (3)
$s(8)=7 - 512a_1+64b_1+8c_1+d_1=7$ (4)
$s(8)=7 - 512a_2+64b_2+8c_2+d_2=7$ (5)
$s(10)=9 - 1000a_2+100b_2+10c_2+d_2=9$ (6)
$s(10)=9 - 1000a_3+100b_3+10c_3+d_3=9$ (7)
$s(12)=15 - 1728a_3+144b_3+12c_3+d_3=15$ (8)

* Por la 1era derivada

$$s'(x) = \begin{cases} 3a_0x^2+2b_0x+c_0 & x\in[6,7] \\ 3a_1x^2+2b_1x+c_1 & x\in[7,8] \\ 3a_2x^2+2b_2x+c_2 & x\in[8,10] \\ 3a_3x^2+2b_3+c_3 & x\in[10,12] \end{cases}$$

Teniendo: $x = 7$

$$3(7)^2 a_0 - 2(7) b_0 + c_0 = 3(7)^2 a_1 + 2(7) b_1 + c_1$$

(9) $$147 a_0 + 14 b_0 + c_0 = 147 a_1 + 14 b_1 + c_1$$

$x = 8$

$$3(8)^2 a_1 + 2(8) b_1 + c_1 = 3(8)^2 a_2 + 2(8) b_2 + c_2$$

(10) $$192 a_1 + 16 b_1 + c_1 = 192 a_2 + 16 b_2 + c_2$$

$x = 10$

$$3(10)^2 a_2 + 20 b_2 + c_2 = 3(10)^2 a_3 + 20 b_3 + c_3$$

(11) $$300 a_2 + 20 b_2 + c_2 = 300 a_3 + 20 b_3 + c_3$$

* Por 2da derivada

$$S''(x) = \begin{cases} 6a_0 x + 2b_0 & x \in [6,7] \\ 6a_1 x + 2b_1 & x \in [7,8] \\ 6a_2 x + 2b_2 & x \in [8,10] \\ 6a_3 x + 2b_3 & x \in [10,12] \end{cases}$$

(12) $$42 a_0 + 2 b_0 = 42 a_1 + 2 b_1$$

(13) $$48 a_1 \; 2 b_1 = 48 a_1 + 2 b_2$$

(14) $$60 a_2 + 2 b_2 = 60 a_3 + 2 b_3$$

Según los 2 gl por:

$S''(x_0)=0$; $S''(x_n)=0$

$6(6)a_0 + 2b_0 = 0$ $\qquad$ $6(12)a_0 + 2b_0 = 0$

$36a_0 + 2b_0 = 0$ (15) $\qquad$ $72a_0 + 2b_0 = 0$ (16)

$216a_0 + 36b_0 + 6c_0 + d_0 = 3$ (1)

$343a_0 + 49b_0 + 7c_0 + d_0 = 4$ (2)

$343a_1 + 49b_1 + 7c_1 + d_1 = 4$ (3)

$512a_1 + 64b_1 + 8c_1 + d_1 = 7$ (4)

$512a_2 + 64b_2 + 8c_2 + d_2 = 7$ (5)

$1000a_2 + 100b_2 + 10c_2 + d_2 = 9$ (6)

$1000a_3 + 100b_3 + 10c_3 + d_3 = 9$ (7)

$1728a_3 + 144b_3 + 12c_3 + d_3 = 15$ (8)

$147a_0 + 14b_0 + c_0 = 147a_1 + 14b_1 + c_1$ (9)

$192a_1 + 16b_1 + c_1 = 192a_2 + 16b_2 + c_2$ (10)

$300a_2 + 20b_2 + c_2 = 300a_3 + 20b_3 + c_3$ (11)

$42a_0 + 2b_0 = 42a_1 + 2b_1$ (12)

$48a_1 + 2b_1 = 48a_2 + 2b_2$ (13)

$60a_2 + 2b_2 = 60a_3 + 2b_3$ (14)

$36a_0 + 2b_0 = 0$ (15)

$72a_0 + 2b_0 = 0$ (16)

	a_0	b_0	c_0	d_0	a_1	b_1	c_1	d_1	a_2	b_2	c_2	d_2	a_3	b_2	c_2	d_2			
a_0	216	36	6	1	0	0	0	0	0	0	0	0	0	0	0	0		3	0
b_0	343	49	7	1	0	0	0	0	0	0	0	0	0	0	0	0		4	0
c_0	0	0	0	0	343	49	7	1	0	0	0	0	0	0	0	0		4	1
d_0	0	0	0	0	512	64	8	1	0	0	0	0	0	0	0	0		7	-3
a_1	0	0	0	0	0	0	0	0	512	64	8	1	0	0	0	0		7	2
b_1	0	0	0	0	0	0	0	0	1000	100	10	1	0	0	0	0		9	-42
c_1	0	0	0	0	0	0	0	0	0	0	0	0	1000	100	10	1		9	295
d_1	0	0	0	0	0	0	0	0	0	0	0	0	1728	144	12	1		15	-689
a_2	147	14	1	0	-147	-14	-1	0	0	0	0	0	0	0	0	0	X	0	-9/2
b_2	0	0	0	0	192	16	1	0	-192	-16	-1	0	0	0	0	0		0	114
c_2	0	0	0	0	0	0	0	0	300	20	1	0	-300	-20	-1	0		0	-953
d_2	42	2	0	0	-42	-2	0	0	0	0	0	0	0	0	0	0		0	2639
a_3	0	0	0	0	48	2	0	0	-48	-2	0	0	0	0	0	0		0	17
b_3	0	0	0	0	0	0	0	0	60	2	0	0	-60	-2	0	0		0	-531
c_3	63	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	5497
d_3	72	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	-18,861

$$S(x) = \begin{cases} x-3 & x \in [6,7] \\ 2x^3 - 42x^2 + 295x - 689 & x \in [7,8] \\ -9/2\,x^3 + 114x^2 - 953x + 2639 & x \in [8,10] \\ 17x^3 - 531x^2 + 5497 - 18,861 & x \in [10,12] \end{cases}$$

Ejercicio 2

$f(x) = e^x$; $x = 3$, $h = 0.15$

$f'(x) = ?$

$x =$	3	3.15
$f(x)$	e^3	$e^{3.15}$

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h} = \frac{f(3+0.15) - f(3)}{0.5}$$

$$= \frac{e^{3.15} - e^3}{0.15} = 21.67018439$$

$f'(3.15) = 23.3366458$

$$E\% = \left| \frac{e^{3.5} - 21.6708439}{e^{3.5}} \right| \times 100 = \underline{7.138650929\%}$$

Ejercicio del 4/05/2020

Teniendo $f(x) = \operatorname{sen} x$; $h = 0.15$

$f'(x) = ?$ $\quad x = \pi/4$

$f''(x) = ?$

$f'''(x) = ?$

$f^{IV}(x) = ?$

* Primera diferencia hacia atrás

$$f'(x_0) = \frac{f(x_0) - f(x_0 - h)}{h} = \frac{f(\pi/4) - f(\pi/4 - 0.15)}{0.15}$$

$$= 0.757391758$$

$f'(x) = \cos x$

$$E\% = \left| \frac{\cos(\pi/4) - 0.757391758}{\cos(\pi/4)} \right| \times 100\%$$

$$= 7.11136\ 9692\ \%$$

* Primera diferencia hacia adelante

$$f''(x_0) = \frac{f(x_0 + 2h) - 2f(x_0 + h) + f(x_0)}{h^2}$$

$$= \frac{f(\pi/4 + 2(0.15)) - 2f(\pi/4 + 0.15) + f(\pi/4)}{(0.15)^2}$$

$$= -0.80\ 33.27516$$

$f''(x) = -\operatorname{sen} x$

$$E\% = \left| \frac{-\operatorname{sen}(\pi/4) - (-0.803327516)}{-\operatorname{sen}(\pi/4)} \right| \times 100\ \%$$

$$= 13.60766681\ \%$$

* Primera diferencia hacia adelante:

$$f'''(x) = \frac{f(x_0) - 3f(x_0 - h) + 3f(x_0 - 2h) - f(x_0 - 3h)}{h^3}$$

$$= \frac{f(\pi/4) - 3f(\pi/4 - 0.15) + 3f(\pi/4 - 2(0.15)) - f(\pi/4 - 3(0.15))}{(0.15)^3}$$

$$= -0.844664136$$

$f'''(x) = -\cos x$

$$E\% = \left| \frac{-\cos(\pi/4) - (-0.84664136)}{-\cos \pi/4} \right| \times 100\ \%$$

$$= 19.45354773\ \%$$

* Primera diferencia hacia atrás

$$f^{iv}(x) = \frac{f(x_0) - 4f(x_0 - h) + 6f(x_0 - 2h) - 4f(x_0 - 3h) + f(x_0 - 4h)}{h^4}$$

$$\frac{f(\pi/4) - 4f(\pi/4 - 0.15) + 6f(\pi/4 - 2(0.15)) - 4f(\pi/4 - 3(0.15)) + f(\pi/4 - 4(0.15))}{(0.15)^4}$$

$= -5,403.912262$

$f^{iv} = \operatorname{sen} x$

$$E\% = \left| \frac{\operatorname{sen}(\pi/4) - (-5,403.912262)}{\operatorname{sen} \pi/4} \right| \times 100$$

$= 5440461.9369 \ \%$