University of Navarra

FN-558-E
0-208-037

The Essential Financial Toolkit*

Tool 4 – Diversification and Correlation

This article discusses the issue of diversification, something that academics preach and most investors practice. It is often said that putting all your eggs in one basket is not a good strategy. Diversification, at the end of the day, consists of following that simple advice when building investment portfolios, a process in which the correlation between assets plays a critical role.

Witty Professor (*WP*): We discussed in our last session two measures of risk, standard deviation and beta, and although my plan for today was to discuss the relationship between beta and required returns, I'm going to change that plan a bit.

Insightful Student (*IS*): Why?

WP: Because some of you wanted to discuss the concept of diversification a bit further, and so we'll do that today. And we'll postpone our discussion about the relationship between beta and required returns for our next session.

IS: Sounds good. I actually did have some doubts about the concept of diversification, particularly regarding the role of the correlation coefficient, which you didn't mention and I never seem to quite understand.

* All calculations in this note have been performed in Excel and the results reported are taken directly from the spreadsheet. If you want to reproduce the numbers discussed accurately and avoid rounding errors, you should also perform all calculations in a spreadsheet.

This technical note was prepared by Professor Javier Estrada. Gabriela Giannattasio provided research assistance. September 2008.

Last edited: 10/10/08

WP: That's a very common doubt, and we'll certainly talk about that today. But let's walk before we run, and so first consider the returns of the three hypothetical assets in Table 1. It shows their returns over 10 periods, as well as their arithmetic mean and standard deviation, both of which you know how to calculate by now.

Table 1

Period	Asset 1	Asset 2	Asset 3
1	25.0%	21.3%	32.5%
2	5.0%	24.3%	22.5%
3	22.5%	21.6%	31.3%
4	6.0%	24.1%	23.0%
5	17.5%	22.4%	28.8%
6	4.0%	24.4%	22.0%
7	31.0%	20.4%	35.5%
8	5.5%	24.2%	22.8%
9	24.0%	21.4%	32.0%
10	4.0%	24.4%	22.0%
AM	**14.5%**	**22.9%**	**27.2%**
SD	**10.0%**	**1.5%**	**5.0%**

IS: Yes, we do! So, what's the story with these three assets?

WP: Well, since diversification is all about combining assets into a portfolio, let's consider a combination between assets 1 and 2. Let's assume that given whatever capital we have to invest, we put 13% of our money in asset 1 and 87% in asset 2. How would you calculate, period by period, the return of that portfolio?

IS: That's easy. For period 1, for example, the return of the portfolio (R_p) would be

$$R_p = (0.13)(0.250) + (0.87)(0.213) = 21.8\%$$

and the only thing that would change from period to period would be the return delivered by each asset.

WP: Great. Now, what do you think we would get if we do the same thing for the second period?

IS: I don't know, but we can just calculate it. The return of the portfolio in the second period would be

$$R_p = (0.13)(0.050) + (0.87)(0.243) = 21.8\%$$

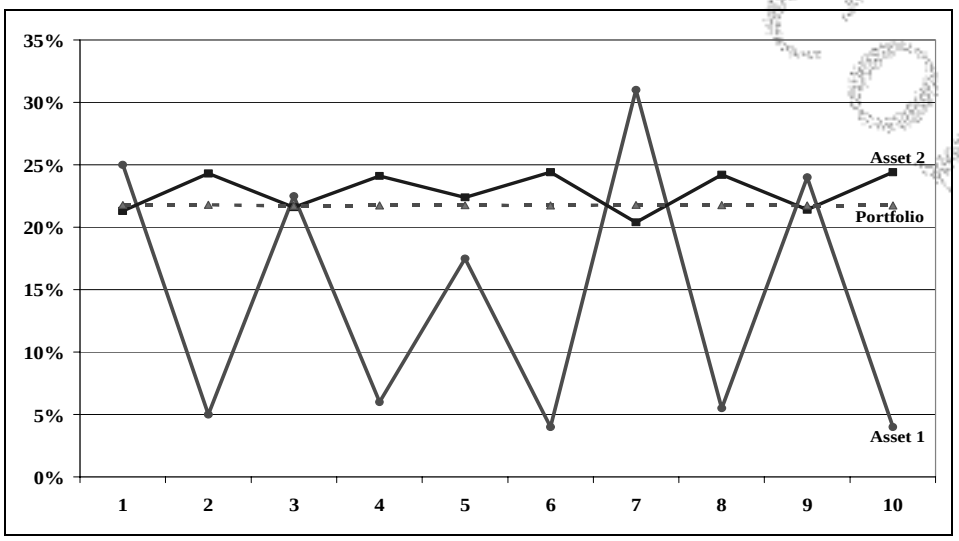
which is kind of funny; that's the same return as in the previous period!

WP: Well, it may be funny, but it's also right. And if you find that curious, let me surprise you: if you calculate period by period the return of a portfolio invested 13% in asset 1 and 87% in asset 2, you will find that the return of the portfolio, give or take a small rounding error, is always ... 21.8%!

IS: No way!!! You're cheating somewhere!

WP: I'm not. You can crunch the numbers yourself if you don't believe me, but let me save you some time. Take a look at Figure 1.

Figure 1



IS: I'm looking but I'm not sure I understand it.

WP: Well, the picture shows the returns of assets 1 and 2 over the ten periods we're considering, as well as the return of the 13-87 portfolio. And as you can see, the return of this portfolio is just a straight line at 21.8%.

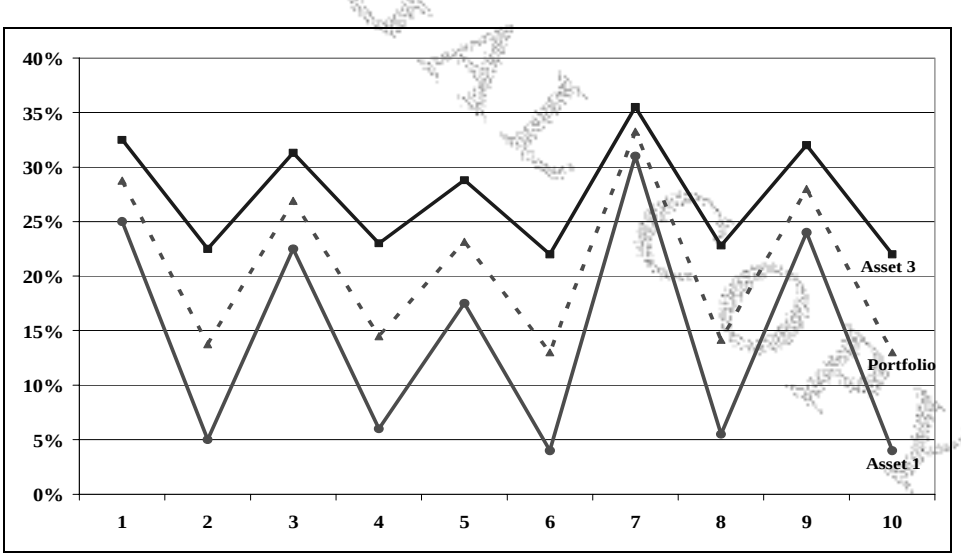
IS: I can hardly believe it! We combine two risky assets and we end up with a riskless portfolio?! What's going on here?!

WP: Well, let's rule out cheating to start with! And before I answer your question, let me ask you to consider another combination of assets. Consider now a portfolio invested 50% in asset 1 and 50% in asset 3. If we were to calculate the return of this portfolio over the ten periods we're considering, what do you think we would get?

IS: Please don't tell me another portfolio with a riskless 21.8% return!

WP: Nope, quite different from that actually. Take a look at Figure 2, which plots the returns of asset 1, asset 3, and the 50-50 portfolio.

Figure 2



IS: That portfolio is far from riskless! It's actually pretty volatile!

WP: It is. And if you're wondering what determines that when combining assets 1 and 2 we end up with a riskless portfolio, and when combining assets 1 and 3 we end up with a volatile portfolio, you're asking the right question. Can you guess what it all comes down to?

IS: No idea!

WP: To the correlation coefficient you have doubts about, so let's talk about that.

IS: Great! Let's see if I can finally understand what that coefficient is all about!

WP: It's actually fairly simple. The correlation coefficient is a magnitude that measures the sign and the strength of the relationship between two variables. When two variables tend to move in the same direction, this coefficient is positive; and when they tend to move in opposite directions, this coefficient is negative.

IS: That's pretty simple. But how do we know if the relationship is weak or strong?

WP: Formally speaking, the correlation coefficient can take any value between –1 and 1. In the first case, the two variables have a perfect negative relationship; in the second case, a perfect positive relationship.

IS: I understand that by a positive relationship you mean that the variables tend to move in the same direction, and by a negative relationship that they tend to move in opposite directions. But what do you mean by a perfect relationship?

WP: I mean that in the extreme cases in which two variables have a correlation of –1 or 1, if I know the value of one variable, I can *exactly* determine the value of the other. If the correlation between any two variables x and y is –1, then they are

linked by a relationship of the type $y = a - b \cdot x$; and if the correlation between them is 1, then they are linked by a relationship of the type $y = a + b \cdot x$. Then, in both cases, if I know x, I can exactly determine y.

IS: I see. So do you mean to say that the correlation coefficient measures the strength and sign of *linear* relationships?

WP: Strictly speaking, yes, although most people seem to forget this. A high and a low correlation are informally referred to as characterizing a strong and a weak relationship, although strictly speaking they characterize a strong and a weak *linear* relationship.

IS: So let me see if I understand. A positive correlation indicates that two variables tend to move in the same direction; a negative correlation that they tend to move in opposite directions; a correlation of –1 that two variables are linked by a perfect, negative linear relationship; a correlation of 1 that they are linked by a perfect, positive linear relationship; and in these last two cases, if I know the value of one variable, I can exactly determine the value of the other. Is that right?

WP: Perfectly correct!

IS: OK, but it seems to me that those correlations of –1 and 1 are rather hypothetical, aren't they? I mean, I cannot imagine any two financial variables that would be related in such a neat, perfect way.

WP: You're absolutely right. But understanding those extreme values is still useful, even if we don't expect to find financial variables that would be characterized by those extreme correlations.

IS: Why?

WP: Because for any two financial variables you may be interested in, the closer their correlation is to –1 or to 1, the stronger the (linear) relationship between them would be. On the other hand, the more their correlation departs from –1 or 1 and the closer it gets to 0, the weaker the (linear) relationship between them would be. And if their correlation is 0, there would simply be no (linear) relationship between them.

IS: So, given what you're saying, just by taking a casual look at Figures 1 and 2, we should be able to conclude that assets 1 and 2 are strongly negatively correlated, and assets 1 and 3 are strongly positively correlated, right?

WP: Exactly! In fact, we can even make a stronger statement. The correlation between assets 1 and 2 is a perfect –1, and the correlation between assets 1 and 3 is a perfect 1.

IS: How do you know that?

WP: Well, I know because those assets are hypothetical, remember? So when I came up with their returns, I made sure that those were the correlations between them.

But if you don't believe me, you can calculate the correlations and check for yourself.

IS: But you haven't told us how to calculate correlations!

WP: You're right. I'll say a couple of things about that later on but right now let me stress something very important that follows from Figures 1 and 2: *The lower the correlation between two assets, the more you gain by combining them.*

IS: Why's that?

WP: Well, we won't get into any math here, but I could formally show you that as long as two assets have a correlation of –1, you can always find a specific proportion of your money to invest in each asset so that you can completely eliminate the risk of your portfolio. And as far as risk reduction goes, it obviously doesn't get any better than that!

IS: But you said before that a correlation of –1 is possible in theory but not really in practice, right?

WP: Right, but it's still useful to know what is the best-case scenario in terms of diversification, isn't it? It's useful to know that when two assets have a correlation of –1, you can always combine them and find a portfolio with zero volatility, even if you can't find two assets with that correlation in practice. Similarly, it is useful to know that when two assets have a correlation of 1, there are no diversification gains from combining them.

IS: Why not?

WP: Well, think about it. If you form a portfolio with two assets that move in the same direction in a fully synchronized way, your portfolio will move in exactly that way. Just take a look at Figure 2 for confirmation. What's the gain from combining assets 1 and 3?

IS: None, I see your point. You're saying that when two assets have a correlation of 1, there's nothing to gain from combining them; and when two assets have a correlation of –1, there's the most to gain from combining them. Then, it necessarily follows that the lower the correlation between two assets, the more diversification benefits we get from combining them, right?

WP: That's right. The closer we get to a correlation of 1, the lower the diversification benefits; and the closer we get to a correlation of –1, the higher the diversification benefits. And importantly, make sure you avoid a common mistake: you don't obtain diversification benefits only when the correlation between two assets is negative; *you obtain diversification benefits as long as the correlation between two assets is lower than 1*. The lower the better, but even positive correlations, as long as they are lower than 1, produce diversification benefits.

IS: I think I'm beginning to understand the 'mysterious' correlation coefficient!

WP: That's good, so let me make another important and related point. Whenever you combine two assets with a correlation equal to 1, the volatility of your portfolio will simply be the weighted average of the volatilities of the two assets in the portfolio.

IS: Wait! What do you mean by a weighted average?

WP: I mean that if we put 30% of our money in asset 1 and 70% of our money in asset 3, then the volatility of our portfolio (SD_p) would be

$$SD_p = (0.30)\cdot SD_1 + (0.70)\cdot SD_3$$

where SD_1 and SD_3 denote the volatility of assets 1 and 3. Similarly, if we put 60% of our money in asset 1 and 40% in asset 3, the volatility of our portfolio would be

$$SD_p = (0.60)\cdot SD_1 + (0.40)\cdot SD_3$$

and the same for any other two weights.

IS: OK, got it. If we combine two assets with a correlation equal to 1, the volatility of the portfolio will simply be equal to the weighted average of the volatilities of the two assets in the portfolio. Please continue.

WP: Well, remember that a correlation of 1 was the worst case scenario in terms of diversification. It then follows that as long as we combine two assets with a correlation lower than 1, then the volatility of the portfolio will be *lower* than the weighted average of the volatilities of the two assets in the portfolio.

IS: That sounds wonderful!

WP: Well, it's just the 'magic' of diversification! As long as two assets are not perfectly, positively correlated, we can combine them and the volatility of the resulting portfolio will be lower than the weighted average of the volatilities of the two assets in the portfolio. And of course, the lower the correlation between two assets, the lower the volatility of the portfolio will be, relative to the weighted average of the volatilities of the two assets.

IS: In other words, if our goal is risk reduction, the lower the correlation between two assets, the more we gain by combining them, right?

WP: That's exactly right, and it brings us to a very important point: don't think of the correlation coefficient as some sort of statistical magnitude with little practical importance. It should be obvious from our discussion that this coefficient is *critically* important from a *practical* point of view. You can't really build a portfolio properly if you ignore the correlations between the assets in the portfolio.

IS: I fully agree with that by now. What I find a bit strange, though, is that we're only talking about diversification and risk reduction, but surely investors are interested in goals beyond risk reduction. Am I wrong about that?

WP: You're not; in fact, you're absolutely right. I framed the whole discussion in terms of risk reduction to avoid mixing different goals. So let me ask you, what do you think would be other plausible goals for investors to have?

IS: Well, I can think of at least one other: to maximize returns for a desired level of risk.

WP: Very good, and here's the good news: everything we said about the relationship between correlations and risk reduction applies to the relationship between correlations and return maximization for a desired level of risk. In short, the lower the correlation between two assets, the higher the returns we can get for a desired level of risk.

IS: What about risk-adjusted returns?

WP: What about them?

IS: Well, it seems to me that investors don't really want to just minimize risk; if they did, they'd put all their money safely in a bank, with zero risk and a very low return; but most investors don't do that. On the other hand, it seems to me that investors don't really want to just maximize returns; if they did, they'd have their portfolios loaded with very risky assets like Internet stocks or emerging markets stocks; but again most investors don't do that. So it seems to me that investors want the best balance between risk and return; they want the highest possible returns at every level of risk. Does that sound plausible?

WP: It's more than plausible; it's absolutely right. We will have a full session to discuss the issue of risk-adjusted returns, so without formally defining them, now let me say this: everything we said about the relationship between correlations and risk reduction also applies to the relationship between correlations and the maximization of risk-adjusted returns. In short, the lower the correlation between two assets, the higher the risk-adjusted returns we can get.

IS: And that means that the lower the correlation between two assets, the higher the returns we can get per unit of risk borne, right?

WP: Right again!

IS: I think I finally understand the correlation coefficient and its relationship to portfolio diversification! But there's one thing I'd like to know a bit more about. You mentioned that correlations of –1 and 1 are hypothetical and not to be expected between financial variables. Can you give us some examples of actual, observed correlations?

WP: I sure can. Take a look at Table 2 which shows the annual stock market returns of the US, Germany, New Zealand, Norway, and the UK, over the 1998-2007 period. The returns of these five markets are given by the MSCI indices, in dollars, and accounting for both capital gains and dividends. As you can see from the last line, the US stock market is positively and strongly related to the

stock markets of Germany and the UK, and positively but rather weakly related to the stock markets of New Zealand and Norway.

Table 2

Year	USA	Germany	New Zealand	Norway	UK
1998	30.7%	29.9%	–21.5%	–29.7%	17.8%
1999	22.4%	20.5%	14.3%	32.4%	12.5%
2000	–12.5%	–15.3%	–33.0%	–0.4%	–11.5%
2001	–12.0%	–22.0%	9.5%	–11.7%	–14.0%
2002	–22.7%	–32.9%	26.1%	–6.7%	–15.2%
2003	29.1%	64.8%	57.8%	49.6%	32.1%
2004	10.7%	16.7%	37.5%	54.5%	19.6%
2005	5.7%	10.5%	3.2%	25.7%	7.4%
2006	15.3%	36.8%	17.7%	46.3%	30.7%
2007	6.0%	35.9%	9.8%	32.4%	8.4%
Correlation		0.89	0.21	0.37	0.89

IS: I see. And the fact that all four correlations are positive indicates that the US and all these four markets tend to move in the same direction, right?

WP: Right. In fact, if you look over a long enough period of time, you will find that all stock markets tend to be positively correlated to each other. Although they may behave erratically, individually and with respect to each other in the short term, in the long term they all tend to go up, which translates into positive correlations.

IS: So you're saying that all financial assets are positively correlated?

WP: No, that's a stronger statement than mine. I'm saying that if you look at several stock markets over a long period of time, they all tend to be positively correlated. Similarly, if you look at stocks within any given market over a long period of time, they all tend to be positively correlated. But that does not apply to all assets. Gold, for example, is known to have been negatively correlated to stock markets over some periods of time. Which actually explains why, when stock markets are weak, some people tend to buy gold; they expect gold to go up when stock markets go down. And although it hasn't worked like that all the time, it has worked like that some of the time.

IS: So, based on our previous discussion, gold would be an ideal addition to a portfolio of stocks, right? Because if most correlations between stocks are positive, but gold is negatively correlated with stocks, at least part of the time, then it would enable us to reduce risk a lot, or to enhance returns substantially for a desired level of risk, or to significantly improve risk-adjusted returns. Is that a fair statement?

WP: It is. In fact, even when the correlation between stocks and gold is not negative, it is positive but fairly low, in which case your previous statement is still true.

IS: I see we're running out of time, so I have one last question. Is it fair to say that investing in mutual funds is popular because they provide diversification?

WP: There are many reasons for buying mutual funds instead of individual stocks, but you're right, obtaining wide diversification at a low cost is one of the main reasons. Just think that by buying one share of a mutual fund, you're buying into a well-diversified portfolio of stocks in any industry, country, or region you may want exposure to.

IS: I see. And may be one last question?! What about the calculation of correlations?

WP: Well, I'm only going to go as far as saying that, as usual, Excel can do that for you in the blink of an eye. The relevant command is "**correl(range1, range2)=**" where 'range1' and 'range2' are the ranges where you have the returns of the two variables you're considering. The Excel Help can give you more details, and since we're running out of time that's as far as we go here.

IS: Wrap up time?

WP: Unless you want to spend your break between classes in class, yes! Here we go. Diversification is one of the cornerstones of modern financial theory. All smart investors should hold widely-diversified portfolios because that's what enables them to maximize risk-adjusted returns. When diversifying portfolios, the correlations between assets play a critical role. The lower the correlations, the higher the diversification benefits; that is, the more we can reduce risk; or the more returns we can get for any desired level of risk; or the higher the risk-adjusted returns we can get. And since the show must go on, it will go on, but in our next class!