CMDA-4654

Project 1

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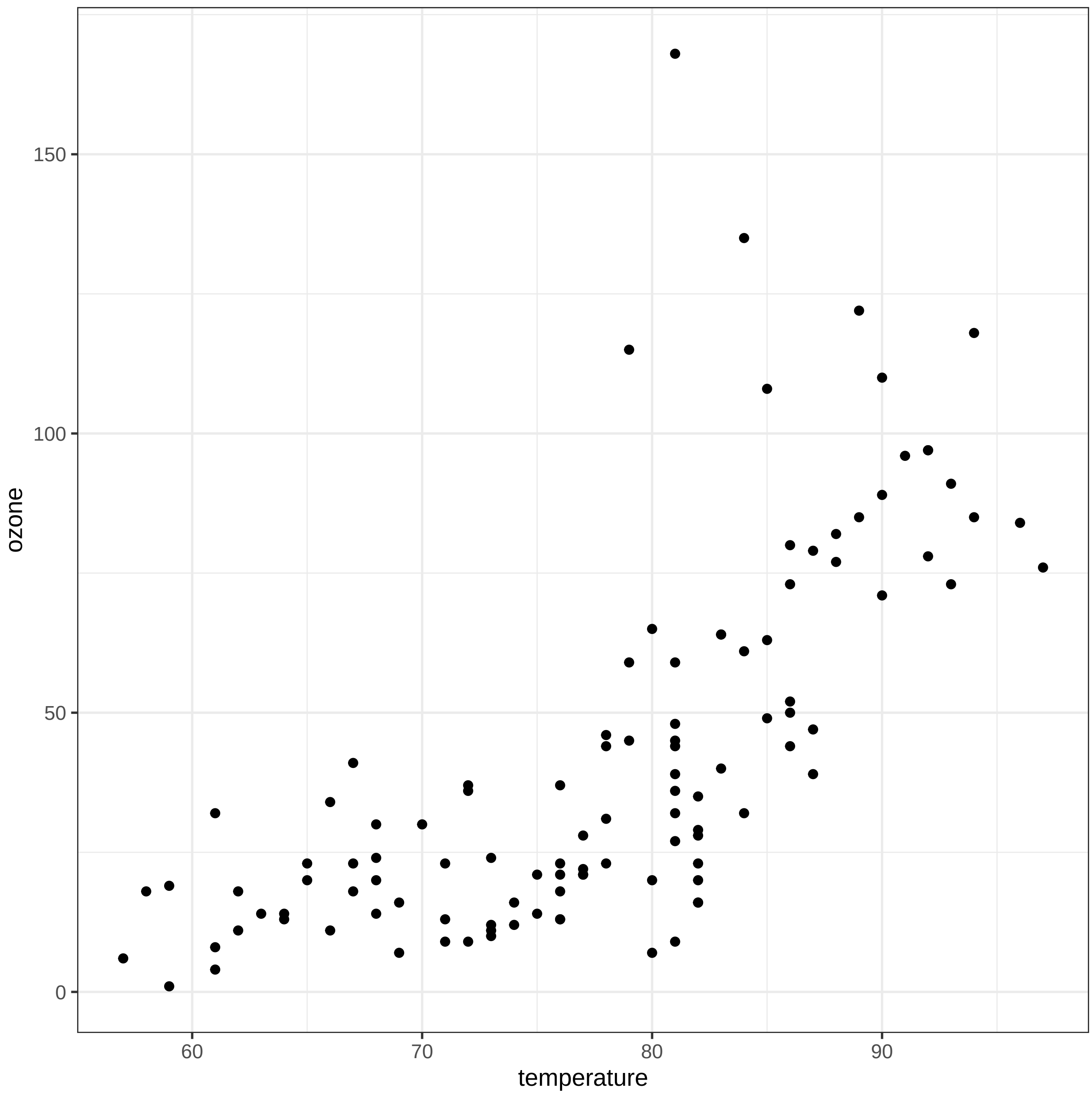
Problem 1

```r
library(dplyr)
library(ggplot2)

#setwd("C:/Users/Spencer/Downloads")
load("ozone.RData")

ggplot(ozone, aes(x = temperature, y = ozone)) + theme_bw() + geom_point()
```



Part a

Fitting polynomials of different degrees

```r
model1 <- lm(ozone ~ poly(radiation,1), data = ozone)
model2 <- lm(ozone ~ poly(radiation,2), data = ozone)
model3 <- lm(ozone ~ poly(radiation,3), data = ozone)
model4 <- lm(ozone ~ poly(radiation,4), data = ozone)
```

```r
model5 <- lm(ozone ~ poly(radiation,5), data = ozone)
model6 <- lm(ozone ~ poly(radiation,6), data = ozone)

summary(model1)
```

```

Call:
lm(formula = ozone ~ poly(radiation, 1), data = ozone)

Residuals:
    Min      1Q  Median      3Q     Max 
-48.292 -21.361  -8.864  16.373 119.136 

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)    
(Intercept)          42.099      2.974   14.15  < 2e-16 ***
poly(radiation, 1)  121.572     31.335    3.88 0.000179 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 31.33 on 109 degrees of freedom
Multiple R-squared:  0.1213,	Adjusted R-squared:  0.1133 
F-statistic: 15.05 on 1 and 109 DF,  p-value: 0.0001793
```

```r
summary(model2)
```

```

Call:
lm(formula = ozone ~ poly(radiation, 2), data = ozone)

Residuals:
    Min      1Q  Median      3Q     Max 
-40.155 -22.793  -6.438  18.061 115.117 

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)    
(Intercept)           42.099      2.832  14.864  < 2e-16 ***
poly(radiation, 2)1  121.572     29.840   4.074 8.84e-05 ***
poly(radiation, 2)2 -104.178     29.840  -3.491 0.000698 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 29.84 on 108 degrees of freedom
Multiple R-squared:  0.2104,	Adjusted R-squared:  0.1958 
F-statistic: 14.39 on 2 and 108 DF,  p-value: 2.875e-06
```

```r
summary(model3)
```

```

Call:
lm(formula = ozone ~ poly(radiation, 3), data = ozone)

Residuals:
   Min     1Q Median     3Q    Max 
-45.05 -19.44  -4.52  15.61 109.95 

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)    
(Intercept)            42.10       2.78  15.141  < 2e-16 ***
poly(radiation, 3)1   121.57      29.29   4.150 6.69e-05 ***
poly(radiation, 3)2  -104.18      29.29  -3.556 0.000562 ***
poly(radiation, 3)3   -65.93      29.29  -2.251 0.026450 *  
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 29.29 on 107 degrees of freedom
Multiple R-squared:  0.2461,	Adjusted R-squared:  0.225
F-statistic: 11.65 on 3 and 107 DF,  p-value: 1.154e-06
```

```r
summary(model4)
```

```
Call:
lm(formula = ozone ~ poly(radiation, 4), data = ozone)

Residuals:
    Min      1Q  Median      3Q     Max
-44.929 -19.519  -4.647  15.700 110.071

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)           42.099      2.793  15.072  < 2e-16 ***
poly(radiation, 4)1  121.572     29.428   4.131 7.23e-05 ***
poly(radiation, 4)2 -104.178     29.428  -3.540 0.000596 ***
poly(radiation, 4)3  -65.932     29.428  -2.240 0.027149 *
poly(radiation, 4)4    4.777     29.428   0.162 0.871356
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 29.43 on 106 degrees of freedom
Multiple R-squared:  0.2463,	Adjusted R-squared:  0.2179
F-statistic: 8.661 on 4 and 106 DF,  p-value: 4.352e-06
```

```r
summary(model5)
```

```
Call:
lm(formula = ozone ~ poly(radiation, 5), data = ozone)

Residuals:
    Min      1Q  Median      3Q     Max
-46.076 -16.978  -5.621  16.472 108.924

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)           42.099      2.801  15.029  < 2e-16 ***
poly(radiation, 5)1  121.572     29.511   4.119 7.59e-05 ***
poly(radiation, 5)2 -104.178     29.511  -3.530 0.000618 ***
poly(radiation, 5)3  -65.932     29.511  -2.234 0.027595 *
poly(radiation, 5)4    4.777     29.511   0.162 0.871718
poly(radiation, 5)5   18.759     29.511   0.636 0.526389
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 29.51 on 105 degrees of freedom
Multiple R-squared:  0.2492,	Adjusted R-squared:  0.2135
F-statistic: 6.971 on 5 and 105 DF,  p-value: 1.168e-05
```

```r
summary(model6)
```

```
Call:
lm(formula = ozone ~ poly(radiation, 6), data = ozone)

Residuals:
```

```
    Min      1Q  Median      3Q     Max
-45.079 -17.204  -5.404  16.483 109.921

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)           42.099      2.812  14.971  < 2e-16 ***
poly(radiation, 6)1  121.572     29.626   4.104 8.11e-05 ***
poly(radiation, 6)2 -104.178     29.626  -3.516  0.00065 ***
poly(radiation, 6)3  -65.932     29.626  -2.225  0.02821 *
poly(radiation, 6)4    4.777     29.626   0.161  0.87221
poly(radiation, 6)5   18.759     29.626   0.633  0.52800
poly(radiation, 6)6  -12.854     29.626  -0.434  0.66527
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 29.63 on 104 degrees of freedom
Multiple R-squared:  0.2506,    Adjusted R-squared:  0.2073
F-statistic: 5.795 on 6 and 104 DF,  p-value: 3.079e-05
```

The polynomial fit that appears to work the best is the one with degree 3.

Part b

Writing the function that carries out LOESS regression

```
# This function has the following inputs.
#
# * x - a numeric input vector
# * y - a numeric response
#
# Note span and degree are shown with their default values.
# * degree should be 1 or 2 only
# * span can be any value in interval (0, 1) non-inclusive.
#
# If show.plot = TRUE then a plot of the final fit is shown

myloess <- function(x, y, span = 0.5, degree = 1, show.plot = TRUE) {

  # Getting range of values
  xrange <- diff(range(x))

  # Checking span meets requirements and setting width
  if (between(span, 0, 1)) {
    width <- span*xrange
  }
  else {
    stop("Span must be between 0 and 1 non-inclusive")
  }

  # Getting total number of points and windows (they'll be the same)
  N_total <- length(x)
  Win_total <- length(x)

  # Allocating space for vector of each window's population
  n_points <- vector(mode = "integer", length = length(x))

  # Allocating space for vector of fitted values
  yhat <- vector(mode = "numeric", length = length(x))

  # Combining our variables in data frame
  mydata <- cbind.data.frame(x, y)
```

```r
  # Fitting each point
  for(x0 in x) {

    # Setting population of window
    sample <- subset(mydata, between(x, x0-width/2, x0+width/2))
    n <- length(sample[,1])

    # Getting weights into diagonal matrix
    weights <- (1-(abs(sample[,1] - x0)/width*2)^3)^3
    W_mat <- diag(weights, n, n)

    # Checking degree and completing our regression accordingly
    if (degree == 1) {

      X_mat <- cbind(rep(1, n), sample[,1])
      betahat <- solve( t(X_mat)%*%W_mat%*%X_mat ) %*% t(X_mat) %*% W_mat %*% sample[,2]

      # Getting fitted value
      yhat[which(x0 == x)] <-  betahat[1] + betahat[2]*x0
    }
    else if (degree == 2) {

      X_mat <- cbind(rep(1, n), sample[,1], sample[,1]^2)
      betahat <- solve( t(X_mat)%*%W_mat%*%X_mat ) %*% t(X_mat) %*% W_mat %*% sample[,2]

      # Getting fitted value
      yhat[which(x0 == x)] <-  betahat[1] + betahat[2]*x0 + betahat[3]*x0^2
    }
    else {
      stop("Degree must be set to either 1 or 2")
    }

    # Getting population of window
    n_points[which(x0 == x)] <- n
  }

  # Calculating SSE, MSE, and residual standard error
  SSE <- sum((y-yhat)^{2})
  MSE <- SSE/(N_total-2)
  RSE <- sqrt(MSE)

  # Creating plot of final fit
  loessplot <- ggplot(mydata, aes(x, y)) +
    geom_point(size = 3, alpha = 0.5, color = "grey") +
    geom_line(aes(x, yhat), color = "red", lty = 1) +
    xlab(deparse(substitute(x))) + ylab(deparse(substitute(y))) +
    ggtitle(paste("LOESS with degree =", degree,"and span =", span, sep = " "))

  # Checking whether to show plot or not
  if (show.plot == T) {
    print(loessplot)
  }

  # Returning named list
  return(invisible(list(span = span, degree = degree, N_total = N_total, Win_total = Win_total,
              n_points = n_points, SSE = SSE, RSE = RSE, loessplot = loessplot)))
}
```

Determining LOESS regression fits on the data

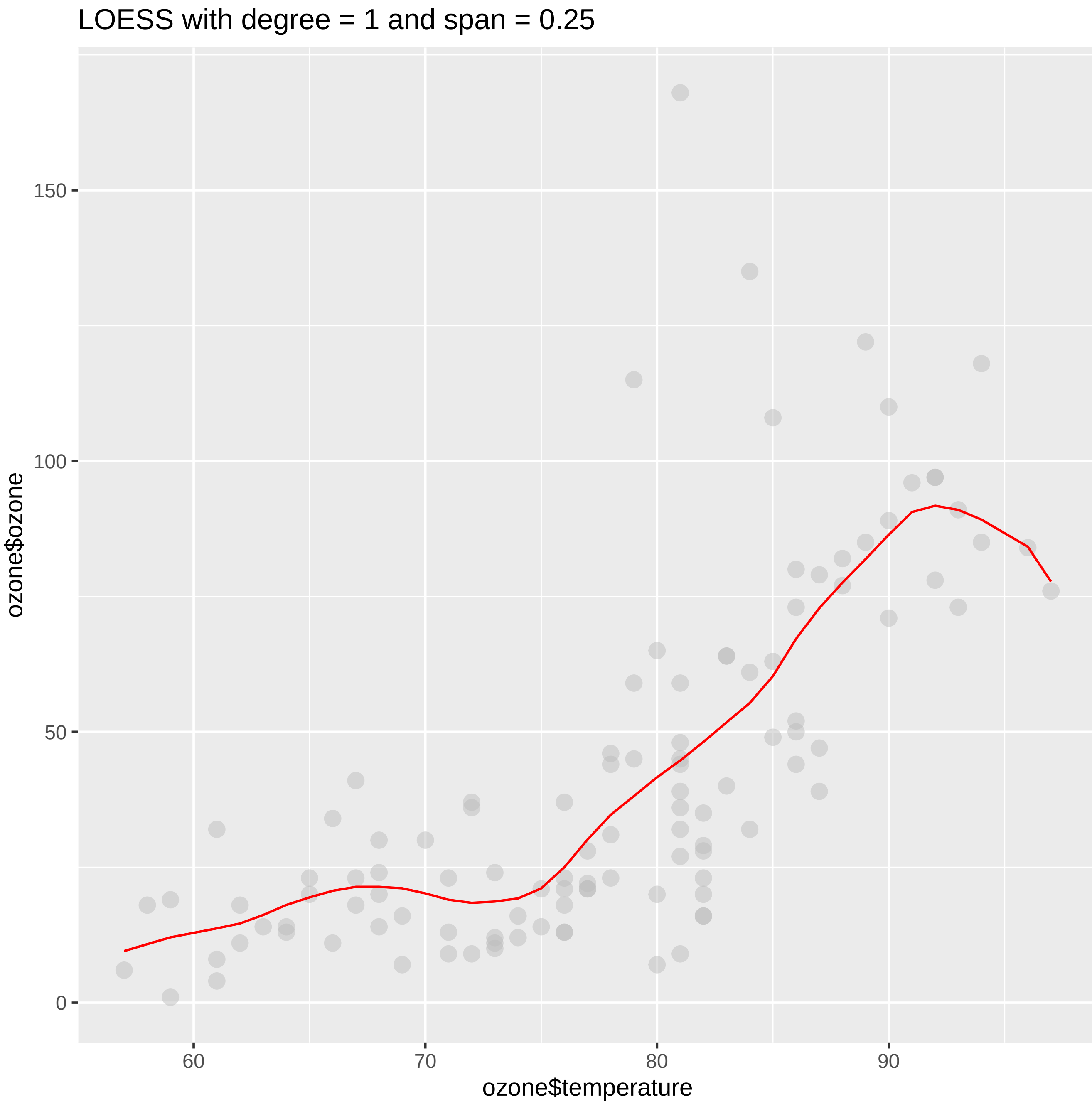
```r
# Creating an empty data frame
fit_table <- data.frame()

# Determining fits and putting info into data frame
for(j in 1:2) {
  for (i in seq(0.25, 0.75, by = 0.05)) {
    fit_table <- rbind(fit_table, c(i, j, myloess(ozone$temperature, ozone$ozone,
                                                  span = i, degree = j, show.plot = F)$RSE))
  }
}

# Changing column names
colnames(fit_table) <- c("Span", "Degree", "RSE")

# Displaying data frame
fit_table
```

```
   Span Degree      RSE
1  0.25      1 21.69723
2  0.30      1 21.84914
3  0.35      1 21.87405
4  0.40      1 21.87005
5  0.45      1 21.88484
6  0.50      1 21.92533
7  0.55      1 21.98316
8  0.60      1 22.05339
9  0.65      1 22.12531
10 0.70      1 22.18786
11 0.75      1 22.24218
12 0.25      2 20.70123
13 0.30      2 21.21788
14 0.35      2 21.65352
15 0.40      2 21.83486
16 0.45      2 21.82088
17 0.50      2 21.75230
18 0.55      2 21.71153
19 0.60      2 21.71786
20 0.65      2 21.76031
21 0.70      2 21.81288
22 0.75      2 21.86516
```

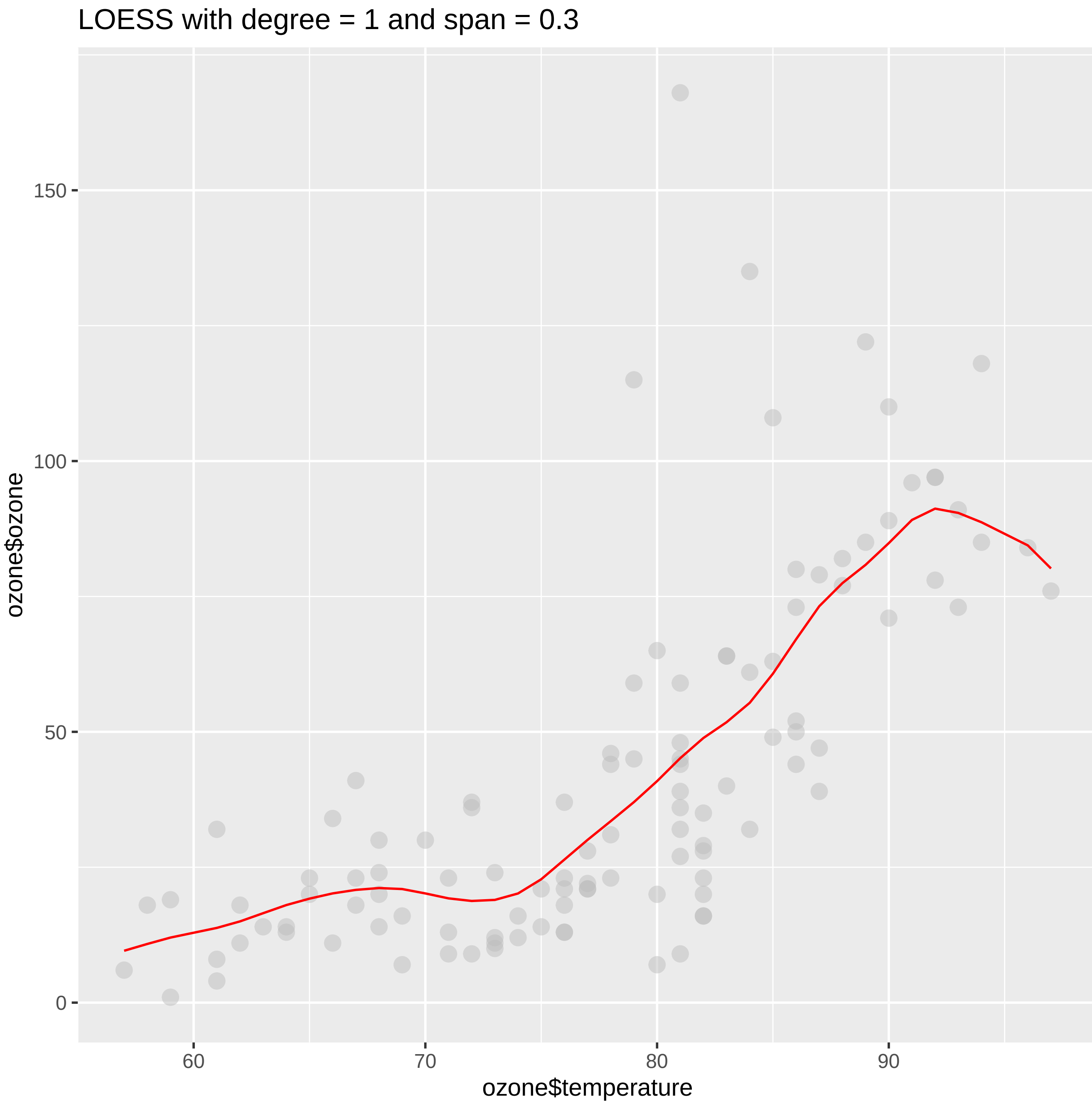
The three "best" fits with degree 1 appear to be span 0.25, 0.30, and 0.40. The three "best" fits with degree 2 appear to be span 0.25, 0.30, and 0.35.

Plotting the best fits found above

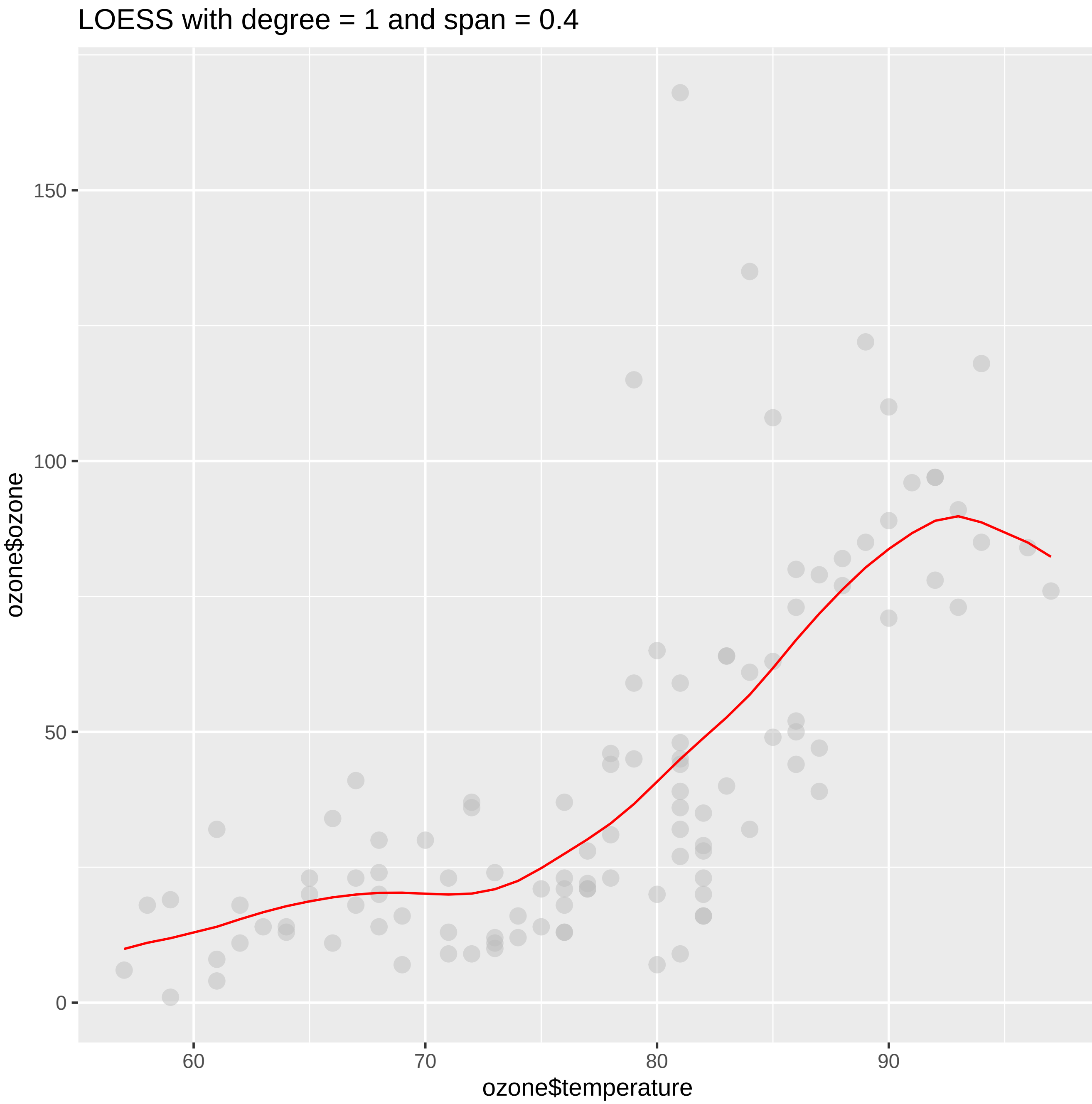
```r
myloess(ozone$temperature, ozone$ozone,
                                                  span = 0.25, degree = 1, show.plot = F)$loessplot
```



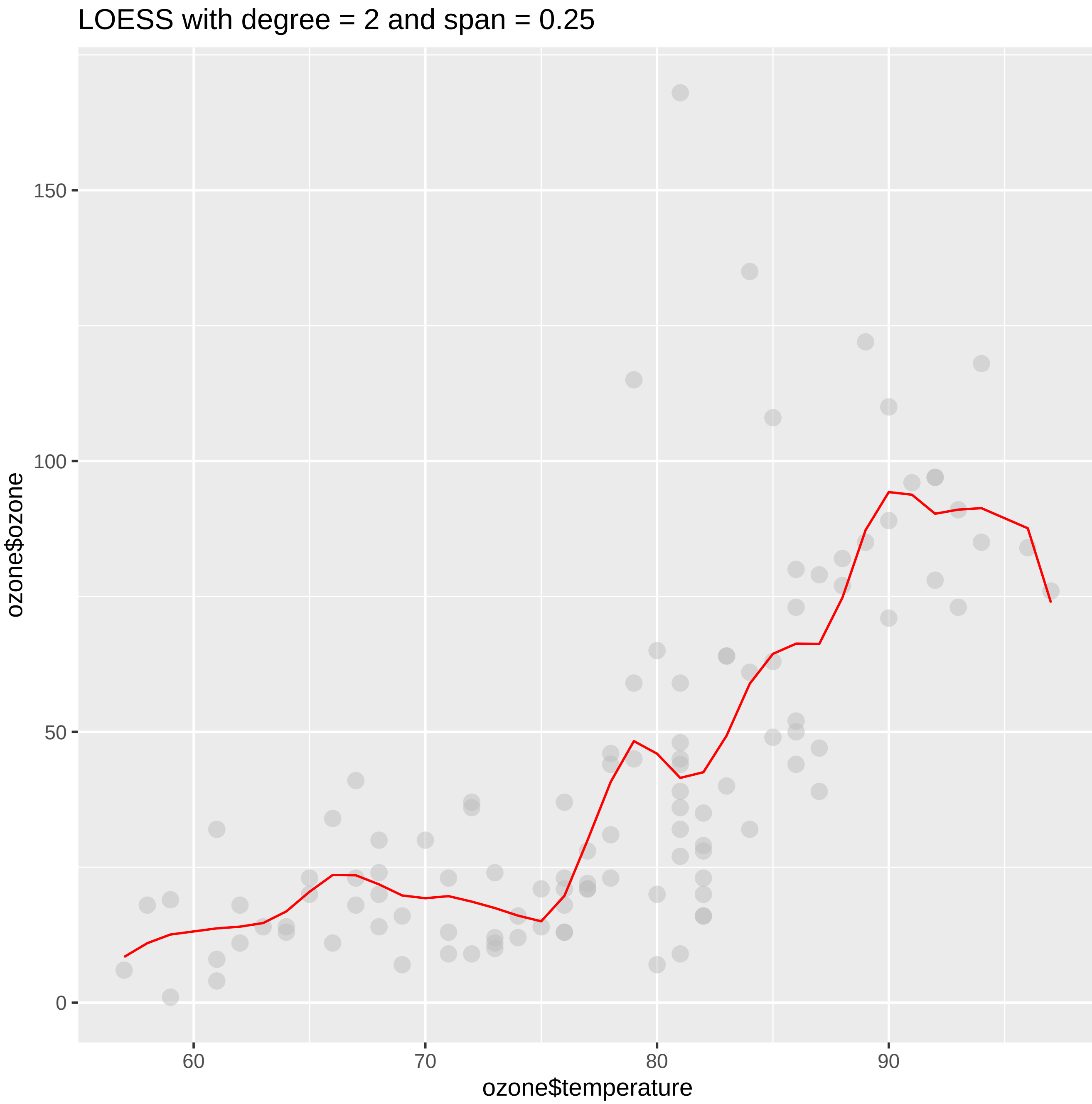

```
myloess(ozone$temperature, ozone$ozone,
                                                   span = 0.30, degree = 1, show.plot = F)$loessplot
```



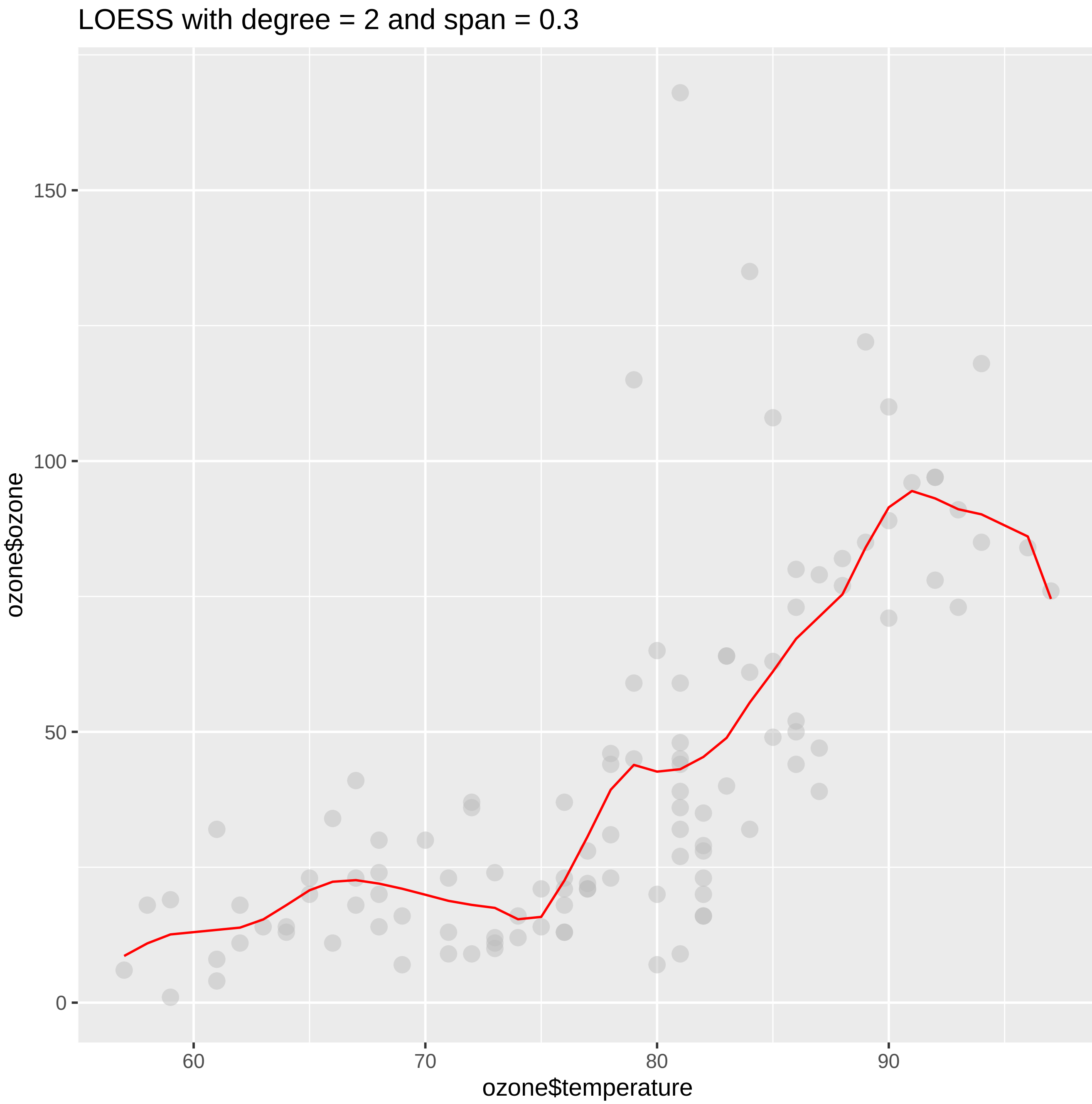

```
myloess(ozone$temperature, ozone$ozone,
                                        span = 0.40, degree = 1, show.plot = F)$loessplot
```



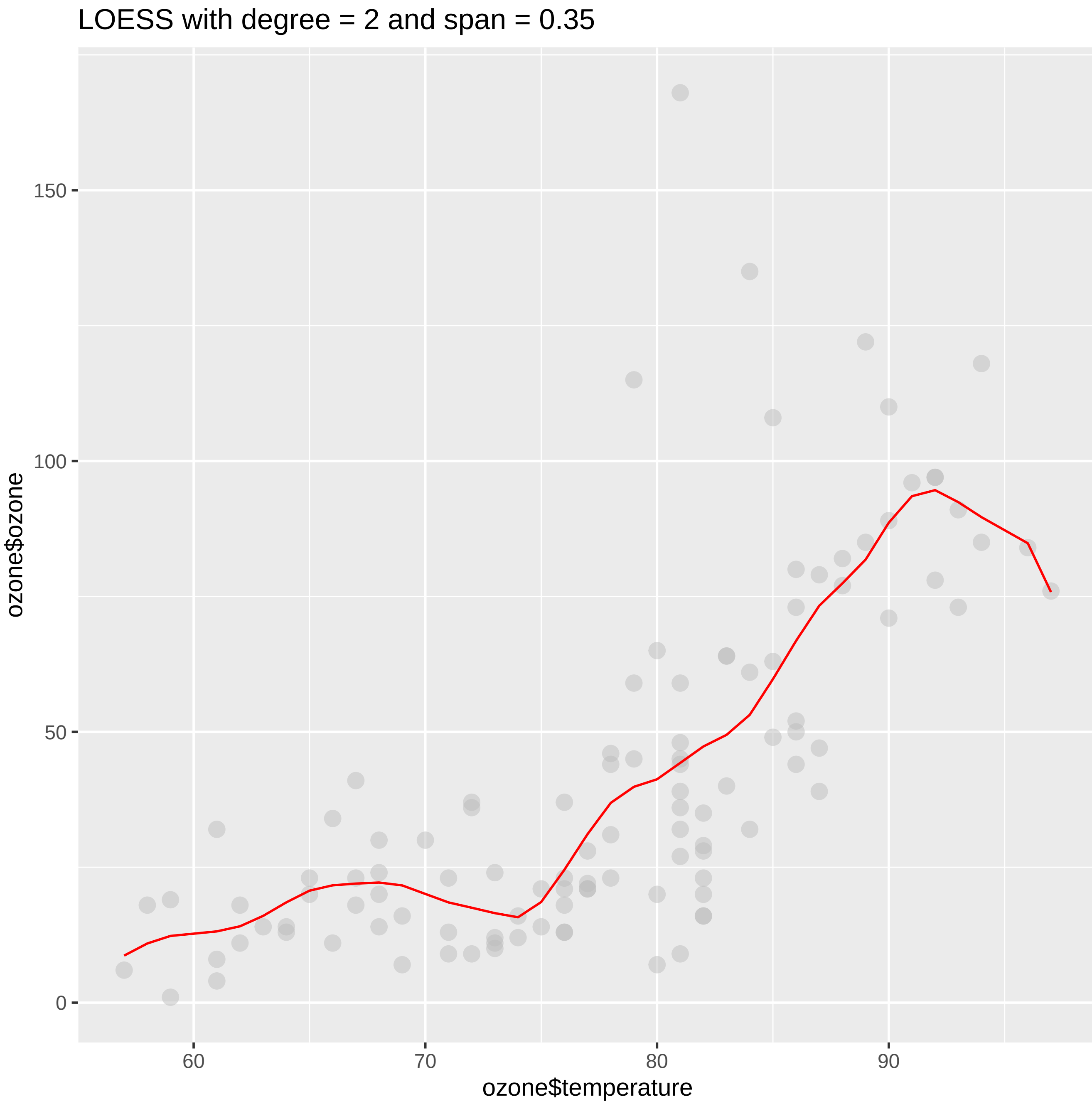

```
myloess(ozone$temperature, ozone$ozone,
                                                span = 0.25, degree = 2, show.plot = F)$loessplot
```




```
myloess(ozone$temperature, ozone$ozone,
                                                span = 0.30, degree = 2, show.plot = F)$loessplot
```




```
myloess(ozone$temperature, ozone$ozone,
                                        span = 0.35, degree = 2, show.plot = F)$loessplot
```

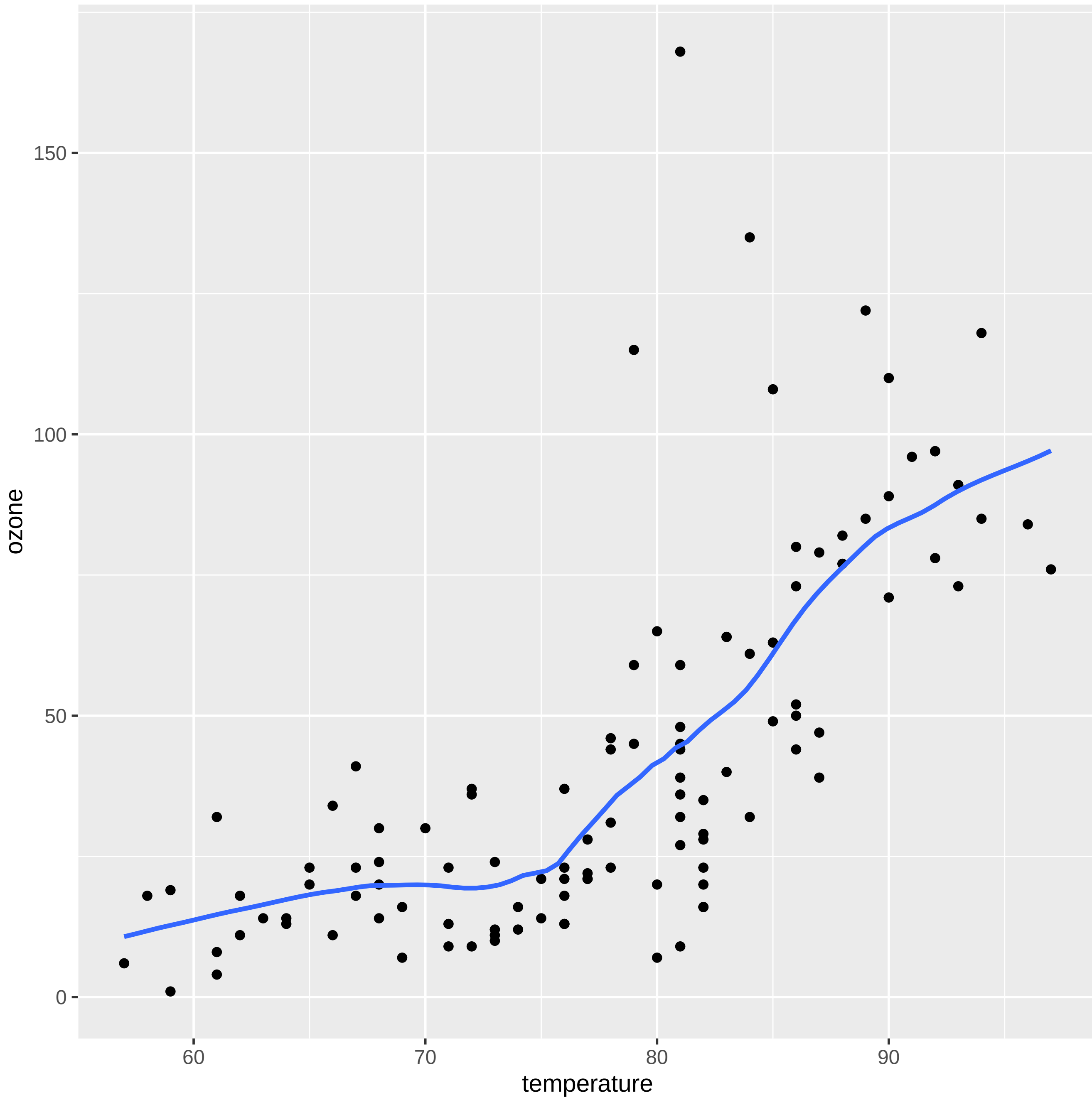


Visually inspecting the best fits compared to the 2nd and 3rd best fits, we do feel we may have over-fit the data, especially for the fits with degree 2. The model with degree 1 and span 0.4 appears to be the "best" fit.

Part c

Comparing results with built-in LOESS function

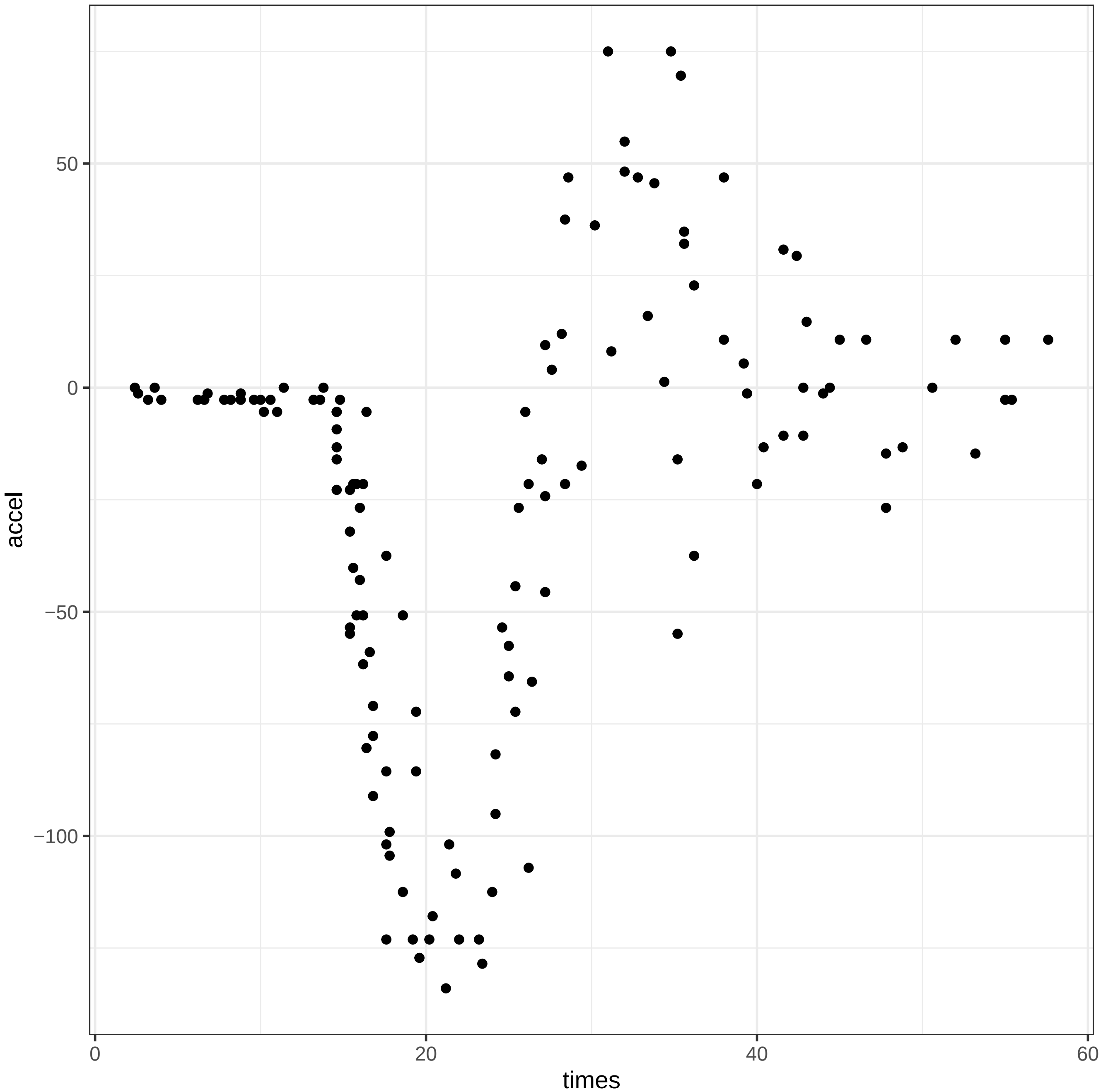
```
ozone %>% ggplot(aes(temperature, ozone)) +
  geom_point() +
  geom_smooth(method = "loess", degree = 1, span = 0.40, se = F, method.args = list(degree=1))
```



The built-in LOESS function results in a fit that does not over-fit the data, but we feel our model provides the better fit.

Problem 2

```
library(MASS)
data("mcycle")

ggplot(mcycle, aes(x = times, y = accel)) + theme_bw() + geom_point()
```



Part a

Determining LOESS regression fits on the data

```
# Creating an empty data frame
fit_table2 <- data.frame()

# Determining fits and putting info into data frame
for(j in 1:2) {
  for (i in seq(0.25, 0.75, by = 0.05)) {
    fit_table2 <- rbind(fit_table2, c(i, j, myloess(mcycle$times, mcycle$accel,
                                                    span = i, degree = j, show.plot = F)$RSE))
  }
}

# Changing column names
colnames(fit_table2) <- c("Span", "Degree", "RSE")
```

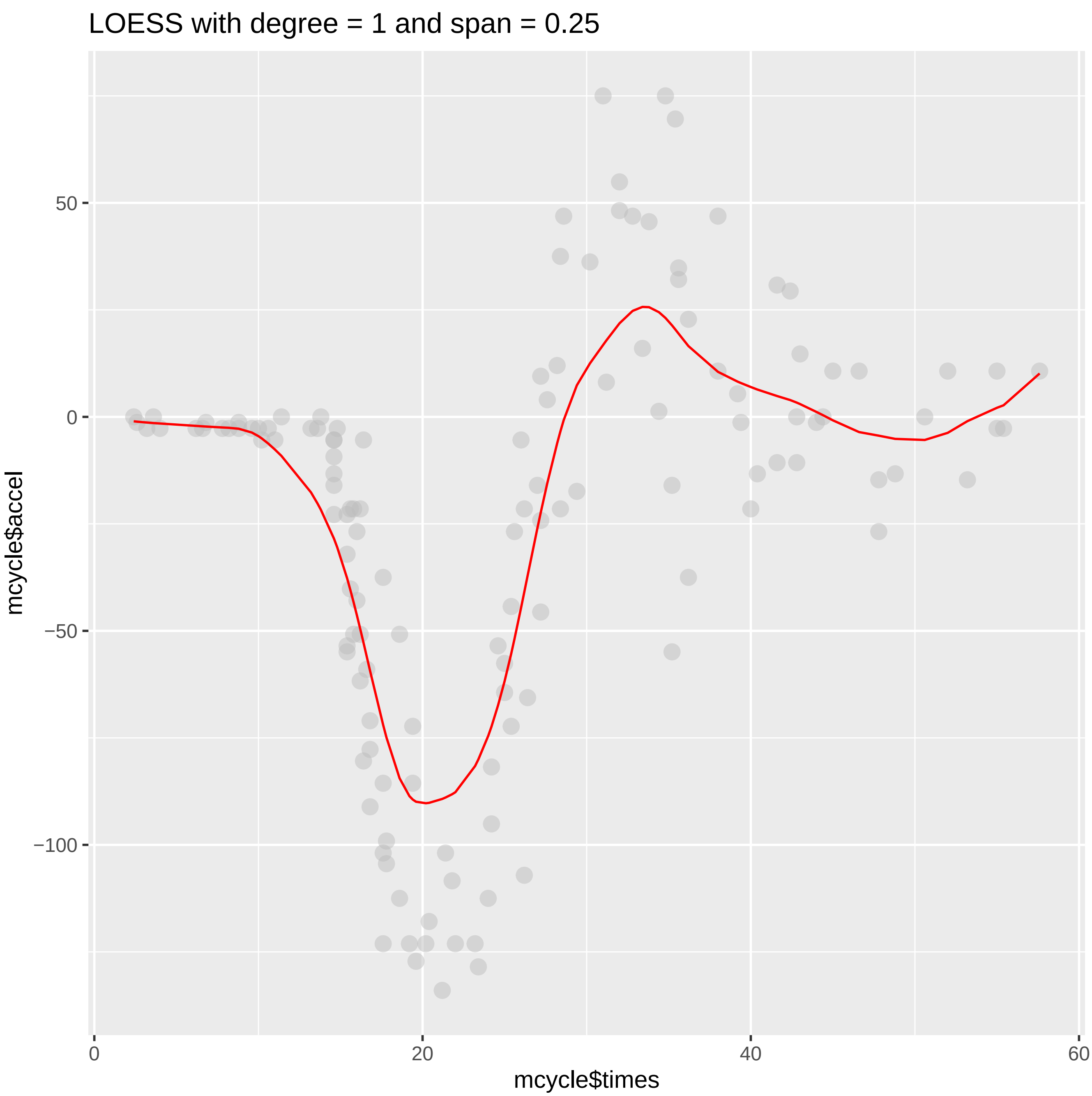
```r
# Displaying data frame
fit_table2
```

```
   Span Degree      RSE
1  0.25      1 24.54703
2  0.30      1 26.63732
3  0.35      1 28.88638
4  0.40      1 30.88313
5  0.45      1 32.57616
6  0.50      1 34.02896
7  0.55      1 35.27381
8  0.60      1 36.36153
9  0.65      1 37.32657
10 0.70      1 38.18843
11 0.75      1 38.96106
12 0.25      2 21.66747
13 0.30      2 21.99463
14 0.35      2 22.42284
15 0.40      2 23.12502
16 0.45      2 24.29479
17 0.50      2 25.87832
18 0.55      2 27.50811
19 0.60      2 29.01310
20 0.65      2 30.39499
21 0.70      2 31.68249
22 0.75      2 32.87589
```

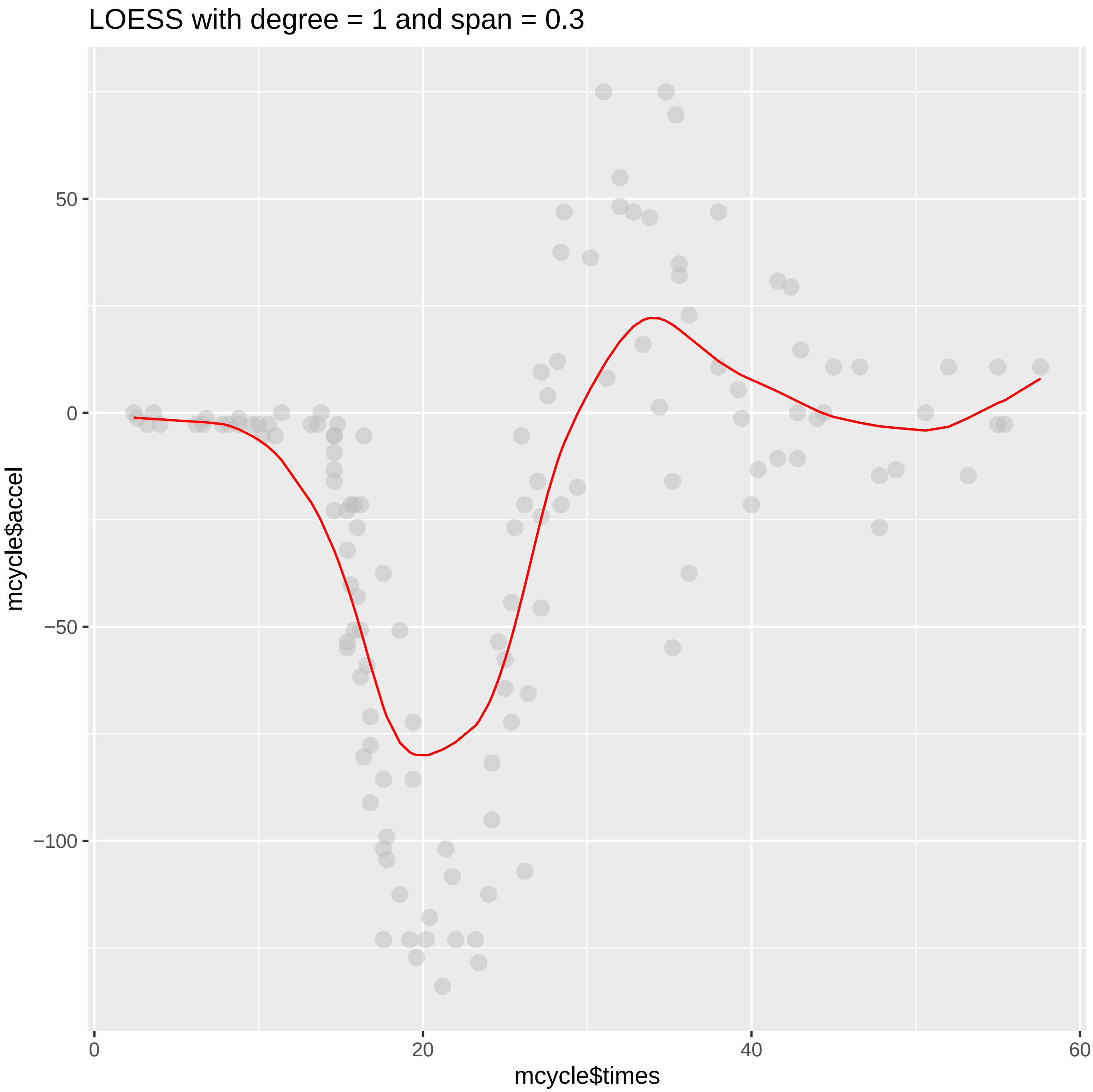
The three "best" fits with degree 1 appear to be span 0.25, 0.30, and 0.35. The three "best" fits with degree 2 appear to be span 0.25, 0.30, and 0.35.

Plotting the best fits found above

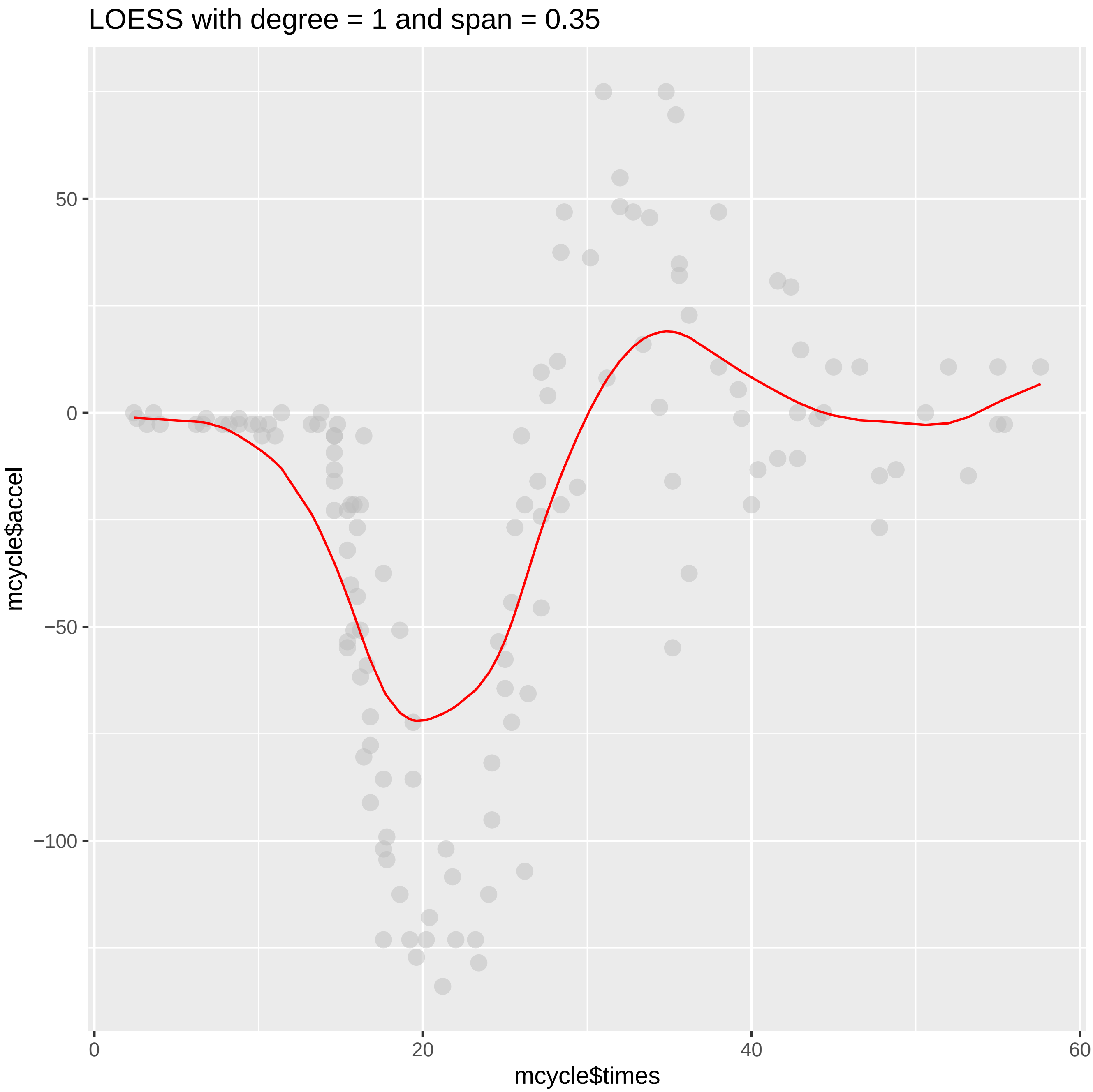
```r
myloess(mcycle$times, mcycle$accel,
                                                  span = 0.25, degree = 1, show.plot = F)$loessplot
```



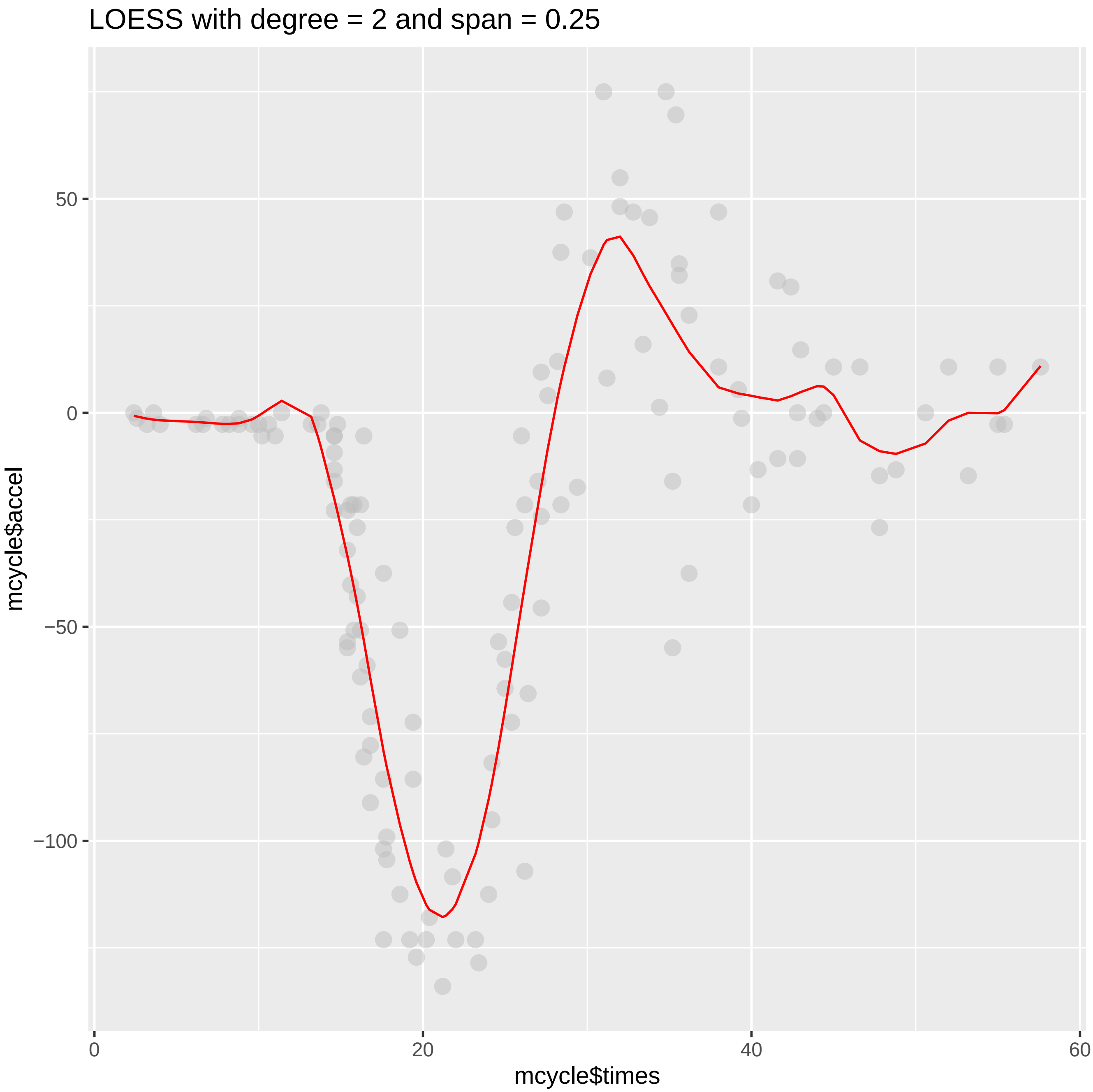

```
myloess(mcycle$times, mcycle$accel,
                                    span = 0.30, degree = 1, show.plot = F)$loessplot
```



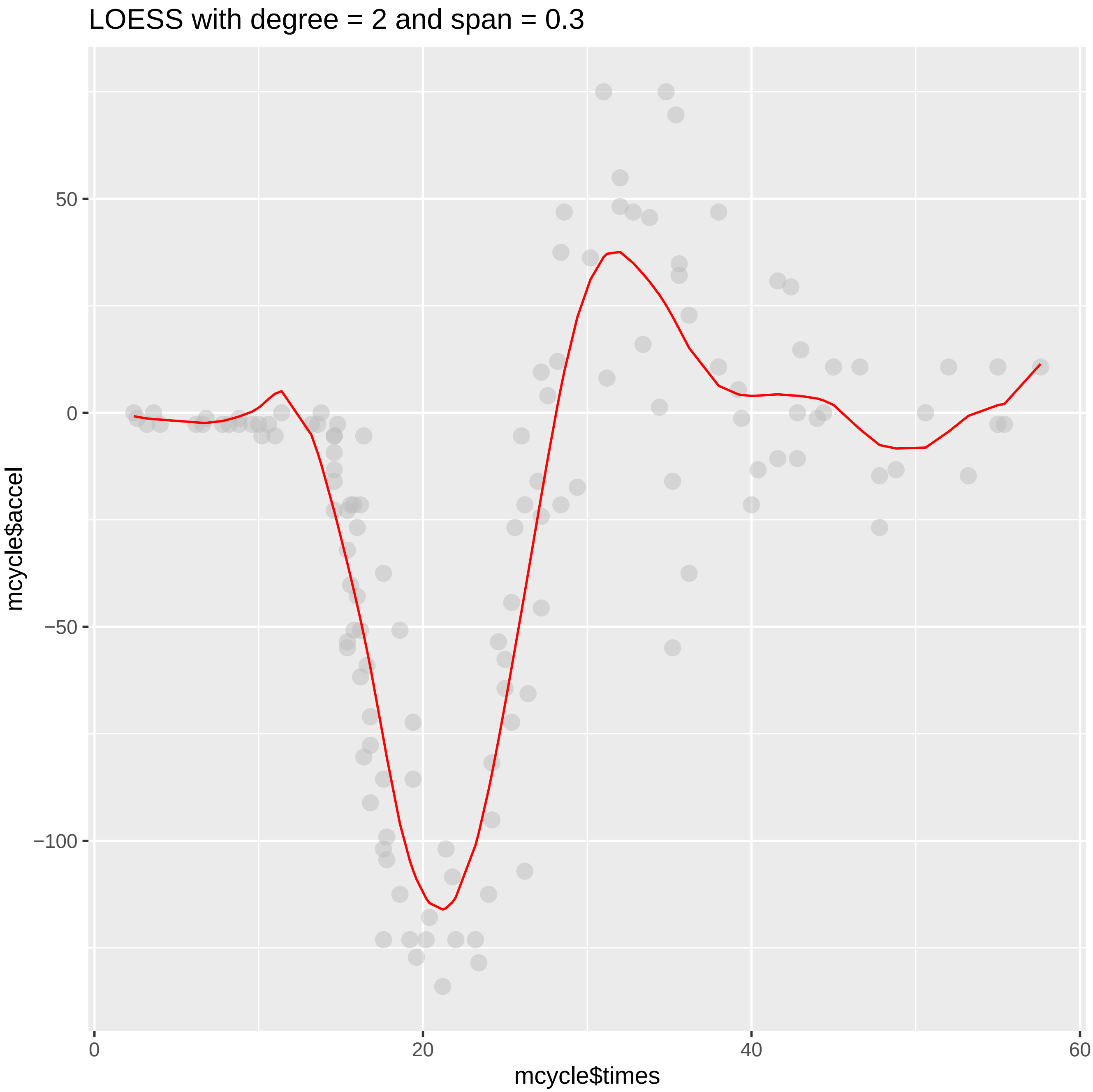

```
myloess(mcycle$times, mcycle$accel,
                                            span = 0.35, degree = 1, show.plot = F)$loessplot
```



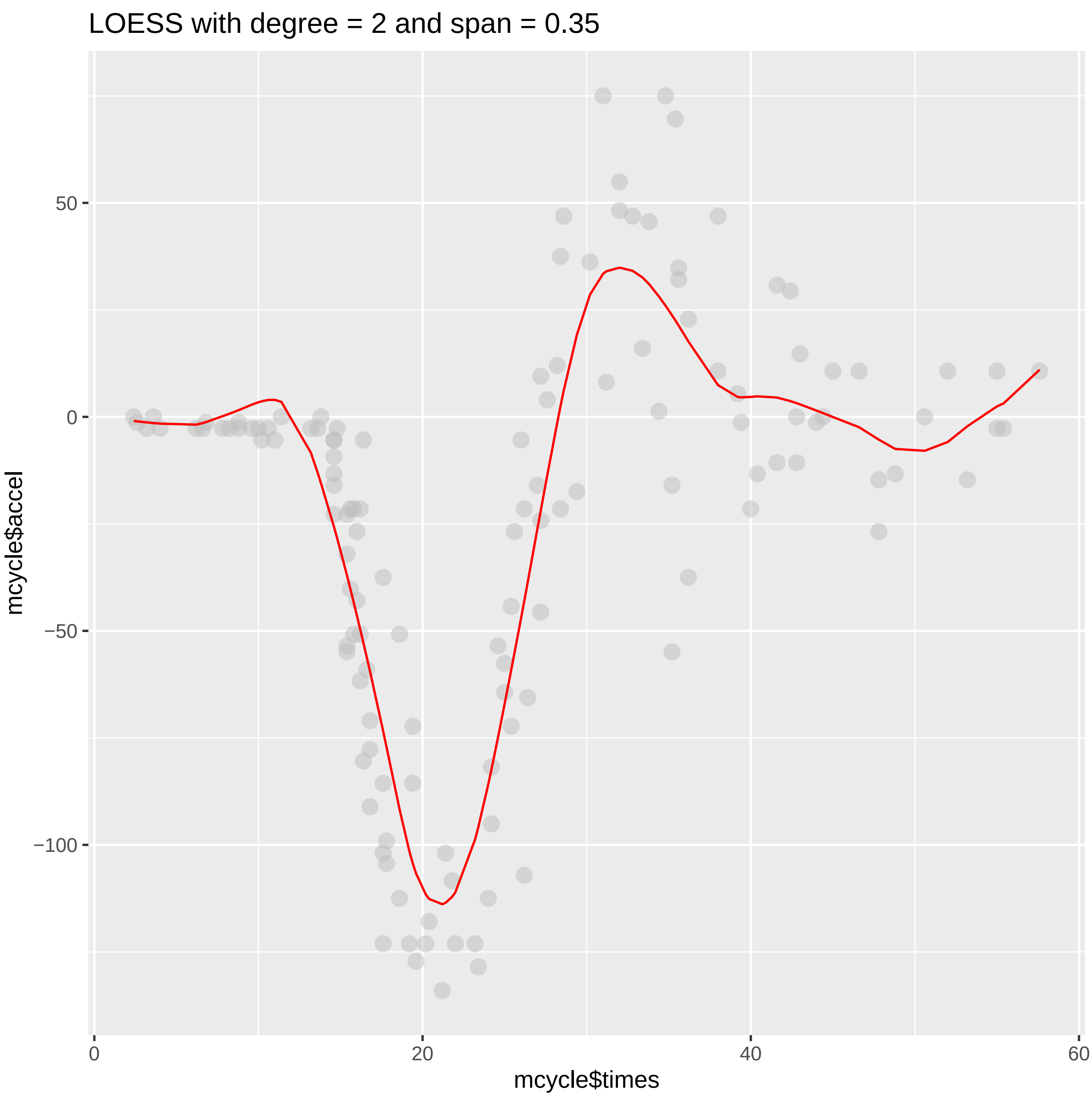

```
myloess(mcycle$times, mcycle$accel,
                              span = 0.25, degree = 2, show.plot = F)$loessplot
```




```
myloess(mcycle$times, mcycle$accel,
                                        span = 0.30, degree = 2, show.plot = F)$loessplot
```




```
myloess(mcycle$times, mcycle$accel,
                                        span = 0.35, degree = 2, show.plot = F)$loessplot
```

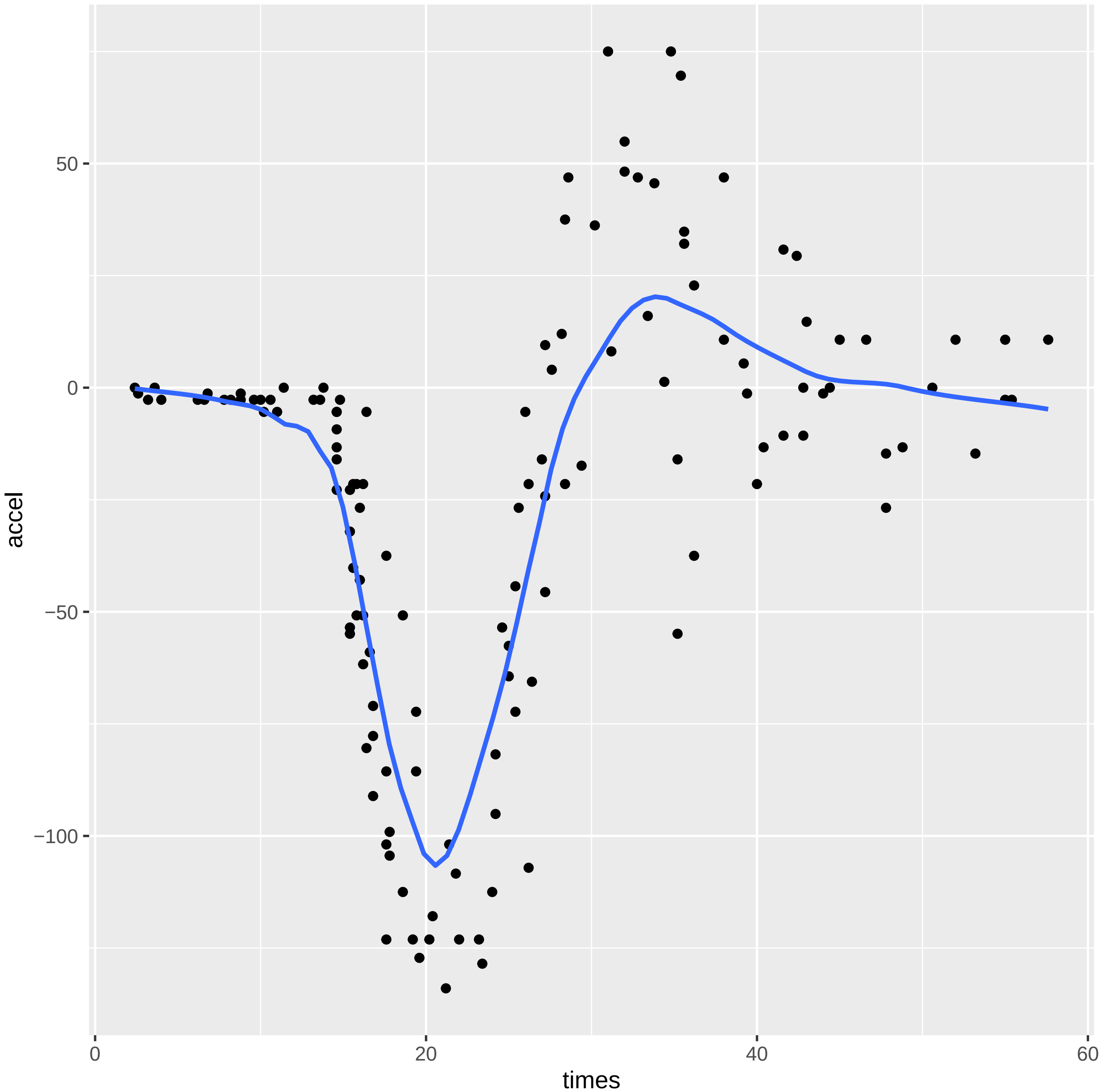


Visually inspecting the models, we believe that the model with degree 2 and span 0.35 provided the "best" fit.

Part b

Comparing results with built-in LOESS function

```
mcycle %>% ggplot(aes(times, accel)) +
  geom_point() +
  geom_smooth(method = "loess", degree = 2, span = 0.35, se = F, method.args = list(degree=1))
```



The built-in LOESS function results in a fit that does not over-fit the data, but we feel our model provides the better fit.

Problem 3

```
# Some pre-processing
library(ISLR)
# Remove the name of the car model and change the origin to categorical with actual name
Auto_new <- Auto[, -9]
# Lookup table
newOrigin <- c("USA", "European", "Japanese")
Auto_new$origin <- factor(newOrigin[Auto_new$origin], newOrigin)

# Look at the first 6 observations to see the final version
head(Auto_new)
```

```
  mpg cylinders displacement horsepower weight acceleration year origin
1  18         8          307        130   3504         12.0   70    USA
```

```
2  15         8          350        165   3693          11.5   70    USA
3  18         8          318        150   3436          11.0   70    USA
4  16         8          304        150   3433          12.0   70    USA
5  17         8          302        140   3449          10.5   70    USA
6  15         8          429        198   4341          10.0   70    USA
```

```r
library(caret)

# This function has the following inputs
#
# * train - matrix or data frame of training set cases
# * test - matrix or data frame of test set cases.
#     (A vector will be interpreted as a row vector for a single case.)
# * y_train - Either a numeric vector, or factor vector for the responses in the training set
# * y_test - Either a numeric vector, or factor vector for the responses in the testing set
# * k - number of neighbors considered, the default value is 3
#
# If weighted = TRUE, then function uses the distance weighted kNN,
#  otherwise it does the default kNN method.

mykNN <- function(train, test, y_train, y_test, k = 3, weighted = TRUE) {

  # Allocating space for vector of fitted values
  yhat <- vector(mode = "numeric", length = length(y_test))

  # Fitting each point
  for (row in 1:nrow(test)) {

    # Getting features
    x0 <- test[row,]

    # Getting Euclidean distance and weights
    distmat <- as.matrix(dist(rbind(x0, train), method = "euclidean", diag = T, upper = T))
    dist <- distmat[1,]
    weights <- 1/dist

    # Getting kth nearest neighbor
    a.order <- order(weights, decreasing = T)[-1]
    last <- weights[a.order[k]]

    # Setting weights to 0 except for kNN
    y_train_sub <- y_train[-which(!is.finite(weights))]
    weights <- weights[-which(!is.finite(weights))]
    weights[weights < last] <- 0

    # Handling
    if(weighted == F) {
      weights[weights != 0] <- 1
    }

    tot_weight <- sum(weights)

    class <- vector(mode = "numeric", length = nlevels(y_test))

    if (is.factor(y_train)) {
      for (i in 1:nlevels(y_test)) {
        class[i] <- sum(weights[which(y_train == levels(y_test)[i])])
      }
      yhat[row] <- levels(y_test)[which(class == max(class))]
    }
```

```
    else {
      yhat[row] <- sum(weights*y_train/tot_weight)
    }
  }

  if (is.factor(y_train)) {
    yhat <- factor(yhat)

    num_correct <- 0

    for (j in 1:length(yhat)) {
      if (y_test[j] == yhat[j]) {
        num_correct <- num_correct + 1
      }
    }
    accuracy <- num_correct / length(yhat)
    error_rate <- 1 - accuracy

    confmat <- confusionMatrix(yhat, y_test)

    return(invisible(list(yhat = yhat, accuracy = accuracy,
                          error_rate = error_rate, confmat = confmat, k = k)))
  }

  else {

    # Calculating SSE, MSE, and residual standard error
    SSE <- sum((y_test-yhat)^{2})
    MSE <- SSE/(nrow(test)-2)
    RSE <- sqrt(MSE)

    residuals <- y_test - yhat

    # Returning named list
    return(invisible(list(yhat = yhat, residuals = residuals, SSE = SSE, k = k)))
  }

}
ntrain <- round(nrow(Auto_new)*0.7)
ntest <- nrow(Auto_new) - ntrain
train <- Auto_new[1:ntrain,1:7]
test <- Auto_new[(ntrain+1):nrow(Auto_new),1:7]
y_train <- Auto_new[1:ntrain,8]
y_test <- Auto_new[(ntrain+1):nrow(Auto_new),8]

trial2 <- mykNN(train, test, y_train, y_test, k = 3)
trial2
```

```
$yhat
 [1] Japanese USA      Japanese USA      USA      USA      USA      USA
 [9] USA      USA      USA      USA      USA      USA      USA      USA
[17] USA      European Japanese European European USA      USA      European
[25] USA      European USA      Japanese USA      USA      USA      USA
[33] USA      USA      European USA      Japanese European European USA
[41] USA      European USA      Japanese European Japanese USA      Japanese
[49] USA      European European European European European USA      European
[57] Japanese Japanese Japanese Japanese Japanese USA      USA      USA
[65] USA      Japanese European Japanese USA      Japanese USA      Japanese
[73] European USA      Japanese European European Japanese European European
[81] European European USA      European USA      USA      USA      USA
```

```
 [89] European European Japanese European Japanese USA      European Japanese
 [97] Japanese European USA      USA      USA      European USA      European
[105] European Japanese USA      USA      USA      USA      European Japanese
[113] European Japanese USA      USA      USA      USA
Levels: European Japanese USA

$accuracy
[1] 0.5762712

$error_rate
[1] 0.4237288

$confmat
Confusion Matrix and Statistics

          Reference
Prediction USA European Japanese
  USA       42        5       10
  European  12       11       12
  Japanese   7        4       15

Overall Statistics

               Accuracy : 0.5763
                 95% CI : (0.4819, 0.6667)
    No Information Rate : 0.5169
    P-Value [Acc > NIR] : 0.11541

                  Kappa : 0.3284

 Mcnemar's Test P-Value : 0.05987

Statistics by Class:

                     Class: USA Class: European Class: Japanese
Sensitivity              0.6885         0.55000          0.4054
Specificity              0.7368         0.75510          0.8642
Pos Pred Value           0.7368         0.31429          0.5769
Neg Pred Value           0.6885         0.89157          0.7609
Prevalence               0.5169         0.16949          0.3136
Detection Rate           0.3559         0.09322          0.1271
Detection Prevalence     0.4831         0.29661          0.2203
Balanced Accuracy        0.7127         0.65255          0.6348

$k
[1] 3
```

```
trial2$residuals
```

```
NULL
```

```
#ncol(train)
```

```
y_test
```

```
  [1] European European European Japanese USA      USA      USA      USA
  [9] USA      USA      USA      USA      USA      USA      USA      USA
 [17] USA      European Japanese USA      USA      European USA      European
 [25] USA      USA      USA      Japanese European USA      USA      USA
 [33] USA      European Japanese USA      Japanese USA      USA      USA
 [41] USA      European Japanese Japanese Japanese Japanese Japanese USA
 [49] Japanese European European European European Japanese Japanese European
```

```
 [57] Japanese Japanese European Japanese USA      USA      USA      USA
 [65] USA      Japanese USA      Japanese Japanese Japanese Japanese Japanese
 [73] USA      USA      USA      European Japanese Japanese Japanese Japanese
 [81] European European Japanese Japanese USA      USA      USA      USA
 [89] USA      USA      USA      USA      USA      USA      USA      European
 [97] Japanese Japanese USA      USA      Japanese Japanese Japanese Japanese
[105] Japanese Japanese USA      USA      USA      USA      Japanese USA
[113] USA      USA      European USA      USA      USA
Levels: USA European Japanese
```

```
trial2$SSE
```

```
NULL
```

```
library(class)


#real <- knn(train, test, y_train, k = 3)
```

```
"ozone4 <- as.data.frame(lapply(ozone[,3], nor))
ozone4 <- cbind(ozone4, ozone$ozone)
index <- sample(1:nrow(ozone4), round(nrow(ozone4) * 0.7))
training_df <- ozone4[index, ]
testing_df <- ozone4[-index, ]
train <- training_df[, 1]
test <- testing_df[, 1]
y_train <- training_df[,2]
y_test <- testing_df[,2]

trial2 <- mykNN2(train, test, y_train, y_test, k = 3)
trial2$residuals"
```

```
[1] "ozone4 <- as.data.frame(lapply(ozone[,3], nor))\nozone4 <- cbind(ozone4, ozone$ozone)\nindex <- sample(1:
```