Assignment 3

Chirag Mehta - AI20BTECH11006

Download all the python codes from

https://github.com/cmaspi/EE3900/tree/main/Assignment−3/code

latex-tikz codes from

https://github.com/cmaspi/EE3900/blob/main/Assignment−3/main.tex

1 Problem

(Construction Q 2.14) Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**

2 Solution

Lemma 1. *The points of contact for the tangent drawn from a point*

$$\mathbf{P} = d\mathbf{e}_1, \text{ where } \mathbf{e}_1 = \begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.1}$$

to the circle are given by

$$\mathbf{x} = \frac{r^2}{d}\mathbf{e}_1 \pm r\sqrt{1-\frac{r^2}{d^2}}\mathbf{e}_2 \tag{2.0.2}$$

Proof. If $\mathbf{x}$ be a point of contact for the tangent from $\mathbf{P}$,

$$(\mathbf{P}-\mathbf{x})^T\mathbf{x} = 0 \tag{2.0.3}$$

$$\implies \mathbf{P}^\top\mathbf{x} = \|\mathbf{x}\|^2 = r^2 \tag{2.0.4}$$

$$\implies \mathbf{e}_1^\top\mathbf{x} = \frac{r^2}{d} \tag{2.0.5}$$

The above equation can be expressed in parametric form as

$$\mathbf{x} = \frac{r^2}{d}\mathbf{e}_1 + \lambda\mathbf{e}_2 \tag{2.0.6}$$

Substituting the above in

$$\|\mathbf{x}\|^2 = r^2, \tag{2.0.7}$$

yields

$$\left\|\frac{r^2}{d}\mathbf{e}_1 + \lambda\mathbf{e}_2\right\|^2 = r^2 \tag{2.0.8}$$

$$\implies \lambda^2 = r^2\left[1-\frac{r^2}{d^2}\right] \tag{2.0.9}$$

$$\text{or, } \lambda = \pm r\sqrt{1-\frac{r^2}{d^2}} \tag{2.0.10}$$

□

Let $\mathbf{A}, \mathbf{B}$ be the corresponding points of tangency from $\mathbf{Q} = 7\mathbf{e_1}$ and $\mathbf{C}, \mathbf{D}$ be the corresponding points of tangency from $\mathbf{P} = -7\mathbf{e_1}$.
Using (2.0.2), we obtain all the points of tangency. A plot for tangents is given below

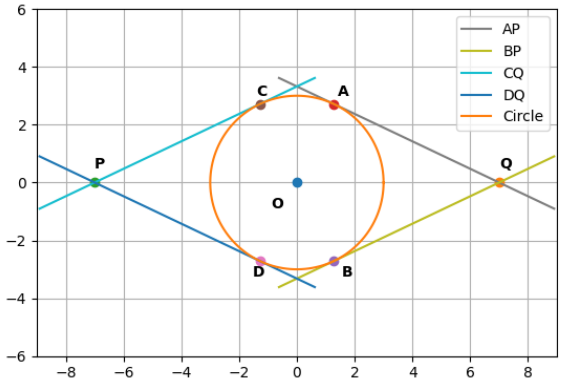

Fig. 0: Plot of the planes