Assignment 5

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Download all the python codes from

https://github.com/cmaspi/EE3900/tree/main/Assignment−5/code

latex-tikz codes from

https://github.com/cmaspi/EE3900/blob/main/Assignment−5/main.tex

1 Problem

(Quadratic forms Q2.66) Find the point at which the line $\begin{pmatrix}-1 & 1\end{pmatrix}\mathbf{x} = 1$ is a tangent to the curve $y^2 = 4x$

2 Solution

The general form of a conic is given by

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

for $y^2 = 4x$

$$\mathbf{V} = \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix},\ \mathbf{u} = \begin{pmatrix}-2\\0\end{pmatrix},\ f = 0 \tag{2.0.2}$$

For any given conic such that $\mathbf{V}$ is non-invertible, the points of tangency are given by

$$\mathbf{V}\mathbf{q} = k\mathbf{n} - u \tag{2.0.3}$$

$$\text{where, } k = \frac{\mathbf{p_1}^T\mathbf{u}}{\mathbf{p_1}^T\mathbf{n}},\ \mathbf{V}\mathbf{p_1} = 0 \tag{2.0.4}$$

Clearly

$$\mathbf{p_1} = \begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.5}$$

$$\implies k = \frac{-2}{-1} = 2 \tag{2.0.6}$$

Using the obtained values

$$\mathbf{V}\mathbf{q} = 2\mathbf{n} - \mathbf{u} \tag{2.0.7}$$

$$\mathbf{V}\mathbf{q} = \begin{pmatrix}-2\\2\end{pmatrix} - \begin{pmatrix}-2\\0\end{pmatrix} = 2\mathbf{e_2} \tag{2.0.8}$$

$$\mathbf{V}(\mathbf{q} - \mathbf{2e_2}) = \mathbf{0} \tag{2.0.9}$$

The basis vector for null space of $\mathbf{V}$ is $\mathbf{e_1}$

$$\therefore \mathbf{q} - 2\mathbf{e_2} = \lambda\mathbf{e_1} \tag{2.0.10}$$

$$\implies \mathbf{q} = 2\mathbf{e_2} + \lambda\mathbf{e_1} \tag{2.0.11}$$

Using (2.0.11) and (2.0.1), we get

$$2\mathbf{q}^T\mathbf{e_2} - 4\mathbf{e_1}^T\mathbf{q} = 0 \tag{2.0.12}$$

Evaluating this, we get

$$4 - \lambda = 0 \tag{2.0.13}$$

$$\implies \mathbf{q} = \begin{pmatrix}1\\2\end{pmatrix} \tag{2.0.14}$$

A plot for the line and parabola is given below

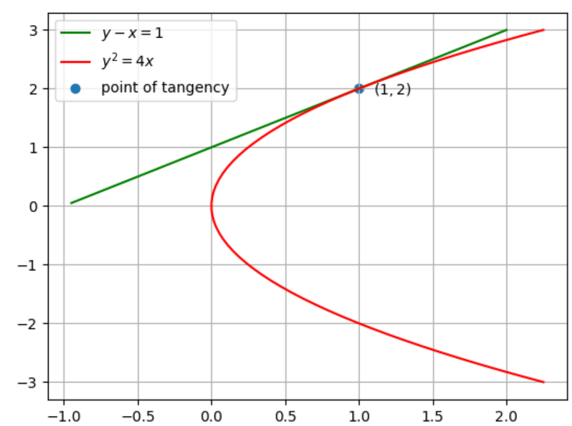

Fig. 0: Plot of the line and parabola