Exam-2

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Download all the python codes from

https://github.com/cmaspi/EE3900/tree/main/exam−2/Codes

latex-tikz codes from

https://github.com/cmaspi/EE3900/blob/main/exam−2/main.tex

1 Problem

(Q 3.1 a,b,c) Determine the $z-$transform, including the region of convergence, for each of the following sequences

(a) $\left(\frac{1}{2}\right)^n u[n]$

(b) $-\left(\frac{1}{2}\right)^n u[-n-1]$

(c) $\left(\frac{1}{2}\right)^n u[-n]$

2 Solution

Definition 1. *The z tansform of a function is defined as*

$$x[n] \overset{z}{\rightleftharpoons} X(z) \tag{2.0.1}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \tag{2.0.2}$$

Definition 2. *The $u[n]$ function is defined as*

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & otherwise \end{cases} \tag{2.0.3}$$

(a) $\left(\frac{1}{2}\right)^n u[n]$

$$x[n] = \left(\frac{1}{2}\right)^n u[n] \tag{2.0.4}$$

Using (1) and (2)

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n] z^{-n} \tag{2.0.5}$$

$$= \sum_{n=0}^{\infty} \left(\frac{z^{-1}}{2}\right)^n \tag{2.0.6}$$

$$= \frac{1}{1-\frac{1}{2}z^{-1}}, \; ROC = \left|\frac{z^{-1}}{2}\right| < 1 \tag{2.0.7}$$

$$= \frac{2}{2-z^{-1}}, \; ROC = |z| > \frac{1}{2} \tag{2.0.8}$$

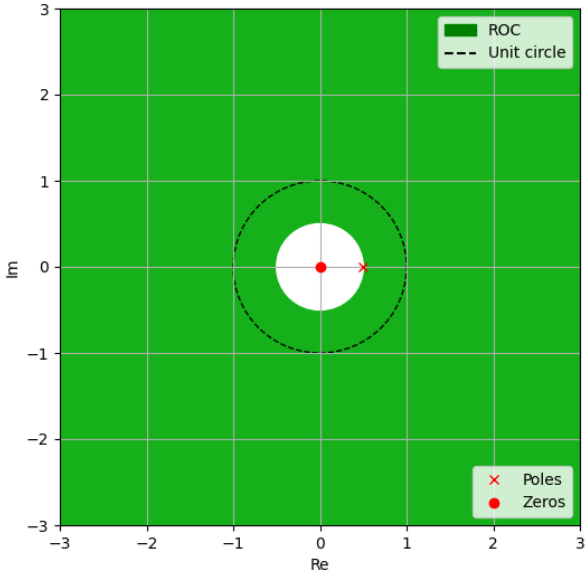

Fig. (a): Pole-zero plot of the system

(b) $-\left(\frac{1}{2}\right)^n u[-n-1]$

$$x[n] = -\left(\frac{1}{2}\right)^n u[-n-1] \tag{2.0.9}$$

Using (1) and (2)

$$X(z) = \sum_{n=-\infty}^{\infty} -\left(\frac{1}{2}\right)^n u[-n-1] \tag{2.0.10}$$

$$= \sum_{n=-\infty}^{-1} -\left(\frac{1}{2}\right)^n z^{-n} \tag{2.0.11}$$

$$= -\sum_{n=1}^{\infty} (2z)^n \tag{2.0.12}$$

$$= \frac{-2z}{1-2z}, \; ROC = |2z| < 1 \tag{2.0.13}$$

$$= \frac{2}{2-z^{-1}}, \; ROC = |z| < \frac{1}{2} \tag{2.0.14}$$

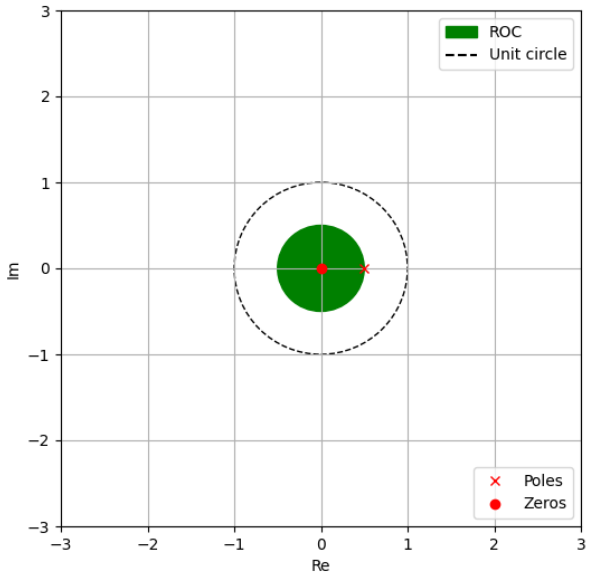

Fig. (b): Pole-zero plot of the system

(c) $\left(\frac{1}{2}\right)^n u[-n]$

$$x[n] = \left(\frac{1}{2}\right)^n u[-n] \tag{2.0.15}$$

Using (1) and (2)

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[-n] z^{-n} \tag{2.0.16}$$

$$= \sum_{n=-\infty}^{0} \left(\frac{1}{2z}\right)^n \tag{2.0.17}$$

$$= \sum_{n=0}^{\infty} (2z)^n \tag{2.0.18}$$

$$= \frac{1}{1-2z}, \; ROC = |2z| < 1 \tag{2.0.19}$$

$$= \frac{1}{1-2z}, \; ROC = |z| < \frac{1}{2} \tag{2.0.20}$$

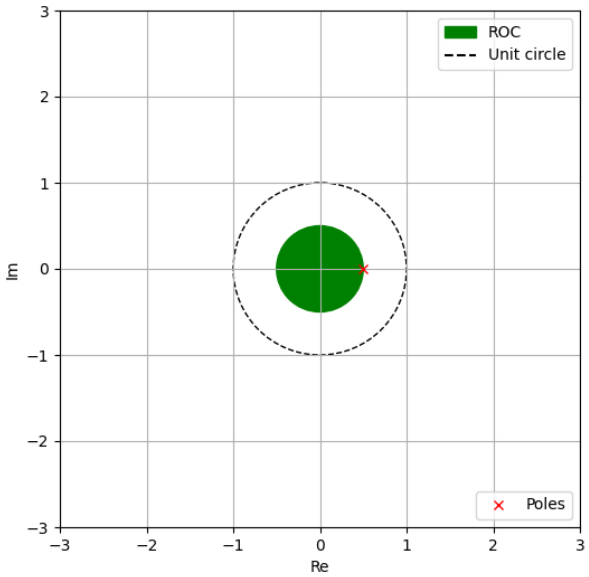

Fig. (c): Pole-zero plot of the system