Gate Assignment - 4

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Download all latex-tikz codes from

https://github.com/cmaspi/EE3900/blob/main/GateAssignment−4/main.tex

1 Problem

(GATE EC 1999 Q1.3) If $[f(t)] = F(s)$, then $[f(t-T)]$ is equal to

1) $e^{sT}F(s)$
2) $e^{-sT}F(s)$
3) $\dfrac{F(s)}{1+e^{sT}}$
4) $\dfrac{F(s)}{1+e^{-sT}}$

2 Solution

Definition 1.

$$[f(t)] = \mathcal{L}\{u(t)f(t)\} \tag{2.0.1}$$

Definition 2 (Laplace Transform)**.** *It is an integral transform that converts a function of a real variable t to a function of a complex variable s. The Laplace transform of $f(t)$ is denoted by $\mathcal{L}\{f(t)\}$ or $F(s)$.*

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st}f(t)dt \tag{2.0.2}$$

Lemma 2.1. *Time Shift Property of Laplace transformation*

$$f(t-T)u(t-T) \overset{\mathcal{L}}{\rightleftharpoons} e^{-sT}F(s) \tag{2.0.3}$$

Proof.

$$\mathcal{L}\{f(t-T)u(t-T)\}$$
$$= \int_0^\infty e^{-st}f(t-T)u(t-T)dt \tag{2.0.4}$$

This can be written as

$$= e^{-sT}\int_{-T}^\infty e^{-s(t-T)}f(t-T)u(t-T)d(t-T) \tag{2.0.5}$$
$$= e^{-sT}\int_0^\infty e^{-s(t-T)}f(t-T)d(t-T) \tag{2.0.6}$$
$$= e^{-sT}F(s) \tag{2.0.7}$$

□

Using (2.1), the correct answer is **Option(2)**