

Assignment 5

Chirag Mehta - AI20BTECH11006

Download all the python codes from

<https://github.com/cmaspi/EE3900/tree/main/Assignment-5/code>

latex-tikz codes from

<https://github.com/cmaspi/EE3900/blob/main/Assignment-5/main.tex>

1 PROBLEM

(Quadratic forms Q2.66) Find the point at which the line $\begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{x} = 1$ is a tangent to the curve $y^2 = 4x$

2 SOLUTION

The equation of line is given by

$$\begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{x} = 1 \quad (2.0.1)$$

$$\mathbf{x} = \mathbf{e}_2 + \lambda \mathbf{1}_2 \quad (2.0.2)$$

General equation of a conic is given by

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.3)$$

for $y^2 = 4x$, the equation can be written as

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 4 \begin{pmatrix} -1 \\ 0 \end{pmatrix}^T \mathbf{x} = 0 \quad (2.0.4)$$

Using the condition of tangency, we get

$$\mathbf{m}^T (\mathbf{V} \mathbf{q} + \mathbf{u}) = 0 \quad (2.0.5)$$

$$\mathbf{1}_2^T \left(\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right) = 0 \quad (2.0.6)$$

$$\mathbf{1}_2^T \begin{pmatrix} -2 \\ y \end{pmatrix} = 0 \quad (2.0.7)$$

$$\implies y = 2, x = 1 \quad (2.0.8)$$

A plot for the line and parabola is given below

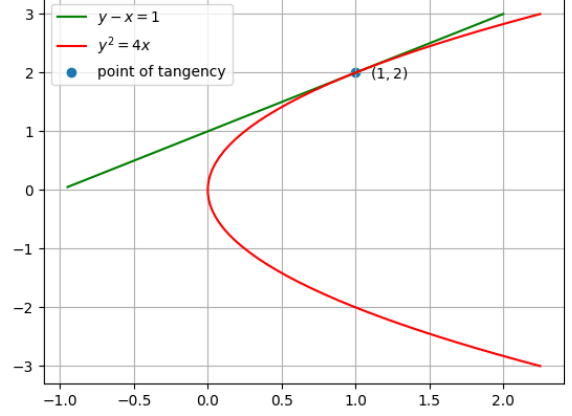


Fig. 0: Plot of the line and parabola