Gate Assignment - 2

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Download all the python codes from

https://github.com/cmaspi/EE3900/tree/main/GateAssignment−2/code

latex-tikz codes from

https://github.com/cmaspi/EE3900/blob/main/GateAssignment−2/main.tex

1 Problem

(GATE EC 2009 Q42) The 4-point discrete fourier transform (DFT) of a discrete time sequence $[1, 0, 2, 3]$ is given by

1) $[0, -2 + 2j, 2, -2 - 2j]$
2) $[2, 2 + j, 6, 2 - 2j]$
3) $[6, 1 - 3j, 2, 1 + 3j]$
4) $[6, -1 + 3j, 0, -1, -3j]$

2 Solution

Lemma 2.1. *The N-point discrete fourier transform is given as* $\mathbf{X} = \mathbf{W}\mathbf{x}$*, where* $\mathbf{x}$ *is the input signal,* $\mathbf{W}$ *is the N-by-N square DFT matrix, and* $\mathbf{X}$ *is the DFT of the signal.*
The transformation matrix $\mathbf{W}$ *is given by*

$$\mathbf{W} = \begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & w & \dots & w^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & w^{N-1} & \dots & w^{(N-1)(N-1)} \end{pmatrix} \tag{2.0.1}$$

where $w = e^{\frac{-2\pi j}{N}}$

The four point DFT matrix is given below

$$W = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix} \tag{2.0.2}$$

Now,

$$\mathbf{X} = \mathbf{W}\mathbf{x} \tag{2.0.3}$$

On multiplying, we get

$$X = \begin{pmatrix} 6 \\ -1 + 3j \\ 0 \\ -1 - 3j \end{pmatrix} \tag{2.0.4}$$

The correct answer is **option D**