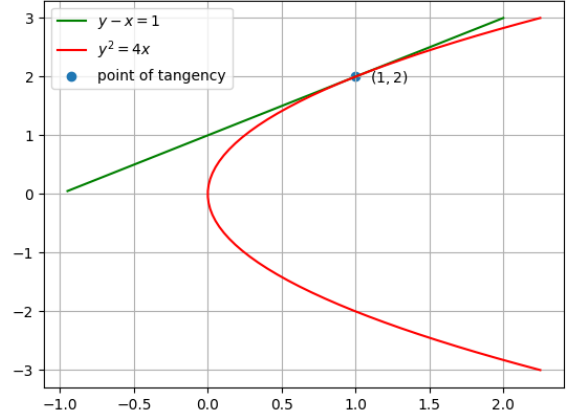# Assignment 5

Chirag Mehta - AI20BTECH11006

Download all the python codes from

latex-tikz codes from

Fig. 0: Plot of the planes

1 PROBLEM

(Quadratic forms Q2.66) Find the point at which the line $\begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{x} = 1$ is a tangent to the curve $y^2 = 4x$

2 SOLUTION

The equation of line is given by

$$\begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{x} = 1 \tag{2.0.1}$$

$$\mathbf{x} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \lambda \\ \lambda + 1 \end{pmatrix} \tag{2.0.2}$$

General equation of a conic is given by

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T x + f = 0 \tag{2.0.3}$$

for $y^2 = 4x$, the equation can be written as

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 4 \begin{pmatrix} -1 \\ 0 \end{pmatrix}^T x = 0 \tag{2.0.4}$$

Using (2.0.2) and (2.0.4), we get

$$\begin{pmatrix} \lambda \\ \lambda + 1 \end{pmatrix}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \lambda \\ \lambda + 1 \end{pmatrix} - 4 \begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \begin{pmatrix} \lambda \\ \lambda + 1 \end{pmatrix} = 0 \tag{2.0.5}$$

$$\begin{pmatrix} 0 & \lambda + 1 \end{pmatrix} \begin{pmatrix} \lambda \\ \lambda + 1 \end{pmatrix} - 4\lambda = 0 \tag{2.0.6}$$

$$\lambda^2 - 2\lambda + 1 = 0 \tag{2.0.7}$$

$$\lambda = 1 \tag{2.0.8}$$

The point of tangency is given by

$$\mathbf{x} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{2.0.9}$$

A plot for the line and parabola is given below