Assignment 3

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Download all the python codes from

https://github.com/cmaspi/EE3900/tree/main/Assignment−3/code

latex-tikz codes from

https://github.com/cmaspi/EE3900/blob/main/Assignment−3/main.tex

1 Problem

(Construction Q 2.14) Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**

2 Solution

The given parameters are listed in Table ??

	Circle
Centre	$\mathbf{O}=\begin{pmatrix}0\\0\end{pmatrix}$
Radius	r=3
Radius	d=7

TABLE 0: Input values

Lemma 1. *The points of contact for the tangent drawn from a point*

$$\mathbf{P} = d\mathbf{e}_1, \text{ where } \mathbf{e}_1 = \begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.1}$$

to the circle are given by

$$\mathbf{x} = \frac{r^2}{d}\mathbf{e}_1 \pm r\sqrt{1-\frac{r^2}{d^2}}\mathbf{e}_2 \tag{2.0.2}$$

If $\mathbf{x}$ be a point of contact for the tangent from $\mathbf{P}$,

$$PR \perp RO \tag{2.0.3}$$

$$\implies (\mathbf{O}-\mathbf{x})^\top(\mathbf{x}-\mathbf{P}) = 0 \tag{2.0.4}$$

$$\text{or, } \mathbf{P}^\top\mathbf{x} = \|\mathbf{x}\|^2 = r^2 \tag{2.0.5}$$

$$\implies \mathbf{e}_1^\top\mathbf{x} = \frac{r^2}{d} \tag{2.0.6}$$

$\because$ $\mathbf{O} = 0$. The above equation can be expressed in parametric form as

$$\mathbf{x} = \frac{r^2}{d}\mathbf{e}_1 + \lambda\mathbf{e}_2 \tag{2.0.7}$$

Substituting the above in

$$\|\mathbf{x}\|^2 = r^2, \tag{2.0.8}$$

yields

$$\|\frac{r^2}{d}\mathbf{e}_1 + \lambda\mathbf{e}_2\|^2 = r^2 \tag{2.0.9}$$

$$\implies \lambda^2 = r^2\left[1-\frac{r^2}{d^2}\right] \tag{2.0.10}$$

$$\text{or, } \lambda = \pm r\sqrt{1-\frac{r^2}{d^2}} \tag{2.0.11}$$

Substituting λ in (**??**) yields (**??**). Fig. **??** shows all possible tangents and their points of contact after substituting the numerical values in (**??**).

Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of 60°.

Solution: The angle between the tangents from **P** is given by

Lemma 2. *Given a circle of radius r and angle θ between the tangents, the intersection of the tangents and points of contact are given by Lemma* **??** *where*

$$\implies d = r\sin\frac{\theta}{2} \tag{2.0.12}$$

Proof. From Fig. **??**,

$$\sin\frac{\theta}{2} = \frac{r}{d} \tag{2.0.13}$$

$$\implies d = r\sin\frac{\theta}{2} \tag{2.0.14}$$

□

Substituting numerical values and plotting, we obtain Fig. **??**.

A plot for the planes is given below