

Assignment 5

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Download all the python codes from

<https://github.com/cmaspi/EE3900/tree/main/Assignment-5/code>

latex-tikz codes from

<https://github.com/cmaspi/EE3900/blob/main/Assignment-5/main.tex>

Using (2.0.9) and (2.0.1), we get

$$2\mathbf{q}^T \mathbf{e}_2 - 4\mathbf{e}_1^T \mathbf{q} = 0 \quad (2.0.10)$$

Evaluating this, we get

$$4 - \lambda = 0 \quad (2.0.11)$$

$$\Rightarrow \mathbf{q} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.12)$$

A plot for the line and parabola is given below

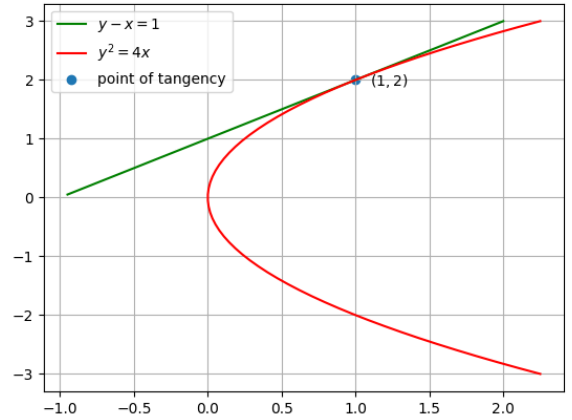


Fig. 0: Plot of the line and parabola

1 PROBLEM

(Quadratic forms Q2.66) Find the point at which the line $\begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{x} = 1$ is a tangent to the curve $y^2 = 4x$

2 SOLUTION

The general form of a conic is given by

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

for $y^2 = 4x$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, f = 0 \quad (2.0.2)$$

For any given conic such that \mathbf{V} is non-invertible, the points of tangency are given by

$$\mathbf{V} \mathbf{q} = k \mathbf{n} - \mathbf{u} \quad (2.0.3)$$

$$\text{where, } k = \frac{\mathbf{p}_1^T \mathbf{u}}{\mathbf{p}_1^T \mathbf{n}}, \mathbf{V} \mathbf{p}_1 = 0 \quad (2.0.4)$$

Clearly

$$\mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.5)$$

$$\Rightarrow k = \frac{-2}{-1} = 2 \quad (2.0.6)$$

Using the obtained values

$$\mathbf{V} \mathbf{q} = 2 \mathbf{n} - \mathbf{u} \quad (2.0.7)$$

$$\mathbf{V} \mathbf{q} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} = 2 \mathbf{e}_2 \quad (2.0.8)$$

$$\Rightarrow \mathbf{q} = 2 \mathbf{e}_2 + \lambda \mathbf{e}_1 \quad (2.0.9)$$