

Assignment 3

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Download all the python codes from

<https://github.com/cmaspi/EE3900/tree/main/Assignment-3/Code>

latex-tikz codes from

<https://github.com/cmaspi/EE3900/blob/main/Assignment-3/main.tex>

yields

$$\left\| \frac{r^2}{d} \mathbf{e}_1 + \lambda \mathbf{e}_2 \right\|^2 = r^2 \quad (2.0.8)$$

$$\Rightarrow \lambda^2 = r^2 \left[1 - \frac{r^2}{d^2} \right] \quad (2.0.9)$$

$$\text{or, } \lambda = \pm r \sqrt{1 - \frac{r^2}{d^2}} \quad (2.0.10)$$

□

1 PROBLEM

(Construction Q 2.14) Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**

2 SOLUTION

Lemma 1. *The points of contact for the tangent drawn from a point*

$$\mathbf{P} = d\mathbf{e}_1, \text{ where } \mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.1)$$

to the circle are given by

$$\mathbf{x} = \frac{r^2}{d} \mathbf{e}_1 \pm r \sqrt{1 - \frac{r^2}{d^2}} \mathbf{e}_2 \quad (2.0.2)$$

Proof. If **x** be a point of contact for the tangent from **P**,

$$(\mathbf{P} - \mathbf{x})^T \mathbf{x} = 0 \quad (2.0.3)$$

$$\Rightarrow \mathbf{P}^T \mathbf{x} = \|\mathbf{x}\|^2 = r^2 \quad (2.0.4)$$

$$\Rightarrow \mathbf{e}_1^T \mathbf{x} = \frac{r^2}{d} \quad (2.0.5)$$

The above equation can be expressed in parametric form as

$$\mathbf{x} = \frac{r^2}{d} \mathbf{e}_1 + \lambda \mathbf{e}_2 \quad (2.0.6)$$

Substituting the above in

$$\|\mathbf{x}\|^2 = r^2, \quad (2.0.7)$$

Let **A, B** be the corresponding points of tangency from **P** = 7 \mathbf{e}_1 and **C, D** be the corresponding points of tangency from **Q** = -7 \mathbf{e}_1 . Using (2.0.2), we obtain all the points of tangency. A plot for tangents is given below

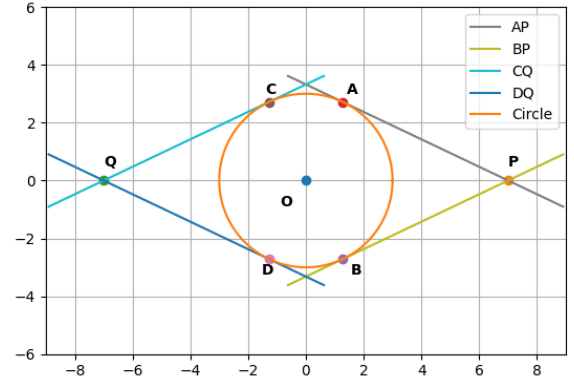


Fig. 0: Plot of the tangents