Assignment 3

Chirag Mehta - AI20BTECH11006

Download all the python codes from

```
https://github.com/cmaspi/EE3900/tree/main/
    Assignment-3/Code
```

latex-tikz codes from

```
https://github.com/cmaspi/EE3900/blob/main/
    Assignment-3/main.tex
```

1 Problem

(Construction Q 2.14) Draw a circle of radius 3 units. Take two points $\mathbf{P}$ and $\mathbf{Q}$ on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points $\mathbf{P}$ and $\mathbf{Q}$

2 Solution

Theorem 2.1. *The points of intersection of the line*

$$L: \quad \mathbf{x} = \mathbf{q} + \mu\mathbf{m} \quad \mu \in \mathbb{R} \tag{2.0.1}$$

with a general conic are given by

$$\mathbf{x}_i = \mathbf{q} + \mu_i\mathbf{m} \tag{2.0.2}$$

where

$$\mu_i = \frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}}\Big(-\mathbf{m}^T\left(\mathbf{V}\mathbf{q}+\mathbf{u}\right) \pm \sqrt{\left[\mathbf{m}^T\left(\mathbf{V}\mathbf{q}+\mathbf{u}\right)\right]^2 - \left(\mathbf{q}^T\mathbf{V}\mathbf{q}+2\mathbf{u}^T\mathbf{q}+f\right)\left(\mathbf{m}^T\mathbf{V}\mathbf{m}\right)}\Big) \tag{2.0.3}$$

The general form of a conic is given by

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.4}$$

for a circle, $\mathbf{V} = \mathbf{I}$ and $\mathbf{u} = -\mathbf{O}$ where $\mathbf{O}$ is the center of the circle.
Let $\mathbf{q}$ be the locus of the point of tangency from point $\mathbf{P}$, the distance of $\mathbf{P}$ from $\mathbf{O}$ is d

$$\mathbf{q} - \mathbf{O}^T\mathbf{q} - \mathbf{P} = 0 \tag{2.0.5}$$

$$\mathbf{q} + \mathbf{u}^T\mathbf{q} - \mathbf{P} = 0 \tag{2.0.6}$$

$$\mathbf{q}^T\mathbf{q} + \mathbf{u}^T\mathbf{q} - \mathbf{u}^T\mathbf{P} - \mathbf{q}^T\mathbf{P} = 0 \tag{2.0.7}$$

Using (2.0.4)

$$(\mathbf{u}+\mathbf{p})^T\mathbf{q} = -f - \mathbf{u}^T\mathbf{P} \tag{2.0.8}$$

Let $\mathbf{n} = \mathbf{u}+\mathbf{P}$ and $c = -f - \mathbf{u}^T\mathbf{P}$ This is equation of a line, let $\mathbf{q} = \mathbf{a}$ be a point that lies on this line

$$\therefore \mathbf{q} = \mathbf{a} + \lambda\begin{pmatrix} -\mathbf{e_1}^T\mathbf{n} \\ \mathbf{e_2}^T\mathbf{n} \end{pmatrix} \tag{2.0.9}$$

We need to find the intersection point of this with the given circle.
Using (2.0.3)

$$\mathbf{q} = \mathbf{a} + \mu_i\mathbf{m} \tag{2.0.10}$$

$$\mu_i = \frac{1}{d^2}\Big(-\mathbf{m}^T\left(\mathbf{a}+\mathbf{u}\right) \pm \sqrt{\left[\mathbf{m}^T\left(\mathbf{a}+\mathbf{u}\right)\right]^2 - \left(\mathbf{a}^T\mathbf{a}+2\mathbf{u}^T\mathbf{a}+f\right)d^2}\Big) \tag{2.0.11}$$

Now, we can confirm the solution by checking for $\mathbf{u} = \mathbf{0}$, $\mathbf{P} = d\mathbf{e}_1$, $f = -r^2$

$$\implies \mathbf{n} = d\mathbf{e}_1,\ \mathbf{m} = d\mathbf{e}_2 \tag{2.0.12}$$

An arbitrary choice of $\mathbf{a}$ could be $\frac{r^2}{d}\mathbf{e}_1$,

$$\mu_i = \pm\frac{1}{d^2}\sqrt{\left(r^2d^2 - r^4\right)} \tag{2.0.13}$$

$$\mathbf{q} = \frac{r^2}{d}\mathbf{e}_1 \pm r\sqrt{1 - \frac{r^2}{d}} \tag{2.0.14}$$

Let $\mathbf{A}, \mathbf{B}$ be the corresponding points of tangency from $\mathbf{P} = 7\mathbf{e_1}$ and $\mathbf{C}, \mathbf{D}$ be the corresponding points of tangency from $\mathbf{Q} = -7\mathbf{e_1}$.
Using (2.0.14), we obtain all the points of tangency. A plot for tangents is given below

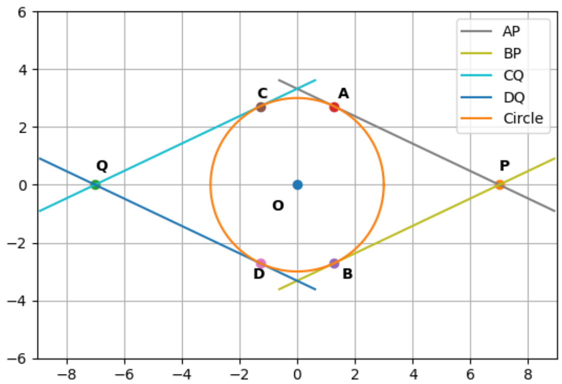

Fig. 0: Plot of the tangents