GATE Assignment

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Download all latex-tikz codes from

https://github.com/cmaspi/EE3900/blob/main/GateAssignment/assignment.tex

1 Problem

(EC-2020/Q.29) A finite duration discrete-time signal $x[n]$ is obtained by sampling the continuous-time signal $x(t) = cos(200\pi t)$ at sampling instants $t = \frac{n}{400}$, $n = 0, 1, ..., 7$. The 8-point discrete Fourier transform (DFT) of $x[n]$ is defined as

$$X[k] = \sum_{n=0}^{7} x[n] e^{-j\frac{\pi k n}{4}}, \quad k = 0, 1, 2, ..., 7 \tag{1.0.1}$$

Which of the following is true?

1) All $X[k]$ are non-zero
2) Only $X[4]$ is non-zero
3) Only $X[2], X[6]$ are non-zero
4) Only $X[2], X[6]$ are non-zero

2 solution

Given,

$$x(t) = \cos(200\pi t) \tag{2.0.1}$$

$$x[n] = \cos\left(\frac{n\pi}{2}\right) \tag{2.0.2}$$

$$X[k] = \sum_{n=0}^{7} x[n] e^{-j\frac{\pi k n}{4}} \tag{2.0.3}$$

Few identities we would use in this solution

$$e^{2jn\pi} = 1 \ \forall \ n \ \in \ \mathbb{N} \tag{2.0.4}$$

$$\sum_{r=0}^{n-1} e^{\frac{2jr\pi}{n}} = 0 \tag{2.0.5}$$

Using (2.0.2) and (2.0.3), we get

$$X[k] = \sum_{n=0}^{7} \cos\left(\frac{n\pi}{2}\right)\left(\cos\left(\frac{n\pi k}{4}\right) + j\sin\left(\frac{n\pi k}{4}\right)\right) \tag{2.0.6}$$

$$X[k] = \sum_{n=0}^{7} \cos\left(\frac{n\pi}{2}\right)\cos\left(\frac{n\pi k}{4}\right) + j\cos\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi k}{4}\right) \tag{2.0.7}$$

Simplifying

$$2X[k] = \sum_{n=0}^{7} \cos\left(\frac{n\pi(k+2)}{4}\right) + \cos\left(\frac{n\pi(k-2)}{4}\right) + j\sin\left(\frac{n\pi(k+2)}{4}\right) - j\sin\left(\frac{n\pi(k-2)}{4}\right) \tag{2.0.8}$$

This can be written as

$$2X[k] = \sum_{n=0}^{7} e^{j\frac{n\pi(k+2)}{4}} + e^{-j\frac{n\pi(k-2)}{4}} \tag{2.0.9}$$

Lets consider the following cases

1) $k = 2$

$$2X[2] = \sum_{n=0}^{7} e^{nj\pi} + 1 = 8 \tag{2.0.10}$$

Using (2.0.5), we get

$$X[2] = 4 \tag{2.0.11}$$

2) $k = 6$

$$2X[6] = \sum_{n=0}^{7} e^{2nj\pi} + e^{nj\pi} = 8 \tag{2.0.12}$$

Using (2.0.4) and (2.0.5)

$$X[6] = 4 \tag{2.0.13}$$

3) $k \in \{0,1,3,4,5,7\}$

$$2X[k] = \sum_{n=0}^{7} \frac{e^{2\pi(k+2)} - 1}{e^{\frac{\pi(k+2)}{4}} - 1} + \frac{e^{-2\pi(k-2)} - 1}{e^{\frac{-\pi(k-2)}{4}} - 1} \tag{2.0.14}$$

Using (2.0.4)

$$\therefore X[k] = 0 \ \forall \ k \in \{0,1,3,4,5,7\} \tag{2.0.15}$$

Therefore, **Option C** is correct