

Gate Assignment - 2

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Download all the python codes from

<https://github.com/cmaspi/EE3900/tree/main/GateAssignment-2/code>

latex-tikz codes from

<https://github.com/cmaspi/EE3900/blob/main/GateAssignment-2/main.tex>

P_N is the permutation matrix defined as

$$\mathbf{P}_N = [a_{ij}]_{N \times N}, i, j \in \{0, 1, \dots, N-1\} \quad (2.0.6)$$

$$a_{ij} = \begin{cases} 1 & j = 2i, i < \frac{N}{2} \\ 1 & j = 2(i - \frac{N}{2}) + 1, i \geq \frac{N}{2} \\ 0 & \text{otherwise} \end{cases} \quad (2.0.7)$$

For $N = 4$

$$\mathbf{F}_4 = \begin{pmatrix} \mathbf{I}_2 & \mathbf{D}_2 \\ \mathbf{I}_2 & -\mathbf{D}_2 \end{pmatrix} \begin{pmatrix} \mathbf{F}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_2 \end{pmatrix} \mathbf{P}_4 \quad (2.0.8)$$

I_2 is 2×2 matrix

$$\mathbf{D}_2 = \begin{pmatrix} w_4^0 & 0 \\ 0 & w_4^1 \end{pmatrix} \quad (2.0.9)$$

$$\mathbf{P}_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (2.0.10)$$

1 PROBLEM

(GATE EC 2009 Q42) The 4-point discrete fourier transform (DFT) of a discrete time sequence $[1, 0, 2, 3]$ is given by

- 1) $[0, -2 + 2j, 2, -2 - 2j]$
- 2) $[2, 2 + j, 6, 2 - 2j]$
- 3) $[6, 1 - 3j, 2, 1 + 3j]$
- 4) $[6, -1 + 3j, 0, -1, -3j]$

2 SOLUTION

The discrete input signal is

$$x(n) = [1, 0, 2, 3] \quad (2.0.1)$$

Let \mathbf{F}_N be the N-point DFT matrix.

Using the property of complex exponentials, we can express \mathbf{F}_N in terms of $\mathbf{F}_{N/2}$

$$\mathbf{F}_N = \begin{pmatrix} \mathbf{I}_{N/2} & \mathbf{D}_{N/2} \\ \mathbf{I}_{N/2} & -\mathbf{D}_{N/2} \end{pmatrix} \begin{pmatrix} \mathbf{F}_{N/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{N/2} \end{pmatrix} \mathbf{P}_N \quad (2.0.2)$$

Where

$$\mathbf{F}_N = \begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & w & \dots & w^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & w^{N-1} & \dots & w^{(N-1)(N-1)} \end{pmatrix} \quad (2.0.3)$$

where $w = e^{-\frac{2\pi j}{N}}$

$$\mathbf{D}_N = \begin{pmatrix} w_{2N}^0 & 0 & \dots \\ 0 & w_{2N}^1 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix} \quad (2.0.4)$$

$$\mathbf{D}_N = \text{diag}(w_{2N}^0, w_{2N}^1, \dots, w_{2N}^{N-1}) \quad (2.0.5)$$

Now,

$$\begin{pmatrix} X(0) \\ X(1) \end{pmatrix} = \begin{pmatrix} X_e(0) \\ X_e(1) \end{pmatrix} + \mathbf{D}_2 \begin{pmatrix} X_o(0) \\ X_o(1) \end{pmatrix} \quad (2.0.11)$$

$$\begin{pmatrix} X(2) \\ X(3) \end{pmatrix} = \begin{pmatrix} X_e(0) \\ X_e(1) \end{pmatrix} - \mathbf{D}_2 \begin{pmatrix} X_o(0) \\ X_o(1) \end{pmatrix} \quad (2.0.12)$$

This results in

$$\begin{pmatrix} X(0) \\ X(1) \end{pmatrix} = \mathbf{F}_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -j \end{pmatrix} \mathbf{F}_2 \begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad (2.0.13)$$

$$\begin{pmatrix} X(0) \\ X(1) \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 3j \end{pmatrix} \quad (2.0.14)$$

$$\begin{pmatrix} X(0) \\ X(1) \end{pmatrix} = \begin{pmatrix} 6 \\ -1 + 3j \end{pmatrix} \quad (2.0.15)$$

and

$$\begin{pmatrix} X(2) \\ X(3) \end{pmatrix} = \mathbf{F}_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & -j \end{pmatrix} \mathbf{F}_2 \begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad (2.0.16)$$

$$\begin{pmatrix} X(2) \\ X(3) \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 3j \end{pmatrix} \quad (2.0.17)$$

$$\begin{pmatrix} X(2) \\ X(3) \end{pmatrix} = \begin{pmatrix} 0 \\ -1 - 3j \end{pmatrix} \quad (2.0.18)$$

The correct answer is **option 4**

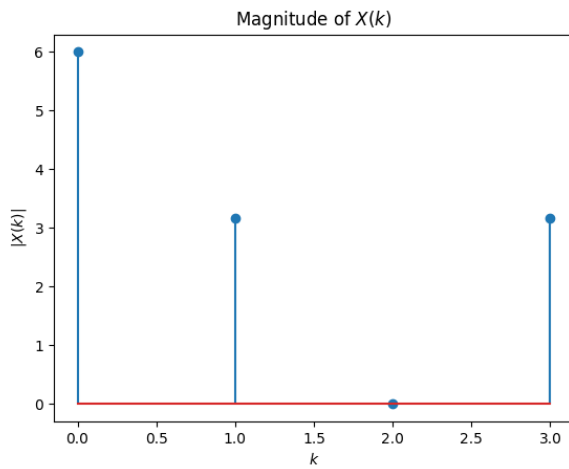


Fig. 4: Magnitude of $X(k)$

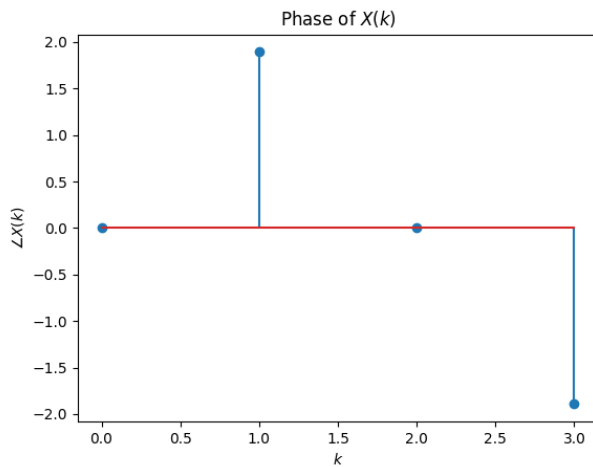


Fig. 4: Phase of $X(k)$

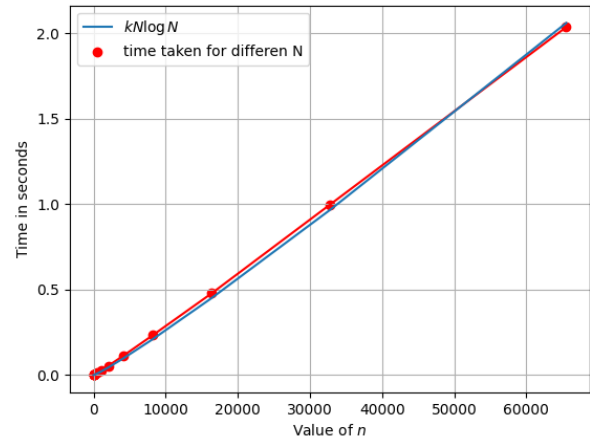


Fig. 4: Time Complexity of fast Fourier transform

The following is a plot for time complexity of fast fourier transform obtained for multiple random sin-gals. The value of k obtained from my computation is $3.25 \times 10^{-6} \pm 2\%$