Assignment 5 Presentation

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Question

Quadratic forms Q2.66

Find the point at which the line $\begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{x} = 1$ is a tangent to the curve $y^2 = 4x$

Solution

The general form of a conic is given by

$$\mathrm{x}^T \mathrm{V} \mathrm{x} + 2\mathrm{u}^T \mathrm{x} + f = 0 \tag{1}$$

for $y^2 = 4x$

$$\mathrm{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \ \mathrm{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, \ f = 0 \tag{2}$$

Solution Contd.

For any given conic such that V is non-invertible, the point of tangency is given by

$$\mathbf{V}\mathbf{q} = k\mathbf{n} - u \tag{3}$$

$$\text{where, } k = \frac{\mathbf{p}_1^T \mathbf{u}}{\mathbf{p}_1^T \mathbf{n}}, \ \mathbf{V}\mathbf{p}_1 = 0 \tag{4}$$

Clearly

$$\mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{5}$$

$$\implies k = \frac{-2}{-1} = 2 \tag{6}$$

Solution Contd.

Using the obtained values

$$\mathsf{V}\mathsf{q} = 2\mathsf{n} - \mathsf{u} \tag{7}$$

$$\mathsf{V}\mathsf{q} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} = 2\mathsf{e}_2 \tag{8}$$

$$\mathsf{V}(\mathsf{q} - 2\mathsf{e}_2) = 0 \tag{9}$$

Solution Contd.

The basis vector for null space of V is e_1

$$\therefore q - 2e_2 = \lambda e_1 \tag{10}$$

$$\implies q = 2e_2 + \lambda e_1 \tag{11}$$

Using (11) and (1), we get

$$2q^T e_2 - 4e_1^T q = 0 \tag{12}$$

Evaluating this, we get

$$4 - \lambda = 0 \tag{13}$$

$$\implies q = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{14}$$

Solution Contd.

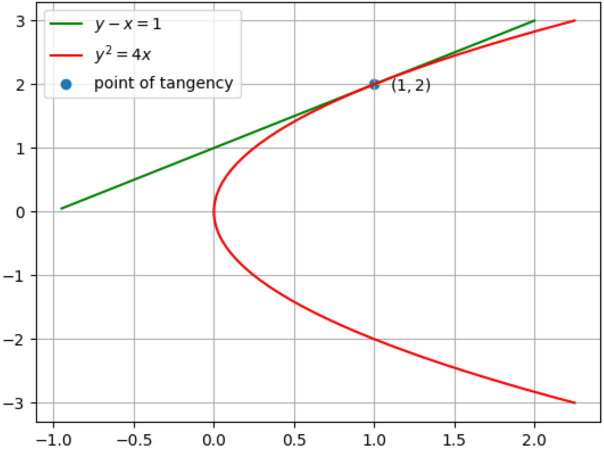

Figure: Plot of the line and parabola

Another approach to solve eq:8

$$q = A^{+}b + [I_n - A^{+}A]w \tag{15}$$

where A^{+} is pseudoinverse of A and w is any $n \times 1$ vector
For idempotent matrix, the pseudoinverse of the matrix is itself.

$$q = Vb + [I - V]w \tag{16}$$
$$q = 2e_2 + \lambda e_1 \tag{17}$$