3

Model Predictive Control

3.1 Predictive Control Methods for Power Converters and Drives

Predictive control covers a very wide class of controllers that have found rather recent application in power converters. A classification for different predictive control methods is shown in Figure 3.1, as proposed in [1].

The main characteristic of predictive control is the use of a model of the system for predicting the future behavior of the controlled variables. This information is used by the controller to obtain the optimal actuation, according to a predefined optimization criterion.

The optimization criterion in hysteresis-based predictive control is to keep the controlled variable within the boundaries of a hysteresis area [2], while in trajectory-based control the variables are forced to follow a predefined trajectory [3]. In deadbeat control, the optimal actuation is the one that makes the error equal to zero in the next sampling instant [4, 5]. A more flexible criterion is used in model predictive control (MPC), expressed as a cost function to be minimized [6].

The difference between these groups of controllers is that deadbeat control and MPC with continuous control set need a modulator in order to generate the required voltage. This will result in having a fixed switching frequency. The other controllers directly generate the switching signals for the converter, do not need a modulator, and will present a variable switching frequency.

One advantage of predictive control is that concepts are very simple and intuitive. Depending on the type of predictive control, implementation can also be simple, as with deadbeat control and finite control set MPC (especially for a two-level converter with horizon $N = 1$). However, some implementations of MPC can be more complex if the continuous control set is considered. Variations of the basic deadbeat control, in order to make it more robust, can also become very complex and difficult to understand.

Using predictive control it is possible to avoid the cascaded structure which is typically used in a linear control scheme, obtaining very fast transient responses. An example of this is speed control using trajectory-based predictive control.

Nonlinearities in the system can be included in the model, avoiding the need to linearize the model for a given operating point, and improving the operation of the system for all

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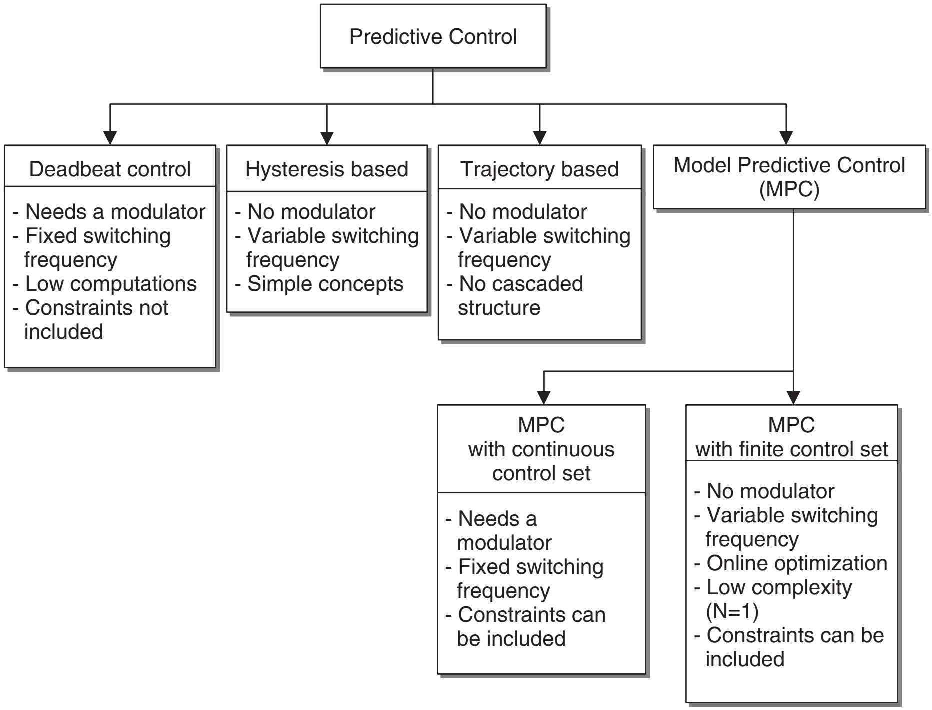

Figure 3.1 Classification of predictive control methods used in power electronics (Cortes *et al.*, 2008 © IEEE)

conditions. It is also possible to include restrictions on some variables when designing the controller. These advantages can be very easily implemented in some control schemes, such as MPC, but are very difficult to obtain in schemes like deadbeat control.

This book will focus on the application of MPC to power converters and drives, considering a finite control set and finite prediction horizon. More details will be found in the following chapters.

3.2 Basic Principles of Model Predictive Control

Among the advanced control techniques, that is, more advanced than standard PID control, MPC is one that has been successfully used in industrial applications [7–9]. Although the ideas of MPC were developed in the 1960s as an application of optimal control theory, industrial interest in these ideas started in the late 1970s [10]. Since then, MPC has been successfully applied in the chemical process industry, where time constants are long enough to perform all the required calculations. Early applications of the ideas of MPC in power electronics can be found from the 1980s considering high-power systems with low switching frequency [2]. The use of higher switching frequencies was not possible at that time due to the large calculation time required for the control algorithm. However, with the development of fast and powerful microprocessors, interest in the application of MPC in power electronics has increased considerably over the last decade.

MPC describes a wide family of controllers, not a specific control strategy [7]. The common elements of this kind of controller are that it uses a model of the system to predict the future behavior of the variables until a predefined horizon in time, and selection of the optimal actuations by minimizing a cost function. This structure has several important advantages:

- Concepts are very intuitive and easy to understand.
- It can be applied to a great variety of systems.
- The multivariable case can be easily considered.
- Dead times can be compensated.
- Easy inclusion of non linearities in the model.
- Simple treatment of constraints.
- The resulting controller is easy to implement.
- This methodology is suitable for the inclusion of modifications and extensions depending on specific applications.

However, some disadvantages have to be mentioned, like the larger number of calculations, compared to classic controllers. The quality of the model has a direct influence on the quality of the resulting controller, and if the parameters of the system change in time, some adaptation or estimation algorithm has to be considered.

The basic ideas present in MPC are:

- The use of a model to predict the future behavior of the variables until a horizon in time.
- A cost function that represents the desired behavior of the system.
- The optimal actuation is obtained by minimizing the cost function.

The model used for prediction is a discrete-time model which can be expressed as a state space model as follows:

$$\mathbf{x}(k+1) = A\mathbf{x}(k) + B\mathbf{u}(k) \tag{3.1}$$

$$\mathbf{y}(k) = C\mathbf{x}(k) + D\mathbf{u}(k) \tag{3.2}$$

A cost function that represents the desired behavior of the system needs to be defined. This function considers the references, future states, and future actuations:

$$J = f(\mathbf{x}(k), \mathbf{u}(k), \ldots, \mathbf{u}(k+N)) \tag{3.3}$$

MPC is an optimization problem that consist of minimizing the cost function J, for a predefined horizon in time N, subject to the model of the system and the restrictions of the system. The result is a sequence of N optimal actuations. The controller will apply only the first element of the sequence

$$\mathbf{u}(k) = [1\ 0\ \cdots\ 0] \arg\min_u J \tag{3.4}$$

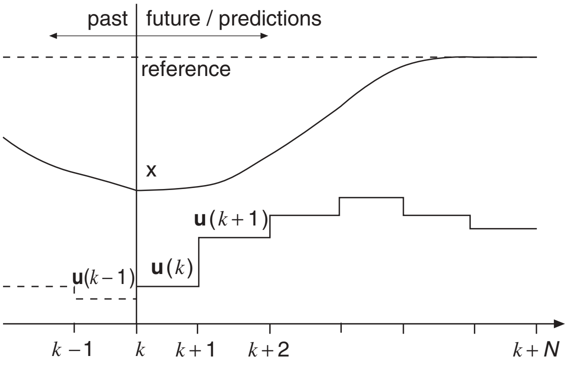

Figure 3.2 Working principle of MPC

where the optimization problem is solved again each sampling instant, using the new measured data and obtaining a new sequence of optimal actuations each time. This is called a *receding horizon* strategy.

The working principle of MPC is summarized in Figure 3.2. The future values of the states of the system are predicted until a predefined horizon in time $k + N$ using the system model and the available information (measurements) until time k. The sequence of optimal actuations is calculated by minimizing the cost function and the first element of this sequence is applied. This whole process is repeated again for each sampling instant considering the new measured data.

3.3 Model Predictive Control for Power Electronics and Drives

Although the theory of MPC was developed in the 1970s, its application in power electronics and drives is more recent due to the fast sampling times that are required in these systems. The fast microcontrollers available in the last decade have triggered research in new control schemes, such as MPC, for power electronics and drives.

As mentioned previously, MPC includes a very wide family of controllers and several different implementations have been proposed. An interesting alternative is the use of generalized predictive control (GPC), which allows solution of the optimization problem analytically, when the system is linear and there are no constraints, providing an explicit control law that can be easily implemented [11, 12]. This control scheme has been used in several power converter [13–15] and drive applications [16–18].

In order to make possible the implementation of MPC in a real system, considering the little time available for calculations due to the fast sampling, it has been proposed to move most of the optimization problem offline using a strategy called explicit MPC. The optimization problem of MPC is solved offline considering the system model, constraints, and objectives, resulting in a look-up table containing the optimal solution as a function of the state of the system. Explicit MPC has been applied for the control of power converters such as DC–DC converters and three-phase inverters [19, 20], and in the control of permanent magnet synchronous motors [21].

Most GPC and explicit MPC schemes approximate the model of the power converter as a linear system by using a modulator. This approximation simplifies the optimization and allows the calculation of an explicit control law, avoiding the need for online optimization. However, this simplification does not take into account the discrete nature of the power converters.

By including the discrete nature of power converters, it is possible to simplify the optimization problem, allowing its online implementation. Considering the finite number of switching states, and the fast microprocessors available today, calculation of the optimal actuation by online evaluation of each switching state is a real possibility. This consideration allows more flexibility and simplicity in the control scheme, as will be explained in subsequent chapters of this book. As the switching states of the power converters allows finite number of possible actuations, this last approach has been called, in some works, finite control set MPC.

3.3.1 Controller Design

In the design stage of finite control set MPC for the control of a power converter, the following steps are identified:

- Modeling of the power converter identifying all possible switching states and its relation to the input or output voltages or currents.
- Defining a cost function that represents the desired behavior of the system.
- Obtaining discrete-time models that allow one to predict the future behavior of the variables to be controlled.

When modeling a converter, the basic element is the power switch, which can be an IGBT, a thyristor, a gate turn-off thyristor (GTO), or others. The simplest model of this power switches considers an ideal switch with only two states: on and off. Therefore, the total number of switching states of a power converter is equal to the number of different combinations of the two switching states of each switch. However, some combinations are not possible, for example, those combinations that short-circuit the DC link.

As a general rule, the number of possible switching states N is

$$N = x^y \tag{3.5}$$

where x is the number of possible states of each leg of the converter, and y is the number of phases (or legs) of the converter. In this way a three-phase, two-level converter has $N = 2^3 = 8$ possible switching states, a three-phase, three-level converter has $N = 3^3 = 27$ switching states, and a five-phase, two-level converter has $N = 2^5 = 32$ switching states. In some multilevel topologies the number of switching states of the converter can be very high, as in a three-phase, nine-level cascaded H-bridge inverter, where the number of switching states is more than 16 million.

Another aspect of the model of the converter is the relation between the switching states and the voltage levels, in the case of single-phase converters, or voltage vectors, in the case of three-phase or multi-phase converters. For current source converters, the possible switching states are related to current vectors instead of voltage vectors. It can be found that, in several cases, two or more switching states generate the same voltage vector.

For example, in a three-phase, two-level converter, the eight switching states generate seven different voltage vectors, with two switching states generating the zero vector. In a three-phase, three-level converter there is a major redundancy, with 27 switching states generating 19 different voltage vectors. Figure 3.3 depicts the relation between switching

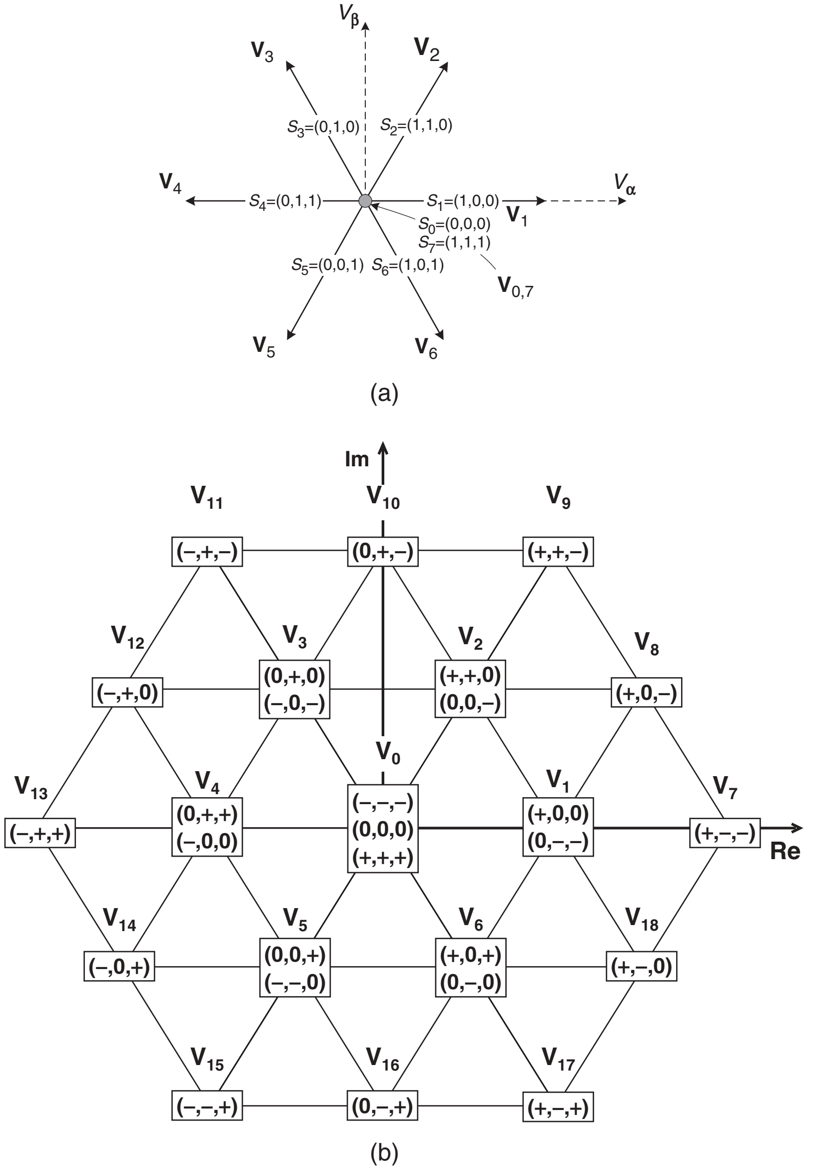

Figure 3.3 Voltage vectors generated by different converters. (a) Three-phase, two-level inverter. (b) Three-phase, three-level inverter

states and voltage vectors for two different converter topologies. In some other topologies, the method of calculating the possible switching states may be different.

Each different application imposes several control requirements on the systems such as current control, torque control, power control, low switching frequency, etc. These requirements can be expressed as a cost function to be minimized. The most basic cost function to be defined is some measure of error between a reference and a predicted variable, for example, load current error, power error, torque error, and others, as will be shown in the following chapters of this book. However, one of the advantages of the predictive control methods is the possibility to control different types of variables and include restrictions on the cost function. In order to deal with the different units and magnitudes of the controlled variables, each term in the cost function is multiplied by a weighting factor that can be used to adjust the importance of each term.

When building the model for prediction, the controlled variables must be considered in order to get discrete-time models that can be used for the prediction of these variables. It is also important to define which variables are measured and which ones are not measured, because in some cases variables that are required for the predictive model are not measured and some kind of estimate will be needed.

To get a discrete-time model it is necessary to use some discretization methods. For first-order systems it is useful, because it is simple, to approximate the derivatives using the Euler forward method, that is, using

$$\frac{dx}{dt} = \frac{x(k+1) - x(k)}{T_s} \tag{3.6}$$

where T_s is the sampling time. However, when the order of the system is higher, the discrete-time model obtained using the Euler method is not so good because the error introduced by this method for higher order systems is significant. For these higher order systems, an exact discretization must be used.

3.3.2 Implementation

When implemented, the controller must consider the following tasks:

- Predict the behavior of the controlled variables for all possible switching states.
- Evaluate the cost function for each prediction.
- Select the switching state that minimizes the cost function.

Implementation of predictive models and a predictive control strategy may encounter different difficulties depending on the type of platform used. When implemented using a fixed-point processor, special attention must be paid to programming in order to get the best accuracy in the fixed-point representation of the variables. On the other hand, when implemented using a floating-point processor, almost the same programming used for simulations can be used in the laboratory.

Depending on the complexity of the controlled system, the number of calculations can be significant and will limit the minimum sampling time. In the simplest case, predictive current control, the calculation time is small, but in other schemes such as torque and flux control, the calculation time is the parameter which determines the allowed sampling time.

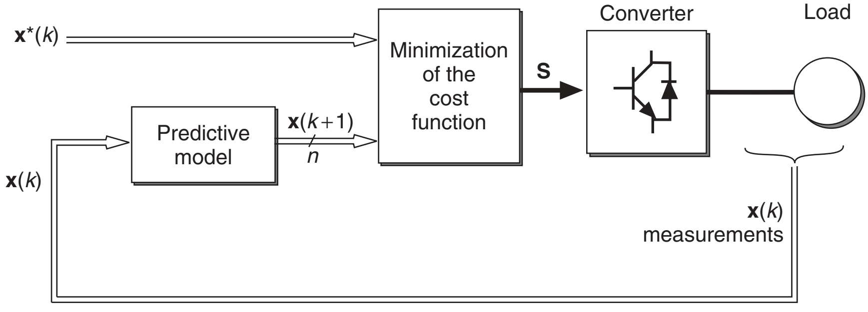

Figure 3.4 General MPC scheme for power converters

To select the switching state which minimizes the cost function, all possible states are evaluated and the optimal value is stored to be applied next. The number of calculations is directly related to the number of possible switching states. In the case of the three-phase, two-level inverter, to calculate predictions for the eight possible switching states is not a problem, but in the case of multi level and multi-phase systems, a different optimization method must be considered in order to reduce the number of calculations.

3.3.3 General Control Scheme

A general control scheme for MPC applied to power converters and drives is presented in Figure 3.4. The power converter can be from any topology and number of phases, while the generic load shown in the figure can represent an electrical machine, the grid, or any other active or passive load. In this scheme measured variables $\mathbf{x}(k)$ are used in the model to calculate predictions $\mathbf{x}(k+1)$ of the controlled variables for each one of the n possible actuations, that is, switching states, voltages, or currents. Then these predictions are evaluated using a cost function which considers the reference values $\mathbf{x}^*(k)$ and restrictions, and the optimal actuation $\mathbf{S}$ is selected and applied in the converter.

3.4 Summary

This chapter presents an overview of different predictive control methods. The basic principles of MPC and its application for power converters and drives are presented. A general control scheme has been introduced in this chapter and will be considered in all applications included in this book.

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