Model Predictive Control of Skeboå Water system

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Abstract

This thesis is a study of model predictive control of water levels and flows in a water system. The water system studied includes five lakes and six dams that are regulated manually by sluice-gates. The water is used in the papermaking process at Holmen Paper Mill in Hallstavik. The aim of this thesis is to find out how to control the water system when all dams are automated and to minimize the discharge of water from the system without risking production stops due to water shortage. To fulfil the aim, a simulation is made during a dry period with low amount of rain. The simulation is then compared to the same period but when the system was manually controlled.

In this thesis two models of the water system are constructed, a simple linear model and a more complex non-linear model. In the linear model the channels between the lakes are assumed to be delays of water flow. In the non-linear model the same channels are described by Saint Venant equations of changes of flow and Manning's equation on how water flow and the cross-section of a channel are related. In both models, the lakes are modelled as the change in volume with respect to time due to inflow to and outflow from the lake.

The non-linear model is verified against measured water levels, flows, sluice-gate heights and precipitation to ensure that the model describes the water system well enough.

The linear model is used in the model predictive controller to calculate the optimal outflow from the dams. The optimal outflows are then converted into optimum gate heights in the dams, which in turn are used as input to the non-linear model. The non-linear model is used to simulate the water system.

The results from the simulation show that the control of the water system can significantly be improved. The conclusion of this thesis is that a lot more water can be saved when the system is automated and that the water levels in the lakes can be kept more stable with respect to a set reference level. The recommendation if only one dam is to be controlled initially is to start with the dam at Närdingen.

Sammanfattning

Denna rapport är en studie i modellbaserad prediktionsreglering av vattennivåer och vattenflöden i ett vattensystem. Vattensystemet innefattar fem sjöar som har tillsammans sex dammar som regleras manuellt med hjälp av dammluckor. Vattnet används för tillverkning av papper i Holmen Papers pappersbruk i Hallstavik. Målsättningen med studien är att komma fram till hur vattensystemet ska regleras när alla dammar är automatiserade och att minimera vattenutsläppet ur systemet utan att riskera produktionsstopp på grund av vattenbrist. För att undersöka det genomförs en simulering av systemet under en torr period med lite nedebörd. Denna simulering jämförs sedan med hur systemet reglerades manuellt under samma tidsperiod.
I studien konstrueras två modeller av vattensystemet, en enklare linjär modell och en mer komplex olinjär modell. I den linjära modellen antas det att vattenkanalerna mellan sjöarna är flödesfördröjningar. I den olinjära modellen beskrivs vattenkanalerna med hjälp av Saint Venants ekvationer om flödesförändringar och Mannings ekvation om hur vattenflödet relaterar till tvärsnittsarean av kanalen. I båda modellerna modelleras sjöarna som förändring av volym beroende på inflöde och utflöde.
Den olinjära modellen verifieras gentemot uppmätta vattennivåer, flöden, dammluckslägen och mängden nedebörd i området för att säkerställa att modellen beskriver systemet tillräckligt bra.
Den linjära modellen används i regleringen för att ta fram optimala flödena från dammarna. De optimala flödena räknas sedan om till optimala luckhöjder i dammarna som i sin tur används som input i den olinjära modellen. Den olinjära modellen används för att simulera vattensystemet.
Resultatet från simuleringen visar på att förbättringar i styrningen av systemet kan göras. Slutsatsen från studien är att vatten kan sparas i systemet och att vattennivåerna i sjöarna kan hållas jämnare. Rekommendationen om en dam automatiseras till en början är att välja dammen vid Närdingens utlopp.

Acknowledgments

First I would like to thank my grandfather Bengt Gabrielsson for inspiring me to write this Master thesis about open water system.
This Master thesis completes my studies for a Master of Science degree in Electrical Engineer at Kungliga Tekniska Högskolan in Stockholm, Sweden.
I would also like to thank my advisor Tobias Janson at Hallsta Paper Mill and my examiner and supervisor Elling Jacobsen at Kungliga Tekniska Högskolan for their comments and tips. I would also thank all the people around the world providing information about open water system and Model predictive control theory.
Finally I would like to thank my girlfriend for her support and encouragement.

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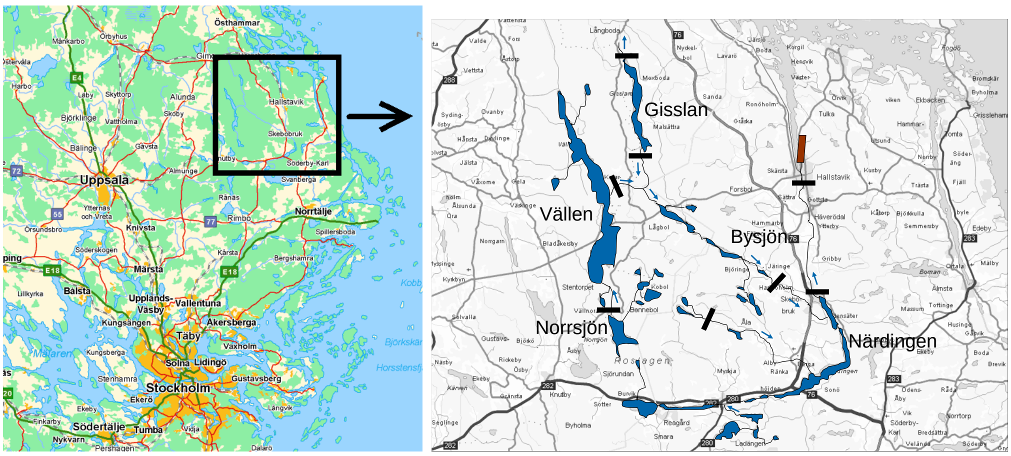
Contents

Chapter 1

Introduction

1.1 Background

Freshwater is essential for the papermaking process. The freshwater that Hallsta Paper mill needs comes from a reservoir system with five larger lakes (see figure 1.1). Hallsta paper mill is located in northern Roslagen, about 100 kilometres north of Stockholm, in a town called Hallstavik.



(a) Large map (b) Map of waterreservoir

Figure 1.1: Both maps show the location of the water system. Figure 1.1a shows where it is located in relation to Stockholm and Uppsala. Figure 1.1b highlights which lakes are in the system, where the dams are located (black rectangles) and in what direction the water flows (blue arrows).

One of the reasons why the paper mill was founded at this location (in 1912) was the large supply of freshwater. It was realised quite early that the flow of water needed to be controlled so that the supply always exceeds the amount of water that the factory needs. In 1923 the Swedish water court decided the boundaries of Skeboå, from the lake Närdingen to Hallstavik and the outflow in the Baltic Sea, for example the lowest water flow as well as the highest and lowest water level in each lake. It also concluded the fines that Holmen AB would have to pay if those constraints where exceeded. Soon after the first sluice-gate dam was constructed at Skebobruk.

In 1955 another water-right judgement was declared, this time including Vällen, Bysjön, Gisslaren, Norrsjön and Stora Mårdsjön as well Närdingen. The boundaries of that judgement will be described later on, in section 3.1.1.

In the 1970's the amount of water that the mill took from Skeboå was 75 $\mathtt{m}^3/\mathtt{min}$ (1.25 $\mathtt{m}^3/\mathtt{s}$)[1]. Since then the processes in the paper mill have been refined and this together with environmental awareness has lead to a decrease in water demand. Today the mill takes between 30 - 55 $\mathtt{m}^3/\mathtt{min}$ (0.5 - 0.92 $\mathtt{m}^3/\mathtt{s}$), however the risk of shortage of freshwater has been high in later years. This has lead to the demand for a better control strategy of the water dams.

1.1.1 Hallsta Paper mill

Here follow a short presentation of numbers about Hallsta Paper mill. The paper mill has three paper machines that produce 700 000 ton paper each year, such as journal paper and paper for pocket books. To make all this paper the mill needs 1 000 000 m^3 of Norway spruce and 350 000 m^3 of wood chips from saw mills each year. For the paper making process the mill needs 2 TWh/year of electric power and 10 000 ton/year of oil. This is as much power as two times the power needed to supply the city of Malmö including its industries during a year. During each minute 0.25 m^3/s of water is used as cooling water and another 0.25 m^3/s of water is used in the papermaking process.

1.2 Problem statement

The main objective of this thesis is to investigate how to control the water system when every dam is automated. The strategy should include the option to manuever the gates manually or automatically in order to change the water level at each gate. Water flow over the gates should be avoided during wintertime if possible because it would allow ice to build up and at the same time make it harder to change the level of the gate. The following objectives are stated for this thesis:

- Suggest a control strategy for the water system which follows the judgements of the water court.
- Ensure sufficient supply of water to the paper mill. It is important not to waste water before and during dry seasons.
- Ensure that the flows between the lakes are not below minimum flow rates specified by the water-rights judgement.

- Recommend which dam that should be automated first.
- The control strategy shall be within Swedish laws and regulations.

1.3 Outline of the thesis

Chapter 1 gives an overview of this thesis as well as a background, the stated objectives and the assumptions of the thesis.
This is followed by chapter 2 that gives an introduction to what an ”Open water system” is. It describes the basic theory for modelling a lake and a channel. It also describes a linear model of a water system with lakes and channels.
Chapter 3 gives a general description of Skeboå water system, followed by how it is modelled and how the channels are identified. That model is then verified using real data. The chapter ends with how the same system was modelled using the linear modelling explained in section 2.4.
Chapter 4 provides the the theory of the control method used, Model Predictive Control, and the various parts of that method.
Chapter 5 explains how the flows in the channels of Skeboå water system were analysed and how the control step time (used in the MPC simulation) was chosen.
Chapter 6 describes the settings for the MPC simulation and how the MPC was tuned.
In chapter 7 the results of the MPC simulation are presented and discussed.
The thesis concludes with chapter 8 which states the conclusions made in this thesis. It also presents suggestions for future work.

1.4 Assumptions of the Thesis

Since the objectives are wide in range some assumptions of the system need to be made. The assumptions made for the models are:

- The flow in the channels is steady and uniform (only friction and gravity are taken into account).
- The shapes of the lake areas are neglected.
- The lake area is the same at every depth of each lake.
- Smaller lakes within the system are disregarded.
- The gates at each dam are counted as one, with the width of the total gate width of that dam.
- The rainfall over the whole runoff area is the same regardless of location.

The control strategy that will be used is Model Predictive Control. The simulations will be done with real start values from 27th of May 2002 and 30 days forward. This period was chosen because it starts after the spring flood and during that summer the supply of water was low.

Chapter 2

Open Water System

2.1 General introduction to an open water system

A water system is a type of flow based system that contains water. Pressure and flow are the quantities that define this type of system. An example that most people can relate to is tap water, which runs in pipes to households so that the residents do not have to go to the local well for it. Such a system relies on water pressure so that the water starts flowing when someone opens the tap. A tap water system is a closed water system because it consists of pipes and tanks. Water pressure in the system is not directly affected by the atmospheric pressure if the tanks are sealed. This characteristic distinguishes closed water systems from open water systems. In contrast to closed systems, open systems are always affected by atmospheric pressure. The open system dealt with in this thesis consists of lakes and channels. In this case the term channel means all kinds of natural streams.
Section 2.2 describes the equations that are used for modelling a lake. The third section describes the equations used for modelling of a natural channel. The fourth section describes a linear model of an open water system.

2.2 Theory for modelling a lake.

The inflow to a lake consists of water flow from upstream channels and rainwater from the area surrounding the lake and upstream channels. The outflow from a lake consist of water passing through the soil and water running out into downstream channels. The inflow, outflow and the water level of the lake can be controlled for example with dams and weirs.
The changes in water volume when the water density stay the same are described by the volume balance equation [2]

$$\frac{dV}{dt} = Q_{in} - Q_{out} \tag{2.1}$$

where V is the volume (m^3), Q_{in} is the inflow (m^3/s) and Q_{out} is the outflow (m^3/s)
.
The following subsections describe the equations for the water flow.

2.2.1 Rainfall

Rainfall can vary significantly both in geographical location and in time. As stated above, the rain is an input to an open water system. As the amount of rain can not be controlled it is considered to be a disturbance [3].
The amount of rainwater that falls within an open water system and its surroundings can estimated from a precipitation forecast. From a forecast an estimate of the inflow can be done e.g. with the rational method [4]. This method is described by the following equation [4]

$$Q = CiA \tag{2.2}$$

where Q is the maximum rate of flow($\mathtt{m}^3/\mathtt{s}$), C is a run-off coefficient representing the fraction of rainfall that becomes inflow, i is the rain intensity ($\mathtt{m/s}$) and A the drainage area($\mathtt{m}^2$). This method will be used in this thesis.

2.2.2 Structures in a open water system

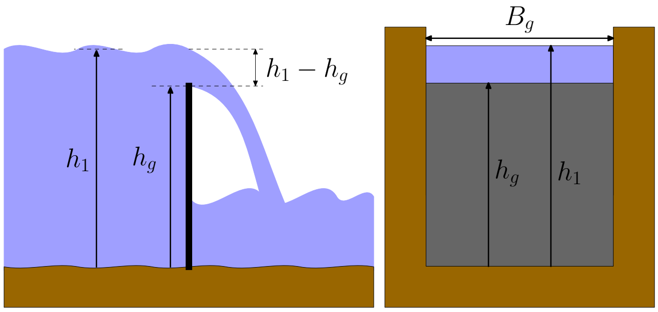
There are several types of structures that can be placed in a open water system to control it. These can be divided into two groups, fixed and non-fixed structures [5].
A weir is structure that allows the water to flow over it once the water level reaches a certain reference level. As the structure is fixed, the reference level cannot be changed without rebuilding the weir. Weirs can be used for measuring discharge, altering the flow characteristics of a channel and to prevent flooding [6]. There are also other types of fixed structures, but they will not be discussed in this thesis.
Non-fixed structures are constructions with movable parts. Some examples of such structures are overshot gates, undershot gates (also called sluice gates) and pumps. These are used for controlling the discharge so that the water system reaches some desired state. Non-fixed constructions are used when there is a need for changing the reference level of a lake or the flow in a channel. The following two parts will describe more in detail the structures that are of interest in this thesis.

Weirs and overshot gates

Overshot gates are adjustable weirs. By adjusting the gate height the water level of the lakes as well as the outflow can be controlled. In this way the reference for the water level can be changed. The flow is described by the general equation (2.3) [6]

$$Q(k) = CLh(k)^{1.5} \tag{2.3}$$

where C is a flow coefficient, L is the length of the weir and h is the height of water over the weir crest.



(a) Sideview (b) Frontview

Figure 2.1: The figure shows an overshot gate dam or a weir, where h_1 is the height of water before the dam, h_g the height of the gate, B_g the width and the difference between h_1 and h_g is the height of water over the weir crest.

In this thesis the cross sectional area of the water flow directly above the dam is assumed to be rectangular. For a rectangular weir [7]

$$C = C_d \frac{2}{3} \sqrt{2g} \tag{2.4}$$

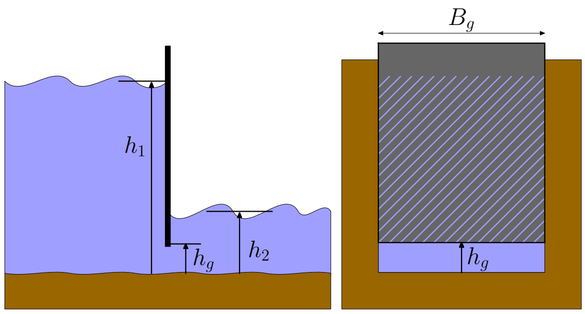
where g is gravitational acceleration and C_d is the discharge coefficient for the weir. Figure 2.1 gives a detailed view of a overshot gate or weir. As seen in Figure 2.1 the height of water over the weir crest is calculated by subtracting the height of the gate from the height of the lake.

Undershot gates

The other structure of interest in this thesis is the undershot gate (also called sluice gate). Instead of letting water flow over it, the undershot gate lets water pass through a gap between the gate and the bottom of the channel. This can be seen in figure 2.2. The flow under the gate can either be free flowing or submerged. Free flowing means that the water level after the dam is the same or lower than the opening height of the gate. On the other hand, submerged flow means that the water level after the gate is higher than the gap made by the gate.
When the flow is submerged it is described by [5]

$$Q(k) = C_g \cdot B_g \cdot \mu_g \cdot h_g(k) \sqrt{2 \cdot g \cdot (h_1(k) - h_2(k)} \tag{2.5}$$

where Q represents the flow under the gate ($\mathtt{m}^3/\mathtt{s}$), C_g the calibration coefficient, B_g the width of the gate ($\mathtt{m}$), μ_g the contraction coefficient, h_1 the upstream water level ($\mathtt{m}$), h_2 the downstream water level ($\mathtt{m}$), h_g the gate height, g the gravitational acceleration (=9.81 $\mathtt{m}/\mathtt{s}^2$). In this thesis, the height of the gap between the gate and the bottom of the channel will be referred to as the gate height. The gap opening area may have another form than rectangular but as it was not possible to find out the exact form of the gap openings that are dealt with in this thesis, all gap openings are assumed to be rectangular. When the flow is free flowing, the water level after the dam, $h_2(k)$, will not affect the flow from the gate.



(a) Sideview (b) Frontview

Figure 2.2: The figure shows a submerged undershotgate dam. Figure 2.2a is from the side and figure 2.2b shows a view from the front of the dam. h_1 is the water level before the dam, h_2 the water level after, h_g the height of the gate opening and B_g the width of the dam.

2.3 Theory for modelling a channel

In a channel, the water flow is driven by the gravitational forces. Once the water is in movement, there are also other forces that affect the flow behaviour of the water. The equation that describes these forces is [4]

$$\underbrace{\frac{1}{A}\frac{\delta Q}{\delta t}}_{(1)} + \underbrace{\frac{1}{A}\frac{\delta}{\delta x}(\frac{Q^2}{A})}_{(2)} + \underbrace{g\frac{\delta h}{\delta x}}_{(3)} - \underbrace{gS_0}_{(4)} + \underbrace{gS_f}_{(5)} = 0 \tag{2.6}$$

where Q is the flow (m^3/s), t is the time (s), A is the cross section area of the channel (m^2), S_0 is the slope of the channel, S_f is the slope of the friction, h is the water level of the channel (m) and g is the gravitational acceleration (=9.81 m/s^2). Equation (2.6) is called the momentum equation and is part of the Saint Venant equations [4]. Equation (2.6) describes the forces that influence the water level and flow in the channel. It can be divided into five parts. The local acceleration term (1), the convective acceleration term (2), the pressure force term (3), the gravity force term (4) and the friction force term (5). The first term describes the change in momentum due to the change in velocity over time and the second term is due to change in velocity along the channel. These two represent the effect of inertial forces on the flow. The third term is proportional to the change in water depth

along the river. The fourth term, the gravity force, is the bed slope and the last term is the friction force, which is the friction between the flowing water and the riverbed (e.g. due to vegetation and stones).
The Saint Venant equations contain two equations, the momentum equation described above and the continuity equation

$$\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} - q = 0 \tag{2.7}$$

where Q is the flow of the channel (m^3/s), t is the time (s), A is the area of the cross section (m^2) and q is the lateral inflow (m^2/s) [4].
The flow in a channel can be classified in many different types. The most common classification is made according to the change in flow depth with respect to time and space [6]. With respect to time, the flow depth can be said to be steady or unsteady. When the flow is unsteady the depth will change during the considered time, while when the flow is steady the depth will not change. The other classification is made with the respect to space. The flow is said to be uniformed or varied. This classification depends on how the flow depth changes along the channel. When the depth is the same in every section of the channel the flow is called uniform but if the depth is different, the flow is called varied [6].
A steady flow can be uniform or varied while an unsteady flow is practically always varied. A uniform unsteady flow is a practically impossible condition for an open water system, because it requires that the water level is parallel to the channel bottom at all times when the flow depth changes. Depending on which type of classification is used, some parts of the channel flow in equation (2.6) will be zero. A natural channel can be assumed to be uniform under normal conditions, as long as there is no flood or any strong varied flows caused by channel irregularities [6]. When the flow is steady and uniform only the friction and gravity parts are non-zero [4], which can be seen in equation (2.8).

$$\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} - q = 0 \tag{2.8a}$$

$$S_o = S_f \tag{2.8b}$$

The momentum equation can be expressed in the form

$$A = \alpha Q^{\beta} \tag{2.9}$$

where A is the area of the cross section (m^2) and Q is the flow (m^3/s) [4]. α and β are constants that depend on how the cross section relates to the flow in each specific channel. One equation in this form is Manning's formula

$$Q = \frac{1.49 S_0^{1/2}}{n P^{2/3}} A^{5/3} \tag{2.10}$$

where Q is the flow of water (m^3/s), S_0 the slope of the channel, n is Manning's coefficient, P is the wetted perimeter of the cross section and A is the area of the

cross section [4]. Manning's formula was developed empirically, which can be seen from Manning's coefficient. The value depends on many factors, for example the shape of the cross section and the state of the vegetation [6]. By comparing equation (2.9) with equation (2.10), the constants α and β are identified as $(\frac{nP^{2/3}}{1.49\sqrt{S_0}})^{3/5}$ and 0.6.
Equation (2.7) contains two dependent variables A and Q. A can be eliminated by taking the time derivative of equation (2.9)

$$\frac{\delta A}{\delta t} = \alpha\beta Q^{\beta-1}\frac{\delta Q}{\delta t} \tag{2.11}$$

and replacing $\delta A/\delta t$ to give an equation with only one dependent variable that describes the flow in a channel

$$\frac{\delta Q(t,x)}{\delta x} + \alpha\beta Q(t,x)^{\beta-1}\Big(\frac{\delta Q(t,x)}{\delta t}\Big) - q(t,x) = 0 \tag{2.12}$$

Then by discretization of equation (2.12) in space using the backward Euler method one gets an equation that is only time dependent

$$\frac{Q(t,x_j) - Q(t,x_{j-1})}{\Delta x} + \alpha\beta Q(t,x)^{\beta-1}\Big(\frac{dQ(t,x)}{dt}\Big) - q(t,x) = 0 \tag{2.13}$$

In equation (2.13) Δx is the length of the discretized channel segment, α and β are the cross sectional constant for that particular channel, $q(t,x)$ is the lateral inflow, $Q(t,x_j)$ is the outflow from the segment j in the channel and $Q(t,x_{j-1})$ is the inflow to the same segment. A long channel can be described by combining many of these equations, each describing one segment of the channel. One such segment can be seen in figure 2.3.

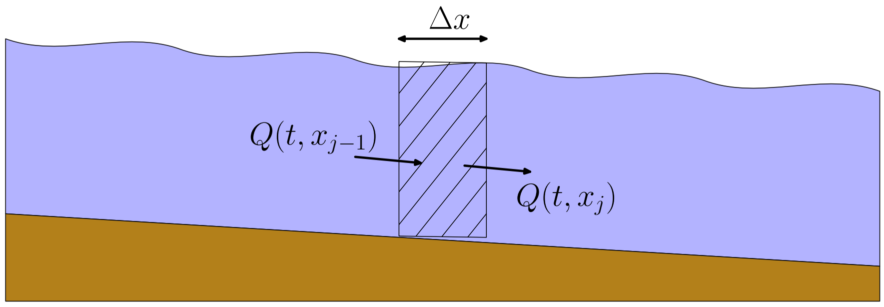


Figure 2.3: The figure show one segment of a channel at the time t with the length Δx, inflow $Q(t,x_{j-1})$ and outflow $Q(t,x_j)$.

2.3.1 Flow delay time

When flow changes at the beginning of a channel, a wave of water forms and travels downstream. As the wave does not instantly reach the end of the channel, delay time occurs and depend on the velocity of the wave and the length of the channel [4]. The time it takes for the wave to reach the end of the channel is described by equation

$$T_d = \frac{L}{c_k} \tag{2.14}$$

where c_k is the wave celerity and L is the length of the channel. The wave celerity is the velocity with which the variation in flow travels along the channel and is calculated by

$$c_k = \frac{dQ}{dA} \tag{2.15}$$

where Q is the flow and A is the area of the cross section [4]. As seen when rearranging equation (2.11), the wave celerity is $\frac{1}{\alpha\beta Q^{\beta-1}}$ and thereby the delay time is calculated by

$$T_d = L\alpha\beta Q^{\beta-1} \tag{2.16}$$

The delay time is modelled for the channel because the friction in a channel is greater than the friction in a reservoir [2].

2.4 Linear model of an open water system

The non linearity of an open water system, as described above, will make a Model Predictive Controller more complicated. Therefore a linear approximated model is required. One way is linearising the Saint Venant equations and getting a model that will simulate the whole dynamics of the open water system. In theory this can be very accurate [2]. However, the model will also be complex and the time it takes to compute the optimal inputs will be long. This will make such a model difficult to use in real-time applications [5].
Instead a simplified model, called Integrator-Delay (ID) model, can be used. This is the most commonly used model for MPC applications in irrigation channels [2]. It is also mathematically easier to handle and capture delay time in the channels and surface area of the lakes. The equation for the ID-model is

$$A\frac{dh(t)}{dt} = Q_{in}(t - T_d) - Q_{out}(t) \tag{2.17}$$

where A is the area of the reservoir (m^2), $h(t)$ is the height of the water level in the reservoir (m), $Q_{in}(t - T_d)$ is the inflow from the dam upstream of the reservoir at the time T_d and $Q_{out}(t)$ is the outflow.
The ID-model is simple to handle and needs less computational time to get the

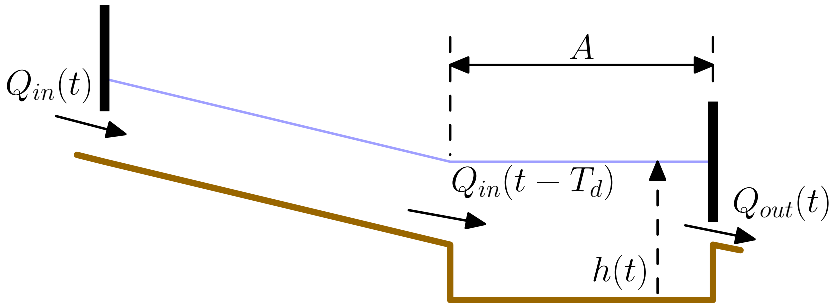


Figure 2.4: Simple interpretation of the Integrator-Delay model. $Q_{in}(t)$ is the inflow from a dam upriver from the reservoir. $Q_{in}(t-T_d)$ is the inflow T_d seconds ago (if t is measured in seconds). $h(t)$ is the height of the water level and $Q_{out}(t)$ is the outflow from the reservoir.

optimal input. The disadvantage is that it will be less accurate than the linearisation of the Saint Venant equations; it cannot model waves and it can only capture the main properties (time delay and surface area for working point). In case the water system contains channels with steep slopes then there is a risk that the closed loop system will be unstable. There are however ways to make the model stable. For example to make multiple models, one for each operating point, or a non-fixed internal model that is time-variant with the operating points. Another way is to filter out the basic frequency with a first order low-pass filter to channels that are sensitive to resonance waves [5].
The ID model contains two parts, a channel part and a reservoir part. The water flows first through the channel, out into the reservoir and then downriver out from the reservoir dam. To use the ID model one needs to make some assumptions of the flow [2]. The first assumption is that the flow in a channel in the water system is of uniform type and the water level in the reservoir is horizontal. The second is that the flow travels from upstream to downstream with some certain delay time in the channel. The third assumption is that all the water taken out of the system is taken from the reservoir. How the model looks like is shown in figure 2.4
The delay time is modelled in the channels because the friction in a channel is greater than the friction in a reservoir [2]. The delay time of flow is calculated as stated in subsection 2.3.1.
The Euler backward method can be used to get the discrete representation of equation (2.17) with the discretization time T

$$h(k+1) = h(k) + \frac{T}{A} \cdot (Q_{in}(k-k_d) - Q_{out}(k)) \tag{2.18}$$

where k is the discrete time step and k_d is the delay time step.

Chapter 3

Skeboå Water system

3.1 General description of the watersystem

Skeboå water system contains five larger lakes that are namned Norrsjön, Vällen, Gisslaren, Bysjön and Närdingen. There are a few others but they are either not controlled or too small to make much of a difference to the system. How these five lakes connect can be seen either in Figure 1.1 or in 3.1. The last block in the figure, Skärbro, is not a lake but a dam close to Hallsta Paper mill. Skärbro is the last dam in the water system before the water reaches the Baltic sea. It is also the first dam that was constructed in Skebo water system. This sluice gate is automated and controlled from the mill and the water inlet to the mill is at the backwater of the gate. If the load of water on the sluice gate is high, it will let the excess continue

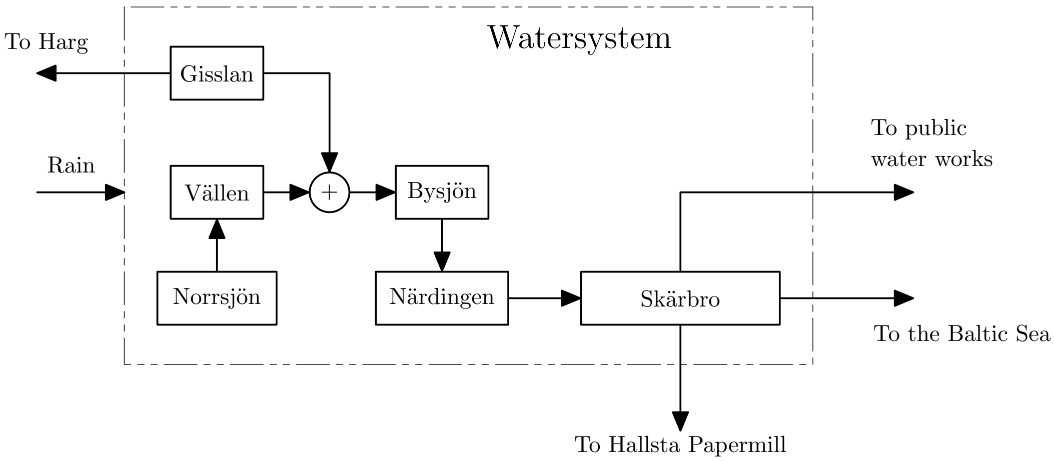


Figure 3.1: A block diagram of the water system. The arrows show which way the water flows and the rectangular solid boxes represent the dams at the lakes of the water system. Skärbro is a dam at the end of Skebo channel to the Baltic sea.

towards the Baltic Sea. The rest of the dams are manually controlled twice or three times a week. Due to the fact that the system is not automated yet, a model of the system needs to be constructed to test out the control method on. This model will not include Skärbro, since as long as the flow to this dam is high enough, there will be a low risk of lowering the production rate because of a freshwater shortage.

3.1.1 Constraints and data of Skeboå watersystem

The constraints on the system are natural boundaries as well as boundaries decided by the Swedish water court. The first water-rights judgement, from 1923, considered only the dams at Skärbro and at Skebo (which is located at the lower end of Närdingen) and the flow in the channel between them. The second water-rights judgement, from 1955, considered all of the lakes as well as one smaller lake, Stora Mårdsjön. Both of these judgements decided on an upper and lower water level limit for each lake and limits on the water flow between the lakes.
As part of the second judgement, the water system was investigated. This was done by other companies, each company investigated a part of the water system. They investigated how the channels and dams looked like and the water flow in the channel in relation to the water height in the beginning of the channel. Their findings where recorded both in drawings of the channels and dams and in graphs of the water bearing (the relationship between the water flow is at a certain water height). The water bearing graphs can be found in Appendix A.1. Each company did their measurements of water levels, water limits and bottom levels according to their own zero benchmarks. The upper and lower water limits as well as the lake area can be seen in table 3.1. The limits are listed according to the zero benchmark level which can be seen in table 3.2. Table 3.2 also present the volume of water that can be controlled within the water level limits, the width of the dams and the bottom level at the dam. The volume of water in each lake means how much water that can be let out from the system when the water level is at the upper limit and

Lake	Upper limit $[m]$	Lower limit $[m]$	Area $[km^2]$
Norrsjön	18.60	17.60	2.2
Vällen	7.95	7.05	9.6
Gisslan	14.20	13.90	2.8
Bysjön	7.15	6.90	1
Närdingen	2.36	1.36	4.3

Table 3.1: This table shows the original data from Hallsta Paper mill for each lake. The first two columns are the upper and lower water level limits of each lake. Both limit columns are presented with the respect to each lakes zero benchmark, presented in table 3.2. The last column is the area size of each lake.

Lake	Volume of water $[Mm^3]$	Dam width $[m]$	Bottom level $[m]$	Zero benchmark $[m]$
Norrsjön	2.2	1.61	17	0.47
Vällen	8.5	6.80	6.67	5.33
Gisslan North	1.5	2.02	13.26	0
Gisslan South	1.5	1.00	13.37	0
Bysjön	0.5	10.17	5.65	5.33
Närdingen	3.6	15.35	0.86	5.96

Table 3.2: This table shows data over each lake according to the findings made by the companies that investigated the system. The first column shows the volume of water that can be controlled. The second column shows the width of each dam. The third column shows the bottom level of each dam according to the benchmark showed in the last column. The zero benchmark level is according to how many meters above sea level the benchmark is.

the volume is zero when the water level reaches the lower limit.
The companies that investigated the water system also did graphs and tables of the water bearing from each dam. These state how much flow there is in the channel at a certain water level on the downstream side of the gate.
The channels in Skeboå water system are of natural type and to model them one can assume that the flow is uniform (as stated in the previous chapter). Floods and strong varied flows in the system have occurred but are not normal conditions of the channels in the system. Because the channel is assumed to be uniform, the water bearing graphs can be assumed to be valid for the whole channel.
To model the flow in a channel with equation (2.12) one needs to know the constants α and β. These can be calculated by numerically fitting equation (2.10) to the graphs of the water bearing. By doing that, the values of the channel slope do not need to be measured and the wetted perimeter and Manning's coefficient do not need to be assumed and calculated. Before fitting the equation to the graph both need to be rewritten to describe the same thing, the flow and the water level.
The shape of the cross section of a natural channel can vary a lot. During time the flow changes the shape of the bottom. How fast and the new shape depend on the state of the flow, the channel slope and the material which the bottom is made up of. In the drawings made from the channel investigations can be found the cross section of the channel of trapezoid shape or near trapezoidal. The water bearing graphs can be assumed to be based on this shape; therefore the cross section of the channel is assumed to be of this shape and is described by the following equation

$$A = h(b + yh) \tag{3.1}$$

where A is the cross section (m^2), h is the water level (m), b is the bottom width (m) and y is a constant for the slope of the side. All drawings made by the investigation

Lakes			Length $[m]$	Width $[m]$	Min. flow $[m^3/s]$	Max. flow $[m^3/s]$
Norrsjön	$\rightarrow$	Vällen	3244	1.61	0.00834	-
Vällen	$\rightarrow$	Bysjön	5801	6.8	0	-
Gisslan N.	$\rightarrow$	Harg	9092	2	0.01	-
Gisslan S.	$\rightarrow$	Bysjön	6638	1	0	0.25
Bysjön	$\rightarrow$	Närdingen	6527	10.17	0.1	-
Närdingen	$\rightarrow$	Hallstavik	10767	15.6	0.25	-

Table 3.3: This table shows the data for each channel. The first lake is where the channel begins and the second is the lake to which the channel runs towards. The lengths are real data from SMHI and the Swedish Water authority but the width is assumed to be the same as the dam at the beginning of the channel.

companies where not found during this thesis; therefore when a drawing of a channel is missing, the constant y will be assumed to be 1.5 and the width is be assumed to be the same as the dam width of the channel above.
To model the linear model of the water system, the ID model, one also needs the α and β as well as the length of the channels to determine the time delay of the water flow. The parameters α and β will be calculated later but other data, such as length and width, can be seen in table 3.3. In that table one can also find the maximum and minimum flow stated in the water court judgement from 1955. The channel length has been confirmed by using the map application at the website of Swedish Meteorological and Hydrological Institute (SMHI) [8] and from the website at the Swedish Water Authority [9].

3.2 Description of the mathematical model

3.2.1 General model

All but one dam of Skebo water system are manually operated. A model of the system is made to simulate the real system; thereby it is possible to test the control strategy on the system without having to be at the gates during the test period or building motors to control the latches at the dams. In this way, the control strategy can be simulated for a test period that has already passed and the manual control strategy can be compared to the MPC strategy that will be described later on.
The lakes that are used to store water will be modelled as big reservoir tanks where the water balance is described by the continuity equation (2.7). It is assumed that the lake area is the same regardless of the water surface elevation. With this assumption the continuity equation (2.7) can be rewritten as the following equation (3.2) for each lake

$$\frac{dh}{dt} = \frac{1}{A}(Q_{in} - Q_{out}) \tag{3.2}$$

where Q_{in} is the inflow to the lake, Q_{out} is the outflow from the lake, h is the water surface elevation and A is the lake area. No lakes in the water system have gates controlling the inflow of water. Therefore the inflow to the lake will be modelled as both the flow from the rainwater as well as the outflow from the channel upstream of the lake ($Q_{in} = Q_{rain} + Q_{Channel\ end}$). At the outflow from the lake there is one or two sluice-gate located, depending on the lake in the system. From figure 1.1b one can see that only Gisslaren has two outflows. One at the north end and one at the south end of the lake.
The dams are of undershot gate type and the water flow under the gate as submerged flow (as described in section 2.2.2). The outflow depend on the surface elevation of the lake and the surface elevation of the channel bellow the dam. Both of these variables, the gate height and water flow has been recorded for a long time (since the water system got the dams) by Hallsta Paper Mill. Therefore they suit as state variables for the model. The flow from one lake to the next one downstream is also needed for the system to be one system were the outflow from one lake influence the rest of the system downstream.

3.2.2 Channel identification

The flow in a channel has two describing general equations. One that describes the change of water flow with respect to time (equation (2.13)) and the other the relationship between the cross section of the channel and the water flow (equation (2.10)). To use equation (2.13) it is necessary to define the relationship between the cross sectional area and the water flow.
The relationship is described by Mannings equation (2.10). Without knowing the friction conditions made by the water against the bottom of the channel one could approximate it on the form of equation (2.9). Then numerically fit equation (2.9) to the water bearing graphs to get the unknown constants α and β. The water bearing graphs state what the water flow is at a specific water level in the channel. Therefore equation (2.9) is rewritten to

$$h = -\frac{b}{2y} + \sqrt{\frac{b^2}{4y^2} + \alpha Q^{\beta}} \tag{3.3}$$

where h is the water level (m), b is the width of the channel (m), y is the slope of the channel side, α and β is the unknown constants and Q is the water flow (m^3/s). There are many ways to fit an equation to data points, the most used method to fit data points to a non linear equation is with Gauss-Newton's algorithm [10]. Therefore this method will be used in this thesis. This algorithm uses Newton's method combined with least square method to determine a solution that minimizes the error sum of squares between the line made by equation (3.3) and the data points. The algorithm is run iteratively until the error sum of squares is is sufficiently low. During each cycle function f and the Jacobian of that function is determined.

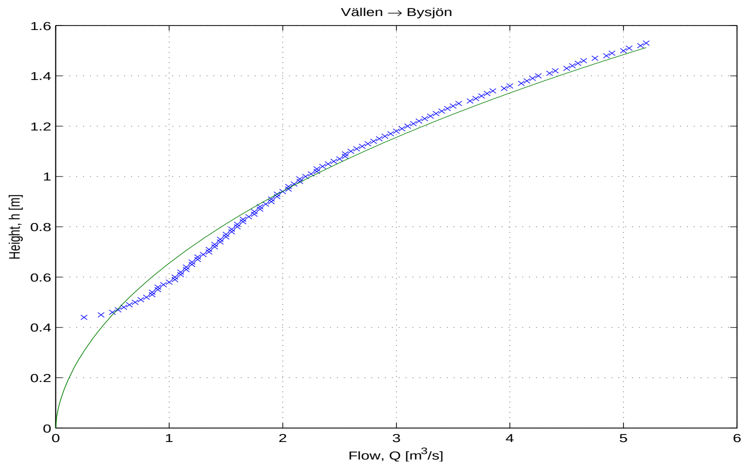
Lakes			α	β	$\alpha_{\beta-f}$
Norrsjön	$\rightarrow$	Vällen	0.8568	0.6550	0.8545
Vällen	$\rightarrow$	Bysjön	3.1906	0.6631	3.4004
Gisslan N.	$\rightarrow$	Harg	2.9471	0.6551	2.9448
Gisslan S.	$\rightarrow$	Bysjön	1.4887	0.8416	1.4092
Bysjön	$\rightarrow$	Närdingen	1.4181	0.7560	1.9525
Närdingen	$\rightarrow$	Hallstavik	3.4460	0.3903	1.9225

Table 3.4: This table shows the calculated values of α and β for both fittings. Column $\alpha_{\beta-f}$ is the value of α when β is fixed to 0.6 and the other columns are when both of the constants are calculated.

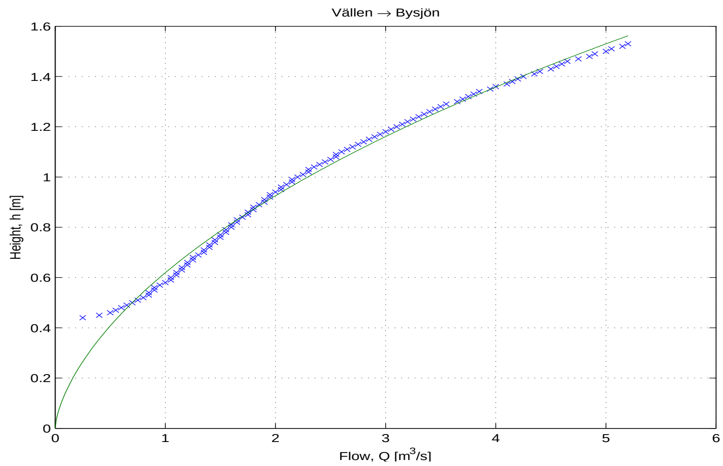
The function f is an over-determined system of equation (3.3), one equation for each data point and the Jacobian is f derived once for α and once for β giving the following equation

$$J = \begin{bmatrix} -Q^{\beta} \\ -\alpha \log(Q) Q^{\beta} \end{bmatrix} \tag{3.4}$$

The fitting was done twice. The first fitting was done to check if β could be set to 0.6, as stated in the previous chapter for Manning's equation, and be a good fit to the water bearing graphs. The second fitting was done to check whether calculation of both α and β gives a better fit than the first fitting with β fixed to 0.6. The fittings for Vällen-Bysjön for both cases are presented in figure 3.2. For the other channels the corresponding graphs can be found in appendix A.2. Table 3.4 show the values for α and β for both fittings.
As one can see, the first fitting (figure 3.2a) is satisfactory and the second fitting is only slightly better. There can be many reasons why α and β differ between the two fittings and why the fittings are not perfect. One can be that Mannings equation does not describe the flow in a perfect way. Another reason can be that the bottom width has changed since the water bearing graphs were made. There are also some uncertainty to how the water bearing graphs where made. Because of that and that the second fitting did not give a better fit for all the water bearing graphs therefore a β value of 0.6 and the α values of the last column presented in table 3.4 will be used.



(a) β fixed to 0.6



(b) Both constants calculated

Figure 3.2: The figures show how equation (2.10) is fitted to real data of the water bearing graph using Gauss-Newtons method. The solid line is the approximation and the "x" are the data. In Figure 3.2a beta is fixed to 0.6 while in Figure 3.2b both constants α and β are calculated. The height in both graphs are from the bottom of the channel.

The next step is to look at the flow dynamics in a channel with respect to time. It is important for the paper mill to know how much time it takes for a change in flow at the beginning of a channel to be seen at the end of that channel. If the delay time is known, the water flow can be controlled and production stops due to fresh water shortage can be avoided.
The flow dynamics are described in section 2.3 by equation (2.13). This equation describes the flow dynamics of one segment. Several segments may be needed to describe the flow dynamics of a whole channel. A group of equations describing sev-

Lakes			$\omega^\star$ [rad/sec]	Time delay (T) [s]	Segments
Norrsjön	$\rightarrow$	Vällen	$0.2739 \cdot 10^{-3}$	3 651	122
Vällen	$\rightarrow$	Bysjön	$0.0792 \cdot 10^{-3}$	14 061	126
Gisslan N.	$\rightarrow$	Harg	$0.0172 \cdot 10^{-3}$	58 215	47
Gisslan S.	$\rightarrow$	Bysjön	$0.0538 \cdot 10^{-3}$	18 603	48
Bysjön	$\rightarrow$	Närdingen	$0.0907 \cdot 10^{-3}$	11 031	90
Närdingen	$\rightarrow$	Hallstavik	$0.0736 \cdot 10^{-3}$	9 576	136

Table 3.5: This table shows the frequencies for low flow, the time delay for low flow and the final amount of segments for each channel model.

eral segments of one channel will from here on be referred to as a channel model. To verify how many segments each channel needs, the channel model for each channel is compared to a time delay function ($G(i\omega) = e^{-i\omega T}$). The reason why to identify by comparing with a time delay function is that such a function describe the main dynamic that the model should describe. With infinite amount of segments equation (2.13) can mimic a time delay function for the desired frequency.
The gain for a time delay function is 1 at all frequencies and the phase is $-\omega T$ [11]. However, the channel model behaves like a low-pass filter at a certain frequency (depending on the length of the channel, the water flow and the channel parameters α and β). Therefore if the channel model gain differs from the time delay function gain by less than 10% and the phase by at most 5 degrees at the desired frequency $\omega^\star$, the model is divided into a sufficient amount of segments, otherwise more are needed. The mathematical expression for this comparison is

$$\begin{gathered} |G(\omega^\star)| \geq 0.90 \\ |\angle G(\omega^\star) - \angle G_{delay}(\omega^\star)| < 5^\circ \Rightarrow |\angle G(\omega^\star) + \omega^\star \cdot T| < 5^\circ \end{gathered} \tag{3.5}$$

where G_{delay} is the time delay function, $G(\omega^\star)$ is the channel model and $\omega^\star$ is the desired angular frequency. If equation (3.5) is satisfied then the channel model behaves like a time delay for frequencies lower then the angular frequency $\omega^\star$. For higher frequencies the channel model behaves like a dampening filter. This attribute is normal for a natural channel [4].
When the channel is divided into n segments, the comparison is executed by linearising the channel model at medium flow. After this, the differences in gain and phase between the time delay function and the channel model is checked. If the comparison does not satisfy equation (3.5), the channel model is divided into *n+1* segments and the same checks are done again. To identify when the flow is low, medium and high, a histogram of each channel is made. How this analysis is made is explained in section 5.1. Table 3.5 presents the time delay for low flow, the angular frequency $\omega^\star$ and the final amount of segments for each channel.

$$
\begin{aligned}
\frac{dQ_{(1,Ch_x)}(t)}{dt} &= \frac{0.6\, b_{Ch_x} H_{(g,Ch_x)} \sqrt{2g\, (h_{(1,XX)}(t) - h_{(2,XX)}(t))} - Q_{(1,Ch_x)}(t)}{\Delta x_{Ch_x} \alpha_{Ch_x} \beta_{Ch_x}} \cdot Q_{(1,Ch_x)}(t)^{1-\beta_{Ch_x}} \\
\frac{dQ_{(2,Ch_x)(t)}}{dt} &= \frac{Q_{(1,Ch_x)}(t) - Q_{(2,Ch_x}(t)}{\Delta x_{Ch_x} \alpha_{Ch_x} \beta_{Ch_x}} \cdot Q_{(2,Ch_x)}(t)^{1-\beta_{Ch_x}} \\
&\vdots \\
\frac{dQ_{(N,Ch_x}(t)}{dt} &= \frac{Q_{(N-1,Ch_x)}(t) - Q_{(N,Ch_x)}(t)}{\Delta x_{Ch_x} \alpha_{Ch_x} \beta_{Ch_x}} \cdot Q_{(N,Ch_x)}(t)^{1-\beta_{Ch_x}}
\end{aligned}
\tag{3.6}
$$

$$
\begin{aligned}
\frac{dh_{(1,No)}(t)}{dt} &= \frac{1}{A_{No}} \Big[\, Q_{in}(t) - 0.6\, b_{No}\, H_{(g,No)} \sqrt{2g\, (h_{(1,No)}(t) - h_{(2,No)}(t)\,)}\, \Big] \\
\frac{dh_{(2,No)}(t)}{dt} &= \frac{1}{A_{No-V}} \Big[\, 0.6\, b_{No}\, H_{(g,No)} \sqrt{2g\, (h_{(1,No)}(t) - h_{(2,No)}(t))} - Q_{(1,No-V)}(t)\, \Big] \\
\frac{dh_{(1,V)}(t)}{dt} &= \frac{1}{A_V} \Big[\, Q_{in}(t) + Q_{(7,No-V)}(t) - 0.6\, b_V\, H_{(g,V)} \sqrt{2g\, (h_{(1,V)}(t) - h_{(2,V)}(t)\,)}\, \Big] \\
\frac{dh_{(2,V)}(t)}{dt} &= \frac{1}{A_{V-B}} \Big[\, 0.6\, b_V\, H_{(g,V)} \sqrt{2g\, (h_{(1,V)}(t) - h_{(2,V)}(t))} - Q_{(1,V-B)}(t)\, \Big] \\
\frac{dh_{1,G}(t)}{dt} &= \frac{1}{A_G} \Big[\, Q_{in}(t) - 0.6\, b_{Gn}\, H_{g,Gn} \sqrt{2g(h_{1,G}(t) - h_{2,Gn}(t)} - \\
&\quad 0.6\, b_{Gs}\, H_{g,Gs} \sqrt{2g(h_{1,G}(t) - h_{2,Gs}}(t)\, \Big] \\
\frac{dh_{2,Gn}(t)}{dt} &= \frac{1}{A_{Gn-Harg}} \Big[\, 0.6\, b_{Gn}\, H_{g,Gn} \sqrt{2g(h_{1,G}(t) - h_{2,Gn}(t)} - Q_{1,Gn-Harg}(t)\, \Big] \\
\frac{dh_{2,Gs}(t)}{dt} &= \frac{1}{A_{Gs-B}} \Big[\, 0.6\, b_{Gs}\, H_{g,Gs} \sqrt{2g\, (h_{1,G}(t) - h_{2,Gs}(t)}\; - \; Q_{1,Gs-B}(t)\, \Big] \\
\frac{dh_{(1,B)}(t)}{dt} &= \frac{1}{A_B} \Big[\, Q_{in}(t) \; + \; Q_{(3,Gs-B)}(t) \; + \; Q_{(6,V-B)} - \\
&\quad 0.6\, b_B\, H_{(g,B)} \sqrt{2g\, (h_{(1,B)}(t) - h_{(2,B)}(t)\,)}\, \Big] \\
\frac{dh_{(2,B)}(t)}{dt} &= \frac{1}{A_{B-Na}} \Big[\, 0.6\, b_B\, H_{(g,B)} \sqrt{2g\, (h_{(1,B)}(t) - h_{(2,B)}(t))} - Q_{(1,B-Na)}(t)\, \Big] \\
\frac{dh_{(1,Na)}(t)}{dt} &= \frac{1}{A_{Na}} \Big[\, Q_{in}(t) - 0.6\, b_{Na}\, H_{(g,Na)} \sqrt{2g\, (h_{(1,Na)}(t) - h_{(2,Na)}(t)\,)}\, \Big] \\
\frac{dh_{(2,Na)}(t)}{dt} &= \frac{1}{A_{Na-Hallsta}} \Big[\, 0.6\, b_{Na}\, H_{(g,Na)} \sqrt{2g\, (h_{(1,Na)}(t) - h_{(2,Na)}(t))} - \\
&\quad Q_{(1,Na-Hallsta)}(t)\, \Big]
\end{aligned}
\tag{3.7}
$$

In conclusion, the model for the real Skeboå water system is summarized by equation (3.7) and equation (3.6). Equation (3.7) describes how the water level changes with respect to time in all the relevant lakes in the water system. The set of equations also shows how the water level after each dam is affected. The notifications (*No, V, G, B, Na*) of equation (3.7) refers to the lakes of the water system (*Norrsjön, Vällen, Gisslaren, Bysjön, Närdingen*).

Equation (3.6) shows the equations for a channel Ch_x. The notification N refers to the number of the final segment of channel Ch_x and the notification X refers to the name of lake X above the channel Ch_x.

3.3 Verification of model

3.3.1 Method

The model of the real Skeboå water system (as described in the above section) needs to be verified so that it describes the system in a satisfactory manner. The verification is done by simulating the model using real inputs. For this, data from year 2002 is chosen, partly because the amount of rain was low and partly due to low water levels in the system during that summer. The last condition is what this thesis should hopefully prevent and control in a better way. If the predicted amount of rain is low and the water level in the lakes is low, it is important to let out just enough water from the system to cover the needs of the paper mill.
Each subsystem is simulated individually in order to facilitate the analysis. The submodels of Norrsjön, Vällen, Bysjön and Närdingen have two inputs (gate height and inflow) and two outputs (water level and outflow). The submodel of Gisslaren has three inputs (gate height of south and north dams and inflow) and three outputs (water level, outflow south and outflow north). The inflow consists of two parts, the flow from the lake above (if there is no such lake this part is zero) and the inflow from the rain. Precipitation data for this period was bought from the Swedish weather institute SMHI and then transformed to inflow using only the lake area as the run-off area. The reason for this simple approximation is that during a year the amount of inflow from the run-off area around each lake is non-linear. During the winter (December to February) most, if not all, rain falls to the ground in the form of snowflakes and stays on the ground instead of contributing to the inflow of water. In spring (March to May) the snow melts and more water flows towards the lake than the rainfall during this period. During summer (June to August) the ground absorbs most of the rain water, if not all. In the last season, during the fall (September to November), more water runs down towards the lakes than during late spring.

3.3.2 Results

The results of the individual verification simulations can be seen in figures 3.3, 3.4, 3.5, 3.6 and 3.7. The x-axis range in the figures indicates days starting January 1st and ending December 31st. The seasons for the year the following, winter from day 335-365 and day 1 to day 59, spring from day 60 to 151, summer from day 152 to 243 and fall from day 244 to 334.

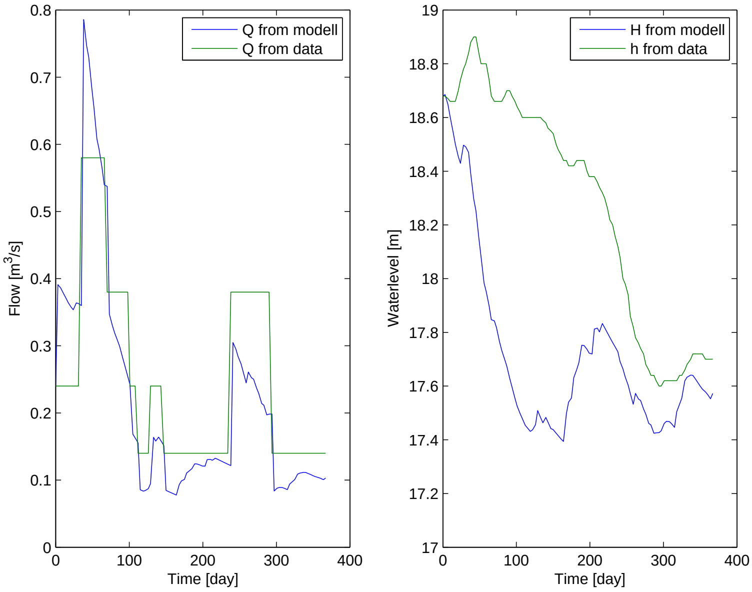


Figure 3.3: The left plot is of the outflow and the right plot is of the waterlevel of Norrsjön.

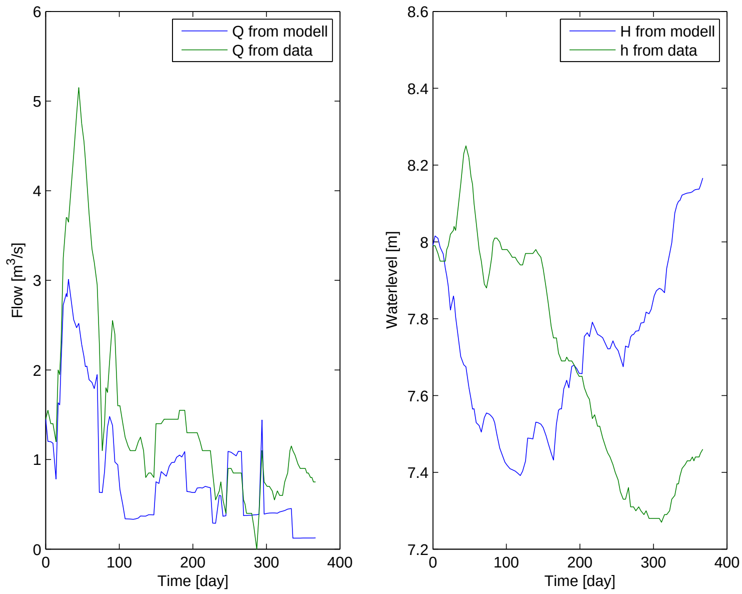


Figure 3.4: The left plot is of the outflow and the right plot is of the water level of Vällen.

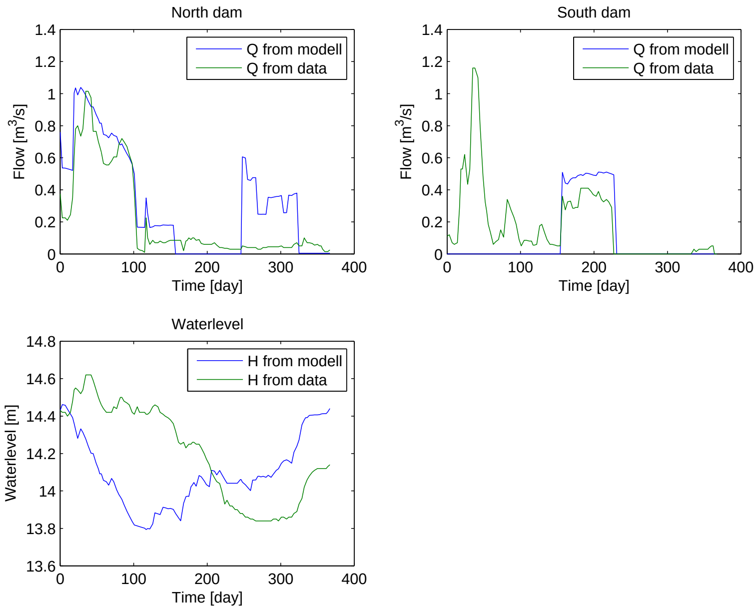


Figure 3.5: The top right and left plots are of the outflow South and North and the bottom plot is of the waterlevel of Gisslaren.

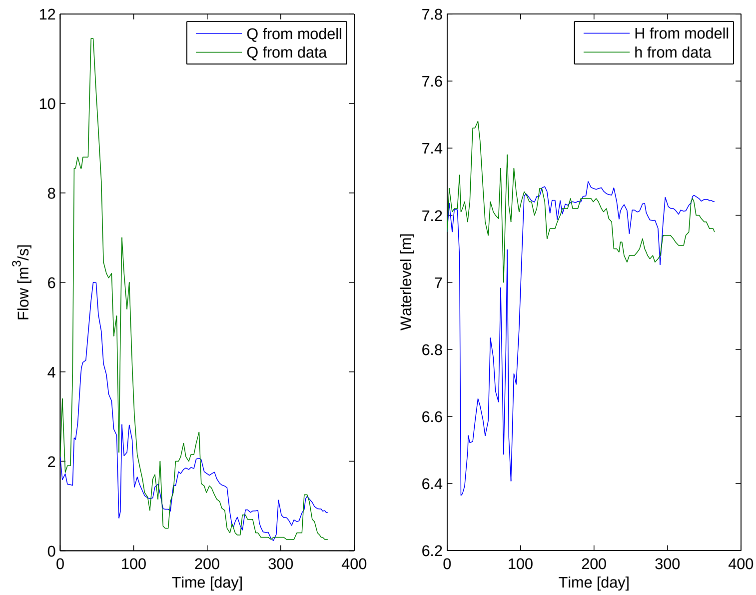


Figure 3.6: The left plot is of the outflow and the right plot is of the water level of Bysjön.

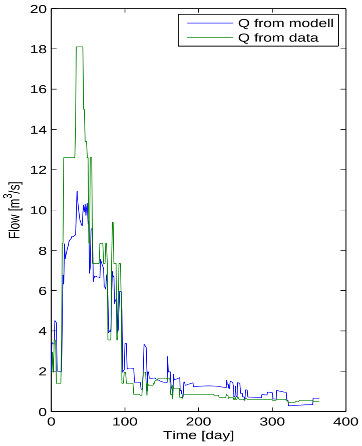


Figure 3.7: The left plot is of the outflow and the right plot is of the water level of Närdingen.

One can see the trends are fairly good; however, there is an offset error during some seasons. The verification plots in figure 3.3 to 3.7 show that the model is not correct for winter and early spring. These trends can be better evaluated by looking at the bode plots of this data. To get a bode plot from data points one can use spectral analysis to get an estimate frequency response and get the bode plot from that. The same method to get the bode plots were done for both the real measured data and for the verification data. The bode plots for the water level as output and gate height as input, can be seen in figures 3.8, 3.9, 3.10, 3.11, 3.12 and 3.13. The figures for the other combinations of input and output can be found in Appendix A.3.

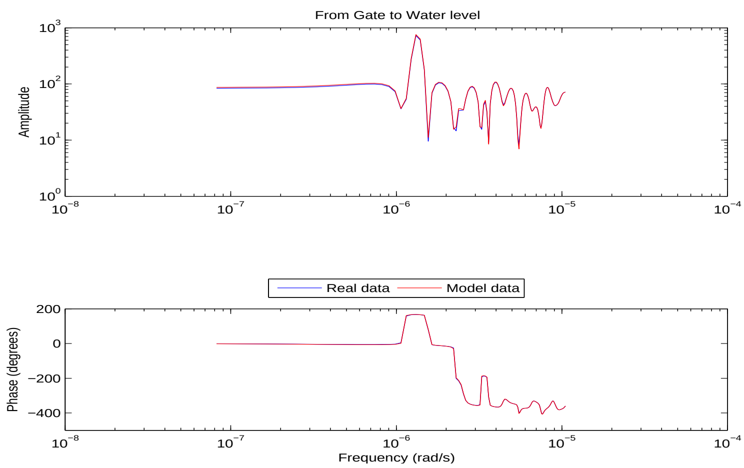


Figure 3.8: The figure show one of the bode plots of Norrsjön. With the dam gate height as input and the waterlevel of Norrsjön as output.

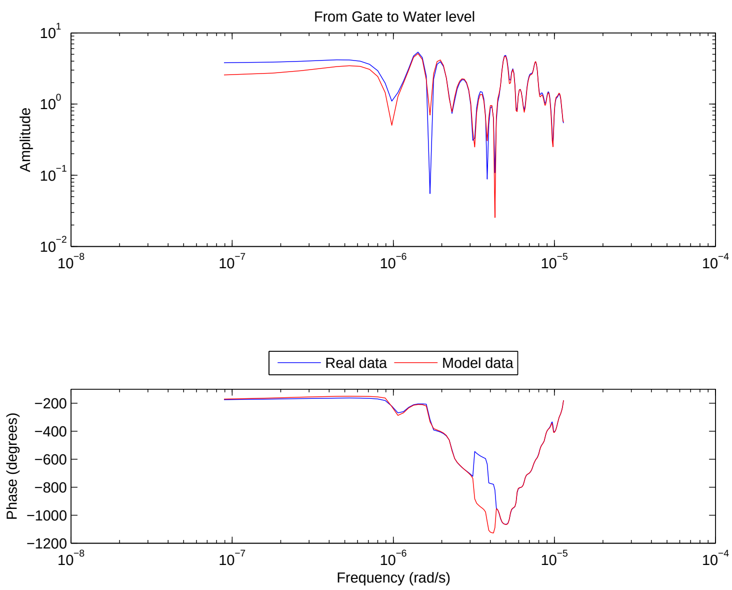


Figure 3.9: The figure show one of the bode plots of Vällen. With the dam gate height as input and the waterlevel of Vällen as output.

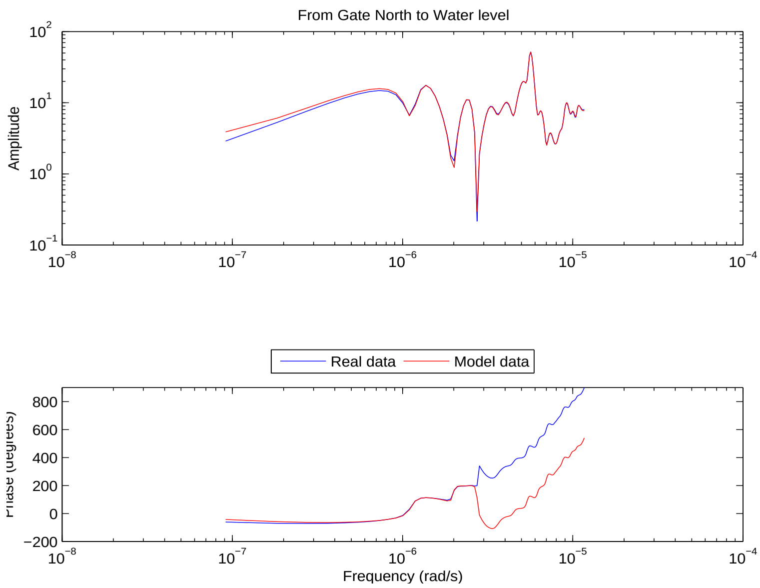


Figure 3.10: The figure show one of the bode plots of Gisslaren. With the gate height of the north dam as input and the water level of Gisslaren as output.

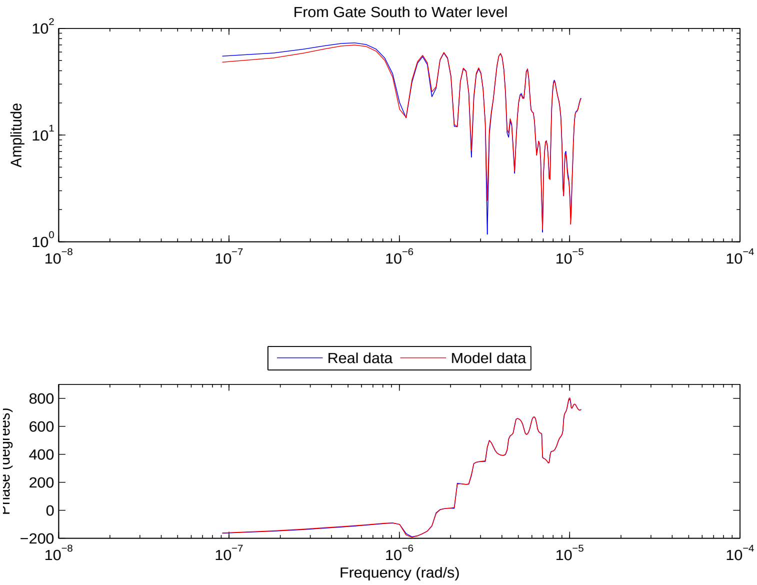


Figure 3.11: The figure show one of the bode plots of Gisslaren. With the gate height of the south dam as input and the water level of Gisslaren as output.

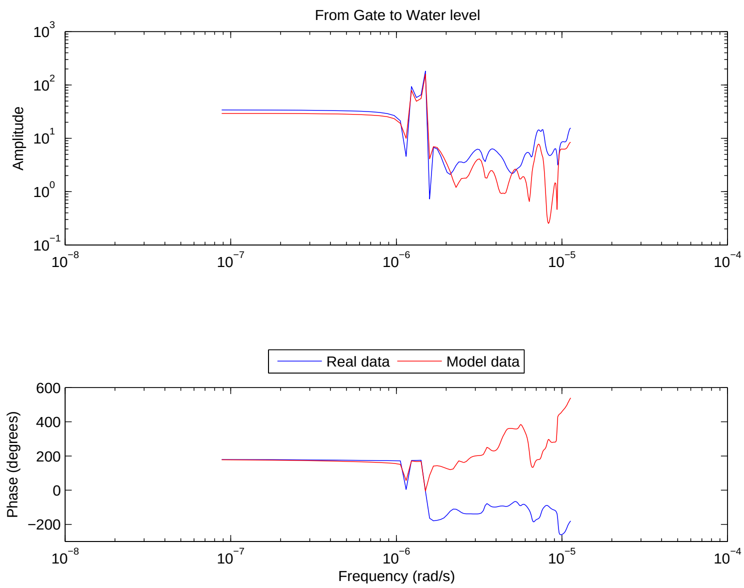


Figure 3.12: The figure show one of the bode plots of Bysjön. With the dam gate height as input and the waterlevel of Närdingen as output.

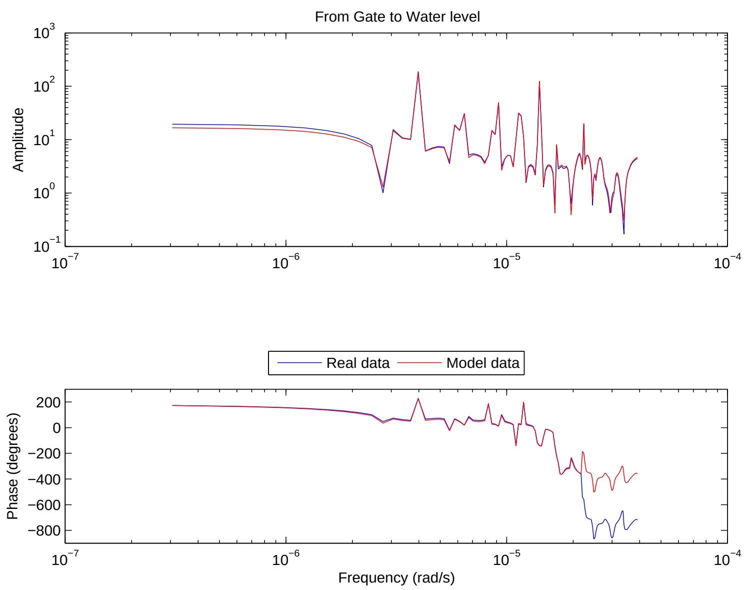


Figure 3.13: The figure show one of the bode plots of Närdingen. With the dam gate height as input and the waterlevel of Närdingen as output.

This non-linear model is good at modelling the water level in the lakes when there is a change in dam gate level or inflow, although it is not as good to in modelling the dynamics of the outflow. The purpose of the model is to describe the water level in a satisfactory manner. It can be concluded that this is the case.

3.4 Integrator Delay model of Skeboå Watersystem

The Integrator Delay model (ID-model) is the model that the model predictive controller uses as an internal model. The ID-model is a simpler and linear model of Skeboå water system. The theory for the ID-model is described in section 2.4.
For the construction of the ID-model two major properties must be known. The first is the area of the lakes and the second is the time delay in the channels. The lake areas where stated earlier in table 3.1 and the time delays for low flow in the channels in table 3.5. The same lake areas and time delays are used for both the ID-model and the model of the real Skeboå water system.
The ID-model is constructed using equation (2.18), time delay constants from table 3.5 and lake areas from table 3.1. Equation (3.8) describes how the water level in each lake changes with respect to discrete time

$$
\begin{aligned}
h_{No}(k+1) &= h_{No}(k) + \tfrac{T_c}{A_{No}}\Big(Q_{rain,No}(k) - Q_{No}(k)\Big)\\
h_{V}(k+1) &= h_{V}(k) + \tfrac{T_c}{A_{V}}\Big(Q_{rain,V}(k) + Q_{No}(k-k_{No}) - Q_{V}(k)\Big)\\
h_{G}(k+1) &= h_{G}(k) + \tfrac{T_c}{A_{G}}\Big(Q_{rain,G}(k) - Q_{Gs}(k) - Q_{Gn}(k)\Big)\\
h_{B}(k+1) &= h_{B}(k) + \tfrac{T_c}{A_{B}}\Big(Q_{rain,B}(k) + Q_{V}(k-k_{V}) + Q_{Gs}(k-k_{Gs}) - Q_{B}(k)\Big)\\
h_{Na}(k+1) &= h_{Na}(k) + \tfrac{T_c}{A_{Na}}\Big(Q_{rain,Na}(k) + Q_{B}(k-k_{B}) - Q_{Na}(k)\Big)
\end{aligned}
\tag{3.8}
$$

where $Q_{rain,*}$ is the disturbance, $Q_*(k)$ is the flow of water out of lake *, $Q_*(k-k_*)$ is the past outflow from lake * and $h_*(k)$ is the water level of the lake * (* represents either *No, V, Gn, Gs, B or Na*). Even though the rain instensity is assumed to be the same over Skeboå water system, the modelled disturbance need to be different for each lake because how much the following flow from the rain is depend on how large the area around the lake is.
This group of equations is the ID-model for Skeboå water system. In the ID model the present outflow from each lake is considered as the input and the past outflows and current water levels are considered as states to the system. To keep track of past inputs, the ID-model is extended to all past inputs until the time step k_*. The amount of states in the final model depends on the discretization time T_c. How this variable is chosen is described in chapter 5. The extended ID-model is described in equation (3.9).

$$
\begin{array}{lcl}
h_{no}(k+1) & = & h_{no}(k) + \frac{T_c}{A_{no}}\Big(Q_{rain,No}(k) - Q_{no}(k)\Big) \\
Q_{no}((k-1)+1) & = & Q_{no}(k) \\
 & \vdots & \\
Q_{no}((k-k_{no})+1) & = & Q_{no}(k-k_{no}+1) \\
h_v(k+1) & = & h_v(k) + \frac{T_c}{A_v}\Big(Q_{rain,V}(k) + Q_{no}(k-k_{no}) - Q_v(k)\Big) \\
Q_v((k-1)+1) & = & Q_v(k) \\
 & \vdots & \\
Q_v((k-k_v)+1) & = & Q_v(k-k_v+1) \\
h_g(k+1) & = & h_g(k) + \frac{T_c}{A_g}\Big(Q_{rain,G}(k) - Q_{gs}(k) - Q_{gn}(k)\Big) \\
Q_{gn}((k-1)+1) & = & Q_{gn}(k) \\
Q_{gs}((k-1)+1) & = & Q_{gs}(k) \\
 & \vdots & \\
Q_{gs}((k-k_{gs})+1) & = & Q_{gs}(k-k_{gs}+1) \\
h_b(k+1) & = & h_b(k) + \frac{T_c}{A_b}\Big(Q_{rain,B}(k) + Q_v(k-k_v) + Q_{gs}(k-k_{gs}) - Q_b(k)\Big) \\
Q_b((k-1)+1) & = & Q_b(k) \\
 & \vdots & \\
Q_{gs}((k-k_b)+1) & = & Q_b(k-k_b+1) \\
h_{na}(k+1) & = & h_{na}(k) + \frac{T_c}{A_{na}}\Big(Q_{rain,Na}(k) + Q_b(k-k_b) - Q_{na}(k)\Big) \\
Q_{na}((k-1)+1) & = & Q_{na}(k) \\
 & \vdots & \\
Q_{na}((k-k_{na})+1) & = & Q_{na}(k-k_{na}+1)
\end{array}
\tag{3.9}
$$

Chapter 4

Model Predictive Control theory

Model Predictive Control (MPC) is an advanced type of controller that emerged from the chemical industry in in the 1970s [5]. It was necessary because of limitations and constraints in the production process. MPC is an optimal controller that uses a model of the real system to calculate the optimum present and future input so that it reaches the setpoint within a given time frame. The difference between MPC and other optimal controllers is that the constraints on the system are part of the calculations.
The first section describes how the internal model is used. Section 4.2 gives a general description of what a state estimator is and how it is used in the MPC. Section 4.3 describes the objective function of the MPC stated for this thesis. Section 4.4 gives a description of how constraints are used in an MPC. Section 4.5 describe how the MPC achieve an optimal solution that minimizes the objective function. In section 4.6 the meaning of horizons are described and how the MPC uses them. The chapter ands with section 4.7 which states how the tuning of the MPC is done.

4.1 Internal model

The internal model is used to predict future states in the real system. The internal model will in this thesis be described as a state-space model but it is possible to use some other type of description (for example a transfer function). The internal model uses present and future inputs to calculate present and future outputs. The inputs to the internal model are the present and future control actions and the present and future disturbances [5]. The control actions are calculated by the optimization algorithm, which will be described later on in this chapter.
A discrete state space model predicts the states for one step ahead, as shown in

equation (4.1).

$$
\begin{aligned}
&\mathbf{x}(k+1) = A\mathbf{x}(k) + B_u\mathbf{u}(k) + B_d\mathbf{d} \\
&\mathbf{y}(k) = C\mathbf{x}(k) \\
&\mathbf{x}(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_n(k) \end{bmatrix} \quad \mathbf{u}(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \\ \vdots \\ u_m(k) \end{bmatrix} \\
&\mathbf{y}(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_l(k) \end{bmatrix} \quad \mathbf{d}(k) = \begin{bmatrix} d_1(k) \\ d_2(k) \\ \vdots \\ d_p(k) \end{bmatrix}
\end{aligned}
\tag{4.1}
$$

where $x(k)$ are the states, $u(k)$ are the inputs $y(k)$ are the outputs and $d(k)$ are the disturbances. For this state space model the number of inputs, outputs, disturbances and states are noted m, l, p and n. The state space model can be used recursively to predict future states and outputs. Equation (4.2a) shows how equation (4.1) is used to predict the states two steps in the future. And equation (4.2b) shows how equation (4.1) and equation (4.2a) is used to predict the states three steps in the future.

$$
\begin{aligned}
\mathbf{x}(k+2) &= A\mathbf{x}(k+1) + B_u\mathbf{u}(k+1) + B_d\mathbf{d}(k+1) \\
&= A\big[A\mathbf{x}(k) + B_u\mathbf{u}(k)\big] + B_u\mathbf{u}(k+1) + B_d\mathbf{d}(k+1) \\
&= A^2\mathbf{x}(k) + AB_u\mathbf{u}(k) + B_u\mathbf{u}(k+1) + \\
&\quad AB_d\mathbf{d}(k) + B_d\mathbf{d}(k+1)
\end{aligned}
\tag{4.2a}
$$

$$
\begin{aligned}
\mathbf{x}(k+3) &= A\mathbf{x}(k+2) + B_u\mathbf{u}(k+2) + B_d\mathbf{d}(k+2) \\
&= A\big[A^2\mathbf{x}(k) + AB_u\mathbf{u}(k) + B_u\mathbf{u}(k+1) + AB_d\mathbf{d}(k) + \\
&\quad B_d\mathbf{d}(k+1)\big] + B_u\mathbf{u}(k+2) + B_d\mathbf{d}(k+2) \\
&= A^3\mathbf{x}(k) + A^2B_u\mathbf{u}(k) + AB_u\mathbf{u}(k+1)B_u\mathbf{u}(k+2) + \\
&\quad A^2B_d\mathbf{d}(k) + B_d\mathbf{d}(k+1)
\end{aligned}
\tag{4.2b}
$$

By using this method, one can easily write the model for any future state ahead, as shown in equation(4.3).

$$
\begin{gathered}
\mathbf{X} = \mathbf{A}x(k) + \mathbf{B}_u \cdot \mathbf{U} + \mathbf{B}_d \cdot \mathbf{D} \\
\mathbf{Y} = \mathbf{C} \cdot \mathbf{X} \\
\mathbf{X} = \begin{bmatrix} \mathbf{x}(k+1) \\ \mathbf{x}(k+2) \\ \vdots \\ \mathbf{x}(k+N_p) \end{bmatrix} \quad \mathbf{U} = \begin{bmatrix} \mathbf{u}(k) \\ \mathbf{u}(k+1) \\ \vdots \\ \mathbf{u}(k+N_c) \end{bmatrix} \\
\mathbf{Y} = \begin{bmatrix} \mathbf{y}(k) \\ \mathbf{y}(k+1) \\ \vdots \\ \mathbf{y}(k+N_p) \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} \mathbf{d}(k) \\ \mathbf{d}(k+1) \\ \vdots \\ \mathbf{d}(k+N_p) \end{bmatrix} \\
\mathbf{A} = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^{N_p} \end{bmatrix} \quad \mathbf{B}_u = \begin{bmatrix} B_u & 0 & \dots & 0 \\ AB_u & B_u & & 0 \\ \vdots & & \ddots & \vdots \\ A^{N_p-1}B_u & A^{N_p-2}B_u & \dots & A^{N_p-N_c}B_u \end{bmatrix} \\
\mathbf{B}_d = \begin{bmatrix} B_d & 0 & \dots & 0 \\ AB_d & B_d & & 0 \\ \vdots & & \ddots & \vdots \\ A^{N_p-1}B_d & A^{N_p-2}B_d & \dots & B_d \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} C & 0 & \dots & 0 \\ 0 & C & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & C \end{bmatrix}
\end{gathered} \tag{4.3}
$$

$\mathbf{D}$ is representing a prediction of the disturbances over the prediction horizon (N_p). In equation (4.3), N_c is the control horizon. The value of the control horizon is the same or lower than the prediction horizon N_p. The control horizon is usually chosen a lot smaller than the prediction horizon [12].

4.2 State estimator

To calculate the optimal inputs to a system one needs to know all the present states of the system. In most applications, this is not the case and some states may need to be estimated. A common way to estimate the states is with a state estimator [12]

$$\hat{x}(k+1) = A\hat{x}(k) + B_u u(k) + K(y(k) - C\hat{x}(k)) \tag{4.4}$$

where $\hat{x}(k)$ are the estimated current states, A, B_u and C are the modelled system matrices, $u(k)$ are the known inputs, $y(k)$ are the measured outputs of the actual system and K is the state estimator. K needs to be chosen so that there is a balance between sensitivity to measurement disturbances and how fast the difference between the measured output y and the estimated output $C\hat{x}$ affects the new estimated value [12].

In this thesis K is calculated using the Kalman theory for discrete time. K is then given by

$$K = (APC^T + B_dR_{12})(CPC^T + R_2)^{-1} \tag{4.5}$$

where R_{12} is the covariance between the state noise and the output noise, R_2 is the intensity of the output noise and P is the covariance matrix for the optimal estimate [12]. The covariance matrix P is calculated by solving the discrete Riccati equation

$$P = APA^T + B_dR_1B_d^T - (APC^T + B_dR_{12})(CPC^T + R_2)^{-1}(APC^T + B_dR_{12})^T \tag{4.6}$$

where R_1 is the intensity of the state noise [12].
This method requires that the noise intensities are known. If that information is not available then the method can be used by regard them as design variables. When regarding the intensities as design variables then there are some principles that are made when tuning the observer. In case the noise in the states and the output noise are independent of each other then R_{12} should be set to zero. If R_2 is set to zero then the measurements are perfect and in case it is set to infinity then the measurements will not be reliable [13].

4.3 Objective function

The objective function is the target that the controller wants to achieve. The goal is to minimize the difference between setpoints and states and the use of control input. There may be a conflict between these objectives and some states may be more important then the others. Therefore weight matrices are used to set the order of importance. A higher value gives that state or input a higher priority in the controller.
An objective function, $\mathbf{V}(U)$, is usually expressed as a quadratic function. The reason for it is that the function does not depend on whether the minimised variable is positive or negative. Another reason why $\mathbf{V}(U)$ is expressed as a quadratic function is that there are plenty of efficient solvers for Quadratic Problems [14]. For this thesis the following function will be used

$$V(U) = (\mathbf{X} - X_{ref})^T Q_x (\mathbf{X} - X_{ref}) + (\mathbf{U} - U_{ref})^T Q_u (\mathbf{U} - U_{ref}) \tag{4.7}$$

where $\mathbf{X}$ are the state variables over the prediction horizon, $\mathbf{U}$ are the inputs over the prediction horizon, X_{ref} are the reference variables of the states for the prediction horizon, U_{ref} are the reference variables of the inputs, Q_x is the weight matrix on the state variables and Q_u is the weight matrix on the control actions. The weight matrices Q_x and Q_u are diagonal [2]. How the weight matrices are chosen is described in section 4.7.
The objective function, described by equation (4.7), is easily rewritten as a general quadratic problem. How this is done is described in section **??**. The form for a general quadratic problem is described by equation (4.8).

$$\min_U U^T H U + f^T U + g \tag{4.8}$$

4.4 Constraints

Constraints are physical and operational limitations of the controlled system [5]. These are used by the optimization algorithm so that the states, inputs and outputs are kept within those boundaries. The constraints can for example be a maximum and minimum water level limit or that a gate cannot go beyond fully opened or fully shut.
Constraints can be divided into two categories [15]. The first category of constraints is called hard constraints and the solution of the MPC must always satisfy these constraints. These constraints should never be violated as they represent fixed values. One example is that the flow downstream could not be negative as it would indicate that the water flows upstream which not possible (as long as the flow is driven by gravity). There may be other hard constraints set for the safety to people or equipment.
The other category is called soft constraints. The optimization solution should satisfy them if possible. Soft constraints can for instance be temperature or pressure limitations to prevent fatigue damage to equipment or to ensure quality. If necessary these constraints can be violated.
The difference in implementation is that hard constraints are used in the optimization as hard limitations to a state or input, while soft constraints are implemented by using slack variables. The slack variables are defined as such that they are nonzero only if the corresponding constraints are violated [16].
The constraints are set up as secondary conditions to the MPC optimization problem (can be seen in equation (4.9)). Equation (4.9a) are the conditions for the inputs and equation (4.9b) are the conditions for the states [12]

$$u_{min} \leq \mathbf{u} \leq u_{max} \tag{4.9a}$$

$$x_{min} \leq \mathbf{x} \leq x_{max} \tag{4.9b}$$

4.5 Optimization

The optimal control is achieved by minimizing the objective function with regard to the constraints. First the objective function (equation 4.7) needs to be rewritten so it can be used in a Quadratic Solver, in the following form

$$\begin{gathered} \min_U V(U) = \min_U \frac{1}{2} \cdot U^T \cdot \mathbf{H} \cdot U + \mathbf{f} \cdot U + g \\ R_u U \leq C_t \end{gathered} \tag{4.10}$$

where $\mathbf{H}$ is the Hessian matrix, $\mathbf{f}$ is the Lagrangian and g is a constant that does not depend on the input. The value of g will not be affected by changing the values of the input. Therefore g can be neglected when minimizing equation (4.10) [5]. R_u and C_t describe how the constraints need to be rewritten to describe the secondary conditions for the optimal input. H f, R_u and C_t are derived to the following form

$$\begin{aligned}
&\mathbf{H} = 2 \cdot (\mathbf{B}_u^T \, Q_x \mathbf{B}_u + Q_u) \\
&\mathbf{f} = 2 \cdot \Big(\, x(k)^T \mathbf{A}^T Q_x + \mathbf{D}^T \mathbf{B}_d^T Q_x - \mathbf{X}_{ref}^T Q_x - \mathbf{U}_{ref}^T Q_u \Big) \\
&R_u = \begin{bmatrix} -B_u \\ B_u \\ -I \\ I \end{bmatrix}, \; C_t = \begin{bmatrix} Ax(k) + \mathbf{B}_d \mathbf{D} - X_{min} \\ -Ax(k) - \mathbf{B}_d \mathbf{D} + X_{max} \\ -U_{min} \\ U_{max} \end{bmatrix}
\end{aligned} \tag{4.11}$$

where U_{min}/U_{max} are input limitations over the prediction horizon, X_{min}/X_{max} are the state limitations over the prediction horizon. A more detailed description of the derivation of equation (4.11) can be found in Appendix A.4.

4.6 Horizons

Horizons, in case of MPC, means how far into the future the MPC predicts the states and the control inputs. The horizon for the future states is called the Prediction horizon. The horizon for the current and future inputs is called the Control horizon. In equation (4.3) the prediction horizon is referred to as N_p and the control horizon is referred to as N_c.

Both the prediction and control horizons are receding horizons. Receding horizon means a horizon that is constantly moving away. This means that when the time moves one step further, both horizons are moving so that they do not include past inputs and past or current states in the optimization.

One cannot say exactly how long each horizon needs to be. The lengths of the horizons are set during tuning of the MPC. The length of the prediction horzion is usually chosen so that all the future states are included in the optimization calculations until the system has settled [12]. If it is set too short, the controller could end up making the system unstable when it tries to reach the reference values.

4.7 Tuning

A MPC usually needs some tuning before it works in a satisfactory way. Unfortunately there are no specific rules for tuning a MPC other than by trial and error procedure. There are many parameters for tuning a MPC. Prediction and control horizons, the values in the weight matrices and the choice of control time step.

A good start value for the prediction horizon is the settling time of the system. The prediction horizon needs, for a water system with long delay times, to be long enough to cover the longest delay time [5]. The control horizon is often chosen

shorter then the prediction horizon [12].
Tuning of the weight matrices are normally done using the trial and error procedure. Their values are a trade-off between minimizing water level errors and minimizing flow errors $(U - U_{ref})$.
The choice of control time step depends the responses of the system. For a large reservoir system, with long response time, one can select a longer control time step. But for a small irrigation canal system should the control time step be short [2].

4.8 Stability

A common constraint on a system is that the closed loop system should be stable. It is however hard to analyse the stability of a MPC controlled system. The difficulties stems from the finite horizon combined with the non-linearities in form of constraints. However there are methods to ensure that the system is stable for example by setting the prediction horizon to infinity or constraints on the final states[13]. Here we address stability mainly through simulation.

Chapter 5

Analysis

This chapter describes the analysis made of Skeboå Water system. In the first section the water flow in the systems channel are analysed. That section also state how high, average and low water flow of each channel is classified. The water flow classification is used when identifying how many channel segments are needed to model the channels of Skeboå Water System.
The second section analyse the how to the prediction horizon and the control step time in the MPC are chosen.

5.1 Histogram analysis of channel flow

A histogram is a graphical representation showing how data is distributed in predefined intervals. The histogram counts how many times the flow has been in a certain interval.
A histogram does not take into account the moment at which the flow was measured or how the conditions where at that time. Therefore data from a whole year is used for this analysis. In this way all the seasons are represented as well as the highest and lowest flows during a year. Because this thesis focuses on minimizing the outflow of water from the system, a year when the amount of rain was low was chosen. Data from year 2006 was therefore used.
In this thesis, histograms are used to identify low, medium and high flow for each channel. It can be assumed that the data in the histogram is divided into three groups, as seen figure 5.1. The most common flow within each group will be used

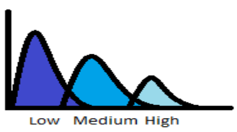


Figure 5.1: A sketch of the how the histograms are expected to turnout.

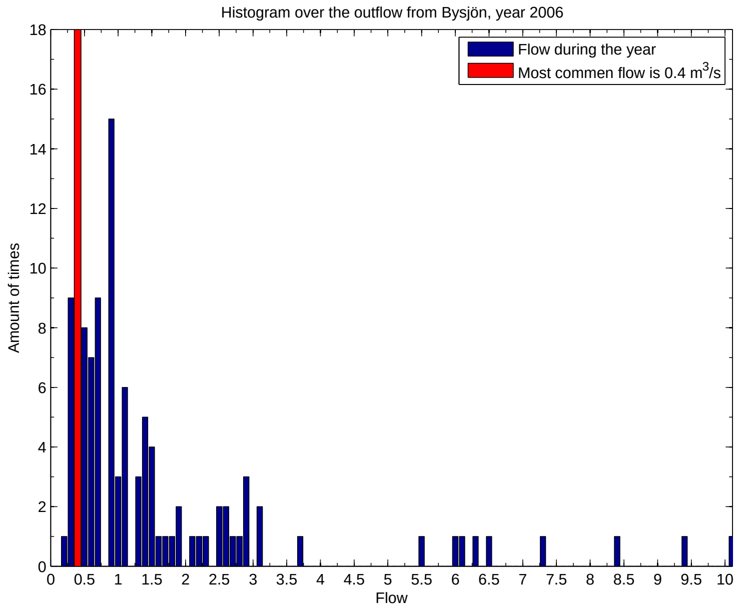


Figure 5.2: The histogram show the flow out of Bysjön the year 2006.

as the flow for that group. For a year with low amount of rain, it can be expected that the most common flow of each histogram is in the ”low flow”-group. This flow will therefore represent the low flow of that channel.
The initial interval was set to a tenth of the maximum flow. Thereafter the interval was refined in steps until three groups could be identified.
Figure 5.2 shows a histogram of the water flow in the channel between Bysjön and Närdingen during year 2006. The interval used in the histogram is 0.05 m^3/s. One can see from figure 5.2 that the majority of the flow was relatively low, which was expected since the data was from a year with little rain. The most common flow in the figure is 0.4 m^3/s which was measured 18 times. This is in the ”low flow”-group and will represent the low flow for the channel downstream of Bysjön. The medium flow of the channel between Bysjön and Närdingen was identified to 2.8 m^3/s. Each of the high flows where only measured once during that year. Because no peak can be identified, the median flow in this group will be used (6.5 m^3/s).
The identification of the channel from Bysjön to Närdingen was straight forward. However, the histogram for the channel from Norrsjön to Vällen was not the expected form (figure 5.3). From the start, the histogram divided the flow into four groups and refining the interval did not change the number of groups. However, refining the interval made the bars more specific for a certain flow.

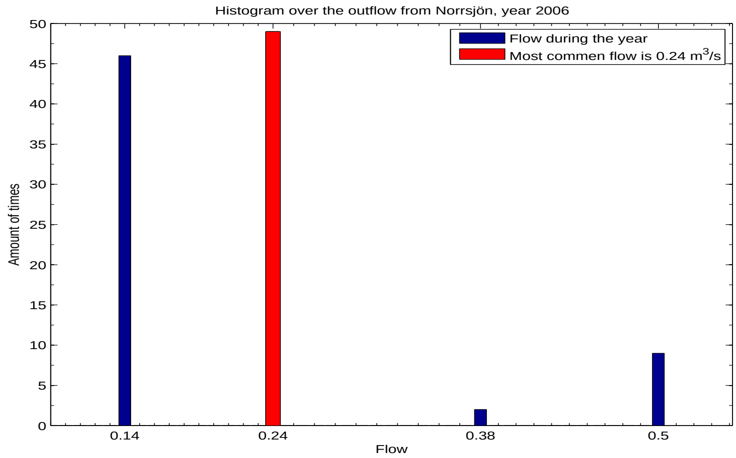


Figure 5.3: The histogram show the flow out of Norrsjön the year 2006.

The interval for the histogram shown in figure 5.4 is 0.005 m^3/s. The most common flow in the figure is 0.24 m^3/s. If this flow is considered to be low flow then the medium flow would be be represented by just 2 of 106 measurements. Therefore in this specific case it is more relevant to consider 0.24 m^3/s as medium flow, 0.14 m^3/s as low flow and 0.5 m^3/s as high flow.
The most common outflow from Gisslaren south is zero, as seen in figure . The reason for not using that as low flow is that it would not be possible to model the channel as described in section 2.3. Instead the second most common flow is used as low flow.

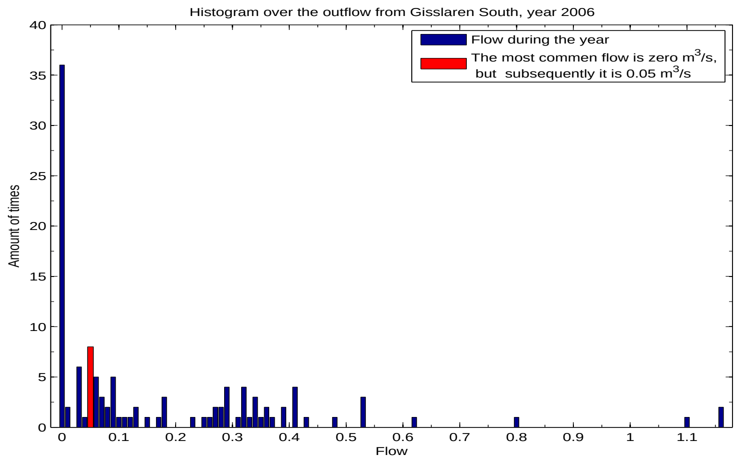


Figure 5.4: The histogram show the flow in the channel from the south dam of Gisslaren to Bysjön the year 2006.

The histograms for the other channels (not presented in this section) can be located in Appendix A.5.
A compilation of the low, average and high flow for each channel can be found in table 5.1.

Lakes			Low $[m^3/s]$	Average $[m^3/s]$	High $[m^3/s]$
Norrsjön	$\rightarrow$	Vällen	0.14	0.24	0.5
Vällen	$\rightarrow$	Bysjön	0.65	1.40	2.9
Gisslan N.	$\rightarrow$	Harg	0.04	0.23	0.8
Gisslan S.	$\rightarrow$	Bysjön	0.05	0.28	0.6
Bysjön	$\rightarrow$	Närdingen	0.40	1.0	6.5
Närdingen	$\rightarrow$	Hallstavik	0.8	1.2	8.4

Table 5.1: This table shows the low, average and high flows for each channel

5.2 Analysis of the Internal model

5.2.1 Prediction horizon

As stated in the MPC theory, the prediction horizon describes how far into the future the MPC predicts the states. The prediction horizon is expressed as the amount of control time steps until the final predicted states are reached. If the control time step is decreased, the prediction horizon needs to be increased in order to predict as far as before the control time step was decreased. Figure 5.5 show an example how to change the prediction horizon in order to predict as far even if the control time step is decreased.
If a change is made at the dam of Norrsjön and the flow in the channels are of low type, this change can be noticed at Hallstavik 38 319 seconds later (10 hours and 38 minutes, see table 3.5). Therefore the prediction horizon should be at least as long as this. How the prediction horizon is set to for this thesis is explained in the next subsection.

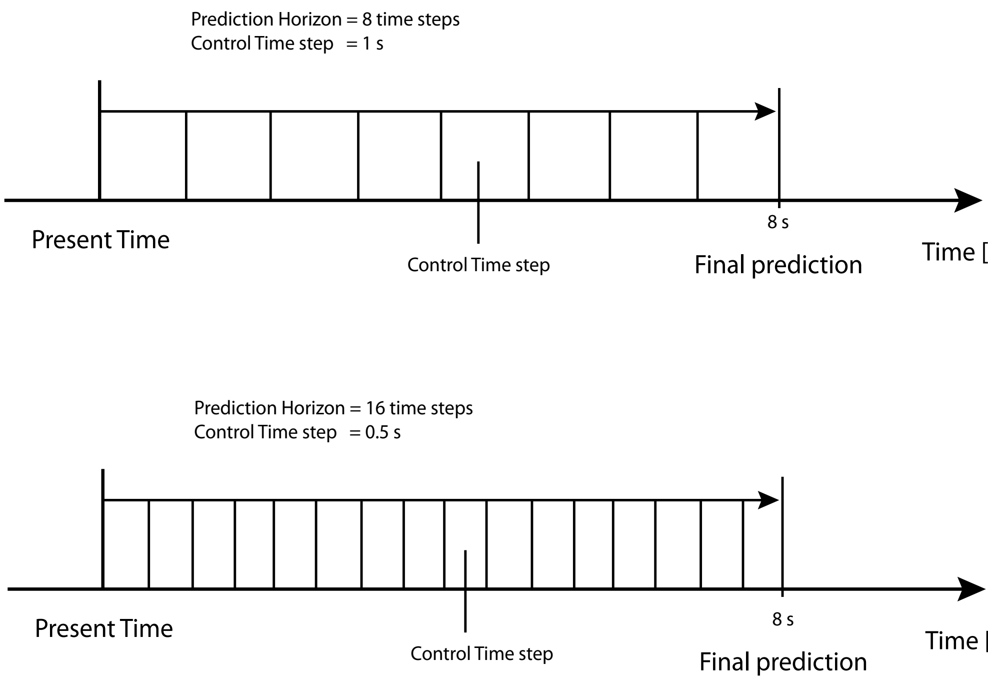


Figure 5.5: The figure show an example of how the prediction horizon needs to be changed when the control step time is decreased.

5.2.2 Control step time

Today Holmen controls the gate height of each dam two or three times in a normal week for most lakes. However, Närdingen is controlled every weekday because it is the last lake before the water reaches Hallstavik and therefore more important to control. When the water levels are high or the predicted precipitation is high (or both), all lakes are controlled more often.
The shortest time between two control occasions of the same dam during 2002 was 6 hours and 30 minutes. This was during spring when the snow melted and the rain caused a large amount of inflow to the lakes.
In order to choose the control step time (T_c) for the MPC simulation one needs to consider the response of the system. The lowest delay time in Skeboå water system is between Norrsjön and Vällen. When the flow is low the delay time is about 1 hour and when the flow is high it is approximately 36 minutes.
The control time step should not be set too low. The reason is that the number of states in the ID model (which for this thesis is the internal model of the MPC) increases when the control time step decreases. More states in the internal model will lead to larger matrices in the optimization algorithm. The reason for this is that the prediction horizon needs to be increased so that the final future states are still predicted. This in turn gives a longer computational time. The time needed for each optimization depends on the hardware of the computer used. The computational time will increase until the computer runs out of memory and then the calculations will not be finished. Therefore the choice of control step time is a trade off between computational time, the amount of states in the internal model, the prediction horizon and the hardware of the computer.
For Skeboå water system the final predicted states of the internal model should be at least 10 hours and 38 minutes in the future, as stated in the section above. With this in mind the control step time chosen for this thesis is 15 minutes or 900 seconds. This sets the first tuning value for the prediction horizon to 43 ($43 \cdot 900 = 38\,700 > 38\,319$) and the number of states of the internal model(equation (3.9)) to 72.
With these values a standard computer bought during recent years should be able to do the optimization calculations within the control time step.

Chapter 6

MPC Simulation of Skeboå water system

6.1 Objectives of the simulation

The main objective of this thesis is to save water using a MPC controller. A simulation instead of a full-scale trial is preformed partly because it is cheaper to try out the controller and partly because the dam gates of the lakes are not automated yet. Another objective of this thesis is to recommend which dam to automate first.
As stated in the introduction, the trial period is 30 days and starts on the 27th of May 2002. This period was chosen because the inflow to the lakes should not be affected by the melt water from the snow and it is before the dry part of the summer. The year 2002 was chosen because the summer that year was very dry.

6.2 Settings for the MPC simulation

The internal model for the MPC is the ID-model (described in section 3.4). This model has six outputs, the water levels of the five lakes and the last water flow in the channel between Närdingen and Hallstavik. The model has six inputs which are the flow of water under each dam.
The reference level for the input is the same as the minimum flow rate for each channel. The reason is that this simulation will try to save water. By choosing the lower limit of the water flow as the reference, the optimization algorithm will try to minimize the flow of water that is let out of the lakes.
The reference level for the water level of the lakes is set to 10 centimetres below the upper limit for that lake. In this way the system has a chance to respond to quick unpredicted changes, such as a lot of rain.
The reference level for the water flows in the all the channels (except the channel between Närdingen and Hallstavik) is set to the minimum flow constraint of each channel. The reference for the water flow between Närdingen and Hallstavik is set

Norrsjön			Bysjön		
Waterlevel	19.02	m	Waterlevel	12.49	m
Gate height	0.0248	m	Gate height	0.0254	m
Outflow	0.135	m^3/s	Outflow	0.5	m^3/s
Reference: Waterlevel	1.5	m	Reference: Waterlevel	1.4	m
Reference: Outflow	0.00833	m^3/s	Reference: Outflow	0.1	m^3/s
Vällen			Närdingen		
Waterlevel	13.29	m	Waterlevel	8.12	m
Gate height	0.075	m	Gate height	0.0417	m
Outflow	0.8	m^3/s	Outflow	1.65	m^3/s
Reference: Waterlevel	1.18	m	Reference: Waterlevel	1.4	m
Reference: Outflow	0.01	m^3/s	Reference: Outflow	1	m^3/s
Gisslaren			MPC parameter		
Waterlevel	14.29	m	Control Step Time	900	s
Gate height south	0.0	m	Prediction Horizon	50	iterations
Gate height north	0.0125	m	Control Horizon	43	iterations
Outflow south	0.055	m^3/s			
Outflow north	0.08	m^3/s			
Reference: Waterlevel	0.84	m			
Reference: Outflow S	0.0	m^3/s			
Reference: Outflow N	0.01	m^3/s			

Table 6.1: This table show the start values for the simulation.

to 1 m^3/s. The paper mill needs 0.5 m^3/s. However, the mill is not the only one that takes fresh water from the channel. Also the local community and farmers need fresh water. Letting out more water should ensure that enough water reaches Hallstavik for the mill. It is assumed that 0.5 m^3/s is enough for the farmers and the community. All the reference levels were decided together with Hallsta Paper mill.

The non-linear model of Skeboå will simulate the real system and the ID-model will be used in the MPC optimization as the internal model. The start values for the states in both models were set according to how they were on the start day. The start value for all states in each of the channels was set to the same value as the water flow under the dam gate before each channel. As an example, all states of the channel between Norrsjön and Vällen were set to 0.135 m^3/s. The reason is because the only flow records in the channels where from the flow at the beginning of the channel. All the start values, as well as the reference values, for each lake can be seen in table 6.1.

$$h_g = \frac{Q_{opt}}{b \cdot C\sqrt{2g(h_1 - h_2)}} \tag{6.1}$$

The MPC calculates the optimum outflow at each dam. However, the non-linear model of Skeboå water system uses the gate height of each dam as the input instead of the outflow. Therefore the optimum outflow needs to be recalculated to the corresponding gate height that gives the optimum outflow. For this, equation (2.5) is rewritten to equation (6.1).
To make the result of the simulation easier to analyse, all water levels and upper and lower water level limits are rewritten to the same zero benchmark (meters above sea level). In this way the water levels of the lakes are easier to compare with each other. The water level limits are presented in table 6.2.

Lakes	Upper water level limit [m]	Lower water level limit [m]
Norrsjön	19.07	18.07
Vällen	13.28	12.38
Gisslaren	14.20	13.90
Bysjön	12.48	12.23
Närdingen	8.32	7.32

Table 6.2: This table shows the water level limits according to meters above sea level.

6.3 Tuning of the MPC

The goal of the MPC tuning is to control the water flow out of each lake in order to maintain the reference level of each lake without letting too much water out of the system. The outflow of water from Skeboå water system is at the north dam of Gisslaren and at the end of the channel between Närdingen and Hallstavik. It is therefore important to keep those two outflows to a minimum so that enough water stays within the system.
Once the matlab files and script where set up, the parameters of the MPC were found iteratively. Each iteration of the simulation ran for 2 simulated days and was then analysed to see if an adjustment was needed. The parameters that where adjusted were the weight matrices (Q_x and Q_u), the prediction horizon and the control horizon. More adjustments where needed if the MPC controller could not find an optimum solution and therefore set the water flow out of each dam to a minimum even if the water level was above the upper limit for that lake. An adjustment was also needed if the system could not get the system within the water level and water flow constraints of each channel and lake. The strategy for the adjustments was that it is more important to hold the output reference level

(reference water outflow from Närdingen and reference water levels of each lake) than minimizing the outflows from the lakes.
After some iterations, the result of the short test was satisfactory. The diagonal elements in the state weight matrix obtained a value of 10 and the diagonal elements of the input weight matrix obtained a value of 0.0001. The prediction horizon was set to 50 and the control horizon to 43. A too large difference between the two horizons actually made the controller worse, it did not find a solution to the optimization problem but it presented the findings in shorter time then if the two horizons were closer to the same value.

6.4 Observer

In the real Skeboå water system, only the water level of the lakes, the water level after the dams and the gate heights are measured. From these three measurements the outflow under each dam can be calculated. From the water level after the dam the flow of water in the channel, at that location, can be calculated using the water bearing graphs and tables.
In the simulation, these parameters are measured in the non-linear model in the beginning of each control step. Thereafter the states of the internal model need to be updated. Not all states of the internal model are measured and must therefore be estimated. For the estimation, a standard Kalman filter is used.
Before the states of the internal model are updated, the Kalman filter is tuned using the matrices of the internal state space model, disturbance intensity R_1 and the measurement noise intensity R_2.
The intensity of the measurement noise is difficult to determine because the real system is also simulated. Measurements in the real system can be affected by water waves due to wind or other movement of the water. For this thesis the intensity of the measurement noise is assumed to be 0.001 for all the water level measurements and water flows. The intensity of the disturbance flow due to rain is harder to estimate. Therefore the value is tuned while tuning the MPC. The intensity values of the disturbances due to rain ($Q_{rain,No}$,$Q_{rain,V}$,$Q_{rain,G}$,$Q_{rain,B}$ and $Q_{rain,Na}$) were found to be 0.1 after the tuning.

Chapter 7

Results

7.1 Result of the MPC simulation

In this chapter we compare the performance of the proposed MPC control scheme with the present manually controlled system through simulations. The simulation ran for 30 days with start the 27th of May 2002. The results of the simulation is then compared to the data of from the system when it was controlled manually.
As stated in the previous chapter the water levels of the lakes in the simulated model were the same as it was when the system was manually controlled. This means that the water level of three lakes, Vällen, Bysjön and Gisslaren, starts above the upper limit which can be seen in figure 7.1, figure 7.2 and figure 7.3.

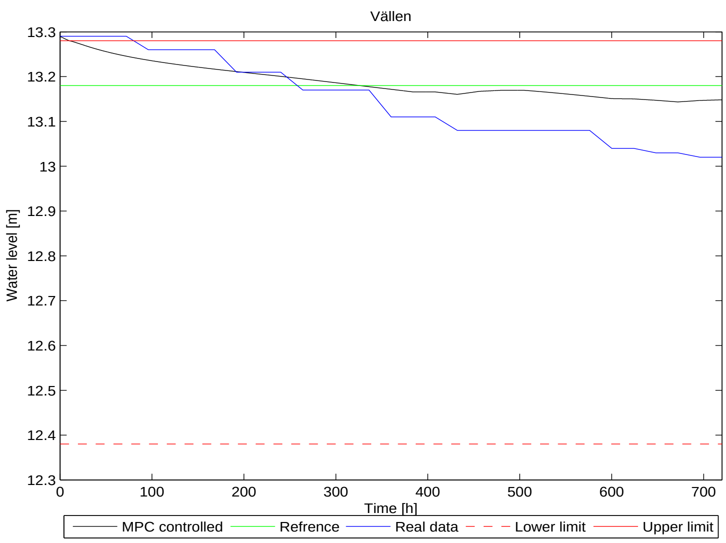


Figure 7.1: This plot shows the simulated water level and the real measured data of the water level in Vällen during the test period (27.5 - 24.6 2002). The water level refers to meters above sea level.

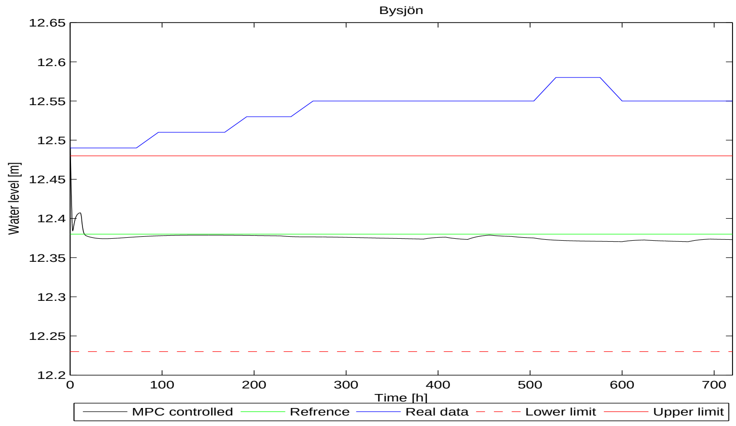


Figure 7.2: This plot shows the simulated water level and the real measured data of the water level in Bysjön during the test period (27.5 - 24.6 2002). The water level refers to meters above sea level.

However the MPC controlled the system to keep the water level within the limits and aimed to and keep the water levels at the reference water levels while minimizing the outflow from each lake.
Bysjön reached the reference water level within a few hours and stayed close to it during the rest of the simulation. It took Gisslaren about 250 hours to reach the reference level but once that happened it also stayed close to it. The water level of Vällen however slowly decreased during the whole simulation with one exception. After about 420 hours the water level started to rise but only after 30 hours it turned back down and continued to slowly decrease.

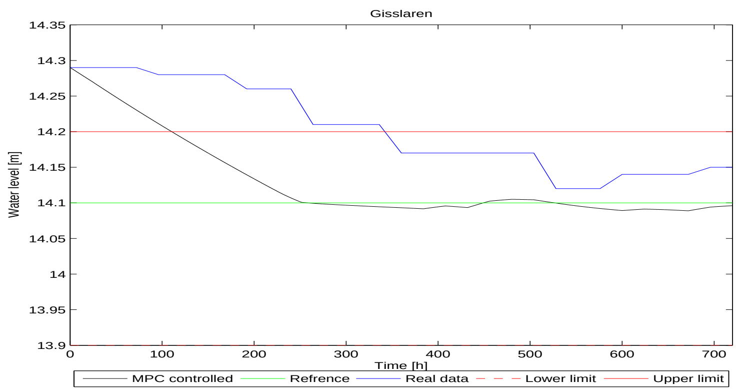


Figure 7.3: This plot shows the simulated water level and the real measured data of the water level in Gisslaren during the test period (27.5 - 24.6 2002). The water level refers to meters above sea level.

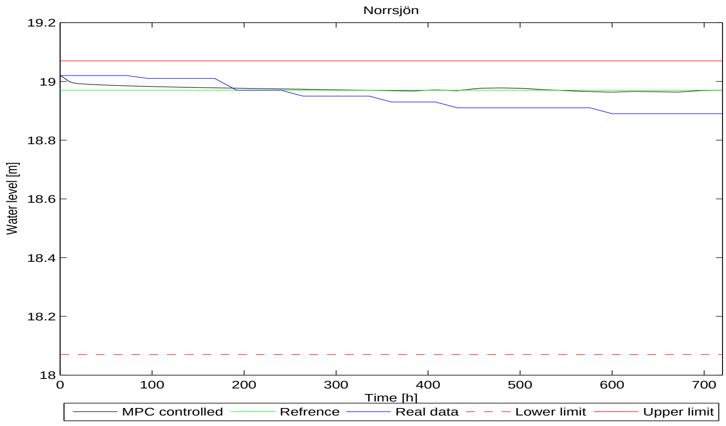


Figure 7.4: This plot shows the simulated water level and the real measured data of the water level in Norrsjön during the test period (27.5 - 24.6 2002). The water level refers to meters above sea level.

The water level of Norrsjön started above the reference level but bellow the upper limit. In the beginning of the simulation it reached the reference level quickly and stayed really close to it throughout the trial period This can be seen in figure 7.4. The water level of Närdingen started 10 centimetres bellow the reference level. During the first 100 hour the water level increased. However the reference level were never reached and after 100 hours the water level started to descrease and at the end of the trail period it was 2 centimetres higher than when the simulation started, at 8.14 meter above sea level. This can be seen in figure 7.5.

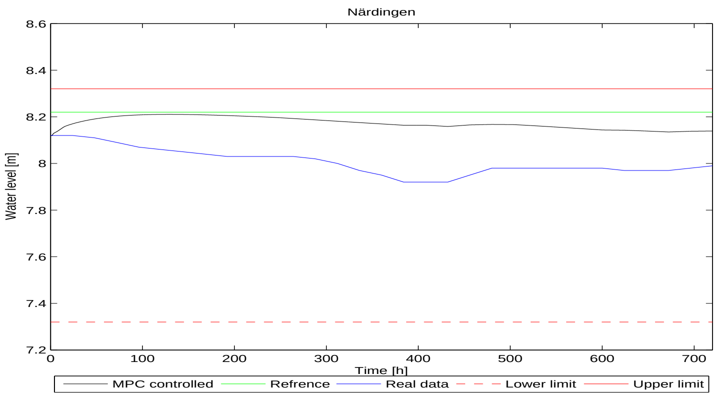


Figure 7.5: This plot shows the simulated water level and the real measured data of the water level in Närdingen during the test period (27.5 - 24.6 2002). The water level refers to meters above sea level.

To understand how well the water system was controlled the control volume of all the lakes are summed together. Control volume means the volume from the lower limit to the current water level. If the water level is at the lower limit then the control volume would be zero. By doing that for both the simulated water system and for the water system when it was manually controlled one can then compare the two ways of controlling the system. From figure 7.6 one can see that the simulation controlled the system in a better way. The blue line represent the volume of the manually controlled water system and the black line represent the volume of the simulated system during the trial period.
The difference between the two volumes are at the end of the trial period was 1.716 Mm^3 of water. That is little less then half of what the manually controlled system let out during the whole trial period, which let out 3.769 Mm^3 of water. The volume of the simulated system was 2.053 Mm^3 of water at the end of the trial period.

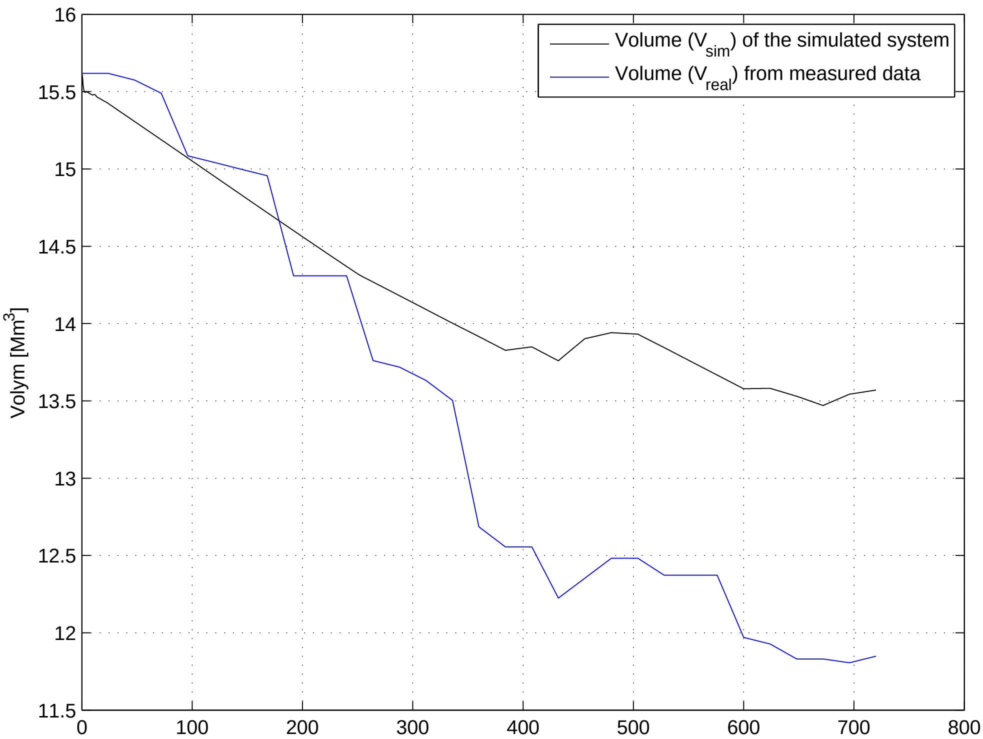


Figure 7.6: This plot shows the total volume of water in the system during the test period. The blue line is from the simulated system and the red line is from the real data.

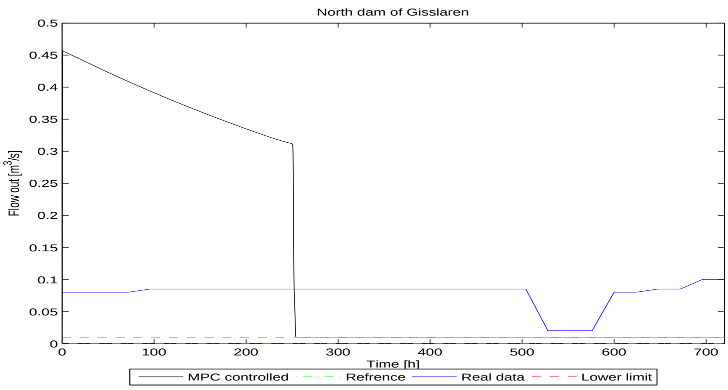


Figure 7.7: The plot shows the flow of water out of the north dam of Gisslaren to Harg during the simulation period (27.5 - 24.6 2002).

The outflow of water from the water system is from the north dam of Gisslaren and from the dam at Närdingen. From the north dam of Gisslaren the outflow started quite high but slowly decrease until the water level is at the reference level. This is an effect of the MPC which aim to keep the lakes water level at the reference level while keeping the outflow to a minimum. This effect can be seen in figure 7.3 and figure 7.7. When the reference level was achieved then the outflow were set to the minimum, only letting 0.01 m^3/s of water pass under the dam gate.
The outflow of Närdingen was keept stable at around 1 m^3/s of water, which can be seen in figure 7.8. This result was excepted as long as the water level was bellow the reference level. As long as the water level in the upstream lakes were above their reference water level Närdingen got more water then it let out.
When the water level in those lakes settled of passed the reference level then they

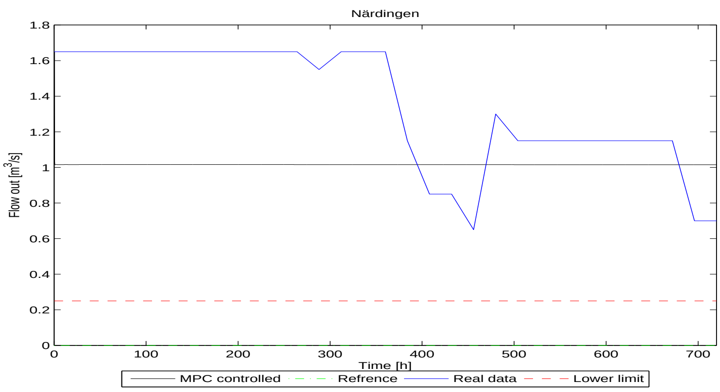


Figure 7.8: The plot shows the flow of water out of the dam at Närdingen to Hallstavik during the simulation period (27.5 - 24.6 2002).

let out just enough to keep the outflow of water from each lake at a minimum. The outflow of water from Vällen slowly settled at around 0.6 m^3/s thereby not draining the lake of water to keep the water level of Närdingen stable at 8.22 m above sea level.
The figures of the other outflows can be found in Appendix A.6. Figures of how the dam gates were managed can also be found in that appendix. Those plots looks like the plots of the outflow but with a different range on the y-axis. That is the reason why no such plots are included in this chapter.

7.2 Discussion of results

During the simulation all water levels stayed within the upper and lower limits once they were within the limits. The final water level for the simulation and and when the system was manually controlled are stated in table 7.1. There one can see that the water level of two lakes in the simulation ended a few centimetres below the manually controlled system. However in the case of Bysjön the water level were above the upper limit when controlled manually during the whole trial period. Bysjön has the lowest area and control span of the lakes in the system and thereby the lowest maximum control volume. The total control volume for the simulation is not significantly affected by the lower water level in Bysjön compared to the measured data throughout the trial period.
The difference in water level in Gisslaren between the simulation and the measured data is not as large as in Bysjön. If the difference would be more then it would probably be noticed when comparing the total control volume of water. The reason is that Gisslaren is the 3rd largest lake in the system. However Gisslaren could only contribute 0.25 m^3/s and could therefore not support Närdingen by it self, if Närdingen needs to let 1 m^3/s of water through to Hallstavik. But during dry seasons it would extend the time until the whole water system is drained if Roslagen would be without rain a longer period of time.

Lake	Water level from MPC simulation [m]	Water level from data [m]	Difference [m]
Norrsjön	18.9698	18.8900	0.0798
Vällen	13.1483	13.0200	0.1283
Gisslan	14.0961	14.1500	-0.0539
Bysjön	12.3730	12.5500	-0.1770
Närdingen	8.1392	7.9900	0.1492

Table 7.1: This table shows the water level at the end of the trial period, both for the simulated system and for the real measured manually controlled system. It also shows with how much they differs from each other.

Chapter 8

Conclusions

8.1 Conclusions

During this project two models and a control strategy where developed, analysed and then simulated. Results from the simulation were compared to real data of the system when the system was manually controlled. From the results of the simulation it can be concluded that when summer approaches, which often means dryer and warmer weather, water can be saved by managing the water in a better way.
The simulation shows that water can be saved. In the simulation, the MPC stored 1.716 Mm^3 of water more in Skeboå water system than when the water system was controlled manually. During the trial period, 27th of May to 24th of June 2002, the water level of two lakes was above the upper limit which meant that Holmen had to pay fines according to the judgements of the Swedish water court. In the simulation that situation was avoided because the water levels stayed within the water level limits.
Setting the reference for the outflow of Närdingen to 1 m^3/s should ensure the demand of freshwater to the paper mill. If the farmers and the local waterworks demand more than 0.5 m^3/s, that reference flow needs to be adjusted. The judgements from the Swedish water court do not contain any information about how much water Holmen has to let out of Närdingen to ensure freshwater to the local population.
Two dams have the potential to be automated first, depending on the preferences of the mill. If it is considered important to keep the flow rate in the channel between Närdingen and Hallstavik constant at a certain reference level, the dam at Närdingen should be automated first. In this case the water flow becomes more steady and reliant. On the other hand, if it is important to keep the water levels of the lakes within their limits, the dam at Bysjön should be automated first. Bysjön is the smallest controlled lake and has the lowest limit span (25 cm). With such a low span and small area, the water level can rise and decrease quickly if Vällen and Bysjön are not controlled properly.
This thesis recommends that the dam at Närdingen should be the first to be au-

tomated. In this way more time can be used for controlling Bysjön and Vällen in such a way that the water levels of these lakes are kept within their limits.

8.2 Future work

Due to time restrictions it was not possible to test whether another model would have been better to use as a model of the real system. Hallsta paper mill has got a lot of data from Skeboå water system. Another model can be a black box or grey-box model created with this data.
If there would have been more time, it would have been good not to compare directly with how the system was controlled in reality. Instead, real gate height values should have been used as input to another simulation using the model of the real system. Then this new simulation should be compared to the simulation presented in this thesis. In this way, the uncertainty due to model error is decreased.
To improve the reliability of the model it would have been useful to have more data of the flow in the channels. If the model needs to be improved, data should be collected at least every day for a longer period, preferably one year in order to cover the seasonal variations.
The objective of this thesis was to recommend which dam to automate first. Because the control strategy used in this thesis automated the whole system, it was hard to give a clear recommendation. However, this thesis gives an indication of which dams should be automated first. To confirm the recommendation presented in the conclusions, another study needs to be made. In that study one dam at a time should be controlled. The first simulations should check what happens to the system if Närdingen or Bysjön is automated.
It can be worth looking into the effects on flow due to vegetation and infiltration of water into the ground. If this has a big effect on the system, it should be taken into account in the model of the system as disturbances to the system.
If the assumption that the local waterworks and farmers use a maximum of 0.5 m^3/s is incorrect, the simulations should be rerun with the reference outflow of Närdingen adjusted accordingly.

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Appendix A

Appendix

A.1 Water bearing graphs

The figures in this section are of the discharge in relation the the water surface elevation after each dam.

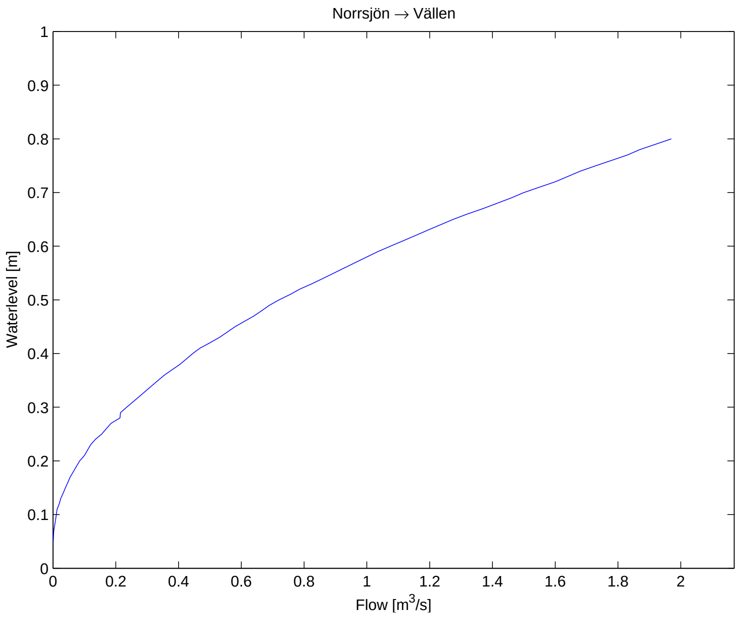

Figure A.1: The figure shows the discharge in relation to the water surface elevation after the dam of Norrsjön.

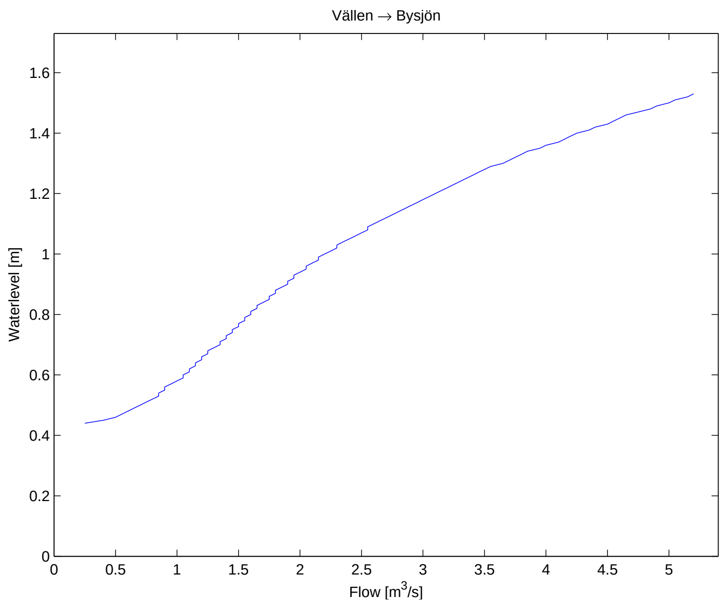

Figure A.2: The figure shows the discharge in relation to the water surface elevation after the dam of Vällen.

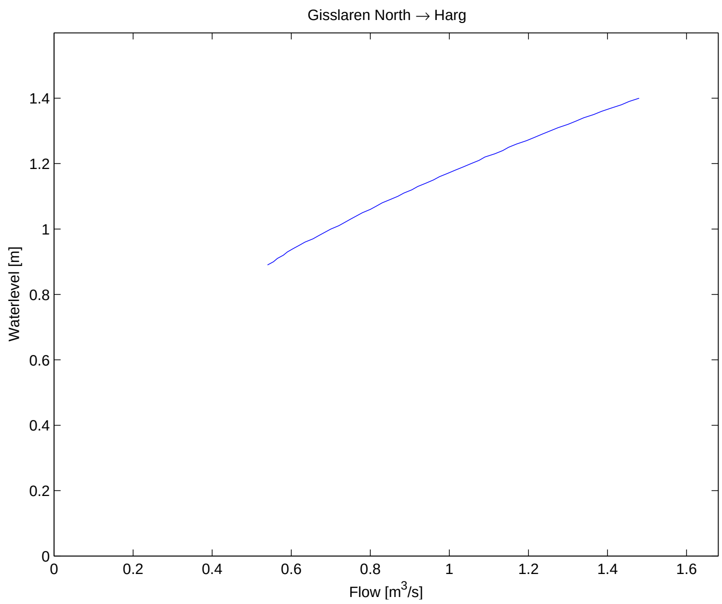

Figure A.3: The figure shows the discharge in relation to the water surface elevation after the the north dam of Gisslaren.

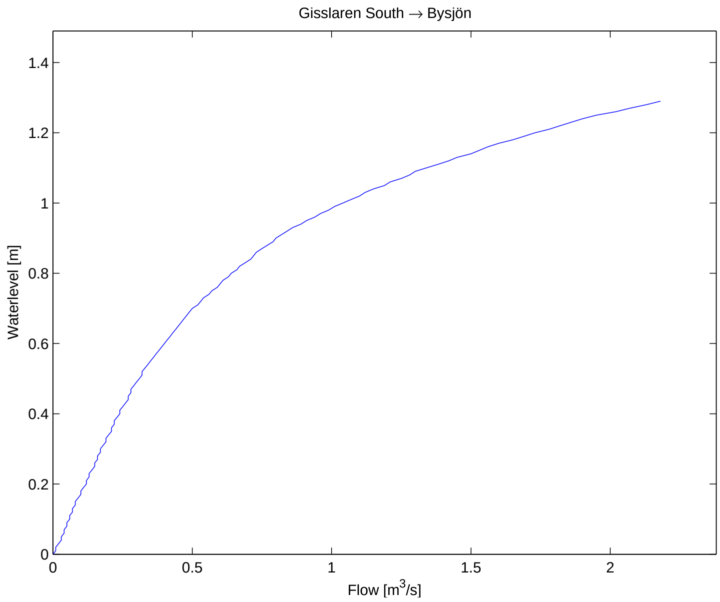

Figure A.4: The figure shows the discharge in relation to the water surface elevation after the south dam of Gisslaren.

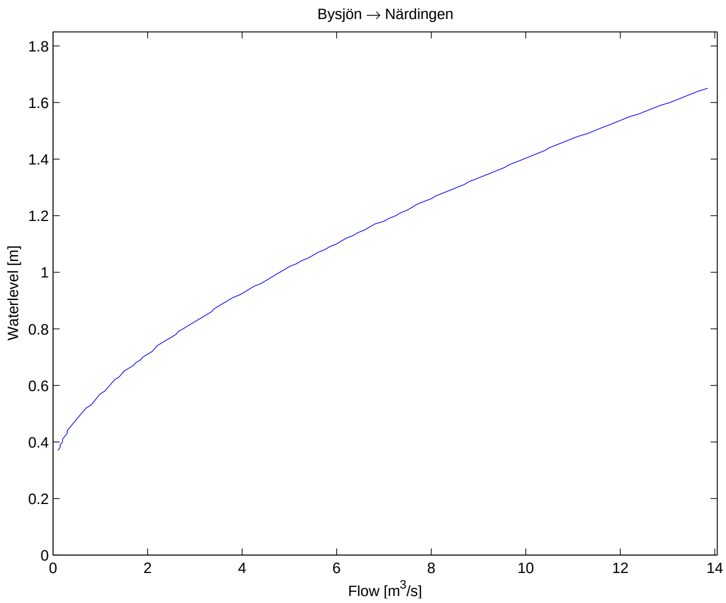

Figure A.5: The figure shows the discharge in relation to the water surface elevation after the dam of Bysjön.

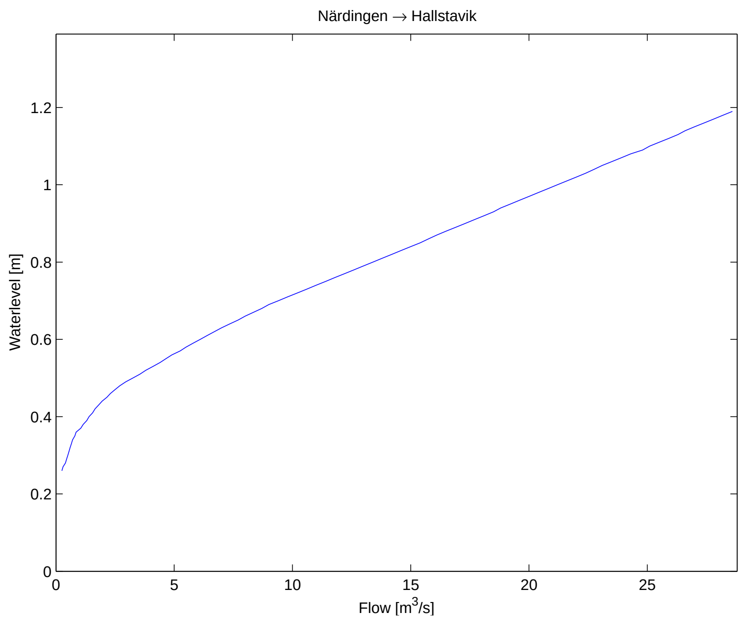

Figure A.6: The figure shows the discharge in relation to the water surface elevation after the dam at Närdingen.

A.2 Fitting graphs

The figures in the first subsection present the best fit when α is iteratively changed and β is set to 0.6, according to Mannings equation (equation (2.10)). The figures in the second subsection present the best fit of when both α and β are iteratively changed. The solid line in all figures is the approximation and the "x" are the data points from the water bearing graphs.

A.2.1 β fixed to 0.6

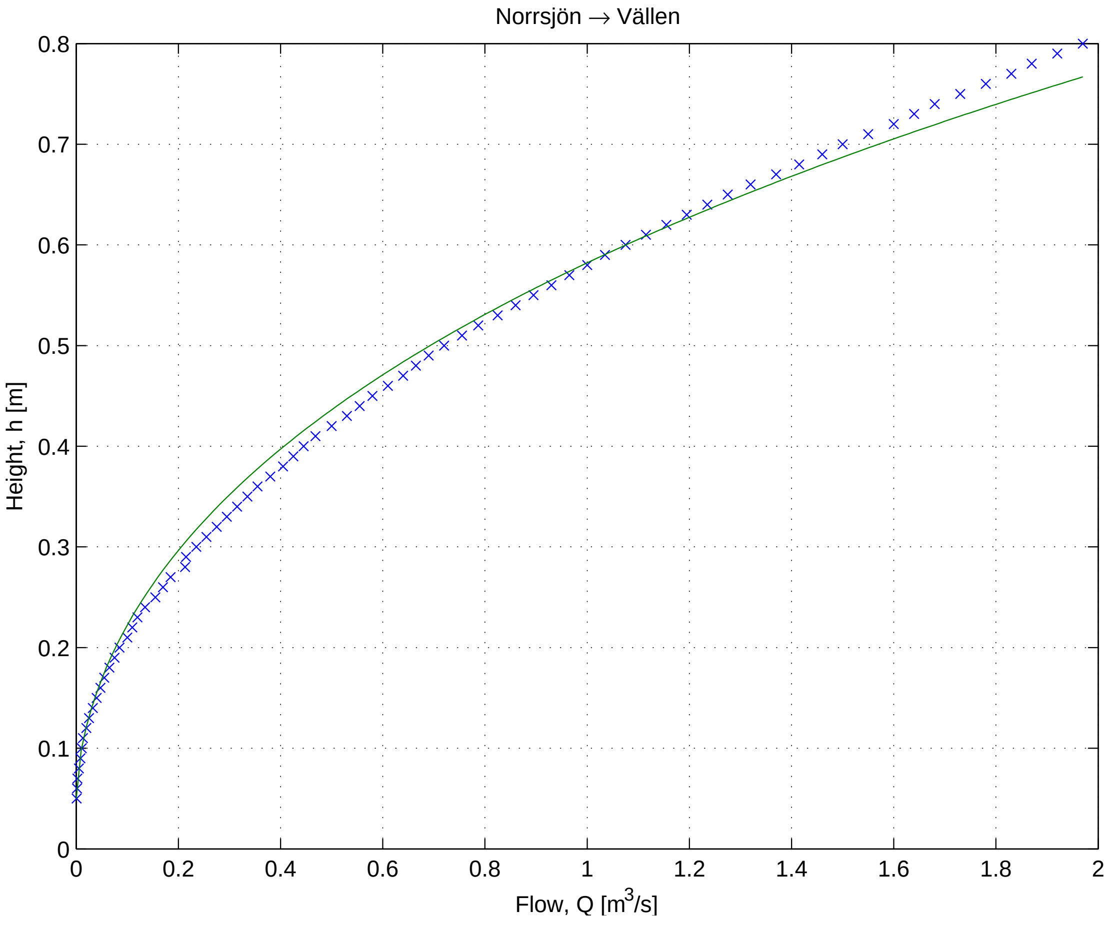

Figure A.7: The figure shows the best fit for equation (2.9) to the data points from figure A.1) for when β fixed to 0.6.

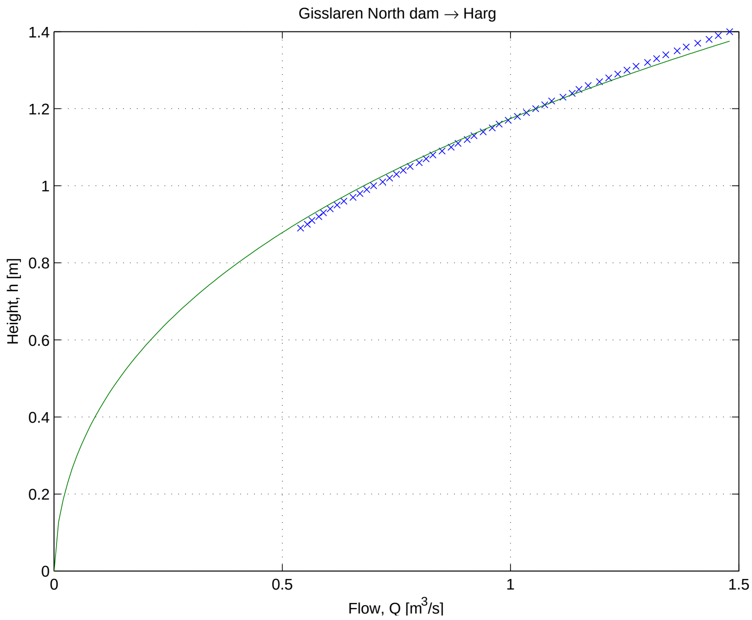

Figure A.8: The figure shows the best fit for equation (2.9) to the data points from figure A.3) for when β fixed to 0.6.

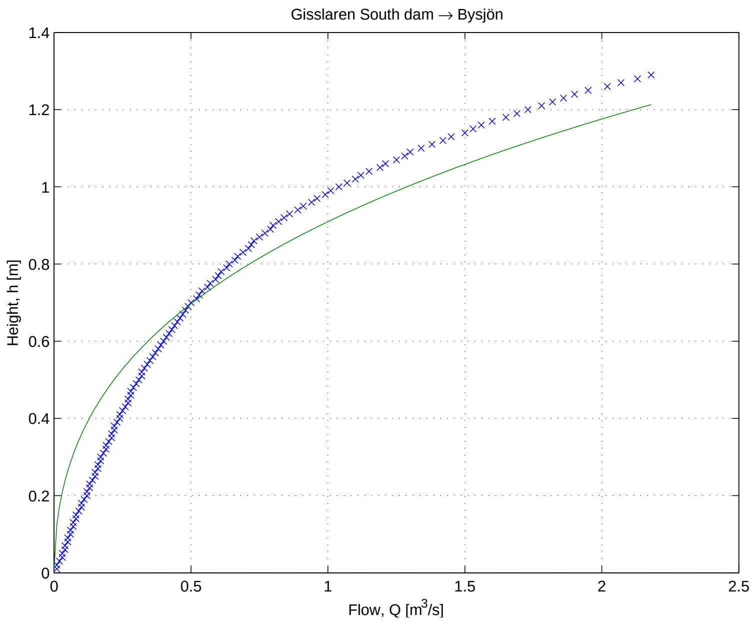

Figure A.9: The figure shows the best fit for equation (2.9) to the data points from figure A.4) for when β fixed to 0.6.

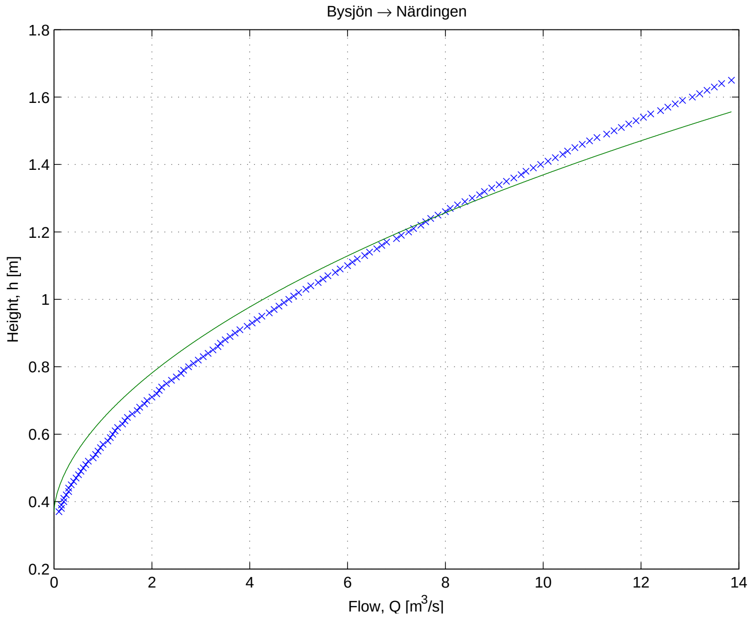

Figure A.10: The figure shows the best fit for equation (2.9) to the data points from figure A.5) for when β fixed to 0.6.

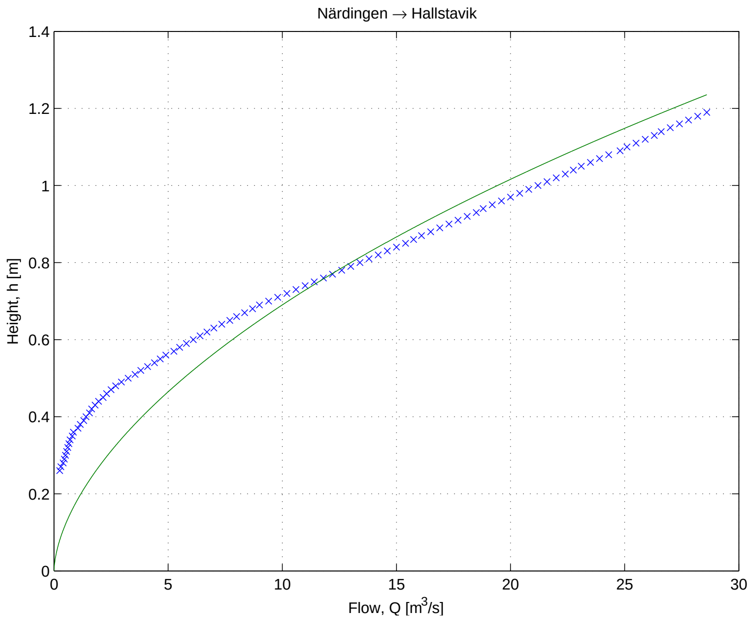

Figure A.11: The figure shows the best fit for equation (2.9) to the data points from figure A.6) for when β fixed to 0.6.

A.2.2 Both α and β are calculated

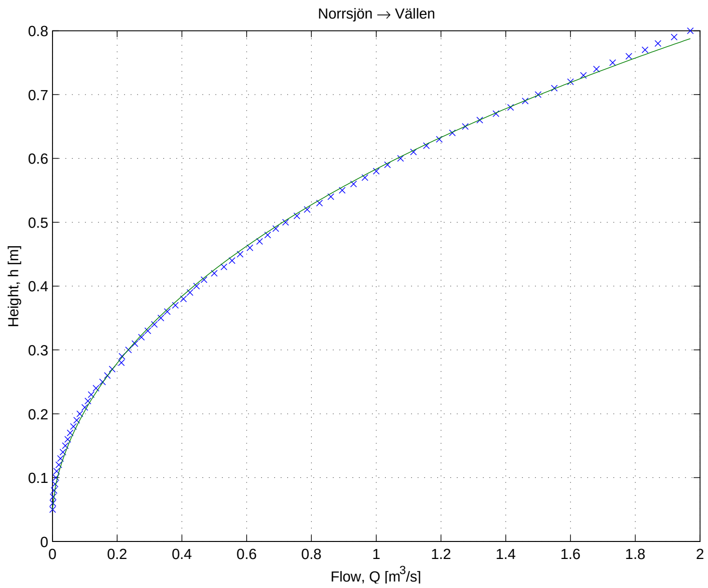

Figure A.12: The figure shows the best fit for equation (2.9) to the data points from figure A.1) when both α and β is calculated

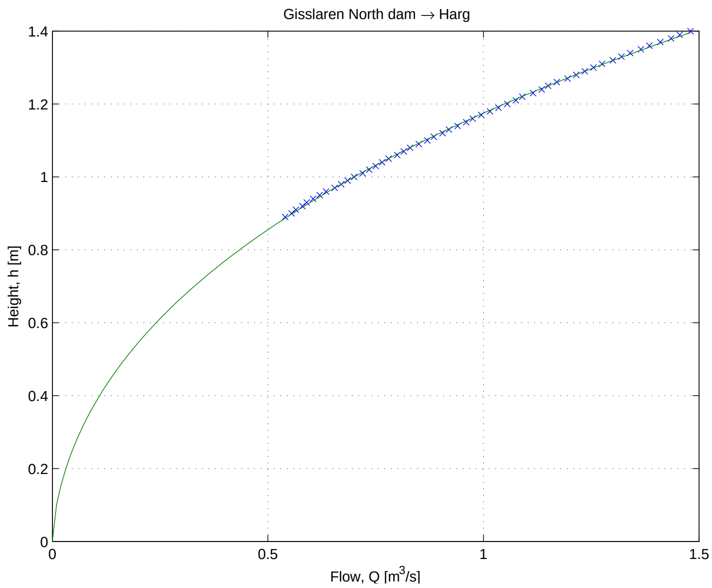

Figure A.13: The figure shows the best fit for equation (2.9) to the data points from figure A.3) when both α and β is calculated

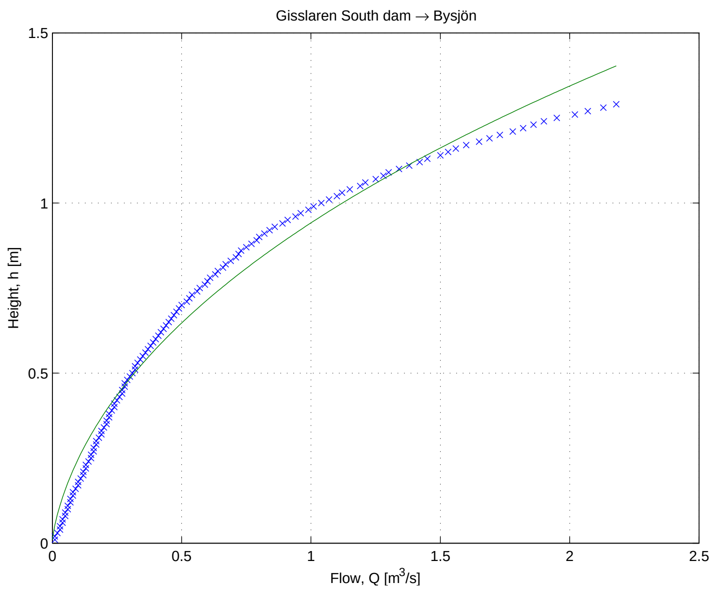

Figure A.14: The figure shows the best fit for equation (2.9) to the data points from figure A.4) when both α and β is calculated

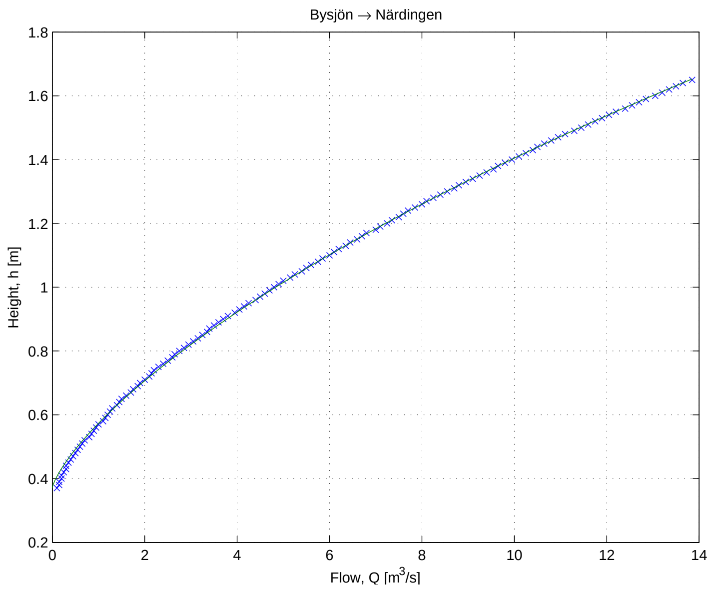

Figure A.15: The figure shows the best fit for equation (2.9) to the data points from figure A.5) when both α and β is calculated

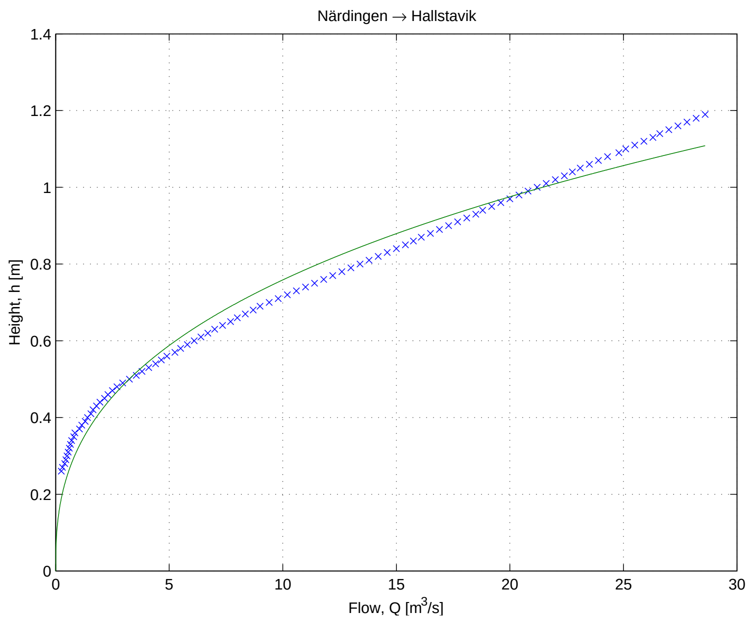

Figure A.16: The figure shows the best fit for equation (2.9) to the data points from figure A.6) when both α and β is calculated

A.3 Verification plots

All the figures in this appendix shows bode plots of data spectrum. The blue line represent the spectrum of the data from Hallsta paper mill and the red line is the spectrum of the data from the verification simulation.

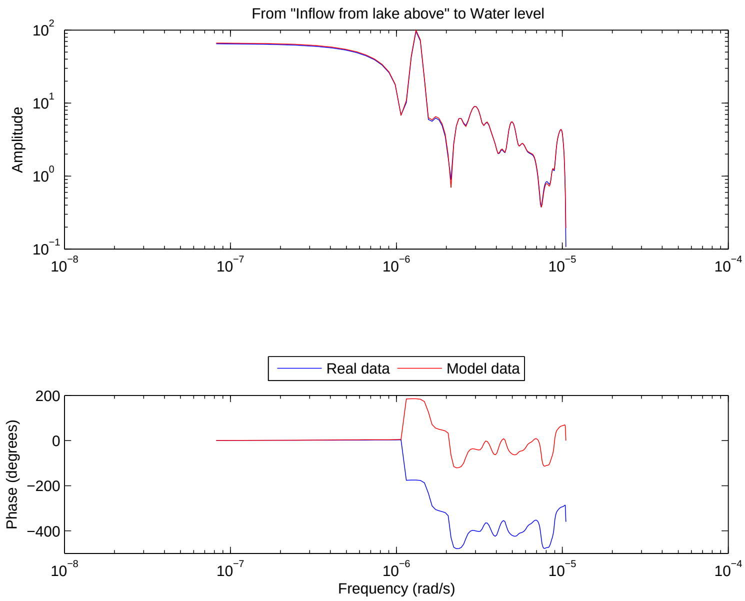

Figure A.17: The figure show one of the bode plots of Norrsjön. With the inflow of water as input and the waterlevel of Norrsjön as output.

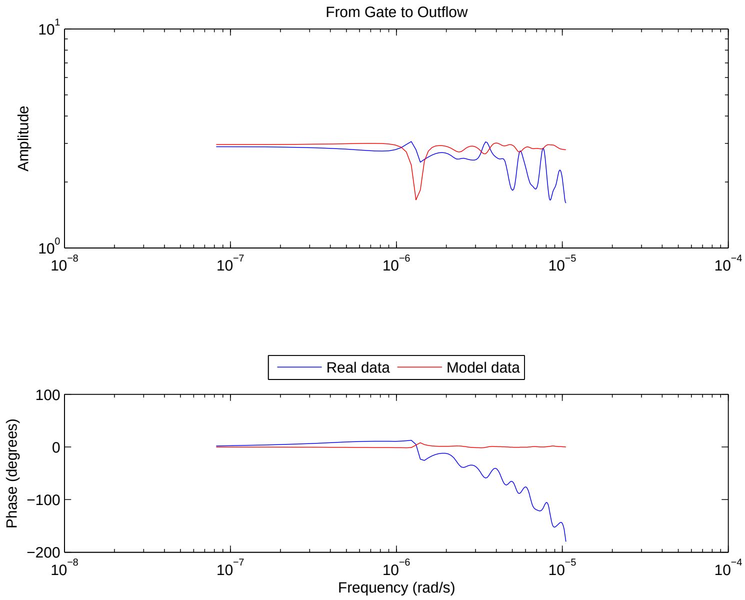

Figure A.18: The figure show one of the bode plots of Norrsjön. With the gate height as input and the outflow of water as output.

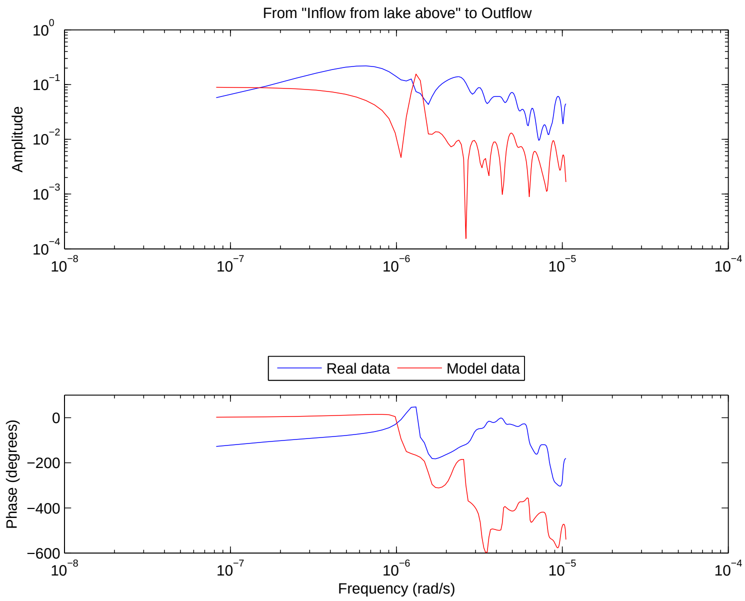

Figure A.19: The figure show one of the bode plots of Norrsjön. With the inflow of water to Norrsjön as input and the outflow of water from Norrsjön as output.

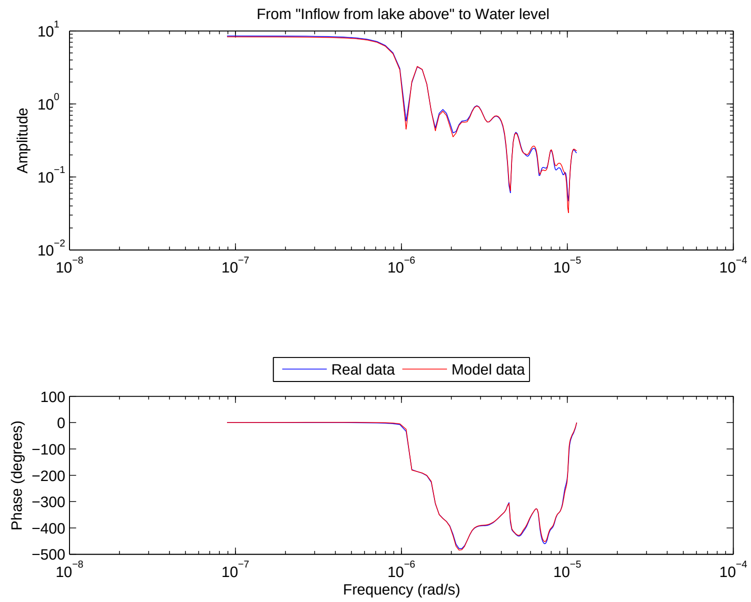

Figure A.20: The figure show one of the bode plots of Vällen. With the inflow of water as input and the waterlevel of Vällen as output.

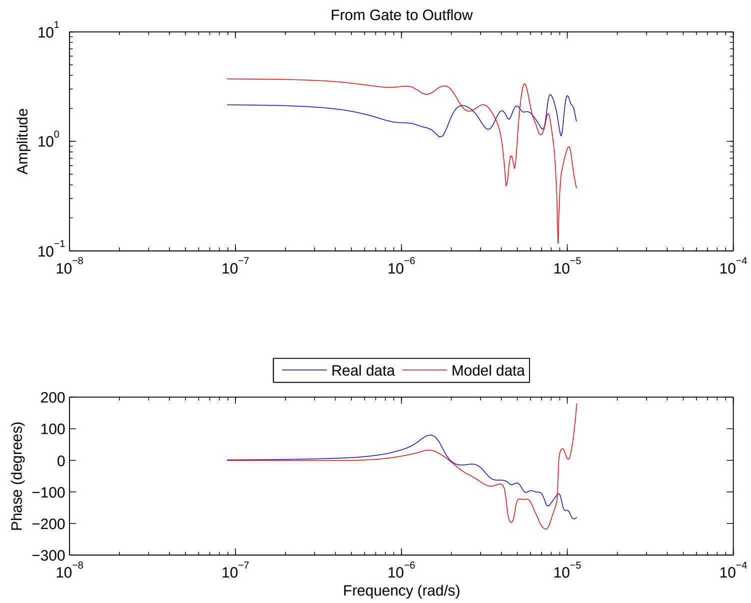

Figure A.21: The figure show one of the bode plots of Vällen. With the gate height as input and the outflow of water as output.

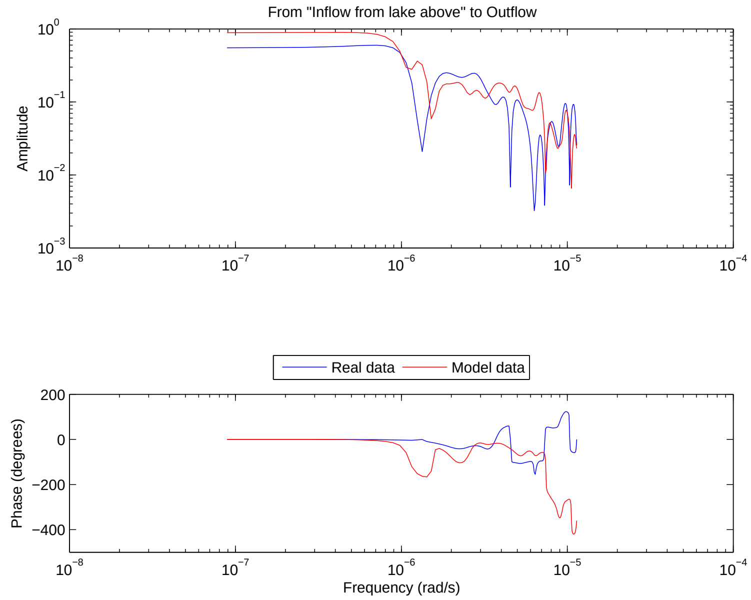

Figure A.22: The figure show one of the bode plots of Vällen. With the inflow of water to Vällen as input and the outflow of water from Vällen as output.

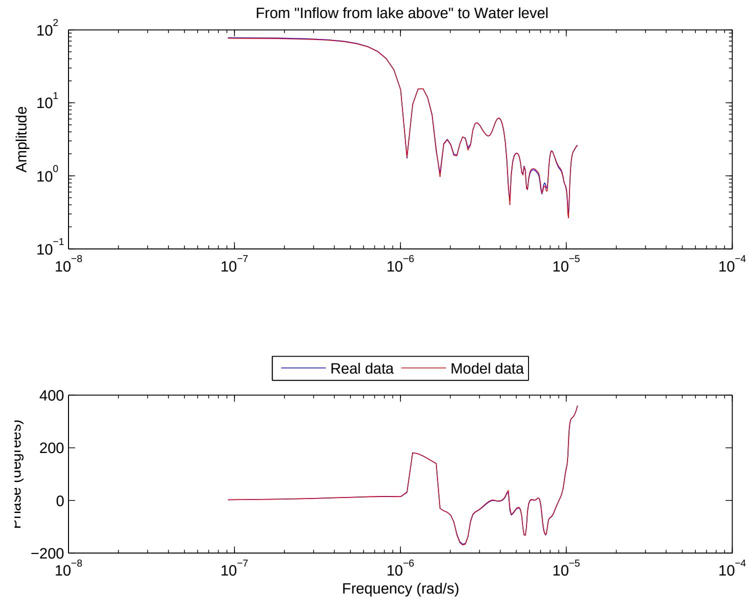

Figure A.23: The figure show one of the bode plots of Gisslaren. With the inflow of water as input and the waterlevel of Gisslaren as output.

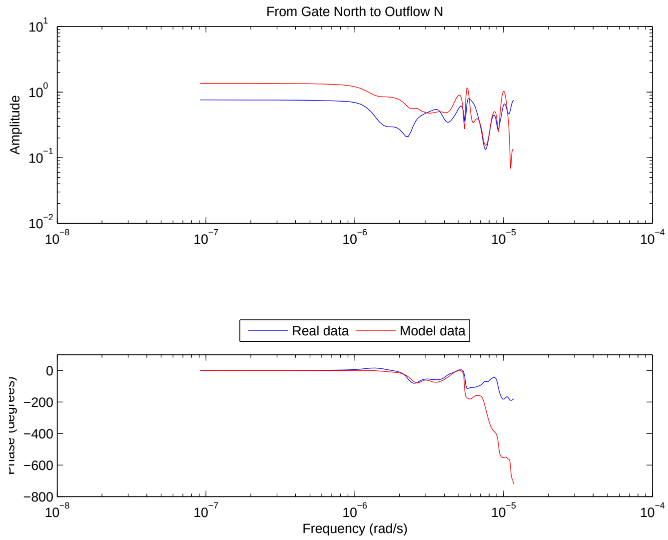

Figure A.24: The figure show one of the bode plots of Gisslaren. With the gate height of the north dam as input and the outflow of water from the north dam as output.

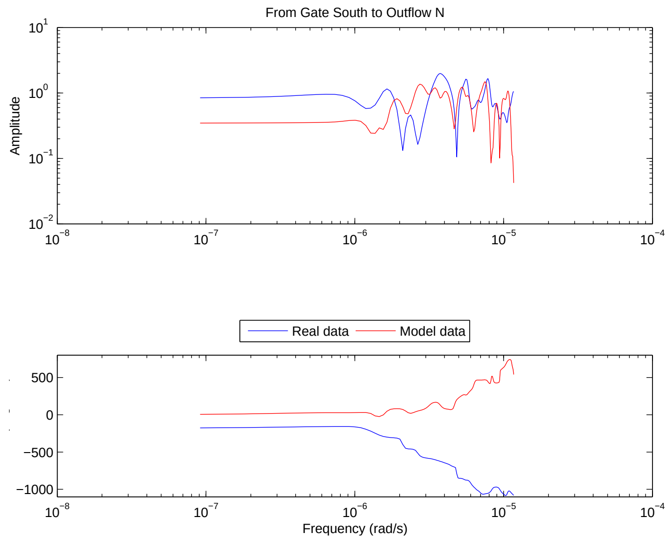

Figure A.25: The figure show one of the bode plots of Gisslaren. With the gate height of the south dam as input and the outflow of water from the south dam as output.

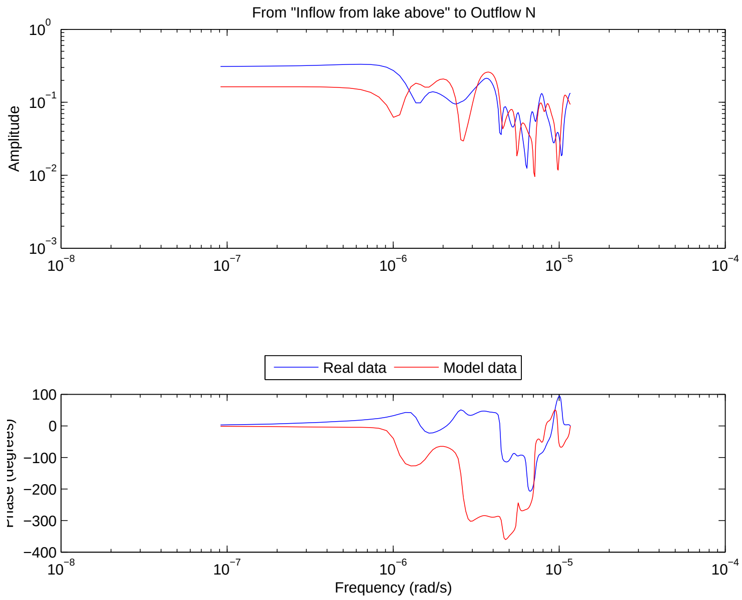

Figure A.26: The figure show one of the bode plots of Gisslaren. With the inflow of water to Gisslaren as input and the outflow of water from the north dam of Gisslaren as output.

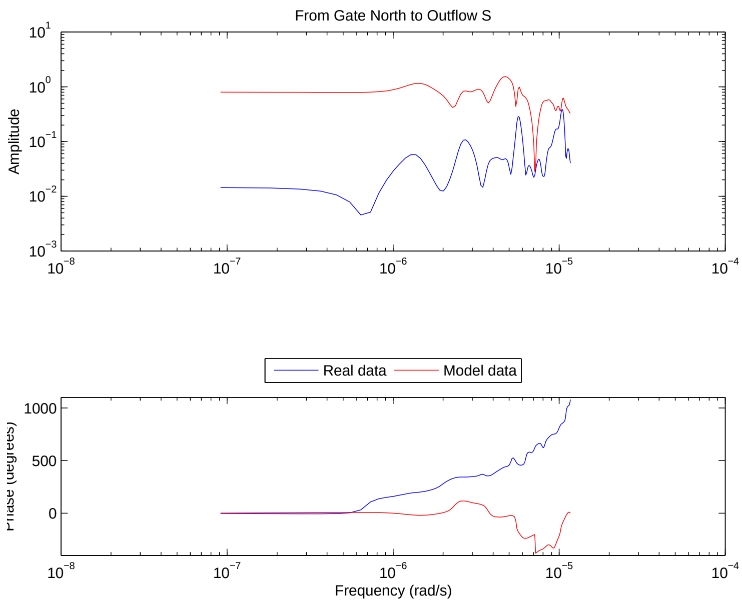

Figure A.27: The figure show one of the bode plots of Gisslaren. With the gate height of the north dam as input and the outflow of water from the south dam as output.

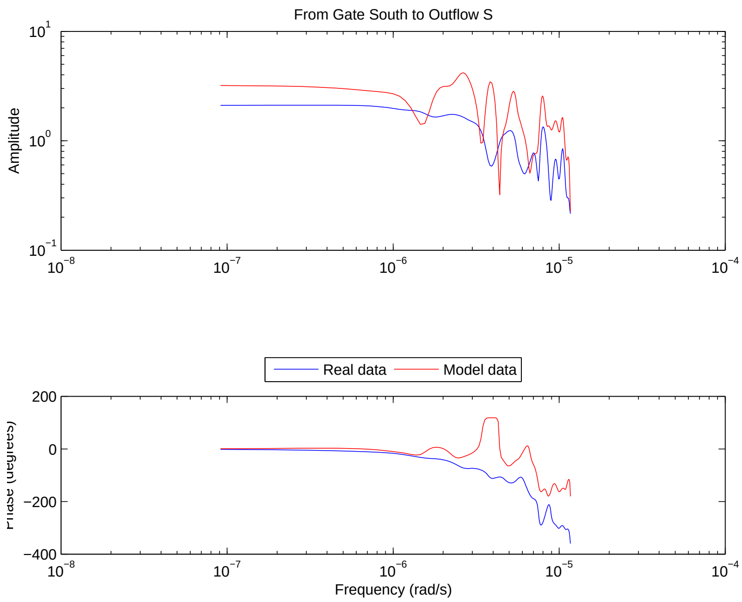

Figure A.28: The figure show one of the bode plots of Gisslaren. With the gate height of the south dam as input and the outflow of water from the north dam as output.

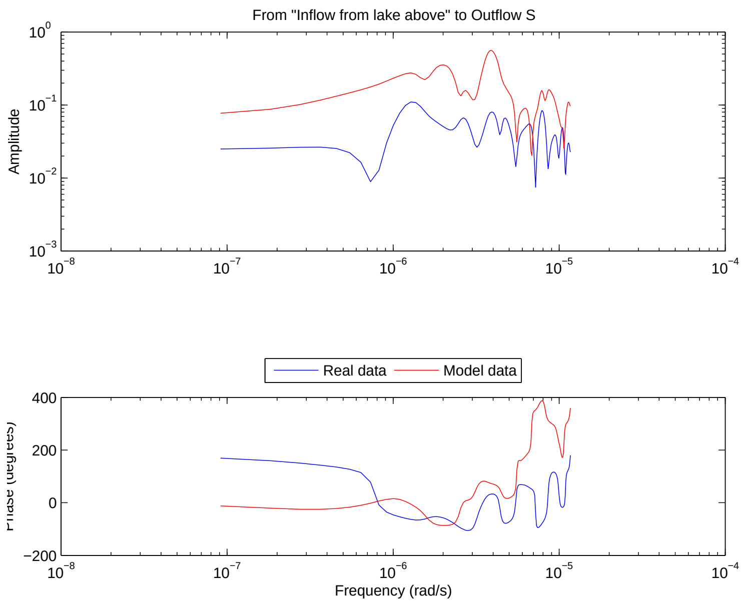

Figure A.29: The figure show one of the bode plots of Gisslaren. With the inflow of water to Gisslaren as input and the outflow of water from the south dam of Gisslaren as output.

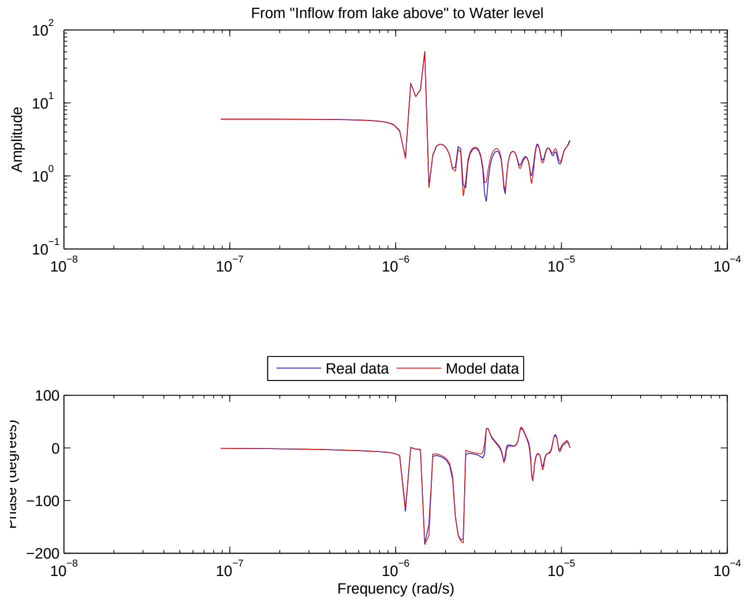

Figure A.30: The figure show one of the bode plots of Bysjön. With the inflow of water as input and the waterlevel of Bysjön as output.

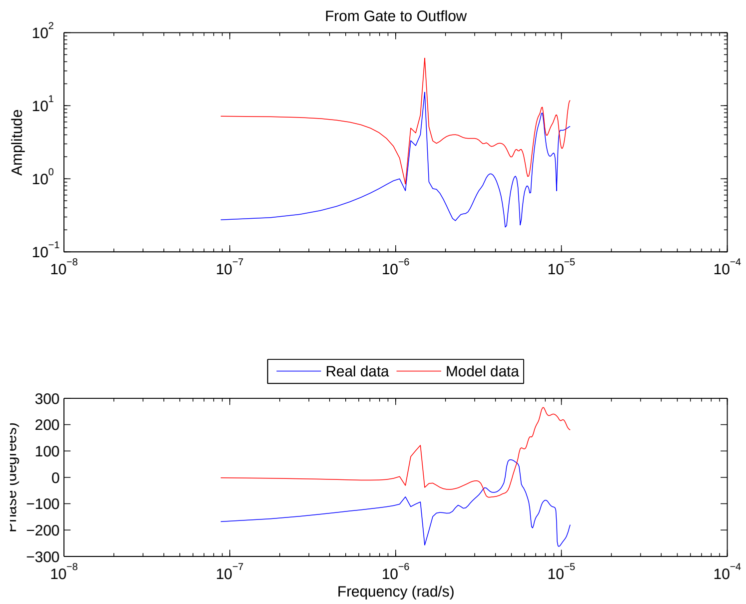

Figure A.31: The figure show one of the bode plots of Bysjön. With the gate height as input and the outflow of water as output.

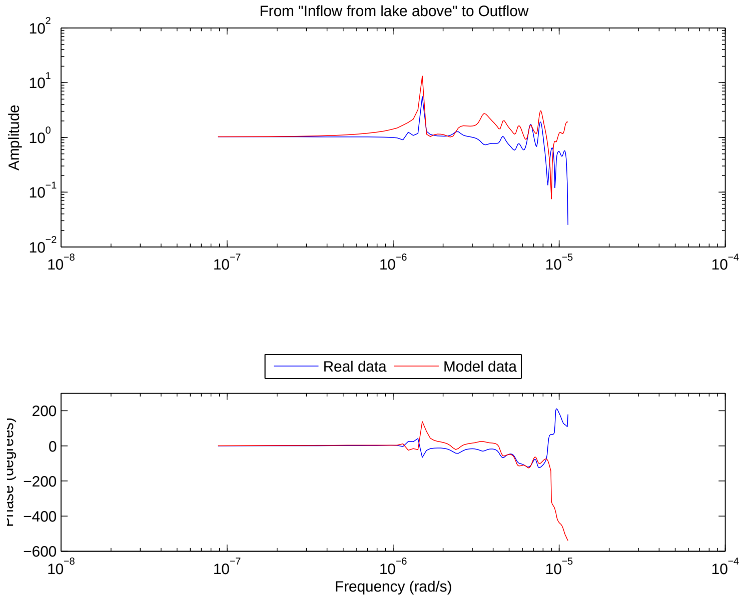

Figure A.32: The figure show one of the bode plots of Bysjön. With the inflow of water to Bysjön as input and the outflow of water from Bysjön as output.

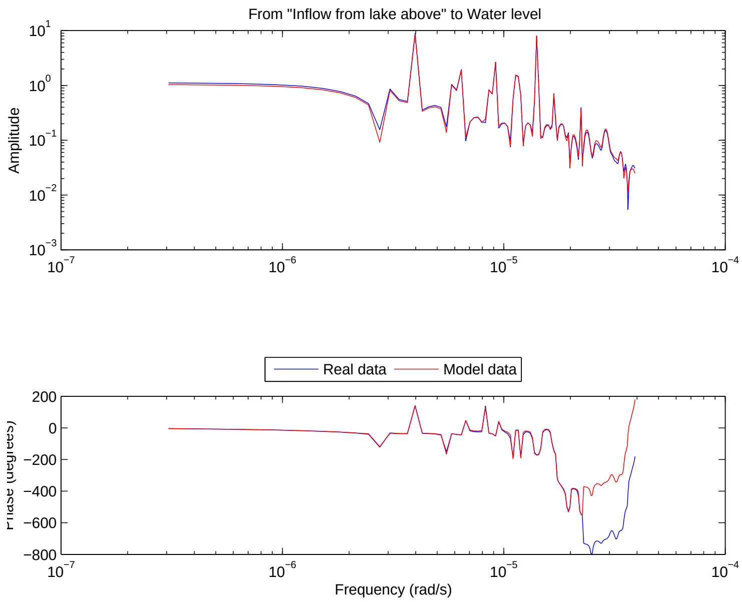

Figure A.33: The figure show one of the bode plots of Närdingen. With the inflow of water as input and the waterlevel of Närdingen as output.

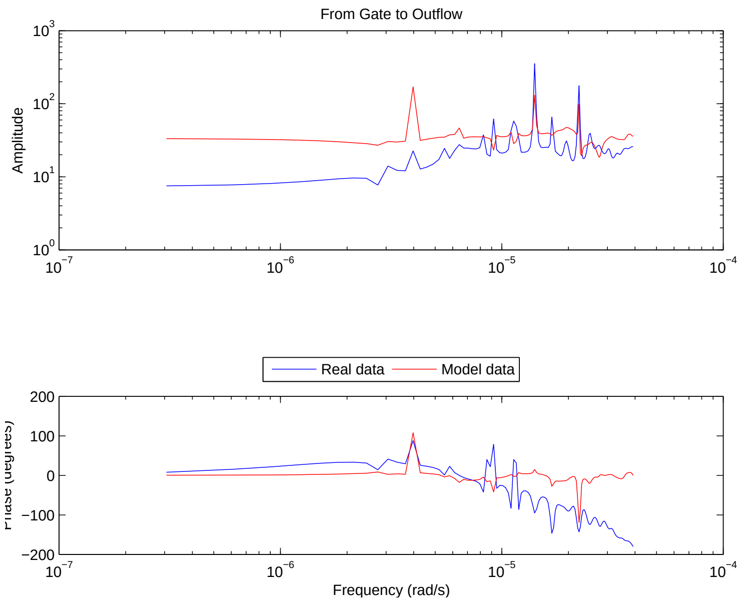

Figure A.34: The figure show one of the bode plots of Närdingen. With the gate height as input and the outflow of water from Närdingen as output.

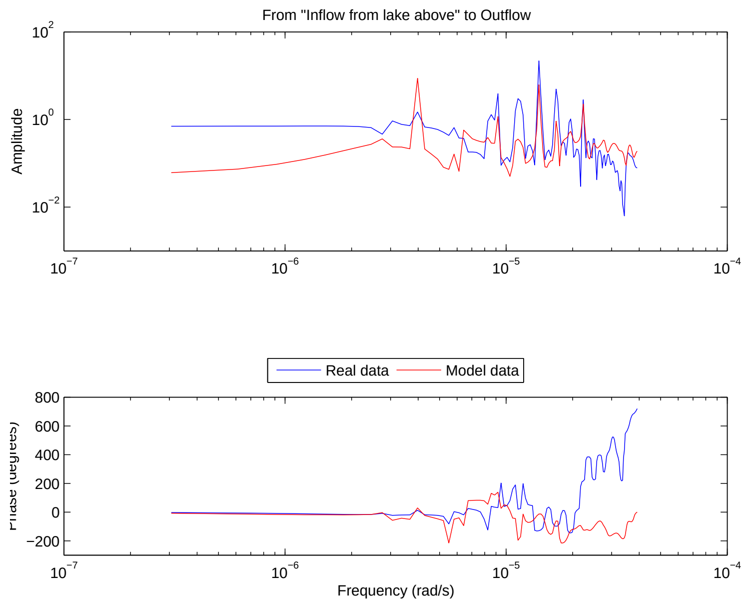

Figure A.35: The figure show one of the bode plots of Närdingen. With the inflow of water to Närdingen as input and the outflow of water from Närdingen as output.

A.4 Optimization equations

The Hessian matrix, **H**, and the Lagrangian, **f**, where derived in the following way:

$$V(U) = (\mathbf{X} - X_{ref})^T Q_x (\mathbf{X} - X_{ref}) + (\mathbf{U} - U_{ref})^T Q_u (\mathbf{U} - U_{ref}) =$$

$$(\mathbf{A}x(k) + \mathbf{B}_u \cdot \mathbf{U} + \mathbf{B}_d \cdot \mathbf{D} - X_{ref})^T \; Q_x \; (\mathbf{A}x(k) + \mathbf{B}_u \cdot \mathbf{U} + \mathbf{B}_d \cdot \mathbf{D} - X_{ref}) + \\ (\mathbf{U}_t - U_{ref})^T \; Q_u \; (\mathbf{U}_t - U_{ref}) =$$

$$\mathbf{U}^T \Big(\mathbf{B}_u^T Q_x \mathbf{B}_u + Q_u \Big) \mathbf{U} + 2 \cdot \Big(\; x(k)^T \mathbf{A}^T Q_x + \mathbf{D}^T \mathbf{B}_d^T Q_x - \mathbf{X}_{ref}^T Q_x - \mathbf{U}_{ref}^T Q_u \Big) \mathbf{U} + \\ x(k)^T \mathbf{A}^T Q_x \mathbf{A} x(k) + \mathbf{D}^T \mathbf{B}_d^T Q_x \mathbf{B}_d \mathbf{D} + 2x(k)^T \mathbf{A}^T Q_x \mathbf{B}_d \mathbf{D} + \mathbf{U}_{ref}^T Q_u \mathbf{U}_{ref} + \\ \mathbf{X}_{ref}^T Q_x \mathbf{X}_{ref} - 2x(k)^T \mathbf{A}^T Q_x \mathbf{X}_{ref} - 2\mathbf{D}^T \mathbf{B}_d^T Q_x \mathbf{X}_{ref} =$$

$$\tfrac{1}{2} \cdot U^T \cdot \mathbf{H} \cdot U + \mathbf{f} \cdot U + g$$

$$\Downarrow$$

$$\left\{ \begin{array}{lcl} \mathbf{H} & = & 2 \cdot (\mathbf{B}_u^T \; Q_x \mathbf{B}_u + Q_u) \\ \mathbf{f} & = & 2 \cdot \Big(\; x(k)^T \mathbf{A}^T Q_x + \mathbf{D}^T \mathbf{B}_d^T Q_x - \mathbf{X}_{ref}^T Q_x - \mathbf{U}_{ref}^T Q_u \Big) \\ g & = & x(k)^T \mathbf{A}^T Q_x \mathbf{A} x(k) + \mathbf{D}^T \mathbf{B}_d^T Q_x \mathbf{B}_d \mathbf{D} + 2x(k)^T \mathbf{A}^T Q_x \mathbf{B}_d \mathbf{D} + \mathbf{U}_{ref}^T Q_u \mathbf{U}_{ref} + \\ & & \mathbf{X}_{ref}^T Q_x \mathbf{X}_{ref} - 2x(k)^T \mathbf{A}^T Q_x \mathbf{X}_{ref} - 2\mathbf{D}^T \mathbf{B}_d^T Q_x \mathbf{X}_{ref} \end{array} \right. \tag{A.1}$$

The constraints are derived in the following way. Instead of constraints on the output **Y** the equations will be derived with constraints on the states **X**.

$$\left. \begin{array}{r} U_{min} \leq \mathbf{U} \leq U_{max} \\ X_{min} \leq \mathbf{X} \leq X_{max} \end{array} \right\} \Rightarrow \begin{pmatrix} -I & 0 \\ I & 0 \\ 0 & -I \\ 0 & I \end{pmatrix} \begin{bmatrix} \mathbf{X} \\ \mathbf{U} \end{bmatrix} \leq \begin{pmatrix} -X_{min} \\ X_{max} \\ -U_{min} \\ U_{max} \end{pmatrix} \Rightarrow \\ \begin{pmatrix} -I & 0 \\ I & 0 \\ 0 & -I \\ 0 & I \end{pmatrix} \begin{bmatrix} \mathbf{A}x(k) + \mathbf{B}_u \mathbf{U} + \mathbf{B}_d \mathbf{D} \\ \mathbf{U} \end{bmatrix} \leq \begin{pmatrix} -X_{min} \\ X_{max} \\ -U_{min} \\ U_{max} \end{pmatrix} \Rightarrow \\ \begin{pmatrix} -\mathbf{B}_u \\ \mathbf{B}_u \\ -I \\ I \end{pmatrix} \mathbf{U} \leq \begin{pmatrix} -X_{min} + \mathbf{A}x(k) + \mathbf{B}_d \mathbf{D} \\ X_{max} - \mathbf{A}x(k) - \mathbf{B}_d \mathbf{D} \\ -U_{min} \\ U_{max} \end{pmatrix} \tag{A.2}$$

A.5 Histograms of the outflow 2006

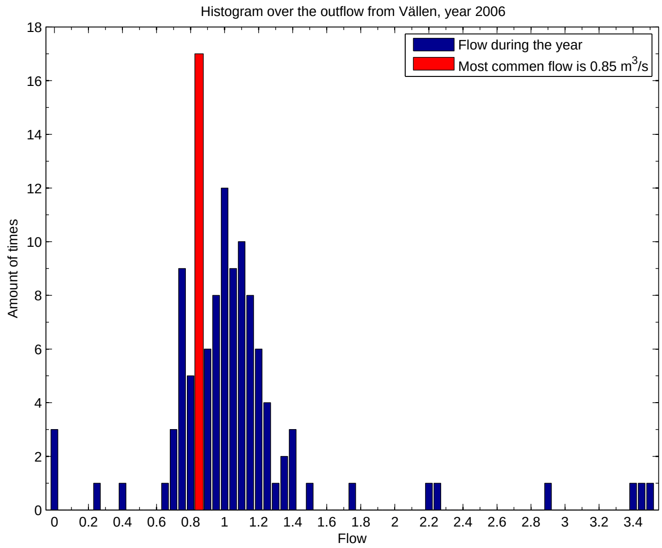

Figure A.36: The histogram show the flow out of Vällen the year 2006. The most common outflow during this year was of 0.85 m^3/s.

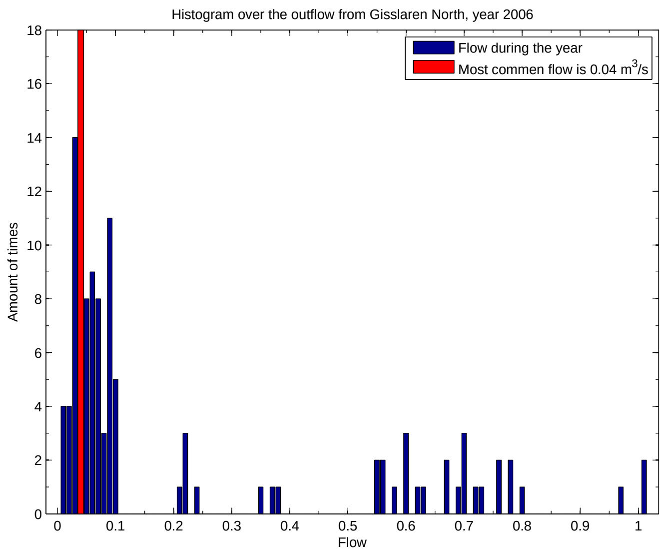

Figure A.37: The histogram show the flow out of the north dam of Gisslaren the year 2006. The most common outflow was of 0.04 m^3/s.

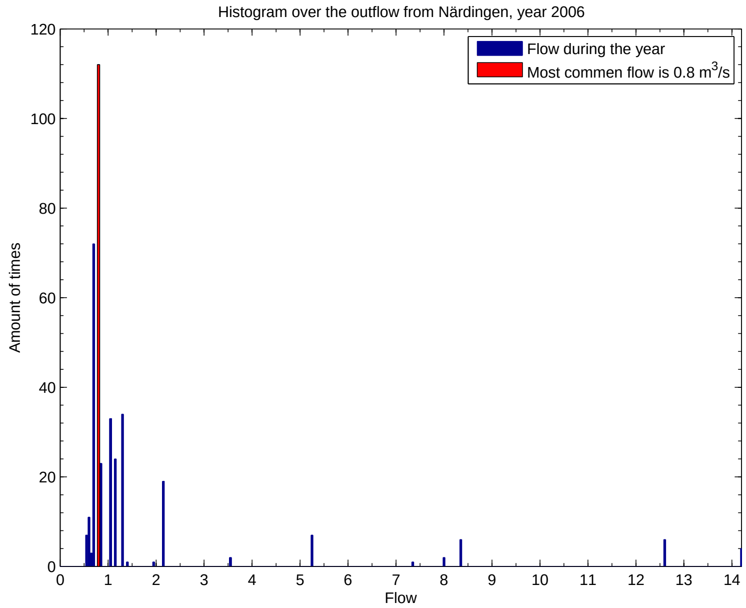

Figure A.38: The histogram show the flow out of Närdingen the year 2006. The outflow that was most common during this year was of 0.8 m^3/s.

A.6 Figures from the MPC simulation

In all figures in this section shows simulated data compared to real measured data during the test period (27.5 - 24.6 2002). The first three figures show the gate height and two figures of the outflow from Norrsjön.

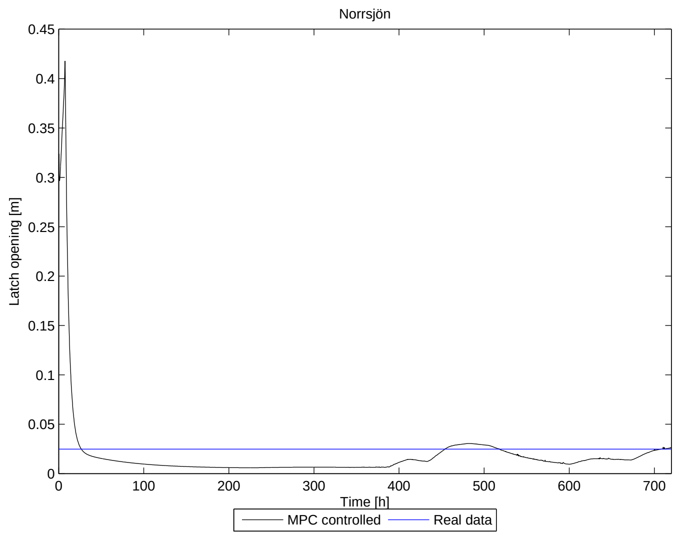

Figure A.39: This plot shows the simulated gate height and the real measured data of the gate height of Norrsjön during the test period (27.5 - 24.6 2002).

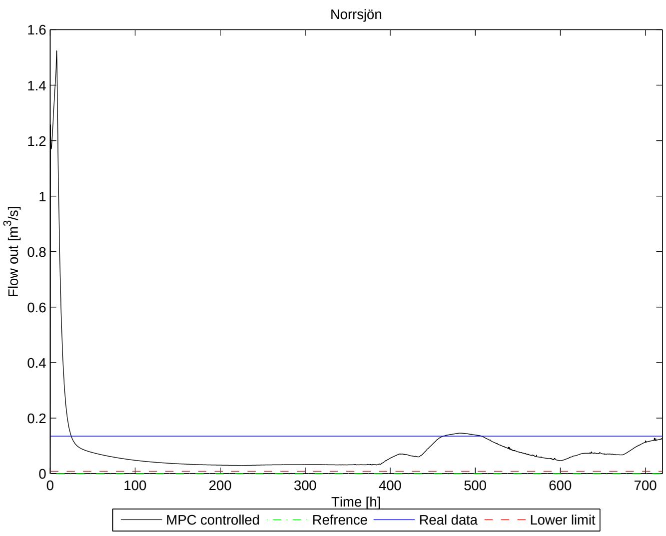

Figure A.40: This plot shows the simulated outflow of water and the real measured data of the water outflow from Norrsjön during the test period (27.5 - 24.6 2002).

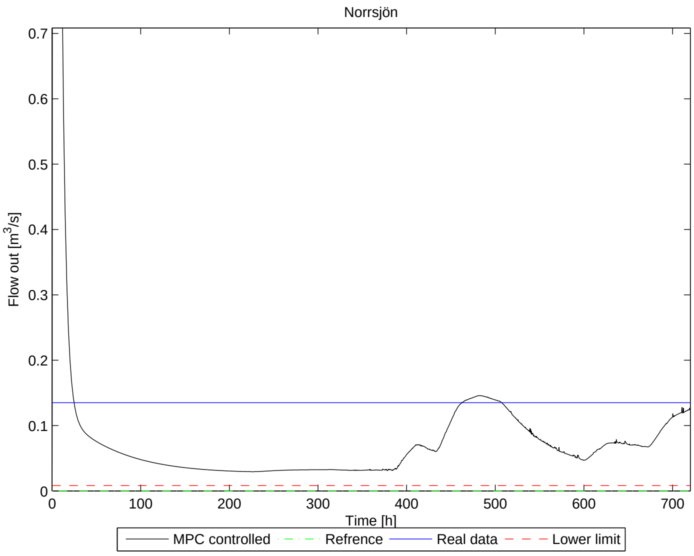

Figure A.41: The figure is the same as figure A.40 though zoomed in to give a better view of how the outflow changed during the test period.

Plots from Vällen:

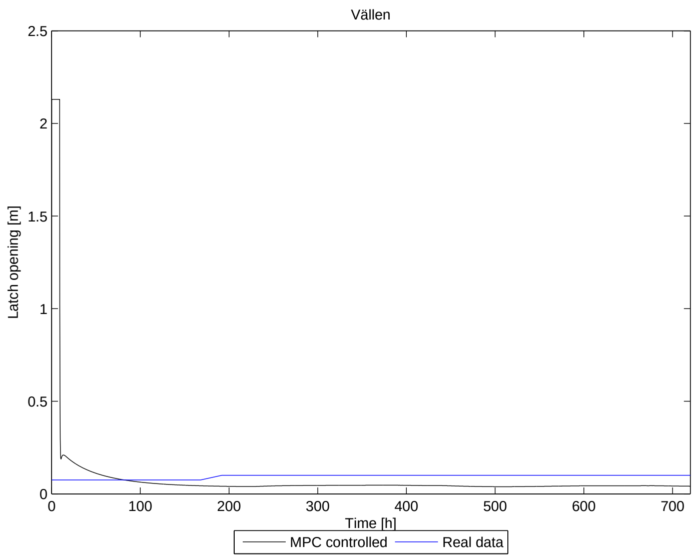

Figure A.42: This plot shows the simulated gate height and the real measured data of the gate height of Vällen during the test period (27.5 - 24.6 2002).

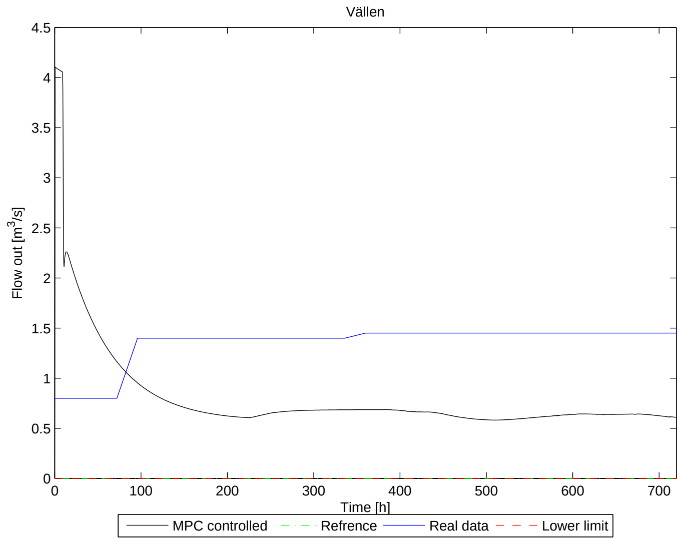

Figure A.43: This plot shows the simulated outflow of water and the real measured data of the water outflow from Vällen during the test period (27.5 - 24.6 2002).

Plots from Gisslaren:

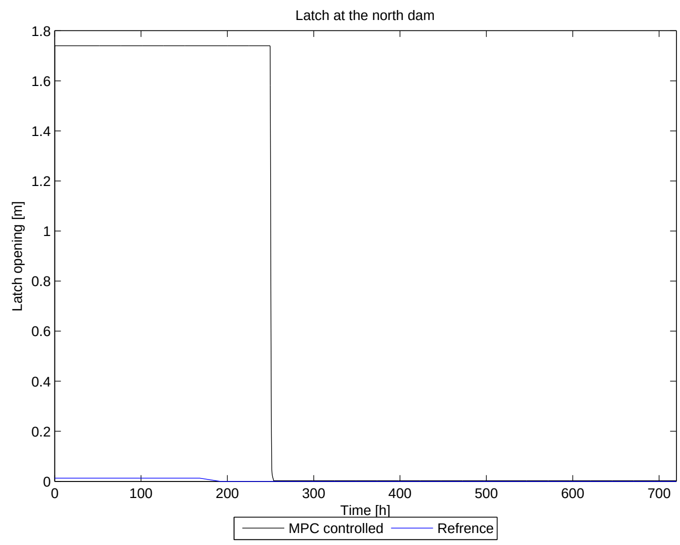

Figure A.44: This plot shows the simulated gate height and the real measured data of the gate height of the north dam of Gisslaren during the test period (27.5 - 24.6 2002).

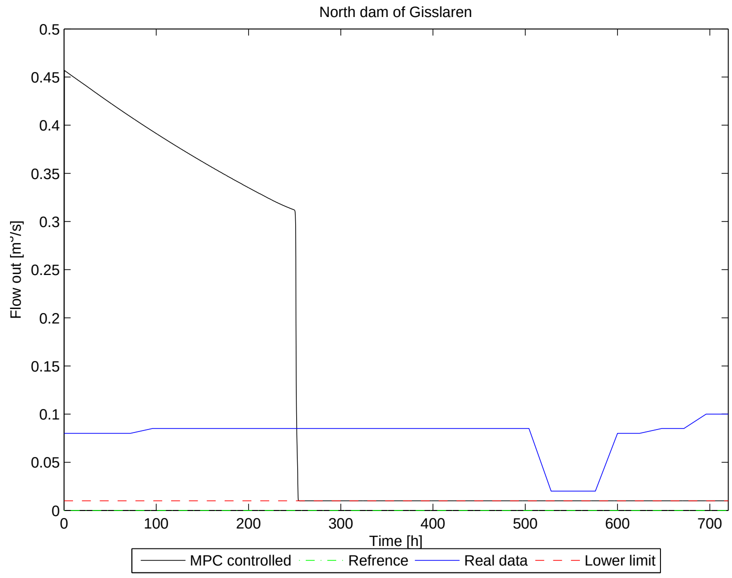

Figure A.45: This plot shows the simulated outflow of water and the real measured data of the water outflow from the north dam of Gisslaren during the test period (27.5 - 24.6 2002).

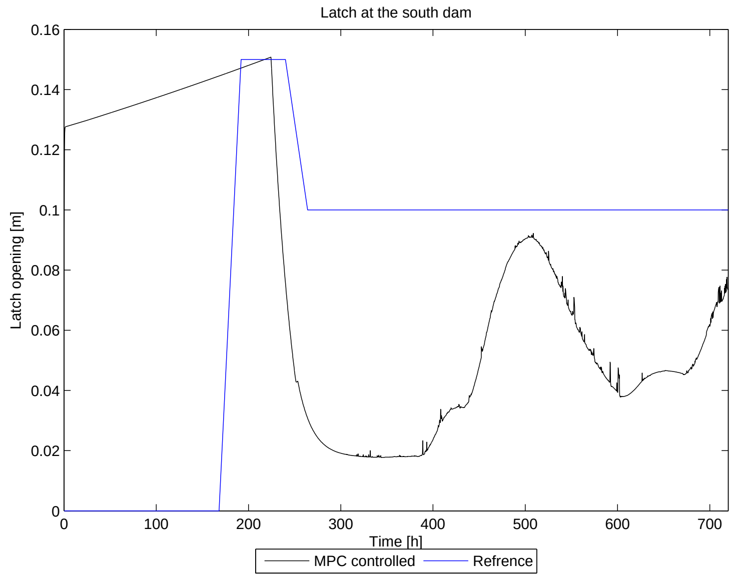

Figure A.46: This plot shows the simulated gate height and the real measured data of the gate height of the south dam of Gisslaren during the test period (27.5 - 24.6 2002)

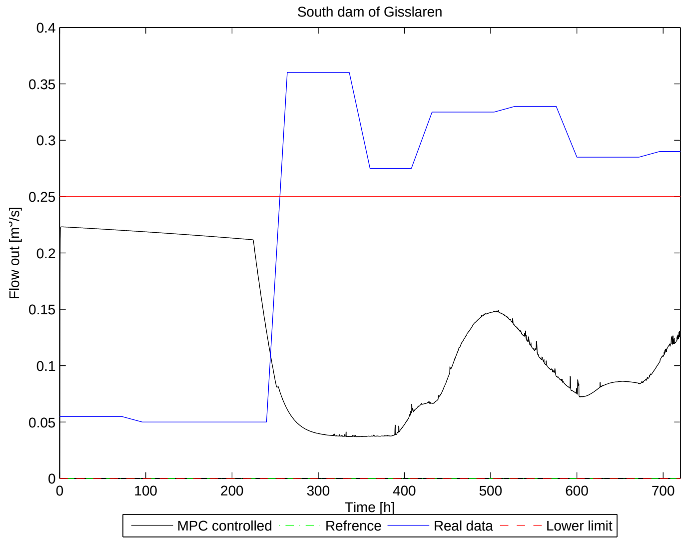

Figure A.47: This plot shows the simulated outflow of water and the real measured data of the water outflow from the south dam of Gisslaren during the test period (27.5 - 24.6 2002).

Plots from Bysjön:

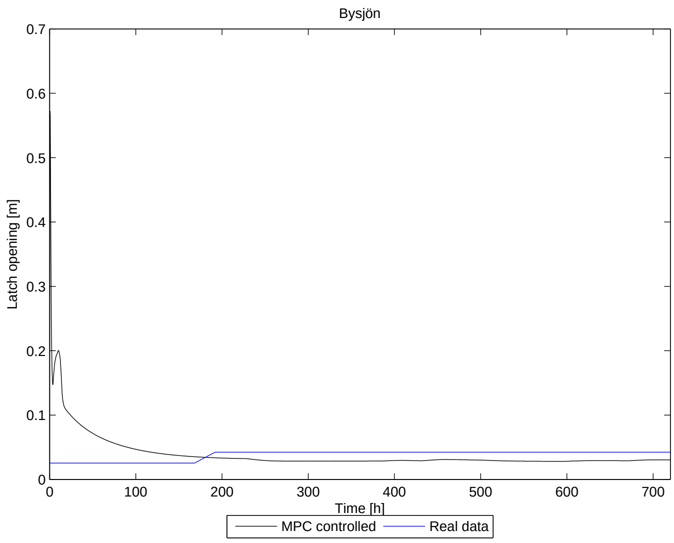

Figure A.48: This plot shows the simulated gate height and the real measured data of the gate height of Bysjön during the test period (27.5 - 24.6 2002).

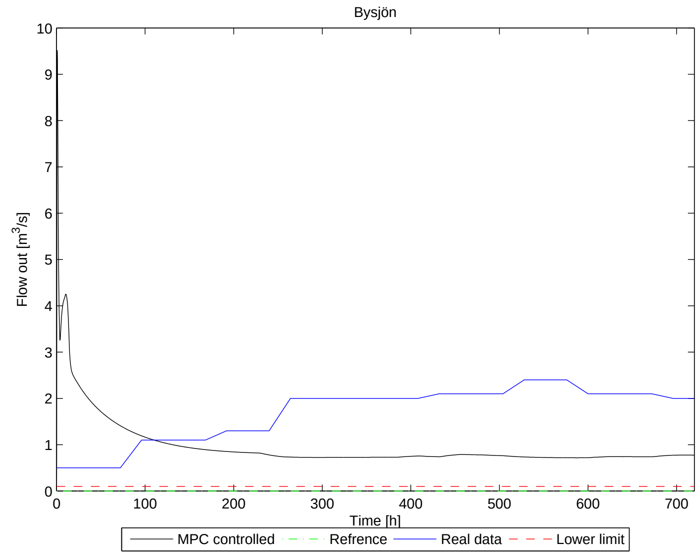

Figure A.49: This plot shows the simulated outflow of water and the real measured data of the water outflow from Bysjön during the test period (27.5 - 24.6 2002).

Plots from Närdingen:

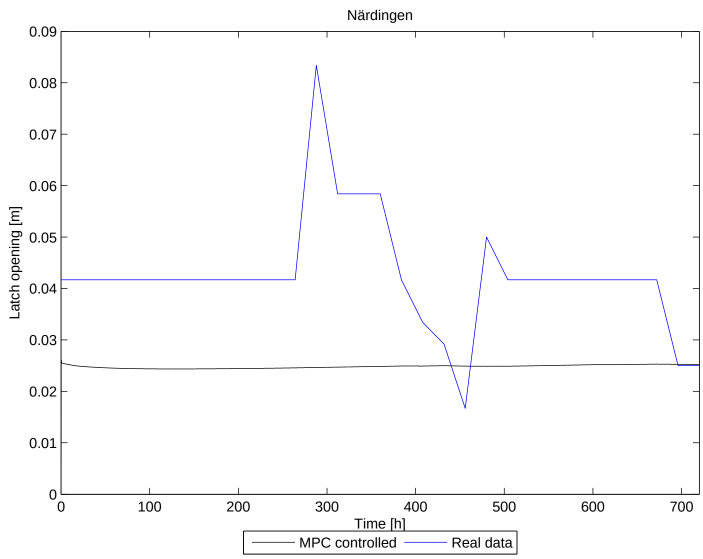

Figure A.50: This plot shows the simulated gate height and the real measured data of the gate height of Närdingen during the test period (27.5 - 24.6 2002)

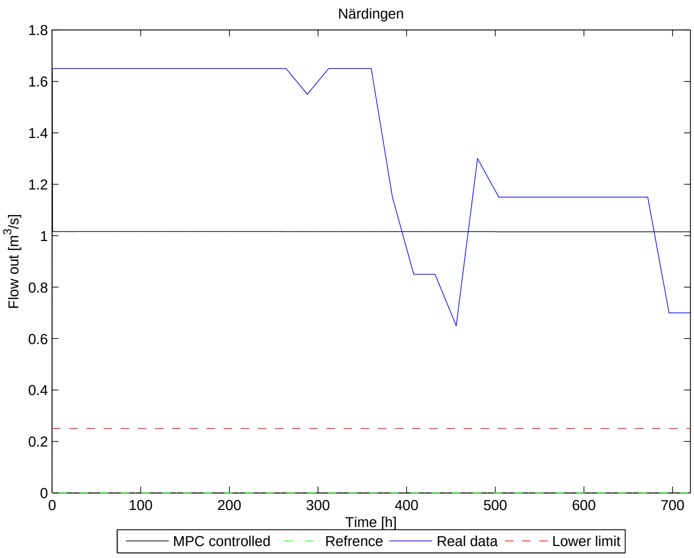

Figure A.51: This plot shows the simulated outflow of water and the real measured data of the water outflow from Närdingen during the test period (27.5 - 24.6 2002).