Project E:
The Hubble Law

In the 1920's, Edwin P. Hubble discovered a curious fact. By measuring both the distances and the redshifts of distant galaxies, he found that all distant galaxies were moving away from the Milky Way. Not only were the galaxies moving away from the Milky Way, but the velocity with which the galaxies were receding was faster in more distant galaxies. The relation that Hubble first identified became known as the "Hubble law". The Hubble law states that the recessional velocity V_r of a galaxy is proportional to its distance D from us. Mathematically, the Hubble law can be written as:

$$V_r = H_0 \times D$$

where H_0 is a number known as the "Hubble Constant". The recessional velocity is traditionally given in km/s, and the distance is given in units of Megaparsecs (Mpc). Therefore, the Hubble constant has units of km/s/Mpc. Historically, the Hubble constant has been thought to lie somewhere between 50 and 100 km/s/ Mpc (although Hubble's initial estimate of his own constant was too large by a factor of 10, due to his mistaking bright clusters of stars in nearby galaxies as single stars). Recent measurements of fluctuations in the cosmic microwave background suggest that the true value of the Hubble constant is closer to 72 km/s/Mpc.

It quickly became clear that Hubble's law did not indicate that galaxies were receding from us because they were moving through space. Instead, the most natural interpretation of the Hubble law is that space itself is expanding, everywhere. The Hubble constant sets the rate of this expansion, and is thus one of the most important numbers in cosmology. First, the Hubble constant sets the size scale of the Universe. Thus, once the Hubble Constant is measured, we can easily measure the distance to any galaxy, simply by measuring the galaxy's recessional velocity V_r. Second, because the Hubble constant limits both the size of the Universe and the rate at which it expands, it also sets the overall age of the Universe. The age can be estimated from the inverse of the Hubble constant, known as a "Hubble time" $t_{Hubble} = 1/H_0$.
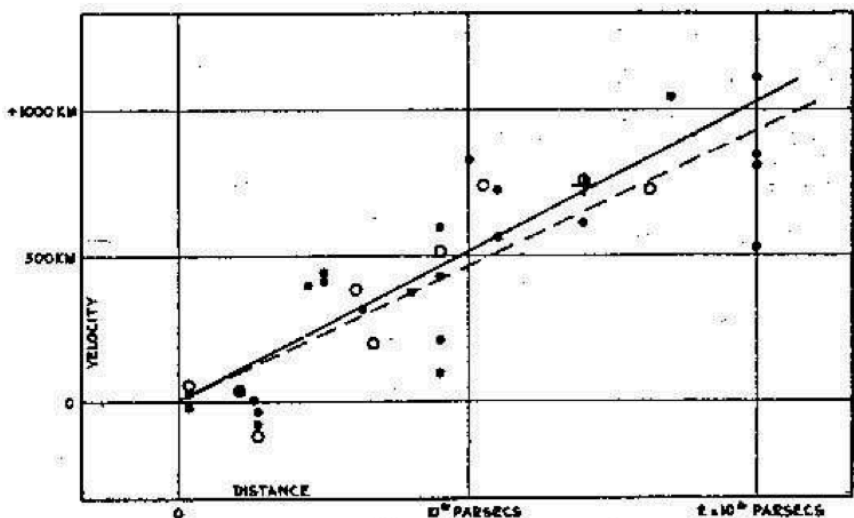

FIGURE 1

Procedure

In this project you will generate a Hubble diagram and use it to measure the value of the Hubble Constant H_0. Generating the Hubble diagram requires measuring redshifts and distances for a large sample of galaxies.

• First, you will measure the redshifts of the galaxies from their spectra. These redshifts will be used to calculate V_r, the speed at which the galaxies are receding from us.

• Second, you will estimate the galaxies' absolute magnitudes from the Tully-Fisher relationship.

• Third, you will measure the distances to the galaxies, by comparing their apparent and absolute magnitudes.

• Finally, you will plot the galaxies' distances versus their recessional velocities, and use the slope of the line to estimate the value of the Hubble Constant.

The Galaxy Sample

For this project, you will be using a sample of inclined late-type spiral galaxies. The rotation speeds and apparent magnitudes of the galaxies can be found in the table below, and images of the galaxies are on the following pages. The tabulated rotation speeds have all been corrected for inclination. As you can see from the table, some of the galaxies are missing measurements of their recessional velocities (V_r) and absolute magnitudes. In the following sections, you will measure these quantities and fill in the remaining data from the table.

Galaxy Data

| Galaxy	V_r (km/s)	Rotation Speed (km/s)	Absolute Magnitude M	Apparent Magnitude m	Distance Modulus $m - M$	Distance (Mpc)
UGC 858	2375	222	-21.9	10.9	32.8	36
UGC 1554	2099	140		11.4		
UGC 1633	4247	239	-22.2	11.4	33.6	52
UGC 3834	2033	152	-20.9	11.4	32.3	29
UGC 4888	2258	166		11.3		
UGC 5183	1336	140		10.3		
UGC 6376	4246	306		10.4		
UGC 6579		165	-21.1	10.4	31.5	20
UGC 7122	1813	141	-20.7	11.8	32.5	32
UGC 7393	4203	193	-21.6	12.4	34.0	63
UGC 7412	1140	138	-20.6	10.2	30.8	14
UGC 8033	2484	180		10.8		
UGC 9358	1903	214	-21.8	10.2	32.0	25
UGC 9366		242		10.0		
UGC 11218	1484	185	-21.4	10.0	31.4	19

Galaxy Images
All images cover the same angular area on the sky
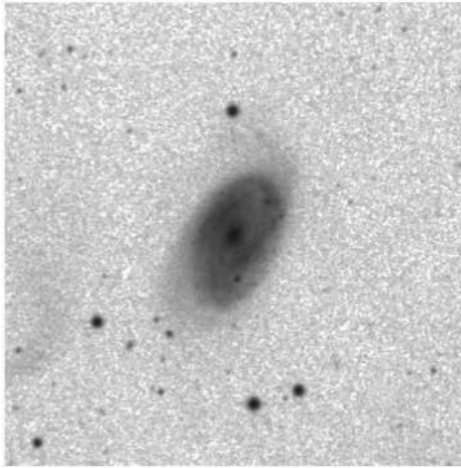
UGC 858
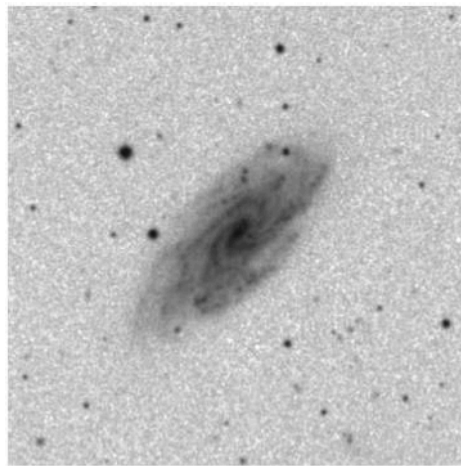$V_r = 2375\,\mathrm{km/s}$
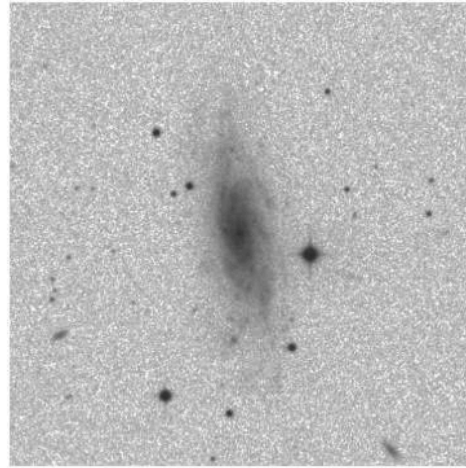
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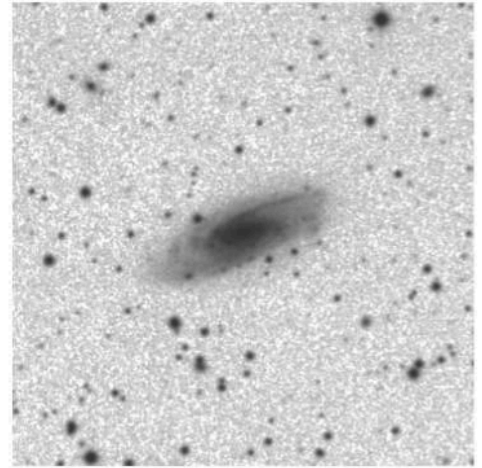
UGC 1633
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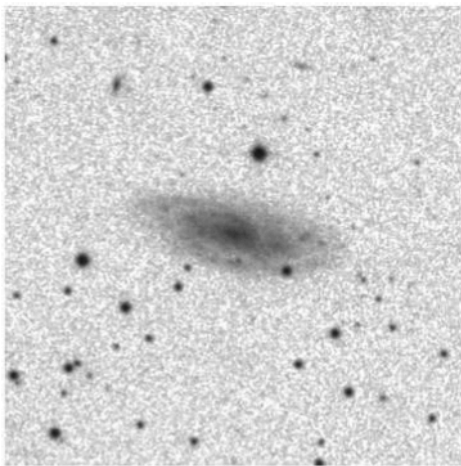
UGC 3834
$V_r = 2033\,\mathrm{km/s}$

UGC 4888
$V_r = 2258\,\mathrm{km/s}$
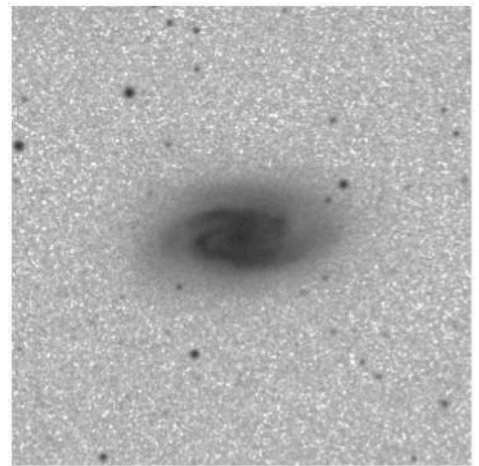
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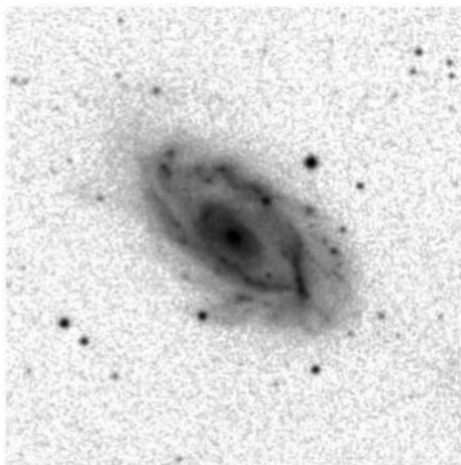
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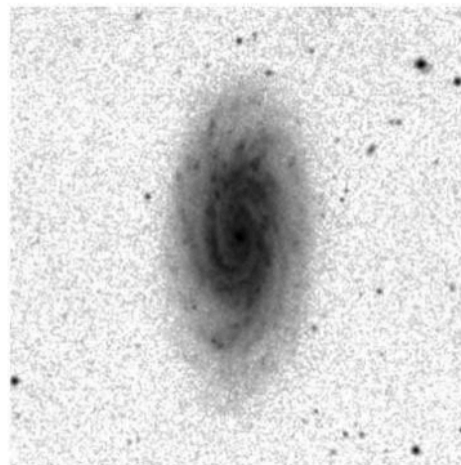
UGC 6579
$V_r = \qquad \mathrm{km/s}$
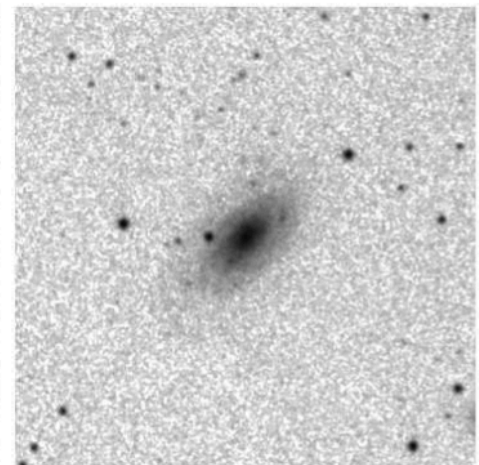
UGC 7122
$V_r = 1813\,\mathrm{km/s}$
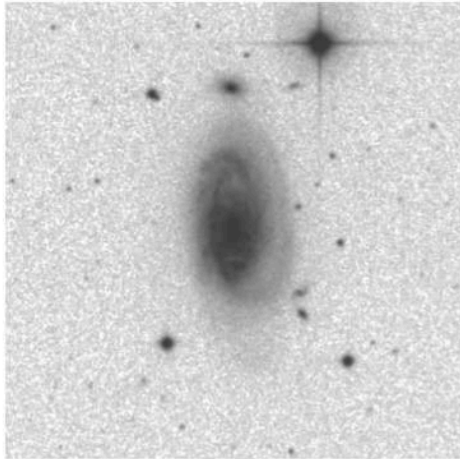
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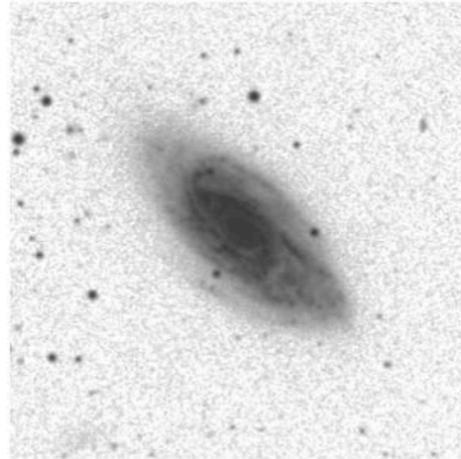
UGC 9366
$V_r = \qquad \mathrm{km/s}$
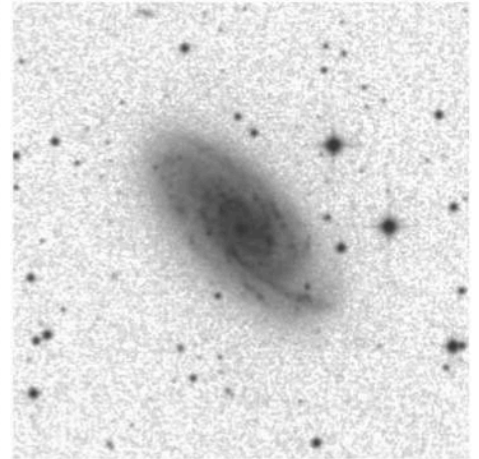
UGC 11218
$V_r = 1484\,\mathrm{km/s}$
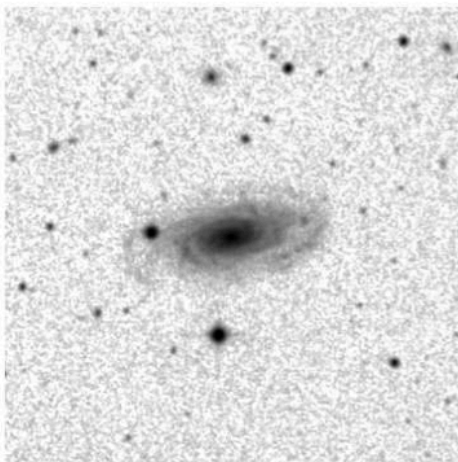
UGC 7393
$V_r = 4203\,\mathrm{km/s}$
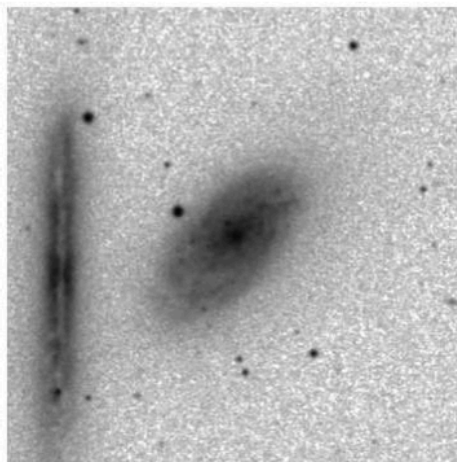
UGC 7412
$V_r = 1140\,\mathrm{km/s}$
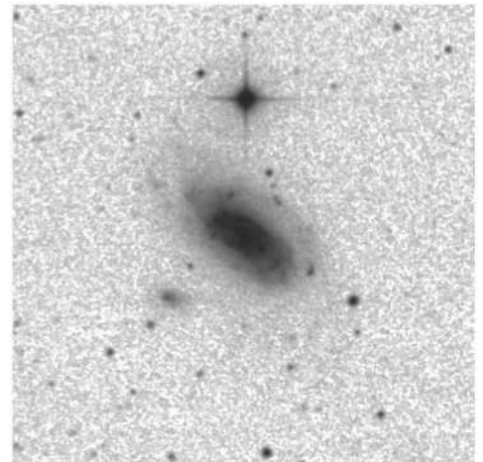
UGC 8033
$V_r = 2484\,\mathrm{km/s}$

Measuring Redshifts
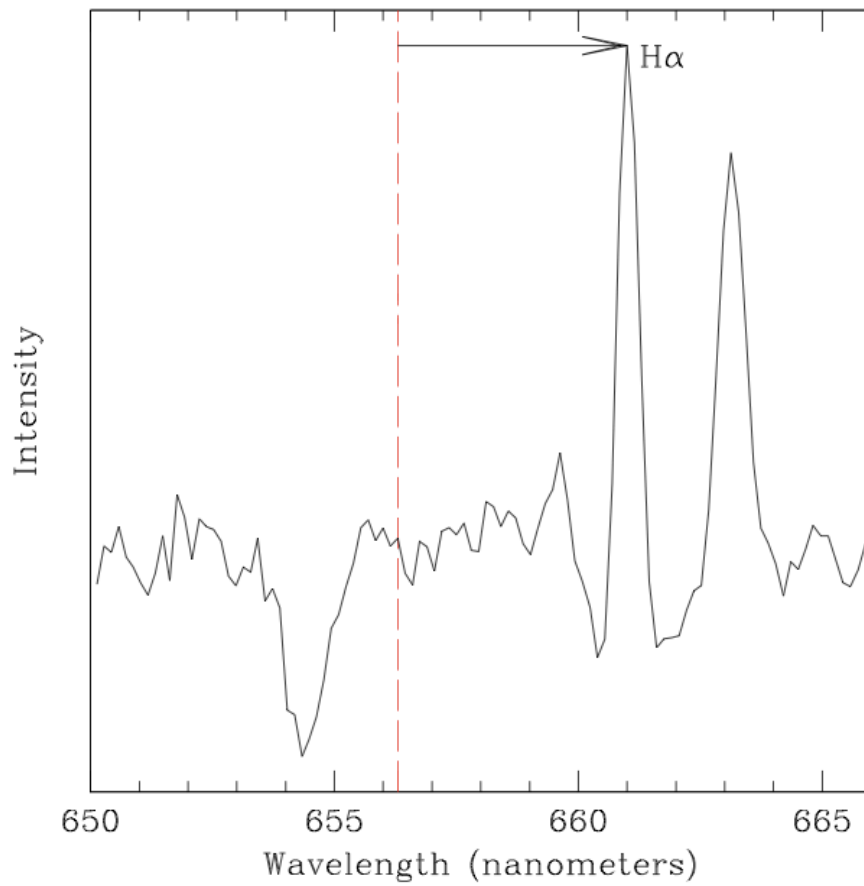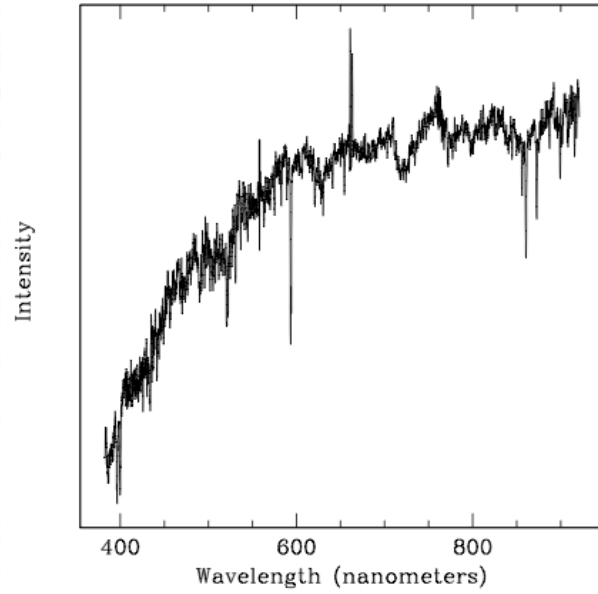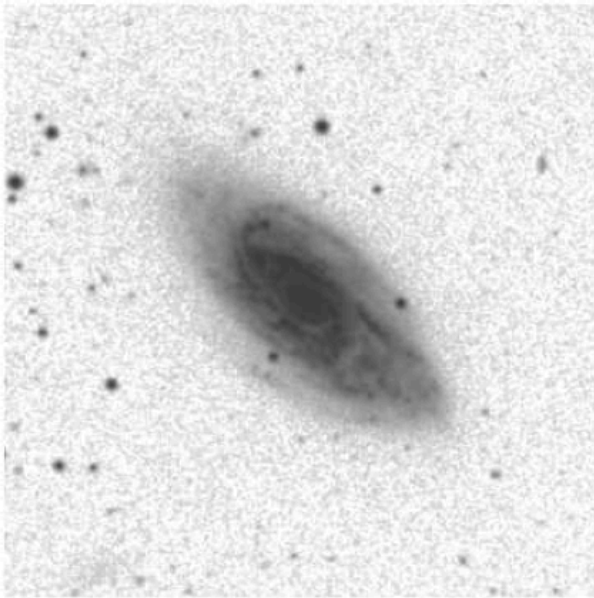
To measure the redshift for a galaxy, you must identify a characteristic emission or absorption line in the galaxy's spectrum and compare the apparent wavelength of the line to the true wavelength (i.e. the wavelength of the line if the galaxy were stationary). For spiral galaxies with bright HII regions, the easiest feature to use is the strong Hα emission line. On the following two pages you will find images and spectra for two galaxies. There are two plots for each spectrum. One shows the entire optical spectrum of the galaxy, from 400nm to 900nm. The second, larger plot shows the spectrum zoomed in on the wavelengths near the Hα emission line near 656 nm. The horizontal arrow on the plot shows how the Hα has redshifted from its true wavelength at λ_{true} = 656.3nm to its longer apparent wavelength $\lambda_{apparent}$. The size of this redshift is proportional to the speed at which the galaxy is receding from us. You will now measure the redshift of these two galaxies, and then derive V_r, the recessional velocity.

1. Using the spectra on the following two pages, read off the apparent wavelength of the peak of the Hα emission line. Enter the values in the table below. Please use a ruler to make your measurement as accurate as possible!!!

2. Calculate the shift in wavelength Δλ between the apparent wavelength and the true wavelength of the Hα emission line. Enter the values in the table below.

3. Calculate the redshift $z = \Delta\lambda/\lambda_{true}$ for each galaxy, and enter your results in the table. Make sure you use at least two significant digits (i.e. write down "0.013" rather than "0.01")

4. The value of the redshift tells you the ratio of a galaxy's recessional velocity to the speed of light (i.e. $z = V_r/c$). Thus, a galaxy that is receding from us with 1% of the speed of light ($V_r \approx$ 3000km/s) has a redshift of $z = 0.01$. Calculate each galaxy's recessional velocity $V_r = c \times z$, and enter the values in the table below. The speed of light is $c = 3 \times 10^5$ km/s, and therefore $V_r = (300000 \text{ km/s}) \times z$.

5. Transfer the recessional velocities you derived for the two galaxies into the appropriate fields in the large table of galaxy data on the previous page.

Galaxy	Apparent Wavelength of Hα	True Wavelength of Hα	$\Delta\lambda = \lambda_{apparent} - \lambda_{true}$	$z = \frac{\Delta\lambda}{\lambda_{true}}$	$V_r = c \times z$
UGC 6579		656.3 nm			
UGC 9366		656.3 nm			

UGC 6579

Hα
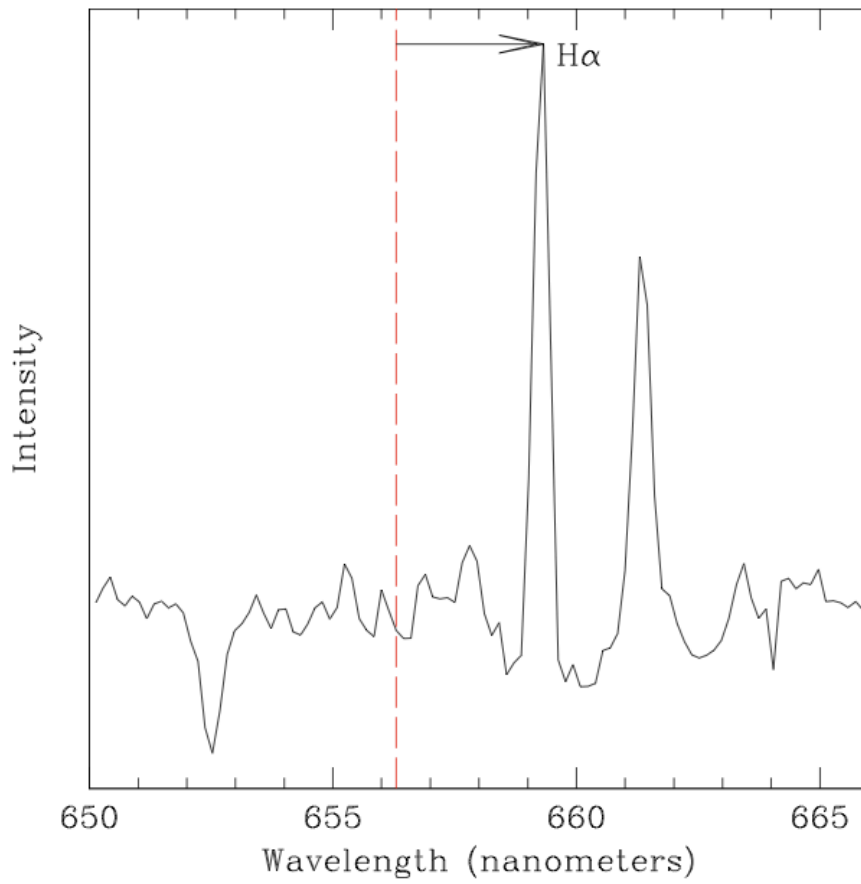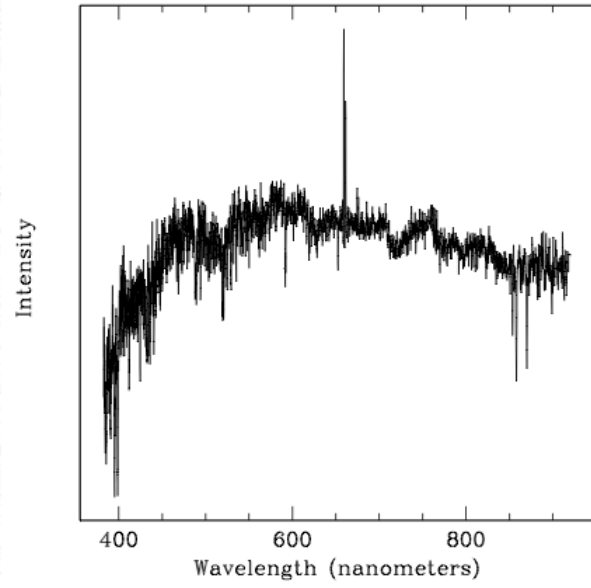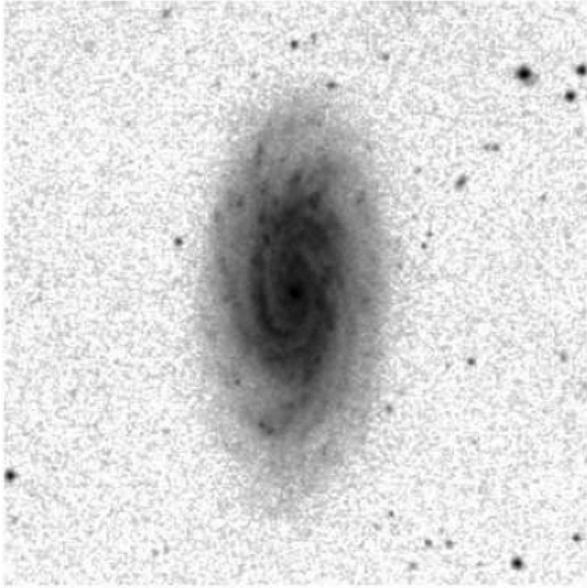
UGC 9366

Measuring Distances

If you know the luminosity of a galaxy, you can derive its distance by measuring how bright the galaxy appears to be. To characterize the luminosity and apparent brightness, as- tronomers frequently use the "absolute magnitude" M and the "apparent magnitude" m. If you know both the apparent and absolute magnitudes, you can derive the distance from Pogson's equation

$$m - M = 5\log(d) + 5$$

For galaxies, it is straightforward to measure the apparent magnitude m from galaxy images. In the large table of galaxy data on the earlier page, we have tabulated the apparent magnitudes for all the galaxies in the sample. Galaxies with larger apparent magnitudes appear fainter on the sky.

Measuring the absolute magnitudes of the galaxies is more tricky. Using galaxy images alone, there is no way to immediately figure out which galaxies are truly less luminous, and which are intrinsically brighter. (You can try this experiment using the galaxy images from earlier in the project!). Instead, one must bring in additional information to figure out which galaxies are more luminous.

To estimate the absolute magnitudes of the galaxies in this project, you will be using the fact that galaxies that are more luminous have faster rotation speeds, on average. This relationship is called the "Tully-Fisher" relation, and is shown in the plot on the following page[1]. As you can see from the plot, galaxies with larger rotation speeds have brighter (i.e. more negative) absolute magnitudes, on average. This relationship is not perfect, however. At any given rotation speed, there are some galaxies that are more or less luminous than the average galaxy with that rotation speed. This scatter around the average Tully-Fisher relation is due to a mix of factors. Some galaxies may have recently had a burst of star formation, making them brighter than average for their rotation speed. Others might be in a more quiescent state, making them less luminous than average. Sometimes, there has simply been an error in the measurement. However, even with all these sources of random errors, the average relation is clear, and can be used to estimate the absolute magnitude of a galaxy whose rotation speed is known.

The scatter in the Tully-Fisher relation will add scatter to your Hubble diagram. You do not know if the individual galaxies in this project are likely to be brighter or fainter than average for their rotation speed. Thus, there will be some unavoidable uncertainty in your estimate of the galaxies' absolute magnitudes, which will translate into an uncertainty in your estimate of the galaxies' distances. However, by measuring distances to a large number of galaxies, you will hopefully have enough galaxies find the right average Hubble law.

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[1]The relationship between rotation speed and absolute magnitude looks more "curved" than you may have seen in your textbook. This difference arises because the data in this project is plotted on a linear plot of rotation speed (to make it easier for you to measure the absolute magnitude). In contrast, the plots of the Tully-Fisher relation you have seen in lecture or in your textbook use a logarithmic plot of rotation speed. If the plot in this project were made with a logarithmic scale along the x-axis, the data would fall along a straight line.

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In the large table of galaxy data on the earlier page, we have tabulated the rotation speeds for all the galaxies in the sample. You will:

• Use these rotation speeds to derive the absolute magnitudes of the galaxies from the Tully-Fisher diagram.

• Combine your estimate of the absolute magnitude with the measured value of the apparent magnitude to derive the distance to each galaxy.
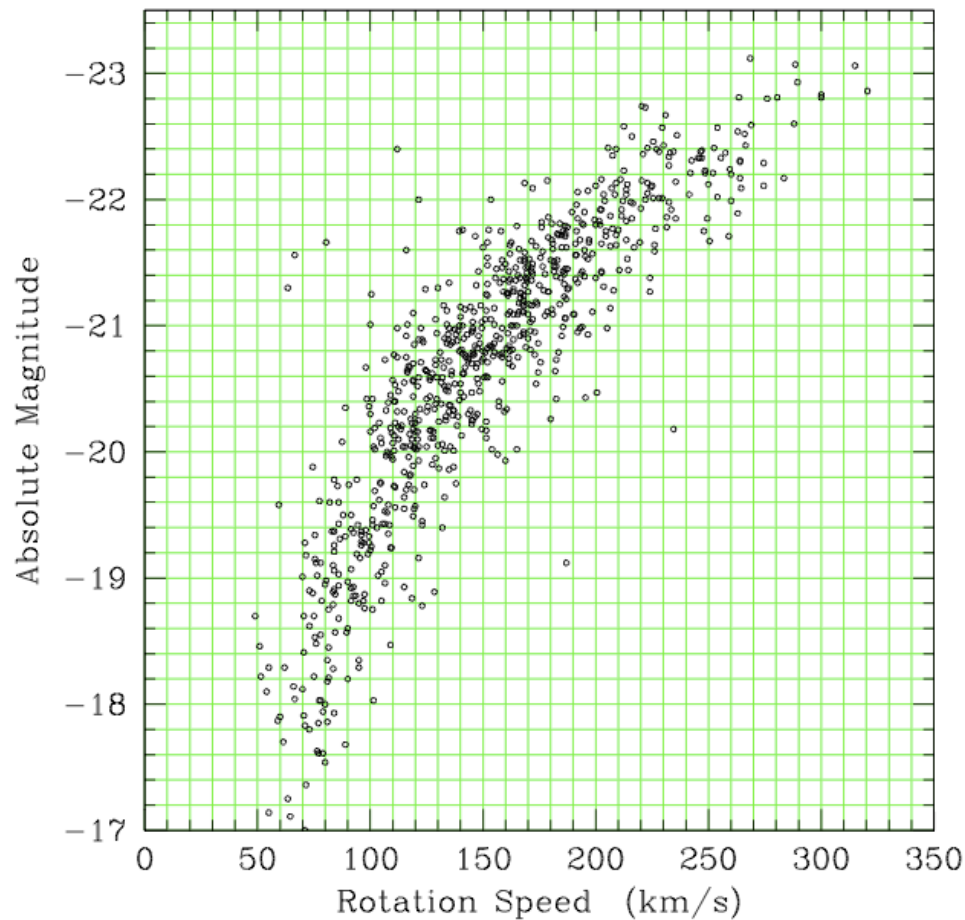

1. On the plot of data for the Tully-Fisher relation, draw a smooth curve that shows the "average" Tully-Fisher relation. To do so, first make a series of dots showing the most typical (or average) absolute magnitude at a number of different rotation speeds, and then draw a smooth, non-wiggly line that goes approximately through the dots.

2. Using your average Tully-Fisher relation and the rotation speeds given in the large galaxy data table, estimate the absolute magnitudes of any galaxies that lack distance measurements. Enter these absolute magnitudes in the galaxy data table. You may wish to check that you would derive similar absolute magnitudes for some of the galaxies that do have absolute magnitudes in the table! Please use a ruler to make your measurements as accurate as possible.

3. Using your estimate of the absolute magnitude, and the apparent magnitude given in the table, calculate the distance modulus (m–M) for each galaxy and record the value in the table.

4. Using the distance modulus, derive the distance to each of the galaxies from Pogson's equation in the previous page. Record your results.

Making a Hubble Diagram

You now have assembled a complete table of recessional velocities and distances for a large sample of galaxies. Thus, you can now create your own Hubble diagram.

1. On the plot below, mark a point for each of the galaxies in the large table of galaxy data.

2. Using a ruler, draw a dark diagonal line that goes through the middle of your points. You want your line to be positioned so that roughly equal numbers of points fall to the left and the right of the line at every recessional velocity. Make sure that your line also goes through zero velocity and zero distance (i.e. the origin of the plot)!
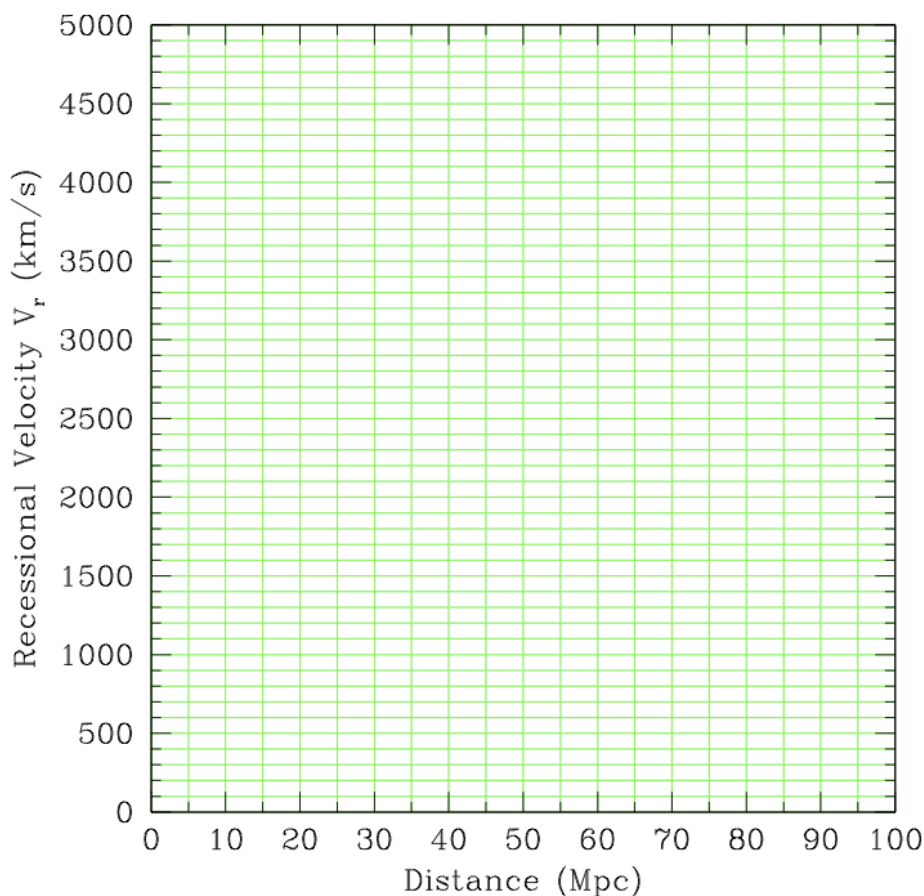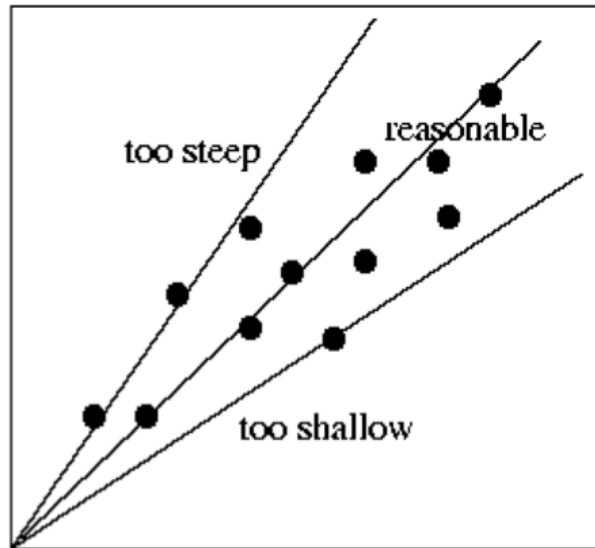
Measuring the Hubble Constant

The Hubble law says that galaxies that are further from us appear to be receding from us at higher velocities. Close to the Milky Way, this law is "linear", such that galaxies that are twice as far away appear to be receding twice as fast. The equation that describes the Hubble law is:

$$V_r = H_0 \times D$$

where D is the distance to the galaxy.

The line that you drew on your Hubble diagram follows the $V_r = H_0 \times D$ equation. The value of H_0 is the slope of the line ($= V_r/D$), such that steeper lines have larger slopes, and thus imply a larger value for the Hubble constant. The ratio V_r/D can be calculated at any point on the line to find the value of H_0.

1. To calculate the slope of the "best-fit" line that you drew through your data on the previous page, first pick a location on your line (i.e. not one of your data points). Enter the velocity and distance of that location in the table on the following page. (Note that you could have chosen any other reference location, and you would get the same final answer for the Hubble constant.)

2. Divide the velocity by the distance at your reference location on the line. This is the value of the Hubble constant! Enter your answer in the table on the following page.

3. The inverse of the Hubble constant ($= 1/H_0$) sets a characteristic age of the Universe, known as the "Hubble Time". Calculate the Hubble time in Gigayears that corresponds to your measurement of the Hubble constant, and enter your answer in the table on the following page. For reference, 1 km/s/ Mpc ≈ 0.001 Gyr_{-1}. Therefore, divide your value of $1/H_0$ by 0.001 to make your final answer in Gigayears!

4. The data in your Hubble diagram probably did not fall perfectly along a straight line. Thus, you probably had to make a guess about where exactly to place your line. This suggests that the slope of your line, and thus the value of the Hubble constant, is somewhat uncertain. To estimate the uncertainty in your measurement of the Hubble constant, draw the steepest and shallowest lines that you think could also be compatible with your data on the plot on the previous page (preferably using another color, or making the lines dashed, to differentiate them from your "best-fit" line). Using the table on the following page, repeat the steps above to estimate the maximum and minimum values of the Hubble constant that are compatible with your data. Use your range to fill out the final line of the table. For example, if your best-fit line gives a Hubble constant of 70km/s/Mpc, your steepest line gives 75km/s/ Mpc and your shallowest line gives 62km/s/Mpc, your final answer should be that $H_0 = 70 \pm 8$ km/s/ Mpc.

|	V_r (km/s)	D at V_r (Mpc)	$H_0 = V_r/D$ (km/s/Mpc)	$t_{Hubble} = 1/H_0$ (Gyrs)
"Best-fit" line				
Steepest line				
Shallowest line				

The Hubble Constant is _____ ± _____ km/s/Mpc

Questions

1. Below you see images of three galaxies that are quite different from those you used to derive the value of the Hubble constant. For each of these three galaxies, explain (1) what type of galaxy it is (e.g. "moderately inclined late-type spiral" would describe all the galaxies you used in the project) and (2) why the galaxy could not have been used to measure the Hubble constant using the techniques you employed in this project.
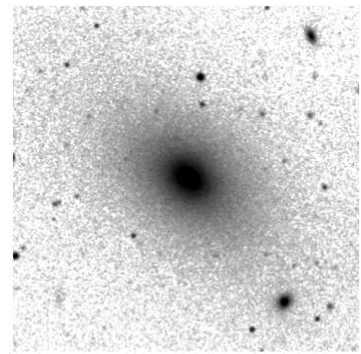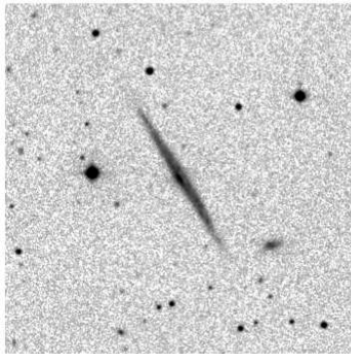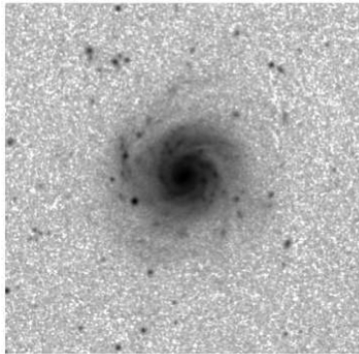
2. Based upon your experience measuring both recessional velocities and distances, which do you think is a larger source of scatter in your Hubble diagram? Briefly explain your reasoning.

3. Suppose that you tried to measure the value of the Hubble constant using only half as many galaxies as you used in this project. Do you think your estimate would be better or worse? Explain your reasoning.

4. Suppose you had the opportunity to add data for five more galaxies to your Hubble diagram. If you want to improve your measurement of the Hubble constant, would you rather add data for five galaxies with recessional velocities of ~ 1000 km/s or ~ 4000 km/s? Explain your thinking.

5. The Earth, the Sun, and the Solar System are roughly 5 billion years old. How old was the Universe when the Earth formed, assuming that your "best-fit" value for Hubble time is a reasonable approximation of the age of the Universe? Compared to the approximate age of the Universe, is this close to the beginning of the Universe, roughly halfway through, or close to the present day?

6. The oldest globular clusters are at least 11 billion years old. How old was the Universe when the first globular clusters formed, assuming that your "best-fit" value for Hubble time is a reasonable approximation of the age of the Universe? Compared to the approximate age of the Universe, is this close to the beginning of the Universe, roughly halfway through, or close to the present day?

7. The Andromeda Galaxy (M31) and the Milky Way are being pulled toward each other. This means that Andromeda has a negative recessional velocity. If we were limited to using only M31 to determine the Hubble constant what would be the most important change in our idea of the current behavior of the Universe, and why?

8. A large fraction of galaxies are found in groups or clusters. These galaxies orbit within the group or cluster, producing additional motions beyond those due to the overall expansion of the Universe. These motions are "extra" components of velocity that are due to the gravitational pull of the group or cluster within which the galaxy happens to be. They have nothing to do with the global expansion of the Universe! These "peculiar velocities" have sizes between 300 km/s in groups up to > 1000 km/s in clusters. Unfortunately, we have no way of knowing how much of any recessional velocity that we measure is due to these peculiar velocities and how much is due to the expansion of the Universe.

a) Can peculiar velocities be negative, positive, or both? Explain your reasoning.

b) Suppose all the peculiar velocities in the Universe were suddenly increased. Would the scatter in your Hubble diagram go up or down? Explain your reasoning.

c) Do peculiar velocities increase or decrease the certainty of your estimate of the Hubble constant? Explain your reasoning.

d) Do you think your measurement of the Hubble constant would be more or less accurate if you used more distant galaxies? Explain your reasoning. It may help you to return to your Hubble diagram, and imagine how a typical peculiar velocity would contribute to the total observed velocity.