

Inference

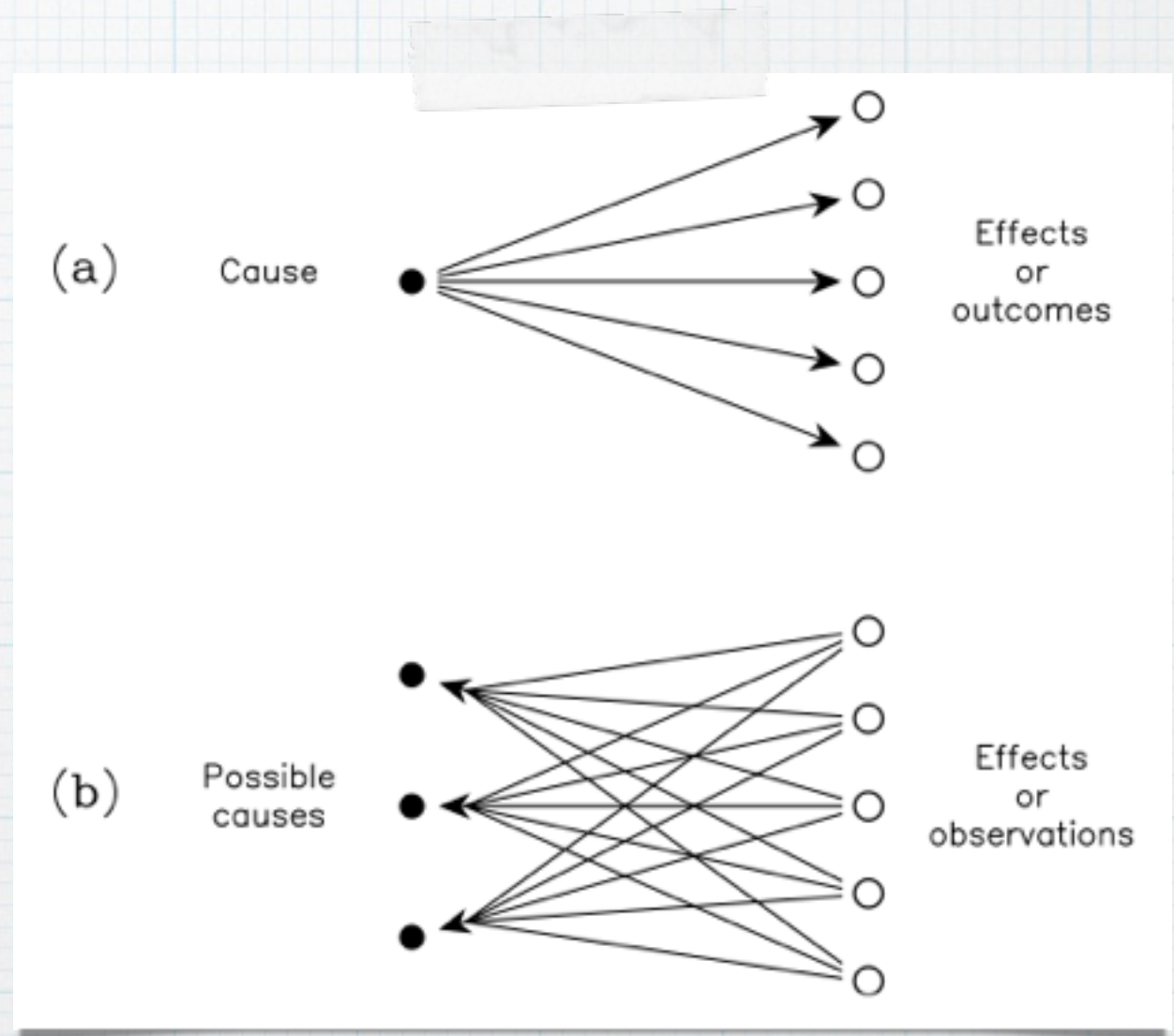
A brief introduction to Bayesian Statistics through Astronomical Applications (Lecture 1)

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Mérida, Venezuela

Universidad de Barcelona
20 de noviembre, 2017

Inference and Deduction

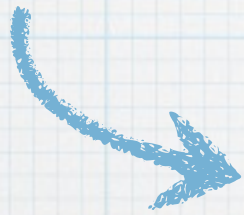
- * The forward problem:
Given a cause predicting the possible effects
- * The inverse problem:
Given a set of effects or observations, inferring the probable causes



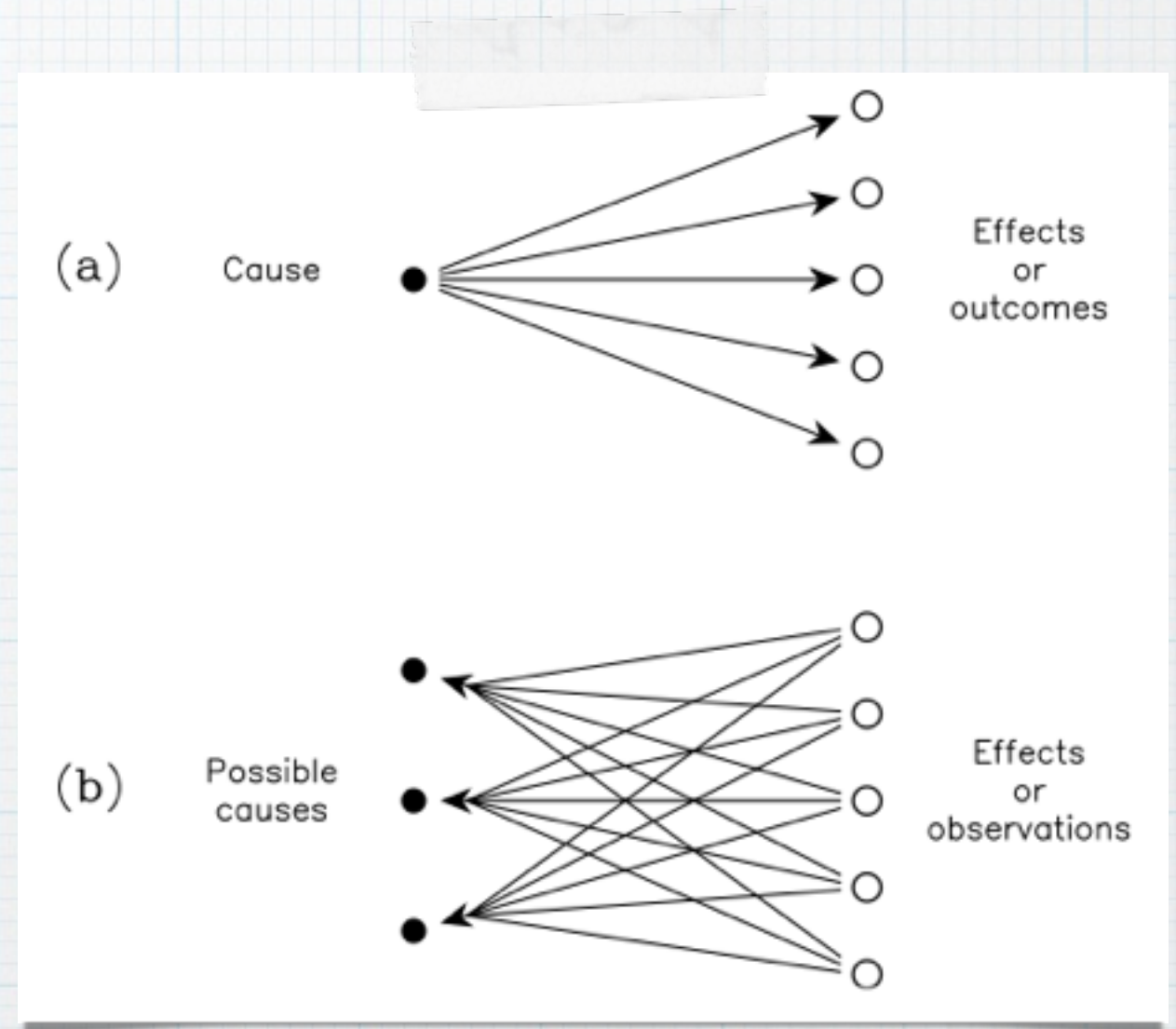
(Sivia & Skilling 1996)

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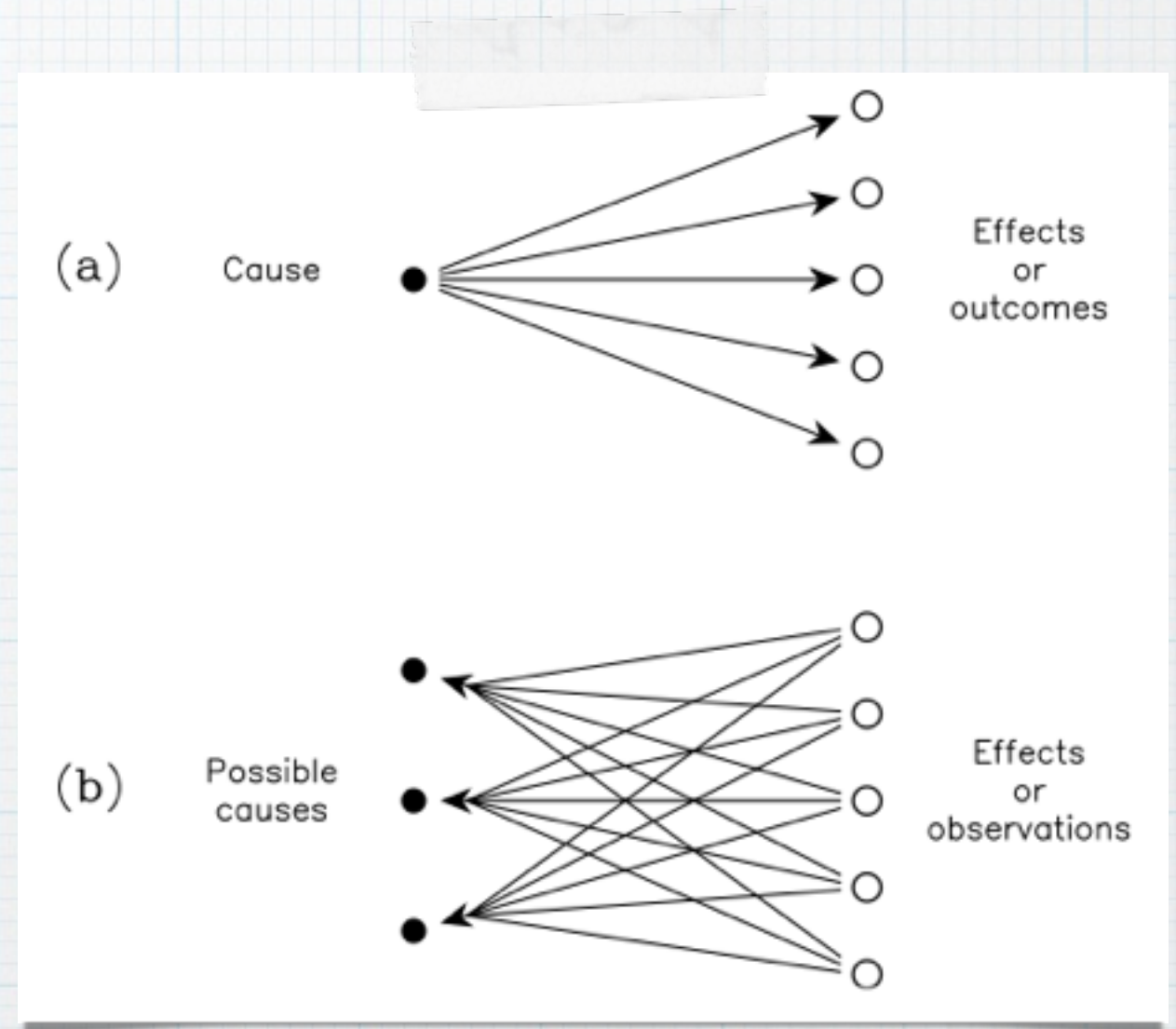
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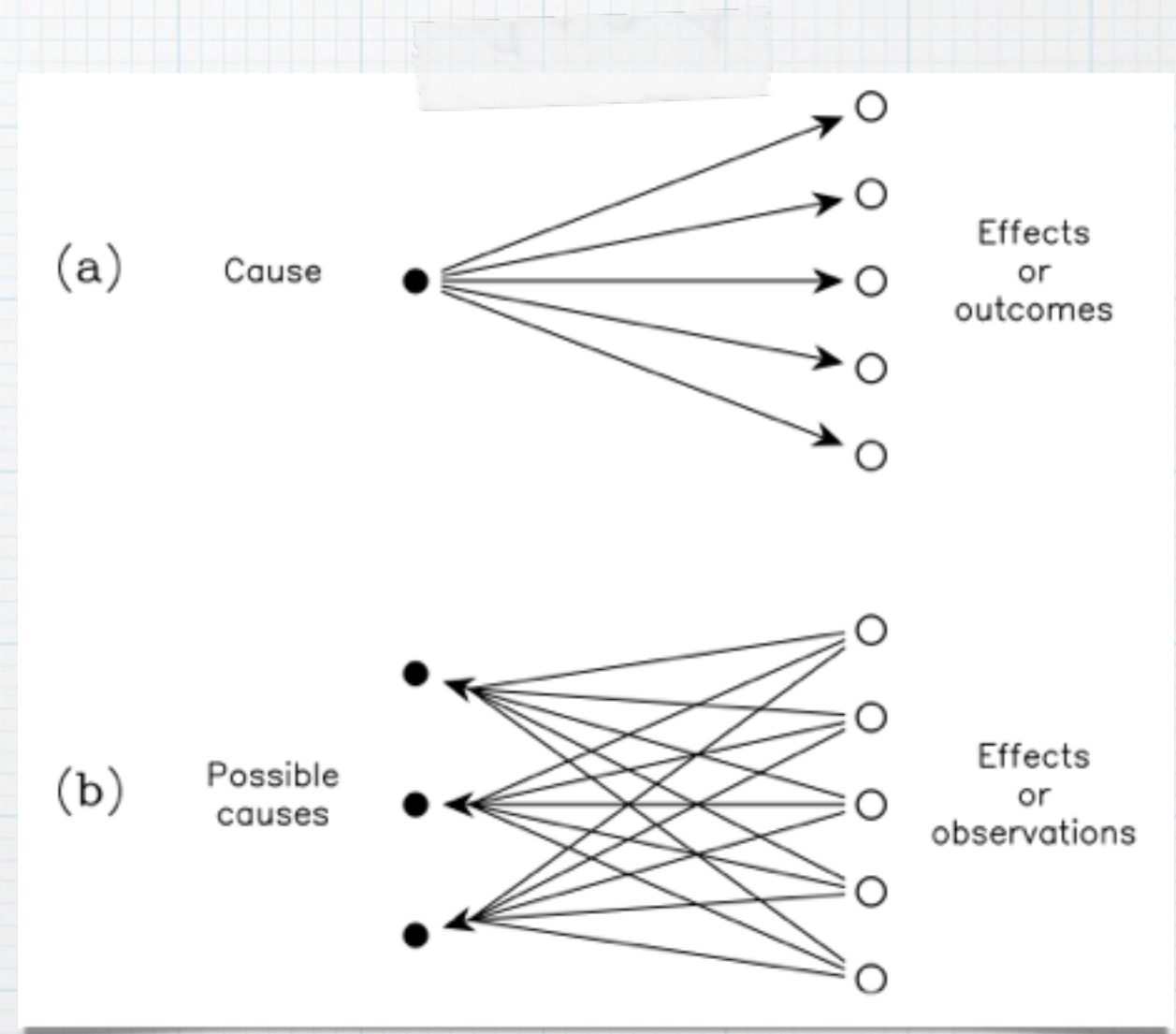
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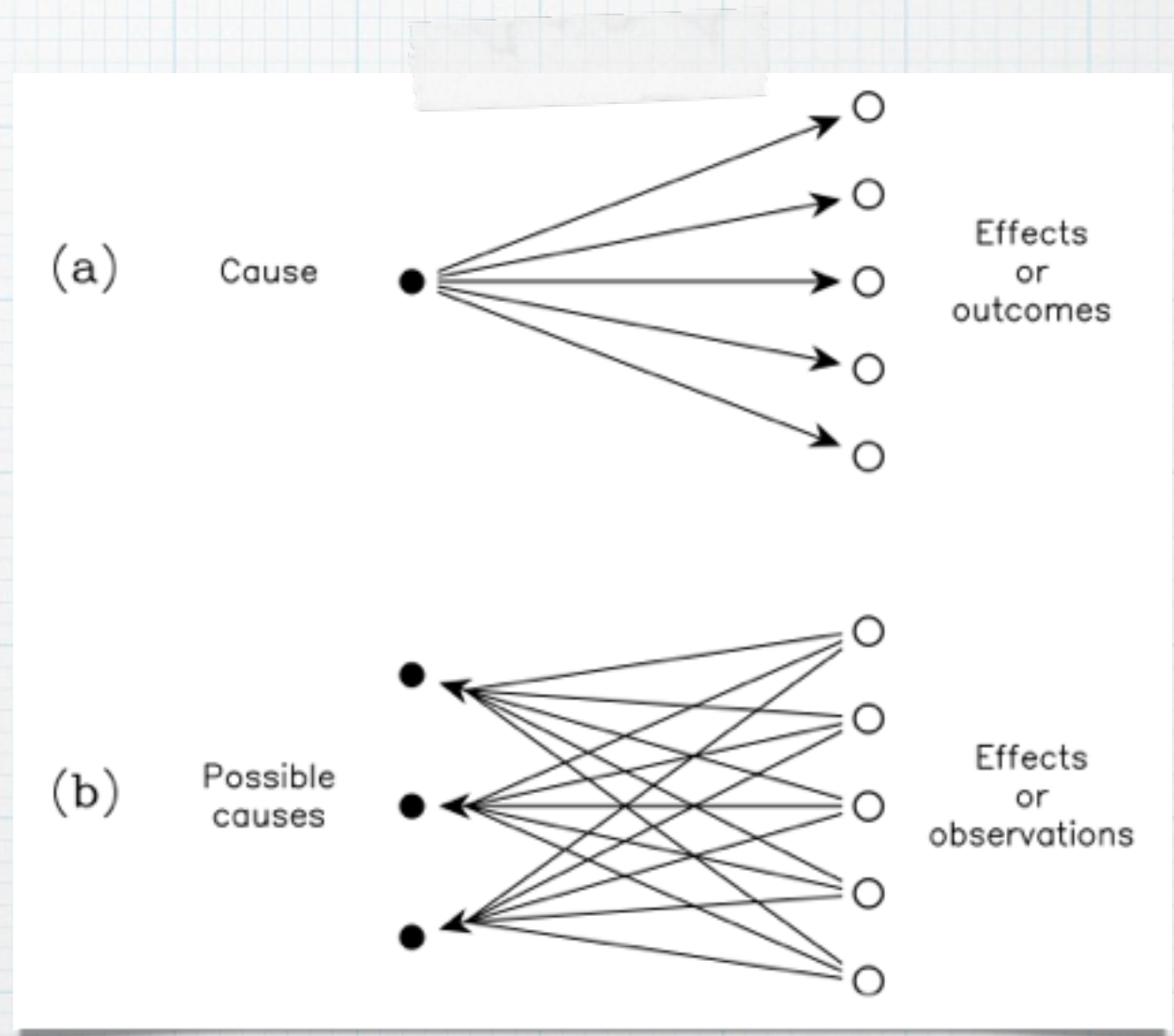
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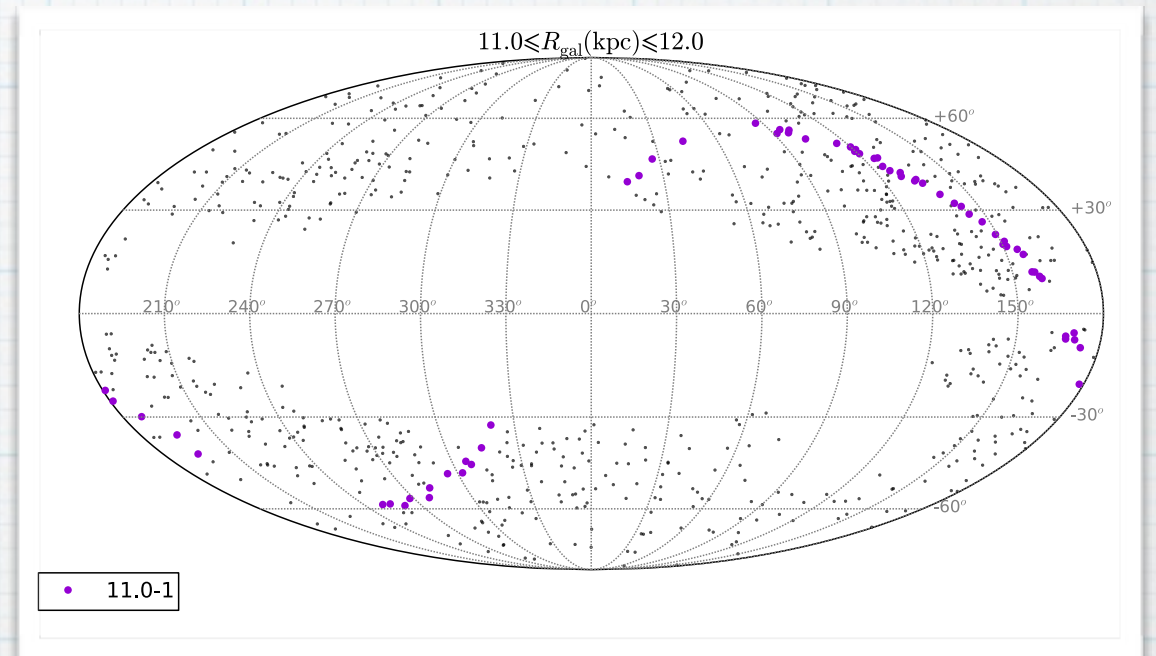
↘ Inference



(Sivia & Skilling 1996)

Parameter Inference

- * Say you're studying substructure in the Galactic Halo and find a stellar stream candidate
- * It has N RR Lyrae stars which you estimate really do belong to the stream
- * what can we say about the luminosity of the stream's progenitor?

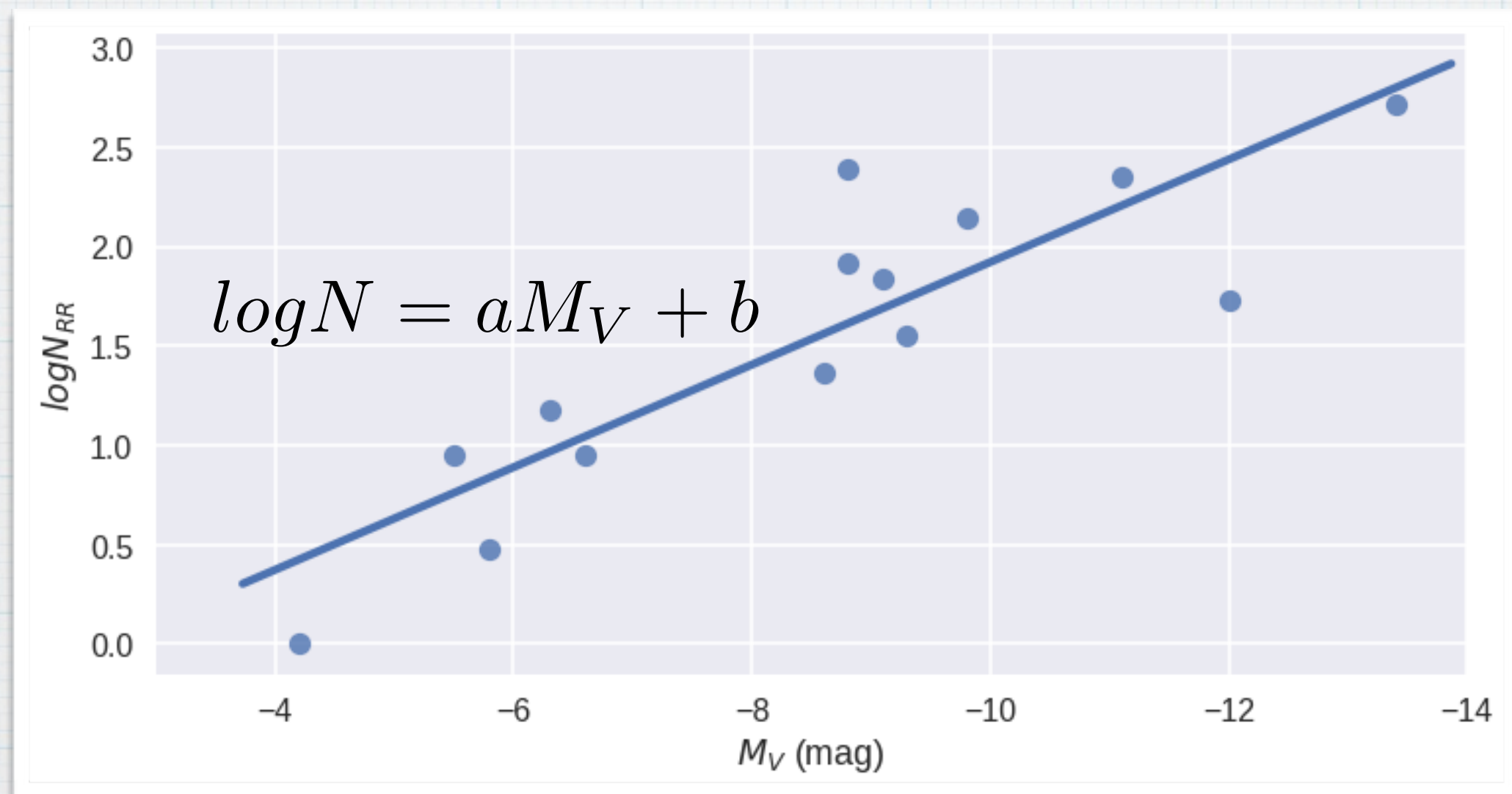


from Mateu, Read & Kawata 2017... what a coincidence, right?

go to blackboard + notebook...

Inference on M_V

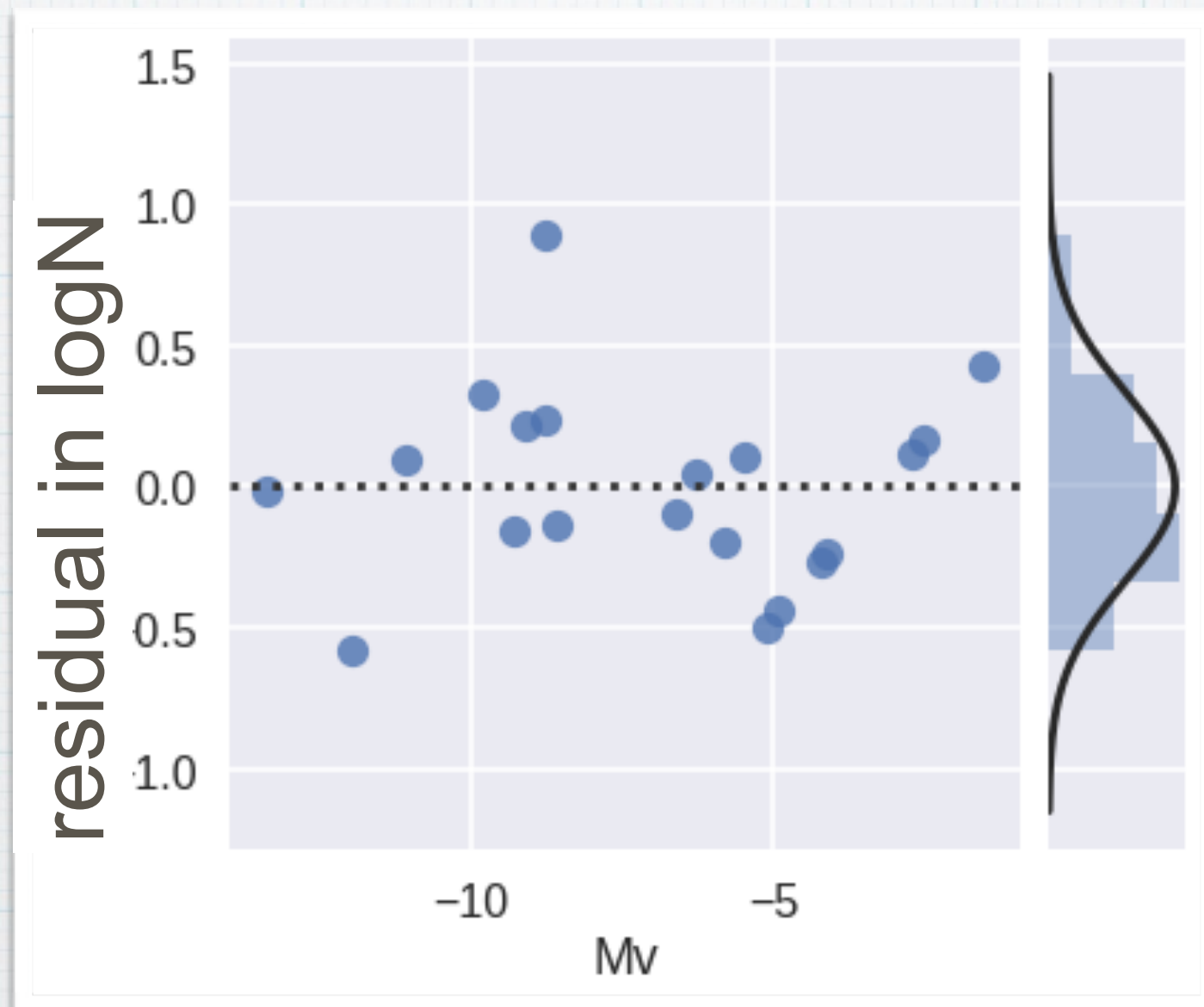
- * We'd expect to more luminous galaxies to host more RR Lyrae



Williams & Baker (2015), Catelan (2009), Harris (1996, 2010)

LogN-Mv

- * The LogN-Mv relationship has approximately Gaussian noise

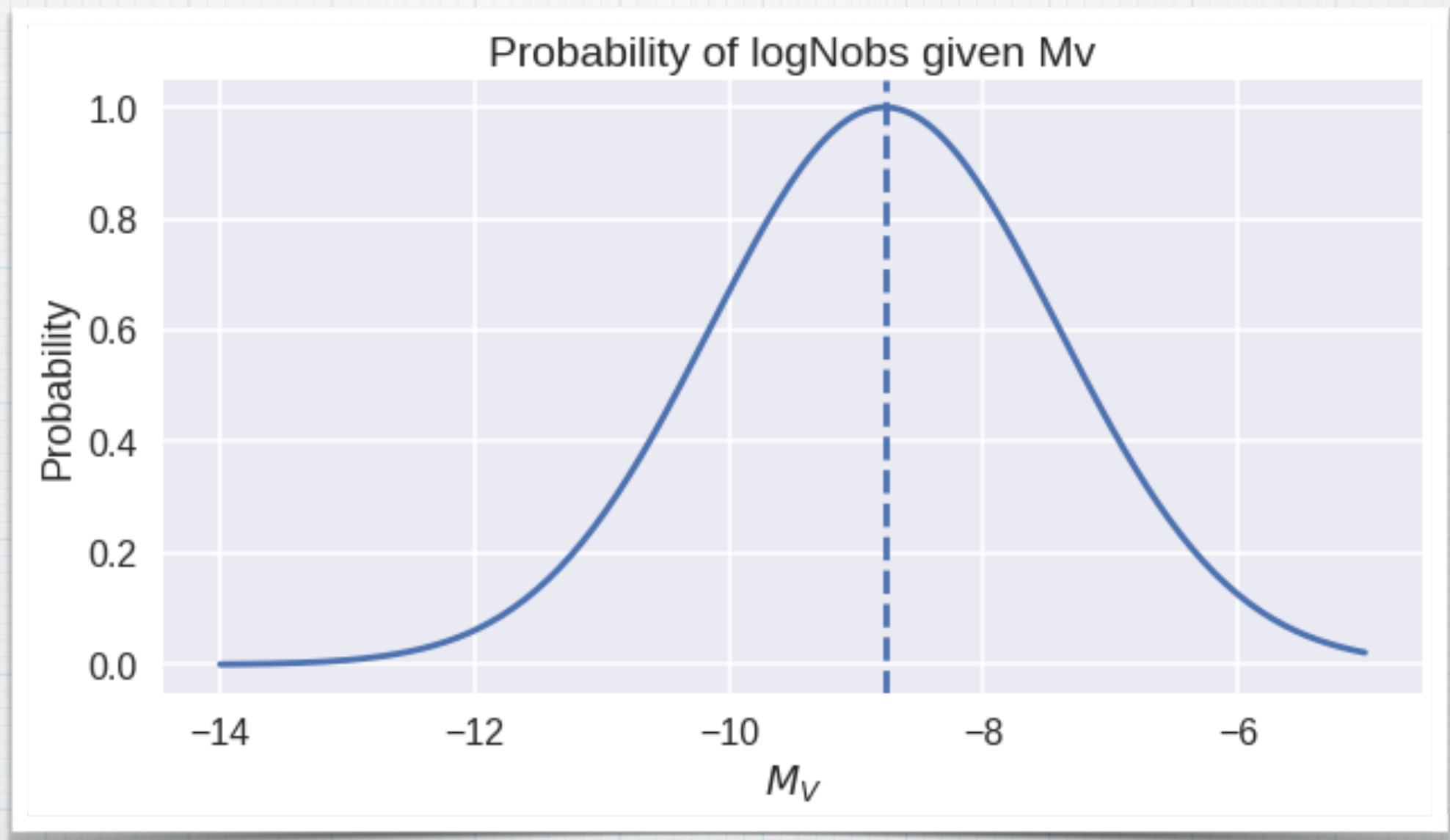


Williams & Baker (2015), Vivas & Zinn (2006)

Inference on M_V

* We will assume a Gaussian Likelihood

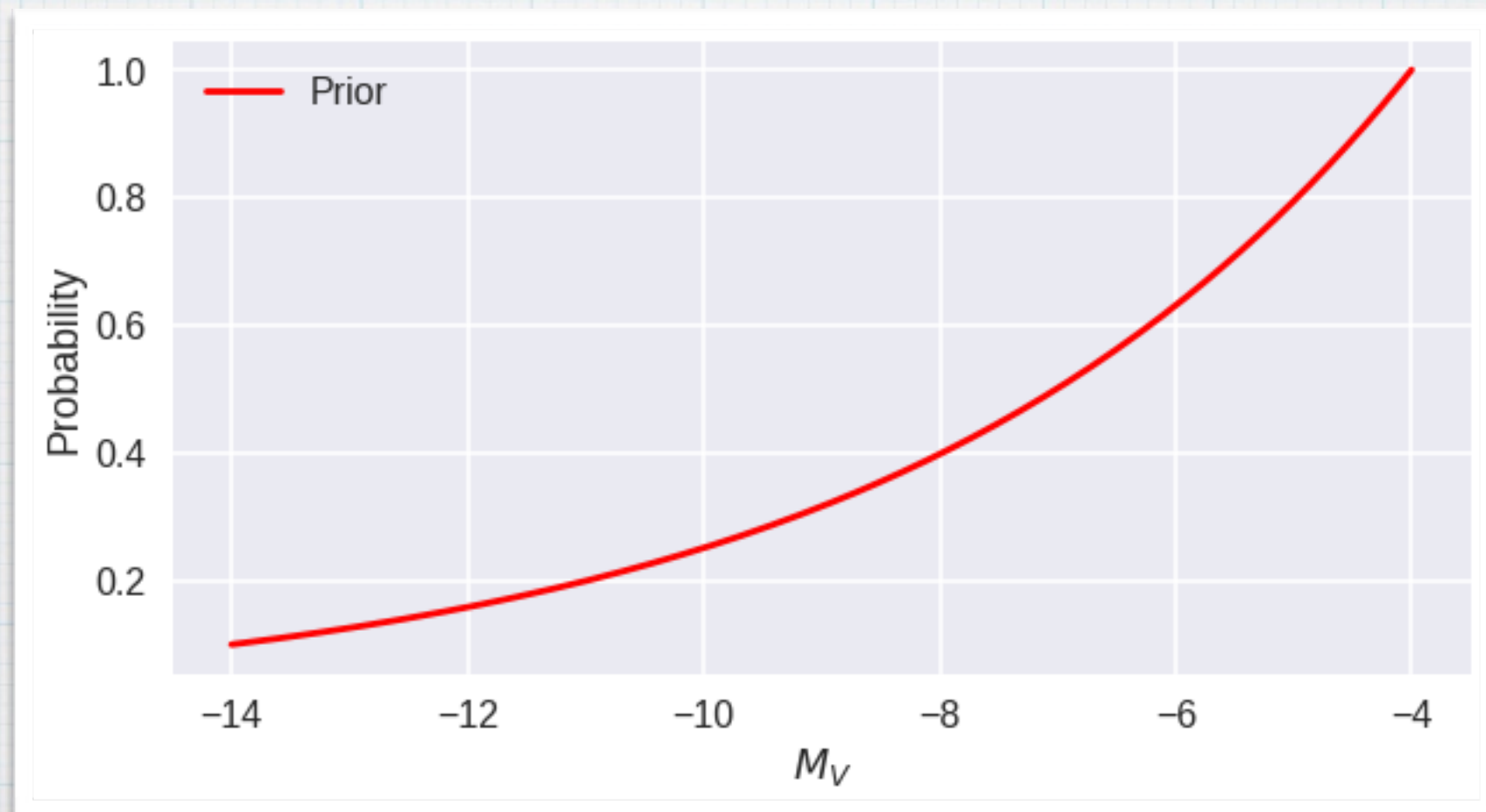
$$P(\log N_{obs} | M_V) = e^{-\frac{(\log N_{obs} - \log N(M_V))^2}{2\sigma^2}}$$



Prior probability for M_V

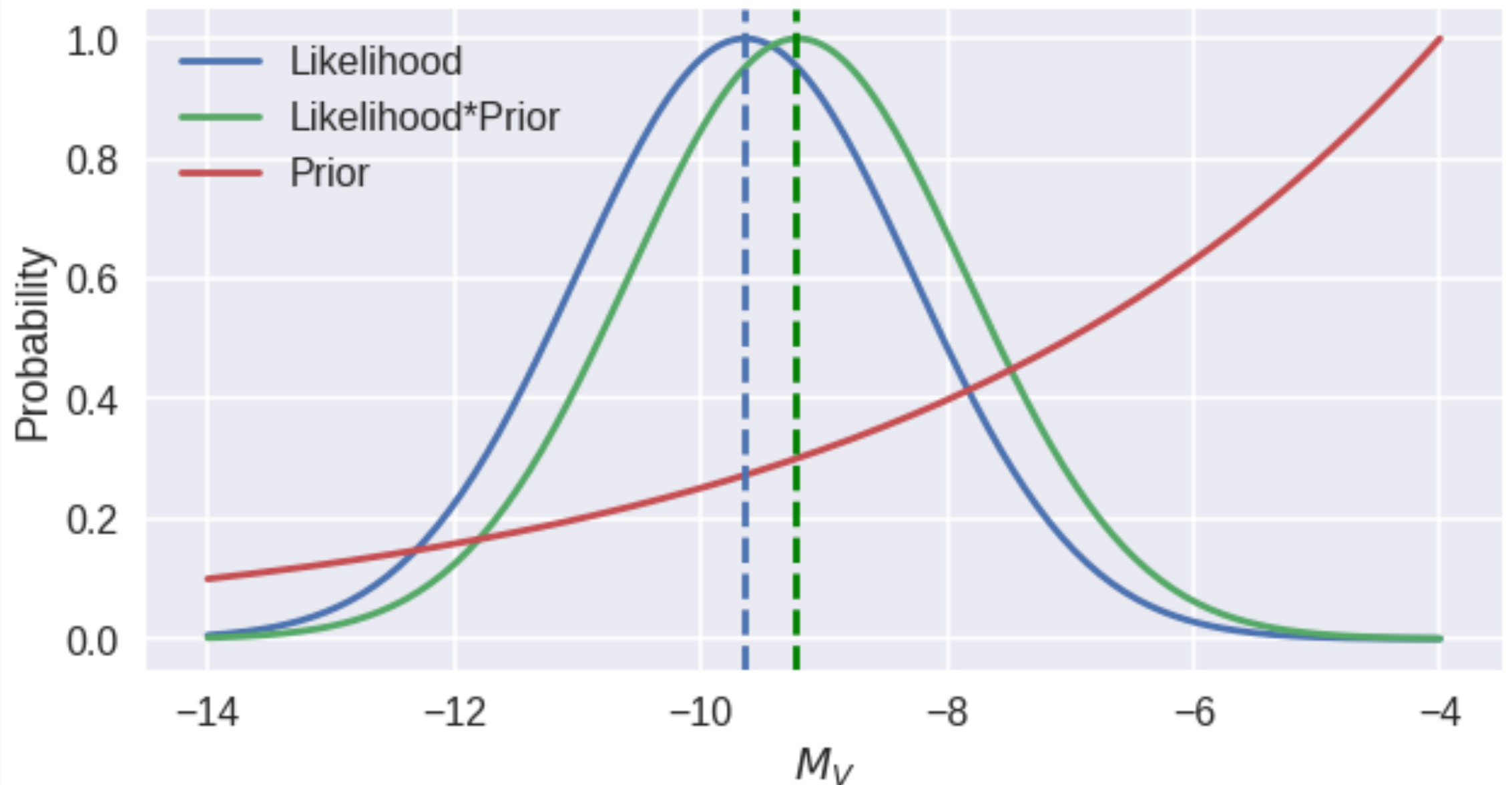
- * But we know the galaxy luminosity function is a power law, hence, less luminous galaxies are more common than more luminous ones
- * We can fold this information in as a "prior" probability

$$P(M_V|I) = 10^{0.1(M_V+5.)}$$



Likelihood*Prior

$$P(\log N_{obs} | M_V) P(M_V | I) = e^{-\frac{(\log N_{obs} - \log N(M_V))^2}{2\sigma^2}} 10^{0.1(M_V + 5.)}$$



back from blackboard + notebook...

The Definition of Probability

"Probability is what everybody knows before going to school and continues to use afterwards, in spite of what one has been taught"

-G. D'Agostini (1998)

The Definition of Probability

The Definition (and interpretation) of Probability

For an event or proposition A , probability is defined as:

* The Frequentist definition:

$P(A)$ is the relative frequency of occurrence of A in a series of Bernoulli trials, as the number of trials tends to infinity

The Definition (and interpretation) of Probability

For an event or proposition A , probability is defined as:

* The Frequentist definition:

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* The Bayesian definition:

$P(A|I)$ is the plausibility (or our degree of belief) that A will occur, given I

I denotes our assumptions (all available info) which in Bayesian statistics must be explicit. No such thing as absolute probabilities, all probabilities are conditional.

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- * $A|B$ means A given B is True
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 $=$ probability of A given B
- * $P(A, B|C) :=$ joint probability $=$ probability of A and B , given C

Probability Rules

- * Our definition of probability + Boolean logic implies that a probability must obey the following rules (see Jaynes 2003):

- * $0 < P < 1$

- * Sum Rule:

$$P(A|I) + P(\text{not-}A|I) = 1$$

- * Product Rule:

$$P(A, B|I) = P(A|B, I) P(B|I)$$

Bayesian Inference

- * We have a set of data D , and a set of hypotheses H (possible causes)
- * We would like to infer the probability for each hypothesis given that we have observed the data, and given all available information I at the time of the experiment

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$$P(H | D, I) = P(D | H, I) P(H | I) / P(D | I)$$

Bayes Theorem

* Bayes' Theorem:

$$P(H|D,I) = P(D|H,I) P(H|I) / P(D|I)$$

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$P(H|D,I)$: Posterior probability

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$P(H|D,I)$: Posterior probability

$P(D|H,I)$: Likelihood

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$P(H|D,I)$: Posterior probability

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$P(H|I)$: Prior probability

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$P(H|D,I)$: Posterior probability

$P(D|H,I)$: Likelihood

$P(H|I)$: Prior probability

$P(D|I)$ = Normalization constant

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- * $P(D|H,I)$: Likelihood

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- * $P(D|I)$ = Normalization constant (called also Bayes factor)

Bayes Theorem

* Bayes' Theorem:

$$P(H|D,I) = P(D|H,I) P(H|I) / P(D|I)$$

translation:

$$\text{Posterior} = \frac{\text{Likelihood} * \text{Prior}}{\text{Constant}}$$

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Some Motivations for Bayesian Inference

- * Bayesian Statistics provides a clear framework for Inference - Hypothesis testing
- * Probability is related to the state of uncertainty in a physical variable/model/theory, not only on the outcome of repeated experiments
- * Our prior knowledge, assumptions, prejudices or lack thereof, must be stated explicitly in our model
- * Propagation of uncertainties follows naturally

The Coin Example

The Coin Example

- * Lets say we're at a casino and see a coin tossed N times, with the following outcome

- * H, T, H, H, H, T, H, T

- * We'd like to know if the coin is biased

The Coin Example

- * Let h be the coin bias, i.e. the probability of getting heads in a single coin toss
- * The probability of having observed N_h heads in N tosses is

$h h h \dots h$ (N_h times)

- * and the probability of getting $(N - N_h)$ tails is

$(1-h)(1-h)(1-h) \dots (1-h)$ ($N - N_h$ times)

The Coin Example

* So, we can write our likelihood function as

$$P(N, N_h | h, I) = h^{N_h} (1-h)^{N-N_h}$$

→ The Binomial Distribution

and the posterior is given by Bayes' Theorem as

$$P(h | N, N_h, I) = C h^{N_h} (1-h)^{N-N_h} P(h | I)$$

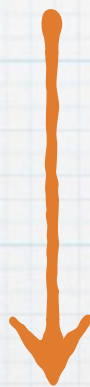
* where C is the normalization constant

The Coin Example

- * Lets recap

- * N and N_h are our data (known)

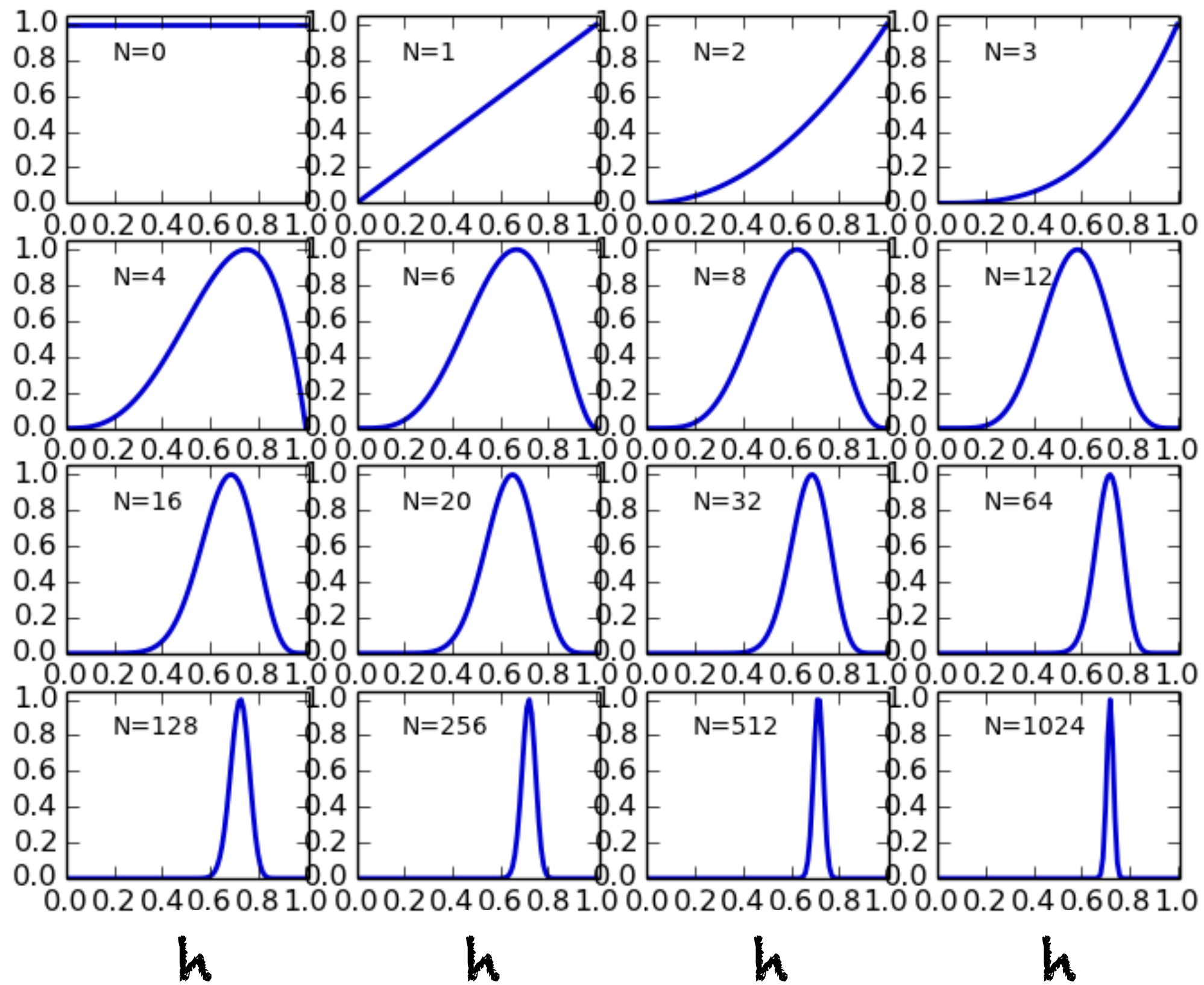
- * Our goal is to get $P(h|N, N_h, I)$ remember this is a function of h



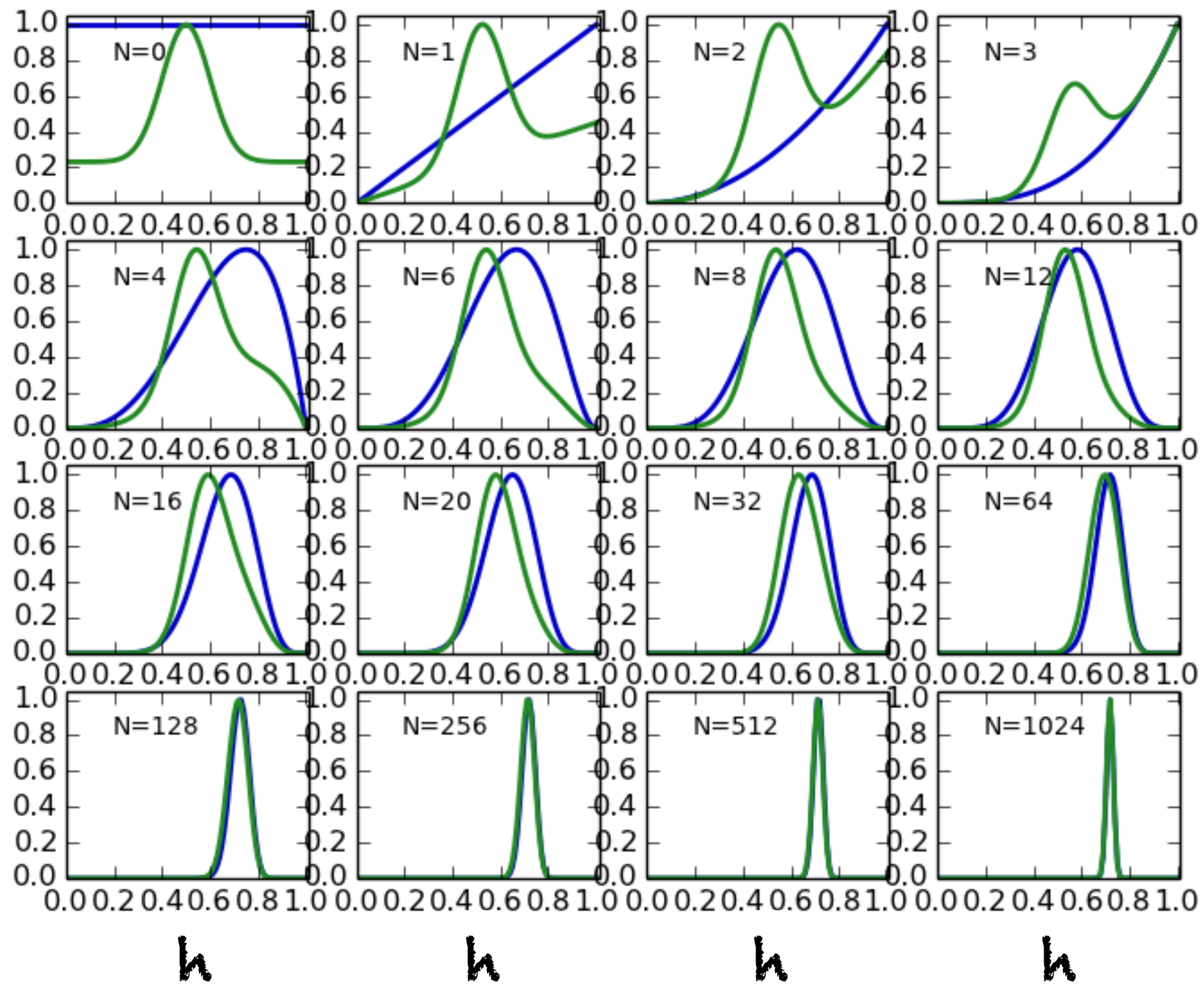
The full posterior IS the answer to our problem

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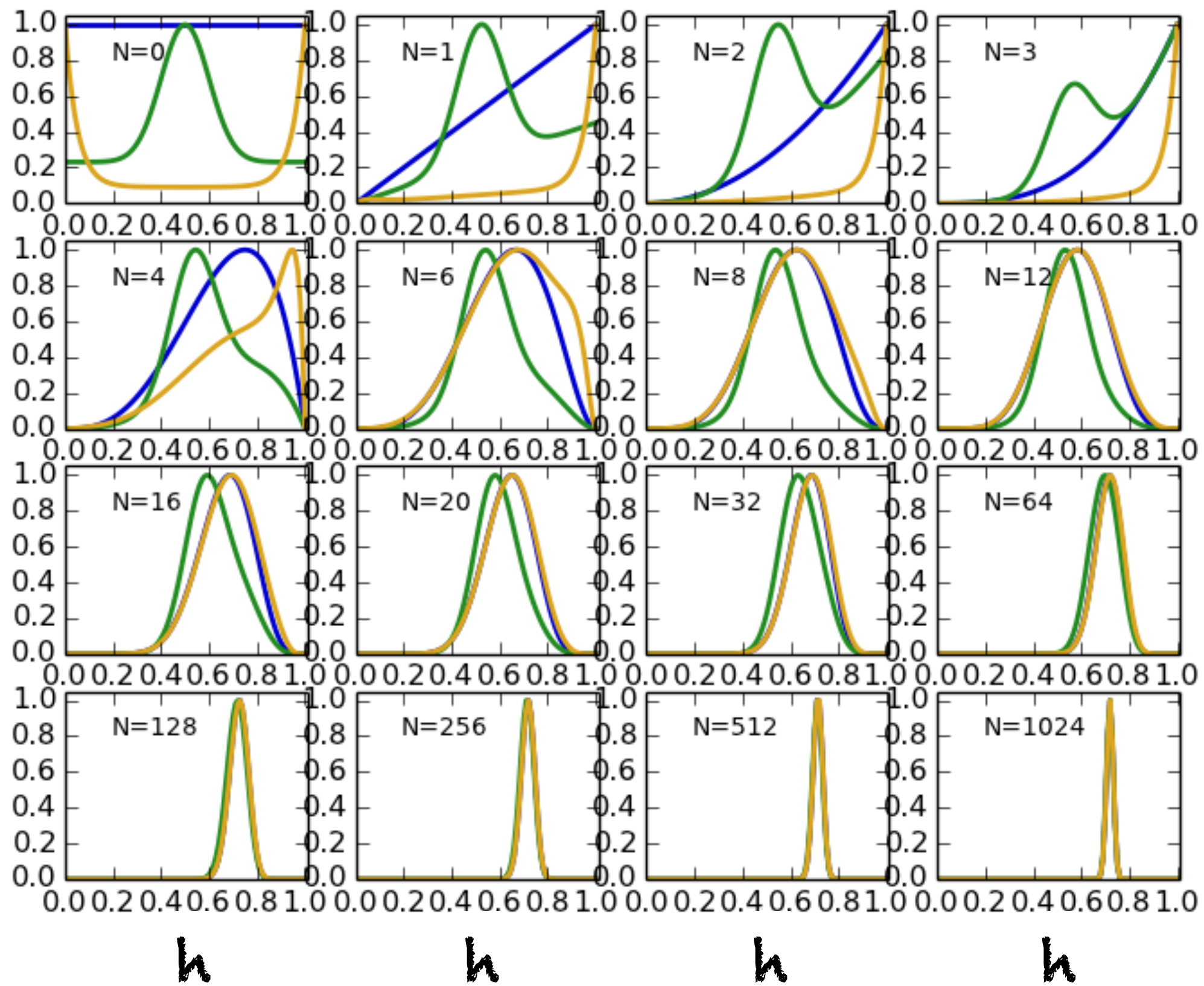
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The full posterior IS the answer to our problem

$$P(h|N, N_h, I) = C h^{N_h} (1-h)^{N-N_h} P(h|I)$$

- * anything else we may want can be calculated from it, e.g.
 - * the most probable value of h
 - * credible regions (Bayesian term for confidence intervals)
 - * The probability that $h > 0.5$
 - * $\int P(h|N, n, I) dh$
 - * ... more on this tomorrow ...

GitHub Repository

- * Classes and programs are available in the following GitHub repository

https://github.com/cmateur/intro_to_bayes_UB

More examples

The full posterior **IS** the answer to our problem

$$P(h|N, N_H, I) \propto h^{N_H} (1 - h)^{N - N_H} P(h|I)$$

More examples

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$$P(h|N, N_H, I) \propto h^{N_H} (1 - h)^{N - N_H} P(h|I)$$

- * The probability that the coin is biased:
 - * Lets say if $0.45 < h < 0.55$ we can safely take the coin as fair

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$$P_{fair} = \int_{0.45}^{0.55} P(h|N, N_H, I) dh$$

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$$P_{fair} = \int_{0.45}^{0.55} P(h|N, N_H, I) dh$$

- * so, the probability that it is biased is $P_{biased} = 1 - P_{fair}$

$$P_{biased} = \int_0^{0.45} P(h|N, N_H, I) dh + \int_{0.55}^1 P(h|N, N_H, I) dh$$

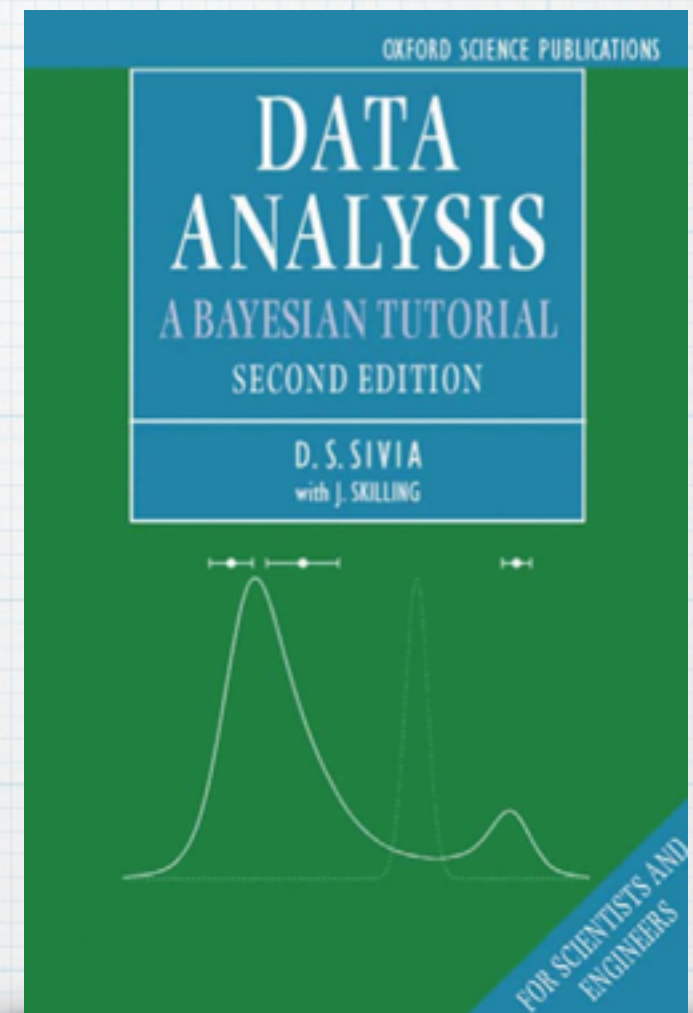
The Importance (or not) of Priors

- * The prior probability reflects our knowledge or ignorance on the problem
- * In practice, for many applications the posterior is dominated by the likelihood
- * If radically different priors are thought to be acceptable and the 'answer' depends strongly on the choice of the prior, it just means the data is not constraining enough! (see Jaynes 2003, D'Agostini 1998)

Very short bibliography

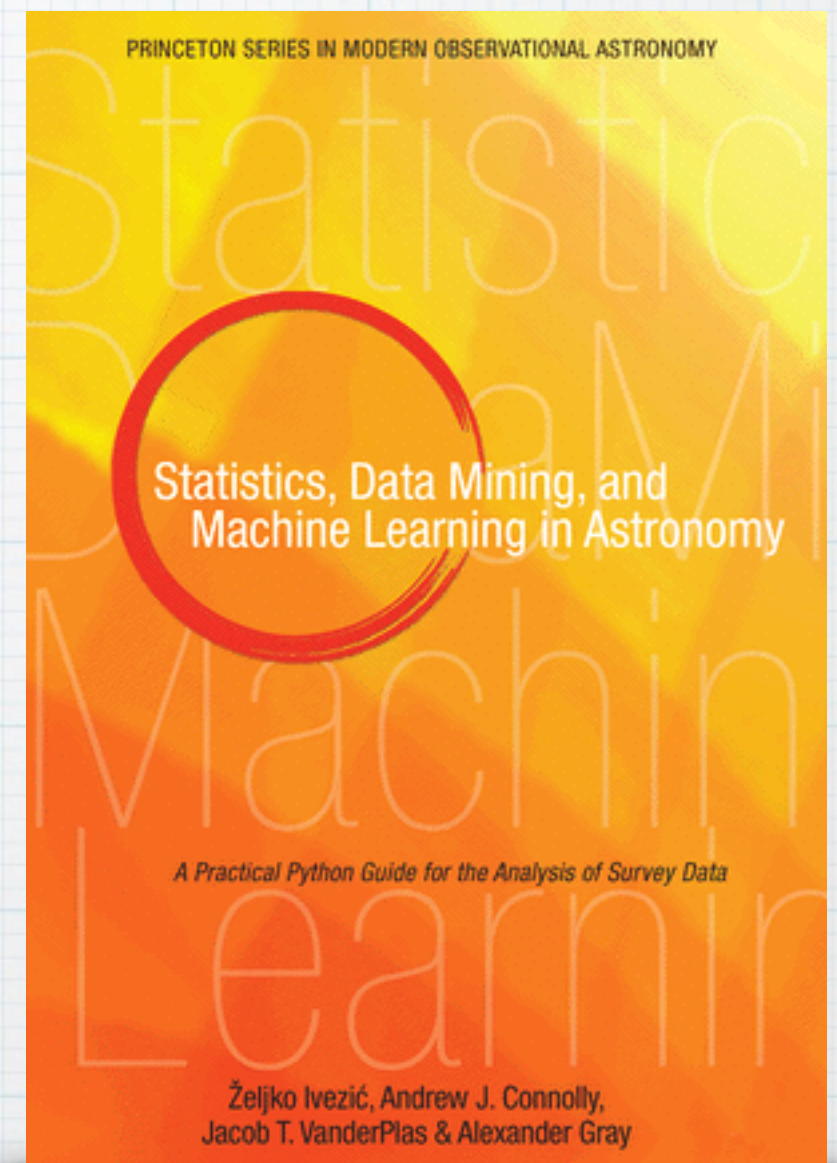
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 - * Sivia & Skilling book
 - * Giulio D'Agostini's notes available at Tom Loredo's BIPS web page:

<http://www.astro.cornell.edu/staff/loredo/bayes/>



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