

1)
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X is an integer.

$x, x + 1$, and $x + 2$ are three consecutive integers

$$N = 3k.$$

$$x + (x + 1) + (x + 2) = 3k$$

Let $k = x + 1$.

$$x + (x + 1) + (x + 2) = 3x + 3 = 3(x + 1)$$

Since x is an integer, $x + 1$ would also be an integer.

Hence $x + (x + 1) + (x + 2) = 3k$.

2)

$$r = n/m \text{ for some integers } n, m.$$

$$5r^2 - 2r + 3$$

$$= 5(n/m)^2 - 2(n/m) + 3$$

$$= 5n^2 / m^2 - 2n/m + 3$$

$$= (5n^2 - 2nm + 3m^2) / m^2$$

Suppose that $p = (5n^2 - 2nm + 3m^2)$ and $q = m^2$

p and q are integers because n and m are integers. So p/q proves that $5r^2 - 2r + 3$ is a rational.

3)

Let P be the proposition that $x^2 (y+3)$ is EVEN

and let Q be the proposition that x is EVEN or y is odd

Contraposition:

IF x is odd or y is even, then $x^2 (y+3)$ is odd

$$x = 2k + 1$$

$$y = 2j$$

Substitute for x and y in the equation...

$$(2k + 1)^2 (2j + 3)$$

$$= (2k + 1)(2k + 1) * (2j + 3)$$

$$= (4k^2 + 2k + 2k + 1) * (2j + 3)$$

$$= (4k^2 + 4k + 1) * (2j + 3)$$

$$= 8kj^2 + 12k^2 + 8kj + 12k + 2j + 3$$

$$2(4kj^2 + 6k^2 + 4kj + 6k + 1j + 1) + 1 = 2(\text{some integer}) + 1 \text{ which is an odd integer.}$$

This proves that $2k + 1 = x^2 (y+3)$ is an odd integer.

4)

P statement: If $m^2 + n^2$ is odd,

Q statement: then m is odd or n is odd.

Contraposition:

If m is even AND n is even, then $m^2 + n^2$ is even

$$m = 2a$$

$$n = 2b$$

$$= 2a^2 + 2b^2$$

$$= 2(a^2 + b^2) = 2 * (\text{some integer}) \text{ is an even integer}$$

so $2(a^2 + b^2) = m^2 + n^2$ which is even. So if m and n are both even, then when $m^2 + n^2$ is odd, then m is odd OR n is odd.

5)

P: If a and b are rational numbers

$b \neq 0$ & x is irrational

Q: then $a + bx$ is irrational.

First I say that " $a + bx$ " is rational. Then " $bx = (a + bx) - a$ " is rational because it's the difference of two rational numbers.

" $x = bx/b$ " which is rational, because it is the quotient of two rational numbers. Remember that b is not equal to 0, so bx/b is rational as a result because the denominator is not 0.

The problem is that x is irrational, but we've assumed that x is rational - therefore we have a contradiction which means that " $a + bx$ " is **irrational**.