

Section 9.4

6a)

There are 6 different remainders that are obtained when an integer is divided by 6.

0, 1, 2, 3, 4, and 5.

Pigeon hole principle: 7 integers are divided by 6 then at least two of them have the same remainder. When divided by 6 there must be two integers among any 7 integers that have the same remainder. The answer to 6a is **Yes**.

6b)

Set of integers 0, 1, 2, 3, 4, 5, and 6. All of these integers have different remainders when divided by 8, so the answer is **No**.

7)

Set $S = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

Partitioning the set: [3, 12] [4, 11] [5, 10] [6, 9] [7, 8]

The sum of these two integers in each partition is 15.

The two distinct ints are 7 and 8 and their set is $\{7, 8\}$ which is summed up to 15.

16)

There are 80 integers between 1 and 100 that are non-multiples of 5. Anything else is a multiple of 5.

If we pick 81 integers, then *at least* one is a multiple of 5. Hence the answer is **81**.

27)

Leap year = 366 days.

General Pigeonhole principle = $366n+1$ which implies $(n+1)$ people would have the same b-day.

$366 * 4 + 1 = 1465$ people. $(n+1) = (4 + 1) = 5$

$366 * 5 + 1 = 1831$ people $(n+1) = (5 + 1) = 6$

$366 * 6 + 1 = 2197$ people $(n+1)=(6+1) = 7$

With 2000 people at least **6** people have the same birthday.