

Set 4.6

12) a and b are rational; $b \neq 0$; r is irrational number

a + br is irrational.

$c = a + br$

Assume that C is also rational so a, b, c are rational and $b \neq 0$

So, $r = (c - a) / b$

(c - a) numerator is rational and b is also rational.

- Contradicts the assumption that r is irrational. So c must be irrational

16)

a, b, c are odd integers

z is rational..... $z = m/n$

Step 1) $a(m/n)^2 + b(m/n) + c = 0$

Step 2) $am^2 + bmn + cn^2 = 0$

ODD NUMBER DEFINITIONS
 $a = 2h+1$; $b = 2k+ 1$; $c= 2j + 1$

Step 3) $(2h + 1)m^2 + (2k+1)mn + (2j+1) n^2 = 0$

Step 4) $(2hm^2 + 2kmn + 2jn^2) + m^2 + mn + n^2 = 0$

Step 5) $2(hm^2 + kmn + jn^2) = - (m^2 + mn + n^2)$

Step 6) $m^2 + mn + n^2 = - 2 (hm^2 + kmn + jn^2)$

When m = 2 and n = 3

$2^2 + (2)(3) + 3^2 = 19$

Contradiction because an odd is not an even. Therefore z is irrational.

28) Proof by Contradiction

p = mn is even

q = n is even, m is even

$p \rightarrow q$ is the equivalent to $\sim q \rightarrow \sim p$

So.... suppose that m and n are odd defined below:

$m = 2k + 1$

$n = 2j + 1$

$mn = (2k + 1)(2j + 1)$

$= 4kj + 2k + 1$

$= 2(2kj + k + 1)$

$= 2n + 1$ which is an odd integer