

5.2  
9)

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$$9) n = 3 \quad 4^3 = 64 = 4 \frac{4^3 - 16}{3}$$

result holds for some integer  $k \geq 3$

$$4^3 + \dots + 4^{k+1} = 4 \frac{4^k - 16}{3} + 4^{k+1} \rightarrow$$

$$= 4 \frac{4^k - 16 + 3 * 4^k}{3} = \frac{4 * 4^k - 16}{3}$$

$$= \frac{4^{k+1} - 16}{3} \quad \text{Result by induction.}$$

5.3  
10)

$$= \frac{4^{k+1} - 16}{3} \quad \text{Result by induction.}$$

5.3

10)  $n^3 - 7n + 3$  is divisible by 3, each integer  $n \geq 0$

For  $n = 0$ ,  $(0)^3 - 7(0) + 3$  which is divisible by 3

For  $n = 0$  the statement is true.

Let  $k$  be any integer  $k \geq 0$  and suppose that

$k^3 - 7k + 3$  is divisible by 3.

This means that  $k^3 - 7k + 3 = 3r$  for some int  $r$ .

$(k+1)^3 - 7(k+1) + 3$  is divisible by 3.  $\rightarrow$

$$\begin{aligned} &= k^3 + 3k^2 - 4k - 3 \\ &= (k^3 - 7k + 3) + 3k^2 + 3k - 6 \\ &= 3k^2 + 3k - 6 + 3r \\ &= 3(k^2 + k - 2 + r) \end{aligned}$$

But  $k^2 + k - 2 + r$  is an int b/c it's the sum of products of integers.

$(k+1)^3 - 7(k+1) + 3$  is divisible by 3

problem is (true)

23.b)

23. b)  $n! > n^2$ , for all int  $n \geq 4$

$$(k+1)! = k!(k+1)$$

$> k^2(k+1) \rightarrow$  assumption

$$> (k+1)(k+1)$$

since  $k^2 > k+1$  for  $k \geq 4$

$n! > n^2$  for all  $n \geq 4$

Set 3, 4

Set 5.4  
2)

2) Let  $P(n) = b_n$  is divisible by 4

1. Anchor

$b_1 = 4$  and  $b_2 = 12$  (both divisible by 4)

Property is valid for  $n=1$  and  $n=2$ .

2. induction

$k > 2$ , the property is valid for all integers  $i$  such that

$1 \leq i \leq k$ , then if it is valid for  $n=k$ , the statement is true.

3. Inductive hypothesis

Let  $b_k > 2$  be in int

$b_i$  is divisible by 4 for all integers  $i$  such that  $1 \leq i \leq k$

If  $b_k$  is divisible by 4, we will have from the statement

$b_k = b_{k-2} + b_{k-1}$  proposition is true.  $\rightarrow$

$$b_k = b_{k-2} + b_{k-1}$$

$b_{k-2}, 0 < k-2 < k, 1 < k-1 < k$

both  $b_{k-2}$  and  $b_{k-1}$  are divisible by 4.

There exists integer  $A$  and  $B$  such that  $b_{k-2} = 4A$ ,

and  $b_{k-1} = 4B$ .

$$b_k = b_{k-2} + b_{k-1} = 4A + 4B = 4(A+B)$$

$b_k = 4(A+B)$  is divisible by 4.

10)

$P(14)$  is true b/c  $14!$  can be obtained with  $3(34)$  and  $1(54)$ .

$$(94 + 54 = 14)$$

$P(15)$  is true b/c it can be obtained by  $5(34) = 154$

$P(16)$  is true b/c ...  $2(34)$  and  $2(54)$ . ( $64 + 104 = 164$ )

$P(17)$  is true as a consequence of  $P(16)$  being true.)

$P(18)$  is true as a consequence of  $P(17)$  being true.

$P(19), (18), (16)$  proves  $n \geq 14$  for  $P(n)$ .