

## Exercise sets for 6.1

12)  $(a - j) \mid$

$$a) = \{x \in \mathbb{R} \mid -3 \leq x < 2\}$$

$$b) = \{x \in \mathbb{R} \mid -1 < x \leq 0\}$$

$$c) = \{x \in \mathbb{R} \mid x < -3 \text{ or } x > 0\}$$

$$d) = \{x \in \mathbb{R} \mid -3 \leq x \leq 0 \text{ or } 6 < x \leq 8\}$$

$$e) = \emptyset$$

$$f) = \{x \in \mathbb{R} \mid x \leq -1 \text{ or } x \geq 2\}$$

$$g) = \{x \in \mathbb{R} \mid x < -3 \text{ or } x \geq 2\}$$

$$h) = \{x \in \mathbb{R} \mid x \leq -1 \text{ or } x > 0\}$$

$$i) = \{x \in \mathbb{R} \mid x \leq -1 \text{ or } x > 0\}$$

$$j) = \{x \in \mathbb{R} \mid x < -3 \text{ or } x \geq 2\}$$

16)  $(a - c)$

a)

$$A \cup (B \cap C) = \{a, b, c\}$$

$$(A \cup B) \cap C = \{b, c\}$$

$$(A \cup B) \cap (A \cup C) = (a, b, c, d) \cap \{a, b, c, e\} = \{a, b, c\}$$

$$A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$$

b)

$$A \cap (B \cup C) = \{a, b, c\} \cap \{b, c, d, e\} = \{b, c\}$$

$$(A \cap B) \cup C = \{b, c\} \cup \{b, c, e\}$$

$$(A \cap B) \cup (A \cap C) = \{b, c\} \cup \{b, c\} = \{b, c\}$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

c)

$$(A - B) - C = \{a\} - \{b, c, e\} = \{a\}$$

$$A - (B - C) = \{a, b, c\} - \{d\} = \{a, b, c\}$$

Sets are not equal.

## Exercises for 6.2

4)  $(a-i)$

$$a) A \cup B \subseteq B$$

$$b) A \cup B$$

$$c) x \in B$$

$$d) A$$

$$e) \text{ or}$$

$$f) B$$

$$g) A$$

$$h) B$$

$$i) B$$

10)

$$- \text{Is } (A - B) \cap (C - B) \subseteq (A \cap C) - B \dots$$

$$\text{Assume } x \in (A - B) \cap (C - B)$$

// Intersection

$$x \in A - B \text{ and } x \in C - B$$

// Difference

$$x \in A \text{ and } x \notin B \text{ and } x \in C \text{ and } x \notin B$$

// Intersection Again

$$x \in A \cap C \text{ and } x \notin B$$

// Difference Again

$$x \in (A \cap C) - B$$

$$(A - B) \cap (C - B) \subseteq (A \cap C) - B \dots \text{ is a valid subset.}$$

$$- \text{Assume } x \in (A \cap C) - B$$

can be written as:

$$x \in A \text{ and } x \notin B \text{ and } x \in C \text{ and } x \notin B$$

So definition and difference kick in which ends up with:

$$x \in A - B \text{ and } x \in C - B.$$

$$\text{By definition of intersection, } x \in (A - B) \cap (C - B).$$

$$\text{So by definition of subset... } (A \cap C) - B \subseteq (A - B) \cap (C - B).$$

14)

A, B, and C are sets and  $A \subseteq B$ .

$$x \in A \cup C$$

// Definition of union

$$x \in A \text{ or } x \in C.$$

In case  $x \in A$  then since  $A \subseteq B$  we have:

$$x \in B \text{ so it's true that}$$

$$x \in B \text{ or } x \in C \text{ and so by union,}$$

$$x \in B \cup C.$$

In case  $x \in C$  then it is true that  $x \in B$  or  $x \in C$ , so by

definition of union  $x \in B \cup C$ .

Therefore...  $x \in B \cup C$

So by definition of subset

$$A \cup C \subseteq B \cup C$$

## Exercises for section 6.3

12)

$$\text{Let } a \in A \cap (B - C)$$

$$\Rightarrow a \in A, a \in B - C$$

$$\Rightarrow a \in A, a \in B, a \notin C$$

$$\Rightarrow a \in A \cap B, a \notin (A \cap C).$$

The left-hand-side(LHS) is a subset of the right-hand-side (RHS).

Converse:

$$\text{Let } a \in (A \cap B) - (A \cap C)$$

$$\Rightarrow a \in A \cap B, a \notin A \cap C$$

$$\Rightarrow a \in A, a \in B, a \notin C$$

$$\Rightarrow a \in A, a \in B - C.$$

The RHS is a subset of the LHS.

37)

Let A and B be sets  $A, B \subseteq U$ .

$$(B^C \cup (B^C - A))^C = (B^C \cup (B^C \cap A))^C$$

$$= (B^C)^C \cap (B^C \cap A)^C \quad // \text{De Morgan's Law}$$

$$= B \cap (B^C \cap A)^C \quad // \text{Double Complement Law}$$

$$= B \cap ((B^C)^C \cup (A^C)^C) \quad // \text{De Morgan's Law}$$

$$= B \cap (B \cup A) \quad // \text{Double complement Law}$$

$$= B \quad // \text{Absorption Law}$$

42)

Let  $A \cap B$  sets,  $A, B \subseteq U$ . // Set Difference Law

$$(A - (A \cap B)) \cap (B - (A \cap B))$$