Programming Project 5

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1 Assignment

In this chapter we studied the bisection method for finding a root of an equation. Another method of finding a root, Newton's method, usually con- verges to a soluction even faster than the bisection method, if it converges at all. Newton's method starts with an initial guess for a root x_0, and then generates successive approximate roots $x_1, x_2, ..., x_j, x_j + 1...$ using the iterative formula

$$x_{j+1} = x_j - \frac{f(x_j)}{f'(x_j)}$$

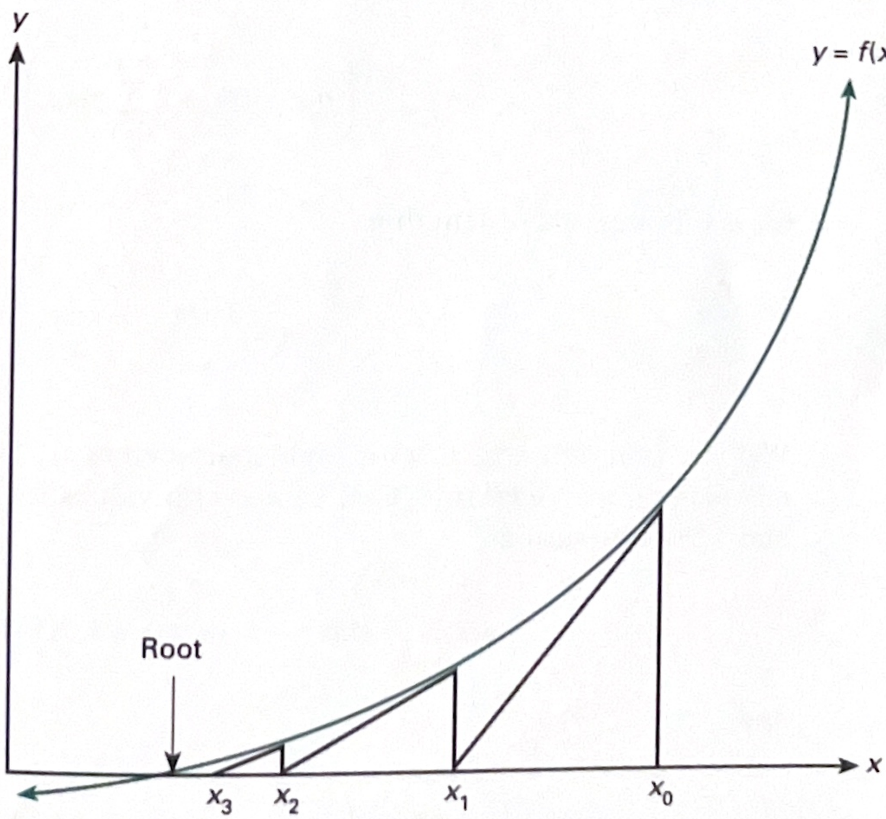
where $f'(x_j)$ is the derivative of function f evaluated at $x = x_j$. The formula generates a new guess, x_{j+1}, from the previous one, x_j. Sometimes Newton's method will fail to converge to a root. In this case, the program should terminate after many trials, perhaps 100.

Figure 7.12 shows the geometric interpretation of Newton's method where x_0, x_1, and x_2 represent successive guesses for the root. At each point x_j, the derivative, $f'(x_j)$, is the slope of the tangent to the curve, $f(x)$. the next guess for the root, x_{j+1}, is the point at which the tangent crosses the x axis.

for geometry, we get the equation

$$\frac{y_{j+1} - y_j}{x_{j+1} - x_j} = m$$

Figure 7.12 Geometric Interpretation of Newton's Method



Where m is the slope of the line between points (x_{j+1}, y_{j+1}) and $(x_j, y_j$. In Fig 7.12, we see that y_{j+1} is zero, y_j is $f(x_j)$, and m is $f'(x_j)$; therfore by substituting and rearranging terms, we get

$$-f(x_j) = f'(x_j) \times (x_{j+1} - x_j)$$

leading to the formula shown at the beginning of this problem.

Write a program that uses Newton's method to approximate the nth roo of a number to six decimal places. If $x^n = c$, then $x^n - c = 0$. Finding a root of the second equation will give you $\sqrt[n]{c}$. Test your program on $\sqrt{2}$, $\sqrt[3]{7}$, and $sqrt[3]{-1}$. Your program could use $c/2$ as it's initial guess.

2 Notes

- It occurs to me, it might be better to start making these notes simply in latex, since I am trying to get better with that anyway. Might be a fun and easy way to get what I want!