

Programming Project 5

Max Reilly, copied from Hanly and Koffman

April 4, 2024

1 Assignment

In this chapter we studied the bisection method for finding a root of an equation. Another method of finding a root, Newton's method, usually converges to a solution even faster than the bisection method, if it converges at all. Newton's method starts with an initial guess for a root x_0 , and then generates successive approximate roots $x_1, x_2, \dots, x_j, x_{j+1} \dots$ using the iterative formula

$$x_{j+1} = x_j - \frac{f(x_j)}{f'(x_j)}$$

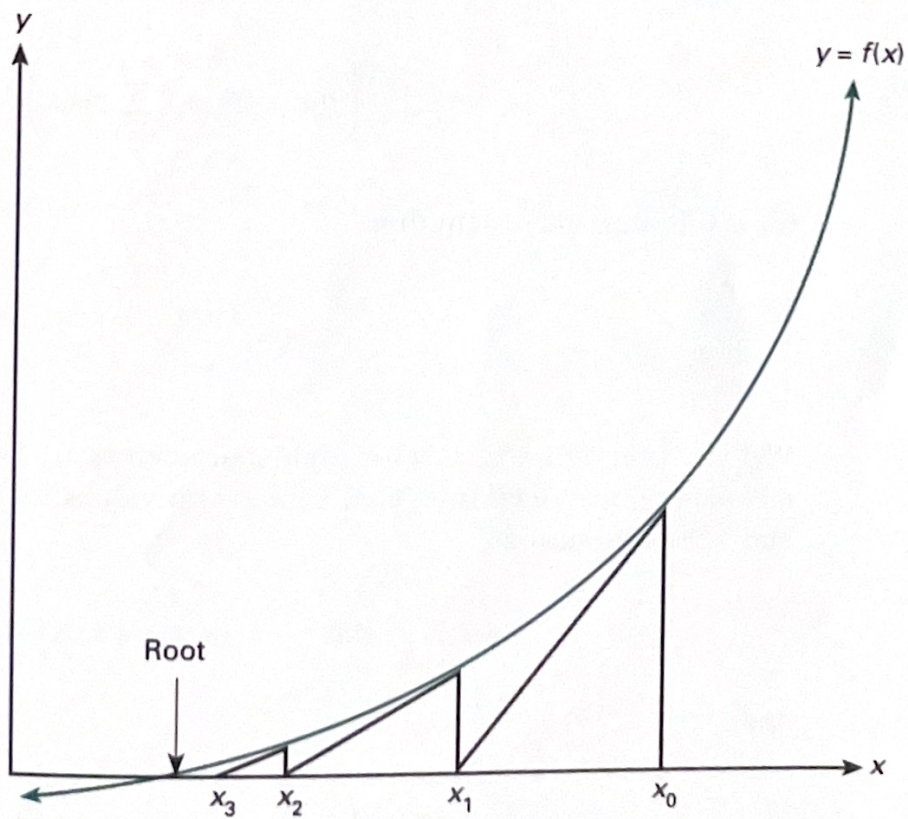
where $f'(x_j)$ is the derivative of function f evaluated at $x = x_j$. The formula generates a new guess, x_{j+1} , from the previous one, x_j . Sometimes Newton's method will fail to converge to a root. In this case, the program should terminate after many trials, perhaps 100.

Figure 7.12 shows the geometric interpretation of Newton's method where x_0, x_1 , and x_2 represent successive guesses for the root. At each point x_j , the derivative, $f'(x_j)$, is the slope of the tangent to the curve, $f(x)$. the next guess for the root, x_{j+1} , is the point at which the tangent crosses the x axis.

for geometry, we get the equation

$$\frac{y_{j+1} - y_j}{x_{j+1} - x_j} = m$$

Figure 7.12
Geometric
Interpretation
of Newton's
Method



Where m is the slope of the line between points (x_{j+1}, y_{j+1}) and (x_j, y_j) . In Fig 7.12, we see that y_{j+1} is zero, y_j is $f(x_j)$, and m is $f'(x_j)$; therefore by substituting and rearranging terms, we get

$$-f(x_j) = f'(x_j) \times (x_{j+1} - x_j)$$

leading to the formula shown at the beginning of this problem.

Write a program that uses Newton's method to approximate the n th root of a number to six decimal places. If $x^n = c$, then $x^n - c = 0$. Finding a root of the second equation will give you $\sqrt[n]{c}$. Test your program on $\sqrt{2}$, $\sqrt[3]{7}$, and $\text{sqrt}[3]-1$. Your program could use $c/2$ as it's initial guess.

2 Notes

- It occurs to me, it might be better to start making these notes simply in latex, since I am trying to get better with that anyway. Might be a fun and easy way to get what I want!