TWELVE-TONE RHYTHMIC STRUCTURE AND THE ELECTRONIC MEDIUM

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TO PROCEED from an assertion of what music has been to an assertion of what music, therefore, must be, is to commit a familiar fallacy; to proceed from an assertion of the properties of the electronic medium to an assertion of what music produced by this medium therefore must be, is not only to commit the same fallacy (and thus do fallacies make strange bedfellows), but to misconstrue that compositional revolution of which the electronic medium has been the enabling instrument. For this revolution has effected, summarily and almost completely, a transfer of the limits of musical composition from the limits of the nonelectronic medium and the human performer, not to the limits of this most extensive and flexible of media but to those more restrictive, more intricate, far less well understood limits: the perceptual and conceptual capacities of the human auditor. Therefore, although every musical composition justifiably may be regarded as an experiment, the embodiment of hypotheses as to certain specific conditions of musical coherence, any electronically realized composition which employs resources singularly obtainable by electronic means, in addition, will incorporate—in that *Gedankenexperiment* which is the mental act of composition—certain premises that are either severely circumscribed by the limited confirmed knowledge of the nature of these capacities or by isolated facts of musical perception, themselves obtained mainly with the assistance of electronic media, for incorporation into the premises of the particular work. Even the composer who employs the RCA Synthesizer, which most conveniently permits the merging and interaction of the "Gedanken" and the "Actual" experiments by allowing immediate aural test of the prescribed events at each stage of compositional realization, cannot employ the medium fluently and efficiently by so doing without a sacrifice of all but the most local points of continuity and interrelationship. If more securely founded and ambitiously structured electronic composition is not, then, to halt to await those perhaps long delayed investigations which may, in turn, produce adequately general results only in an unforeseeably distant future, it probably must hypothesize in a more traditional manner, by incorporating into its postulates widely tested and confirmed statements regarding the perception of music, derived from successful past experiments, that is, from musical compositions. In this consequential respect, electronically produced compositions can differ among themselves and from non-electronically produced compositions in terms of the extent to which the hypotheses they exemplify already have been widely tested and confirmed, that is, the degree to which they incorporate "traditional" laws into the postulates of the work, and also the degree to which these "traditional" laws are incorporated into the scope of the rule of substitution[1] for descriptive terms, founded either on validated properties of similitude or on hypothesized properties of similitude, so that in the latter case these properties are themselves being tested by the composition, while in the former case of validated properties it is rather the significance of the similitudes with regard to a specific property that is being tested. At the extreme of "nontraditionalism" is the selection of an uninterpreted formal system, no interpreted instances of which have been musically validated, along with coordinative rules which, likewise, have not been validated independently. In such a case, the probability that such an unrestricted choice from such a large number of possibilities at both stages will yield a significant result is extremely small, or the result itself is likely to be virtually trivial, that is, hardly to admit nonverification.

In constructing a musical system for an electronically produced work, whether this system be exemplified in but a single work or a body of works, there is a particular temptation to proceed in this "nontraditional" fashion, since one can presume as the values associated with notationally separable components (the range of discrete values that each component of the musical event may assume) those which are obtainable as the result of the medium's providing measurable and regulable values of frequency, intensity, duration, and spectrum to a degree of differentiation far exceeding the, at least, present discriminative capacity of the auditory apparatus under the most generous temporal conditions, and further providing those values at time points whose precise specifications similarly can produce measurably different quantities which surpass the discriminative and memorative abilities of the most appropriately qualified observers. Surely it is in the domain of temporal control that the electronic

[1] The rule of substitution may be regarded informally as providing the transformation of a validated statement into a statement which is less validated by virtue of the incomplete knowledge of the object designated by the substituted terms or of the relations among the objects designated by the terms, for the purpose of testing the second statement.

medium represents the most striking advance over performance instruments, for such control has implications not only for those events which are normally and primarily termed "rhythmic" but for all other notationally apparently independent areas: speed and flexibility of frequency succession, time rate of change of intensity, and important components of what is perceived in conjunction as tone-color, such as envelope—which is merely the time rate of change of intensity during the growth and decay stages—and deviations of spectrum, frequency, and intensity during the quasi or genuinely steady-state. Indeed, it is this imposition of time control upon timbral components which is, at least partially, responsible for the emphasis, the exaggerated emphasis, on the purely sonic possibilities of the electronic medium; but whereas not even the number of relevant dimensions of tone color are generally known (in the sense of reproducing the dimensionality, not the identical characteristics, of non-electronic timbre) the basis of perceived homogeneity of timbre over an extended registral span—fixed, or limitedly variable input signal subjected to the resonance influence of a fixed formant—is known, and is synthetically verifiable and easily obtained electronically. But the precise placement of time points and their associated durations, though easily and exactly specifiable, takes one into the area of rhythm, which is not only of central concern in contemporary compositional thought, non-electronic as well as electronic, but the most refractory and mysterious perceptually. There are very few useful results available concerning the correlation between specified and perceived duration; even specified identity appears to be not necessarily perceptually invariant with regard to a contextual situation, and those bases of similitude of durational succession inferred from traditional contexts—multiplication of the constituent durational values by a positive constant, usually an integer or its reciprocal—are not of general applicability when the associated pitch succession or pitch contour is altered, or the durational succession is not endowed with obvious cues. With so little information of these types to provide the postulates of a rhythmic system, applicable to nonelectronic music and extrapolable to the electronic realm, it is more fruitful to examine a musical pitch-class system, one which by now can be regarded as "traditional," which incorporates qualitative time properties into its very rule of formation. For, in the extensive discussions which have surrounded the twelve-tone system, be they those which have concerned themselves with inferring or imposing rhythmic schemata, "serial" or non-"serial," from or upon twelve-tone compositions, or those questioning the associative, articulative role of rhythm as a function of characteristics of the pitch structure, there has appeared to be little awareness of or concern with the immanently temporal nature of the twelve-tone pitch-class system. To the end of examining this temporal nature and deriving a quantitative temporal interpretation of the system, I propose to consider a few structural properties of the system which incorporate different modes of dependency upon temporal factors.

A twelve-tone set can be characterized as a collection of twelve different pitch-classes[2] (or, more conveniently, as the integers from 0 to 11 inclusive, denoting these classes) ordered by the relation of temporal precedence (designated $<$) or, equivalently, temporal antecedence (designated $>$). The collection is strict simply ordered with regard to this relation; that is, the relation is asymmetric, transitive, and connected (and, of course, irreflexive)[3] in the collection, and—therefore—is indeed a relation which induces a "serial" ordering. (It should be emphasized that this is the total meaning of the term "serial": it implies nothing with regard to the operations upon such an ordering, or the nature of the elements ordered.) This ordering is the basis for the assignment of order numbers to the pitch class numbers, for these integers of order (0-11 inclusive) are strict simply ordered with regard to the usual interpretation of $<$ as less than, and $>$ as greater than.

At this point, as a means of informally evaluating the temporal constraints imposed by the principle of formation of a twelve-tone set, I shall assume on purely empirical grounds that there are eleven qualitatively significant temporal relationships which can hold between two musical (say, pitch) events. Let x and y designate these events, and let a left parenthesis signify the time point initiation of the event and a right parenthesis signify the time point termination. Then, the eleven relationships are:

1. $x) < (y.$ [that is, the termination of x precedes the initiation of y]
2. $(x < (y;\ \ x) < y);\ \ \text{but } x) \nless (y.$
3. $(x < (y;\ \ x) \nless y);\ \ y) \nless x).$

[2] For terms and notation not fully explained here and later in this article, see my articles: "Twelve-Tone Invariants as Compositional Determinants," *The Musical Quarterly*, April 1960, pp. 246-259 (henceforth abbreviated: TT); "Set Structure as a Compositional Determinant," *Journal of Music Theory*, April 1961, pp. 72-94 (henceforth abbreviated: SS).

[3] A relation R is irreflexive on a collection: if for any element x of the collection, xRx does not hold, i.e. x cannot have the relation (such as temporal precedence) to itself.

R is transitive: if for any elements x, y, z: if xRy and yRz, then xRz.

R is connected: if between any two elements of the collection the relation can be said to hold or not hold.

R is asymmetric: if xRy, then yRx cannot hold.

4. $(x < (y;\ \ y) < x)$.
5. 6. 7. 8. are derived from 1. 2. 3. 4. respectively by substituting x for y, and y for x.
9. $(x \nless (y;\ \ (y \nless (x;\ \ x) \nless y);\ \ y) \nless x)$.
10. $(x \nless (y;\ \ (y \nless (x;\ \ x) < y)$.
11. $(x \nless (y;\ \ (y \nless (x;\ \ y) < x)$.

Given any two pitch classes of a twelve-tone set, only four of the eleven possibilities can be made to hold without violating the meaning of the order numbers.

The collection of ordered pairs: order number, pitch-class number, each of which uniquely determines an element of a set, is a function, in which the order numbers may be considered as defining the domain, and the pitch-class numbers the values of the domains; in this sense, pitch-class is, not inconsequentially, a "function" of a relative time point, designated by the order number. The function is biunique,[4] possessing—therefore—a unique inverse, which defines a twelve-tone set, and the ordered pairs associated with the elements are alternatively to be considered as defining a mapping[5] of the ordered chromatic scale onto itself in terms of either order numbers or pitch-class numbers (the two are equivalent in this, and only this, case). The pitch-class numbers themselves can be regarded as interval numbers, numerically characterizing the interval class by the unique integer arrived at by subtracting (mod. 12) the pitch-class number associated with order number 0 from the pitch-class number of the element in question. This definition of interval is extended to define the directed pitch class distance between any two elements, and when applied to successive elements (those elements whose order numbers differ by one) yields the familiar interval class number succession, which when associated with the appropriate order numbers is also strict simply ordered. Conventionally, the order number associated with the interval class number determined by the larger of the two order numbers associated with the pitch class numbers involved in the determination of the interval will be that interval's order number. This collection of ordered pairs (order number, interval class number) is, similarly a function, but only in the case of an all-interval set is it a biunique function; in all other cases, it is a mapping of the integers 1-11 into, rather than onto, themselves. It is this succession which is preserved under transposition, but before considering the usual twelve-tone operations upon the defined relations, I shall examine

[4] In the sense of a one-to-one relationship between order number and pitch number.
[5] A mapping is a law that associates with any element of a domain an element of another domain, which may be itself, as in this instance.

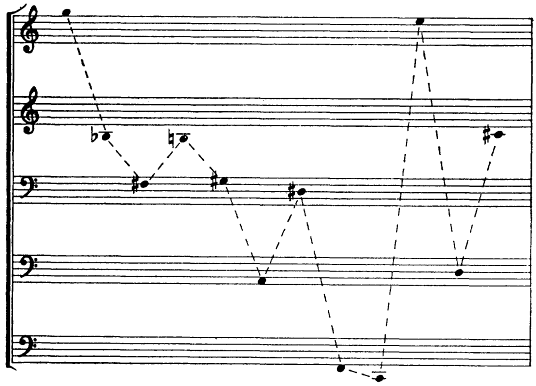
further the structure of the set in terms of temporal precedence and antecedence relations. In strict analogy with pitch-class or interval class interval numbers, one may speak of order interval numbers, defining the directed temporal distance between pitch-classes, or interval classes, in terms of the number of intervening pitch or interval classes. Although the succession of numbers so determined by successive set elements is trivially a succession of 1's for any form of the set, the order interval number associated with a particular pair of pitch-classes or interval classes is significant for a large number of used and useful compositional techniques. Consider first a given set, presented at least twice, on each occasion partitioned (instrumentally or registrally or dynamically or etc.) on the principle of identical extracted interval sequences. Total pitch order is preserved, and the succession in each part preserves ordered interval content, but the durational rhythm in each part is not, in general, preserved or preservable. The very fact and the nature of the number of different ways of such a presentation are entirely determined by the ordering of the particular set.

These are (3^2) (2^3) partitions of a set,[6] with the constituent parts represented linearly. The case of such an ordering which preserves the order interval numbers of each extracted part is obtainable only when the set has been constructed originally through the identification of order intervals with pitch-class intervals, not merely those determined by successive pitch-class elements. The rhythmic implications of such interval associative procedures are as obvious as is their dependence upon the order number interval fulfilling the role of a metric.

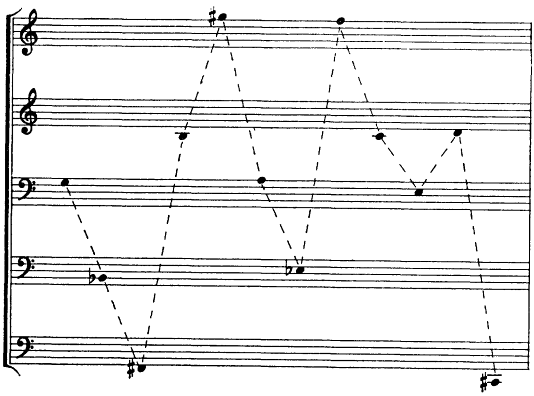
Invariants under transposition similarly involve temporal order criteria. While pitch interval succession is preserved, and no order number, pitch number couple can be, one of the most general, and least obvious of such invariants is the equal number of order inversions associated with transposed sets whose transposition numbers are complementary. This measure of the derangement of the order numbers of sets in relation to a reference set also serves as a reminder that, for any given set, a transposition of that set can be represented equivalently as a permutation of pitch-class numbers or order numbers, for indeed a permutation, in this context, is an operation on relative temporal positions.

The identification of different transpositional forms of a set through identity of extracted pitch sequences creates, as in the related case

[6] Two 3-element and three 2-element partitions.

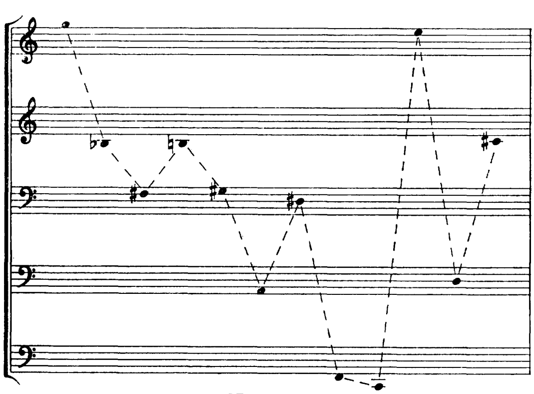


Ex. 1a



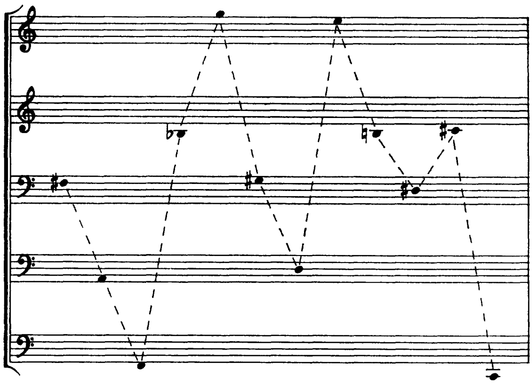
Ex. 1b

of interval sequences, a rhythmic situation, with the temporal attributes of the set determining the precise nature of such identities.



Ex. 2a

is the same partitioning as Ex. 1a, with



Ex. 2b

the transposition at 11, identically partitioned with regard to pitch sequences.

Somewhat different in nature, but familiarly consequential, is the temporal role of the combinational identification of a collection within a set with a content identical collection in another transposition of the set. This property can be considered either by observing those collections whose content is held fixed by a transposition number (that is, as a general invariant) or by considering the specific set structure required to enable such collections to remain fixed in pre-assigned order positions. For a simple instance, any collection of four pitch-classes containing two tritones must map into itself under transposition by 6. Thus (Ex. 3a), the pitch-class collections repre-

Ex. 3a

sented in each set form by order numbers 0, 2, 8, 11, are identical while the temporal permutation of the four elements within the collection—of 0, 2, 8, 11 into 11, 8, 2, 0—can be represented cyclically: (1 4) (2 3); similarly the collections represented by order numbers 1, 3, 7, 9 and 4, 5, 6, 10. The similarity of permutations within the collections can be seen to be a result of set structure, and is not a general invariant.

The order interval number associated with any given pitch dyad of a set undergoes, under transposition, an alteration—or non-alteration—of value depending upon the specific pitch ordering of the set. Thus, in the set of Ex. 1a, the undirected order interval number associated with the pitch dyad G-C♯ for each transposition number is: $t_0 = 11$, $t_1 = 6$, $t_2 = 4$, $t_3 = 8$, $t_4 = 1$, $t_5 = 6$, $t_6 = 11$, $t_7 = 6$, $t_8 = 4$, etc. The temporal aspects of such properties of set structure, particularly in their more complex, but easily inferred, extensions are as manifold as they are inescapable, but since the desire here is to examine selected, representative manifestations of a rhythmic nature rather than to attempt to exhaust the subject, I turn to those, in some respects strongly dissimilar, temporal factors involved in some of the primary invariants associated with the operation of inversion.

The fundamental interdependence of temporal order and inversion can be inferred immediately from the possibility of defining the inversion of a linear dyad in terms of interchange of order numbers; any pitch dyad can be mapped into itself under inversion and transposition by the interval determined by the dyad in its original order. Stated in another way, the intervallic result of reversing the order of pitch-class elements is complementation, which is of course the intervallic result of inversion.

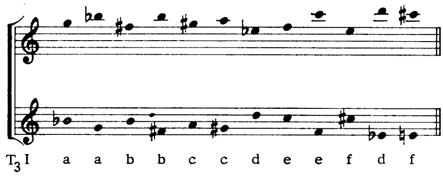
The most familiar invariant under inversion, made so by its constant and varied application by Webern, is that which necessarily obtains under transposition by an even transposition number: the retention of those, and only those, order number, pitch number couples of the S which are a tritone apart and whose pitch numbers are equal to one half of the transposition number (TT, p. 254). The generalization of this criterion provides a means of defining 12 equivalence classes[7] of 12 inversionally related set pairs each, among the 144 so related pairs; each pair within a so-determined class establishes a succession of pitch dyads consisting of pitch-classes of the same order number, which is a permutation of the similarly arrived at succession of each of the other pairs of the class. Obviously, such a given pitch dyad appears in one and only one equivalence class, so that such a class is uniquely identified by a single so-constructed pitch dyad. The 12 pairs of sets of such a class are closely analogous to those 12 pairs of transposed related sets which produce a succession of dyadically determined interval numbers, all of which are equal; in the present case, the sums of the pitch numbers of each pair are equal. This unique (for each equivalence class) number is called the "index" of the equivalence class.

[7] An equivalence relation R is a relation that is reflexive, transitive, symmetric, and connected.

In contradistinction is that partitioning into equivalence classes of the 144 pairs with regard to the 12 total transpositions of a set pair which preserve the succession of intervals determined by pitch-classes of the same order number, or, perhaps more simply, the coupling of a fixed, so computed interval number with a duplicated order number. Exactly two pairs from each of six such equivalence classes here belong to the same equivalence class formed according to the previous criterion.

All such 144 pairs reveal the same *pattern* of intervallic repetition, that is, the order intervals determined by repeated intervals are invariant, since this property is determined by the tritone related pitch-classes in S. If we let the first six letters of the alphabet denote the six intervals which can occur (even, when the transposition interval between the components of the pair is even; odd, when it is odd), then the pattern of repetition of the set of Ex. 3a is: a, b, c, d, e, f, f, d, c, b, e, a. The order interval 11, for instance, is associated with a different pitch interval for each set pair in an equivalence class determined by the first criterion above, that dependent on the sum of the pitch-numbers, and—naturally—is associated with the same pitch interval for each set pair in an equivalence class determined by the second criterion, that dependent on total transposition. But the pattern of pitch dyad repetition, and, therefore, the associated order intervals, are determined not by the structure of the set, but by the interval between pitch numbers of the same order number. For the set of Ex. 3, the pattern of identical dyad repetition for transposition interval 3 (as determined by elements of order number 0) is: a, a, b, b, c, c, d, e, e, f, d, f; for interval 9 it is: a, b, c, d, e, f, e, c, d, a, f, b (Ex. 3b and 3c). The pattern is identical for pairs belonging to the same equivalence class in the second sense (although the actual pitch content of these intervals is different in each case); the pattern is the same for exactly two pairs in each equivalence class in the first sense.

Closely related to these characteristics which impose rhythmic patterns of repetition with associated qualitative values of duration on pairs (and, by simple extension, to any number) of inversionally, and—therefore—transpositionally related sets, is a property which is difficult to characterize informally in its most general application. (See TT, pp. 256-257.) However, both for purposes of later discussion and for its significance as a temporal aspect of the twelve-tone system an instance of this property must be displayed.

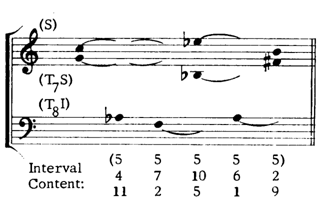


Ex. 3a

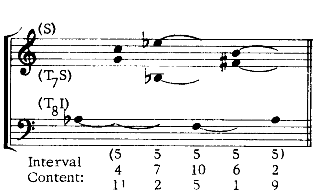
Ex. 3b

Ex. 4b is a duplication of the succession of three-part simultaneities of Ex. 4a, although the temporal relation of the transpositionally related dyads and the inversionally related single line has been exactly interchanged; this is an instance of intervallic structure invariance under a prescribed alteration of the temporal order and a consequent alteration of the pitch content of the successive simultaneities. The property generalizes to any number of inversionally related components, with any number of transpositionally related lines constituting a component, and—of course—to any total transposition.

The identification of inversionally related sets through extracted interval or pitch sequences is again dependent on the temporal structure of the set and the statement of conditions for such a representation in a pre-defined number of parts with a specified number of



Ex. 4a



Ex. 4b

elements in each part must incorporate a condition upon the order relations of S.

That the representative properties that have been and shall be discussed are not to be construed as compositional imperatives or prescriptions to the end of securing temporal characteristics from the properties of the twelve-tone system, but as temporal attributes which inhere in the system and must, therefore, be manifest compositionally, is most apparent in the next case: the interval succession of retrograde-inverted related sets is identical to within complete order reversal. I shall forego further discussion of this property beyond the indication that the succession of simultaneously formed intervals determined by RI related sets symmetrically ordered is intervallically symmetrical about the midinterval (or intervals). The following is a characteristic pair:

Ex. 5

The means of formation of equivalence classes of pairs by the application of this principle is obvious.

As a final observation, in order to indicate the dependence of a concept which is not normally regarded as temporal in character upon temporal considerations, I shall use the concept of combinatoriality. (See SS, pp. 74ff.) Whatever the number of set forms contained in an aggregate, and whatever the number of pitch elements each form contributes to the aggregate, qualitative temporal constraints are necessarily involved. In the simple case of hexachordal inversional combinatoriality, so often encountered in Schönberg's music, the formation of an aggregate by two inversionally related hexachords requires the statement that the pitch-class with order number 6 in one hexachord may not be stated until after the statement of the pitch-class with order number 5 in the other hexachord, and vice versa; this is a necessary condition for such a hexachordal construction.

The license of simultaneous statement of pitch-classes whose order interval number is 1, which has been stated verbally and employed compositionally is—most strikingly—a temporal condition. Such a statement of a set can be regarded as a strict partial ordering (a serial ordering minus the property of connectivity) with regard to the relation of $<$, or a simple ordering with regard to $\leqq$ (the equals sign here denoting simultaneity). The admission of this possibility increases from four to seven the number of temporal relationships which can obtain between two events, from the total of 11 listed above, and allows a twelve-tone set to be stated totally in terms of any number of order numbers ranging integrally from 0 to 11 inclusive. With relation to the serial ordering of the set, such a procedure can be regarded as a mapping of the order numbers into themselves, subject to the condition that if order number m maps into order number n (with $n < m$) then order number $m + 1$ maps into n or $n + 1$. The absence of any constraint on spatial ordering makes possible, as in the related case of the spatial distribution of a set or aggregate in any time distribution, the representation—spatially—of any set by any other set; this overidentification suggests that a serious study of this question is crucially necessary. (David Lewin's article elsewhere in this periodical does present a solution to one aspect of this problem, and suggests paths for future investigation.—*Ed.*)

The construction of a quantitative temporal system by interpreting pitch numbers as temporal values, since order numbers themselves are "ordinal" temporal values, and thus constructing a "twelve-tone rhythmic system" can be viewed either as a reinterpretation of pitch numbers so as to assure isomorphism between the two systems, or as assigning temporal interpretations to the uninterpreted terms of the finite numerical equal difference structure of which both the pitch and rhythmic systems will be exemplifications. It seems reasonable to require, in the light of the preceding discussion, that such an interpretation satisfy a number of general conditions. It must not reduce the possibilities or range of applicability of such qualitative temporal characteristics as those discussed above; it should provide only a substitution for the relation of precedence and antecedence of a relation of measured precedence and antecedence. It must interpret the entire extensional meaning of pitch-class numbers and those concepts which are formulated in terms of pitch-class numbers. It must provide for such concepts being endowed with an interpretation tenable in terms of musical perception, so that the system so constructed will be autonomously closed, not merely by formal analogy with the pitch-classes, so that the totality of, at most, 48 temporally founded sets which can be formed from a given set will be justifiably separable from the 12! permutations of the temporal equivalents of pitch-class numbers, and so that the invariants associated with the transformations of the pitch system will have independent analogs in the temporal system.

Manifestly, the interpretation of set numbers as multiples of a durational unit does not satisfy these minimal conditions, however acceptable duration may appear as the primitive constituent of a description of temporal perception. There is no apparent basis for constructing duration classes by designating as elements of the same durational equivalence class those durations which differ by a multiple of 12 or any other number. The temporal analog of pitch interval is translatable only as "the difference between durations." Even without arguing the dubious perceptual status of this notion, the ordered succession of such differences remains invariant under transposition if and only if one assumes difference classes as a result of applying transposition modularly, and therefore embracing the assumption of duration classes in its most unrealistic form, so that the succession of, say, a quarter-note duration followed by a dotted quarter-note duration creates an "interval" equal to that created by a dotted half-note followed by a sixteenth. Naturally, the same result is obtained whether one ostensibly avoids this dilemma by interpreting a given transposition as a permutation of order numbers, or faces it fully by modularly adding to each duration a duration equal to the unit duration multiplied by the transposition number. (This is precisely what has been done compositionally and stated verbally by those who most vehemently and precipitously have since renounced the twelve-tone system as "old-fashioned serialism" and "exhausted." They have revealed significantly their profound comprehension of the nature of the system by "inventing" the notion of "double series" to accomplish what has just been described. The "two" series are totally equivalent representations of a set, one in terms of pitch numbers, the other in terms of order numbers.)

This unsatisfactory analog of interval, in all of the ramifications derivable from the earlier discussion of invariants, should suffice to close the discussion of this interpretation. But a few other deficiencies perhaps should be noted briefly. The analogy of pitch properties dependent upon correlation of equal order numbers of two (or more) set forms cannot, in general, be fulfilled under this interpretation. Combinatorially related durational set forms must depend upon equality of the sum of durations of the constituent set segments, and therefore combinatoriality, almost contradictorily, does not hold in general under total transposition of the component set elements.

The apparent insistence upon the necessity of the temporal interpretation translating completely the attributes of the pitch system into temporal terms must not be regarded as an insistence upon complete analogy for analogy's sake or as an unawareness of the differences between temporal and pitch elements. On the contrary, it is to justify the construction of a system which will impose constraints upon the temporal elements of a composition with a comparable effect upon the nature and extent of the inter-event influence in the rhythmic domain to that of the pitch system in its domain. As a system, it should possess unique properties independent of pitch association, as the pitch system possesses properties independent of quantitative temporal values.

To this end, since duration is a measure of distance between time points, as interval is a measure of distance between pitch points, we begin by interpreting interval as duration. Then, pitch number is interpretable as the point of initiation of a temporal event, that is, as a time-point number. If this number is to be further interpretable as a representative of an equivalence class of time points and the durational interval with regard to the first such element, it is necessary merely to imbed it in a metrical unit, a measure in the usual musical metrical sense, so that a recurrence of succession of time points is achieved, while the notion of meter is made an essential part of the systematic structure. The equivalence relation is statable as "occurring at the same time point with relation to the measure." The "ascending" ordered "chromatic scale" of twelve time points, then, is a measure divided into twelve equally spaced time points, with the metrical signature probably determined by the internal structure of the time-point set, and with the measure now corresponding in function to the octave in the pitch-class system. A time-point set, then, is a serial ordering of time points with regard to $<$. At the outset, I do not wish to attempt to avoid the manifest differences between the elements of the pitch system and those of the time-point system, that is, perceptual—not formal—differences. A pitch representative of a pitch-class system is identifiable in isolation; a time-point representative cannot conceivably be, by its purely dispositional character. But an examination of a time-point set will clarify the systematic meanings, and the reasonable musical meanings associated with these new concepts.

Ex. 6

Ex. 6 is a time-point set analog of the pitch set of Ex. 1, whose numerical representation as number couples is indicated. The metrical signature is chosen in the light of the hexachordal combinatorial structure of the set. Since duration is simply the directed distance between time points, the notated durations are not obligatorily the "actual" durations of the event, be it represented in terms of pitch or register or timbre or dynamics, etc., initiated at a time point; the notated duration, under this interpretation, may represent an actual duration followed by a rest to complete the duration between time points.

Ex. 6 is but one possible representation of this set, the unique representation in the minimal total temporal duration. Obviously, the minimal number of measures required for the statement of a given time-point set is determined by the non-modular sum of the intervals divided by 12, and is equal to the number of octaves required for a statement of the analogous pitch set stated as an ordered simultaneity. (The minimal statement of a pitch set as a succession is trivially the same for all sets, the interval of 11.)

In strict conformity with the present interpretation, the initial three measures of the set are presented in two different ways in the following examples:

Ex. 7a

Ex. 7b

Both preserve the order of time points of Ex. 6. The first durational interval of Ex. 6 is 3 (interval units; the unit here being the duration of $\frac{1}{12}$ of the measure); of Ex. 7a it is 15, which equals 3 (mod. 12). A durational interval, then, represents a class of intervals equivalent to within an integral number of measures difference, and there can be no unique maximal statement of a time-point set. In Ex. 7b the first time point is repeated before the statement of the second time point, thus creating a first interval equal to the measure (this resource of "octave" statement can serve compositionally to present the meter of reference employed). All three representations must be regarded as "all-interval"; no durational class other than the 0 class is repeated.

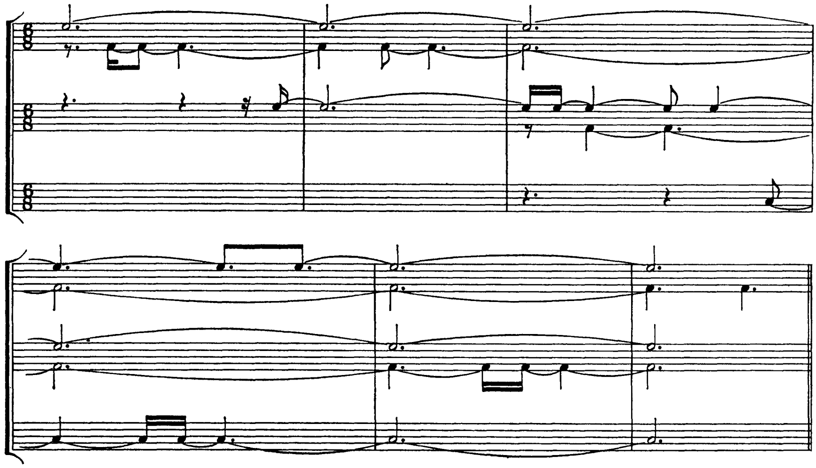
The differences between Exx. 7a and 7b suggest the necessity for examining the nature of repetition in a time-point set, and it may be easily—too easily—assumed that the repeated time points of Ex. 7b derive their "justification" from the principle and practice of permitted repetitions of a single pitch in the pitch system. However, it should be observed that pitch repetition is not a pitch procedure, but a temporal procedure, independent of considerations of the pitch system, and, if a time-point system is assumed, the temporal placements of such pitch repetitions are determined by the time-point structure, not by pitch considerations. Therefore, the repeated time points of Ex. 7b must not be regarded as analogous with pitch repetitions; only real time duplication of time points (simultaneous statements of the same time point) is analogous, for the absolute interval between the first time point of Ex. 7b and the third is 15, between the second and third, 3. Pitch repetition does not alter the absolute pitch interval between the pitches repeated and the eventual successive pitch. The repetitions of Ex. 7b are analogous to the representation of a pitch class by different "registral" members of the class. It should be recalled that the concept of register in the twelve-tone pitch system with regard to all available pitches, founded merely on the assumption that no two non-identical elements of the same pitch-class can be regarded as in the same register, and that transposition which preserves absolute intervals must be regarded as preserving the registral relations among all the constituent pitch elements, is characterizable as irreflexive, symmetric, nontransitive, and not connected, with regard to the relation of "is not a member of the same register as"; so, too, is time-point "register" in the above sense.

The time-point analogs of Ex. 1a and 1b (Exx. 8a and 8b) indicate not only the results of interval (here, durational) sequence extraction but, necessarily, the meaning of "transposition" of time-point sets. Each transposition preserves the duration class succession, while effecting a particular permutation of the twelve time-point classes, and may be thought of as a translation of each time point a number of time-point units (sixteenths in these examples) to the right (i.e. as notated) equal to the transposition number. The result is metric reorientation of the set.

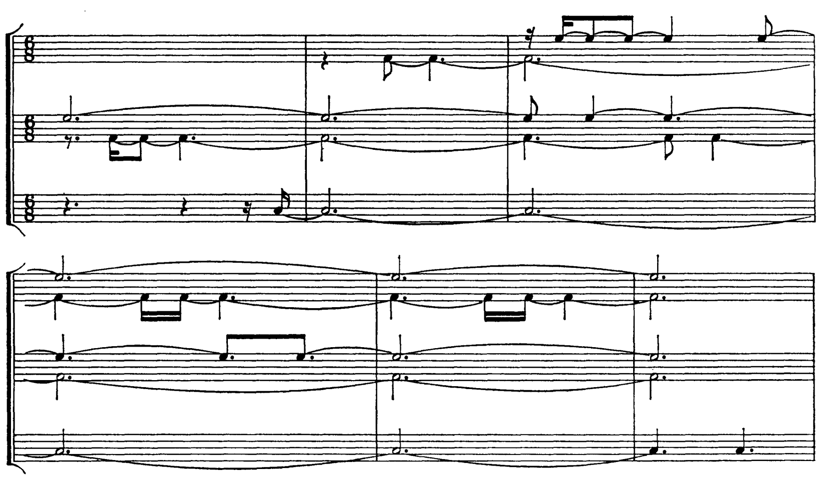
Ex. 9 is a transposition of the set of Ex. 6 with $t = 6$; the properties revealed in Ex. 3 are here evident. It follows that the number of order inversions of time points for complementary transpositions is equal, and that—indeed—all the properties of pitch transposition are translated into equally apparent properties of rhythmic transposition.

The inversion of a time-point set maps durations into their complements, mod. the measure. To verify that a time-point dyad maps into itself with interchange of order number, under inversion, the first two time points of Ex. 6 are displayed with the set inverted, and $t = 3$, in Ex. 10.

The preservation of order number, time-point number couples



Ex. 8a

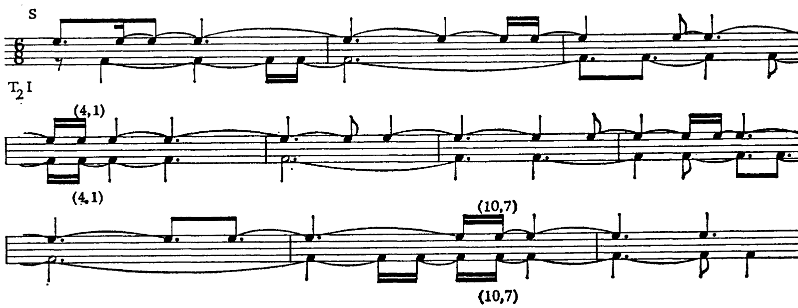


Ex. 8b

Ex. 9

Ex. 10

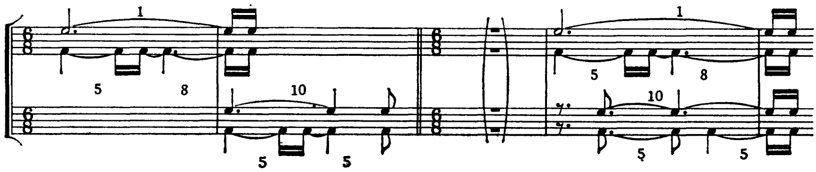
under inversion and an even transposition number, in this case, t = 2, is exhibited:



Ex. 11

The duplicated time-point numbers are, again necessarily, 1 and 7. Ex. 12 displays the resultant rhythm created by these two set forms. It indicates that such a rhythm created by set pairs belonging to the same equivalence class of pairs, under the criterion of equal sums associated with the time-point numbers of the same order number, will be permutations of the durations formed by disjunct time point dyads (beginning with the first); the resultant rhythms created by set pairs which are members of the equivalence class determined by total transposition are simply metrical displacements of one another. In Ex. 12 the succession of durations is to be regarded as: 2, 8, 4, 6, 0, 10, 10, 6, 4, 8, 0, 2. If the first time-point number of the set were—say—4(T_4S), and thus the associated inversion's first time-point number were 10(T_{10}I), the succession would be: 6, 0, 8, 10, 4, 2, 2, 10, 8, 0, 4, 6. This demonstrates also that the distribution of equal durations in the resultant rhythm depends upon the distribution of time points in the set which are related by the time-point interval (duration) 6, while the specific durational values associated with these equal pairs are determined by the transpositional relationship between the two sets; since the distribution of complementary durations bounded by the same time points is determined by the transposition number, this distribution is the same for pairs belonging to the same equivalence class by total transposition.

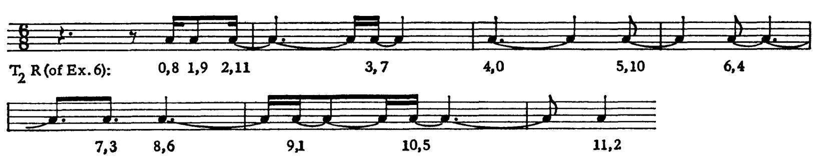
The further exemplification in time-point and durational terms of inversional invariants, and the quantification of qualitative order properties so arrived at may seem superfluous in an introductory discussion such as this. But the temporal analog of Exx. 4a and 4b may not be immediately obvious, since the property involves invariance under temporal alteration; that the resultant durational successions associated with such presentations can be identical is demonstrated in the following example,



Ex. 13a Ex. 13b

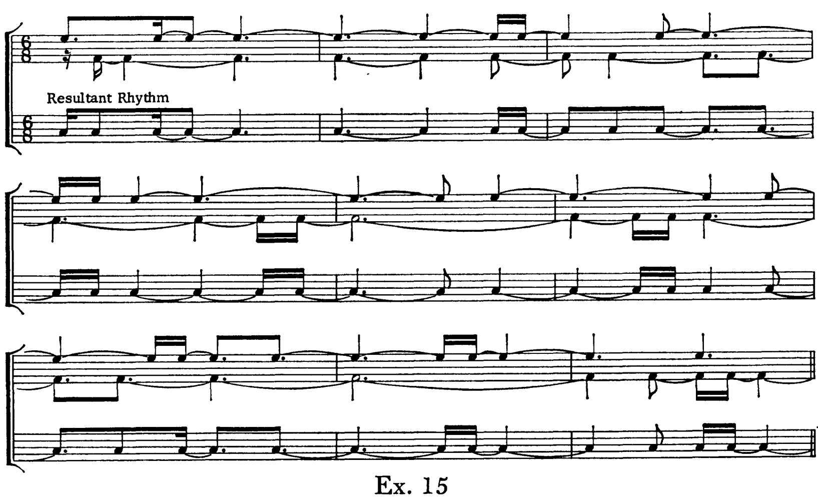
where the first two "interval" complexes of the succession are displayed in linear redistributions, with durationally equivalent components stated side by side. The complements of the intervals of Exx. 4a and 4b are displayed here, for the sake of simplicity.

The following example displays a retrograde form of the set of Ex. 6, with t = 2:



Ex. 14

The lower line of the following example

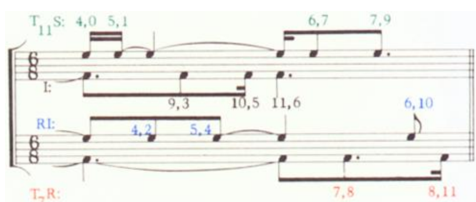


Ex. 15

displays a retrograde-inversional form, with t = 7. Both forms present the durations of their respective RI related sets in reverse order. The upper line of Ex. 15 is Ex. 6 repeated (mod. the measure), and the

S: 0,0 1,3 2,11 3,4
Duration Numbers:
T_2I: 0,2 1,11 2,3 3,10
2 8 4 6
4,1 5,2 6,8 7,10 8,5
0 10 5,0 10 6,6 6 7,4 4 8,9
9,9 10,7 11,6
8 9,5 0 10,7 2 11,8

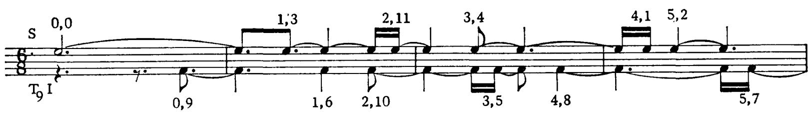
Ex. 12



T_{11}S: 4,0 5,1 6,7 7,9
I:
9,3 10,5 11,6
RI:
4,2 5,4 6,10
T_7R:
7,8 8,11

Ex. 19

two RI related lines display, in their resultant rhythm, a symmetry of intervallic occurrences such as that discussed with regard to Ex. 5. It now should be apparent that the "twelve time-point class system" is structurally isomorphic with the twelve-tone pitch-class system. It can be regarded as an interpretation of that permutation group of order 48 at most, where the group elements are the permutations of the twelve time-point classes specified by the transformation of the system, or by the associated permutation matrices, with the group operation, matrix multiplication. Therefore, such special characteristics of set structure as those permitting, say, the "combinational" concept of combinatoriality, in time-point terms, simply involves the construction of time-point aggregates by the components of appropriately related set forms. The structural characteristics of the set that assures such resources are directly translatable from the pitch domain. The first hexachord of Ex. 6 and the first hexachord of the combinatorially related inversion are so translated:



Ex. 16

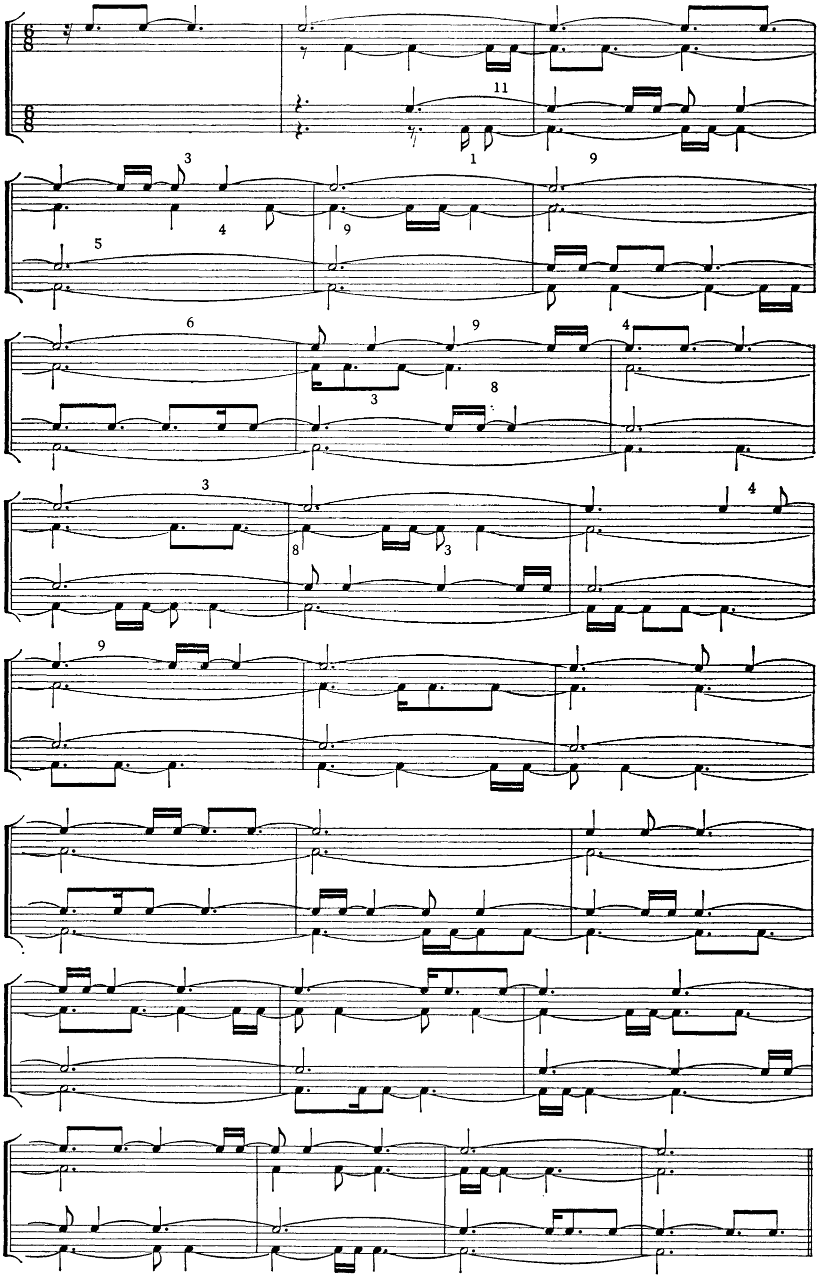
Each time point occurs once and only once; beyond this contrapuntal condition, the time-point aggregate—as the pitch aggregate—has no unique representation. Indeed, even a minimal representation is not, in general, unique. For example, in Ex. 16, the inversional component requires four measures for its minimal representation, while the set component requires only three; therefore this latter component can be presented in a number of different ways within this totally minimal representation. As in the analogous pitch situation, the time point 0 can be re-presented at the beginning of measure two without altering the order structure of the component; then the aggregate, if presented linearly, as a resultant, contains a non-immediate repetition of time points. This is but an instance of the extensive compositional issue arising from the circumstance that there are an unlimited number of representations of an aggregate, both in the time-point and pitch domains. If constituent pitch set segments are undifferentiated compositionally—presented, for example, as a single line in a single instrument without registral differentiation—then the set origins of the aggregate are made ambiguous to the point of virtual undetectability. In compositional practice, the constituents normally are differentiated timbrally and/or registrally and/or dynamically and/or rhythmically;

similar methods clearly are required in the case of time-point aggregates.

The extension of time-point combinatoriality to all types and orders, to partitions of the aggregate into more than two equal parts, unequal parts, as well as to incomplete aggregates or weighted aggregates is immediate (SS, pp. 80-83). The basis of compositional decisions as to the length of a time-point aggregate—which may itself function as a large scale rhythmic unit, the exact distribution of time-point components within an aggregate, and the temporal progression of such aggregates is beyond the scope of this article, but I conclude this part of the discussion with Ex. 17, in which each part consists of two derived sets,[8] creating secondary sets (SS, p. 86), the upper two parts derived, through the operations of the system, from the first three time-point classes of Ex. 6, the bottom two from the succeeding three classes. Each of these pairs of lines is a rhythmic canon by inversion; the total rhythmic progression is in disjunct aggregates. Other properties, clearly revealed in the resultant rhythm and familiarly encountered in the pitch domain, will be apparent to the experienced observer. One property, however, should be mentioned, since it is a particularly significant temporal equivalent of a characteristic of inversionally related sets (SS, p. 91). For each of the inversionally related lines, the set numbers of elements of the same order number sum to 3; therefore any elements of the reference set (Ex. 6) whose set numbers sum to 3 will appear here as elements of the same order number. Thus, for example, the time-point succession associated with order numbers 5 and 6 in Ex. 6 appears here as a resultant rhythm formed by the upper two voices in measure eight, followed by the time points associated with order numbers 4 and 3, followed by those associated with order numbers 1 and 2. Many of the techniques of delinearization of a linear rhythm (the distribution of time points associated with a single set form among two or more linear representations of that form), and linearization of a resultant rhythm reside in this property, which—in turn—is but another facet of the inversional equivalence class property of the first kind mentioned above (p. 67); the modes of applicability for a given composition depend wholly on the structure of the compositional set.

It must not be inferred that this time-point system merely because it is equivalent to the twelve-tone pitch system, and for purposes of explanatory simplicity has been described by analogical reference to

[8] For an explanation of derived sets in the pitch sense, see my article: "Some Aspects of Twelve-Tone Composition." *The Score and I.M.A. Magazine*, June 1955, p. 59.

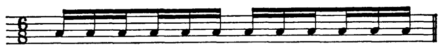


Ex. 17

the pitch system, implies a one-to-one compositional application of the two systems. The rhythmic system is closed, and as its structure is independent of pitch clarification, it can be applied as independently as the pitch system. Thus, a time point of a set can represent the point of initiation of a single pitch, the repetition of a pitch, or a pitch simultaneity, but it can fulfill also this function with regard to timbre, register, dynamic level, etc. Indeed, it is the polyphonic structure, not the simple coordination, between the pitch system and the time-point system that the formulation of this latter makes most valuable, and the structured rhythmic counterpoint of these dimensions is a question of compositional applications, and is a subject for, at least, another article. The brief, incomplete exposition of the system as here presented is merely to suggest a traditional premise for a temporal approach to the electronic domain.

It might be asserted that, although the principles of formation and transformation of the time-point system could have been suggested entirely by the appropriate formal system and adopted by virtue of the properties which maintain under this interpretation, the assumption of "twelve" time points is an arbitrary derivative of the pitch system. Obviously, the time-point system is applicable to any number of set elements, and has been applied compositionally to a smaller number; the pitch system did suggest the number twelve. But having suggested it, it is a suggestion well worth adopting independently, for many of the resources of the system (the time point, as well as the pitch) arise from the properties of the number 12, particularly the property of integral factorization by so large a number of integers, represented by the totient of 12 being equal to 4 (1, 5, 7, 11). Nor is it surprising or irrelevant that the compositions which apply this temporal system employ, as the time-point set, the exact analog of the pitch set of the composition; one might say, with equal justice, that the pitch set is the analog of the time-point set. So, such a composition is the point of conjunction and presentation of the two independently coherent yet deeply related structures.

The temporal constraints imposed by the rhythmic system, the degree and extent of the inter-event influence so determined, depend upon—at least—two contextual considerations: the particular temporal phenomenon desired, and the structure of the specific time-point set. With regard to the paradigm of the preceding examples, the composer might desire, and could achieve, any one of the 2^{11} possible compositional representations of the twelve time-point measure as a resultant rhythm, but the means of arriving at and departing from a predefined measure depend upon set structure. Consider that simplest of measure representations:



Ex. 18

which, nevertheless, was not arrived at by any of the combinations of Ex. 17. If the set of the work were the time-point "ascending chromatic scale," this measure would be easily available, and would impose no conditions on approach and departure by virtue of set structure. But if this measure were to be arrived at from the set of Ex. 6, then the simultaneous statement of a number of set forms is required.

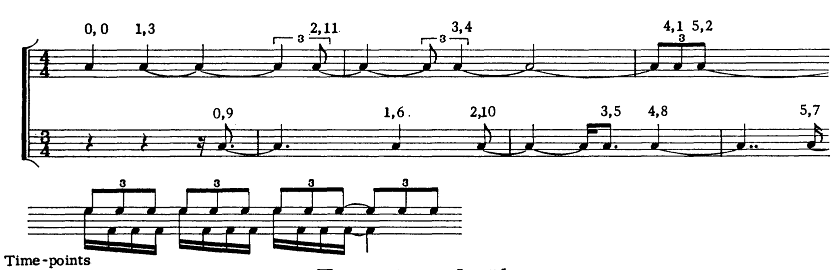
Ex. 19 is one such presentation; the linear components, reading from top to bottom, are the fifth, sixth, seventh, and eighth elements of $T_{11}S$, the tenth, eleventh, and twelfth elements of I, the fifth, sixth, and seventh elements of RI, and the—say—eighth and ninth elements of T_7R. This presentation then imposes specific conditions on the time points preceding and following this aggregate. (The further implications for the total rhythmic structure of this conjunction of sets are well worth considering, for all that they cannot be discussed here.) In short, any rhythmic configuration is "possible," but any such state of the composition must influence, to a greater or lesser extent, other states of the composition. The unavoidable inference that not everything is possible independently at every state of the composition is merely to observe that the system is not constructed to induce, in a relatively strict sense of the word, "randomness": the absence of inter-event influence.

But, it may be asked, how can "any" possible rhythmic event be made to occur in a system which assumes a minimum duration between successive time points, and admits no durations other than those which are integral multiples of this unit duration? I shall combine the answer to this question with the answer to another: what does this rhythmic system have to do with the electronic medium, particularly since it has been employed in, and is—therefore—applicable to, nonelectronic works? Clearly, the system crucially depends upon the maintenance of an isochronous durational unit and its multiple, the measure, the modular unit. To secure this, with nonelectronic media, is not only to court the terrifying and cumulative hazards associated with the presentation of ensemble rhythms of any complexity, but to be obliged to assume a quite coarse quantization of the temporal continuum. But, with the electronic medium, the maintenance of the isochronous unit is assured mechanically, and the accuracy of the ensemble rhythm is obtainable to any degree of exactness; in addition, the fineness of quantization available answers the first question above. To be sure, the examples so far presented do not contain, and could not have contained, triplet, quintuplet, and similar subdivisions in the usual sense; but such notational means are required only when the practical exigencies of rhythmic notation and tempo indication prevent the rhythmic structure from being notated in terms of a least common multiple durational unit. But, if it be assumed that each 16th note duration in the above examples represents a time duration of $1/32$ of a second, a common unit in Synthesizer programming, then the tempo of the examples would be: ♩. = 320. If the resultant of the combination of Ex. 16 is notated in terms of the reasonable tempo, ♩. = 80, the result would be:

Ex. 20

More extended answers to the two questions scarcely seem demanded if one requires merely the assurance that satisfactory answers exist.

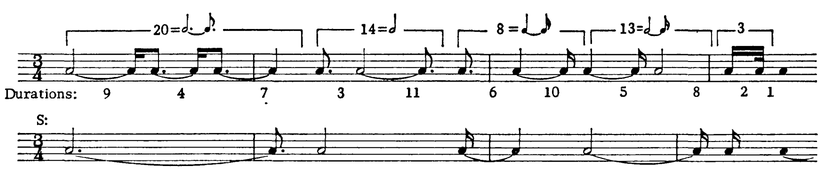
Nevertheless, it is possible to answer the first question by showing another technique within the system. One of the fundamental empirical differences between the pitch and time-point systems is that the "octave" of the time-point system is determined only contextually, by metric signature and tempo indication; therefore, without altering the meaning of "octave" in this systematic sense, the two set components of Ex. 16 can be represented in their unique minimal form by equating, as total durations, three measures of the S component and four measures of the T_9I component:



Exx. 21a and 21b

Here, however, there is no aggregate construction; if the concept of aggregate is applicable at all, it must be in terms of the twenty different time-point values available in the resultant measure (Ex. 21b). Also, different time-point values in the components become identical values in the resultant. Although this technique does overcome some of the difficulties associated with the quantization minimum of nonelectronic music, its most fruitful application is the achieving of different modular units in association with different interpretations of the rhythmic structure, and where—therefore—aggregate structure has no particular relevance.

The use of the aggregate as the unit of temporal progression (in a sense similar to that in which Schönberg employed the pitch aggregate) makes the single, total set representation a constituent of a multiple of such aggregate units as in Ex. 17, with the multiple dependent upon the number of parts in the partitioning of the aggregate complex. This suggests that the compositional time-point set need not (in this case cannot) appear as the explicit, foreground rhythm. The determination of still another level of foreground, derived directly from the unique characteristics of the set, by the imbedding of new time points through the subdivision of the set-determined durations, is demonstrated:



Ex. 22

The first temporal hexachord of Ex. 6 (now displayed in a 3/4 meter) is here subdivided into eleven durations, which represent the durational succession of the inversion. The first three of these durations are placed between the first two time points of S, and each successive pair of durations occurs between the successive time-point pairs of S. The effect is that of changing the modular unit with each successive time point of S, with the critical requirement being that no new, so arrived at time point occupy a temporal position corresponding to that of the fundamental division of the measure, here, 16th-note durations. It is this avoidance of ambiguity that creates the appearance of complexity in conventional notation, but for all this forbidding appearance it is easily recognized that any such imbedded succession is merely a "diminution" or "augmentation" of a segment of S, and should be readily perceptible as such when associated with identities or similarities in other dimensions. The avoidance of auditory confusion of such derived time points with time points of forms of S is a matter of compositional clarification, solvable usually by the availability of superposed comparison of the two levels of time points (like a length comparison by superposition), accomplished by assigning different timbral or registral lines to the different levels of time points; the limits of such differentiation are then at the limits of discrimination between attacks (or, in general, initiations), making them therefore dependent upon such phenomena as envelope characteristics, absolute and relative frequencies, spectra, and dynamics. These limits, then, need not be determined by essentially memorative considerations.

If even so cursory a discussion of temporal levels suggests unavoidably the need for electronic realization, and a host of questions regarding temporal perception, it also suggests—perhaps less obviously—a brief excursion into the domain of frequency. This excursion is prompted by (and it shall not be allowed to exceed the immediate implications of this prompting) the fact that the examination of temporal systematization began with and originated from the traditional principles of the twelve-tone pitch-class system; the results of this examination return us to the area of pitch by a comparable mode of reasoning by analogy. Certainly, the electronic availability of the frequency continuum does not entail the imperative that this continuum be totally employed, any more than does the fact of the similar availability on the violin. But, likewise, it does not entail the consequence that it not be employed, particularly since new selections from this continuum need not be derived from independent premises, but from the attributes of traditional systems, or extensions of these systems. A combinational system (such as the traditional tonal system), founded on the selection of an unordered sub-collection from the total pitch-class collection, but with a *prime* number of equal parts to an octave, possesses the possibility of generating the complete collection of pitch classes by any non-zero interval, whereas this collection, in the familiar twelve-part division, can be generated only by intervals 1, 11, 5, and 7: numbers prime to twelve. Every such interval, then, will generate a maximal sub-collection (the number of pitch-classes in this sub-collection will be $n/2 + 1/2$, for primes greater than 2, where n is the total number of pitch classes) in which each non-complementary interval occurs with unique multiplicity; thus, a "circle" generated by each interval is obtainable, defining a unique hierarchy among the transpositions of the so generated sub-collections (transposed by the generating interval), a hierarchy founded on the traditional criterion of intersection of pitch class content between such sub-collections. In the usual system of temperament, only the content corresponding to the major or minor scales, and to the half chromatic scale, can be so generated. A division of the octave into, for example, 11 equal parts would yield five such non-complementary sub-collections of six pitch classes each.

The means of providing the physical basis of a permutational system by the extension of octave division to multiples of twelve equal parts will not be discussed; on the one hand, the properties of the twelve-tone system are extendable immediately to this new collection, while—on the other hand—the regarding of the traditional twelve-tone system as a permutational sub-system within such a total collection is too large and complex a subject to be discussed here.

A contextually determined division of the octave, thus varying from work to work and within a work, is suggested by the pitch analog of the final rhythmic procedure discussed (Ex. 22). Within the fixed, twelve part equal temperament, the interval between any two such pitch elements is divisible into an interval succession of the pitch set, with the similar restriction that there be no ambiguity of identification of a frequency so derived with a frequency of the fixed temperament.

The most elaborate extrapolation from the principle of construction of the twelve-tone pitch set is to a twelve-tone frequency scale, in which the frequencies chosen for an individual composition make available an exact reflection of the interval structure of the compositional set. For example, corresponding to the set of Ex. 1, such a "frequency set" would be (say, in the fifth octave): 262.6 (cps); 273.5; 302.6; 320.8; 353.5; 357.1; 378.8; 386.1; 411.5; 426.0; 462.3; 502.3; (525.2). If a twelve-tone set is now formed by serially ordering the classes of which these frequencies are representatives, an examination of the simplest invariants under the normal rules of transformation will reveal surprisingly new consequences; I shall mention merely that transposition is not, in general, interval-preserving, assuming that "interval" has its usual designation. Under this system, the properties associated with the normal systematic operations are dependent on the *frequency* materials of the particular work.

If there can be little question that such pitch and rhythmic extensions of the twelve-tone system carry music to the point of purely electronic feasibility, there still remain large questions that return this discussion to its beginning. Do such extensions maintain those characteristics of differentiation and identification which endow the principles of formation and transformation with their empirical justification in the traditional system? And, on the other hand, do such constraints, however extended their domain of application, not eliminate, unnecessarily and undesirably, certain electronically available resources?

The answer to the first question is, obviously, that such extensions well might not, in a significant sense. Even though one is not prepared to state general laws with regard to those complex, multiple correlations between the acoustic and auditory domains, so that the precise auditory effect of a particular acoustical specification may be difficult of useful prediction, electronic instruments have made certain specific kinds of consequences predictable. Even with regard to only the normal frequency materials of conventional temperament, the identification of interval succession appears impossible beyond, electronically, quite modest speeds, to the point where even the mere number of different intervals cannot be identified; two, specified as equal, frequencies will be heard as different pitches beyond a certain durational minimum, with the minimum determined by the associated spectra, among other things. Here, then, the bases of traditional musical hearing disappear, for both the tonal and twelve-tone systems rest upon the assumptions of pitch invariance with regard to time, timbre, loudness, and duration, and of intervallic invariance with regard to transposition under similar conditions of alteration of other dimensions. The ordering principle of the twelve-tone system, which embodies the "new" memorative demands of the system (although the very conception of "theme" or "motive" in any music assumes the significance of order, but not as a primitive of the system), is also made inapplicable when a pitch succession, whose internal pitch ordering is clearly identifiable in a certain contour presentation, is altered in contour and registral range, so that the order becomes not completely identifiable. Similarly, a succession of pitches, clearly identifiable at a particular speed, cannot be perceived as containing even the same number of pitches at a critically greater speed, this critical point being dependent upon not only the acoustical characteristics of the components, both individually and in relation to one another, but the compositional context of the event.

But all of this is merely to say that the necessary characteristics of the system must be preserved in the auditory domain, and not merely in the domain of notational specification. Indeed, it is the fact of systematic presuppositions that makes it possible for the composer to determine the acceptability of a presented event, independent of its electronic specification. To say that, for example, a specified frequency lasting $1/8$ of a second does not represent the same pitch as the same specified frequency lasting $1/32$ of a second is merely to assert that the two different pitches have different notations with regard to duration. Any electronically specified event will have its aural correlate, even if this correlate be silence or a click; the compositional question is simply whether silence, or a click, is what is required at that moment. The relation between notation and aural event has never been one-to-one traditionally, and the increase of the values of the "many" in the many-to-one relationship does not alter the fundamental situation. If the properties crucial to a composition's being perceived as an instance of a particular system are embodied only in the input specifications of the composition, these properties may be destroyed by perceptual "limitations," for—indeed—one can speak of such "limitations," as opposed to "characteristics," only with regard to systematic presuppositions; that a perceived alternation of pitches 30 times a second in the eighth octave becomes a perceived repetition of a single pitch in the third octave cannot be termed a perceptual "limitation" without an initial assumption of intervallic invariance under transposition. Systematically determined similarity relations, particularly when reinforced by identity of other components, are in fact powerful perceptual aids; two isolated events, specified as similar but (for a reason such as those stated above) perceived as dissimilar, may be perceived as similar when made components of larger contexts whose relationship as totalities is inferrable under the presented constraints of the system.

As for the second question as to whether systematic constraints do not eliminate the use of available resources, it should be clear from the preceding that the twelve-tone extensions into the electronic domain do not necessarily eliminate any auditory event or complex of events. In any case, a collection of available physical materials—in this case, the area of materials made available electronically—does not entail a particular system. If musical structure can be presumed to address itself to the "ear," and to be founded on criteria of relatedness, purely "contextual" electronic music must either deny all past experience and criteria of similitude, or disallow it as irrelevant on the grounds that each event is unique by virtue of its (non-modular) time-point value; but even uniqueness is a relational property, for it assumes criteria of differentness, and—thus—relatedness. A musical system can provide only the possibility of musical coherence in its own terms; the question of the perceptual and conceptual significance of these terms is the issue with which we began. Perhaps a system founded on the unique resources of the electronic medium, and on premises hitherto unknown and not as yet even foreseeable, will be discovered and vindicated. Meanwhile, if it is only meanwhile, there is still an unforeseeably extensive domain in which the electronic medium uniquely can enrich and extend the musical systems whose premises have been tested, and whose resources barely have been tapped.