3000788 Intro to Comp Molec Biol

Lecture 27: Supervised learning

November 17, 2022

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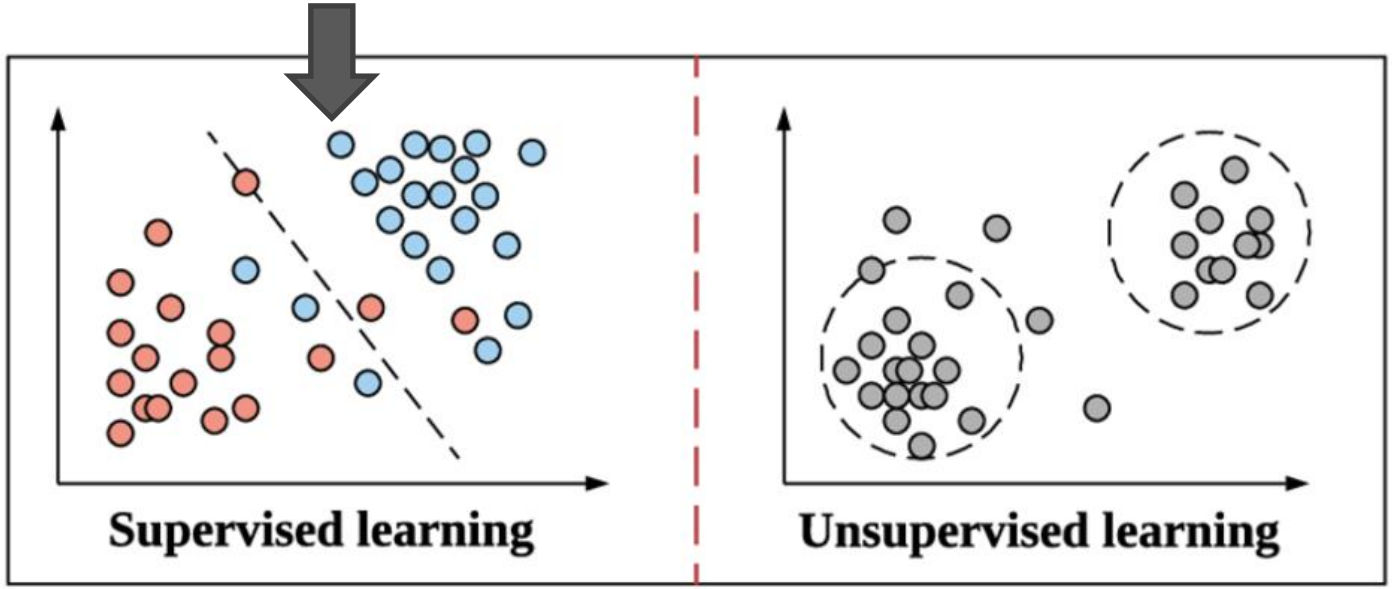
- Research Affairs
- Center of Excellence in Computational Molecular Biology (CMB)
- Center for Artificial Intelligence in Medicine (CU-AIM)

Machine learning paradigms

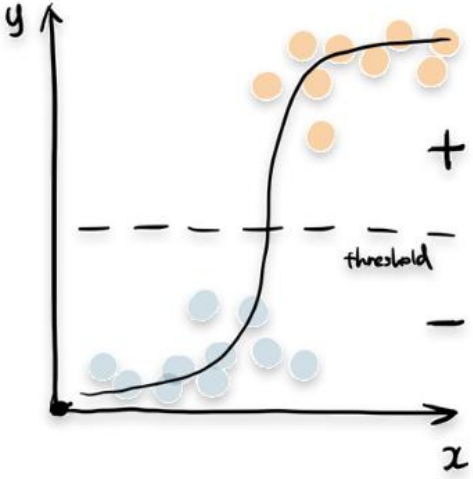


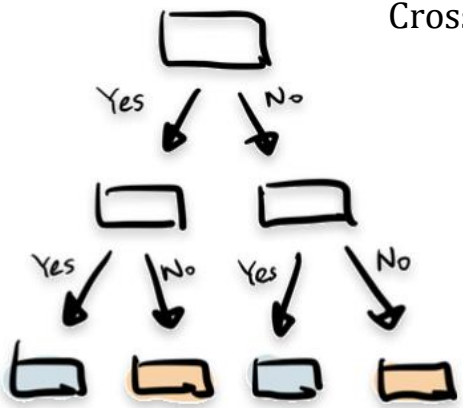
Qian, B. et al. "Orchestrating the Development Lifecycle of Machine Learning-Based IoT Applications: A Taxonomy and Survey"

- Identify robust patterns that can be generalized to new data

The cores of supervised learning

Model





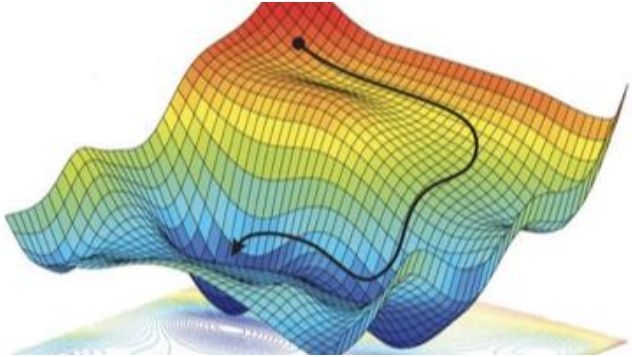
https://towardsdatascience.com/top-machine-learning-algorithms-for-classification-2197870ff501

Objective / Loss Function

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \qquad \text{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\frac{|y_i - \hat{y}_i|}{y_i} \times 100$$

$$\text{Crossentropy} = -\frac{1}{n}\sum_{i=1}^{n} y_i \ln(\hat{y}_i) + (1 - y_i)\ln(1 - \hat{y}_i)$$

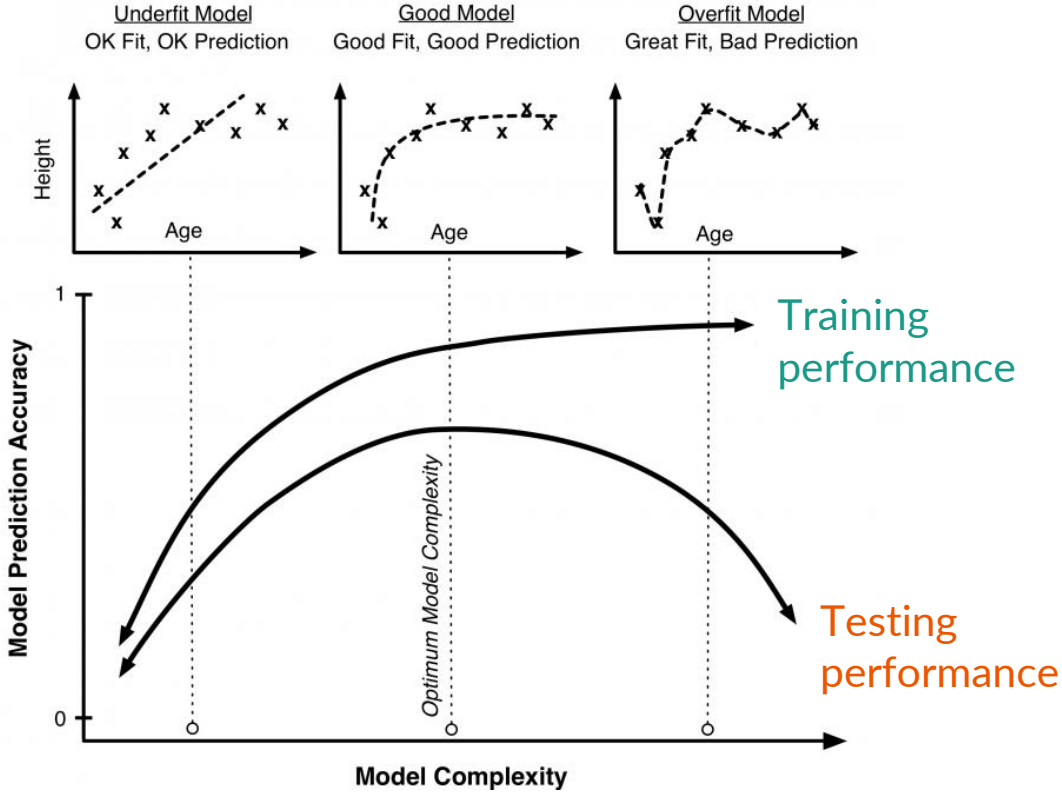
Learning Algorithm



https://medium.com/analytics-vidhya/gradient-descent-b0dc1af33517

Supervised learning is all about control

https://en.wikipedia.org/wiki/Bull_riding



https://realkm.com/2018/04/23/optimization-and-complexity-the-cost-of-complexity-systems-thinking-modelling-series/

Likelihood

- **Likelihood**: Probability of observing data x from a model with parameters θ

- Probability of getting two heads in a row, given a fair coin
  - $P(HH \mid p_H = 0.5) = 0.5 \times 0.5 = 0.25$

- Probability of getting two heads in a row, given a biased coin
  - $P(HH \mid p_H = 0.8) = 0.8 \times 0.8 = 0.64$

- **Maximum Likelihood Estimate (MLE)**: find θ that maximize the likelihood

Statistical control of overfitting

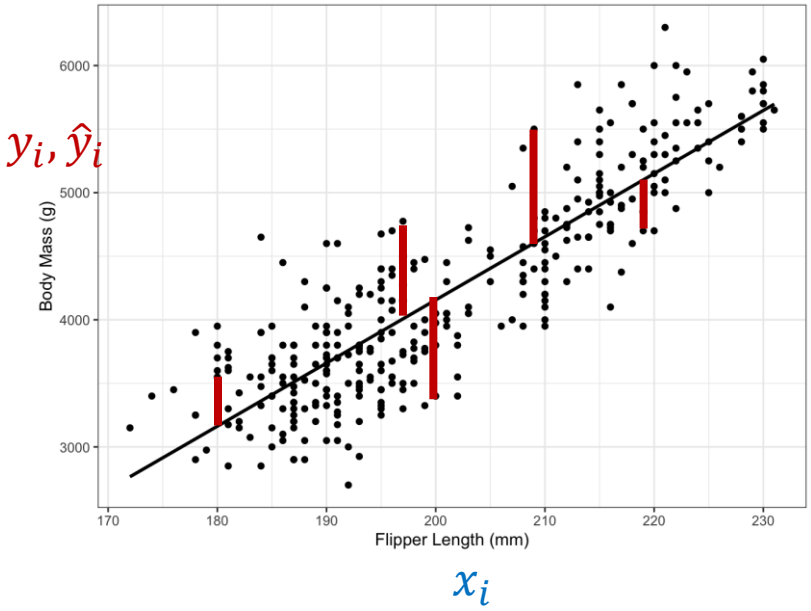
- Better model achieves higher likelihood
- Complex model has more parameters

- Information Criterion
  - Akaike (AIC) $= 2k - 2 \cdot \ln(\hat{L})$, where $\hat{L}$ is the likelihood
  - Bayesian (BIC) $= \ln(n)\, k - 2 \cdot \ln(\hat{L})$, where n is the sample size

- Nested model testing
  - Simple model has n parameters, fit the data with likelihood $\widehat{L_1}$
  - Complex model has $m > n$ parameters, fit the data with likelihood $\widehat{L_2} > \widehat{L_1}$
  - Is the improvement $\frac{\widehat{L_2}}{\widehat{L_1}}$ worth the increase in $m - n$ parameters?

Linear and logistic regression

Linear regression (Ordinary Least Square)



- Model: $\hat{y}_i = b_0 + b_1 x_i$
- Minimize MSE: $\frac{1}{n}\sum_i (y_i - [b_0 + b_1 x_i])^2$

- $\frac{\delta MSE}{\delta b_o} = -2\sum_i y_i - 2b_1 \sum_i x_i - 2nb_0$
- $\frac{\delta MSE}{\delta b_1} = -2\sum_i x_i y_i - 2b_1 \sum_i x_i^2 - 2b_0 \sum_i x_i$

- $b_0 = \frac{\sum xy \sum x - \sum x^2 \sum y}{(\sum x)^2 - n\sum x^2}$
- $b_1 = \frac{\sum y \sum x - n\sum xy}{(\sum x)^2 - n\sum x^2}$

Ordinary Least Square interpretation

- Observed value = True value + Normally-distributed noise
- **Assumption**: Noises are identical and independent across samples

- Model: $(y_i - \hat{y}_i) \sim N(0, \sigma^2)$
- Density: $\mathrm{P}(y_i - \hat{y}_i = \varepsilon_i \mid \sigma^2) \propto e^{\frac{-\varepsilon_i{}^2}{2\sigma^2}}$
- Likelihood: $\prod_i \mathrm{P}(y_i - \hat{y}_i = \varepsilon_i \mid \sigma^2) \propto e^{\frac{-\sum_i \varepsilon_i{}^2}{2\sigma^2}}$

- MSE: $\frac{1}{n}\sum_i (y_i - \hat{y}_i)^2 = \frac{1}{n}\sum_i \varepsilon_i{}^2$
- Minimizing MSE is the same as maximizing likelihood

Logistic regression

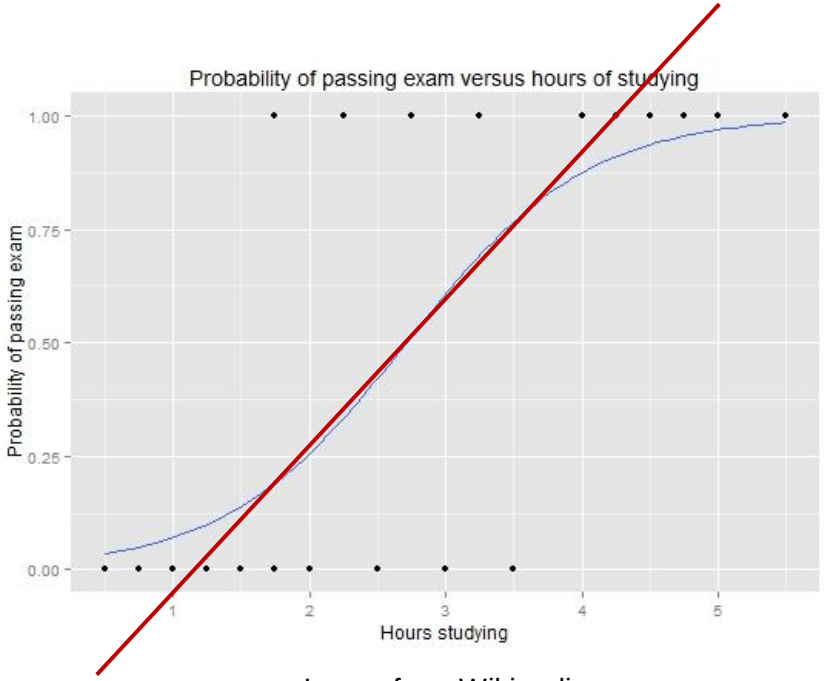


Image from Wikipedia

- Classification output = 0 or 1
- Linear regression outputs $-\infty$ to ∞

- Probability of success p
- Log odd: $\ln\left(\frac{p}{1-p}\right)$
  - $\ln\left(\frac{p}{1-p}\right) \to -\infty$ as $p \to 0$
  - $\ln\left(\frac{p}{1-p}\right) \to \infty$ as $p \to 1$

- Transform linear regression output with log odd!

Logistic regression

- Model: $\ln\left(\frac{\widehat{y_i}}{1-\widehat{y_i}}\right) = f(x_i) = b_0 + b_1 x_{i,1} + \cdots + b_n x_{i,n}$
- $\widehat{y_i} = \frac{e^{b_0+b_1 x_{i,1}+\cdots+b_n x_{i,n}}}{1+e^{b_0+b_1 x_{i,1}+\cdots+b_n x_{i,n}}}$
  - When $x_i \to \infty$, $\widehat{y_i} \to 1$
  - When $x_i \to -\infty$, $\widehat{y_i} \to 0$

- Can we keep using MSE as the loss function?
  - Brier score $= \frac{1}{N}\sum_i (y_i - \widehat{y_i})^2$
  - But this does not interpret logistic output as probability

Likelihood for logistic regression

- Likelihood: $\mathrm{P}(y_i \mid x_i) = \hat{y}_i^{\,y_i}(1-\hat{y}_i)^{1-y_i}$
  - y_i is either 0 or 1
  - When y_i is 0, the likelihood is $1-\hat{y}_i$
  - When y_i is 1, the likelihood is $\hat{y}_i$

- Log likelihood: $y_i \ln(\hat{y}_i) + (1-y_i)\ln(1-\hat{y}_i)$
  - This is the cross-entropy loss function!
  - Maximizing likelihood is the same as minimizing cross-entropy

Impact of large coefficients

- **Model:** $\widehat{y_i} = b_0 + b_1 x_{i,1} + \cdots + b_n x_{i,n}$

- If there are two models (two sets of $b_j's$) that achieve similar performance, we should prefer the model with smaller magnitudes of $b_j's$

- Prevent overfitting to an input feature
  - Magnitude of b_k = influence of the kth input feature on the model

- Robustness to future inputs
  - Measurement error of 1 unit in $x_{i,k}$ will be amplified to b_k units in $\widehat{y_i}$

Regularization of linear model

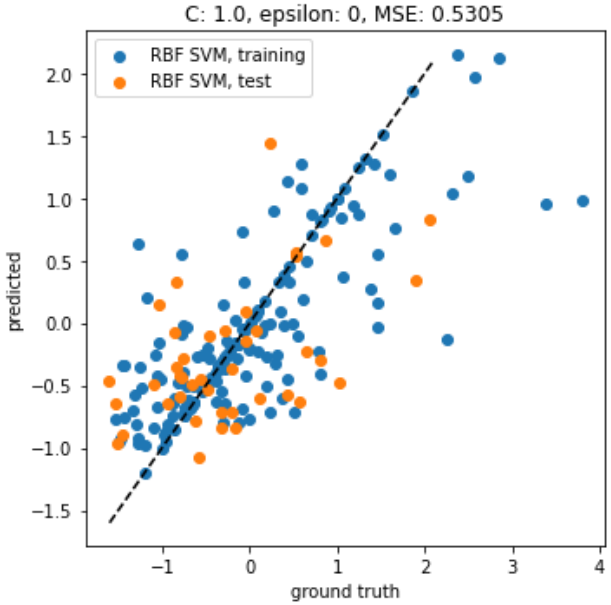
- L1 regularization (LASSO): MSE + $\alpha \sum_k |b_k|$
- L2 regularization (Ridge): MSE + $\alpha \sum_k {b_k}^2$

- α is the hyperparameter that controls the regularization strength

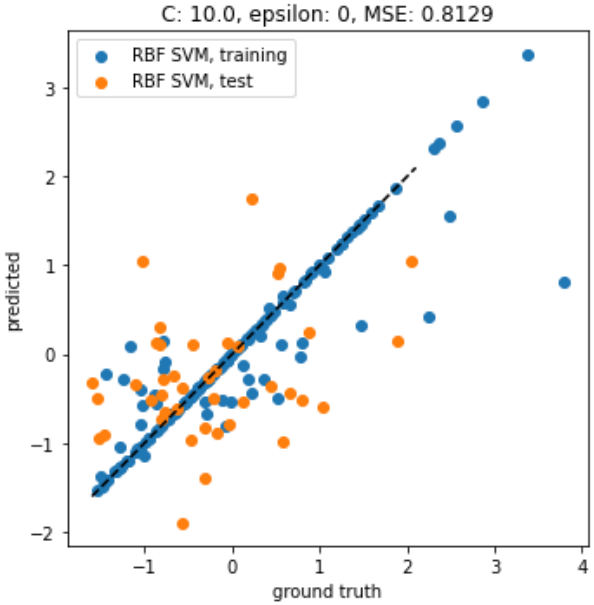
- Hyperparameter must be tuned
  - Split data into **Training-Validation-Test**
  - Try many values of α while training the model on the **Training** set
  - Select α that results in highest performance on the **Validation** set
  - Report model performance with selected α on the **Test** set

Tuning regularization strength

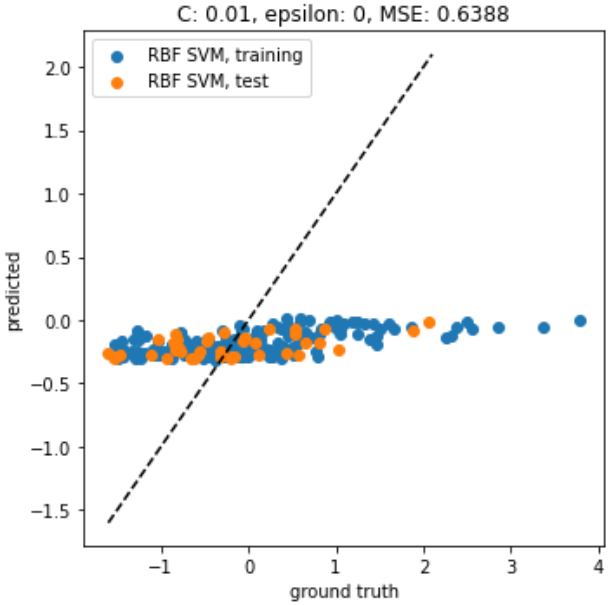
Just right



Too weak

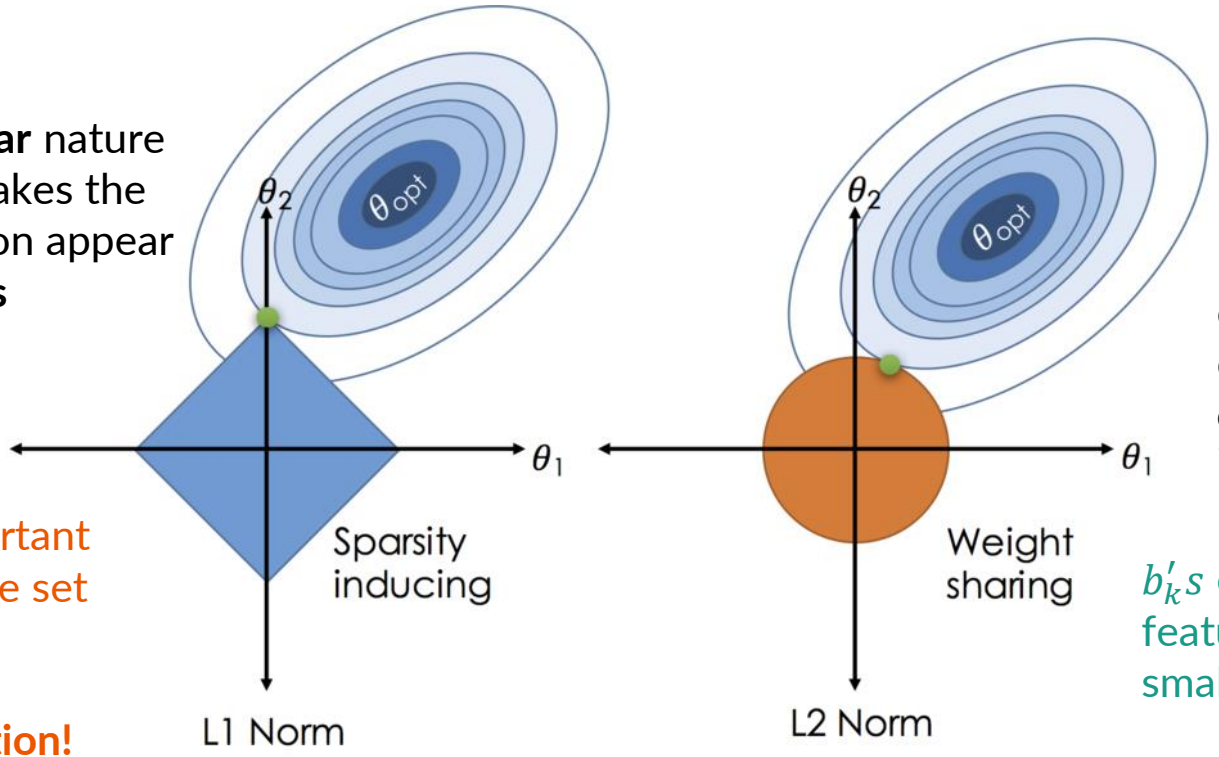


Too strong



L1 versus L2 regularization

The **rectangular** nature of $|b_0 + b_1|$ makes the optimal solution appear on the **corners**



For $(b_0 + b_1)^2$, the optimal solution can be **anywhere on the boundary**

$b_k's$ of unimportant features will be set to zeroes

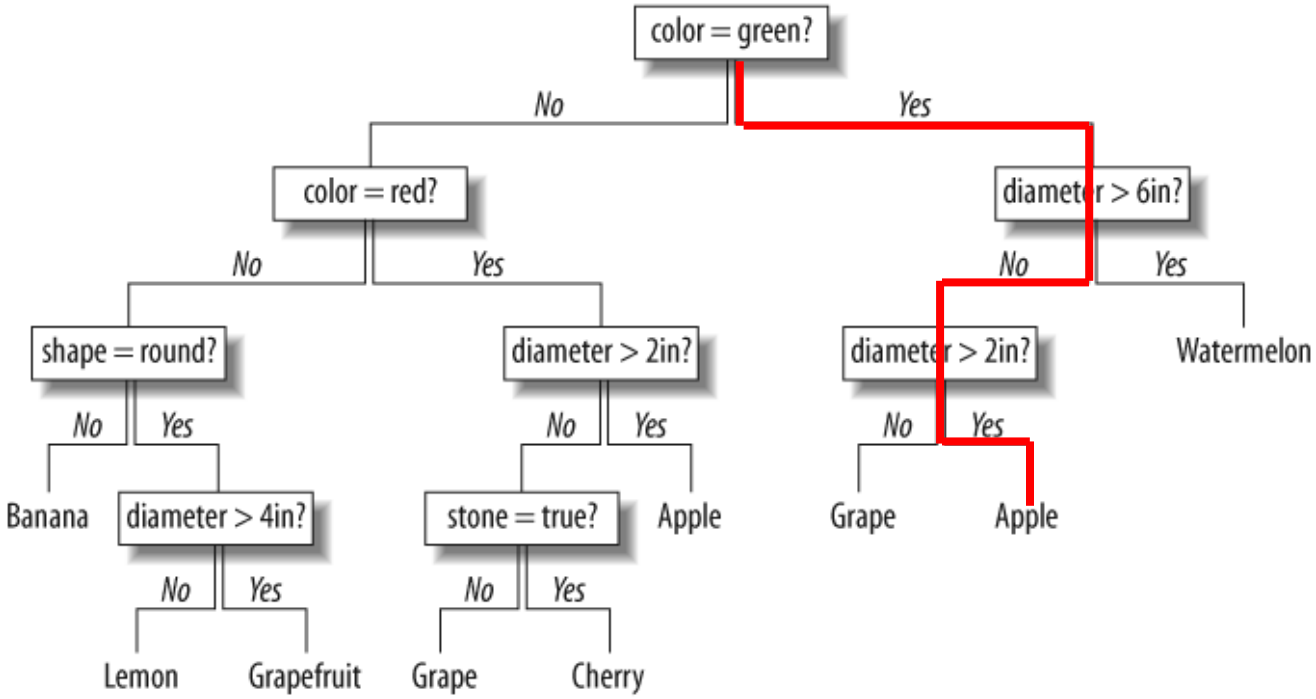
Feature selection!

$b_k's$ of unimportant features will be small but not zeroes

https://www.linkedin.com/pulse/regularization-episode-1-amr-mahmoud/

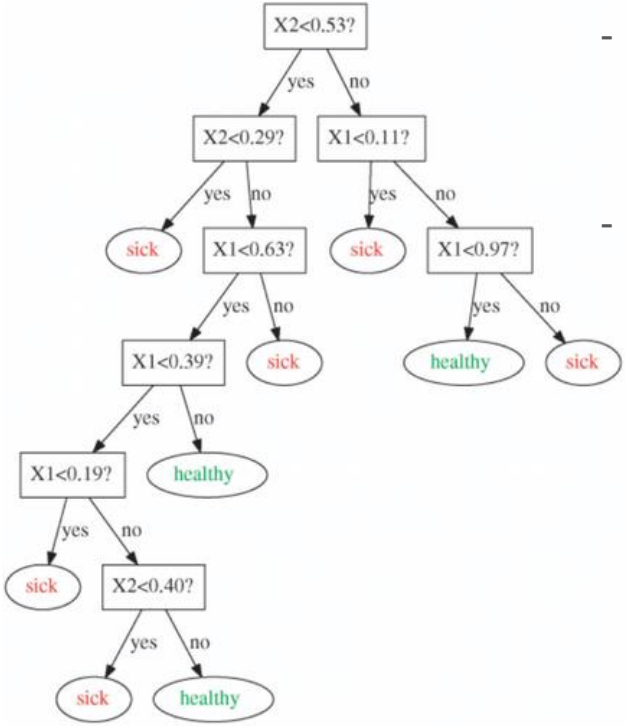
Decision tree

Decision tree

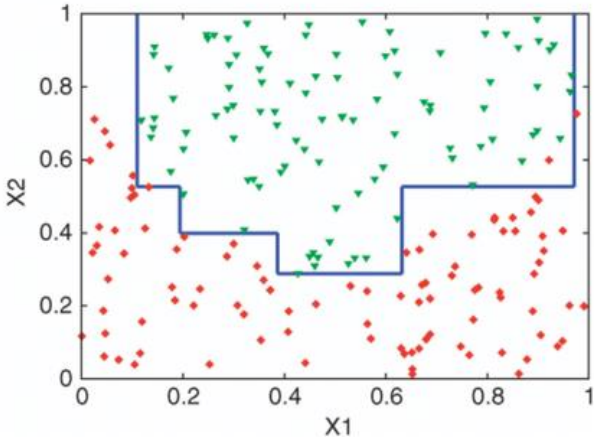


Source: Programming collective intelligence by Toby Segaran

Decision tree behaviors

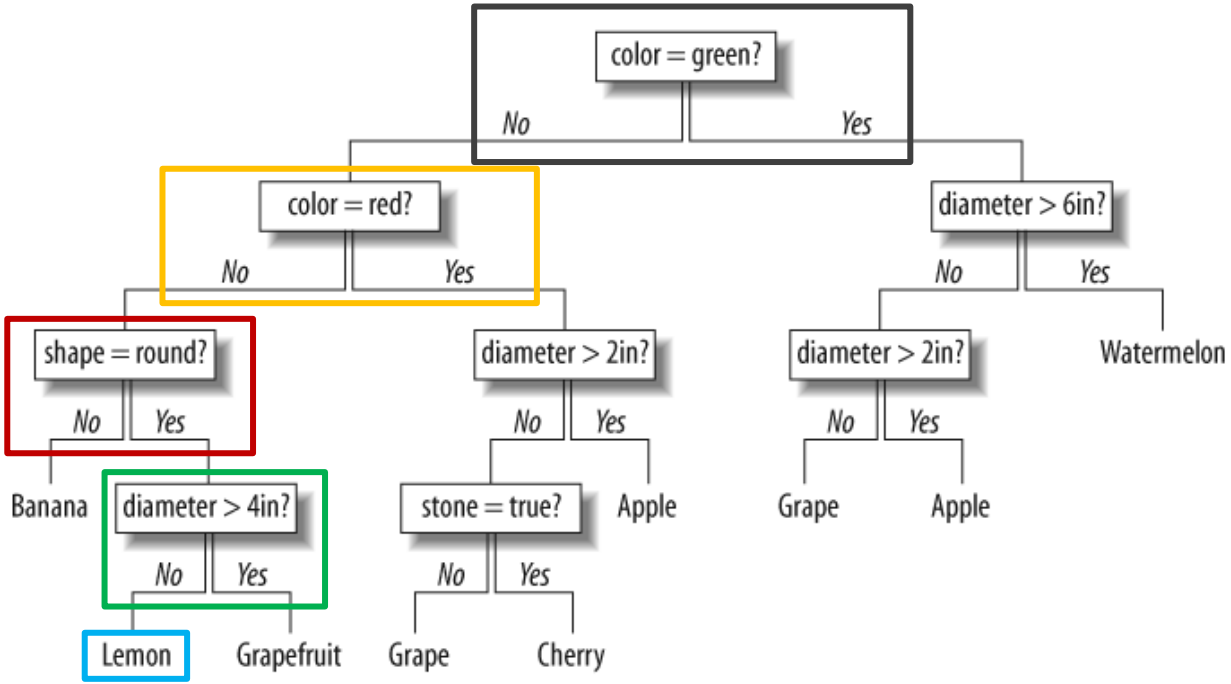


- Each decision is a threshold on each feature
  - Piecewise linear
  - Parallel to an axis
- Good for criteria-based classification



Miller, C. “Screening meter data: Characterization of temporal energy data from large groups of non-residential buildings”

Tree building



- Pick a feature & a criterion, but how?

Splitting quality

100 healthy, 100 sick

85 healthy, 5 sick

15 healthy, 95 sick

45 healthy, 51 sick

55 healthy, 49 sick

- Gini impurity: $\sum p(1-p)$
- Entropy: $-\sum p \ln(p)$
  - Minimal at p = 0 or 1 → Perfect split
  - Maximal at p = 0.5 → 50-50 split
- Search for feature and cutoff that yield lowest impurity or entropy

Control mechanisms for tree building

1. Too few samples to make a split

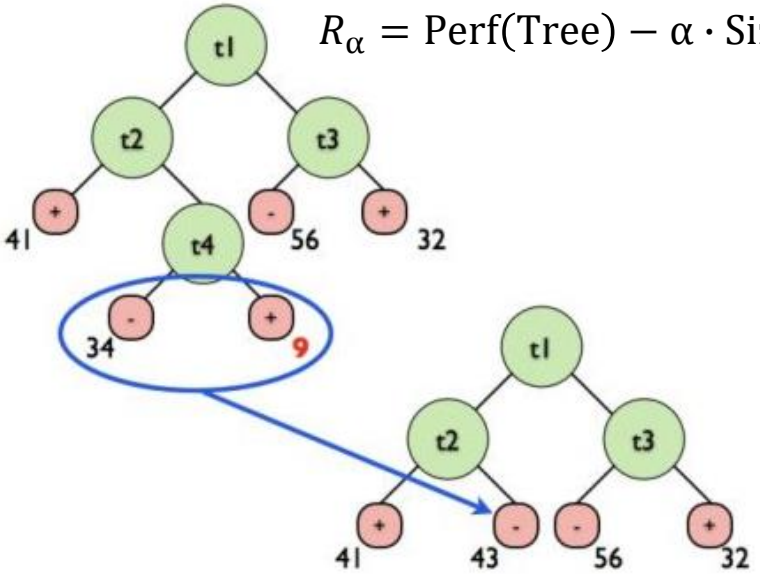
3. Impurity or entropy does not change much after the split

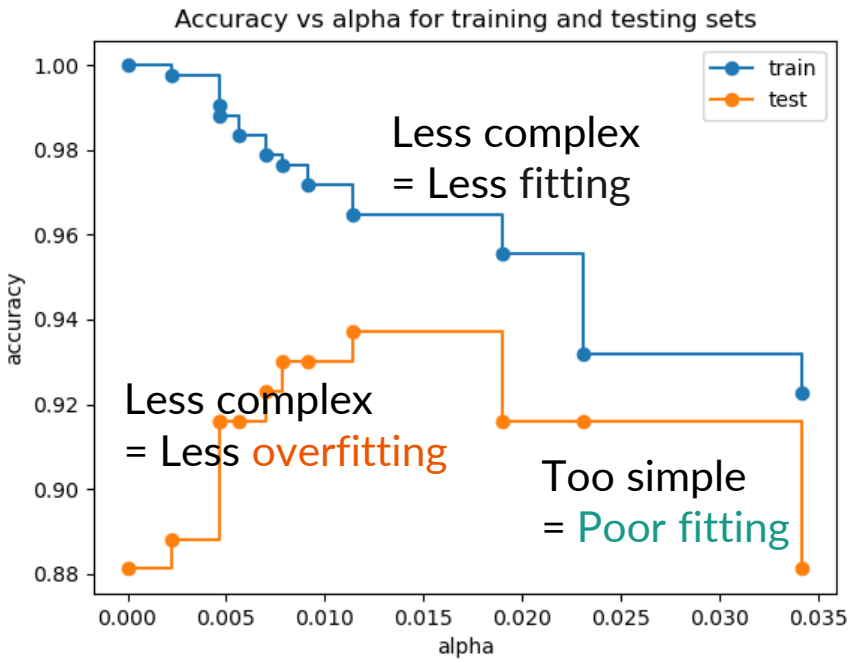
2. Too few samples on either branch

- Limit the tree size
- Limit the improvement in quality
- Limit the number of samples that support a split

Tree pruning (post-processing)

$$R_\alpha = \text{Perf(Tree)} - \alpha \cdot \text{Size}$$





Patel, N. and Upadhyay, S. “Study of Various Decision Tree Pruning Methods with their Empirical Comparison in WEKA” IJCA 2012

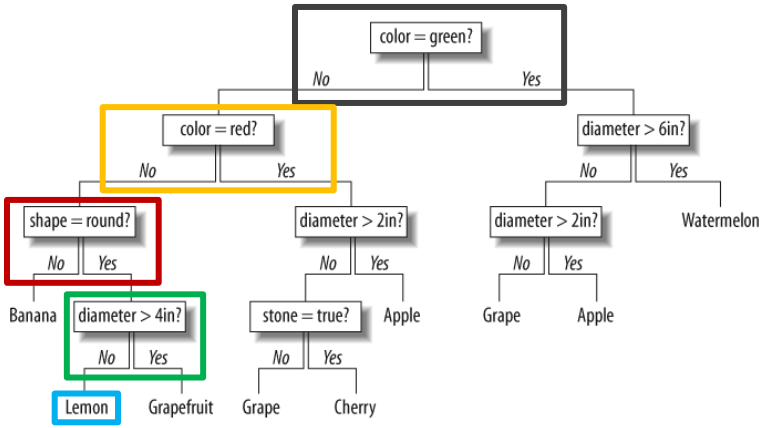
Image from scikit-learn,org

Regularization on features

- **Linear model:** $\hat{y}_i = b_0 + b_1 x_{i,1} + \cdots + b_n x_{i,n}$
  - LASSO

- **Tree model:**
  - Repeatedly using the same feature
  - Early decision affects the rest

- Feature bagging
  - Look at only *N* features at each step
  - Force model to use diverse features



Regression tree

Predictors | Target

Outlook	Temp.	Humidity	Windy	Hours Played
Rainy	Hot	High	False	26
Rainy	Hot	High	True	30
Overcast	Hot	High	False	48
Sunny	Mild	High	False	46
Sunny	Cool	Normal	False	62
Sunny	Cool	Normal	True	23
Overcast	Cool	Normal	True	43
Rainy	Mild	High	False	36
Rainy	Cool	Normal	False	38
Sunny	Mild	Normal	False	48
Rainy	Mild	Normal	True	48
Overcast	Mild	High	True	52
Overcast	Hot	Normal	False	44
Sunny	Mild	High	True	30

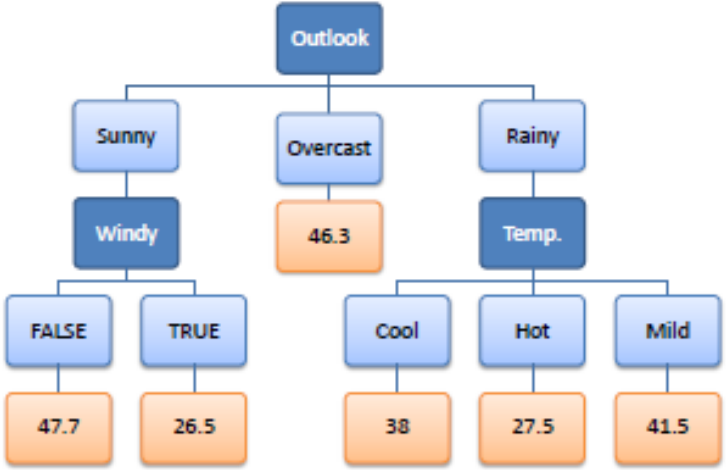


Image from saedsayad.com

- Use decision tree to group data points and predict the average

Regression tree in action

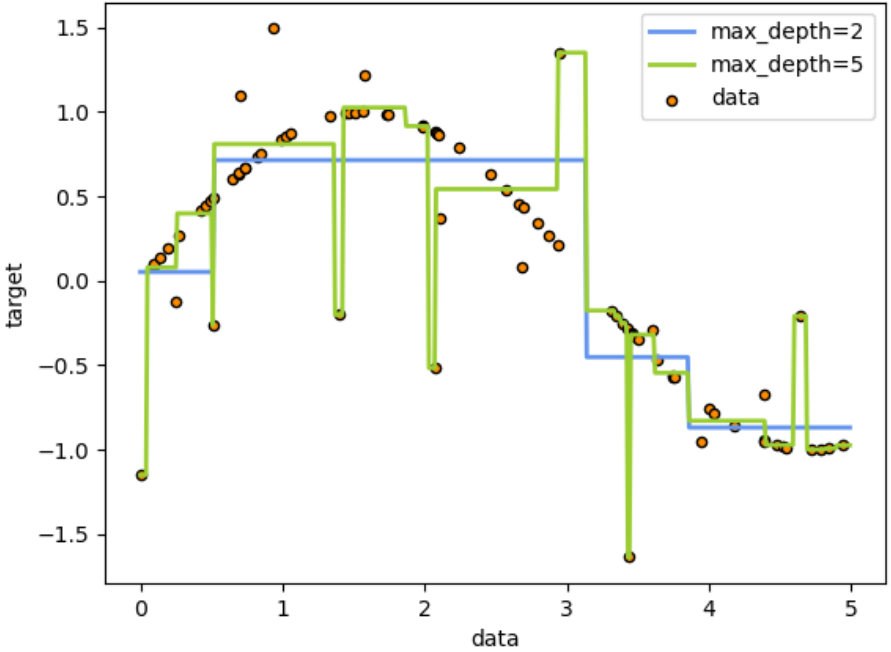
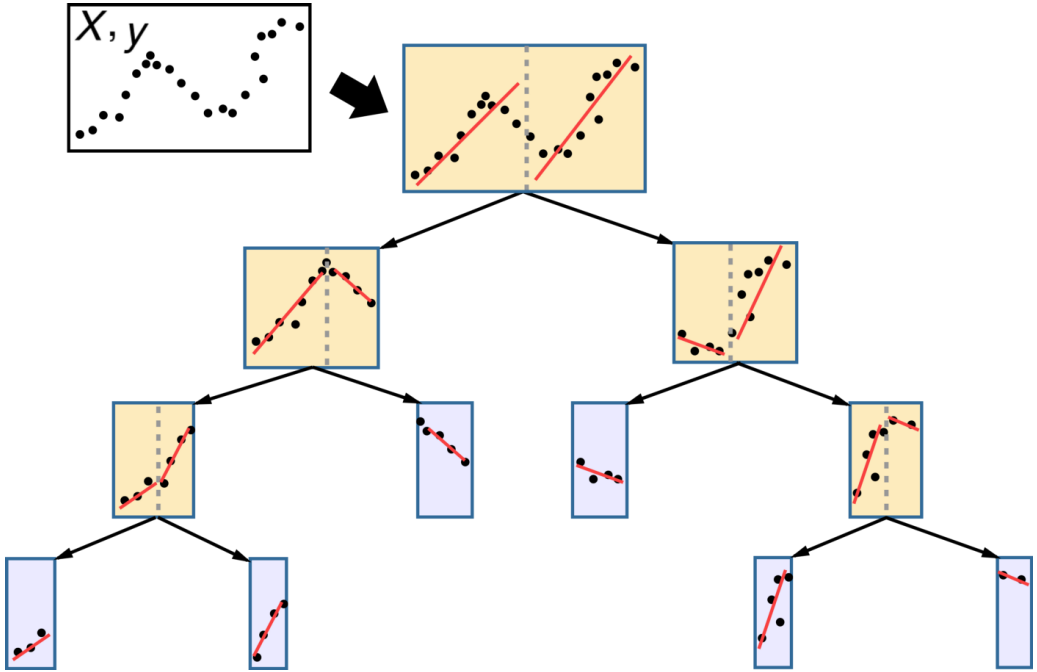


Image from scikit-learn,org

- Low depth = less complex = smoother prediction values
- High depth can lead to overfitting

Decision tree with regression model

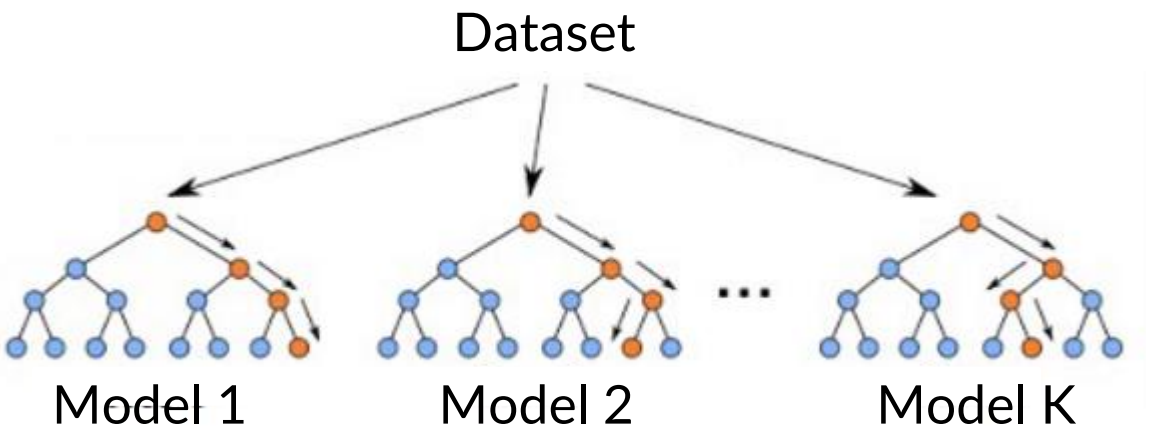


https://towardsdatascience.com/introduction-to-model-trees-6e396259379a

- For each group of samples, use the data to fit a regression model

Ensemble approaches

Training and aggregating multiple models

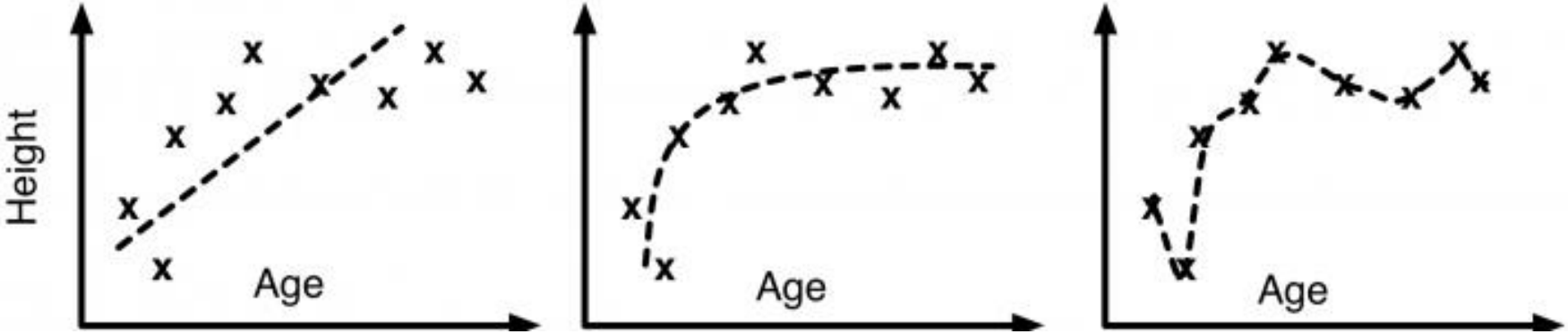


- Bagging: Generate random sets of samples to train multiple models
- Boosting: The *k*-th model address the errors made by earlier models

Impact of ensemble

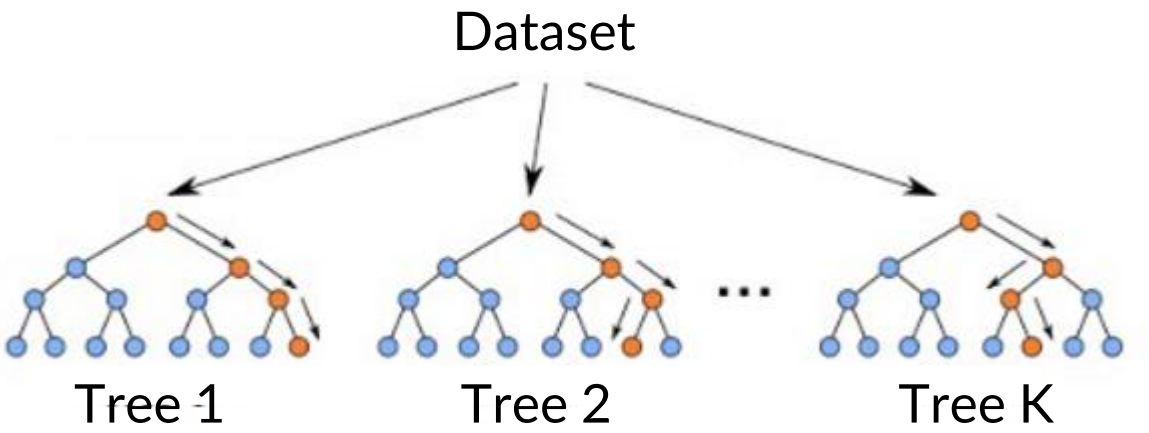
Boosting solves underfitting

Bagging prevent overfitting



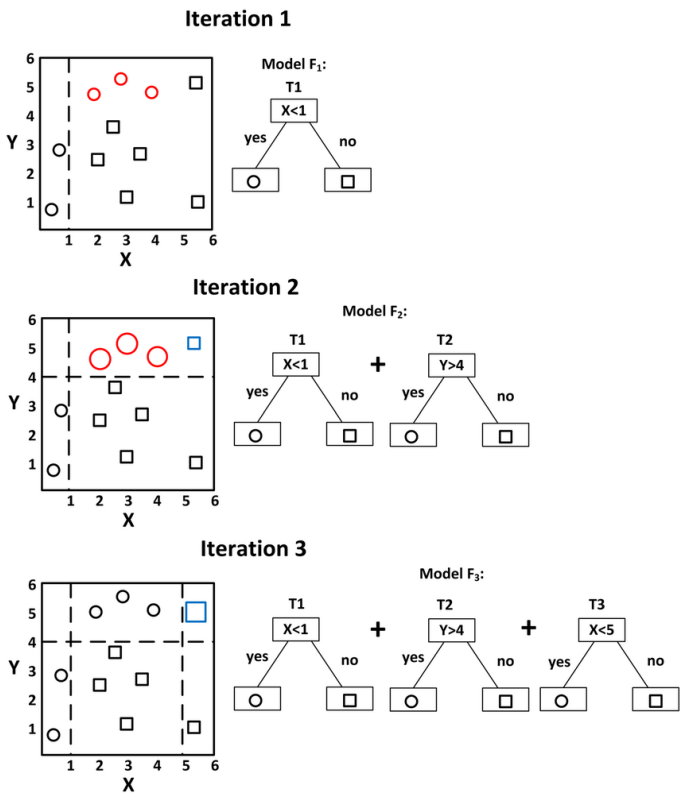
https://realkm.com/2018/04/23/optimization-and-complexity-the-cost-of-complexity-systems-thinking-modelling-series/

Random forest



- Sample 80% of the dataset to train each decision tree
- Each tree may overfit to different part of the dataset
- But the consensus should be correct

Boosting for classification = weight the error



- Ensemble predictor = weighted average
  - $C_n(x) = \alpha_1 f_1(x) + \alpha_2 f_2(x) + \cdots + \alpha_n f_n(x)$
- **Adaptive Boosting** (AdaBoost)
- **Exponential loss:** $\sum_{y_i \neq f_n(x_i)} e^{-y_i C_{n-1}(x_i)}$
  - Weight error made by *n*-th model using the error made by the first *n*−1 models
- α_n is based on the performance of $f_n(x)$

Boosting for regression = fit the residual directly

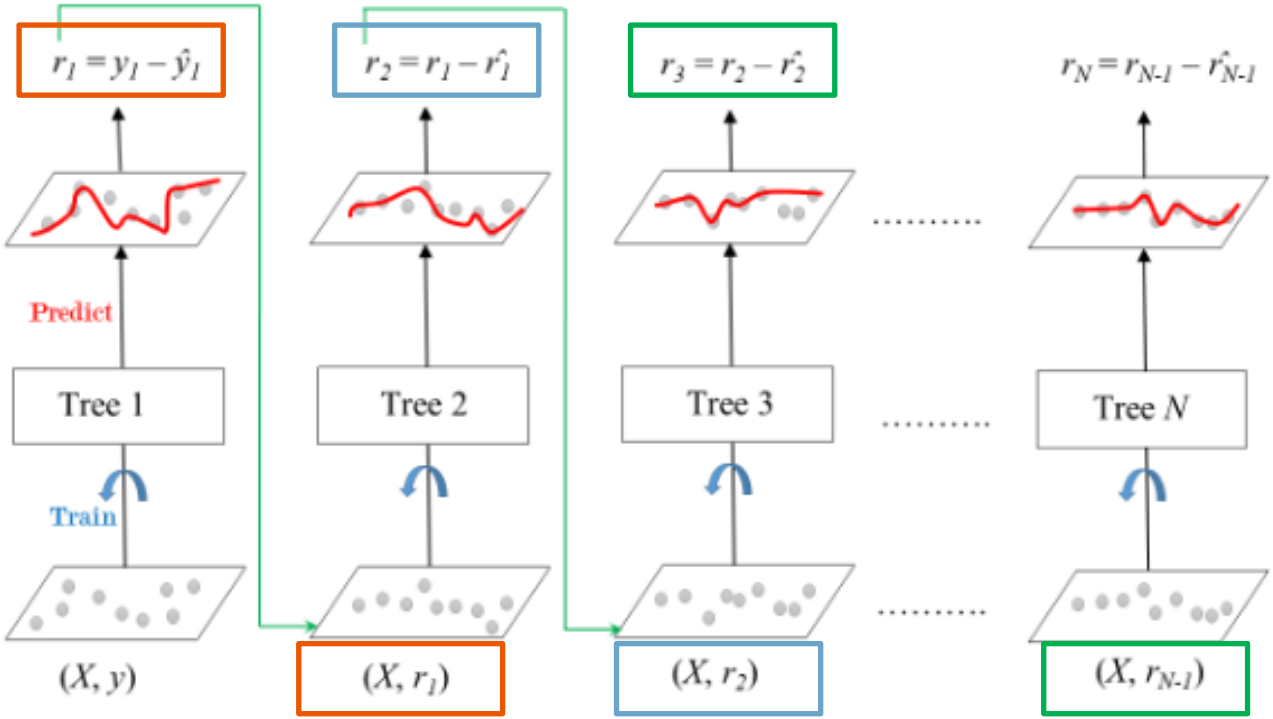
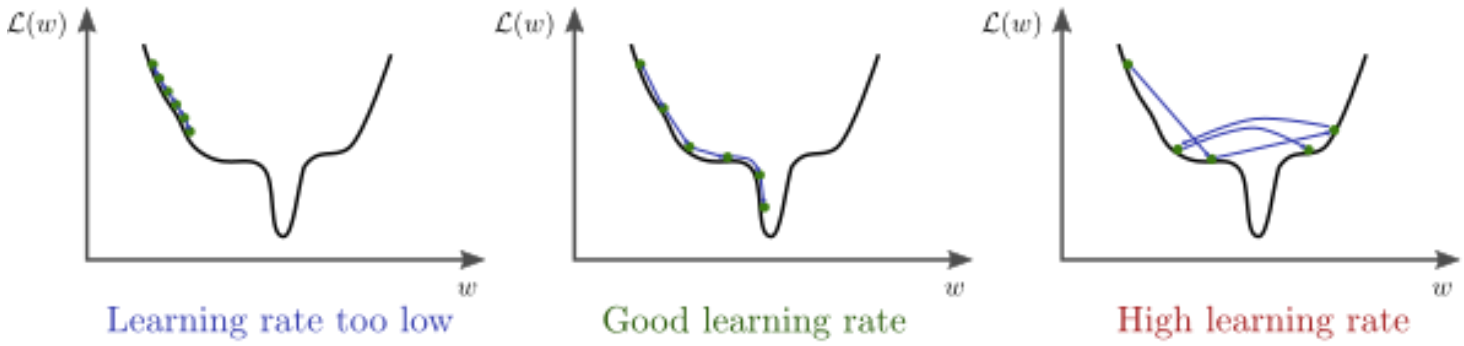


Image from geeksforgeeks.org

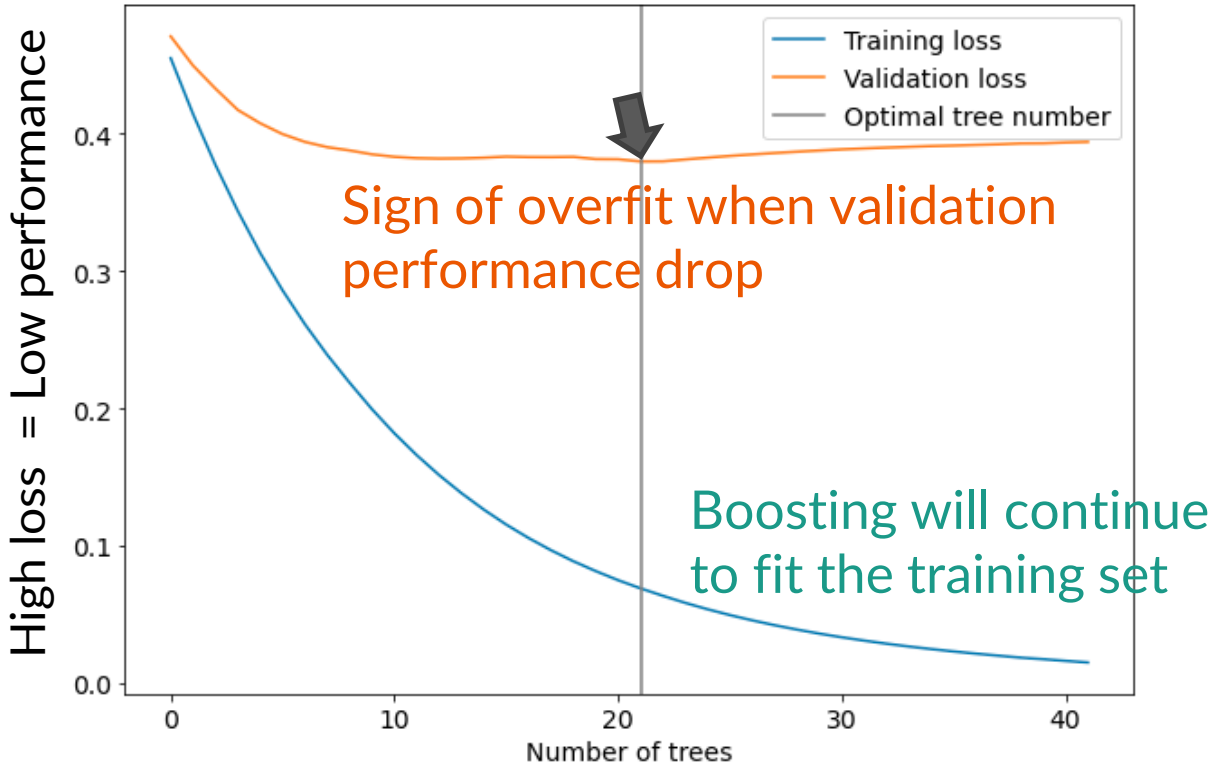
Controlling the boosting process



http://www.bdhammel.com/learning-rates/

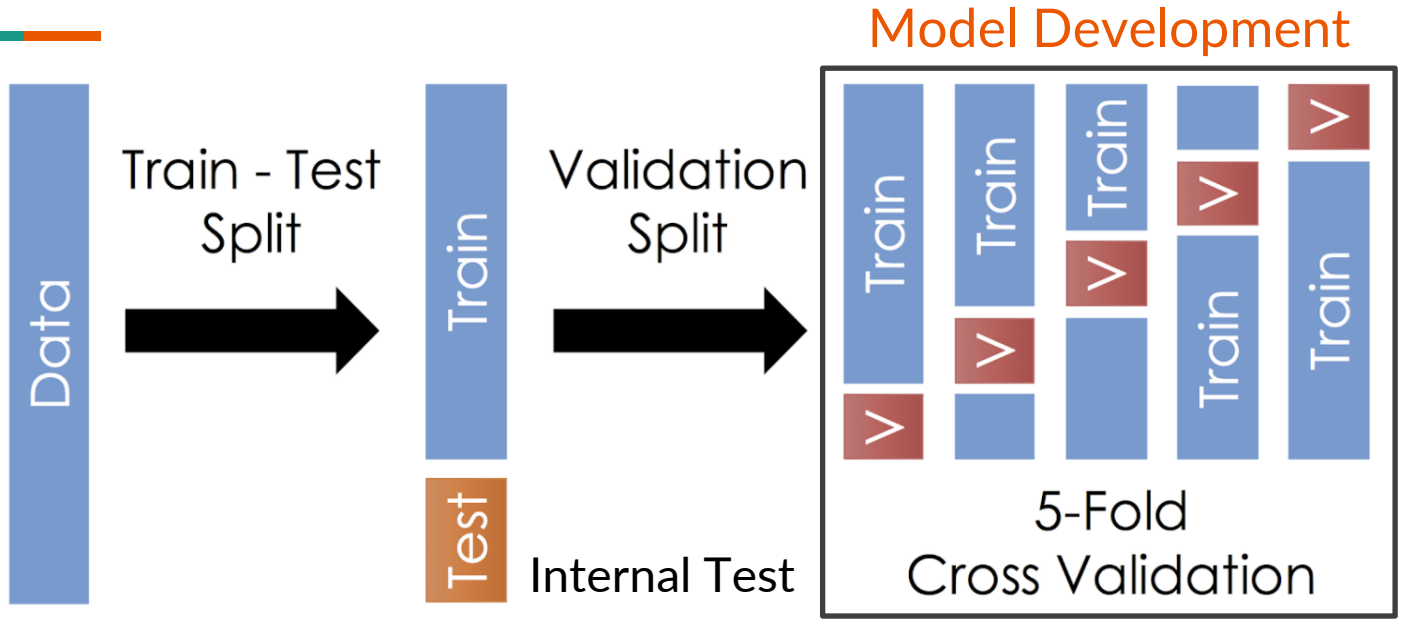
- Learning rate: how much to trust the next update
  - $C_n(x) = \alpha_1 f_1(x) + \alpha_2 f_2(x) + \cdots + \alpha_n f_n(x)$
- Too low → slow training process → many models → computational cost
- Too high → overshoot the optimal point

Early stopping with validation set



Model validation

Train-Val-Test



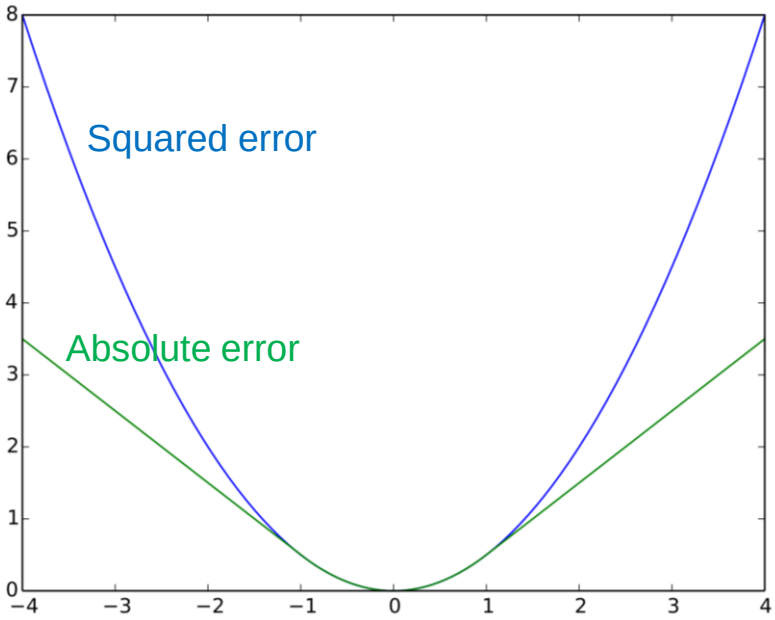
Source: medium.com

- **Training** data determines the best coefficients / weights
- **Validation** data determine the best hyperparameters
- **Test** data determine performance on new datasets

Regression metrics

- Mean Square Error
- Mean Absolute Error
- Mean Absolute Percentage Error
- R^2 (Coefficient of Determination)

- Select to match use case
  - MAPE = 15%
  - MAE = 1 unit
  - MSE penalize outliers more



Classification metrics

		Predicted	
		Negative	**Positive**
Actual	**Negative**	True Negative	False Positive
	Positive	False Negative	True Positive
		Predicted < 0.5	Predicted > 0.5

- Accuracy = (TN + TP) / total
- Precision = TP / (TP + FP) = Positive predictive value

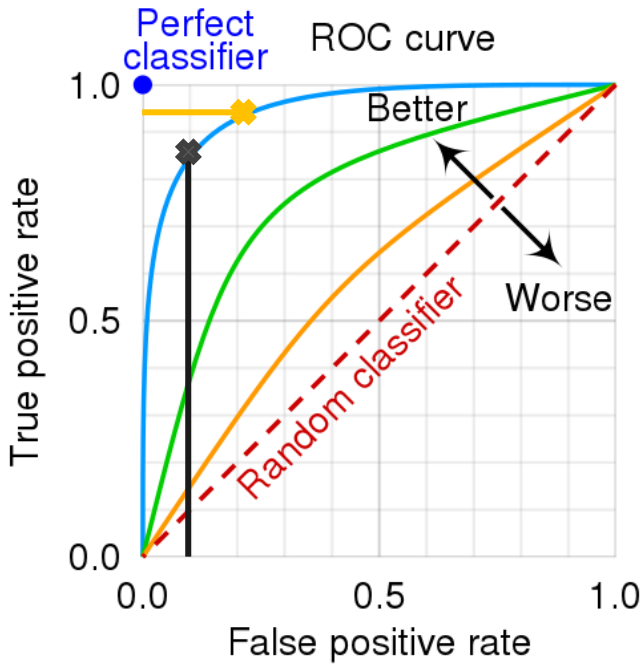
- Recall = TP / (TP + FN) = Sensitivity
- Specificity = TN / (TN + FP)

Classification metric use cases

- Screening for secondary inspection
  - Recall: Missed samples cannot be recovered
  - Improve precision during secondary inspection

- Taking action based on prediction
  - Precision
    - Whether to perform surgery
  - Negative-class precision
    - Whether to send patient home
    - Whether the patient will be allergic to drug

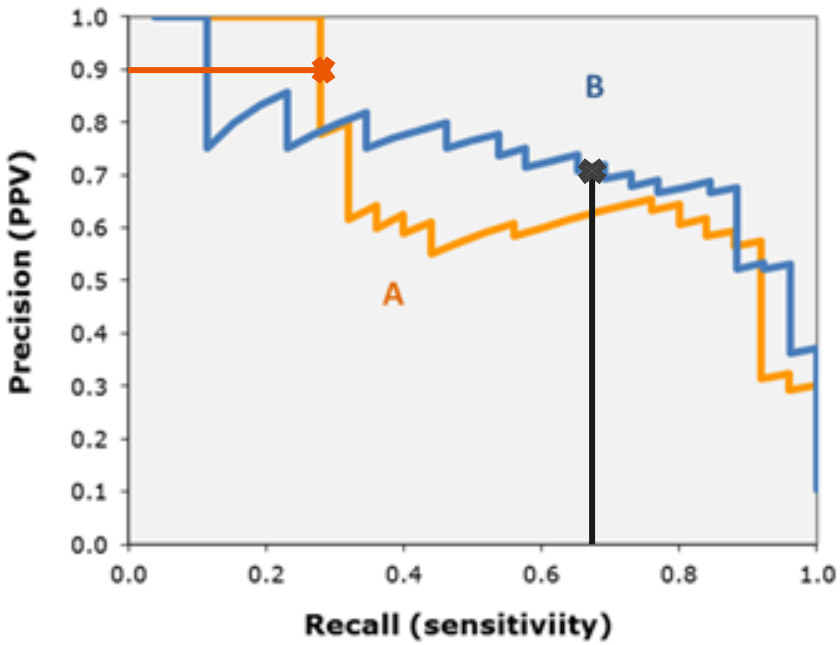
Threshold-free metrics



https://commons.wikimedia.org/wiki/File:Roc_curve.svg

- Sensitivity-specificity at every output threshold
- Area under the ROC curve (AUROC, AUC)
  - Random guess = 0.5
  - Perfect model = 1.0
- Pick threshold based on use case
  - Specificity >0.9
  - Sensitivity >0.9

Precision-Recall curve

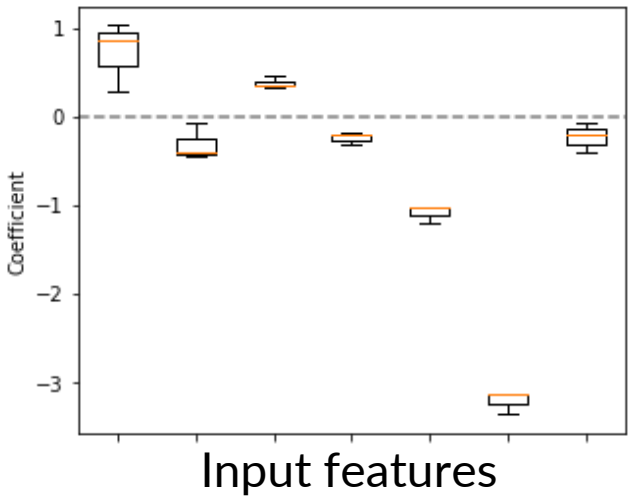


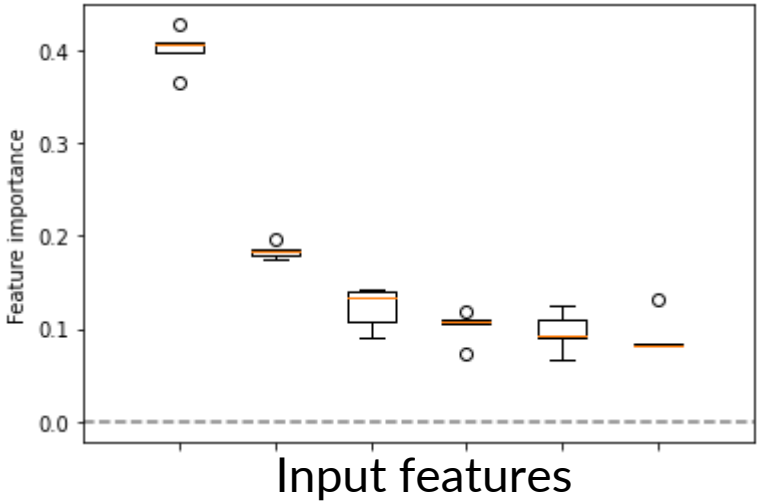
https://acutecaretesting.org/en/articles/precision-recall-curves-what-are-they-and-how-are-they-used

- The best model can depend on use case

Explainability / Interpretability

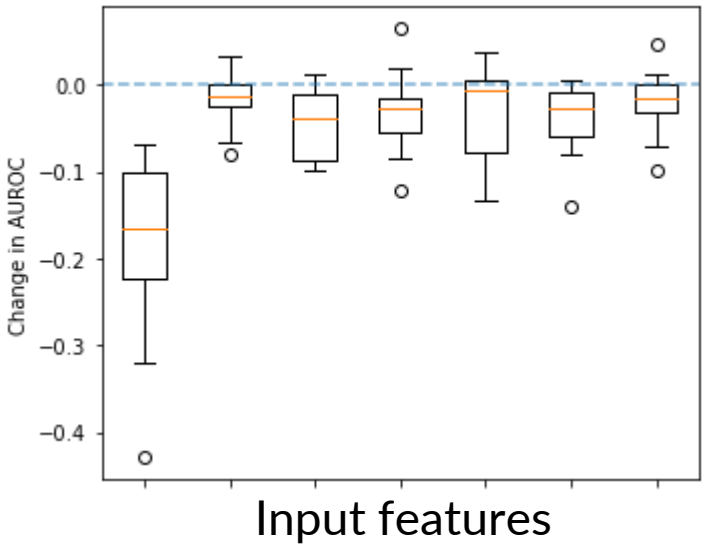
Feature importance





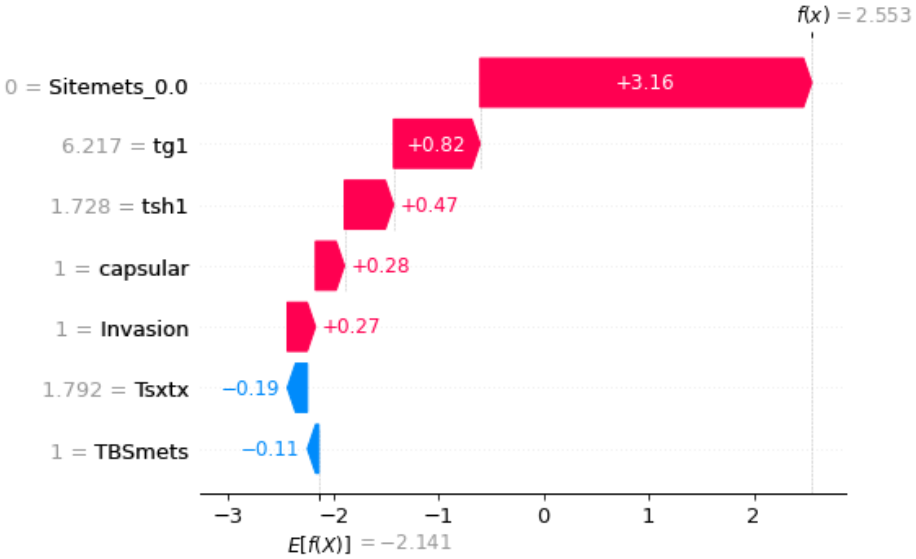
- Coefficients of linear, logistic, and SVM models
- Average improvement in impurity or entropy in tree models
- Model-level explanation

Change in performance after dropping a feature



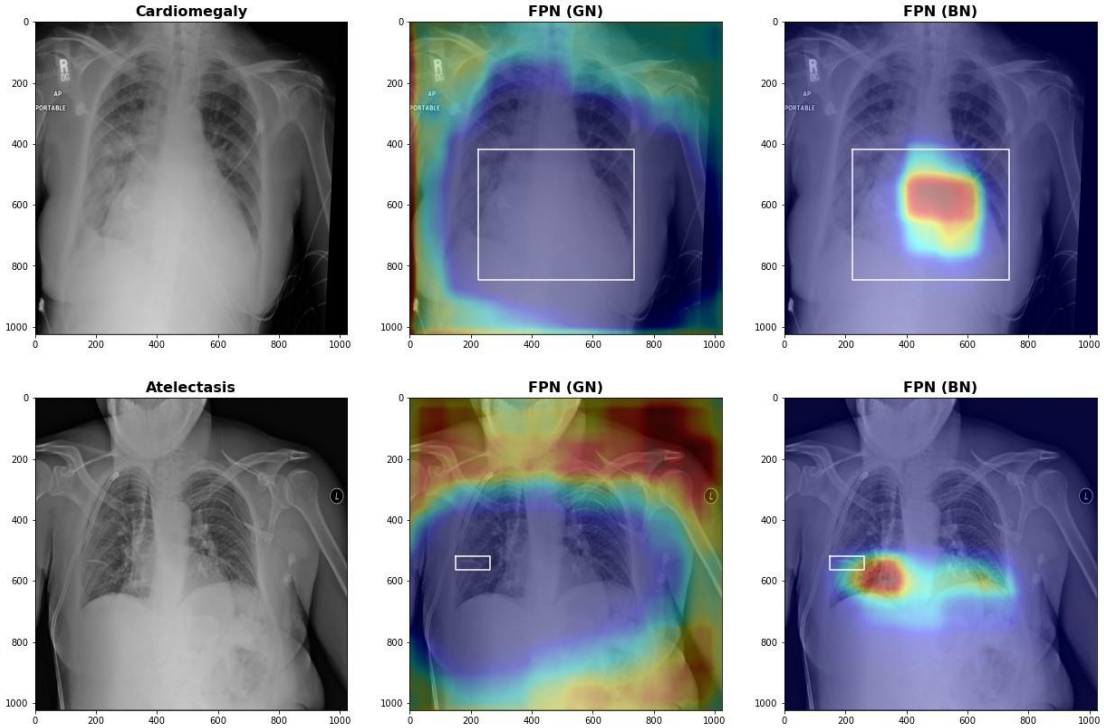
- Compare performance with and without each input feature
- Big drop = important

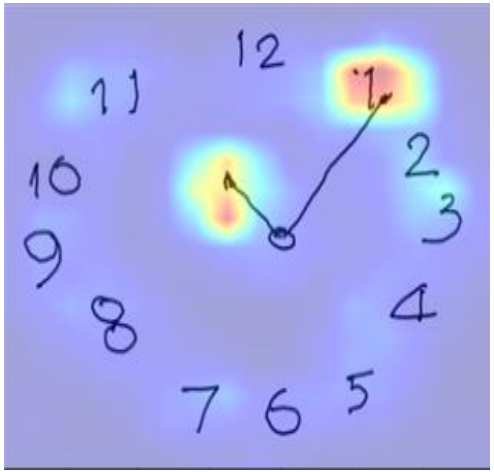
Shapley value



- Change in predicted value from adding a feature *i*
  - $\varphi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n-|S|-1)!}{n!} [v(S \cup \{i\}) - v(S)]$
- Sample-level explanation

Explainability is needed for complex model





Overall summary

- Machine learning vs statistics = data driven vs hypothesis driven
- ML components = **model architecture** + **objective** + **learning algorithm**
- Heart of ML = balancing between data fitting and generalization
- Explainability/interpretability is key

Any question?

- See you next week on November 22nd 9-10:30am