OpenGeoProver Output for conjecture "Chou 335 (Converse of Ceva's Theorem)"

Wu's method used

February 20, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Free point C
- Line BC through two points B and C
- Random point D from line BC
- Line CA through two points C and A
- Random point E from line CA
- Generalized segment division point F of segment AB with respect to ratio product (CD/DB)*(AE/EC) and coefficient 1.0
- Line AD through two points A and D
- Line BE through two points B and E
- Line CF through two points C and F

Theorem statement:

- Lines AD, BE, CF are concurrent

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

2.2 Transformation of point B:

- Point B has been assigned following coordinates: $(0, u_1)$

2.3 Transformation of point C:

- Point C has been assigned following coordinates: (u_2, u_3)

2.4 Transformation of point D:

- Point D has been assigned following coordinates: (u_4, x_1)
- Polynomial that point D has to satisfy is:

$$p = u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1)$$

- Processing of polynomial

$$p = u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1)$$

Info: Polynomial

$$p = u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of point E:

- Point E has been assigned following coordinates: (u_5, x_2)
- Polynomial that point E has to satisfy is:

$$p = u_2x_2 - u_5u_3$$

- Processing of polynomial

$$p = u_2x_2 - u_5u_3$$

Info: Polynomial

$$p = u_2x_2 - u_5u_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point F:

- Point F has been assigned following coordinates: (x_3, x_4)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_3$$

Info: Will try to rename X coordinate of point F

Info: Y coordinate of point F will be replaced by X coordinate

Info: X coordinate of point F renamed by zero

- Point F has been renamed. Point F has been assigned following coordinates: (0, x_3)
- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = (u_5u_4 - 0.5u_5u_2 - 0.5u_4u_2)x_3 + (-0.5u_5u_4u_1 + 0.5u_5u_2u_1)$$

Info: Polynomial

$$p = (u_5u_4 - 0.5u_5u_2 - 0.5u_4u_2)x_3 + (-0.5u_5u_4u_1 + 0.5u_5u_2u_1)$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = (u_5u_4 - 0.5u_5u_2 - 0.5u_4u_2)x_3 + (-0.5u_5u_4u_1 + 0.5u_5u_2u_1)$$

is added to polynomial system

Transformation of Theorem statement

2.7 Transformation of point intersectPoint-BE.CF:

- Point intersectPoint-BE.CF has been assigned following coordinates: (x_4, x_5)
- Polynomial that point intersectPoint-BE.CF has to satisfy is:

$$p = u_5x_5 - x_4x_2 + u_1x_4 - u_5u_1$$

- Processing of polynomial

$$p = u_5x_5 - x_4x_2 + u_1x_4 - u_5u_1$$

Info: Polynomial

$$p = u_5x_5 - x_4x_2 + u_1x_4 - u_5u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point intersectPoint-BE.CF has to satisfy is:

$$p = u_2x_5 + x_4x_3 - u_3x_4 - u_2x_3$$

- Processing of polynomial

$$p = u_2x_5 + x_4x_3 - u_3x_4 - u_2x_3$$

Info: Polynomial

$$p = u_2x_5 + x_4x_3 - u_3x_4 - u_2x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial for theorem statement:

$$p = u_4x_5 - x_4x_1$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.09 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1) \\
p_2 &= u_2x_2 - u_5u_3 \\
p_3 &= (u_5u_4 - 0.5u_5u_2 - 0.5u_4u_2)x_3 + (-0.5u_5u_4u_1 + 0.5u_5u_2u_1) \\
p_4 &= u_5x_5 - x_4x_2 + u_1x_4 - u_5u_1 \\
p_5 &= u_2x_5 + x_4x_3 - u_3x_4 - u_2x_3
\end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_5 from all other polynomials by reducing them with polynomial p_4 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1) \\
p_2 &= u_2x_2 - u_5u_3 \\
p_3 &= (u_5u_4 - 0.5u_5u_2 - 0.5u_4u_2)x_3 + (-0.5u_5u_4u_1 + 0.5u_5u_2u_1) \\
p_4 &= u_5x_4x_3 + u_2x_4x_2 + (-u_5u_3 - u_2u_1)x_4 - u_5u_2x_3 + u_5u_2u_1 \\
p_5 &= u_5x_5 - x_4x_2 + u_1x_4 - u_5u_1
\end{aligned}
$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1) \\
p_2 &= u_2x_2 - u_5u_3 \\
p_3 &= (u_5u_4 - 0.5u_5u_2 - 0.5u_4u_2)x_3 + (-0.5u_5u_4u_1 + 0.5u_5u_2u_1) \\
p_4 &= u_5x_4x_3 + u_2x_4x_2 + (-u_5u_3 - u_2u_1)x_4 - u_5u_2x_3 + u_5u_2u_1 \\
p_5 &= u_5x_5 - x_4x_2 + u_1x_4 - u_5u_1
\end{aligned}
$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 335 (Converse of Ceva's Theorem)

Calculating final remainder of the conclusion:

$$g = u_4x_5 - x_4x_1$$

with respect to the triangular system.

1. Pseudo remainder with p_5 over variable x_5:

$$g = u_4x_4x_2 - u_5x_4x_1 - u_4u_1x_4 + u_5u_4u_1$$

2. Pseudo remainder with p_4 over variable x_4:

$$
\begin{aligned}
g = \; & u_5u_4u_2x_3x_2 - u_5^2u_2x_3x_1 + \\
& (u_5^2u_4u_1 - u_5u_4u_2u_1)x_3 + u_5^2u_2u_1x_1 \\
& -u_5^2u_4u_3u_1
\end{aligned}
$$

3. Pseudo remainder with p_3 over variable x_3:

$$
\begin{aligned}
g = \; & (0.5u_5^2u_4^2u_2u_1 - 0.5u_5^2u_4u_2^2u_1)x_2 + \\
& (0.5u_5^3u_4u_2u_1 - 0.5u_5^2u_4u_2^2u_1)x_1 + \\
& (-u_5^3u_4^2u_3u_1 + 0.5u_5^3u_4^2u_1^2 + \\
& 0.5u_5^3u_4u_3u_2u_1 - 0.5u_5^3u_4u_2u_1^2 + \\
& 0.5u_5^2u_4^2u_3u_2u_1 - 0.5u_5^2u_4^2u_2u_1^2 + \\
& 0.5u_5^2u_4u_2^2u_1^2)
\end{aligned}
$$

4. Pseudo remainder with p_2 over variable x_2:

$$
\begin{aligned}
g &= (0.5u_5^3u_4u_2^2u_1 - 0.5u_5^2u_4u_2^3u_1)x_1 + \\
&\quad (-0.5u_5^3u_4^2u_3u_2u_1 + 0.5u_5^3u_4^2u_2u_1^2 \\
&\quad -0.5u_5^3u_4u_2^2u_1^2 + 0.5u_5^2u_4^2u_3u_2^2u_1 \\
&\quad -0.5u_5^2u_4^2u_2^2u_1^2 + 0.5u_5^2u_4u_2^3u_1^2)
\end{aligned}
$$

5. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 5 terms.

Time Complexity: Time spent by the prover is 0.07 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points F, A, B and C are not collinear
- Points D, E, B and C are not collinear
- Line through points E and B is not parallel with line through points F and C
- Points E and B are not identical

Time spent for processing NDG Conditions

- 0.928 seconds