OpenGeoProver Output for RC-Constructibility problem "Wernick 124 (Ha, Hb, Hc — A)"

Used algebraic method (with triangulation)

July 14, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point Ha
- Free point Hb
- Free point Hc
- Angle bisector hb of angle HaHbHc
- Angle bisector hc of angle HbHcHa
- Line b through point Hb perpendicular to line hb
- Line c through point Hc perpendicular to line hc
- Intersection point A of point sets b and c

Free points:

- Ha
- Hb
- Hc

Points to be constructed:

- A

Info: Attempting to add the construction of new random point tempPoint-394hb necessary for completion of construction of line b

Warrning: Generated new random point tempPoint-394hb on line hb in order to complete the construction of perpendicular line b

Info: Attempting to add the construction of new random point tempPoint-154hc necessary for completion of construction of line c

Warrning: Generated new random point tempPoint-154hc on line hc in order to complete the construction of perpendicular line c

Validation result: Construction protocol is valid.

2 Instantiation of points with symbolic variables

- Point A has been assigned following coordinates: (x_1, x_2)
- Point Ha has been assigned following coordinates: (u_1, u_2)
- Point Hb has been assigned following coordinates: (u_3, u_4)
- Point Hc has been assigned following coordinates: (u_5, u_6)
- Point tempPoint-394hb has been assigned following coordinates: (u_7, x_3)
- Point tempPoint-154hc has been assigned following coordinates: (u_8, x_4)

3 Transformation of geometry conditions for points to polynomial form

3.1 Transformation, step 1

Point to transform: Ha

Polynomial condition(s): N/A - free point

3.2 Transformation, step 2

Point to transform: Hb

Polynomial condition(s): N/A - free point

3.3 Transformation, step 3

Point to transform: Hc

Polynomial condition(s): N/A - free point

3.4 Transformation, step 4

Point to transform: tempPoint-394hb

Polynomial condition(s): One polynomial

•

$$
\begin{aligned}
p \;=\;& (-u_6u_3 + u_6u_1 - u_5u_4 + u_5u_2 + 2u_4u_3 - u_4u_1 - u_3u_2)x_3^2 + \\
& (2u_7u_6u_4 - 2u_7u_6u_2 - 2u_7u_5u_3 + 2u_7u_5u_1 - 2u_7u_4^2 + \\
& 2u_7u_4u_2 + 2u_7u_3^2 - 2u_7u_3u_1 - 2u_6u_4u_1 + 2u_6u_3u_2 + \\
& 2u_5u_4^2 - 2u_5u_4u_2 + 2u_5u_3^2 - 2u_5u_3u_1 - 2u_4^2u_3 + \\
& 2u_4^2u_1 - 2u_3^3 + 2u_3^2u_1) \\
& x_3 \\
& + \\
& (u_7^2u_6u_3 - u_7^2u_6u_1 + u_7^2u_5u_4 - u_7^2u_5u_2 \\
& -2u_7^2u_4u_3 + u_7^2u_4u_1 + u_7^2u_3u_2 - 2u_7u_6u_4^2 + \\
& 2u_7u_6u_4u_2 - 2u_7u_6u_3^2 + 2u_7u_6u_3u_1 - 2u_7u_5u_4u_1 + \\
& 2u_7u_5u_3u_2 + 2u_7u_4^3 - 2u_7u_4^2u_2 + 2u_7u_4u_3^2 \\
& -2u_7u_3^2u_2 + u_6u_4^2u_3 + u_6u_4^2u_1 - 2u_6u_4u_3u_2 + \\
& u_6u_3^3 - u_6u_3^2u_1 - u_5u_4^3 + u_5u_4^2u_2 - u_5u_4u_3^2 + \\
& 2u_5u_4u_3u_1 - u_5u_3^2u_2 - u_4^3u_1 + u_4^2u_3u_2 \\
& -u_4u_3^2u_1 + u_3^3u_2)
\end{aligned}
$$

3.5 Transformation, step 5

Point to transform: tempPoint-154hc

Polynomial condition(s): One polynomial

•

$$
\begin{aligned}
p \;=\;& (2u_6u_5 - u_6u_3 - u_6u_1 - u_5u_4 - u_5u_2 + u_4u_1 + u_3u_2)x_4^2 + \\
& (-2u_8u_6^2 + 2u_8u_6u_4 + 2u_8u_6u_2 + 2u_8u_5^2 - 2u_8u_5u_3 \\
& -2u_8u_5u_1 - 2u_8u_4u_2 + 2u_8u_3u_1 - 2u_6^2u_5 + 2u_6^2u_3 + \\
& 2u_6^2u_1 - 2u_6u_4u_1 - 2u_6u_3u_2 - 2u_5^3 + 2u_5^2u_3 + 2u_5^2u_1 + \\
& 2u_5u_4u_2 - 2u_5u_3u_1) \\
& x_4 \\
& + \\
& (-2u_8^2u_6u_5 + u_8^2u_6u_3 + u_8^2u_6u_1 + u_8^2u_5u_4 + \\
& u_8^2u_5u_2 - u_8^2u_4u_1 - u_8^2u_3u_2 + 2u_8u_6^3 \\
& -2u_8u_6^2u_4 - 2u_8u_6^2u_2 + 2u_8u_6u_5^2 + 2u_8u_6u_4u_2 \\
& -2u_8u_6u_3u_1 - 2u_8u_5^2u_4 - 2u_8u_5^2u_2 + 2u_8u_5u_4u_1 + \\
& 2u_8u_5u_3u_2 - u_6^3u_3 - u_6^3u_1 + u_6^2u_5u_4 + u_6^2u_5u_2 + \\
& u_6^2u_4u_1 + u_6^2u_3u_2 - u_6u_5^2u_3 - u_6u_5^2u_1 \\
& -2u_6u_5u_4u_2 + 2u_6u_5u_3u_1 + u_5^3u_4 + u_5^3u_2 \\
& -u_5^2u_4u_1 - u_5^2u_3u_2)
\end{aligned}
$$

3.6 Transformation, step 6

Point to transform: A

Polynomial condition(s): Two polynomials

- $$p = x_3x_2 - u_4x_3 - u_4x_2 + (u_7 - u_3)x_1 + (-u_7u_3 + u_4^2 + u_3^2)$$

- $$p = x_4x_2 - u_6x_4 - u_6x_2 + (u_8 - u_5)x_1 + (-u_8u_5 + u_6^2 + u_5^2)$$

4 Triangulation of polynomial system

The input system is:

$$\begin{aligned}
p_1 = {} & (-u_6u_3 + u_6u_1 - u_5u_4 + u_5u_2 + 2u_4u_3 - u_4u_1 - u_3u_2)x_3^2 + \\
& (2u_7u_6u_4 - 2u_7u_6u_2 - 2u_7u_5u_3 + 2u_7u_5u_1 - 2u_7u_4^2 + \\
& 2u_7u_4u_2 + 2u_7u_3^2 - 2u_7u_3u_1 - 2u_6u_4u_1 + 2u_6u_3u_2 + \\
& 2u_5u_4^2 - 2u_5u_4u_2 + 2u_5u_3^2 - 2u_5u_3u_1 - 2u_4^2u_3 + \\
& 2u_4^2u_1 - 2u_3^3 + 2u_3^2u_1) \\
& x_3 \\
& + \\
& (u_7^2u_6u_3 - u_7^2u_6u_1 + u_7^2u_5u_4 - u_7^2u_5u_2 \\
& -2u_7^2u_4u_3 + u_7^2u_4u_1 + u_7^2u_3u_2 - 2u_7u_6u_4^2 + \\
& 2u_7u_6u_4u_2 - 2u_7u_6u_3^2 + 2u_7u_6u_3u_1 - 2u_7u_5u_4u_1 + \\
& 2u_7u_5u_3u_2 + 2u_7u_4^3 - 2u_7u_4^2u_2 + 2u_7u_4u_3^2 \\
& -2u_7u_3^2u_2 + u_6u_4^2u_3 + u_6u_4^2u_1 - 2u_6u_4u_3u_2 + \\
& u_6u_3^3 - u_6u_3^2u_1 - u_5u_4^3 + u_5u_4^2u_2 - u_5u_4u_3^2 + \\
& 2u_5u_4u_3u_1 - u_5u_3^2u_2 - u_4^3u_1 + u_4^2u_3u_2 \\
& -u_4u_3^2u_1 + u_3^3u_2)
\end{aligned}$$

$$\begin{aligned}
p_2 = {} & (2u_6u_5 - u_6u_3 - u_6u_1 - u_5u_4 - u_5u_2 + u_4u_1 + u_3u_2)x_4^2 + \\
& (-2u_8u_6^2 + 2u_8u_6u_4 + 2u_8u_6u_2 + 2u_8u_5^2 - 2u_8u_5u_3 \\
& -2u_8u_5u_1 - 2u_8u_4u_2 + 2u_8u_3u_1 - 2u_6^2u_5 + 2u_6^2u_3 + \\
& 2u_6^2u_1 - 2u_6u_4u_1 - 2u_6u_3u_2 - 2u_5^3 + 2u_5^2u_3 + 2u_5^2u_1 + \\
& 2u_5u_4u_2 - 2u_5u_3u_1) \\
& x_4 \\
& + \\
& (-2u_8^2u_6u_5 + u_8^2u_6u_3 + u_8^2u_6u_1 + u_8^2u_5u_4 + \\
& u_8^2u_5u_2 - u_8^2u_4u_1 - u_8^2u_3u_2 + 2u_8u_6^3 \\
& -2u_8u_6^2u_4 - 2u_8u_6^2u_2 + 2u_8u_6u_5^2 + 2u_8u_6u_4u_2
\end{aligned}$$

$$
\begin{aligned}
&-2u_8u_6u_3u_1 - 2u_8u_5^2u_4 - 2u_8u_5^2u_2 + 2u_8u_5u_4u_1 + \\
&2u_8u_5u_3u_2 - u_6^3u_3 - u_6^3u_1 + u_6^2u_5u_4 + u_6^2u_5u_2 + \\
&u_6^2u_4u_1 + u_6^2u_3u_2 - u_6u_5^2u_3 - u_6u_5^2u_1 \\
&-2u_6u_5u_4u_2 + 2u_6u_5u_3u_1 + u_5^3u_4 + u_5^3u_2 \\
&-u_5^2u_4u_1 - u_5^2u_3u_2)
\end{aligned}
$$

$$
\begin{aligned}
p_3 &= x_3x_2 - u_4x_3 - u_4x_2 + (u_7 - u_3)x_1 + (-u_7u_3 + u_4^2 + u_3^2) \\
p_4 &= x_4x_2 - u_6x_4 - u_6x_2 + (u_8 - u_5)x_1 + (-u_8u_5 + u_6^2 + u_5^2)
\end{aligned}
$$

4.1 Triangulation, step 1

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 2.

Minimal degrees: 1 polynomial(s) with degree 1 and 1 polynomial(s) with degree 2.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_4 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (-u_6u_3 + u_6u_1 - u_5u_4 + u_5u_2 + 2u_4u_3 - u_4u_1 - u_3u_2)x_3^2 + \\
&(2u_7u_6u_4 - 2u_7u_6u_2 - 2u_7u_5u_3 + 2u_7u_5u_1 - 2u_7u_4^2 + \\
&2u_7u_4u_2 + 2u_7u_3^2 - 2u_7u_3u_1 - 2u_6u_4u_1 + 2u_6u_3u_2 + \\
&2u_5u_4^2 - 2u_5u_4u_2 + 2u_5u_3^2 - 2u_5u_3u_1 - 2u_4^2u_3 + \\
&2u_4^2u_1 - 2u_3^3 + 2u_3^2u_1) \\
&x_3 \\
&+ \\
&(u_7^2u_6u_3 - u_7^2u_6u_1 + u_7^2u_5u_4 - u_7^2u_5u_2 \\
&-2u_7^2u_4u_3 + u_7^2u_4u_1 + u_7^2u_3u_2 - 2u_7u_6u_4^2 + \\
&2u_7u_6u_4u_2 - 2u_7u_6u_3^2 + 2u_7u_6u_3u_1 - 2u_7u_5u_4u_1 + \\
&2u_7u_5u_3u_2 + 2u_7u_4^3 - 2u_7u_4^2u_2 + 2u_7u_4u_3^2 \\
&-2u_7u_3^2u_2 + u_6u_4^2u_3 + u_6u_4^2u_1 - 2u_6u_4u_3u_2 + \\
&u_6u_3^3 - u_6u_3^2u_1 - u_5u_4^3 + u_5u_4^2u_2 - u_5u_4u_3^2 + \\
&2u_5u_4u_3u_1 - u_5u_3^2u_2 - u_4^3u_1 + u_4^2u_3u_2 \\
&-u_4u_3^2u_1 + u_3^3u_2)
\end{aligned}
$$

$$
\begin{aligned}
p_2 &= x_3x_2 - u_4x_3 - u_4x_2 + (u_7 - u_3)x_1 + (-u_7u_3 + u_4^2 + u_3^2) \\
p_3 &= \dots \\
p_4 &= x_4x_2 - u_6x_4 - u_6x_2 + (u_8 - u_5)x_1 + (-u_8u_5 + u_6^2 + u_5^2)
\end{aligned}
$$

4.2 Triangulation, step 2

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 2.

Minimal degrees: 1 polynomial(s) with degree 1 and 1 polynomial(s) with degree 2.

Polynomial with linear degree: Removing variable x_3 from all other polynomials by reducing them with polynomial p_2 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_1 &= \dots \\ p_2 &= \dots \\ p_3 &= x_3x_2 - u_4x_3 - u_4x_2 + (u_7 - u_3)x_1 + (-u_7u_3 + u_4^2 + u_3^2) \\ p_4 &= x_4x_2 - u_6x_4 - u_6x_2 + (u_8 - u_5)x_1 + (-u_8u_5 + u_6^2 + u_5^2) \end{aligned}$$

4.3 Triangulation, step 3

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 2.

No linear degree polynomials: Reducing polynomial p_2 (of degree 2) with p_1 (of degree 2).

Finished a triangulation step, the current system is:

$$\begin{aligned} p_1 &= \dots \\ p_2 &= \dots \\ p_3 &= x_3x_2 - u_4x_3 - u_4x_2 + (u_7 - u_3)x_1 + (-u_7u_3 + u_4^2 + u_3^2) \\ p_4 &= x_4x_2 - u_6x_4 - u_6x_2 + (u_8 - u_5)x_1 + (-u_8u_5 + u_6^2 + u_5^2) \end{aligned}$$

4.4 Triangulation, step 4

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_1 &= \ldots \\
p_2 &= \ldots \\
p_3 &= x_3x_2 - u_4x_3 - u_4x_2 + (u_7 - u_3)x_1 + (-u_7u_3 + u_4^2 + u_3^2) \\
p_4 &= x_4x_2 - u_6x_4 - u_6x_2 + (u_8 - u_5)x_1 + (-u_8u_5 + u_6^2 + u_5^2)
\end{aligned}$$

List of leading monic terms from triangular polynomial system

$$p_1 = x_1^4$$

$$p_2 = x_2x_1$$

$$p_3 = x_3x_2$$

$$p_4 = x_4x_2$$

5 Result of transformation of RC-constructibility problem to polynomial form

Success Message: Successful completion.

Space Complexity: The biggest polynomial obtained during application execution contains 7 terms.

Time Complexity: Time spent in execution is 163.779 seconds.