OpenGeoProver Output for RC-Constructibility problem "Wernick 124 (Ha, Hb, Hc —— A)"

Used algebraic method (with triangulation)

July 14, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Free point C
- Line a through two points B and C
- Line b through two points C and A
- Line c through two points A and B
- Line ha through point A perpendicular to line a
- Line hb through point B perpendicular to line b
- Intersection point Ha of point sets a and ha
- Intersection point Hb of point sets b and hb
- Intersection point H of point sets ha and hb
- Line hc through two points C and H
- Intersection point Hc of point sets c and hc

Free points:

- Ha
- Hb
- Hc

Points to be constructed:

- A

Validation result: Construction protocol is valid.

2 Instantiation of points with symbolic variables

- Point A has been assigned following coordinates: (x_1, x_2)
- Point Ha has been assigned following coordinates: (u_1, u_2)
- Point Hb has been assigned following coordinates: (u_3, u_4)
- Point Hc has been assigned following coordinates: (u_5, u_6)
- Point B has been assigned following coordinates: (x_3, x_4)
- Point C has been assigned following coordinates: (x_5, x_6)
- Point H has been assigned following coordinates: (x_7, x_8)

3 Transformation of geometry conditions for points to polynomial form

3.1 Transformation, step 1

Point to transform: A

Polynomial condition(s): N/A - free point

3.2 Transformation, step 2

Point to transform: B

Polynomial condition(s): N/A - free point

3.3 Transformation, step 3

Point to transform: C

Polynomial condition(s): N/A - free point

3.4 Transformation, step 4

Point to transform: Ha

Polynomial condition(s): Two polynomials

- $$p = -x_6x_3 + u_1x_6 + x_5x_4 - u_2x_5 - u_1x_4 + u_2x_3$$
- $$p = -x_6x_2 + u_2x_6 - x_5x_1 + u_1x_5 + x_4x_2 - u_2x_4 + x_3x_1 - u_1x_3$$

3.5 Transformation, step 5

Point to transform: Hb

Polynomial condition(s): Two polynomials

- $$p = x_6x_1 - u_3x_6 - x_5x_2 + u_4x_5 + u_3x_2 - u_4x_1$$
- $$p = x_6x_4 - u_4x_6 + x_5x_3 - u_3x_5 - x_4x_2 - x_3x_1 + u_4x_2 + u_3x_1$$

3.6 Transformation, step 6

Point to transform: H

Polynomial condition(s): Two polynomials

- $$p = x_8x_6 - x_8x_4 + x_7x_5 - x_7x_3 - x_6x_2 - x_5x_1 + x_4x_2 + x_3x_1$$
- $$p = -x_8x_6 + x_8x_2 - x_7x_5 + x_7x_1 + x_6x_4 + x_5x_3 - x_4x_2 - x_3x_1$$

3.7 Transformation, step 7

Point to transform: Hc

Polynomial condition(s): Two polynomials

- $$p = -x_4x_1 + u_5x_4 + x_3x_2 - u_6x_3 - u_5x_2 + u_6x_1$$
- $$p = -x_8x_5 + u_5x_8 + x_7x_6 - u_6x_7 - u_5x_6 + u_6x_5$$

4 Triangulation of polynomial system

The input system is:

$$\begin{aligned}
p_1 &= -x_6x_3 + u_1x_6 + x_5x_4 - u_2x_5 - u_1x_4 + u_2x_3 \\
p_2 &= -x_6x_2 + u_2x_6 - x_5x_1 + u_1x_5 + x_4x_2 - u_2x_4 + x_3x_1 - u_1x_3 \\
p_3 &= x_6x_1 - u_3x_6 - x_5x_2 + u_4x_5 + u_3x_2 - u_4x_1 \\
p_4 &= x_6x_4 - u_4x_6 + x_5x_3 - u_3x_5 - x_4x_2 - x_3x_1 + u_4x_2 + u_3x_1 \\
p_5 &= x_8x_6 - x_8x_4 + x_7x_5 - x_7x_3 - x_6x_2 - x_5x_1 + x_4x_2 + x_3x_1 \\
p_6 &= -x_8x_6 + x_8x_2 - x_7x_5 + x_7x_1 + x_6x_4 + x_5x_3 - x_4x_2 - x_3x_1 \\
p_7 &= -x_4x_1 + u_5x_4 + x_3x_2 - u_6x_3 - u_5x_2 + u_6x_1 \\
p_8 &= -x_8x_5 + u_5x_8 + x_7x_6 - u_6x_7 - u_5x_6 + u_6x_5
\end{aligned}$$

4.1 Triangulation, step 1

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_8 from all other polynomials by reducing them with polynomial p_5 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_6x_3 + u_1x_6 + x_5x_4 - u_2x_5 - u_1x_4 + u_2x_3 \\
p_2 &= -x_6x_2 + u_2x_6 - x_5x_1 + u_1x_5 + x_4x_2 - u_2x_4 + x_3x_1 - u_1x_3 \\
p_3 &= x_6x_1 - u_3x_6 - x_5x_2 + u_4x_5 + u_3x_2 - u_4x_1 \\
p_4 &= x_6x_4 - u_4x_6 + x_5x_3 - u_3x_5 - x_4x_2 - x_3x_1 + u_4x_2 + u_3x_1 \\
p_5 &= -x_4x_1 + u_5x_4 + x_3x_2 - u_6x_3 - u_5x_2 + u_6x_1 \\
p_6 &= -x_7x_6x_3 + x_7x_6x_1 + x_7x_5x_4 - x_7x_5x_2 - x_7x_4x_1 + \\
&\quad x_7x_3x_2 + x_6^2x_4 - x_6^2x_2 + x_6x_5x_3 - x_6x_5x_1 - x_6x_4^2 + \\
&\quad x_6x_2^2 - x_5x_4x_3 + x_5x_2x_1 + x_4^2x_2 + x_4x_3x_1 - x_4x_2^2 \\
&\quad -x_3x_2x_1 \\
p_7 &= x_7x_6^2 - x_7x_6x_4 - u_6x_7x_6 + x_7x_5^2 - x_7x_5x_3 \\
&\quad -u_5x_7x_5 + u_6x_7x_4 + u_5x_7x_3 - u_5x_6^2 - x_6x_5x_2 + \\
&\quad u_6x_6x_5 + u_5x_6x_4 + u_5x_6x_2 - x_5^2x_1 + x_5x_4x_2 \\
&\quad -u_6x_5x_4 + x_5x_3x_1 + u_5x_5x_1 - u_5x_4x_2 - u_5x_3x_1 \\
p_8 &= x_8x_6 - x_8x_4 + x_7x_5 - x_7x_3 - x_6x_2 - x_5x_1 + x_4x_2 + x_3x_1
\end{aligned}
$$

4.2 Triangulation, step 2

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_7 from all other polynomials by reducing them with polynomial p_6 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_6x_3 + u_1x_6 + x_5x_4 - u_2x_5 - u_1x_4 + u_2x_3 \\
p_2 &= -x_6x_2 + u_2x_6 - x_5x_1 + u_1x_5 + x_4x_2 - u_2x_4 + x_3x_1 - u_1x_3 \\
p_3 &= x_6x_1 - u_3x_6 - x_5x_2 + u_4x_5 + u_3x_2 - u_4x_1 \\
p_4 &= x_6x_4 - u_4x_6 + x_5x_3 - u_3x_5 - x_4x_2 - x_3x_1 + u_4x_2 + u_3x_1
\end{aligned}
$$

$$
\begin{aligned}
p_5 &= -x_4x_1 + u_5x_4 + x_3x_2 - u_6x_3 - u_5x_2 + u_6x_1 \\
p_6 &= \ldots \\
p_7 &= -x_7x_6x_3 + x_7x_6x_1 + x_7x_5x_4 - x_7x_5x_2 - x_7x_4x_1 + \\
&\quad x_7x_3x_2 + x_6^2x_4 - x_6^2x_2 + x_6x_5x_3 - x_6x_5x_1 - x_6x_4^2 + \\
&\quad x_6x_2^2 - x_5x_4x_3 + x_5x_2x_1 + x_4^2x_2 + x_4x_3x_1 - x_4x_2^2 \\
&\quad -x_3x_2x_1 \\
p_8 &= x_8x_6 - x_8x_4 + x_7x_5 - x_7x_3 - x_6x_2 - x_5x_1 + x_4x_2 + x_3x_1
\end{aligned}
$$

4.3 Triangulation, step 3

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 5.

Minimal degrees: 4 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_6 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4x_1 + u_5x_4 + x_3x_2 - u_6x_3 - u_5x_2 + u_6x_1 \\
p_2 &= x_5x_4x_2 - u_2x_5x_4 + x_5x_3x_1 - u_1x_5x_3 - u_2x_5x_2 \\
&\quad -u_1x_5x_1 + (u_2^2 + u_1^2)x_5 - x_4x_3x_2 + u_2x_4x_3 - x_3^2x_1 + \\
&\quad u_1x_3^2 + u_2x_3x_2 + u_1x_3x_1 + (-u_2^2 - u_1^2)x_3 \\
p_3 &= -x_5x_4x_1 + u_3x_5x_4 + x_5x_3x_2 - u_4x_5x_3 - u_1x_5x_2 + \\
&\quad u_2x_5x_1 + (u_4u_1 - u_3u_2)x_5 + u_1x_4x_1 - u_3u_1x_4 - u_3x_3x_2 + \\
&\quad (u_4 - u_2)x_3x_1 + u_3u_2x_3 + u_3u_1x_2 - u_4u_1x_1 \\
p_4 &= -x_5x_4^2 + (u_4 + u_2)x_5x_4 - x_5x_3^2 + (u_3 + u_1)x_5x_3 + \\
&\quad (-u_4u_2 - u_3u_1)x_5 + u_1x_4^2 + x_4x_3x_2 - u_2x_4x_3 - u_1x_4x_2 \\
&\quad -u_4u_1x_4 + x_3^2x_1 - u_4x_3x_2 + (-u_3 - u_1)x_3x_1 + u_4u_2x_3 + \\
&\quad u_4u_1x_2 + u_3u_1x_1 \\
p_5 &= \ldots \\
p_6 &= -x_6x_3 + u_1x_6 + x_5x_4 - u_2x_5 - u_1x_4 + u_2x_3 \\
p_7 &= -x_7x_6x_3 + x_7x_6x_1 + x_7x_5x_4 - x_7x_5x_2 - x_7x_4x_1 + \\
&\quad x_7x_3x_2 + x_6^2x_4 - x_6^2x_2 + x_6x_5x_3 - x_6x_5x_1 - x_6x_4^2 + \\
&\quad x_6x_2^2 - x_5x_4x_3 + x_5x_2x_1 + x_4^2x_2 + x_4x_3x_1 - x_4x_2^2 \\
&\quad -x_3x_2x_1 \\
p_8 &= x_8x_6 - x_8x_4 + x_7x_5 - x_7x_3 - x_6x_2 - x_5x_1 + x_4x_2 + x_3x_1
\end{aligned}
$$

4.4 Triangulation, step 4

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 4.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_5 from all other polynomials by reducing them with polynomial p_2 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4x_1 + u_5x_4 + x_3x_2 - u_6x_3 - u_5x_2 + u_6x_1 \\
p_2 &= -x_4^2x_3x_2x_1 + u_3x_4^2x_3x_2 + u_2x_4^2x_3x_1 \\
&\quad -u_3u_2x_4^2x_3 + u_1x_4^2x_2x_1 - u_3u_1x_4^2x_2 \\
&\quad -u_2u_1x_4^2x_1 + u_3u_2u_1x_4^2 + x_4x_3^2x_2^2 + \\
&\quad (-u_4 - u_2)x_4x_3^2x_2 - x_4x_3^2x_1^2 + (u_3 + u_1)x_4x_3^2x_1 + \\
&\quad (u_4u_2 - u_3u_1)x_4x_3^2 + (-u_3 - u_1)x_4x_3x_2^2 + \\
&\quad (u_4 + u_2)x_4x_3x_2x_1 + (u_4u_1 + u_2u_1)x_4x_3x_2 + \\
&\quad 2u_1x_4x_3x_1^2 + (-u_4u_2 - 2u_3u_1 - u_2^2 - 2u_1^2)x_4x_3x_1 + \\
&\quad (-u_4u_2u_1 + u_3u_2^2 + 2u_3u_1^2)x_4x_3 + u_3u_1x_4x_2^2 + \\
&\quad (-u_4u_1 - u_2u_1)x_4x_2x_1 - u_1^2x_4x_1^2 + \\
&\quad (u_4u_2u_1 + u_3u_1^2 + u_2^2u_1 + u_1^3)x_4x_1 + \\
&\quad (-u_3u_2^2u_1 - u_3u_1^3)x_4 + x_3^3x_2x_1 - u_1x_3^3x_2 \\
&\quad -u_4x_3^3x_1 + u_4u_1x_3^3 - u_2x_3^2x_2^2 + \\
&\quad (-u_3 - 2u_1)x_3^2x_2x_1 + \\
&\quad (u_4u_2 + u_3u_1 + u_2^2 + 2u_1^2)x_3^2x_2 + u_4x_3^2x_1^2 + \\
&\quad u_4u_1x_3^2x_1 + (-u_4u_2^2 - 2u_4u_1^2)x_3^2 + \\
&\quad (u_3u_2 + u_2u_1)x_3x_2^2 + (-u_4u_2 + 2u_3u_1 + u_1^2)x_3x_2x_1 + \\
&\quad (-u_4u_2u_1 - u_3u_2^2 - 2u_3u_1^2 - u_2^2u_1 - u_1^3)x_3x_2 \\
&\quad -2u_4u_1x_3x_1^2 + (u_4u_2^2 + u_4u_1^2)x_3x_1 + \\
&\quad (u_4u_2^2u_1 + u_4u_1^3)x_3 - u_3u_2u_1x_2^2 + \\
&\quad (u_4u_2u_1 - u_3u_1^2)x_2x_1 + (u_3u_2^2u_1 + u_3u_1^3)x_2 + \\
&\quad u_4u_1^2x_1^2 + (-u_4u_2^2u_1 - u_4u_1^3)x_1 \\
p_3 &= -x_4^3x_3x_2 + u_2x_4^3x_3 + u_1x_4^3x_2 - u_2u_1x_4^3 \\
&\quad -x_4^2x_3^2x_1 + u_1x_4^2x_3^2 + x_4^2x_3x_2^2 + \\
&\quad u_4x_4^2x_3x_2 + 2u_1x_4^2x_3x_1 + \\
&\quad (-u_4u_2 - u_2^2 - 2u_1^2)x_4^2x_3 - u_1x_4^2x_2^2 \\
&\quad -u_4u_1x_4^2x_2 - u_1^2x_4^2x_1 + \\
&\quad (u_4u_2u_1 + u_2^2u_1 + u_1^3)x_4^2 - x_4x_3^3x_2 + u_2x_4x_3^3 + \\
&\quad 2x_4x_3^2x_2x_1 + u_3x_4x_3^2x_2 + (u_4 - u_2)x_4x_3^2x_1 + \\
&\quad (-u_4u_1 - u_3u_2 - u_2u_1)x_4x_3^2 + (-u_4 - u_2)x_4x_3x_2^2 + \\
&\quad (-u_3 - 3u_1)x_4x_3x_2x_1 + (-u_3u_1 + u_2^2 + 2u_1^2)x_4x_3x_2 + \\
&\quad (-2u_4u_1 + u_3u_2 + u_2u_1)x_4x_3x_1 +
\end{aligned}
$$

$$
\begin{aligned}
& (u_4u_2^2 + 2u_4u_1^2 + u_3u_2u_1)x_4x_3 + \\
& (u_4u_1 + u_2u_1)x_4x_2^2 + (u_3u_1 + u_1^2)x_4x_2x_1 + \\
& (-u_2^2u_1 - u_1^3)x_4x_2 + (u_4u_1^2 - u_3u_2u_1)x_4x_1 + \\
& (-u_4u_2^2u_1 - u_4u_1^3)x_4 - x_3^4x_1 + u_1x_3^4 + u_2x_3^3x_2 + \\
& x_3^3x_1^2 + (u_3 + u_1)x_3^3x_1 + (-u_3u_1 - u_2^2 - 2u_1^2)x_3^3 + \\
& (-u_4 - u_2)x_3^2x_2x_1 + (u_4u_1 - u_3u_2 - u_2u_1)x_3^2x_2 + \\
& (-u_3 - 2u_1)x_3^2x_1^2 + (-u_3u_1 + u_2^2 + u_1^2)x_3^2x_1 + \\
& (u_3u_2^2 + 2u_3u_1^2 + u_2^2u_1 + u_1^3)x_3^2 + u_4u_2x_3x_2^2 + \\
& (2u_4u_1 + u_3u_2 + u_2u_1)x_3x_2x_1 + \\
& (-u_4u_2^2 - 2u_4u_1^2 + u_3u_2u_1)x_3x_2 + \\
& (2u_3u_1 + u_1^2)x_3x_1^2 + \\
& (-u_3u_2^2 - u_3u_1^2 - u_2^2u_1 - u_1^3)x_3x_1 + \\
& (-u_3u_2^2u_1 - u_3u_1^3)x_3 - u_4u_2u_1x_2^2 + \\
& (-u_4u_1^2 - u_3u_2u_1)x_2x_1 + (u_4u_2^2u_1 + u_4u_1^3)x_2 \\
& -u_3u_1^2x_1^2 + (u_3u_2^2u_1 + u_3u_1^3)x_1 \\
p_4 \;=\; & 0 \\
p_5 \;=\; & x_5x_4x_2 - u_2x_5x_4 + x_5x_3x_1 - u_1x_5x_3 - u_2x_5x_2 \\
& -u_1x_5x_1 + (u_2^2 + u_1^2)x_5 - x_4x_3x_2 + u_2x_4x_3 - x_3^2x_1 + \\
& u_1x_3^2 + u_2x_3x_2 + u_1x_3x_1 + (-u_2^2 - u_1^2)x_3 \\
p_6 \;=\; & -x_6x_3 + u_1x_6 + x_5x_4 - u_2x_5 - u_1x_4 + u_2x_3 \\
p_7 \;=\; & -x_7x_6x_3 + x_7x_6x_1 + x_7x_5x_4 - x_7x_5x_2 - x_7x_4x_1 + \\
& x_7x_3x_2 + x_6^2x_4 - x_6^2x_2 + x_6x_5x_3 - x_6x_5x_1 - x_6x_4^2 + \\
& x_6x_2^2 - x_5x_4x_3 + x_5x_2x_1 + x_4^2x_2 + x_4x_3x_1 - x_4x_2^2 \\
& -x_3x_2x_1 \\
p_8 \;=\; & x_8x_6 - x_8x_4 + x_7x_5 - x_7x_3 - x_6x_2 - x_5x_1 + x_4x_2 + x_3x_1
\end{aligned}
$$

4.5 Triangulation, step 5

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 3.

Minimal degrees: 1 polynomial(s) with degree 1 and 1 polynomial(s) with degree 2.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 0 \\
p_2 &= \dots \\
p_3 &= \dots
\end{aligned}
$$

$$
\begin{aligned}
p_4 &= -x_4x_1 + u_5x_4 + x_3x_2 - u_6x_3 - u_5x_2 + u_6x_1 \\
p_5 &= x_5x_4x_2 - u_2x_5x_4 + x_5x_3x_1 - u_1x_5x_3 - u_2x_5x_2 \\
&\quad -u_1x_5x_1 + (u_2^2 + u_1^2)x_5 - x_4x_3x_2 + u_2x_4x_3 - x_3^2x_1 + \\
&\quad u_1x_3^2 + u_2x_3x_2 + u_1x_3x_1 + (-u_2^2 - u_1^2)x_3 \\
p_6 &= -x_6x_3 + u_1x_6 + x_5x_4 - u_2x_5 - u_1x_4 + u_2x_3 \\
p_7 &= -x_7x_6x_3 + x_7x_6x_1 + x_7x_5x_4 - x_7x_5x_2 - x_7x_4x_1 + \\
&\quad x_7x_3x_2 + x_6^2x_4 - x_6^2x_2 + x_6x_5x_3 - x_6x_5x_1 - x_6x_4^2 + \\
&\quad x_6x_2^2 - x_5x_4x_3 + x_5x_2x_1 + x_4^2x_2 + x_4x_3x_1 - x_4x_2^2 \\
&\quad -x_3x_2x_1 \\
p_8 &= x_8x_6 - x_8x_4 + x_7x_5 - x_7x_3 - x_6x_2 - x_5x_1 + x_4x_2 + x_3x_1
\end{aligned}
$$

4.6 Triangulation, step 6

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 2.

Minimal degrees: 1 polynomial(s) with degree 3 and 1 polynomial(s) with degree 4.

No linear degree polynomials: Reducing polynomial p_2 (of degree 4) with p_1 (of degree 3).

Error: Two polynomials have common factor.Failed to triangulate system of polynomials

5 Result of transformation of RC-constructibility problem to polynomial form

Success Message: Failed to transform the RC-constructibility problem to polynomial form - find more details in log file.

Space Complexity: The biggest polynomial obtained during application execution contains 1344 terms.

Time Complexity: Time spent in execution is 5.652 seconds.