OpenGeoProver Output for conjecture "Chou 019 (Brianchon's theorem for circle)"

Wu's method used

February 14, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point O
- Free point A
- Circle c with center O and one point A
- Random point B from circle c
- Random point C from circle c
- Random point D from circle c
- Random point E from circle c
- Random point F from circle c
- Line ra through two points O and A
- Line rb through two points O and B
- Line rc through two points O and C
- Line rd through two points O and D
- Line re through two points O and E
- Line rf through two points O and F
- Line ta through point A perpendicular to line ra
- Line tb through point B perpendicular to line rb
- Line tc through point C perpendicular to line rc
- Line td through point D perpendicular to line rd
- Line te through point E perpendicular to line re
- Line tf through point F perpendicular to line rf

- Intersection point A1 of point sets ta and tb
- Intersection point B1 of point sets tb and tc
- Intersection point C1 of point sets tc and td
- Intersection point D1 of point sets td and te
- Intersection point E1 of point sets te and tf
- Intersection point F1 of point sets tf and ta
- Line A1D1 through two points A1 and D1
- Line B1E1 through two points B1 and E1
- Line C1F1 through two points C1 and F1

Theorem statement:

- Lines A1D1, B1E1, C1F1 are concurrent

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point O:

- Point O has been assigned following coordinates: (0, 0)

2.2 Transformation of point A:

- Point A has been assigned following coordinates: (0, u_1)

2.3 Transformation of point B:

- Point B has been assigned following coordinates: (u_2, x_1)
- Polynomial that point B has to satisfy is:

$$p \quad = \quad x_1^2 + (u_2^2 - u_1^2)$$

- Processing of polynomial

$$p \quad = \quad x_1^2 + (u_2^2 - u_1^2)$$

Info: Polynomial

$$p \quad = \quad x_1^2 + (u_2^2 - u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.4 Transformation of point C:

- Point C has been assigned following coordinates: $(u_3,\ x_2)$
- Polynomial that point C has to satisfy is:

$$p \quad = \quad x_2^2 + (u_3^2 - u_1^2)$$

- Processing of polynomial

$$p \quad = \quad x_2^2 + (u_3^2 - u_1^2)$$

Info: Polynomial

$$p \quad = \quad x_2^2 + (u_3^2 - u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of point D:

- Point D has been assigned following coordinates: $(u_4,\ x_3)$
- Polynomial that point D has to satisfy is:

$$p \quad = \quad x_3^2 + (u_4^2 - u_1^2)$$

- Processing of polynomial

$$p \quad = \quad x_3^2 + (u_4^2 - u_1^2)$$

Info: Polynomial

$$p \quad = \quad x_3^2 + (u_4^2 - u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point E:

- Point E has been assigned following coordinates: $(u_5,\ x_4)$
- Polynomial that point E has to satisfy is:

$$p \quad = \quad x_4^2 + (u_5^2 - u_1^2)$$

- Processing of polynomial

$$p \quad = \quad x_4^2 + (u_5^2 - u_1^2)$$

Info: Polynomial

$$p \quad = \quad x_4^2 + (u_5^2 - u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point F:

- Point F has been assigned following coordinates: (u_6, x_5)
- Polynomial that point F has to satisfy is:

$$p = x_5^2 + (u_6^2 - u_1^2)$$

- Processing of polynomial

$$p = x_5^2 + (u_6^2 - u_1^2)$$

Info: Polynomial

$$p = x_5^2 + (u_6^2 - u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.8 Transformation of point A1:

- Point A1 has been assigned following coordinates: (x_6, x_7)
- Polynomial that point A1 has to satisfy is:

$$p = x_7 - u_1$$

- Processing of polynomial

$$p = x_7 - u_1$$

Info: Will try to rename Y coordinate of point A1

Info: Y coordinate of point A1 renamed by independent variable u_1

- Point A1 has been renamed. Point A1 has been assigned following coordinates: (x_6, u_1)
- Polynomial that point A1 has to satisfy is:

$$p = u_2 x_6 - x_1^2 + u_1 x_1 - u_2^2$$

- Processing of polynomial

$$p = u_2 x_6 - x_1^2 + u_1 x_1 - u_2^2$$

Info: Polynomial

$$p = u_2 x_6 - x_1^2 + u_1 x_1 - u_2^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.9 Transformation of point B1:

- Point B1 has been assigned following coordinates: (x_7, x_8)
- Polynomial that point B1 has to satisfy is:

$$p = x_8x_1 + u_2x_7 - x_1^2 - u_2^2$$

- Processing of polynomial

$$p = x_8x_1 + u_2x_7 - x_1^2 - u_2^2$$

Info: Polynomial

$$p = x_8x_1 + u_2x_7 - x_1^2 - u_2^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point B1 has to satisfy is:

$$p = x_8x_2 + u_3x_7 - x_2^2 - u_3^2$$

- Processing of polynomial

$$p = x_8x_2 + u_3x_7 - x_2^2 - u_3^2$$

Info: Polynomial

$$p = x_8x_2 + u_3x_7 - x_2^2 - u_3^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.10 Transformation of point C1:

- Point C1 has been assigned following coordinates: (x_9, x_{10})
- Polynomial that point C1 has to satisfy is:

$$p = x_{10}x_2 + u_3x_9 - x_2^2 - u_3^2$$

- Processing of polynomial

$$p = x_{10}x_2 + u_3x_9 - x_2^2 - u_3^2$$

Info: Polynomial

$$p = x_{10}x_2 + u_3x_9 - x_2^2 - u_3^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point C1 has to satisfy is:

$$p = x_{10}x_3 + u_4x_9 - x_3^2 - u_4^2$$

- Processing of polynomial

$$p = x_{10}x_3 + u_4x_9 - x_3^2 - u_4^2$$

Info: Polynomial

$$p = x_{10}x_3 + u_4x_9 - x_3^2 - u_4^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.11 Transformation of point D1:

- Point D1 has been assigned following coordinates: (x_{11}, x_{12})
- Polynomial that point D1 has to satisfy is:

$$p = x_{12}x_3 + u_4x_{11} - x_3^2 - u_4^2$$

- Processing of polynomial

$$p = x_{12}x_3 + u_4x_{11} - x_3^2 - u_4^2$$

Info: Polynomial

$$p = x_{12}x_3 + u_4x_{11} - x_3^2 - u_4^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point D1 has to satisfy is:

$$p = x_{12}x_4 + u_5x_{11} - x_4^2 - u_5^2$$

- Processing of polynomial

$$p = x_{12}x_4 + u_5x_{11} - x_4^2 - u_5^2$$

Info: Polynomial

$$p = x_{12}x_4 + u_5x_{11} - x_4^2 - u_5^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.12 Transformation of point E1:

- Point E1 has been assigned following coordinates: (x_{13}, x_{14})
- Polynomial that point E1 has to satisfy is:

$$p = x_{14}x_4 + u_5x_{13} - x_4^2 - u_5^2$$

- Processing of polynomial

$$p = x_{14}x_4 + u_5x_{13} - x_4^2 - u_5^2$$

Info: Polynomial

$$p = x_{14}x_4 + u_5x_{13} - x_4^2 - u_5^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point E1 has to satisfy is:

$$p = x_{14}x_5 + u_6x_{13} - x_5^2 - u_6^2$$

- Processing of polynomial

$$p = x_{14}x_5 + u_6x_{13} - x_5^2 - u_6^2$$

Info: Polynomial

$$p = x_{14}x_5 + u_6x_{13} - x_5^2 - u_6^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.13 Transformation of point F1:

- Point F1 has been assigned following coordinates: (x_{15}, x_{16})
- Polynomial that point F1 has to satisfy is:

$$p = x_{16}x_5 + u_6x_{15} - x_5^2 - u_6^2$$

- Processing of polynomial

$$p = x_{16}x_5 + u_6x_{15} - x_5^2 - u_6^2$$

Info: Polynomial

$$p = x_{16}x_5 + u_6x_{15} - x_5^2 - u_6^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point F1 has to satisfy is:

$$p = x_{16} - u_1$$

- Processing of polynomial

$$p = x_{16} - u_1$$

Info: Will try to rename Y coordinate of point F1

Info: Y coordinate of point F1 renamed by independent variable u_1

- Point F1 has been renamed. Point F1 has been assigned following coordinates: (x_{15}, u_1)
- Repeating instantiation of first condition of this point, after its coordinate has been renamed
- Polynomial that point F1 has to satisfy is:

$$p = u_6 x_{15} - x_5^2 + u_1 x_5 - u_6^2$$

- Processing of polynomial

$$p = u_6 x_{15} - x_5^2 + u_1 x_5 - u_6^2$$

Info: Polynomial

$$p = u_6 x_{15} - x_5^2 + u_1 x_5 - u_6^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

2.14 Transformation of point intersectPoint-A1D1.B1E1:

- Point intersectPoint-A1D1.B1E1 has been assigned following coordinates: (x_{16}, x_{17})
- Polynomial that point intersectPoint-A1D1.B1E1 has to satisfy is:

$$p = x_{17}x_{11} - x_{17}x_6 - x_{16}x_{12} + u_1 x_{16} + x_{12}x_6 - u_1 x_{11}$$

- Processing of polynomial

$$p = x_{17}x_{11} - x_{17}x_6 - x_{16}x_{12} + u_1 x_{16} + x_{12}x_6 - u_1 x_{11}$$

Info: Polynomial

$$p = x_{17}x_{11} - x_{17}x_6 - x_{16}x_{12} + u_1 x_{16} + x_{12}x_6 - u_1 x_{11}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point intersectPoint-A1D1.B1E1 has to satisfy is:

$$p = x_{17}x_{13} - x_{17}x_7 - x_{16}x_{14} + x_{16}x_8 + x_{14}x_7 - x_{13}x_8$$

- Processing of polynomial

$$p = x_{17}x_{13} - x_{17}x_7 - x_{16}x_{14} + x_{16}x_8 + x_{14}x_7 - x_{13}x_8$$

Info: Polynomial

$$p = x_{17}x_{13} - x_{17}x_7 - x_{16}x_{14} + x_{16}x_8 + x_{14}x_7 - x_{13}x_8$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial for theorem statement:

$$p = x_{17}x_{15} - x_{17}x_9 + x_{16}x_{10} - u_1x_{16} - x_{15}x_{10} + u_1x_9$$

Time spent for transformation of Construction Protocol to algebraic form

- 1.186 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= x_1^2 + (u_2^2 - u_1^2) \\
p_2 &= x_2^2 + (u_3^2 - u_1^2) \\
p_3 &= x_3^2 + (u_4^2 - u_1^2) \\
p_4 &= x_4^2 + (u_5^2 - u_1^2) \\
p_5 &= x_5^2 + (u_6^2 - u_1^2) \\
p_6 &= u_2x_6 - x_1^2 + u_1x_1 - u_2^2 \\
p_7 &= x_8x_1 + u_2x_7 - x_1^2 - u_2^2 \\
p_8 &= x_8x_2 + u_3x_7 - x_2^2 - u_3^2 \\
p_9 &= x_{10}x_2 + u_3x_9 - x_2^2 - u_3^2 \\
p_{10} &= x_{10}x_3 + u_4x_9 - x_3^2 - u_4^2 \\
p_{11} &= x_{12}x_3 + u_4x_{11} - x_3^2 - u_4^2 \\
p_{12} &= x_{12}x_4 + u_5x_{11} - x_4^2 - u_5^2 \\
p_{13} &= x_{14}x_4 + u_5x_{13} - x_4^2 - u_5^2 \\
p_{14} &= x_{14}x_5 + u_6x_{13} - x_5^2 - u_6^2 \\
p_{15} &= u_6x_{15} - x_5^2 + u_1x_5 - u_6^2 \\
p_{16} &= x_{17}x_{11} - x_{17}x_6 - x_{16}x_{12} + u_1x_{16} + x_{12}x_6 - u_1x_{11} \\
p_{17} &= x_{17}x_{13} - x_{17}x_7 - x_{16}x_{14} + x_{16}x_8 + x_{14}x_7 - x_{13}x_8
\end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{17} from all other polynomials by reducing them with polynomial p_{16} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= x_1^2 + (u_2^2 - u_1^2) \\
p_2 &= x_2^2 + (u_3^2 - u_1^2) \\
p_3 &= x_3^2 + (u_4^2 - u_1^2) \\
p_4 &= x_4^2 + (u_5^2 - u_1^2) \\
p_5 &= x_5^2 + (u_6^2 - u_1^2) \\
p_6 &= u_2x_6 - x_1^2 + u_1x_1 - u_2^2 \\
p_7 &= x_8x_1 + u_2x_7 - x_1^2 - u_2^2 \\
p_8 &= x_8x_2 + u_3x_7 - x_2^2 - u_3^2 \\
p_9 &= x_{10}x_2 + u_3x_9 - x_2^2 - u_3^2 \\
p_{10} &= x_{10}x_3 + u_4x_9 - x_3^2 - u_4^2 \\
p_{11} &= x_{12}x_3 + u_4x_{11} - x_3^2 - u_4^2 \\
p_{12} &= x_{12}x_4 + u_5x_{11} - x_4^2 - u_5^2 \\
p_{13} &= x_{14}x_4 + u_5x_{13} - x_4^2 - u_5^2 \\
p_{14} &= x_{14}x_5 + u_6x_{13} - x_5^2 - u_6^2 \\
p_{15} &= u_6x_{15} - x_5^2 + u_1x_5 - u_6^2 \\
p_{16} &= -x_{16}x_{14}x_{11} + x_{16}x_{14}x_6 + x_{16}x_{13}x_{12} - u_1x_{16}x_{13} - x_{16}x_{12}x_7 + \\
&\quad x_{16}x_{11}x_8 - x_{16}x_8x_6 + u_1x_{16}x_7 + x_{14}x_{11}x_7 - x_{14}x_7x_6 \\
&\quad -x_{13}x_{12}x_6 - x_{13}x_{11}x_8 + u_1x_{13}x_{11} + x_{13}x_8x_6 + x_{12}x_7x_6 \\
&\quad -u_1x_{11}x_7 \\
p_{17} &= x_{17}x_{11} - x_{17}x_6 - x_{16}x_{12} + u_1x_{16} + x_{12}x_6 - u_1x_{11}
\end{aligned}
$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{16}. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{15}. No reduction needed.

The triangular system has not been changed.

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{14} from all other polynomials by reducing them with polynomial p_{13} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= x_1^2 + (u_2^2 - u_1^2) \\
p_2 &= x_2^2 + (u_3^2 - u_1^2) \\
p_3 &= x_3^2 + (u_4^2 - u_1^2) \\
p_4 &= x_4^2 + (u_5^2 - u_1^2) \\
p_5 &= x_5^2 + (u_6^2 - u_1^2) \\
p_6 &= u_2 x_6 - x_1^2 + u_1 x_1 - u_2^2 \\
p_7 &= x_8 x_1 + u_2 x_7 - x_1^2 - u_2^2 \\
p_8 &= x_8 x_2 + u_3 x_7 - x_2^2 - u_3^2 \\
p_9 &= x_{10} x_2 + u_3 x_9 - x_2^2 - u_3^2 \\
p_{10} &= x_{10} x_3 + u_4 x_9 - x_3^2 - u_4^2 \\
p_{11} &= x_{12} x_3 + u_4 x_{11} - x_3^2 - u_4^2 \\
p_{12} &= x_{12} x_4 + u_5 x_{11} - x_4^2 - u_5^2 \\
p_{13} &= -u_5 x_{13} x_5 + u_6 x_{13} x_4 - x_5^2 x_4 + x_5 x_4^2 + u_5^2 x_5 \\
&\quad -u_6^2 x_4 \\
p_{14} &= x_{14} x_4 + u_5 x_{13} - x_4^2 - u_5^2 \\
p_{15} &= u_6 x_{15} - x_5^2 + u_1 x_5 - u_6^2 \\
p_{16} &= -x_{16} x_{14} x_{11} + x_{16} x_{14} x_6 + x_{16} x_{13} x_{12} - u_1 x_{16} x_{13} - x_{16} x_{12} x_7 + \\
&\quad x_{16} x_{11} x_8 - x_{16} x_8 x_6 + u_1 x_{16} x_7 + x_{14} x_{11} x_7 - x_{14} x_7 x_6 \\
&\quad -x_{13} x_{12} x_6 - x_{13} x_{11} x_8 + u_1 x_{13} x_{11} + x_{13} x_8 x_6 + x_{12} x_7 x_6 \\
&\quad -u_1 x_{11} x_7 \\
p_{17} &= x_{17} x_{11} - x_{17} x_6 - x_{16} x_{12} + u_1 x_{16} + x_{12} x_6 - u_1 x_{11}
\end{aligned}
$$

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{13}. No reduction needed.

The triangular system has not been changed.

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{12} from all other polynomials by reducing them with polynomial p_{11} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= x_1^2 + (u_2^2 - u_1^2) \\
p_2 &= x_2^2 + (u_3^2 - u_1^2) \\
p_3 &= x_3^2 + (u_4^2 - u_1^2) \\
p_4 &= x_4^2 + (u_5^2 - u_1^2) \\
p_5 &= x_5^2 + (u_6^2 - u_1^2) \\
p_6 &= u_2 x_6 - x_1^2 + u_1 x_1 - u_2^2 \\
p_7 &= x_8 x_1 + u_2 x_7 - x_1^2 - u_2^2 \\
p_8 &= x_8 x_2 + u_3 x_7 - x_2^2 - u_3^2 \\
p_9 &= x_{10} x_2 + u_3 x_9 - x_2^2 - u_3^2 \\
p_{10} &= x_{10} x_3 + u_4 x_9 - x_3^2 - u_4^2 \\
p_{11} &= -u_4 x_{11} x_4 + u_5 x_{11} x_3 - x_4^2 x_3 + x_4 x_3^2 + u_4^2 x_4 \\
&\quad -u_5^2 x_3 \\
p_{12} &= x_{12} x_3 + u_4 x_{11} - x_3^2 - u_4^2 \\
p_{13} &= -u_5 x_{13} x_5 + u_6 x_{13} x_4 - x_5^2 x_4 + x_5 x_4^2 + u_5^2 x_5 \\
&\quad -u_6^2 x_4 \\
p_{14} &= x_{14} x_4 + u_5 x_{13} - x_4^2 - u_5^2 \\
p_{15} &= u_6 x_{15} - x_5^2 + u_1 x_5 - u_6^2 \\
p_{16} &= -x_{16} x_{14} x_{11} + x_{16} x_{14} x_6 + x_{16} x_{13} x_{12} - u_1 x_{16} x_{13} - x_{16} x_{12} x_7 + \\
&\quad x_{16} x_{11} x_8 - x_{16} x_8 x_6 + u_1 x_{16} x_7 + x_{14} x_{11} x_7 - x_{14} x_7 x_6 \\
&\quad -x_{13} x_{12} x_6 - x_{13} x_{11} x_8 + u_1 x_{13} x_{11} + x_{13} x_8 x_6 + x_{12} x_7 x_6 \\
&\quad -u_1 x_{11} x_7 \\
p_{17} &= x_{17} x_{11} - x_{17} x_6 - x_{16} x_{12} + u_1 x_{16} + x_{12} x_6 - u_1 x_{11}
\end{aligned}
$$

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{11}. No reduction needed.

The triangular system has not been changed.

3.8 Triangulation, step 8

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{10} from all other polynomials by reducing them with polynomial p_9 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= x_1^2 + (u_2^2 - u_1^2) \\
p_2 &= x_2^2 + (u_3^2 - u_1^2) \\
p_3 &= x_3^2 + (u_4^2 - u_1^2) \\
p_4 &= x_4^2 + (u_5^2 - u_1^2) \\
p_5 &= x_5^2 + (u_6^2 - u_1^2) \\
p_6 &= u_2 x_6 - x_1^2 + u_1 x_1 - u_2^2 \\
p_7 &= x_8 x_1 + u_2 x_7 - x_1^2 - u_2^2 \\
p_8 &= x_8 x_2 + u_3 x_7 - x_2^2 - u_3^2 \\
p_9 &= -u_3 x_9 x_3 + u_4 x_9 x_2 - x_3^2 x_2 + x_3 x_2^2 + u_3^2 x_3 - u_4^2 x_2 \\
p_{10} &= x_{10} x_2 + u_3 x_9 - x_2^2 - u_3^2 \\
p_{11} &= -u_4 x_{11} x_4 + u_5 x_{11} x_3 - x_4^2 x_3 + x_4 x_3^2 + u_4^2 x_4 \\
&\quad -u_5^2 x_3 \\
p_{12} &= x_{12} x_3 + u_4 x_{11} - x_3^2 - u_4^2 \\
p_{13} &= -u_5 x_{13} x_5 + u_6 x_{13} x_4 - x_5^2 x_4 + x_5 x_4^2 + u_5^2 x_5 \\
&\quad -u_6^2 x_4 \\
p_{14} &= x_{14} x_4 + u_5 x_{13} - x_4^2 - u_5^2 \\
p_{15} &= u_6 x_{15} - x_5^2 + u_1 x_5 - u_6^2 \\
p_{16} &= -x_{16} x_{14} x_{11} + x_{16} x_{14} x_6 + x_{16} x_{13} x_{12} - u_1 x_{16} x_{13} - x_{16} x_{12} x_7 + \\
&\quad x_{16} x_{11} x_8 - x_{16} x_8 x_6 + u_1 x_{16} x_7 + x_{14} x_{11} x_7 - x_{14} x_7 x_6 \\
&\quad -x_{13} x_{12} x_6 - x_{13} x_{11} x_8 + u_1 x_{13} x_{11} + x_{13} x_8 x_6 + x_{12} x_7 x_6 \\
&\quad -u_1 x_{11} x_7 \\
p_{17} &= x_{17} x_{11} - x_{17} x_6 - x_{16} x_{12} + u_1 x_{16} + x_{12} x_6 - u_1 x_{11}
\end{aligned}
$$

3.9 Triangulation, step 9

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_9. No reduction needed.

The triangular system has not been changed.

3.10 Triangulation, step 10

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_8 from all other polynomials by reducing them with polynomial p_7 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= x_1^2 + (u_2^2 - u_1^2) \\
p_2 &= x_2^2 + (u_3^2 - u_1^2) \\
p_3 &= x_3^2 + (u_4^2 - u_1^2) \\
p_4 &= x_4^2 + (u_5^2 - u_1^2) \\
p_5 &= x_5^2 + (u_6^2 - u_1^2) \\
p_6 &= u_2x_6 - x_1^2 + u_1x_1 - u_2^2 \\
p_7 &= -u_2x_7x_2 + u_3x_7x_1 - x_2^2x_1 + x_2x_1^2 + u_2^2x_2 - u_3^2x_1 \\
p_8 &= x_8x_1 + u_2x_7 - x_1^2 - u_2^2 \\
p_9 &= -u_3x_9x_3 + u_4x_9x_2 - x_3^2x_2 + x_3x_2^2 + u_3^2x_3 - u_4^2x_2 \\
p_{10} &= x_{10}x_2 + u_3x_9 - x_2^2 - u_3^2 \\
p_{11} &= -u_4x_{11}x_4 + u_5x_{11}x_3 - x_4^2x_3 + x_4x_3^2 + u_4^2x_4 \\
&\quad -u_5^2x_3 \\
p_{12} &= x_{12}x_3 + u_4x_{11} - x_3^2 - u_4^2 \\
p_{13} &= -u_5x_{13}x_5 + u_6x_{13}x_4 - x_5^2x_4 + x_5x_4^2 + u_5^2x_5 \\
&\quad -u_6^2x_4 \\
p_{14} &= x_{14}x_4 + u_5x_{13} - x_4^2 - u_5^2 \\
p_{15} &= u_6x_{15} - x_5^2 + u_1x_5 - u_6^2 \\
p_{16} &= -x_{16}x_{14}x_{11} + x_{16}x_{14}x_6 + x_{16}x_{13}x_{12} - u_1x_{16}x_{13} - x_{16}x_{12}x_7 + \\
&\quad x_{16}x_{11}x_8 - x_{16}x_8x_6 + u_1x_{16}x_7 + x_{14}x_{11}x_7 - x_{14}x_7x_6 \\
&\quad -x_{13}x_{12}x_6 - x_{13}x_{11}x_8 + u_1x_{13}x_{11} + x_{13}x_8x_6 + x_{12}x_7x_6 \\
&\quad -u_1x_{11}x_7 \\
p_{17} &= x_{17}x_{11} - x_{17}x_6 - x_{16}x_{12} + u_1x_{16} + x_{12}x_6 - u_1x_{11}
\end{aligned}
$$

3.11 Triangulation, step 11

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7. No reduction needed.

The triangular system has not been changed.

3.12 Triangulation, step 12

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

3.13 Triangulation, step 13

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

3.14 Triangulation, step 14

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

3.15 Triangulation, step 15

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.16 Triangulation, step 16

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.17 Triangulation, step 17

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= x_1^2 + (u_2^2 - u_1^2) \\
p_2 &= x_2^2 + (u_3^2 - u_1^2) \\
p_3 &= x_3^2 + (u_4^2 - u_1^2) \\
p_4 &= x_4^2 + (u_5^2 - u_1^2) \\
p_5 &= x_5^2 + (u_6^2 - u_1^2) \\
p_6 &= u_2 x_6 - x_1^2 + u_1 x_1 - u_2^2 \\
p_7 &= -u_2 x_7 x_2 + u_3 x_7 x_1 - x_2^2 x_1 + x_2 x_1^2 + u_2^2 x_2 - u_3^2 x_1 \\
p_8 &= x_8 x_1 + u_2 x_7 - x_1^2 - u_2^2 \\
p_9 &= -u_3 x_9 x_3 + u_4 x_9 x_2 - x_3^2 x_2 + x_3 x_2^2 + u_3^2 x_3 - u_4^2 x_2 \\
p_{10} &= x_{10} x_2 + u_3 x_9 - x_2^2 - u_3^2 \\
p_{11} &= -u_4 x_{11} x_4 + u_5 x_{11} x_3 - x_4^2 x_3 + x_4 x_3^2 + u_4^2 x_4 \\
&\quad -u_5^2 x_3
\end{aligned}
$$

$$
\begin{aligned}
p_{12} &= x_{12}x_3 + u_4x_{11} - x_3^2 - u_4^2 \\
p_{13} &= -u_5x_{13}x_5 + u_6x_{13}x_4 - x_5^2x_4 + x_5x_4^2 + u_5^2x_5 \\
&\quad -u_6^2x_4 \\
p_{14} &= x_{14}x_4 + u_5x_{13} - x_4^2 - u_5^2 \\
p_{15} &= u_6x_{15} - x_5^2 + u_1x_5 - u_6^2 \\
p_{16} &= -x_{16}x_{14}x_{11} + x_{16}x_{14}x_6 + x_{16}x_{13}x_{12} - u_1x_{16}x_{13} - x_{16}x_{12}x_7 + \\
&\quad x_{16}x_{11}x_8 - x_{16}x_8x_6 + u_1x_{16}x_7 + x_{14}x_{11}x_7 - x_{14}x_7x_6 \\
&\quad -x_{13}x_{12}x_6 - x_{13}x_{11}x_8 + u_1x_{13}x_{11} + x_{13}x_8x_6 + x_{12}x_7x_6 \\
&\quad -u_1x_{11}x_7 \\
p_{17} &= x_{17}x_{11} - x_{17}x_6 - x_{16}x_{12} + u_1x_{16} + x_{12}x_6 - u_1x_{11}
\end{aligned}
$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 019 (Brianchon's theorem for circle)

Calculating final remainder of the conclusion:

$$g = x_{17}x_{15} - x_{17}x_9 + x_{16}x_{10} - u_1x_{16} - x_{15}x_{10} + u_1x_9$$

with respect to the triangular system.

1. Pseudo remainder with p_{17} over variable x_{17}:

$$
\begin{aligned}
g &= x_{16}x_{15}x_{12} - u_1x_{16}x_{15} - x_{16}x_{12}x_9 + x_{16}x_{11}x_{10} - u_1x_{16}x_{11} \\
&\quad -x_{16}x_{10}x_6 + u_1x_{16}x_9 + u_1x_{16}x_6 - x_{15}x_{12}x_6 - x_{15}x_{11}x_{10} + \\
&\quad u_1x_{15}x_{11} + x_{15}x_{10}x_6 + x_{12}x_9x_6 - u_1x_9x_6
\end{aligned}
$$

2. Pseudo remainder with p_{16} over variable x_{16}:
   Polynomial too big for output (text size is 2433 *characters, number of terms is* 82*)*
3. Pseudo remainder with p_{15} over variable x_{15}:
   Polynomial too big for output (text size is 4856 *characters, number of terms is* 142*)*
4. Pseudo remainder with p_{14} over variable x_{14}:
   Polynomial too big for output (text size is 9082 *characters, number of terms is* 236*)*
5. Pseudo remainder with p_{13} over variable x_{13}:
   Polynomial too big for output (number of terms is 364*)*
6. Pseudo remainder with p_{12} over variable x_{12}:
   Polynomial too big for output (number of terms is 657*)*

7. Pseudo remainder with p_{11} over variable x_{11}:

   Polynomial too big for output (number of terms is 1439*)*

8. Pseudo remainder with p_{10} over variable x_{10}:

   Polynomial too big for output (number of terms is 2347*)*

9. Pseudo remainder with p_9 over variable x_9:

   Polynomial too big for output (number of terms is 2264*)*

10. Pseudo remainder with p_8 over variable x_8:

    Polynomial too big for output (number of terms is 3570*)*

11. Pseudo remainder with p_7 over variable x_7:

    Polynomial too big for output (number of terms is 3828*)*

12. Pseudo remainder with p_6 over variable x_6:

    Polynomial too big for output (number of terms is 2844*)*

13. Pseudo remainder with p_5 over variable x_5:

    Polynomial too big for output (number of terms is 1554*)*

14. Pseudo remainder with p_4 over variable x_4:

    Polynomial too big for output (number of terms is 666*)*

15. Pseudo remainder with p_3 over variable x_3:

    Polynomial too big for output (text size is 433710 *characters, number of terms is* 226*)*

16. Pseudo remainder with p_2 over variable x_2:

    Polynomial too big for output (text size is 234290 *characters, number of terms is* 98*)*

17. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3828 terms.

Time Complexity: Time spent by the prover is 4.586 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points A, B and O are not collinear
- Points B, C and O are not collinear
- Line through points A and O is not perpendicular to line through points O and B
- Points D, C and O are not collinear
- Line through points A and O is not perpendicular to line through points O and C
- Points D, E and O are not collinear
- Line through points D and O is not perpendicular to line through points O and A
- Points E, F and O are not collinear
- Line through points E and O is not perpendicular to line through points O and A
- Points F and O are not identical
- Line through points A1 and D1 is not parallel with line through points E1 and B1
- Line through points A1 and D1 is not parallel with line through points A and O

Time spent for processing NDG Conditions

- 1.123 seconds