OpenGeoProver Output for conjecture "Circumcenter Theorem"

Wu's method used

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1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Free point C
- Perpendicular bisector ma of segment BC
- Perpendicular bisector mb of segment CA
- Intersection point O of point sets ma and mb
- Perpendicular bisector mc of segment AB

Theorem statement:

- Point O lies on set of points mc

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

2.2 Transformation of point B:

- Point B has been assigned following coordinates: (0, u_1)

2.3 Transformation of point C:

- Point C has been assigned following coordinates: (u_2, u_3)

2.4 Transformation of point O:

- Point O has been assigned following coordinates: (x_1, x_2)
- Polynomial that point O has to satisfy is:

$$p = (u_3 - u_1)x_2 + u_2x_1 + (-0.5u_3^2 - 0.5u_2^2 + 0.5u_1^2)$$

- Processing of polynomial

$$p = (u_3 - u_1)x_2 + u_2x_1 + (-0.5u_3^2 - 0.5u_2^2 + 0.5u_1^2)$$

Info: Polynomial

$$p = (u_3 - u_1)x_2 + u_2x_1 + (-0.5u_3^2 - 0.5u_2^2 + 0.5u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point O has to satisfy is:

$$p = u_3x_2 + u_2x_1 + (-0.5u_3^2 - 0.5u_2^2)$$

- Processing of polynomial

$$p = u_3x_2 + u_2x_1 + (-0.5u_3^2 - 0.5u_2^2)$$

Info: Polynomial

$$p = u_3x_2 + u_2x_1 + (-0.5u_3^2 - 0.5u_2^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_2 - 0.5u_1$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.031 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned} p_1 &= (u_3 - u_1)x_2 + u_2x_1 + (-0.5u_3^2 - 0.5u_2^2 + 0.5u_1^2) \\ p_2 &= u_3x_2 + u_2x_1 + (-0.5u_3^2 - 0.5u_2^2) \end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_2 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_1 &= -u_2u_1x_1 + (0.5u_3^2u_1 - 0.5u_3u_1^2 + 0.5u_2^2u_1) \\ p_2 &= (u_3 - u_1)x_2 + u_2x_1 + (-0.5u_3^2 - 0.5u_2^2 + 0.5u_1^2) \end{aligned}$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned} p_1 &= -u_2u_1x_1 + (0.5u_3^2u_1 - 0.5u_3u_1^2 + 0.5u_2^2u_1) \\ p_2 &= (u_3 - u_1)x_2 + u_2x_1 + (-0.5u_3^2 - 0.5u_2^2 + 0.5u_1^2) \end{aligned}$$

4 Final Remainder

4.1 Final remainder for conjecture Circumcenter Theorem

Calculating final remainder of the conclusion:

$$g = x_2 - 0.5u_1$$

with respect to the triangular system.

1. Pseudo remainder with p_2 over variable x_2:

$$g = -u_2x_1 + (0.5u_3^2 - 0.5u_3u_1 + 0.5u_2^2)$$

2. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.016 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points B and C are not identical
- Points A and B are not identical
- Points B and C are not identical

Time spent for processing NDG Conditions

- 0.078 seconds