

# OpenGeoProver Output for conjecture “Chou 336 (Gergonne’s point theorem)”

Wu’s method used

February 23, 2012

## 1 Validation of Construction Protocol

### Construction steps:

- Free point P
- Free point Q
- Free point R
- Circumscribed circle k around triangle PQR
- Tangent line a through point P of set of points k
- Tangent line b through point R of set of points k
- Tangent line c through point Q of set of points k
- Intersection point A of point sets b and c
- Intersection point B of point sets a and c
- Intersection point C of point sets a and b
- Line ap through two points A and P
- Line br through two points B and R
- Intersection point G of point sets ap and br

### Theorem statement:

- Points C, Q, G are collinear

**Validation result:** Construction protocol is valid.

## 2 Transformation of Construction Protocol to algebraic form

### Transformation of Construction steps

#### 2.1 Transformation of point P:

- Point P has been assigned following coordinates:  $(0, 0)$

#### 2.2 Transformation of point Q:

- Point Q has been assigned following coordinates:  $(0, u_1)$

#### 2.3 Transformation of point R:

- Point R has been assigned following coordinates:  $(u_2, u_3)$

#### 2.4 Transformation of point A:

- Point A has been assigned following coordinates:  $(x_1, x_2)$
- Polynomial that point A has to satisfy is:

$$p = (u_3u_2 - 0.5u_2u_1)x_2 + (-0.5u_3^2 + 0.5u_3u_1 + 0.5u_2^2)x_1 + (-0.5u_3^2u_2 - 0.5u_2^3)$$

- Processing of polynomial

$$p = (u_3u_2 - 0.5u_2u_1)x_2 + (-0.5u_3^2 + 0.5u_3u_1 + 0.5u_2^2)x_1 + (-0.5u_3^2u_2 - 0.5u_2^3)$$

**Info:** Polynomial

$$p = (u_3u_2 - 0.5u_2u_1)x_2 + (-0.5u_3^2 + 0.5u_3u_1 + 0.5u_2^2)x_1 + (-0.5u_3^2u_2 - 0.5u_2^3)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point A has to satisfy is:

$$p = u_2u_1x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_1 - u_2u_1^2$$

- Processing of polynomial

$$p = u_2u_1x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_1 - u_2u_1^2$$

**Info:** Polynomial

$$p = u_2u_1x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_1 - u_2u_1^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.5 Transformation of point B:

- Point B has been assigned following coordinates:  $(x_3, x_4)$
- Polynomial that point B has to satisfy is:

$$p = u_2u_1x_4 + (u_3^2 - u_3u_1 + u_2^2)x_3$$

- Processing of polynomial

$$p = u_2u_1x_4 + (u_3^2 - u_3u_1 + u_2^2)x_3$$

**Info:** Polynomial

$$p = u_2u_1x_4 + (u_3^2 - u_3u_1 + u_2^2)x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point B has to satisfy is:

$$p = u_2u_1x_4 + (-u_3^2 + u_3u_1 - u_2^2)x_3 - u_2u_1^2$$

- Processing of polynomial

$$p = u_2u_1x_4 + (-u_3^2 + u_3u_1 - u_2^2)x_3 - u_2u_1^2$$

**Info:** Polynomial

$$p = u_2u_1x_4 + (-u_3^2 + u_3u_1 - u_2^2)x_3 - u_2u_1^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.6 Transformation of point C:

- Point C has been assigned following coordinates:  $(x_5, x_6)$
- Polynomial that point C has to satisfy is:

$$p = u_2u_1x_6 + (u_3^2 - u_3u_1 + u_2^2)x_5$$

- Processing of polynomial

$$p = u_2u_1x_6 + (u_3^2 - u_3u_1 + u_2^2)x_5$$

**Info:** Polynomial

$$p = u_2u_1x_6 + (u_3^2 - u_3u_1 + u_2^2)x_5$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point C has to satisfy is:

$$p = (u_3u_2 - 0.5u_2u_1)x_6 + (-0.5u_3^2 + 0.5u_3u_1 + 0.5u_2^2)x_5 + (-0.5u_3^2u_2 - 0.5u_2^3)$$

- Processing of polynomial

$$p = (u_3u_2 - 0.5u_2u_1)x_6 + (-0.5u_3^2 + 0.5u_3u_1 + 0.5u_2^2)x_5 + (-0.5u_3^2u_2 - 0.5u_2^3)$$

**Info:** Polynomial

$$p = (u_3u_2 - 0.5u_2u_1)x_6 + (-0.5u_3^2 + 0.5u_3u_1 + 0.5u_2^2)x_5 + (-0.5u_3^2u_2 - 0.5u_2^3)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.7 Transformation of point G:

- Point G has been assigned following coordinates:  $(x_7, x_8)$
- Polynomial that point G has to satisfy is:

$$p = x_8x_1 - x_7x_2$$

- Processing of polynomial

$$p = x_8x_1 - x_7x_2$$

**Info:** Polynomial

$$p = x_8x_1 - x_7x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point G has to satisfy is:

$$p = x_8x_3 - u_2x_8 - x_7x_4 + u_3x_7 + u_2x_4 - u_3x_3$$

- Processing of polynomial

$$p = x_8x_3 - u_2x_8 - x_7x_4 + u_3x_7 + u_2x_4 - u_3x_3$$

**Info:** Polynomial

$$p = x_8x_3 - u_2x_8 - x_7x_4 + u_3x_7 + u_2x_4 - u_3x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_8x_5 - x_7x_6 + u_1x_7 - u_1x_5$$

## Time spent for transformation of Construction Protocol to algebraic form

- 0.324 seconds

## 3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned} p_1 &= (u_3u_2 - 0.5u_2u_1)x_2 + (-0.5u_3^2 + 0.5u_3u_1 + 0.5u_2^2)x_1 + \\ &\quad (-0.5u_3^2u_2 - 0.5u_2^3) \\ p_2 &= u_2u_1x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_1 - u_2u_1^2 \\ p_3 &= u_2u_1x_4 + (u_3^2 - u_3u_1 + u_2^2)x_3 \\ p_4 &= u_2u_1x_4 + (-u_3^2 + u_3u_1 - u_2^2)x_3 - u_2u_1^2 \\ p_5 &= u_2u_1x_6 + (u_3^2 - u_3u_1 + u_2^2)x_5 \\ p_6 &= (u_3u_2 - 0.5u_2u_1)x_6 + (-0.5u_3^2 + 0.5u_3u_1 + 0.5u_2^2)x_5 + \\ &\quad (-0.5u_3^2u_2 - 0.5u_2^3) \\ p_7 &= x_8x_1 - x_7x_2 \\ p_8 &= x_8x_3 - u_2x_8 - x_7x_4 + u_3x_7 + u_2x_4 - u_3x_3 \end{aligned}$$

### 3.1 Triangulation, step 1

**Choosing variable:** Trying the variable with index 8.

**Variable  $x_8$  selected:** The number of polynomials with this variable, with indexes from 1 to 8, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_8$  from all other polynomials by reducing them with polynomial  $p_7$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_1 &= (u_3u_2 - 0.5u_2u_1)x_2 + (-0.5u_3^2 + 0.5u_3u_1 + 0.5u_2^2)x_1 + \\ &\quad (-0.5u_3^2u_2 - 0.5u_2^3) \\ p_2 &= u_2u_1x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_1 - u_2u_1^2 \\ p_3 &= u_2u_1x_4 + (u_3^2 - u_3u_1 + u_2^2)x_3 \\ p_4 &= u_2u_1x_4 + (-u_3^2 + u_3u_1 - u_2^2)x_3 - u_2u_1^2 \end{aligned}$$

$$\begin{aligned}
p_5 &= u_2u_1x_6 + (u_3^2 - u_3u_1 + u_2^2)x_5 \\
p_6 &= (u_3u_2 - 0.5u_2u_1)x_6 + (-0.5u_3^2 + 0.5u_3u_1 + 0.5u_2^2)x_5 + \\
&\quad (-0.5u_3^2u_2 - 0.5u_2^3) \\
p_7 &= -x_7x_4x_1 + x_7x_3x_2 - u_2x_7x_2 + u_3x_7x_1 + u_2x_4x_1 \\
&\quad - u_3x_3x_1 \\
p_8 &= x_8x_1 - x_7x_2
\end{aligned}$$

### 3.2 Triangulation, step 2

**Choosing variable:** Trying the variable with index 7.

**Variable  $x_7$  selected:** The number of polynomials with this variable, with indexes from 1 to 7, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_7$ . No reduction needed.

The triangular system has not been changed.

### 3.3 Triangulation, step 3

**Choosing variable:** Trying the variable with index 6.

**Variable  $x_6$  selected:** The number of polynomials with this variable, with indexes from 1 to 6, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_6$  from all other polynomials by reducing them with polynomial  $p_5$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_3u_2 - 0.5u_2u_1)x_2 + (-0.5u_3^2 + 0.5u_3u_1 + 0.5u_2^2)x_1 + \\
&\quad (-0.5u_3^2u_2 - 0.5u_2^3) \\
p_2 &= u_2u_1x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_1 - u_2u_1^2 \\
p_3 &= u_2u_1x_4 + (u_3^2 - u_3u_1 + u_2^2)x_3 \\
p_4 &= u_2u_1x_4 + (-u_3^2 + u_3u_1 - u_2^2)x_3 - u_2u_1^2 \\
p_5 &= (-u_3^3u_2 + u_3^2u_2u_1 - u_3u_2^3 + u_2^3u_1)x_5 + \\
&\quad (-0.5u_3^2u_2^2u_1 - 0.5u_2^4u_1) \\
p_6 &= u_2u_1x_6 + (u_3^2 - u_3u_1 + u_2^2)x_5 \\
p_7 &= -x_7x_4x_1 + x_7x_3x_2 - u_2x_7x_2 + u_3x_7x_1 + u_2x_4x_1 \\
&\quad - u_3x_3x_1 \\
p_8 &= x_8x_1 - x_7x_2
\end{aligned}$$

### 3.4 Triangulation, step 4

**Choosing variable:** Trying the variable with index 5.

**Variable  $x_5$  selected:** The number of polynomials with this variable, with indexes from 1 to 5, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_5$ . No reduction needed.

The triangular system has not been changed.

### 3.5 Triangulation, step 5

**Choosing variable:** Trying the variable with index 4.

**Variable  $x_4$  selected:** The number of polynomials with this variable, with indexes from 1 to 4, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_4$  from all other polynomials by reducing them with polynomial  $p_3$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_1 &= (u_3u_2 - 0.5u_2u_1)x_2 + (-0.5u_3^2 + 0.5u_3u_1 + 0.5u_2^2)x_1 + \\ &\quad (-0.5u_3^2u_2 - 0.5u_2^3) \\ p_2 &= u_2u_1x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_1 - u_2u_1^2 \\ p_3 &= (-2u_3^2u_2u_1 + 2u_3u_2u_1^2 - 2u_2^3u_1)x_3 - u_2^2u_1^3 \\ p_4 &= u_2u_1x_4 + (u_3^2 - u_3u_1 + u_2^2)x_3 \\ p_5 &= (-u_3^3u_2 + u_3^2u_2u_1 - u_3u_2^3 + u_2^3u_1)x_5 + \\ &\quad (-0.5u_3^2u_2^2u_1 - 0.5u_2^4u_1) \\ p_6 &= u_2u_1x_6 + (u_3^2 - u_3u_1 + u_2^2)x_5 \\ p_7 &= -x_7x_4x_1 + x_7x_3x_2 - u_2x_7x_2 + u_3x_7x_1 + u_2x_4x_1 \\ &\quad - u_3x_3x_1 \\ p_8 &= x_8x_1 - x_7x_2 \end{aligned}$$

### 3.6 Triangulation, step 6

**Choosing variable:** Trying the variable with index 3.

**Variable  $x_3$  selected:** The number of polynomials with this variable, with indexes from 1 to 3, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_3$ . No reduction needed.

The triangular system has not been changed.

### 3.7 Triangulation, step 7

**Choosing variable:** Trying the variable with index 2.

**Variable  $x_2$  selected:** The number of polynomials with this variable, with indexes from 1 to 2, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_2$  from all other polynomials by reducing them with polynomial  $p_1$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (-u_3^3 u_2 + 2u_3^2 u_2 u_1 - u_3 u_2^3 - u_3 u_2 u_1^2) x_1 + \\
&\quad (0.5u_3^2 u_2^2 u_1 - u_3 u_2^2 u_1^2 + 0.5u_2^4 u_1 + 0.5u_2^2 u_1^3) \\
p_2 &= (u_3 u_2 - 0.5u_2 u_1) x_2 + (-0.5u_3^2 + 0.5u_3 u_1 + 0.5u_2^2) x_1 + \\
&\quad (-0.5u_3^3 u_2 - 0.5u_2^3) \\
p_3 &= (-2u_3^2 u_2 u_1 + 2u_3 u_2 u_1^2 - 2u_2^3 u_1) x_3 - u_2^2 u_1^3 \\
p_4 &= u_2 u_1 x_4 + (u_3^2 - u_3 u_1 + u_2^2) x_3 \\
p_5 &= (-u_3^3 u_2 + u_3^2 u_2 u_1 - u_3 u_2^3 + u_2^3 u_1) x_5 + \\
&\quad (-0.5u_3^2 u_2^2 u_1 - 0.5u_2^4 u_1) \\
p_6 &= u_2 u_1 x_6 + (u_3^2 - u_3 u_1 + u_2^2) x_5 \\
p_7 &= -x_7 x_4 x_1 + x_7 x_3 x_2 - u_2 x_7 x_2 + u_3 x_7 x_1 + u_2 x_4 x_1 \\
&\quad - u_3 x_3 x_1 \\
p_8 &= x_8 x_1 - x_7 x_2
\end{aligned}$$

### 3.8 Triangulation, step 8

**Choosing variable:** Trying the variable with index 1.

**Variable  $x_1$  selected:** The number of polynomials with this variable, with indexes from 1 to 1, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_1$ . No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_1 &= (-u_3^3 u_2 + 2u_3^2 u_2 u_1 - u_3 u_2^3 - u_3 u_2 u_1^2) x_1 + \\
&\quad (0.5u_3^2 u_2^2 u_1 - u_3 u_2^2 u_1^2 + 0.5u_2^4 u_1 + 0.5u_2^2 u_1^3) \\
p_2 &= (u_3 u_2 - 0.5u_2 u_1) x_2 + (-0.5u_3^2 + 0.5u_3 u_1 + 0.5u_2^2) x_1 + \\
&\quad (-0.5u_3^3 u_2 - 0.5u_2^3)
\end{aligned}$$



$$\begin{aligned}
p_3 &= (-2u_3^2u_2u_1 + 2u_3u_2u_1^2 - 2u_2^3u_1)x_3 - u_2^2u_1^3 \\
p_4 &= u_2u_1x_4 + (u_3^2 - u_3u_1 + u_2^2)x_3 \\
p_5 &= (-u_3^3u_2 + u_3^2u_2u_1 - u_3u_2^3 + u_2^3u_1)x_5 + \\
&\quad (-0.5u_3^2u_2^2u_1 - 0.5u_2^4u_1) \\
p_6 &= u_2u_1x_6 + (u_3^2 - u_3u_1 + u_2^2)x_5 \\
p_7 &= -x_7x_4x_1 + x_7x_3x_2 - u_2x_7x_2 + u_3x_7x_1 + u_2x_4x_1 \\
&\quad - u_3x_3x_1 \\
p_8 &= x_8x_1 - x_7x_2
\end{aligned}$$

## 4 Final Remainder

### 4.1 Final remainder for conjecture Chou 336 (Gergonne's point theorem)

Calculating final remainder of the conclusion:

$$g = x_8x_5 - x_7x_6 + u_1x_7 - u_1x_5$$

with respect to the triangular system.

1. Pseudo remainder with  $p_8$  over variable  $x_8$ :

$$g = -x_7x_6x_1 + x_7x_5x_2 + u_1x_7x_1 - u_1x_5x_1$$

2. Pseudo remainder with  $p_7$  over variable  $x_7$ :

$$\begin{aligned}
g &= u_2x_6x_4x_1^2 - u_3x_6x_3x_1^2 - u_2x_5x_4x_2x_1 + \\
&\quad u_1x_5x_4x_1^2 + (u_3 - u_1)x_5x_3x_2x_1 + u_2u_1x_5x_2x_1 \\
&\quad - u_3u_1x_5x_1^2 - u_2u_1x_4x_1^2 + u_3u_1x_3x_1^2
\end{aligned}$$

3. Pseudo remainder with  $p_6$  over variable  $x_6$ :

$$\begin{aligned}
g &= -u_2^2u_1x_5x_4x_2x_1 + \\
&\quad (-u_3^2u_2 + u_3u_2u_1 - u_2^3 + u_2u_1^2)x_5x_4x_1^2 + \\
&\quad (u_3u_2u_1 - u_2u_1^2)x_5x_3x_2x_1 + \\
&\quad (u_3^3 - u_3^2u_1 + u_3u_2^2)x_5x_3x_1^2 + \\
&\quad u_2^2u_1^2x_5x_2x_1 - u_3u_2u_1^2x_5x_1^2 \\
&\quad - u_2^2u_1^2x_4x_1^2 + u_3u_2u_1^2x_3x_1^2
\end{aligned}$$

4. Pseudo remainder with  $p_5$  over variable  $x_5$ :

$$\begin{aligned}
g = & (-0.5u_3^2u_2^4u_1^2 - 0.5u_2^6u_1^2)x_4x_2x_1 + \\
& (-0.5u_3^4u_2^3u_1 + 1.5u_3^3u_2^3u_1^2 - u_3^2u_2^5u_1 \\
& -0.5u_3^2u_2^3u_1^3 + 1.5u_3u_2^5u_1^2 - 0.5u_2^7u_1 \\
& -0.5u_2^5u_1^3) \\
& x_4x_1^2 \\
& + \\
& (0.5u_3^3u_2^3u_1^2 - 0.5u_3^2u_2^3u_1^3 + 0.5u_3u_2^5u_1^2 \\
& -0.5u_2^5u_1^3) \\
& x_3x_2x_1 \\
& + \\
& (0.5u_3^5u_2^2u_1 - 1.5u_3^4u_2^2u_1^2 + u_3^3u_2^4u_1 + \\
& u_3^3u_2^2u_1^3 - 1.5u_3^2u_2^4u_1^2 + 0.5u_3u_2^6u_1 + \\
& u_3u_2^4u_1^3) \\
& x_3x_1^2 \\
& + (0.5u_3^2u_2^4u_1^3 + 0.5u_2^6u_1^3)x_2x_1 + \\
& (-0.5u_3^3u_2^3u_1^3 - 0.5u_3u_2^5u_1^3)x_1^2
\end{aligned}$$

5. Pseudo remainder with  $p_4$  over variable  $x_4$ :

$$\begin{aligned}
g = & (0.5u_3^4u_2^4u_1^2 + u_3^2u_2^6u_1^2 - 0.5u_3^2u_2^4u_1^4 + \\
& 0.5u_2^8u_1^2 - 0.5u_2^6u_1^4) \\
& x_3x_2x_1 \\
& + \\
& (0.5u_3^6u_2^3u_1 - 1.5u_3^5u_2^3u_1^2 + 1.5u_3^4u_2^5u_1 + \\
& 0.5u_3^4u_2^3u_1^3 - 3u_3^3u_2^5u_1^2 + 0.5u_3^3u_2^3u_1^4 + \\
& 1.5u_3^2u_2^7u_1 + u_3^2u_2^5u_1^3 - 1.5u_3u_2^7u_1^2 + \\
& 0.5u_3u_2^5u_1^4 + 0.5u_2^9u_1 + 0.5u_2^7u_1^3) \\
& x_3x_1^2 \\
& + (0.5u_3^2u_2^5u_1^4 + 0.5u_2^7u_1^4)x_2x_1 + \\
& (-0.5u_3^3u_2^4u_1^4 - 0.5u_3u_2^6u_1^4)x_1^2
\end{aligned}$$

6. Pseudo remainder with  $p_3$  over variable  $x_3$ :

$$\begin{aligned}
g = & (-0.5u_3^4u_2^6u_1^5 + u_3^3u_2^6u_1^6 - u_3^2u_2^8u_1^5 \\
& -0.5u_3^2u_2^6u_1^7 + u_3u_2^8u_1^6 - 0.5u_2^{10}u_1^5 \\
& -0.5u_2^8u_1^7) \\
& x_2x_1 \\
& +
\end{aligned}$$

$$\begin{aligned}
& (0.5u_3^6u_2^5u_1^4 - 0.5u_3^5u_2^5u_1^5 + \\
& 1.5u_3^4u_2^7u_1^4 - 0.5u_3^4u_2^5u_1^6 - u_3^3u_2^7u_1^5 + \\
& 0.5u_3^3u_2^5u_1^7 + 1.5u_3^2u_2^9u_1^4 - 0.5u_3u_2^9u_1^5 + \\
& 0.5u_3u_2^7u_1^7 + 0.5u_2^{11}u_1^4 + 0.5u_2^9u_1^6) \\
& x_1^2
\end{aligned}$$

7. Pseudo remainder with  $p_2$  over variable  $x_2$ :

$$\begin{aligned}
g = & (0.5u_3^7u_2^6u_1^4 - u_3^6u_2^6u_1^5 + 1.5u_3^5u_2^8u_1^4 + \\
& 0.5u_3^5u_2^6u_1^6 - 2u_3^4u_2^8u_1^5 + 1.5u_3^3u_2^{10}u_1^4 + \\
& u_3^3u_2^8u_1^6 - u_3^2u_2^{10}u_1^5 + 0.5u_3u_2^{12}u_1^4 + \\
& 0.5u_3u_2^{10}u_1^6) \\
& x_1^2 \\
& + \\
& (-0.25u_3^6u_2^7u_1^5 + 0.5u_3^5u_2^7u_1^6 \\
& -0.75u_3^4u_2^9u_1^5 - 0.25u_3^4u_2^7u_1^7 + \\
& u_3^3u_2^9u_1^6 - 0.75u_3^2u_2^{11}u_1^5 \\
& -0.5u_3^2u_2^9u_1^7 + 0.5u_3u_2^{11}u_1^6 - 0.25u_2^{13}u_1^5 \\
& -0.25u_2^{11}u_1^7) \\
& x_1
\end{aligned}$$

8. Pseudo remainder with  $p_1$  over variable  $x_1$ :

$$g = 0$$

## 5 Prover results

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.089 seconds.

## 6 NDG Conditions

### NDG Conditions in readable form

- Points Q, P and R are not collinear
- Points Q, P and R are not collinear
- Points Q, P and R are not collinear
- Points Q, P and R are not collinear

- Points Q and P are not identical
- Points Q, P and R are not collinear
- Points Q, P and R are not collinear
- Line through points P and A is not parallel with line through points B and R
- Points P and A are not identical

**Time spent for processing NDG Conditions**

- 1.025 seconds