# OpenGeoProver Output for conjecture "Chou 007 (Converse of Pascal's Theorem)"

Wu's method used

February 12, 2012

## 1 Validation of Construction Protocol

### Construction steps:

- General Conic Section c
- Random point A from general conic c
- Random point B from general conic c
- Random point C from general conic c
- Random point D from general conic c
- Random point E from general conic c
- Line AB through two points A and B
- Line DE through two points D and E
- Intersection point P of point sets AB and DE
- Line BC through two points B and C
- Random point Q from line BC
- Line PQ through two points P and Q
- Line CD through two points C and D
- Intersection point R of point sets PQ and CD
- Line EQ through two points E and Q
- Line RA through two points R and A
- Intersection point F of point sets EQ and RA

### Theorem statement:

- Point F lies on set of points c

**Validation result:** Construction protocol is valid.

# 2 Transformation of Construction Protocol to algebraic form

### Transformation of Construction steps

## 2.1 Transformation of general conic section c:

List of parametric points

- Point Ac has been assigned following coordinates: $(u_1, 0)$
- Point Bc has been assigned following coordinates: $(u_2, 0)$
- Point Cc has been assigned following coordinates: $(u_3, 0)$
- Point Dc has been assigned following coordinates: $(u_4, 0)$
- Point Ec has been assigned following coordinates: $(u_5, 0)$
- Condition for point X($x_1$, $x_2$) to belong to this conic section is following equation:

$$p = u_3x_2^2 + u_2x_2x_1 + u_5x_2 + u_1x_1^2 + u_4x_1$$

## 2.2 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

## 2.3 Transformation of point B:

- Point B has been assigned following coordinates: $(0, x_1)$
- Polynomial that point B has to satisfy is:

$$p = u_3x_1^2 + u_5x_1$$

- Processing of polynomial

$$p = u_3x_1^2 + u_5x_1$$

**Info:** Polynomial

$$p = u_3x_1^2 + u_5x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.4 Transformation of point C:

- Point C has been assigned following coordinates: ($u_6$, $x_2$)
- Polynomial that point C has to satisfy is:

$$p = u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)$$

- Processing of polynomial

$$p = u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)$$

**Info:** Polynomial

$$p = u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.5 Transformation of point D:

- Point D has been assigned following coordinates: ($u_7$, $x_3$)
- Polynomial that point D has to satisfy is:

$$p = u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4)$$

- Processing of polynomial

$$p = u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4)$$

**Info:** Polynomial

$$p = u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.6 Transformation of point E:

- Point E has been assigned following coordinates: ($u_8$, $x_4$)
- Polynomial that point E has to satisfy is:

$$p = u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)$$

- Processing of polynomial

$$p = u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)$$

**Info:** Polynomial

$$p = u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.7 Transformation of point P:

- Point P has been assigned following coordinates: ($x_5$, $x_6$)
- Polynomial that point P has to satisfy is:

$$p = x_5x_1$$

- Processing of polynomial

$$p = x_5x_1$$

**Info:** Polynomial

$$p = x_5x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point P has to satisfy is:

$$p = (u_8 - u_7)x_6 - x_5x_4 + x_5x_3 + u_7x_4 - u_8x_3$$

- Processing of polynomial

$$p = (u_8 - u_7)x_6 - x_5x_4 + x_5x_3 + u_7x_4 - u_8x_3$$

**Info:** Polynomial

$$p = (u_8 - u_7)x_6 - x_5x_4 + x_5x_3 + u_7x_4 - u_8x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.8 Transformation of point Q:

- Point Q has been assigned following coordinates: ($u_9$, $x_7$)
- Polynomial that point Q has to satisfy is:

$$p = u_6x_7 - u_9x_2 + (u_9 - u_6)x_1$$

- Processing of polynomial

$$p = u_6x_7 - u_9x_2 + (u_9 - u_6)x_1$$

**Info:** Polynomial

$$p = u_6x_7 - u_9x_2 + (u_9 - u_6)x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.9 Transformation of point R:

- Point R has been assigned following coordinates: ($x_8$, $x_9$)
- Polynomial that point R has to satisfy is:

$$p = x_9x_5 - u_9x_9 + x_8x_7 - x_8x_6 - x_7x_5 + u_9x_6$$

- Processing of polynomial

$$p = x_9x_5 - u_9x_9 + x_8x_7 - x_8x_6 - x_7x_5 + u_9x_6$$

**Info:** Polynomial

$$p = x_9x_5 - u_9x_9 + x_8x_7 - x_8x_6 - x_7x_5 + u_9x_6$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point R has to satisfy is:

$$p = (u_7 - u_6)x_9 - x_8x_3 + x_8x_2 + u_6x_3 - u_7x_2$$

- Processing of polynomial

$$p = (u_7 - u_6)x_9 - x_8x_3 + x_8x_2 + u_6x_3 - u_7x_2$$

**Info:** Polynomial

$$p = (u_7 - u_6)x_9 - x_8x_3 + x_8x_2 + u_6x_3 - u_7x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.10 Transformation of point F:

- Point F has been assigned following coordinates: ($x_{10}$, $x_{11}$)
- Polynomial that point F has to satisfy is:

$$p = (u_9 - u_8)x_{11} - x_{10}x_7 + x_{10}x_4 + u_8x_7 - u_9x_4$$

- Processing of polynomial

$$p = (u_9 - u_8)x_{11} - x_{10}x_7 + x_{10}x_4 + u_8x_7 - u_9x_4$$

**Info:** Polynomial

$$p = (u_9 - u_8)x_{11} - x_{10}x_7 + x_{10}x_4 + u_8x_7 - u_9x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point F has to satisfy is:

$$p = x_{11}x_8 - x_{10}x_9$$

- Processing of polynomial

$$p = x_{11}x_8 - x_{10}x_9$$

**Info:** Polynomial

$$p = x_{11}x_8 - x_{10}x_9$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

### Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = u_3x_{11}^2 + u_2x_{11}x_{10} + u_5x_{11} + u_1x_{10}^2 + u_4x_{10}$$

### Time spent for transformation of Construction Protocol to algebraic form

- 0.193 seconds

## 3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= x_5x_1 \\
p_6 &= (u_8 - u_7)x_6 - x_5x_4 + x_5x_3 + u_7x_4 - u_8x_3 \\
p_7 &= u_6x_7 - u_9x_2 + (u_9 - u_6)x_1 \\
p_8 &= x_9x_5 - u_9x_9 + x_8x_7 - x_8x_6 - x_7x_5 + u_9x_6 \\
p_9 &= (u_7 - u_6)x_9 - x_8x_3 + x_8x_2 + u_6x_3 - u_7x_2 \\
p_{10} &= (u_9 - u_8)x_{11} - x_{10}x_7 + x_{10}x_4 + u_8x_7 - u_9x_4 \\
p_{11} &= x_{11}x_8 - x_{10}x_9
\end{aligned}$$

### 3.1 Triangulation, step 1

**Choosing variable:** Trying the variable with index 11.

**Variable $x_{11}$ selected:** The number of polynomials with this variable, with indexes from 1 to 11, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable $x_{11}$ from all other polynomials by reducing them with polynomial $p_{10}$ from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= x_5x_1 \\
p_6 &= (u_8 - u_7)x_6 - x_5x_4 + x_5x_3 + u_7x_4 - u_8x_3 \\
p_7 &= u_6x_7 - u_9x_2 + (u_9 - u_6)x_1 \\
p_8 &= x_9x_5 - u_9x_9 + x_8x_7 - x_8x_6 - x_7x_5 + u_9x_6 \\
p_9 &= (u_7 - u_6)x_9 - x_8x_3 + x_8x_2 + u_6x_3 - u_7x_2 \\
p_{10} &= (-u_9 + u_8)x_{10}x_9 + x_{10}x_8x_7 - x_{10}x_8x_4 - u_8x_8x_7 + u_9x_8x_4 \\
p_{11} &= (u_9 - u_8)x_{11} - x_{10}x_7 + x_{10}x_4 + u_8x_7 - u_9x_4
\end{aligned}
$$

### 3.2 Triangulation, step 2

**Choosing variable:** Trying the variable with index 10.

**Variable $x_{10}$ selected:** The number of polynomials with this variable, with indexes from 1 to 10, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_{10}$. No reduction needed.

The triangular system has not been changed.

### 3.3 Triangulation, step 3

**Choosing variable:** Trying the variable with index 9.

**Variable $x_9$ selected:** The number of polynomials with this variable, with indexes from 1 to 9, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable $x_9$ from all other polynomials by reducing them with polynomial $p_8$ from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= x_5x_1 \\
p_6 &= (u_8 - u_7)x_6 - x_5x_4 + x_5x_3 + u_7x_4 - u_8x_3 \\
p_7 &= u_6x_7 - u_9x_2 + (u_9 - u_6)x_1 \\
p_8 &= (-u_7 + u_6)x_8x_7 + (u_7 - u_6)x_8x_6 - x_8x_5x_3 + x_8x_5x_2 + \\
&\quad u_9x_8x_3 - u_9x_8x_2 + (u_7 - u_6)x_7x_5 + (-u_9u_7 + u_9u_6)x_6 + \\
&\quad u_6x_5x_3 - u_7x_5x_2 - u_9u_6x_3 + u_9u_7x_2 \\
p_9 &= x_9x_5 - u_9x_9 + x_8x_7 - x_8x_6 - x_7x_5 + u_9x_6 \\
p_{10} &= (-u_9 + u_8)x_{10}x_9 + x_{10}x_8x_7 - x_{10}x_8x_4 - u_8x_8x_7 + u_9x_8x_4 \\
p_{11} &= (u_9 - u_8)x_{11} - x_{10}x_7 + x_{10}x_4 + u_8x_7 - u_9x_4
\end{aligned}
$$

## 3.4 Triangulation, step 4

**Choosing variable:** Trying the variable with index 8.

**Variable $x_8$ selected:** The number of polynomials with this variable, with indexes from 1 to 8, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_8$. No reduction needed.

The triangular system has not been changed.

## 3.5 Triangulation, step 5

**Choosing variable:** Trying the variable with index 7.

**Variable $x_7$ selected:** The number of polynomials with this variable, with indexes from 1 to 7, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_7$. No reduction needed.

The triangular system has not been changed.

## 3.6 Triangulation, step 6

**Choosing variable:** Trying the variable with index 6.

**Variable $x_6$ selected:** The number of polynomials with this variable, with indexes from 1 to 6, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_6$. No reduction needed.

The triangular system has not been changed.

### 3.7 Triangulation, step 7

**Choosing variable:** Trying the variable with index 5.

**Variable $x_5$ selected:** The number of polynomials with this variable, with indexes from 1 to 5, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_5$. No reduction needed.

The triangular system has not been changed.

### 3.8 Triangulation, step 8

**Choosing variable:** Trying the variable with index 4.

**Variable $x_4$ selected:** The number of polynomials with this variable, with indexes from 1 to 4, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_4$. No reduction needed.

The triangular system has not been changed.

### 3.9 Triangulation, step 9

**Choosing variable:** Trying the variable with index 3.

**Variable $x_3$ selected:** The number of polynomials with this variable, with indexes from 1 to 3, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_3$. No reduction needed.

The triangular system has not been changed.

### 3.10 Triangulation, step 10

**Choosing variable:** Trying the variable with index 2.

**Variable $x_2$ selected:** The number of polynomials with this variable, with indexes from 1 to 2, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_2$. No reduction needed.

The triangular system has not been changed.

### 3.11 Triangulation, step 11

**Choosing variable:** Trying the variable with index 1.

**Variable $x_1$ selected:** The number of polynomials with this variable, with indexes from 1 to 1, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_1$. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= x_5x_1 \\
p_6 &= (u_8 - u_7)x_6 - x_5x_4 + x_5x_3 + u_7x_4 - u_8x_3 \\
p_7 &= u_6x_7 - u_9x_2 + (u_9 - u_6)x_1 \\
p_8 &= (-u_7 + u_6)x_8x_7 + (u_7 - u_6)x_8x_6 - x_8x_5x_3 + x_8x_5x_2 + \\
&\quad u_9x_8x_3 - u_9x_8x_2 + (u_7 - u_6)x_7x_5 + (-u_9u_7 + u_9u_6)x_6 + \\
&\quad u_6x_5x_3 - u_7x_5x_2 - u_9u_6x_3 + u_9u_7x_2 \\
p_9 &= x_9x_5 - u_9x_9 + x_8x_7 - x_8x_6 - x_7x_5 + u_9x_6 \\
p_{10} &= (-u_9 + u_8)x_{10}x_9 + x_{10}x_8x_7 - x_{10}x_8x_4 - u_8x_8x_7 + u_9x_8x_4 \\
p_{11} &= (u_9 - u_8)x_{11} - x_{10}x_7 + x_{10}x_4 + u_8x_7 - u_9x_4
\end{aligned}
$$

# 4 Final Remainder

## 4.1 Final remainder for conjecture Chou 007 (Converse of Pascal's Theorem)

Calculating final remainder of the conclusion:

$$g = u_3x_{11}^2 + u_2x_{11}x_{10} + u_5x_{11} + u_1x_{10}^2 + u_4x_{10}$$

with respect to the triangular system.

1. Pseudo remainder with $p_{11}$ over variable $x_{11}$:

$$
\begin{aligned}
g &= u_3x_{10}^2x_7^2 - 2u_3x_{10}^2x_7x_4 + (u_9u_2 - u_8u_2)x_{10}^2x_7 + \\
&\quad u_3x_{10}^2x_4^2 + (-u_9u_2 + u_8u_2)x_{10}^2x_4 + \\
&\quad (u_9^2u_1 - 2u_9u_8u_1 + u_8^2u_1)x_{10}^2 - 2u_8u_3x_{10}x_7^2 + \\
&\quad (2u_9u_3 + 2u_8u_3)x_{10}x_7x_4 +
\end{aligned}
$$

$$
\begin{array}{l}
(-u_9u_8u_2 + u_9u_5 + u_8^2u_2 - u_8u_5)x_{10}x_7 \\
-2u_9u_3x_{10}x_4^2 + \\
(u_9^2u_2 - u_9u_8u_2 - u_9u_5 + u_8u_5)x_{10}x_4 + \\
(u_9^2u_4 - 2u_9u_8u_4 + u_8^2u_4)x_{10} + u_8^2u_3x_7^2 \\
-2u_9u_8u_3x_7x_4 + (-u_9u_8u_5 + u_8^2u_5)x_7 + u_9^2u_3x_4^2 + \\
(u_9^2u_5 - u_9u_8u_5)x_4
\end{array}
$$

2. Pseudo remainder with $p_{10}$ over variable $x_{10}$:

$$
\begin{array}{rcl}
g & = & (u_9^2u_8^2u_3 - 2u_9u_8^3u_3 + u_8^4u_3)x_9^2x_7^2 + \\
 & & (-2u_9^3u_8u_3 + 4u_9^2u_8^2u_3 - 2u_9u_8^3u_3)x_9^2x_7x_4 + \\
 & & (-u_9^3u_8u_5 + 3u_9^2u_8^2u_5 - 3u_9u_8^3u_5 + u_8^4u_5) \\
 & & x_9^2x_7 \\
 & & +(u_9^4u_3 - 2u_9^3u_8u_3 + u_9^2u_8^2u_3)x_9^2x_4^2 + \\
 & & (u_9^4u_5 - 3u_9^3u_8u_5 + 3u_9^2u_8^2u_5 - u_9u_8^3u_5) \\
 & & x_9^2x_4 \\
 & & + \\
 & & (u_9^2u_8^2u_2 + u_9^2u_8u_5 - 2u_9u_8^3u_2 - 2u_9u_8^2u_5 + \\
 & & u_8^4u_2 + u_8^3u_5) \\
 & & x_9x_8x_7^2 \\
 & & + \\
 & & (-2u_9^3u_8u_2 - u_9^3u_5 + 4u_9^2u_8^2u_2 + u_9^2u_8u_5 \\
 & & -2u_9u_8^3u_2 + u_9u_8^2u_5 - u_8^3u_5) \\
 & & x_9x_8x_7x_4 \\
 & & + \\
 & & (-u_9^3u_8u_4 + 3u_9^2u_8^2u_4 - 3u_9u_8^3u_4 + u_8^4u_4) \\
 & & x_9x_8x_7 \\
 & & + \\
 & & (u_9^4u_2 - 2u_9^3u_8u_2 + u_9^3u_5 + u_9^2u_8^2u_2 \\
 & & -2u_9^2u_8u_5 + u_9u_8^2u_5) \\
 & & x_9x_8x_4^2 \\
 & & + \\
 & & (u_9^4u_4 - 3u_9^3u_8u_4 + 3u_9^2u_8^2u_4 - u_9u_8^3u_4) \\
 & & x_9x_8x_4 \\
 & & + \\
 & & (u_9^2u_8^2u_1 + u_9^2u_8u_4 - 2u_9u_8^3u_1 - 2u_9u_8^2u_4 + \\
 & & u_8^4u_1 + u_8^3u_4) \\
 & & x_8^2x_7^2 \\
 & & + \\
 & & (-2u_9^3u_8u_1 - u_9^3u_4 + 4u_9^2u_8^2u_1 + u_9^2u_8u_4
\end{array}
$$

$$
\begin{aligned}
&-2u_9u_8^3u_1 + u_9u_8^2u_4 - u_8^3u_4) \\
&x_8^2x_7x_4 \\
&+ \\
&(u_9^4u_1 - 2u_9^3u_8u_1 + u_9^3u_4 + u_9^2u_8^2u_1 \\
&-2u_9^2u_8u_4 + u_9u_8^2u_4) \\
&x_8^2x_4^2
\end{aligned}
$$

3. Pseudo remainder with $p_9$ over variable $x_9$:

   *Polynomial too big for output (text size is* 11057 *characters, number of terms is* 85*)*

4. Pseudo remainder with $p_8$ over variable $x_8$:

   *Polynomial too big for output (number of terms is* 429*)*

5. Pseudo remainder with $p_7$ over variable $x_7$:

   *Polynomial too big for output (number of terms is* 861*)*

6. Pseudo remainder with $p_6$ over variable $x_6$:

   *Polynomial too big for output (number of terms is* 713*)*

7. Pseudo remainder with $p_5$ over variable $x_5$:

   *Polynomial too big for output (text size is* 211509 *characters, number of terms is* 138*)*

8. Pseudo remainder with $p_4$ over variable $x_4$:

   *Polynomial too big for output (text size is greater than* 2000 *characters, number of terms is* 120*)*

9. Pseudo remainder with $p_3$ over variable $x_3$:

   *Polynomial too big for output (text size is greater than* 2000 *characters, number of terms is* 76*)*

10. Pseudo remainder with $p_2$ over variable $x_2$:

    *Polynomial too big for output (text size is greater than* 2000 *characters, number of terms is* 36*)*

11. Pseudo remainder with $p_1$ over variable $x_1$:

$$g = 0$$

# 5 Prover results

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 861 terms.

**Time Complexity:** Time spent by the prover is 7.021 seconds.

# 6 NDG Conditions

## NDG Conditions in readable form

- Conic Section is not in degenerate form
- Points A and B are not identical
- Points D and E are not identical
- Points A and C are not identical
- Line through points D and C is not parallel with line through points Q and P
- Segment with endpoints Q and A is not collinear and congruent with segment with endpoints P and B
- Line through points E and Q is not parallel with line through points A and R
- Points E and Q are not identical

## Time spent for processing NDG Conditions

- 1.592 seconds