OpenGeoProver Output for conjecture “Chou 288 (Simson’s Theorem)”

Wu’s method used

February 23, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Free point C
- Line c through two points A and B
- Line a through two points B and C
- Line b through two points C and A
- Circumscribed circle k around triangle ABC
- Random point M from circle k
- Line footPointPerpLine961 through point M perpendicular to line a
- Intersection point A’ of point sets footPointPerpLine961 and a
- Line footPointPerpLine468 through point M perpendicular to line b
- Intersection point B’ of point sets footPointPerpLine468 and b
- Line footPointPerpLine455 through point M perpendicular to line c
- Intersection point C’ of point sets footPointPerpLine455 and c

Theorem statement:

- Points A’, B’, C’ are collinear

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

2.2 Transformation of point B:

- Point B has been assigned following coordinates: $(0,\, u_1)$

2.3 Transformation of point C:

- Point C has been assigned following coordinates: $(u_2,\, u_3)$

2.4 Transformation of point M:

- Point M has been assigned following coordinates: $(u_4,\, x_1)$
- Polynomial that point M has to satisfy is:

$$\begin{aligned} p &= u_2x_1^2 - u_2u_1x_1 + \\ &\quad (u_4^2u_2 - u_4u_3^2 + u_4u_3u_1 - u_4u_2^2) \end{aligned}$$

- Processing of polynomial

$$\begin{aligned} p &= u_2x_1^2 - u_2u_1x_1 + \\ &\quad (u_4^2u_2 - u_4u_3^2 + u_4u_3u_1 - u_4u_2^2) \end{aligned}$$

Info: Polynomial

$$\begin{aligned} p &= u_2x_1^2 - u_2u_1x_1 + \\ &\quad (u_4^2u_2 - u_4u_3^2 + u_4u_3u_1 - u_4u_2^2) \end{aligned}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of point A':

- Point A' has been assigned following coordinates: $(x_2,\, x_3)$
- Polynomial that point A' has to satisfy is:

$$p = (u_3 - u_1)x_3 + u_2x_2 + (-u_3 + u_1)x_1 - u_4u_2$$

- Processing of polynomial

$$p = (u_3 - u_1)x_3 + u_2x_2 + (-u_3 + u_1)x_1 - u_4u_2$$

Info: Polynomial

$$p = (u_3 - u_1)x_3 + u_2x_2 + (-u_3 + u_1)x_1 - u_4u_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point A' has to satisfy is:

$$p = u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1$$

- Processing of polynomial

$$p = u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1$$

Info: Polynomial

$$p = u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point B':

- Point B' has been assigned following coordinates: (x_4, x_5)
- Polynomial that point B' has to satisfy is:

$$p = u_3x_5 + u_2x_4 - u_3x_1 - u_4u_2$$

- Processing of polynomial

$$p = u_3x_5 + u_2x_4 - u_3x_1 - u_4u_2$$

Info: Polynomial

$$p = u_3x_5 + u_2x_4 - u_3x_1 - u_4u_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point B' has to satisfy is:

$$p = u_2x_5 - u_3x_4$$

- Processing of polynomial

$$p = u_2x_5 - u_3x_4$$

Info: Polynomial

$$p = u_2x_5 - u_3x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point C':

- Point C' has been assigned following coordinates: (x_6, x_7)
- Polynomial that point C' has to satisfy is:

$$p = x_7 - x_1$$

- Processing of polynomial

$$p = x_7 - x_1$$

Info: Will try to rename Y coordinate of point C'

Info: Y coordinate of point C' renamed by dependent variable x_1

- Point C' has been renamed. Point C' has been assigned following coordinates: (x_6, x_1)
- Polynomial that point C' has to satisfy is:

$$p = x_6$$

- Processing of polynomial

$$p = x_6$$

Info: Will try to rename X coordinate of point C'

Info: X coordinate of point C' renamed by zero

- Point C' has been renamed. Point C' has been assigned following coordinates: (0, x_1)

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_5x_2 - x_4x_3 + x_4x_1 - x_2x_1$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.08 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= u_2x_1^2 - u_2u_1x_1 + \\
&\quad (u_4^2u_2 - u_4u_3^2 + u_4u_3u_1 - u_4u_2^2) \\
p_2 &= (u_3 - u_1)x_3 + u_2x_2 + (-u_3 + u_1)x_1 - u_4u_2 \\
p_3 &= u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1 \\
p_4 &= u_3x_5 + u_2x_4 - u_3x_1 - u_4u_2 \\
p_5 &= u_2x_5 - u_3x_4
\end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_5 from all other polynomials by reducing them with polynomial p_4 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_2x_1^2 - u_2u_1x_1 + \\
&\quad (u_4^2u_2 - u_4u_3^2 + u_4u_3u_1 - u_4u_2^2) \\
p_2 &= (u_3 - u_1)x_3 + u_2x_2 + (-u_3 + u_1)x_1 - u_4u_2 \\
p_3 &= u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1 \\
p_4 &= (-u_3^2 - u_2^2)x_4 + u_3u_2x_1 + u_4u_2^2 \\
p_5 &= u_3x_5 + u_2x_4 - u_3x_1 - u_4u_2
\end{aligned}
$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_3 from all other polynomials by reducing them with polynomial p_2 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_2x_1^2 - u_2u_1x_1 + \\
&\quad (u_4^2u_2 - u_4u_3^2 + u_4u_3u_1 - u_4u_2^2) \\
p_2 &= (-u_3^2 + 2u_3u_1 - u_2^2 - u_1^2)x_2 + (u_3u_2 - u_2u_1)x_1 +
\end{aligned}
$$

$$
\begin{aligned}
& (u_4u_2^2 - u_3u_2u_1 + u_2u_1^2) \\
p_3 &= (u_3 - u_1)x_3 + u_2x_2 + (-u_3 + u_1)x_1 - u_4u_2 \\
p_4 &= (-u_3^2 - u_2^2)x_4 + u_3u_2x_1 + u_4u_2^2 \\
p_5 &= u_3x_5 + u_2x_4 - u_3x_1 - u_4u_2
\end{aligned}
$$

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= u_2x_1^2 - u_2u_1x_1 + \\
& (u_4^2u_2 - u_4u_3^2 + u_4u_3u_1 - u_4u_2^2) \\
p_2 &= (-u_3^2 + 2u_3u_1 - u_2^2 - u_1^2)x_2 + (u_3u_2 - u_2u_1)x_1 + \\
& (u_4u_2^2 - u_3u_2u_1 + u_2u_1^2) \\
p_3 &= (u_3 - u_1)x_3 + u_2x_2 + (-u_3 + u_1)x_1 - u_4u_2 \\
p_4 &= (-u_3^2 - u_2^2)x_4 + u_3u_2x_1 + u_4u_2^2 \\
p_5 &= u_3x_5 + u_2x_4 - u_3x_1 - u_4u_2
\end{aligned}
$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 288 (Simson's Theorem)

Calculating final remainder of the conclusion:

$$g = x_5x_2 - x_4x_3 + x_4x_1 - x_2x_1$$

with respect to the triangular system.

1. Pseudo remainder with p_5 over variable x_5:

$$g = -u_3x_4x_3 - u_2x_4x_2 + u_3x_4x_1 + u_4u_2x_2$$

2. Pseudo remainder with p_4 over variable x_4:

$$\begin{aligned} g = \; & u_3^2u_2x_3x_1 + u_4u_3u_2^2x_3 + u_3u_2^2x_2x_1 \\ & -u_4u_3^2u_2x_2 - u_3^2u_2x_1^2 - u_4u_3u_2^2x_1 \end{aligned}$$

3. Pseudo remainder with p_3 over variable x_3:

$$\begin{aligned} g = \; & -u_3u_2^2u_1x_2x_1 + \\ & (-u_4u_3^3u_2 + u_4u_3^2u_2u_1 - u_4u_3u_2^3)x_2 + \\ & u_4u_3^2u_2^2x_1 + u_4^2u_3u_2^3 \end{aligned}$$

4. Pseudo remainder with p_2 over variable x_2:

$$\begin{aligned} g = \; & (u_3^2u_2^3u_1 - u_3u_2^3u_1^2)x_1^2 + \\ & (-u_3^2u_2^3u_1^2 + u_3u_2^3u_1^3)x_1 + \\ & (u_4^2u_3^2u_2^3u_1 - u_4^2u_3u_2^3u_1^2 \\ & -u_4u_3^4u_2^2u_1 + 2u_4u_3^3u_2^2u_1^2 \\ & -u_4u_3^2u_2^4u_1 - u_4u_3^2u_2^2u_1^3 + u_4u_3u_2^4u_1^2) \end{aligned}$$

5. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

Time Complexity: Time spent by the prover is 0.08 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points C' and C are not identical
- Points C', A, B and C are not collinear
- Points C', A, B and C are not collinear
- Points C' and C are not identical
- Points C' and C are not identical

Time spent for processing NDG Conditions

- 0.346 seconds