# OpenGeoProver Output for conjecture "Polar 001"

Wu's method used

February 12, 2012

## 1 Validation of Construction Protocol

### Construction steps:

- General Conic Section c
- Free point P
- Random point polarA1317 from general conic c
- Random point polarB1521 from general conic c
- Line polara613 through two points P and polarA1317
- Line polarb496 through two points P and polarB1521
- Intersection point polarA2343 of point sets c and polara613
- Intersection point polarB2302 of point sets c and polarb496
- Tangent line polara1940 through point polarA1317 of set of points c
- Tangent line polara2313 through point polarA2343 of set of points c
- Tangent line polarb1629 through point polarB1521 of set of points c
- Tangent line polarb221 through point polarB2302 of set of points c
- Intersection point polarA390 of point sets polara1940 and polara2313
- Intersection point polarB665 of point sets polarb1629 and polarb221
- Line p through two points polarA390 and polarB665
- Random point A from general conic c
- Line s through two points P and A
- Intersection point B of point sets s and c
- Intersection point Q of point sets s and p

### Theorem statement:

- Pair of points A and B is in harmonic conjunction with pair of points P and Q

**Validation result:** Construction protocol is valid.

# 2 Transformation of Construction Protocol to algebraic form

### Transformation of Construction steps

## 2.1 Transformation of general conic section c:

List of parametric points

- Point Ac has been assigned following coordinates: ($u_1$, 0)
- Point Bc has been assigned following coordinates: ($u_2$, 0)
- Point Cc has been assigned following coordinates: ($u_3$, 0)
- Point Dc has been assigned following coordinates: ($u_4$, 0)
- Point Ec has been assigned following coordinates: ($u_5$, 0)
- Condition for point X($x_1$, $x_2$) to belong to this conic section is following equation:

$$p = u_3x_2^2 + u_2x_2x_1 + u_5x_2 + u_1x_1^2 + u_4x_1$$

## 2.2 Transformation of point P:

- Point P has been assigned following coordinates: ($u_6$, $u_7$)

## 2.3 Transformation of point polarA1317:

- Point polarA1317 has been assigned following coordinates: (0, 0)

## 2.4 Transformation of point polarB1521:

- Point polarB1521 has been assigned following coordinates: (0, $x_1$)
- Polynomial that point polarB1521 has to satisfy is:

$$p = u_3x_1^2 + u_5x_1$$

- Processing of polynomial

$$p = u_3x_1^2 + u_5x_1$$

**Info:** Polynomial

$$p = u_3x_1^2 + u_5x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.5 Transformation of point polarA2343:

- Point polarA2343 has been assigned following coordinates: ($x_2$, $x_3$)
- Polynomial that point polarA2343 has to satisfy is:

$$p = u_3x_3^2 + u_2x_3x_2 + u_5x_3 + u_1x_2^2 + u_4x_2$$

- Processing of polynomial

$$p = u_3x_3^2 + u_2x_3x_2 + u_5x_3 + u_1x_2^2 + u_4x_2$$

**Info:** Polynomial

$$p = u_3x_3^2 + u_2x_3x_2 + u_5x_3 + u_1x_2^2 + u_4x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point polarA2343 has to satisfy is:

$$p = u_6x_3 - u_7x_2$$

- Processing of polynomial

$$p = u_6x_3 - u_7x_2$$

**Info:** Polynomial

$$p = u_6x_3 - u_7x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.6 Transformation of point polarB2302:

- Point polarB2302 has been assigned following coordinates: ($x_4$, $x_5$)
- Polynomial that point polarB2302 has to satisfy is:

$$p = u_3x_5^2 + u_2x_5x_4 + u_5x_5 + u_1x_4^2 + u_4x_4$$

- Processing of polynomial

$$p = u_3x_5^2 + u_2x_5x_4 + u_5x_5 + u_1x_4^2 + u_4x_4$$

**Info:** Polynomial

$$p = u_3x_5^2 + u_2x_5x_4 + u_5x_5 + u_1x_4^2 + u_4x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point polarB2302 has to satisfy is:

$$p = u_6x_5 + x_4x_1 - u_7x_4 - u_6x_1$$

- Processing of polynomial

$$p = u_6x_5 + x_4x_1 - u_7x_4 - u_6x_1$$

**Info:** Polynomial

$$p = u_6x_5 + x_4x_1 - u_7x_4 - u_6x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.7 Transformation of point polarA390:

- Point polarA390 has been assigned following coordinates: ($x_6$, $x_7$)
- Polynomial that point polarA390 has to satisfy is:

$$p = u_5x_7 + u_4x_6$$

- Processing of polynomial

$$p = u_5x_7 + u_4x_6$$

**Info:** Polynomial

$$p = u_5x_7 + u_4x_6$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point polarA390 has to satisfy is:

$$\begin{aligned} p = {} & u_3x_7x_3 + 0.5u_2x_7x_2 + 0.5u_5x_7 + 0.5u_2x_6x_3 + u_1x_6x_2 + \\ & 0.5u_4x_6 - u_3x_3^2 - u_2x_3x_2 - 0.5u_5x_3 - u_1x_2^2 - 0.5u_4x_2 \end{aligned}$$

- Processing of polynomial

$$\begin{aligned} p = {} & u_3x_7x_3 + 0.5u_2x_7x_2 + 0.5u_5x_7 + 0.5u_2x_6x_3 + u_1x_6x_2 + \\ & 0.5u_4x_6 - u_3x_3^2 - u_2x_3x_2 - 0.5u_5x_3 - u_1x_2^2 - 0.5u_4x_2 \end{aligned}$$

**Info:** Polynomial

$$\begin{aligned} p = {} & u_3x_7x_3 + 0.5u_2x_7x_2 + 0.5u_5x_7 + 0.5u_2x_6x_3 + u_1x_6x_2 + \\ & 0.5u_4x_6 - u_3x_3^2 - u_2x_3x_2 - 0.5u_5x_3 - u_1x_2^2 - 0.5u_4x_2 \end{aligned}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.8 Transformation of point polarB665:

- Point polarB665 has been assigned following coordinates: ($x_8$, $x_9$)
- Polynomial that point polarB665 has to satisfy is:

$$p = u_3x_9x_1 + 0.5u_5x_9 + 0.5u_2x_8x_1 + 0.5u_4x_8 - u_3x_1^2 - 0.5u_5x_1$$

- Processing of polynomial

$$p = u_3x_9x_1 + 0.5u_5x_9 + 0.5u_2x_8x_1 + 0.5u_4x_8 - u_3x_1^2 - 0.5u_5x_1$$

**Info:** Polynomial

$$p = u_3x_9x_1 + 0.5u_5x_9 + 0.5u_2x_8x_1 + 0.5u_4x_8 - u_3x_1^2 - 0.5u_5x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point polarB665 has to satisfy is:

$$\begin{aligned} p = {} & u_3x_9x_5 + 0.5u_2x_9x_4 + 0.5u_5x_9 + 0.5u_2x_8x_5 + u_1x_8x_4 + \\ & 0.5u_4x_8 - u_3x_5^2 - u_2x_5x_4 - 0.5u_5x_5 - u_1x_4^2 - 0.5u_4x_4 \end{aligned}$$

- Processing of polynomial

$$\begin{aligned} p = {} & u_3x_9x_5 + 0.5u_2x_9x_4 + 0.5u_5x_9 + 0.5u_2x_8x_5 + u_1x_8x_4 + \\ & 0.5u_4x_8 - u_3x_5^2 - u_2x_5x_4 - 0.5u_5x_5 - u_1x_4^2 - 0.5u_4x_4 \end{aligned}$$

**Info:** Polynomial

$$\begin{aligned} p = {} & u_3x_9x_5 + 0.5u_2x_9x_4 + 0.5u_5x_9 + 0.5u_2x_8x_5 + u_1x_8x_4 + \\ & 0.5u_4x_8 - u_3x_5^2 - u_2x_5x_4 - 0.5u_5x_5 - u_1x_4^2 - 0.5u_4x_4 \end{aligned}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.9 Transformation of point A:

- Point A has been assigned following coordinates: ($u_8$, $x_{10}$)
- Polynomial that point A has to satisfy is:

$$p = u_3x_{10}^2 + (u_8u_2 + u_5)x_{10} + (u_8^2u_1 + u_8u_4)$$

- Processing of polynomial

$$p = u_3x_{10}^2 + (u_8u_2 + u_5)x_{10} + (u_8^2u_1 + u_8u_4)$$

**Info:** Polynomial

$$p = u_3x_{10}^2 + (u_8u_2 + u_5)x_{10} + (u_8^2u_1 + u_8u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.10 Transformation of point B:

- Point B has been assigned following coordinates: ($x_{11}$, $x_{12}$)
- Polynomial that point B has to satisfy is:

$$p = (u_8 - u_6)x_{12} - x_{11}x_{10} + u_7x_{11} + u_6x_{10} - u_8u_7$$

- Processing of polynomial

$$p = (u_8 - u_6)x_{12} - x_{11}x_{10} + u_7x_{11} + u_6x_{10} - u_8u_7$$

**Info:** Polynomial

$$p = (u_8 - u_6)x_{12} - x_{11}x_{10} + u_7x_{11} + u_6x_{10} - u_8u_7$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point B has to satisfy is:

$$p = u_3x_{12}^2 + u_2x_{12}x_{11} + u_5x_{12} + u_1x_{11}^2 + u_4x_{11}$$

- Processing of polynomial

$$p = u_3x_{12}^2 + u_2x_{12}x_{11} + u_5x_{12} + u_1x_{11}^2 + u_4x_{11}$$

**Info:** Polynomial

$$p = u_3x_{12}^2 + u_2x_{12}x_{11} + u_5x_{12} + u_1x_{11}^2 + u_4x_{11}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.11 Transformation of point Q:

- Point Q has been assigned following coordinates: ($x_{13}$, $x_{14}$)
- Polynomial that point Q has to satisfy is:

$$p = (u_8 - u_6)x_{14} - x_{13}x_{10} + u_7x_{13} + u_6x_{10} - u_8u_7$$

- Processing of polynomial

$$p = (u_8 - u_6)x_{14} - x_{13}x_{10} + u_7x_{13} + u_6x_{10} - u_8u_7$$

**Info:** Polynomial

$$p = (u_8 - u_6)x_{14} - x_{13}x_{10} + u_7x_{13} + u_6x_{10} - u_8u_7$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point Q has to satisfy is:

$$p = x_{14}x_8 - x_{14}x_6 - x_{13}x_9 + x_{13}x_7 + x_9x_6 - x_8x_7$$

- Processing of polynomial

$$p = x_{14}x_8 - x_{14}x_6 - x_{13}x_9 + x_{13}x_7 + x_9x_6 - x_8x_7$$

**Info:** Polynomial

$$p = x_{14}x_8 - x_{14}x_6 - x_{13}x_9 + x_{13}x_7 + x_9x_6 - x_8x_7$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

### Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_{13}x_{11} + (u_8 - 2u_6)x_{13} + (-2u_8 + u_6)x_{11} + u_8u_6$$

### Time spent for transformation of Construction Protocol to algebraic form

- 0.154 seconds

## 3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_3^2 + u_2x_3x_2 + u_5x_3 + u_1x_2^2 + u_4x_2 \\
p_3 &= u_6x_3 - u_7x_2 \\
p_4 &= u_3x_5^2 + u_2x_5x_4 + u_5x_5 + u_1x_4^2 + u_4x_4 \\
p_5 &= u_6x_5 + x_4x_1 - u_7x_4 - u_6x_1 \\
p_6 &= u_5x_7 + u_4x_6 \\
p_7 &= u_3x_7x_3 + 0.5u_2x_7x_2 + 0.5u_5x_7 + 0.5u_2x_6x_3 + u_1x_6x_2 + \\
&\quad 0.5u_4x_6 - u_3x_3^2 - u_2x_3x_2 - 0.5u_5x_3 - u_1x_2^2 - 0.5u_4x_2 \\
p_8 &= u_3x_9x_1 + 0.5u_5x_9 + 0.5u_2x_8x_1 + 0.5u_4x_8 - u_3x_1^2 - 0.5u_5x_1 \\
p_9 &= u_3x_9x_5 + 0.5u_2x_9x_4 + 0.5u_5x_9 + 0.5u_2x_8x_5 + u_1x_8x_4 + \\
&\quad 0.5u_4x_8 - u_3x_5^2 - u_2x_5x_4 - 0.5u_5x_5 - u_1x_4^2 - 0.5u_4x_4 \\
p_{10} &= u_3x_{10}^2 + (u_8u_2 + u_5)x_{10} + (u_8^2u_1 + u_8u_4) \\
p_{11} &= (u_8 - u_6)x_{12} - x_{11}x_{10} + u_7x_{11} + u_6x_{10} - u_8u_7 \\
p_{12} &= u_3x_{12}^2 + u_2x_{12}x_{11} + u_5x_{12} + u_1x_{11}^2 + u_4x_{11} \\
p_{13} &= (u_8 - u_6)x_{14} - x_{13}x_{10} + u_7x_{13} + u_6x_{10} - u_8u_7 \\
p_{14} &= x_{14}x_8 - x_{14}x_6 - x_{13}x_9 + x_{13}x_7 + x_9x_6 - x_8x_7
\end{aligned}$$

## 3.1 Triangulation, step 1

**Choosing variable:** Trying the variable with index 14.

**Variable $x_{14}$ selected:** The number of polynomials with this variable, with indexes from 1 to 14, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable $x_{14}$ from all other polynomials by reducing them with polynomial $p_{13}$ from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_3^2 + u_2x_3x_2 + u_5x_3 + u_1x_2^2 + u_4x_2 \\
p_3 &= u_6x_3 - u_7x_2 \\
p_4 &= u_3x_5^2 + u_2x_5x_4 + u_5x_5 + u_1x_4^2 + u_4x_4 \\
p_5 &= u_6x_5 + x_4x_1 - u_7x_4 - u_6x_1 \\
p_6 &= u_5x_7 + u_4x_6 \\
p_7 &= u_3x_7x_3 + 0.5u_2x_7x_2 + 0.5u_5x_7 + 0.5u_2x_6x_3 + u_1x_6x_2 + \\
&\quad 0.5u_4x_6 - u_3x_3^2 - u_2x_3x_2 - 0.5u_5x_3 - u_1x_2^2 - 0.5u_4x_2 \\
p_8 &= u_3x_9x_1 + 0.5u_5x_9 + 0.5u_2x_8x_1 + 0.5u_4x_8 - u_3x_1^2 - 0.5u_5x_1 \\
p_9 &= u_3x_9x_5 + 0.5u_2x_9x_4 + 0.5u_5x_9 + 0.5u_2x_8x_5 + u_1x_8x_4 + \\
&\quad 0.5u_4x_8 - u_3x_5^2 - u_2x_5x_4 - 0.5u_5x_5 - u_1x_4^2 - 0.5u_4x_4 \\
p_{10} &= u_3x_{10}^2 + (u_8u_2 + u_5)x_{10} + (u_8^2u_1 + u_8u_4) \\
p_{11} &= (u_8 - u_6)x_{12} - x_{11}x_{10} + u_7x_{11} + u_6x_{10} - u_8u_7 \\
p_{12} &= u_3x_{12}^2 + u_2x_{12}x_{11} + u_5x_{12} + u_1x_{11}^2 + u_4x_{11} \\
p_{13} &= x_{13}x_{10}x_8 - x_{13}x_{10}x_6 + (-u_8 + u_6)x_{13}x_9 - u_7x_{13}x_8 + \\
&\quad (u_8 - u_6)x_{13}x_7 + u_7x_{13}x_6 - u_6x_{10}x_8 + u_6x_{10}x_6 + \\
&\quad (u_8 - u_6)x_9x_6 + (-u_8 + u_6)x_8x_7 + u_8u_7x_8 - u_8u_7x_6 \\
p_{14} &= (u_8 - u_6)x_{14} - x_{13}x_{10} + u_7x_{13} + u_6x_{10} - u_8u_7
\end{aligned}
$$

## 3.2 Triangulation, step 2

**Choosing variable:** Trying the variable with index 13.

**Variable $x_{13}$ selected:** The number of polynomials with this variable, with indexes from 1 to 13, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_{13}$. No reduction needed.

The triangular system has not been changed.

### 3.3 Triangulation, step 3

**Choosing variable:** Trying the variable with index 12.

**Variable $x_{12}$ selected:** The number of polynomials with this variable, with indexes from 1 to 12, is 2.

**Minimal degrees:** 1 polynomial(s) with degree 1 and 1 polynomial(s) with degree 2.

**Polynomial with linear degree:** Removing variable $x_{12}$ from all other polynomials by reducing them with polynomial $p_{11}$ from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_3^2 + u_2x_3x_2 + u_5x_3 + u_1x_2^2 + u_4x_2 \\
p_3 &= u_6x_3 - u_7x_2 \\
p_4 &= u_3x_5^2 + u_2x_5x_4 + u_5x_5 + u_1x_4^2 + u_4x_4 \\
p_5 &= u_6x_5 + x_4x_1 - u_7x_4 - u_6x_1 \\
p_6 &= u_5x_7 + u_4x_6 \\
p_7 &= u_3x_7x_3 + 0.5u_2x_7x_2 + 0.5u_5x_7 + 0.5u_2x_6x_3 + u_1x_6x_2 + \\
&\quad 0.5u_4x_6 - u_3x_3^2 - u_2x_3x_2 - 0.5u_5x_3 - u_1x_2^2 - 0.5u_4x_2 \\
p_8 &= u_3x_9x_1 + 0.5u_5x_9 + 0.5u_2x_8x_1 + 0.5u_4x_8 - u_3x_1^2 - 0.5u_5x_1 \\
p_9 &= u_3x_9x_5 + 0.5u_2x_9x_4 + 0.5u_5x_9 + 0.5u_2x_8x_5 + u_1x_8x_4 + \\
&\quad 0.5u_4x_8 - u_3x_5^2 - u_2x_5x_4 - 0.5u_5x_5 - u_1x_4^2 - 0.5u_4x_4 \\
p_{10} &= u_3x_{10}^2 + (u_8u_2 + u_5)x_{10} + (u_8^2u_1 + u_8u_4) \\
p_{11} &= u_3x_{11}^2x_{10}^2 + (u_8u_2 - 2u_7u_3 - u_6u_2)x_{11}^2x_{10} + \\
&\quad (u_8^2u_1 - u_8u_7u_2 - 2u_8u_6u_1 + u_7^2u_3 + u_7u_6u_2 + u_6^2u_1) \\
&\quad x_{11}^2 \\
&\quad -2u_6u_3x_{11}x_{10}^2 + \\
&\quad (2u_8u_7u_3 - u_8u_6u_2 + u_8u_5 + 2u_7u_6u_3 + u_6^2u_2 - u_6u_5) \\
&\quad x_{11}x_{10} \\
&\quad + \\
&\quad (u_8^2u_7u_2 + u_8^2u_4 - 2u_8u_7^2u_3 - u_8u_7u_6u_2 \\
&\quad -u_8u_7u_5 - 2u_8u_6u_4 + u_7u_6u_5 + u_6^2u_4) \\
&\quad x_{11} \\
&\quad +u_6^2u_3x_{10}^2 + (-2u_8u_7u_6u_3 - u_8u_6u_5 + u_6^2u_5)x_{10} + \\
&\quad (u_8^2u_7^2u_3 + u_8^2u_7u_5 - u_8u_7u_6u_5) \\
p_{12} &= (u_8 - u_6)x_{12} - x_{11}x_{10} + u_7x_{11} + u_6x_{10} - u_8u_7 \\
p_{13} &= x_{13}x_{10}x_8 - x_{13}x_{10}x_6 + (-u_8 + u_6)x_{13}x_9 - u_7x_{13}x_8 + \\
&\quad (u_8 - u_6)x_{13}x_7 + u_7x_{13}x_6 - u_6x_{10}x_8 + u_6x_{10}x_6 + \\
&\quad (u_8 - u_6)x_9x_6 + (-u_8 + u_6)x_8x_7 + u_8u_7x_8 - u_8u_7x_6 \\
p_{14} &= (u_8 - u_6)x_{14} - x_{13}x_{10} + u_7x_{13} + u_6x_{10} - u_8u_7
\end{aligned}
$$

## 3.4 Triangulation, step 4

**Choosing variable:** Trying the variable with index 11.

**Variable $x_{11}$ selected:** The number of polynomials with this variable, with indexes from 1 to 11, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_{11}$. No reduction needed.

The triangular system has not been changed.

## 3.5 Triangulation, step 5

**Choosing variable:** Trying the variable with index 10.

**Variable $x_{10}$ selected:** The number of polynomials with this variable, with indexes from 1 to 10, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_{10}$. No reduction needed.

The triangular system has not been changed.

## 3.6 Triangulation, step 6

**Choosing variable:** Trying the variable with index 9.

**Variable $x_9$ selected:** The number of polynomials with this variable, with indexes from 1 to 9, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable $x_9$ from all other polynomials by reducing them with polynomial $p_8$ from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_3^2 + u_2x_3x_2 + u_5x_3 + u_1x_2^2 + u_4x_2 \\
p_3 &= u_6x_3 - u_7x_2 \\
p_4 &= u_3x_5^2 + u_2x_5x_4 + u_5x_5 + u_1x_4^2 + u_4x_4 \\
p_5 &= u_6x_5 + x_4x_1 - u_7x_4 - u_6x_1 \\
p_6 &= u_5x_7 + u_4x_6 \\
p_7 &= u_3x_7x_3 + 0.5u_2x_7x_2 + 0.5u_5x_7 + 0.5u_2x_6x_3 + u_1x_6x_2 + \\
&\quad 0.5u_4x_6 - u_3x_3^2 - u_2x_3x_2 - 0.5u_5x_3 - u_1x_2^2 - 0.5u_4x_2 \\
p_8 &= (0.25u_5u_2 - 0.5u_4u_3)x_8x_5 + (u_3u_1 - 0.25u_2^2)x_8x_4x_1 + \\
&\quad (0.5u_5u_1 - 0.25u_4u_2)x_8x_4 + (-0.25u_5u_2 + 0.5u_4u_3)x_8x_1 \\
&\quad -u_3^2x_5^2x_1 - 0.5u_5u_3x_5^2 - u_3u_2x_5x_4x_1 \\
&\quad -0.5u_5u_2x_5x_4 + u_3^2x_5x_1^2 - 0.25u_5^2x_5
\end{aligned}
$$

$$
\begin{aligned}
& -u_3u_1x_4^2x_1 - 0.5u_5u_1x_4^2 + 0.5u_3u_2x_4x_1^2 + \\
& (0.25u_5u_2 - 0.5u_4u_3)x_4x_1 - 0.25u_5u_4x_4 + 0.5u_5u_3x_1^2 + \\
& 0.25u_5^2x_1 \\
p_9 \;=\;& u_3x_9x_1 + 0.5u_5x_9 + 0.5u_2x_8x_1 + 0.5u_4x_8 - u_3x_1^2 - 0.5u_5x_1 \\
p_{10} \;=\;& u_3x_{10}^2 + (u_8u_2 + u_5)x_{10} + (u_8^2u_1 + u_8u_4) \\
p_{11} \;=\;& u_3x_{11}^2x_{10}^2 + (u_8u_2 - 2u_7u_3 - u_6u_2)x_{11}^2x_{10} + \\
& (u_8^2u_1 - u_8u_7u_2 - 2u_8u_6u_1 + u_7^2u_3 + u_7u_6u_2 + u_6^2u_1) \\
& x_{11}^2 \\
& -2u_6u_3x_{11}x_{10}^2 + \\
& (2u_8u_7u_3 - u_8u_6u_2 + u_8u_5 + 2u_7u_6u_3 + u_6^2u_2 - u_6u_5) \\
& x_{11}x_{10} \\
& + \\
& (u_8^2u_7u_2 + u_8^2u_4 - 2u_8u_7^2u_3 - u_8u_7u_6u_2 \\
& -u_8u_7u_5 - 2u_8u_6u_4 + u_7u_6u_5 + u_6^2u_4) \\
& x_{11} \\
& +u_6^2u_3x_{10}^2 + (-2u_8u_7u_6u_3 - u_8u_6u_5 + u_6^2u_5)x_{10} + \\
& (u_8^2u_7^2u_3 + u_8^2u_7u_5 - u_8u_7u_6u_5) \\
p_{12} \;=\;& (u_8 - u_6)x_{12} - x_{11}x_{10} + u_7x_{11} + u_6x_{10} - u_8u_7 \\
p_{13} \;=\;& x_{13}x_{10}x_8 - x_{13}x_{10}x_6 + (-u_8 + u_6)x_{13}x_9 - u_7x_{13}x_8 + \\
& (u_8 - u_6)x_{13}x_7 + u_7x_{13}x_6 - u_6x_{10}x_8 + u_6x_{10}x_6 + \\
& (u_8 - u_6)x_9x_6 + (-u_8 + u_6)x_8x_7 + u_8u_7x_8 - u_8u_7x_6 \\
p_{14} \;=\;& (u_8 - u_6)x_{14} - x_{13}x_{10} + u_7x_{13} + u_6x_{10} - u_8u_7
\end{aligned}
$$

## 3.7 Triangulation, step 7

**Choosing variable:** Trying the variable with index 8.

**Variable $x_8$ selected:** The number of polynomials with this variable, with indexes from 1 to 8, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_8$. No reduction needed.

The triangular system has not been changed.

## 3.8 Triangulation, step 8

**Choosing variable:** Trying the variable with index 7.

**Variable $x_7$ selected:** The number of polynomials with this variable, with indexes from 1 to 7, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable $x_7$ from all other polynomials by reducing them with polynomial $p_6$ from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_3^2 + u_2x_3x_2 + u_5x_3 + u_1x_2^2 + u_4x_2 \\
p_3 &= u_6x_3 - u_7x_2 \\
p_4 &= u_3x_5^2 + u_2x_5x_4 + u_5x_5 + u_1x_4^2 + u_4x_4 \\
p_5 &= u_6x_5 + x_4x_1 - u_7x_4 - u_6x_1 \\
p_6 &= (0.5u_5u_2 - u_4u_3)x_6x_3 + (u_5u_1 - 0.5u_4u_2)x_6x_2 \\
&\quad -u_5u_3x_3^2 - u_5u_2x_3x_2 - 0.5u_5^2x_3 - u_5u_1x_2^2 \\
&\quad -0.5u_5u_4x_2 \\
p_7 &= u_5x_7 + u_4x_6 \\
p_8 &= (0.25u_5u_2 - 0.5u_4u_3)x_8x_5 + (u_3u_1 - 0.25u_2^2)x_8x_4x_1 + \\
&\quad (0.5u_5u_1 - 0.25u_4u_2)x_8x_4 + (-0.25u_5u_2 + 0.5u_4u_3)x_8x_1 \\
&\quad -u_3^2x_5^2x_1 - 0.5u_5u_3x_5^2 - u_3u_2x_5x_4x_1 \\
&\quad -0.5u_5u_2x_5x_4 + u_3^2x_5x_1^2 - 0.25u_5^2x_5 \\
&\quad -u_3u_1x_4^2x_1 - 0.5u_5u_1x_4^2 + 0.5u_3u_2x_4x_1^2 + \\
&\quad (0.25u_5u_2 - 0.5u_4u_3)x_4x_1 - 0.25u_5u_4x_4 + 0.5u_5u_3x_1^2 + \\
&\quad 0.25u_5^2x_1 \\
p_9 &= u_3x_9x_1 + 0.5u_5x_9 + 0.5u_2x_8x_1 + 0.5u_4x_8 - u_3x_1^2 - 0.5u_5x_1 \\
p_{10} &= u_3x_{10}^2 + (u_8u_2 + u_5)x_{10} + (u_8^2u_1 + u_8u_4) \\
p_{11} &= u_3x_{11}^2x_{10}^2 + (u_8u_2 - 2u_7u_3 - u_6u_2)x_{11}^2x_{10} + \\
&\quad (u_8^2u_1 - u_8u_7u_2 - 2u_8u_6u_1 + u_7^2u_3 + u_7u_6u_2 + u_6^2u_1) \\
&\quad x_{11}^2 \\
&\quad -2u_6u_3x_{11}x_{10}^2 + \\
&\quad (2u_8u_7u_3 - u_8u_6u_2 + u_8u_5 + 2u_7u_6u_3 + u_6^2u_2 - u_6u_5) \\
&\quad x_{11}x_{10} \\
&\quad + \\
&\quad (u_8^2u_7u_2 + u_8^2u_4 - 2u_8u_7^2u_3 - u_8u_7u_6u_2 \\
&\quad -u_8u_7u_5 - 2u_8u_6u_4 + u_7u_6u_5 + u_6^2u_4) \\
&\quad x_{11} \\
&\quad +u_6^2u_3x_{10}^2 + (-2u_8u_7u_6u_3 - u_8u_6u_5 + u_6^2u_5)x_{10} + \\
&\quad (u_8^2u_7^2u_3 + u_8^2u_7u_5 - u_8u_7u_6u_5) \\
p_{12} &= (u_8 - u_6)x_{12} - x_{11}x_{10} + u_7x_{11} + u_6x_{10} - u_8u_7 \\
p_{13} &= x_{13}x_{10}x_8 - x_{13}x_{10}x_6 + (-u_8 + u_6)x_{13}x_9 - u_7x_{13}x_8 + \\
&\quad (u_8 - u_6)x_{13}x_7 + u_7x_{13}x_6 - u_6x_{10}x_8 + u_6x_{10}x_6 + \\
&\quad (u_8 - u_6)x_9x_6 + (-u_8 + u_6)x_8x_7 + u_8u_7x_8 - u_8u_7x_6 \\
p_{14} &= (u_8 - u_6)x_{14} - x_{13}x_{10} + u_7x_{13} + u_6x_{10} - u_8u_7
\end{aligned}
$$

## 3.9 Triangulation, step 9

**Choosing variable:** Trying the variable with index 6.

**Variable $x_6$ selected:** The number of polynomials with this variable, with indexes from 1 to 6, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_6$. No reduction needed.

The triangular system has not been changed.

## 3.10 Triangulation, step 10

**Choosing variable:** Trying the variable with index 5.

**Variable $x_5$ selected:** The number of polynomials with this variable, with indexes from 1 to 5, is 2.

**Minimal degrees:** 1 polynomial(s) with degree 1 and 1 polynomial(s) with degree 2.

**Polynomial with linear degree:** Removing variable $x_5$ from all other polynomials by reducing them with polynomial $p_5$ from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_3^2 + u_2x_3x_2 + u_5x_3 + u_1x_2^2 + u_4x_2 \\
p_3 &= u_6x_3 - u_7x_2 \\
p_4 &= u_3x_4^2x_1^2 + (-2u_7u_3 - u_6u_2)x_4^2x_1 + \\
&\quad (u_7^2u_3 + u_7u_6u_2 + u_6^2u_1)x_4^2 - 2u_6u_3x_4x_1^2 + \\
&\quad (2u_7u_6u_3 + u_6^2u_2 - u_6u_5)x_4x_1 + \\
&\quad (u_7u_6u_5 + u_6^2u_4)x_4 + u_6^2u_3x_1^2 + u_6^2u_5x_1 \\
p_5 &= u_6x_5 + x_4x_1 - u_7x_4 - u_6x_1 \\
p_6 &= (0.5u_5u_2 - u_4u_3)x_6x_3 + (u_5u_1 - 0.5u_4u_2)x_6x_2 \\
&\quad -u_5u_3x_3^2 - u_5u_2x_3x_2 - 0.5u_5^2x_3 - u_5u_1x_2^2 \\
&\quad -0.5u_5u_4x_2 \\
p_7 &= u_5x_7 + u_4x_6 \\
p_8 &= (0.25u_5u_2 - 0.5u_4u_3)x_8x_5 + (u_3u_1 - 0.25u_2^2)x_8x_4x_1 + \\
&\quad (0.5u_5u_1 - 0.25u_4u_2)x_8x_4 + (-0.25u_5u_2 + 0.5u_4u_3)x_8x_1 \\
&\quad -u_3^2x_5^2x_1 - 0.5u_5u_3x_5^2 - u_3u_2x_5x_4x_1 \\
&\quad -0.5u_5u_2x_5x_4 + u_3^2x_5x_1^2 - 0.25u_5^2x_5 \\
&\quad -u_3u_1x_4^2x_1 - 0.5u_5u_1x_4^2 + 0.5u_3u_2x_4x_1^2 + \\
&\quad (0.25u_5u_2 - 0.5u_4u_3)x_4x_1 - 0.25u_5u_4x_4 + 0.5u_5u_3x_1^2 + \\
&\quad 0.25u_5^2x_1 \\
p_9 &= u_3x_9x_1 + 0.5u_5x_9 + 0.5u_2x_8x_1 + 0.5u_4x_8 - u_3x_1^2 - 0.5u_5x_1 \\
p_{10} &= u_3x_{10}^2 + (u_8u_2 + u_5)x_{10} + (u_8^2u_1 + u_8u_4) \\
p_{11} &= u_3x_{11}^2x_{10}^2 + (u_8u_2 - 2u_7u_3 - u_6u_2)x_{11}^2x_{10} + \\
&\quad (u_8^2u_1 - u_8u_7u_2 - 2u_8u_6u_1 + u_7^2u_3 + u_7u_6u_2 + u_6^2u_1)
\end{aligned}
$$

$$
\begin{aligned}
& x_{11}^2 \\
& -2u_6u_3x_{11}x_{10}^2 + \\
& (2u_8u_7u_3 - u_8u_6u_2 + u_8u_5 + 2u_7u_6u_3 + u_6^2u_2 - u_6u_5) \\
& x_{11}x_{10} \\
& + \\
& (u_8^2u_7u_2 + u_8^2u_4 - 2u_8u_7^2u_3 - u_8u_7u_6u_2 \\
& -u_8u_7u_5 - 2u_8u_6u_4 + u_7u_6u_5 + u_6^2u_4) \\
& x_{11} \\
& +u_6^2u_3x_{10}^2 + (-2u_8u_7u_6u_3 - u_8u_6u_5 + u_6^2u_5)x_{10} + \\
& (u_8^2u_7^2u_3 + u_8^2u_7u_5 - u_8u_7u_6u_5)
\end{aligned}
$$

$$
\begin{aligned}
p_{12} &= (u_8 - u_6)x_{12} - x_{11}x_{10} + u_7x_{11} + u_6x_{10} - u_8u_7 \\
p_{13} &= x_{13}x_{10}x_8 - x_{13}x_{10}x_6 + (-u_8 + u_6)x_{13}x_9 - u_7x_{13}x_8 + \\
&\quad (u_8 - u_6)x_{13}x_7 + u_7x_{13}x_6 - u_6x_{10}x_8 + u_6x_{10}x_6 + \\
&\quad (u_8 - u_6)x_9x_6 + (-u_8 + u_6)x_8x_7 + u_8u_7x_8 - u_8u_7x_6 \\
p_{14} &= (u_8 - u_6)x_{14} - x_{13}x_{10} + u_7x_{13} + u_6x_{10} - u_8u_7
\end{aligned}
$$

## 3.11 Triangulation, step 11

**Choosing variable:** Trying the variable with index 4.

**Variable $x_4$ selected:** The number of polynomials with this variable, with indexes from 1 to 4, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_4$. No reduction needed.

The triangular system has not been changed.

## 3.12 Triangulation, step 12

**Choosing variable:** Trying the variable with index 3.

**Variable $x_3$ selected:** The number of polynomials with this variable, with indexes from 1 to 3, is 2.

**Minimal degrees:** 1 polynomial(s) with degree 1 and 1 polynomial(s) with degree 2.

**Polynomial with linear degree:** Removing variable $x_3$ from all other polynomials by reducing them with polynomial $p_3$ from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= (u_7^2u_3 + u_7u_6u_2 + u_6^2u_1)x_2^2 + \\
&\quad (u_7u_6u_5 + u_6^2u_4)x_2 \\
p_3 &= u_6x_3 - u_7x_2
\end{aligned}
$$

$$
\begin{aligned}
p_4 &= u_3x_4^2x_1^2 + (-2u_7u_3 - u_6u_2)x_4^2x_1 + \\
&\quad (u_7^2u_3 + u_7u_6u_2 + u_6^2u_1)x_4^2 - 2u_6u_3x_4x_1^2 + \\
&\quad (2u_7u_6u_3 + u_6^2u_2 - u_6u_5)x_4x_1 + \\
&\quad (u_7u_6u_5 + u_6^2u_4)x_4 + u_6^2u_3x_1^2 + u_6^2u_5x_1 \\
p_5 &= u_6x_5 + x_4x_1 - u_7x_4 - u_6x_1 \\
p_6 &= (0.5u_5u_2 - u_4u_3)x_6x_3 + (u_5u_1 - 0.5u_4u_2)x_6x_2 \\
&\quad -u_5u_3x_3^2 - u_5u_2x_3x_2 - 0.5u_5^2x_3 - u_5u_1x_2^2 \\
&\quad -0.5u_5u_4x_2 \\
p_7 &= u_5x_7 + u_4x_6 \\
p_8 &= (0.25u_5u_2 - 0.5u_4u_3)x_8x_5 + (u_3u_1 - 0.25u_2^2)x_8x_4x_1 + \\
&\quad (0.5u_5u_1 - 0.25u_4u_2)x_8x_4 + (-0.25u_5u_2 + 0.5u_4u_3)x_8x_1 \\
&\quad -u_3^2x_5^2x_1 - 0.5u_5u_3x_5^2 - u_3u_2x_5x_4x_1 \\
&\quad -0.5u_5u_2x_5x_4 + u_3^2x_5x_1^2 - 0.25u_5^2x_5 \\
&\quad -u_3u_1x_4^2x_1 - 0.5u_5u_1x_4^2 + 0.5u_3u_2x_4x_1^2 + \\
&\quad (0.25u_5u_2 - 0.5u_4u_3)x_4x_1 - 0.25u_5u_4x_4 + 0.5u_5u_3x_1^2 + \\
&\quad 0.25u_5^2x_1 \\
p_9 &= u_3x_9x_1 + 0.5u_5x_9 + 0.5u_2x_8x_1 + 0.5u_4x_8 - u_3x_1^2 - 0.5u_5x_1 \\
p_{10} &= u_3x_{10}^2 + (u_8u_2 + u_5)x_{10} + (u_8^2u_1 + u_8u_4) \\
p_{11} &= u_3x_{11}^2x_{10}^2 + (u_8u_2 - 2u_7u_3 - u_6u_2)x_{11}^2x_{10} + \\
&\quad (u_8^2u_1 - u_8u_7u_2 - 2u_8u_6u_1 + u_7^2u_3 + u_7u_6u_2 + u_6^2u_1) \\
&\quad x_{11}^2 \\
&\quad -2u_6u_3x_{11}x_{10}^2 + \\
&\quad (2u_8u_7u_3 - u_8u_6u_2 + u_8u_5 + 2u_7u_6u_3 + u_6^2u_2 - u_6u_5) \\
&\quad x_{11}x_{10} \\
&\quad + \\
&\quad (u_8^2u_7u_2 + u_8^2u_4 - 2u_8u_7^2u_3 - u_8u_7u_6u_2 \\
&\quad -u_8u_7u_5 - 2u_8u_6u_4 + u_7u_6u_5 + u_6^2u_4) \\
&\quad x_{11} \\
&\quad +u_6^2u_3x_{10}^2 + (-2u_8u_7u_6u_3 - u_8u_6u_5 + u_6^2u_5)x_{10} + \\
&\quad (u_8^2u_7^2u_3 + u_8^2u_7u_5 - u_8u_7u_6u_5) \\
p_{12} &= (u_8 - u_6)x_{12} - x_{11}x_{10} + u_7x_{11} + u_6x_{10} - u_8u_7 \\
p_{13} &= x_{13}x_{10}x_8 - x_{13}x_{10}x_6 + (-u_8 + u_6)x_{13}x_9 - u_7x_{13}x_8 + \\
&\quad (u_8 - u_6)x_{13}x_7 + u_7x_{13}x_6 - u_6x_{10}x_8 + u_6x_{10}x_6 + \\
&\quad (u_8 - u_6)x_9x_6 + (-u_8 + u_6)x_8x_7 + u_8u_7x_8 - u_8u_7x_6 \\
p_{14} &= (u_8 - u_6)x_{14} - x_{13}x_{10} + u_7x_{13} + u_6x_{10} - u_8u_7
\end{aligned}
$$

## 3.13 Triangulation, step 13

**Choosing variable:** Trying the variable with index 2.

**Variable $x_2$ selected:** The number of polynomials with this variable, with indexes from 1 to 2, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_2$. No reduction needed.

The triangular system has not been changed.

## 3.14 Triangulation, step 14

**Choosing variable:** Trying the variable with index 1.

**Variable $x_1$ selected:** The number of polynomials with this variable, with indexes from 1 to 1, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_1$. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= (u_7^2u_3 + u_7u_6u_2 + u_6^2u_1)x_2^2 + \\
&\quad (u_7u_6u_5 + u_6^2u_4)x_2 \\
p_3 &= u_6x_3 - u_7x_2 \\
p_4 &= u_3x_4^2x_1^2 + (-2u_7u_3 - u_6u_2)x_4^2x_1 + \\
&\quad (u_7^2u_3 + u_7u_6u_2 + u_6^2u_1)x_4^2 - 2u_6u_3x_4x_1^2 + \\
&\quad (2u_7u_6u_3 + u_6^2u_2 - u_6u_5)x_4x_1 + \\
&\quad (u_7u_6u_5 + u_6^2u_4)x_4 + u_6^2u_3x_1^2 + u_6^2u_5x_1 \\
p_5 &= u_6x_5 + x_4x_1 - u_7x_4 - u_6x_1 \\
p_6 &= (0.5u_5u_2 - u_4u_3)x_6x_3 + (u_5u_1 - 0.5u_4u_2)x_6x_2 \\
&\quad -u_5u_3x_3^2 - u_5u_2x_3x_2 - 0.5u_5^2x_3 - u_5u_1x_2^2 \\
&\quad -0.5u_5u_4x_2 \\
p_7 &= u_5x_7 + u_4x_6 \\
p_8 &= (0.25u_5u_2 - 0.5u_4u_3)x_8x_5 + (u_3u_1 - 0.25u_2^2)x_8x_4x_1 + \\
&\quad (0.5u_5u_1 - 0.25u_4u_2)x_8x_4 + (-0.25u_5u_2 + 0.5u_4u_3)x_8x_1 \\
&\quad -u_3^2x_5^2x_1 - 0.5u_5u_3x_5^2 - u_3u_2x_5x_4x_1 \\
&\quad -0.5u_5u_2x_5x_4 + u_3^2x_5x_1^2 - 0.25u_5^2x_5 \\
&\quad -u_3u_1x_4^2x_1 - 0.5u_5u_1x_4^2 + 0.5u_3u_2x_4x_1^2 + \\
&\quad (0.25u_5u_2 - 0.5u_4u_3)x_4x_1 - 0.25u_5u_4x_4 + 0.5u_5u_3x_1^2 + \\
&\quad 0.25u_5^2x_1 \\
p_9 &= u_3x_9x_1 + 0.5u_5x_9 + 0.5u_2x_8x_1 + 0.5u_4x_8 - u_3x_1^2 - 0.5u_5x_1 \\
p_{10} &= u_3x_{10}^2 + (u_8u_2 + u_5)x_{10} + (u_8^2u_1 + u_8u_4) \\
p_{11} &= u_3x_{11}^2x_{10}^2 + (u_8u_2 - 2u_7u_3 - u_6u_2)x_{11}^2x_{10} + \\
&\quad (u_8^2u_1 - u_8u_7u_2 - 2u_8u_6u_1 + u_7^2u_3 + u_7u_6u_2 + u_6^2u_1) \\
&\quad x_{11}^2 \\
&\quad -2u_6u_3x_{11}x_{10}^2 +
\end{aligned}
$$

$$
\begin{aligned}
& (2u_8u_7u_3 - u_8u_6u_2 + u_8u_5 + 2u_7u_6u_3 + u_6^2u_2 - u_6u_5) \\
& x_{11}x_{10} \\
& + \\
& (u_8^2u_7u_2 + u_8^2u_4 - 2u_8u_7^2u_3 - u_8u_7u_6u_2 \\
& -u_8u_7u_5 - 2u_8u_6u_4 + u_7u_6u_5 + u_6^2u_4) \\
& x_{11} \\
& +u_6^2u_3x_{10}^2 + (-2u_8u_7u_6u_3 - u_8u_6u_5 + u_6^2u_5)x_{10} + \\
& (u_8^2u_7^2u_3 + u_8^2u_7u_5 - u_8u_7u_6u_5)
\end{aligned}
$$

$$
\begin{aligned}
p_{12} &= (u_8 - u_6)x_{12} - x_{11}x_{10} + u_7x_{11} + u_6x_{10} - u_8u_7 \\
p_{13} &= x_{13}x_{10}x_8 - x_{13}x_{10}x_6 + (-u_8 + u_6)x_{13}x_9 - u_7x_{13}x_8 + \\
& (u_8 - u_6)x_{13}x_7 + u_7x_{13}x_6 - u_6x_{10}x_8 + u_6x_{10}x_6 + \\
& (u_8 - u_6)x_9x_6 + (-u_8 + u_6)x_8x_7 + u_8u_7x_8 - u_8u_7x_6 \\
p_{14} &= (u_8 - u_6)x_{14} - x_{13}x_{10} + u_7x_{13} + u_6x_{10} - u_8u_7
\end{aligned}
$$

# 4 Final Remainder

## 4.1 Final remainder for conjecture Polar 001

Calculating final remainder of the conclusion:

$$g = x_{13}x_{11} + (u_8 - 2u_6)x_{13} + (-2u_8 + u_6)x_{11} + u_8u_6$$

with respect to the triangular system.

1. Pseudo remainder with $p_{14}$ over variable $x_{14}$:

$$g = x_{13}x_{11} + (u_8 - 2u_6)x_{13} + (-2u_8 + u_6)x_{11} + u_8u_6$$

2. Pseudo remainder with $p_{13}$ over variable $x_{13}$:

$$
\begin{aligned}
g = & (-2u_8 + 2u_6)x_{11}x_{10}x_8 + (2u_8 - 2u_6)x_{11}x_{10}x_6 + \\
& (-u_8 + u_6)x_{11}x_9x_6 + (2u_8^2 - 3u_8u_6 + u_6^2)x_{11}x_9 + \\
& (u_8 - u_6)x_{11}x_8x_7 + (u_8u_7 - u_7u_6)x_{11}x_8 + \\
& (-2u_8^2 + 3u_8u_6 - u_6^2)x_{11}x_7 + (-u_8u_7 + u_7u_6)x_{11}x_6 + \\
& (2u_8u_6 - 2u_6^2)x_{10}x_8 + (-2u_8u_6 + 2u_6^2)x_{10}x_6 + \\
& (-u_8^2 + 3u_8u_6 - 2u_6^2)x_9x_6 + (-u_8^2u_6 + u_8u_6^2)x_9 + \\
& (u_8^2 - 3u_8u_6 + 2u_6^2)x_8x_7 + (-u_8^2u_7 + u_8u_7u_6)x_8 + \\
& (u_8^2u_6 - u_8u_6^2)x_7 + (u_8^2u_7 - u_8u_7u_6)x_6
\end{aligned}
$$

3. Pseudo remainder with $p_{12}$ over variable $x_{12}$:

$$
\begin{aligned}
g \;=\;& (-2u_8+2u_6)x_{11}x_{10}x_8+(2u_8-2u_6)x_{11}x_{10}x_6+\\
& (-u_8+u_6)x_{11}x_9x_6+(2u_8^2-3u_8u_6+u_6^2)x_{11}x_9+\\
& (u_8-u_6)x_{11}x_8x_7+(u_8u_7-u_7u_6)x_{11}x_8+\\
& (-2u_8^2+3u_8u_6-u_6^2)x_{11}x_7+(-u_8u_7+u_7u_6)x_{11}x_6+\\
& (2u_8u_6-2u_6^2)x_{10}x_8+(-2u_8u_6+2u_6^2)x_{10}x_6+\\
& (-u_8^2+3u_8u_6-2u_6^2)x_9x_6+(-u_8^2u_6+u_8u_6^2)x_9+\\
& (u_8^2-3u_8u_6+2u_6^2)x_8x_7+(-u_8^2u_7+u_8u_7u_6)x_8+\\
& (u_8^2u_6-u_8u_6^2)x_7+(u_8^2u_7-u_8u_7u_6)x_6
\end{aligned}
$$

4. Pseudo remainder with $p_{11}$ over variable $x_{11}$:

$$
\begin{aligned}
g \;=\;& (-2u_8+2u_6)x_{11}x_{10}x_8+(2u_8-2u_6)x_{11}x_{10}x_6+\\
& (-u_8+u_6)x_{11}x_9x_6+(2u_8^2-3u_8u_6+u_6^2)x_{11}x_9+\\
& (u_8-u_6)x_{11}x_8x_7+(u_8u_7-u_7u_6)x_{11}x_8+\\
& (-2u_8^2+3u_8u_6-u_6^2)x_{11}x_7+(-u_8u_7+u_7u_6)x_{11}x_6+\\
& (2u_8u_6-2u_6^2)x_{10}x_8+(-2u_8u_6+2u_6^2)x_{10}x_6+\\
& (-u_8^2+3u_8u_6-2u_6^2)x_9x_6+(-u_8^2u_6+u_8u_6^2)x_9+\\
& (u_8^2-3u_8u_6+2u_6^2)x_8x_7+(-u_8^2u_7+u_8u_7u_6)x_8+\\
& (u_8^2u_6-u_8u_6^2)x_7+(u_8^2u_7-u_8u_7u_6)x_6
\end{aligned}
$$

5. Pseudo remainder with $p_{10}$ over variable $x_{10}$:

$$
\begin{aligned}
g \;=\;& (-2u_8+2u_6)x_{11}x_{10}x_8+(2u_8-2u_6)x_{11}x_{10}x_6+\\
& (-u_8+u_6)x_{11}x_9x_6+(2u_8^2-3u_8u_6+u_6^2)x_{11}x_9+\\
& (u_8-u_6)x_{11}x_8x_7+(u_8u_7-u_7u_6)x_{11}x_8+\\
& (-2u_8^2+3u_8u_6-u_6^2)x_{11}x_7+(-u_8u_7+u_7u_6)x_{11}x_6+\\
& (2u_8u_6-2u_6^2)x_{10}x_8+(-2u_8u_6+2u_6^2)x_{10}x_6+\\
& (-u_8^2+3u_8u_6-2u_6^2)x_9x_6+(-u_8^2u_6+u_8u_6^2)x_9+\\
& (u_8^2-3u_8u_6+2u_6^2)x_8x_7+(-u_8^2u_7+u_8u_7u_6)x_8+\\
& (u_8^2u_6-u_8u_6^2)x_7+(u_8^2u_7-u_8u_7u_6)x_6
\end{aligned}
$$

6. Pseudo remainder with $p_9$ over variable $x_9$:

   *Polynomial too big for output (text size is* 2247 *characters, number of terms is* 34*)*

7. Pseudo remainder with $p_8$ over variable $x_8$:

   *Polynomial too big for output (text size is* 25385 *characters, number of terms is* 174*)*

8. Pseudo remainder with $p_7$ over variable $x_7$:

   *Polynomial too big for output (text size is* 23331 *characters, number of terms is* 126*)*

9. Pseudo remainder with $p_6$ over variable $x_6$:

   *Polynomial too big for output (number of terms is* 282*)*

10. Pseudo remainder with $p_5$ over variable $x_5$:

    *Polynomial too big for output (text size is* 132822 *characters, number of terms is* 152*)*

11. Pseudo remainder with $p_4$ over variable $x_4$:

    *Polynomial too big for output (text size is* 390096 *characters, number of terms is* 206*)*

12. Pseudo remainder with $p_3$ over variable $x_3$:

    *Polynomial too big for output (text size is* 289569 *characters, number of terms is* 86*)*

13. Pseudo remainder with $p_2$ over variable $x_2$:

    *Polynomial too big for output (text size is greater than* 2000 *characters, number of terms is* 48*)*

14. Pseudo remainder with $p_1$ over variable $x_1$:

    *Polynomial too big for output (text size is greater than* 2000 *characters, number of terms is* 4*)*

# 5 Prover results

**Status:** Theorem can't be neither proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 282 terms.

**Time Complexity:** Time spent by the prover is 2.539 seconds.