OpenGeoProver Output for conjecture "Chou 012"

Wu's method used

February 12, 2012

1 Validation of Construction Protocol

Construction steps:

- General Conic Section c
- Random point A0 from general conic c
- Random point A1 from general conic c
- Random point A2 from general conic c
- Random point A3 from general conic c
- Random point A4 from general conic c
- Random point A5 from general conic c
- Line A0A1 through two points A0 and A1
- Line A2A3 through two points A2 and A3
- Intersection point A of point sets A0A1 and A2A3
- Line A4A5 through two points A4 and A5
- Intersection point B of point sets A0A1 and A4A5
- Line A0A2 through two points A0 and A2
- Line A3A4 through two points A3 and A4
- Intersection point C of point sets A0A2 and A3A4
- Line A0A3 through two points A0 and A3
- Line A2A5 through two points A2 and A5
- Intersection point D of point sets A0A3 and A2A5
- Line A1A4 through two points A1 and A4
- Intersection point E of point sets A1A4 and A2A5

- Line A1A5 through two points A1 and A5

- Intersection point F of point sets A1A5 and A3A4

- General Conic Section c1 which contains points A, B, C, D and E

Theorem statement:

- Point F lies on set of points c1

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of general conic section c:

List of parametric points

- Point Ac has been assigned following coordinates: $(u_1, 0)$

- Point Bc has been assigned following coordinates: $(u_2, 0)$

- Point Cc has been assigned following coordinates: $(u_3, 0)$

- Point Dc has been assigned following coordinates: $(u_4, 0)$

- Point Ec has been assigned following coordinates: $(u_5, 0)$

- Condition for point X(x_1, x_2) to belong to this conic section is following equation:

$$p \quad = \quad u_3 x_2^2 + u_2 x_2 x_1 + u_5 x_2 + u_1 x_1^2 + u_4 x_1$$

2.2 Transformation of point A0:

- Point A0 has been assigned following coordinates: $(0, 0)$

2.3 Transformation of point A1:

- Point A1 has been assigned following coordinates: $(0, x_1)$

- Polynomial that point A1 has to satisfy is:

$$p \quad = \quad u_3 x_1^2 + u_5 x_1$$

- Processing of polynomial

$$p \quad = \quad u_3 x_1^2 + u_5 x_1$$

  Info: Polynomial

$$p \quad = \quad u_3 x_1^2 + u_5 x_1$$

  added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.4 Transformation of point A2:

- Point A2 has been assigned following coordinates: $(u_6,\ x_2)$
- Polynomial that point A2 has to satisfy is:

$$p \quad = \quad u_3 x_2^2 + (u_6 u_2 + u_5)x_2 + (u_6^2 u_1 + u_6 u_4)$$

- Processing of polynomial

$$p \quad = \quad u_3 x_2^2 + (u_6 u_2 + u_5)x_2 + (u_6^2 u_1 + u_6 u_4)$$

Info: Polynomial

$$p \quad = \quad u_3 x_2^2 + (u_6 u_2 + u_5)x_2 + (u_6^2 u_1 + u_6 u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of point A3:

- Point A3 has been assigned following coordinates: $(u_7,\ x_3)$
- Polynomial that point A3 has to satisfy is:

$$p \quad = \quad u_3 x_3^2 + (u_7 u_2 + u_5)x_3 + (u_7^2 u_1 + u_7 u_4)$$

- Processing of polynomial

$$p \quad = \quad u_3 x_3^2 + (u_7 u_2 + u_5)x_3 + (u_7^2 u_1 + u_7 u_4)$$

Info: Polynomial

$$p \quad = \quad u_3 x_3^2 + (u_7 u_2 + u_5)x_3 + (u_7^2 u_1 + u_7 u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point A4:

- Point A4 has been assigned following coordinates: $(u_8,\ x_4)$
- Polynomial that point A4 has to satisfy is:

$$p \quad = \quad u_3 x_4^2 + (u_8 u_2 + u_5)x_4 + (u_8^2 u_1 + u_8 u_4)$$

- Processing of polynomial

$$p \quad = \quad u_3 x_4^2 + (u_8 u_2 + u_5)x_4 + (u_8^2 u_1 + u_8 u_4)$$

Info: Polynomial

$$p \quad = \quad u_3 x_4^2 + (u_8 u_2 + u_5)x_4 + (u_8^2 u_1 + u_8 u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point A5:

- Point A5 has been assigned following coordinates: (u_9, x_5)

- Polynomial that point A5 has to satisfy is:

$$p = u_3 x_5^2 + (u_9 u_2 + u_5) x_5 + (u_9^2 u_1 + u_9 u_4)$$

- Processing of polynomial

$$p = u_3 x_5^2 + (u_9 u_2 + u_5) x_5 + (u_9^2 u_1 + u_9 u_4)$$

  Info: Polynomial

$$p = u_3 x_5^2 + (u_9 u_2 + u_5) x_5 + (u_9^2 u_1 + u_9 u_4)$$

  added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.8 Transformation of point A:

- Point A has been assigned following coordinates: (x_6, x_7)

- Polynomial that point A has to satisfy is:

$$p = x_6 x_1$$

- Processing of polynomial

$$p = x_6 x_1$$

  Info: Polynomial

$$p = x_6 x_1$$

  added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point A has to satisfy is:

$$p = (u_7 - u_6) x_7 - x_6 x_3 + x_6 x_2 + u_6 x_3 - u_7 x_2$$

- Processing of polynomial

$$p = (u_7 - u_6) x_7 - x_6 x_3 + x_6 x_2 + u_6 x_3 - u_7 x_2$$

  Info: Polynomial

$$p = (u_7 - u_6) x_7 - x_6 x_3 + x_6 x_2 + u_6 x_3 - u_7 x_2$$

  added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.9 Transformation of point B:

- Point B has been assigned following coordinates: (x_8, x_9)
- Polynomial that point B has to satisfy is:

$$p = x_8 x_1$$

- Processing of polynomial

$$p = x_8 x_1$$

  Info: Polynomial

$$p = x_8 x_1$$

  added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point B has to satisfy is:

$$p = (u_9 - u_8)x_9 - x_8 x_5 + x_8 x_4 + u_8 x_5 - u_9 x_4$$

- Processing of polynomial

$$p = (u_9 - u_8)x_9 - x_8 x_5 + x_8 x_4 + u_8 x_5 - u_9 x_4$$

  Info: Polynomial

$$p = (u_9 - u_8)x_9 - x_8 x_5 + x_8 x_4 + u_8 x_5 - u_9 x_4$$

  added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.10 Transformation of point C:

- Point C has been assigned following coordinates: (x_{10}, x_{11})
- Polynomial that point C has to satisfy is:

$$p = u_6 x_{11} - x_{10} x_2$$

- Processing of polynomial

$$p = u_6 x_{11} - x_{10} x_2$$

  Info: Polynomial

$$p = u_6 x_{11} - x_{10} x_2$$

  added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point C has to satisfy is:

$$p \quad = \quad (u_8 - u_7)x_{11} - x_{10}x_4 + x_{10}x_3 + u_7x_4 - u_8x_3$$

- Processing of polynomial

$$p \quad = \quad (u_8 - u_7)x_{11} - x_{10}x_4 + x_{10}x_3 + u_7x_4 - u_8x_3$$

  Info: Polynomial

$$p \quad = \quad (u_8 - u_7)x_{11} - x_{10}x_4 + x_{10}x_3 + u_7x_4 - u_8x_3$$

  added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.11 Transformation of point D:

- Point D has been assigned following coordinates: (x_{12}, x_{13})
- Polynomial that point D has to satisfy is:

$$p \quad = \quad u_7x_{13} - x_{12}x_3$$

- Processing of polynomial

$$p \quad = \quad u_7x_{13} - x_{12}x_3$$

  Info: Polynomial

$$p \quad = \quad u_7x_{13} - x_{12}x_3$$

  added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point D has to satisfy is:

$$p \quad = \quad (u_9 - u_6)x_{13} - x_{12}x_5 + x_{12}x_2 + u_6x_5 - u_9x_2$$

- Processing of polynomial

$$p \quad = \quad (u_9 - u_6)x_{13} - x_{12}x_5 + x_{12}x_2 + u_6x_5 - u_9x_2$$

  Info: Polynomial

$$p \quad = \quad (u_9 - u_6)x_{13} - x_{12}x_5 + x_{12}x_2 + u_6x_5 - u_9x_2$$

  added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.12   Transformation of point E:

- Point E has been assigned following coordinates: $(x_{14},\ x_{15})$
- Polynomial that point E has to satisfy is:

$$p \quad = \quad u_8 x_{15} - x_{14} x_4 + x_{14} x_1 - u_8 x_1$$

- Processing of polynomial

$$p \quad = \quad u_8 x_{15} - x_{14} x_4 + x_{14} x_1 - u_8 x_1$$

Info: Polynomial

$$p \quad = \quad u_8 x_{15} - x_{14} x_4 + x_{14} x_1 - u_8 x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point E has to satisfy is:

$$p \quad = \quad (u_9 - u_6) x_{15} - x_{14} x_5 + x_{14} x_2 + u_6 x_5 - u_9 x_2$$

- Processing of polynomial

$$p \quad = \quad (u_9 - u_6) x_{15} - x_{14} x_5 + x_{14} x_2 + u_6 x_5 - u_9 x_2$$

Info: Polynomial

$$p \quad = \quad (u_9 - u_6) x_{15} - x_{14} x_5 + x_{14} x_2 + u_6 x_5 - u_9 x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.13   Transformation of point F:

- Point F has been assigned following coordinates: $(x_{16},\ x_{17})$
- Polynomial that point F has to satisfy is:

$$p \quad = \quad u_9 x_{17} - x_{16} x_5 + x_{16} x_1 - u_9 x_1$$

- Processing of polynomial

$$p \quad = \quad u_9 x_{17} - x_{16} x_5 + x_{16} x_1 - u_9 x_1$$

Info: Polynomial

$$p \quad = \quad u_9 x_{17} - x_{16} x_5 + x_{16} x_1 - u_9 x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point F has to satisfy is:

$$p \quad = \quad (u_8 - u_7)x_{17} - x_{16}x_4 + x_{16}x_3 + u_7x_4 - u_8x_3$$

- Processing of polynomial

$$p \quad = \quad (u_8 - u_7)x_{17} - x_{16}x_4 + x_{16}x_3 + u_7x_4 - u_8x_3$$

  Info: Polynomial

$$p \quad = \quad (u_8 - u_7)x_{17} - x_{16}x_4 + x_{16}x_3 + u_7x_4 - u_8x_3$$

  added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.14 Transformation of general conic section c1:

List of parametric points

- Point Bc1 has been assigned following coordinates: $(x_{18}, 0)$

- Point Cc1 has been assigned following coordinates: $(x_{19}, 0)$

- Point Dc1 has been assigned following coordinates: $(x_{20}, 0)$

- Point Ec1 has been assigned following coordinates: $(x_{21}, 0)$

- Point Fc1 has been assigned following coordinates: $(x_{22}, 0)$

- Condition for point X(x_1, x_2) to belong to this conic section is following equation:

$$p \quad = \quad x_{22} + x_{21}x_2 + x_{20}x_1 + x_{19}x_2^2 + x_{18}x_2x_1 + x_1^2$$

- Polynomial condition for point A to belong to conic section c1 is:

$$p \quad = \quad x_{22} + x_{21}x_7 + x_{20}x_6 + x_{19}x_7^2 + x_{18}x_7x_6 + x_6^2$$

- Polynomial condition for point B to belong to conic section c1 is:

$$p \quad = \quad x_{22} + x_{21}x_9 + x_{20}x_8 + x_{19}x_9^2 + x_{18}x_9x_8 + x_8^2$$

- Polynomial condition for point C to belong to conic section c1 is:

$$p \quad = \quad x_{22} + x_{21}x_{11} + x_{20}x_{10} + x_{19}x_{11}^2 + x_{18}x_{11}x_{10} + x_{10}^2$$

- Polynomial condition for point D to belong to conic section c1 is:

$$p \quad = \quad x_{22} + x_{21}x_{13} + x_{20}x_{12} + x_{19}x_{13}^2 + x_{18}x_{13}x_{12} + x_{12}^2$$

- Polynomial condition for point E to belong to conic section c1 is:

$$p \quad = \quad x_{22} + x_{21}x_{15} + x_{20}x_{14} + x_{19}x_{15}^2 + x_{18}x_{15}x_{14} + x_{14}^2$$

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_{22} + x_{21}x_{17} + x_{20}x_{16} + x_{19}x_{17}^2 + x_{18}x_{17}x_{16} + x_{16}^2$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.21 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$
\begin{aligned}
p_1 &= u_3 x_1^2 + u_5 x_1 \\
p_2 &= u_3 x_2^2 + (u_6 u_2 + u_5) x_2 + (u_6^2 u_1 + u_6 u_4) \\
p_3 &= u_3 x_3^2 + (u_7 u_2 + u_5) x_3 + (u_7^2 u_1 + u_7 u_4) \\
p_4 &= u_3 x_4^2 + (u_8 u_2 + u_5) x_4 + (u_8^2 u_1 + u_8 u_4) \\
p_5 &= u_3 x_5^2 + (u_9 u_2 + u_5) x_5 + (u_9^2 u_1 + u_9 u_4) \\
p_6 &= x_6 x_1 \\
p_7 &= (u_7 - u_6) x_7 - x_6 x_3 + x_6 x_2 + u_6 x_3 - u_7 x_2 \\
p_8 &= x_8 x_1 \\
p_9 &= (u_9 - u_8) x_9 - x_8 x_5 + x_8 x_4 + u_8 x_5 - u_9 x_4 \\
p_{10} &= u_6 x_{11} - x_{10} x_2 \\
p_{11} &= (u_8 - u_7) x_{11} - x_{10} x_4 + x_{10} x_3 + u_7 x_4 - u_8 x_3 \\
p_{12} &= u_7 x_{13} - x_{12} x_3 \\
p_{13} &= (u_9 - u_6) x_{13} - x_{12} x_5 + x_{12} x_2 + u_6 x_5 - u_9 x_2 \\
p_{14} &= u_8 x_{15} - x_{14} x_4 + x_{14} x_1 - u_8 x_1 \\
p_{15} &= (u_9 - u_6) x_{15} - x_{14} x_5 + x_{14} x_2 + u_6 x_5 - u_9 x_2 \\
p_{16} &= u_9 x_{17} - x_{16} x_5 + x_{16} x_1 - u_9 x_1 \\
p_{17} &= (u_8 - u_7) x_{17} - x_{16} x_4 + x_{16} x_3 + u_7 x_4 - u_8 x_3 \\
p_{18} &= x_{22} + x_{21} x_7 + x_{20} x_6 + x_{19} x_7^2 + x_{18} x_7 x_6 + x_6^2 \\
p_{19} &= x_{22} + x_{21} x_9 + x_{20} x_8 + x_{19} x_9^2 + x_{18} x_9 x_8 + x_8^2 \\
p_{20} &= x_{22} + x_{21} x_{11} + x_{20} x_{10} + x_{19} x_{11}^2 + x_{18} x_{11} x_{10} + x_{10}^2 \\
p_{21} &= x_{22} + x_{21} x_{13} + x_{20} x_{12} + x_{19} x_{13}^2 + x_{18} x_{13} x_{12} + x_{12}^2 \\
p_{22} &= x_{22} + x_{21} x_{15} + x_{20} x_{14} + x_{19} x_{15}^2 + x_{18} x_{15} x_{14} + x_{14}^2
\end{aligned}
$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 22.

Variable x_{22} selected: The number of polynomials with this variable, with indexes from 1 to 22, is 5.

Minimal degrees: 5 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{22} from all other polynomials by reducing them with polynomial p_{18} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3 x_1^2 + u_5 x_1 \\
p_2 &= u_3 x_2^2 + (u_6 u_2 + u_5) x_2 + (u_6^2 u_1 + u_6 u_4) \\
p_3 &= u_3 x_3^2 + (u_7 u_2 + u_5) x_3 + (u_7^2 u_1 + u_7 u_4) \\
p_4 &= u_3 x_4^2 + (u_8 u_2 + u_5) x_4 + (u_8^2 u_1 + u_8 u_4) \\
p_5 &= u_3 x_5^2 + (u_9 u_2 + u_5) x_5 + (u_9^2 u_1 + u_9 u_4) \\
p_6 &= x_6 x_1 \\
p_7 &= (u_7 - u_6) x_7 - x_6 x_3 + x_6 x_2 + u_6 x_3 - u_7 x_2 \\
p_8 &= x_8 x_1 \\
p_9 &= (u_9 - u_8) x_9 - x_8 x_5 + x_8 x_4 + u_8 x_5 - u_9 x_4 \\
p_{10} &= u_6 x_{11} - x_{10} x_2 \\
p_{11} &= (u_8 - u_7) x_{11} - x_{10} x_4 + x_{10} x_3 + u_7 x_4 - u_8 x_3 \\
p_{12} &= u_7 x_{13} - x_{12} x_3 \\
p_{13} &= (u_9 - u_6) x_{13} - x_{12} x_5 + x_{12} x_2 + u_6 x_5 - u_9 x_2 \\
p_{14} &= u_8 x_{15} - x_{14} x_4 + x_{14} x_1 - u_8 x_1 \\
p_{15} &= (u_9 - u_6) x_{15} - x_{14} x_5 + x_{14} x_2 + u_6 x_5 - u_9 x_2 \\
p_{16} &= u_9 x_{17} - x_{16} x_5 + x_{16} x_1 - u_9 x_1 \\
p_{17} &= (u_8 - u_7) x_{17} - x_{16} x_4 + x_{16} x_3 + u_7 x_4 - u_8 x_3 \\
p_{18} &= x_{21} x_9 - x_{21} x_7 + x_{20} x_8 - x_{20} x_6 + x_{19} x_9^2 - x_{19} x_7^2 + \\
&\quad x_{18} x_9 x_8 - x_{18} x_7 x_6 + x_8^2 - x_6^2 \\
p_{19} &= x_{21} x_{11} - x_{21} x_7 + x_{20} x_{10} - x_{20} x_6 + x_{19} x_{11}^2 - x_{19} x_7^2 + \\
&\quad x_{18} x_{11} x_{10} - x_{18} x_7 x_6 + x_{10}^2 - x_6^2 \\
p_{20} &= x_{21} x_{13} - x_{21} x_7 + x_{20} x_{12} - x_{20} x_6 + x_{19} x_{13}^2 - x_{19} x_7^2 + \\
&\quad x_{18} x_{13} x_{12} - x_{18} x_7 x_6 + x_{12}^2 - x_6^2 \\
p_{21} &= x_{21} x_{15} - x_{21} x_7 + x_{20} x_{14} - x_{20} x_6 + x_{19} x_{15}^2 - x_{19} x_7^2 + \\
&\quad x_{18} x_{15} x_{14} - x_{18} x_7 x_6 + x_{14}^2 - x_6^2 \\
p_{22} &= x_{22} + x_{21} x_7 + x_{20} x_6 + x_{19} x_7^2 + x_{18} x_7 x_6 + x_6^2
\end{aligned}
$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 21.

Variable x_{21} selected: The number of polynomials with this variable, with indexes from 1 to 21, is 4.

Minimal degrees: 4 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{21} from all other polynomials by reducing them with polynomial p_{18} from previous step.

Finished a triangulation step, the current system is:

$$p_1 = u_3 x_1^2 + u_5 x_1$$

$$p_2 = u_3 x_2^2 + (u_6 u_2 + u_5) x_2 + (u_6^2 u_1 + u_6 u_4)$$

$$p_3 = u_3 x_3^2 + (u_7 u_2 + u_5) x_3 + (u_7^2 u_1 + u_7 u_4)$$

$$p_4 = u_3 x_4^2 + (u_8 u_2 + u_5) x_4 + (u_8^2 u_1 + u_8 u_4)$$

$$p_5 = u_3 x_5^2 + (u_9 u_2 + u_5) x_5 + (u_9^2 u_1 + u_9 u_4)$$

$$p_6 = x_6 x_1$$

$$p_7 = (u_7 - u_6) x_7 - x_6 x_3 + x_6 x_2 + u_6 x_3 - u_7 x_2$$

$$p_8 = x_8 x_1$$

$$p_9 = (u_9 - u_8) x_9 - x_8 x_5 + x_8 x_4 + u_8 x_5 - u_9 x_4$$

$$p_{10} = u_6 x_{11} - x_{10} x_2$$

$$p_{11} = (u_8 - u_7) x_{11} - x_{10} x_4 + x_{10} x_3 + u_7 x_4 - u_8 x_3$$

$$p_{12} = u_7 x_{13} - x_{12} x_3$$

$$p_{13} = (u_9 - u_6) x_{13} - x_{12} x_5 + x_{12} x_2 + u_6 x_5 - u_9 x_2$$

$$p_{14} = u_8 x_{15} - x_{14} x_4 + x_{14} x_1 - u_8 x_1$$

$$p_{15} = (u_9 - u_6) x_{15} - x_{14} x_5 + x_{14} x_2 + u_6 x_5 - u_9 x_2$$

$$p_{16} = u_9 x_{17} - x_{16} x_5 + x_{16} x_1 - u_9 x_1$$

$$p_{17} = (u_8 - u_7) x_{17} - x_{16} x_4 + x_{16} x_3 + u_7 x_4 - u_8 x_3$$

$$\begin{aligned}
p_{18} = {} & -x_{20} x_{11} x_8 + x_{20} x_{11} x_6 + x_{20} x_{10} x_9 - x_{20} x_{10} x_7 - x_{20} x_9 x_6 + \\
& x_{20} x_8 x_7 + x_{19} x_{11}^2 x_9 - x_{19} x_{11}^2 x_7 - x_{19} x_{11} x_9^2 + \\
& x_{19} x_{11} x_7^2 + x_{19} x_9^2 x_7 - x_{19} x_9 x_7^2 + x_{18} x_{11} x_{10} x_9 \\
& -x_{18} x_{11} x_{10} x_7 - x_{18} x_{11} x_9 x_8 + x_{18} x_{11} x_7 x_6 + x_{18} x_9 x_8 x_7 \\
& -x_{18} x_9 x_7 x_6 - x_{11} x_8^2 + x_{11} x_6^2 + x_{10}^2 x_9 - x_{10}^2 x_7 \\
& -x_9 x_6^2 + x_8^2 x_7
\end{aligned}$$

$$\begin{aligned}
p_{19} = {} & -x_{20} x_{13} x_8 + x_{20} x_{13} x_6 + x_{20} x_{12} x_9 - x_{20} x_{12} x_7 - x_{20} x_9 x_6 + \\
& x_{20} x_8 x_7 + x_{19} x_{13}^2 x_9 - x_{19} x_{13}^2 x_7 - x_{19} x_{13} x_9^2 + \\
& x_{19} x_{13} x_7^2 + x_{19} x_9^2 x_7 - x_{19} x_9 x_7^2 + x_{18} x_{13} x_{12} x_9 \\
& -x_{18} x_{13} x_{12} x_7 - x_{18} x_{13} x_9 x_8 + x_{18} x_{13} x_7 x_6 + x_{18} x_9 x_8 x_7 \\
& -x_{18} x_9 x_7 x_6 - x_{13} x_8^2 + x_{13} x_6^2 + x_{12}^2 x_9 - x_{12}^2 x_7 \\
& -x_9 x_6^2 + x_8^2 x_7
\end{aligned}$$

$$\begin{aligned}
p_{20} = {} & -x_{20} x_{15} x_8 + x_{20} x_{15} x_6 + x_{20} x_{14} x_9 - x_{20} x_{14} x_7 - x_{20} x_9 x_6 + \\
& x_{20} x_8 x_7 + x_{19} x_{15}^2 x_9 - x_{19} x_{15}^2 x_7 - x_{19} x_{15} x_9^2 + \\
& x_{19} x_{15} x_7^2 + x_{19} x_9^2 x_7 - x_{19} x_9 x_7^2 + x_{18} x_{15} x_{14} x_9 \\
& -x_{18} x_{15} x_{14} x_7 - x_{18} x_{15} x_9 x_8 + x_{18} x_{15} x_7 x_6 + x_{18} x_9 x_8 x_7 \\
& -x_{18} x_9 x_7 x_6 - x_{15} x_8^2 + x_{15} x_6^2 + x_{14}^2 x_9 - x_{14}^2 x_7 \\
& -x_9 x_6^2 + x_8^2 x_7
\end{aligned}$$

$$\begin{aligned}
p_{21} = {} & x_{21} x_9 - x_{21} x_7 + x_{20} x_8 - x_{20} x_6 + x_{19} x_9^2 - x_{19} x_7^2 + \\
& x_{18} x_9 x_8 - x_{18} x_7 x_6 + x_8^2 - x_6^2
\end{aligned}$$

$$p_{22} = x_{22} + x_{21} x_7 + x_{20} x_6 + x_{19} x_7^2 + x_{18} x_7 x_6 + x_6^2$$

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 20.

Variable x_{20} selected: The number of polynomials with this variable, with indexes from 1 to 20, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{20} from all other polynomials by reducing them with polynomial p_{18} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3 x_1^2 + u_5 x_1 \\
p_2 &= u_3 x_2^2 + (u_6 u_2 + u_5)x_2 + (u_6^2 u_1 + u_6 u_4) \\
p_3 &= u_3 x_3^2 + (u_7 u_2 + u_5)x_3 + (u_7^2 u_1 + u_7 u_4) \\
p_4 &= u_3 x_4^2 + (u_8 u_2 + u_5)x_4 + (u_8^2 u_1 + u_8 u_4) \\
p_5 &= u_3 x_5^2 + (u_9 u_2 + u_5)x_5 + (u_9^2 u_1 + u_9 u_4) \\
p_6 &= x_6 x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6 x_3 + x_6 x_2 + u_6 x_3 - u_7 x_2 \\
p_8 &= x_8 x_1 \\
p_9 &= (u_9 - u_8)x_9 - x_8 x_5 + x_8 x_4 + u_8 x_5 - u_9 x_4 \\
p_{10} &= u_6 x_{11} - x_{10} x_2 \\
p_{11} &= (u_8 - u_7)x_{11} - x_{10} x_4 + x_{10} x_3 + u_7 x_4 - u_8 x_3 \\
p_{12} &= u_7 x_{13} - x_{12} x_3 \\
p_{13} &= (u_9 - u_6)x_{13} - x_{12} x_5 + x_{12} x_2 + u_6 x_5 - u_9 x_2 \\
p_{14} &= u_8 x_{15} - x_{14} x_4 + x_{14} x_1 - u_8 x_1 \\
p_{15} &= (u_9 - u_6)x_{15} - x_{14} x_5 + x_{14} x_2 + u_6 x_5 - u_9 x_2 \\
p_{16} &= u_9 x_{17} - x_{16} x_5 + x_{16} x_1 - u_9 x_1 \\
p_{17} &= (u_8 - u_7)x_{17} - x_{16} x_4 + x_{16} x_3 + u_7 x_4 - u_8 x_3 \\
p_{18} &= \ldots \\
p_{19} &= \ldots \\
p_{20} &= -x_{20} x_{11} x_8 + x_{20} x_{11} x_6 + x_{20} x_{10} x_9 - x_{20} x_{10} x_7 - x_{20} x_9 x_6 + \\
&\quad x_{20} x_8 x_7 + x_{19} x_{11}^2 x_9 - x_{19} x_{11}^2 x_7 - x_{19} x_{11} x_9^2 + \\
&\quad x_{19} x_{11} x_7^2 + x_{19} x_9^2 x_7 - x_{19} x_9 x_7^2 + x_{18} x_{11} x_{10} x_9 \\
&\quad -x_{18} x_{11} x_{10} x_7 - x_{18} x_{11} x_9 x_8 + x_{18} x_{11} x_7 x_6 + x_{18} x_9 x_8 x_7 \\
&\quad -x_{18} x_9 x_7 x_6 - x_{11} x_8^2 + x_{11} x_6^2 + x_{10}^2 x_9 - x_{10}^2 x_7 \\
&\quad -x_9 x_6^2 + x_8^2 x_7 \\
p_{21} &= x_{21} x_9 - x_{21} x_7 + x_{20} x_8 - x_{20} x_6 + x_{19} x_9^2 - x_{19} x_7^2 + \\
&\quad x_{18} x_9 x_8 - x_{18} x_7 x_6 + x_8^2 - x_6^2 \\
p_{22} &= x_{22} + x_{21} x_7 + x_{20} x_6 + x_{19} x_7^2 + x_{18} x_7 x_6 + x_6^2
\end{aligned}
$$

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 19.

Variable x_{19} selected: The number of polynomials with this variable, with indexes from 1 to 19, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{19} from all other polynomials by reducing them with polynomial p_{18} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3 x_1^2 + u_5 x_1 \\
p_2 &= u_3 x_2^2 + (u_6 u_2 + u_5)x_2 + (u_6^2 u_1 + u_6 u_4) \\
p_3 &= u_3 x_3^2 + (u_7 u_2 + u_5)x_3 + (u_7^2 u_1 + u_7 u_4) \\
p_4 &= u_3 x_4^2 + (u_8 u_2 + u_5)x_4 + (u_8^2 u_1 + u_8 u_4) \\
p_5 &= u_3 x_5^2 + (u_9 u_2 + u_5)x_5 + (u_9^2 u_1 + u_9 u_4) \\
p_6 &= x_6 x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6 x_3 + x_6 x_2 + u_6 x_3 - u_7 x_2 \\
p_8 &= x_8 x_1 \\
p_9 &= (u_9 - u_8)x_9 - x_8 x_5 + x_8 x_4 + u_8 x_5 - u_9 x_4 \\
p_{10} &= u_6 x_{11} - x_{10} x_2 \\
p_{11} &= (u_8 - u_7)x_{11} - x_{10} x_4 + x_{10} x_3 + u_7 x_4 - u_8 x_3 \\
p_{12} &= u_7 x_{13} - x_{12} x_3 \\
p_{13} &= (u_9 - u_6)x_{13} - x_{12} x_5 + x_{12} x_2 + u_6 x_5 - u_9 x_2 \\
p_{14} &= u_8 x_{15} - x_{14} x_4 + x_{14} x_1 - u_8 x_1 \\
p_{15} &= (u_9 - u_6)x_{15} - x_{14} x_5 + x_{14} x_2 + u_6 x_5 - u_9 x_2 \\
p_{16} &= u_9 x_{17} - x_{16} x_5 + x_{16} x_1 - u_9 x_1 \\
p_{17} &= (u_8 - u_7)x_{17} - x_{16} x_4 + x_{16} x_3 + u_7 x_4 - u_8 x_3 \\
p_{18} &= \ldots \\
p_{19} &= \ldots \\
p_{20} &= -x_{20}x_{11}x_8 + x_{20}x_{11}x_6 + x_{20}x_{10}x_9 - x_{20}x_{10}x_7 - x_{20}x_9x_6 + \\
&\quad x_{20}x_8x_7 + x_{19}x_{11}^2 x_9 - x_{19}x_{11}^2 x_7 - x_{19}x_{11}x_9^2 + \\
&\quad x_{19}x_{11}x_7^2 + x_{19}x_9^2 x_7 - x_{19}x_9x_7^2 + x_{18}x_{11}x_{10}x_9 \\
&\quad -x_{18}x_{11}x_{10}x_7 - x_{18}x_{11}x_9x_8 + x_{18}x_{11}x_7x_6 + x_{18}x_9x_8x_7 \\
&\quad -x_{18}x_9x_7x_6 - x_{11}x_8^2 + x_{11}x_6^2 + x_{10}^2 x_9 - x_{10}^2 x_7 \\
&\quad -x_9x_6^2 + x_8^2 x_7 \\
p_{21} &= x_{21}x_9 - x_{21}x_7 + x_{20}x_8 - x_{20}x_6 + x_{19}x_9^2 - x_{19}x_7^2 + \\
&\quad x_{18}x_9x_8 - x_{18}x_7x_6 + x_8^2 - x_6^2 \\
p_{22} &= x_{22} + x_{21}x_7 + x_{20}x_6 + x_{19}x_7^2 + x_{18}x_7x_6 + x_6^2
\end{aligned}
$$

3.5   Triangulation, step 5

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{18}. No reduction needed.

The triangular system has not been changed.

3.6   Triangulation, step 6

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{17} from all other polynomials by reducing them with polynomial p_{16} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3 x_1^2 + u_5 x_1 \\
p_2 &= u_3 x_2^2 + (u_6 u_2 + u_5)x_2 + (u_6^2 u_1 + u_6 u_4) \\
p_3 &= u_3 x_3^2 + (u_7 u_2 + u_5)x_3 + (u_7^2 u_1 + u_7 u_4) \\
p_4 &= u_3 x_4^2 + (u_8 u_2 + u_5)x_4 + (u_8^2 u_1 + u_8 u_4) \\
p_5 &= u_3 x_5^2 + (u_9 u_2 + u_5)x_5 + (u_9^2 u_1 + u_9 u_4) \\
p_6 &= x_6 x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6 x_3 + x_6 x_2 + u_6 x_3 - u_7 x_2 \\
p_8 &= x_8 x_1 \\
p_9 &= (u_9 - u_8)x_9 - x_8 x_5 + x_8 x_4 + u_8 x_5 - u_9 x_4 \\
p_{10} &= u_6 x_{11} - x_{10} x_2 \\
p_{11} &= (u_8 - u_7)x_{11} - x_{10} x_4 + x_{10} x_3 + u_7 x_4 - u_8 x_3 \\
p_{12} &= u_7 x_{13} - x_{12} x_3 \\
p_{13} &= (u_9 - u_6)x_{13} - x_{12} x_5 + x_{12} x_2 + u_6 x_5 - u_9 x_2 \\
p_{14} &= u_8 x_{15} - x_{14} x_4 + x_{14} x_1 - u_8 x_1 \\
p_{15} &= (u_9 - u_6)x_{15} - x_{14} x_5 + x_{14} x_2 + u_6 x_5 - u_9 x_2 \\
p_{16} &= (u_8 - u_7)x_{16} x_5 - u_9 x_{16} x_4 + u_9 x_{16} x_3 + (-u_8 + u_7)x_{16} x_1 + \\
&\quad u_9 u_7 x_4 - u_9 u_8 x_3 + (u_9 u_8 - u_9 u_7)x_1 \\
p_{17} &= u_9 x_{17} - x_{16} x_5 + x_{16} x_1 - u_9 x_1 \\
p_{18} &= \ldots \\
p_{19} &= \ldots \\
p_{20} &= -x_{20} x_{11} x_8 + x_{20} x_{11} x_6 + x_{20} x_{10} x_9 - x_{20} x_{10} x_7 - x_{20} x_9 x_6 +
\end{aligned}
$$

$$x_{20}x_8x_7 + x_{19}x_{11}^2x_9 - x_{19}x_{11}^2x_7 - x_{19}x_{11}x_9^2 +$$
$$x_{19}x_{11}x_7^2 + x_{19}x_9^2x_7 - x_{19}x_9x_7^2 + x_{18}x_{11}x_{10}x_9$$
$$-x_{18}x_{11}x_{10}x_7 - x_{18}x_{11}x_9x_8 + x_{18}x_{11}x_7x_6 + x_{18}x_9x_8x_7$$
$$-x_{18}x_9x_7x_6 - x_{11}x_8^2 + x_{11}x_6^2 + x_{10}^2x_9 - x_{10}^2x_7$$
$$-x_9x_6^2 + x_8^2x_7$$

$$p_{21} = x_{21}x_9 - x_{21}x_7 + x_{20}x_8 - x_{20}x_6 + x_{19}x_9^2 - x_{19}x_7^2 +$$
$$x_{18}x_9x_8 - x_{18}x_7x_6 + x_8^2 - x_6^2$$

$$p_{22} = x_{22} + x_{21}x_7 + x_{20}x_6 + x_{19}x_7^2 + x_{18}x_7x_6 + x_6^2$$

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{16}. No reduction needed.

The triangular system has not been changed.

3.8 Triangulation, step 8

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{15} from all other polynomials by reducing them with polynomial p_{14} from previous step.

Finished a triangulation step, the current system is:

$$p_1 = u_3x_1^2 + u_5x_1$$
$$p_2 = u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)$$
$$p_3 = u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4)$$
$$p_4 = u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)$$
$$p_5 = u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4)$$
$$p_6 = x_6x_1$$
$$p_7 = (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2$$
$$p_8 = x_8x_1$$
$$p_9 = (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4$$
$$p_{10} = u_6x_{11} - x_{10}x_2$$
$$p_{11} = (u_8 - u_7)x_{11} - x_{10}x_4 + x_{10}x_3 + u_7x_4 - u_8x_3$$
$$p_{12} = u_7x_{13} - x_{12}x_3$$

$$
\begin{aligned}
p_{13} &= (u_9 - u_6)x_{13} - x_{12}x_5 + x_{12}x_2 + u_6x_5 - u_9x_2 \\
p_{14} &= -u_8x_{14}x_5 + (u_9 - u_6)x_{14}x_4 + u_8x_{14}x_2 + (-u_9 + u_6)x_{14}x_1 + \\
&\quad u_8u_6x_5 - u_9u_8x_2 + (u_9u_8 - u_8u_6)x_1 \\
p_{15} &= u_8x_{15} - x_{14}x_4 + x_{14}x_1 - u_8x_1 \\
p_{16} &= (u_8 - u_7)x_{16}x_5 - u_9x_{16}x_4 + u_9x_{16}x_3 + (-u_8 + u_7)x_{16}x_1 + \\
&\quad u_9u_7x_4 - u_9u_8x_3 + (u_9u_8 - u_9u_7)x_1 \\
p_{17} &= u_9x_{17} - x_{16}x_5 + x_{16}x_1 - u_9x_1 \\
p_{18} &= \ldots \\
p_{19} &= \ldots \\
p_{20} &= -x_{20}x_{11}x_8 + x_{20}x_{11}x_6 + x_{20}x_{10}x_9 - x_{20}x_{10}x_7 - x_{20}x_9x_6 + \\
&\quad x_{20}x_8x_7 + x_{19}x_{11}^2x_9 - x_{19}x_{11}^2x_7 - x_{19}x_{11}x_9^2 + \\
&\quad x_{19}x_{11}x_7^2 + x_{19}x_9^2x_7 - x_{19}x_9x_7^2 + x_{18}x_{11}x_{10}x_9 \\
&\quad -x_{18}x_{11}x_{10}x_7 - x_{18}x_{11}x_9x_8 + x_{18}x_{11}x_7x_6 + x_{18}x_9x_8x_7 \\
&\quad -x_{18}x_9x_7x_6 - x_{11}x_8^2 + x_{11}x_6^2 + x_{10}^2x_9 - x_{10}^2x_7 \\
&\quad -x_9x_6^2 + x_8^2x_7 \\
p_{21} &= x_{21}x_9 - x_{21}x_7 + x_{20}x_8 - x_{20}x_6 + x_{19}x_9^2 - x_{19}x_7^2 + \\
&\quad x_{18}x_9x_8 - x_{18}x_7x_6 + x_8^2 - x_6^2 \\
p_{22} &= x_{22} + x_{21}x_7 + x_{20}x_6 + x_{19}x_7^2 + x_{18}x_7x_6 + x_6^2
\end{aligned}
$$

3.9 Triangulation, step 9

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14}. No reduction needed.

The triangular system has not been changed.

3.10 Triangulation, step 10

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{13} from all other polynomials by reducing them with polynomial p_{12} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)
\end{aligned}
$$

$$\begin{aligned}
p_3 &= u_3 x_3^2 + (u_7 u_2 + u_5) x_3 + (u_7^2 u_1 + u_7 u_4) \\
p_4 &= u_3 x_4^2 + (u_8 u_2 + u_5) x_4 + (u_8^2 u_1 + u_8 u_4) \\
p_5 &= u_3 x_5^2 + (u_9 u_2 + u_5) x_5 + (u_9^2 u_1 + u_9 u_4) \\
p_6 &= x_6 x_1 \\
p_7 &= (u_7 - u_6) x_7 - x_6 x_3 + x_6 x_2 + u_6 x_3 - u_7 x_2 \\
p_8 &= x_8 x_1 \\
p_9 &= (u_9 - u_8) x_9 - x_8 x_5 + x_8 x_4 + u_8 x_5 - u_9 x_4 \\
p_{10} &= u_6 x_{11} - x_{10} x_2 \\
p_{11} &= (u_8 - u_7) x_{11} - x_{10} x_4 + x_{10} x_3 + u_7 x_4 - u_8 x_3 \\
p_{12} &= -u_7 x_{12} x_5 + (u_9 - u_6) x_{12} x_3 + u_7 x_{12} x_2 + u_7 u_6 x_5 - u_9 u_7 x_2 \\
p_{13} &= u_7 x_{13} - x_{12} x_3 \\
p_{14} &= -u_8 x_{14} x_5 + (u_9 - u_6) x_{14} x_4 + u_8 x_{14} x_2 + (-u_9 + u_6) x_{14} x_1 + \\
&\quad u_8 u_6 x_5 - u_9 u_8 x_2 + (u_9 u_8 - u_8 u_6) x_1 \\
p_{15} &= u_8 x_{15} - x_{14} x_4 + x_{14} x_1 - u_8 x_1 \\
p_{16} &= (u_8 - u_7) x_{16} x_5 - u_9 x_{16} x_4 + u_9 x_{16} x_3 + (-u_8 + u_7) x_{16} x_1 + \\
&\quad u_9 u_7 x_4 - u_9 u_8 x_3 + (u_9 u_8 - u_9 u_7) x_1 \\
p_{17} &= u_9 x_{17} - x_{16} x_5 + x_{16} x_1 - u_9 x_1 \\
p_{18} &= \ldots \\
p_{19} &= \ldots \\
p_{20} &= -x_{20} x_{11} x_8 + x_{20} x_{11} x_6 + x_{20} x_{10} x_9 - x_{20} x_{10} x_7 - x_{20} x_9 x_6 + \\
&\quad x_{20} x_8 x_7 + x_{19} x_{11}^2 x_9 - x_{19} x_{11}^2 x_7 - x_{19} x_{11} x_9^2 + \\
&\quad x_{19} x_{11} x_7^2 + x_{19} x_9^2 x_7 - x_{19} x_9 x_7^2 + x_{18} x_{11} x_{10} x_9 \\
&\quad -x_{18} x_{11} x_{10} x_7 - x_{18} x_{11} x_9 x_8 + x_{18} x_{11} x_7 x_6 + x_{18} x_9 x_8 x_7 \\
&\quad -x_{18} x_9 x_7 x_6 - x_{11} x_8^2 + x_{11} x_6^2 + x_{10}^2 x_9 - x_{10}^2 x_7 \\
&\quad -x_9 x_6^2 + x_8^2 x_7 \\
p_{21} &= x_{21} x_9 - x_{21} x_7 + x_{20} x_8 - x_{20} x_6 + x_{19} x_9^2 - x_{19} x_7^2 + \\
&\quad x_{18} x_9 x_8 - x_{18} x_7 x_6 + x_8^2 - x_6^2 \\
p_{22} &= x_{22} + x_{21} x_7 + x_{20} x_6 + x_{19} x_7^2 + x_{18} x_7 x_6 + x_6^2
\end{aligned}$$

3.11 Triangulation, step 11

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{12}. No reduction needed.

The triangular system has not been changed.

3.12 Triangulation, step 12

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{11} from all other polynomials by reducing them with polynomial p_{10} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3 x_1^2 + u_5 x_1 \\
p_2 &= u_3 x_2^2 + (u_6 u_2 + u_5)x_2 + (u_6^2 u_1 + u_6 u_4) \\
p_3 &= u_3 x_3^2 + (u_7 u_2 + u_5)x_3 + (u_7^2 u_1 + u_7 u_4) \\
p_4 &= u_3 x_4^2 + (u_8 u_2 + u_5)x_4 + (u_8^2 u_1 + u_8 u_4) \\
p_5 &= u_3 x_5^2 + (u_9 u_2 + u_5)x_5 + (u_9^2 u_1 + u_9 u_4) \\
p_6 &= x_6 x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6 x_3 + x_6 x_2 + u_6 x_3 - u_7 x_2 \\
p_8 &= x_8 x_1 \\
p_9 &= (u_9 - u_8)x_9 - x_8 x_5 + x_8 x_4 + u_8 x_5 - u_9 x_4 \\
p_{10} &= -u_6 x_{10} x_4 + u_6 x_{10} x_3 + (u_8 - u_7)x_{10} x_2 + u_7 u_6 x_4 - u_8 u_6 x_3 \\
p_{11} &= u_6 x_{11} - x_{10} x_2 \\
p_{12} &= -u_7 x_{12} x_5 + (u_9 - u_6)x_{12} x_3 + u_7 x_{12} x_2 + u_7 u_6 x_5 - u_9 u_7 x_2 \\
p_{13} &= u_7 x_{13} - x_{12} x_3 \\
p_{14} &= -u_8 x_{14} x_5 + (u_9 - u_6)x_{14} x_4 + u_8 x_{14} x_2 + (-u_9 + u_6)x_{14} x_1 + \\
      &\quad u_8 u_6 x_5 - u_9 u_8 x_2 + (u_9 u_8 - u_8 u_6)x_1 \\
p_{15} &= u_8 x_{15} - x_{14} x_4 + x_{14} x_1 - u_8 x_1 \\
p_{16} &= (u_8 - u_7)x_{16} x_5 - u_9 x_{16} x_4 + u_9 x_{16} x_3 + (-u_8 + u_7)x_{16} x_1 + \\
      &\quad u_9 u_7 x_4 - u_9 u_8 x_3 + (u_9 u_8 - u_9 u_7)x_1 \\
p_{17} &= u_9 x_{17} - x_{16} x_5 + x_{16} x_1 - u_9 x_1 \\
p_{18} &= \ldots \\
p_{19} &= \ldots \\
p_{20} &= -x_{20} x_{11} x_8 + x_{20} x_{11} x_6 + x_{20} x_{10} x_9 - x_{20} x_{10} x_7 - x_{20} x_9 x_6 + \\
      &\quad x_{20} x_8 x_7 + x_{19} x_{11}^2 x_9 - x_{19} x_{11}^2 x_7 - x_{19} x_{11} x_9^2 + \\
      &\quad x_{19} x_{11} x_7^2 + x_{19} x_9^2 x_7 - x_{19} x_9 x_7^2 + x_{18} x_{11} x_{10} x_9 \\
      &\quad -x_{18} x_{11} x_{10} x_7 - x_{18} x_{11} x_9 x_8 + x_{18} x_{11} x_7 x_6 + x_{18} x_9 x_8 x_7 \\
      &\quad -x_{18} x_9 x_7 x_6 - x_{11} x_8^2 + x_{11} x_6^2 + x_{10}^2 x_9 - x_{10}^2 x_7 \\
      &\quad -x_9 x_6^2 + x_8^2 x_7 \\
p_{21} &= x_{21} x_9 - x_{21} x_7 + x_{20} x_8 - x_{20} x_6 + x_{19} x_9^2 - x_{19} x_7^2 + \\
      &\quad x_{18} x_9 x_8 - x_{18} x_7 x_6 + x_8^2 - x_6^2 \\
p_{22} &= x_{22} + x_{21} x_7 + x_{20} x_6 + x_{19} x_7^2 + x_{18} x_7 x_6 + x_6^2
\end{aligned}
$$

3.13 Triangulation, step 13

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10}. No reduction needed.

The triangular system has not been changed.

3.14   Triangulation, step 14

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_9. No reduction needed.

The triangular system has not been changed.

3.15   Triangulation, step 15

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8. No reduction needed.

The triangular system has not been changed.

3.16   Triangulation, step 16

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7. No reduction needed.

The triangular system has not been changed.

3.17   Triangulation, step 17

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

3.18 Triangulation, step 18

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

3.19 Triangulation, step 19

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

3.20 Triangulation, step 20

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.21 Triangulation, step 21

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.22 Triangulation, step 22

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= u_3 x_1^2 + u_5 x_1 \\
p_2 &= u_3 x_2^2 + (u_6 u_2 + u_5)x_2 + (u_6^2 u_1 + u_6 u_4) \\
p_3 &= u_3 x_3^2 + (u_7 u_2 + u_5)x_3 + (u_7^2 u_1 + u_7 u_4) \\
p_4 &= u_3 x_4^2 + (u_8 u_2 + u_5)x_4 + (u_8^2 u_1 + u_8 u_4) \\
p_5 &= u_3 x_5^2 + (u_9 u_2 + u_5)x_5 + (u_9^2 u_1 + u_9 u_4) \\
p_6 &= x_6 x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6 x_3 + x_6 x_2 + u_6 x_3 - u_7 x_2 \\
p_8 &= x_8 x_1 \\
p_9 &= (u_9 - u_8)x_9 - x_8 x_5 + x_8 x_4 + u_8 x_5 - u_9 x_4 \\
p_{10} &= -u_6 x_{10} x_4 + u_6 x_{10} x_3 + (u_8 - u_7)x_{10} x_2 + u_7 u_6 x_4 - u_8 u_6 x_3 \\
p_{11} &= u_6 x_{11} - x_{10} x_2 \\
p_{12} &= -u_7 x_{12} x_5 + (u_9 - u_6)x_{12} x_3 + u_7 x_{12} x_2 + u_7 u_6 x_5 - u_9 u_7 x_2 \\
p_{13} &= u_7 x_{13} - x_{12} x_3 \\
p_{14} &= -u_8 x_{14} x_5 + (u_9 - u_6)x_{14} x_4 + u_8 x_{14} x_2 + (-u_9 + u_6)x_{14} x_1 + \\
&\quad u_8 u_6 x_5 - u_9 u_8 x_2 + (u_9 u_8 - u_8 u_6)x_1 \\
p_{15} &= u_8 x_{15} - x_{14} x_4 + x_{14} x_1 - u_8 x_1 \\
p_{16} &= (u_8 - u_7)x_{16} x_5 - u_9 x_{16} x_4 + u_9 x_{16} x_3 + (-u_8 + u_7)x_{16} x_1 + \\
&\quad u_9 u_7 x_4 - u_9 u_8 x_3 + (u_9 u_8 - u_9 u_7)x_1 \\
p_{17} &= u_9 x_{17} - x_{16} x_5 + x_{16} x_1 - u_9 x_1 \\
p_{18} &= \ldots \\
p_{19} &= \ldots \\
p_{20} &= -x_{20} x_{11} x_8 + x_{20} x_{11} x_6 + x_{20} x_{10} x_9 - x_{20} x_{10} x_7 - x_{20} x_9 x_6 + \\
&\quad x_{20} x_8 x_7 + x_{19} x_{11}^2 x_9 - x_{19} x_{11}^2 x_7 - x_{19} x_{11} x_9^2 + \\
&\quad x_{19} x_{11} x_7^2 + x_{19} x_9^2 x_7 - x_{19} x_9 x_7^2 + x_{18} x_{11} x_{10} x_9 \\
&\quad -x_{18} x_{11} x_{10} x_7 - x_{18} x_{11} x_9 x_8 + x_{18} x_{11} x_7 x_6 + x_{18} x_9 x_8 x_7 \\
&\quad -x_{18} x_9 x_7 x_6 - x_{11} x_8^2 + x_{11} x_6^2 + x_{10}^2 x_9 - x_{10}^2 x_7 \\
&\quad -x_9 x_6^2 + x_8^2 x_7 \\
p_{21} &= x_{21} x_9 - x_{21} x_7 + x_{20} x_8 - x_{20} x_6 + x_{19} x_9^2 - x_{19} x_7^2 + \\
&\quad x_{18} x_9 x_8 - x_{18} x_7 x_6 + x_8^2 - x_6^2 \\
p_{22} &= x_{22} + x_{21} x_7 + x_{20} x_6 + x_{19} x_7^2 + x_{18} x_7 x_6 + x_6^2
\end{aligned}
$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 012

Calculating final remainder of the conclusion:

$$g = x_{22} + x_{21}x_{17} + x_{20}x_{16} + x_{19}x_{17}^2 + x_{18}x_{17}x_{16} + x_{16}^2$$

with respect to the triangular system.

1. Pseudo remainder with p_{22} over variable x_{22}:

$$\begin{aligned} g = {} & x_{21}x_{17} - x_{21}x_7 + x_{20}x_{16} - x_{20}x_6 + x_{19}x_{17}^2 - x_{19}x_7^2 + \\ & x_{18}x_{17}x_{16} - x_{18}x_7x_6 + x_{16}^2 - x_6^2 \end{aligned}$$

2. Pseudo remainder with p_{21} over variable x_{21}:

$$\begin{aligned} g = {} & -x_{20}x_{17}x_8 + x_{20}x_{17}x_6 + x_{20}x_{16}x_9 - x_{20}x_{16}x_7 - x_{20}x_9x_6 + \\ & x_{20}x_8x_7 + x_{19}x_{17}^2x_9 - x_{19}x_{17}^2x_7 - x_{19}x_{17}x_9^2 + \\ & x_{19}x_{17}x_7^2 + x_{19}x_9^2x_7 - x_{19}x_9x_7^2 + x_{18}x_{17}x_{16}x_9 \\ & -x_{18}x_{17}x_{16}x_7 - x_{18}x_{17}x_9x_8 + x_{18}x_{17}x_7x_6 + x_{18}x_9x_8x_7 \\ & -x_{18}x_9x_7x_6 - x_{17}x_8^2 + x_{17}x_6^2 + x_{16}^2x_9 - x_{16}^2x_7 \\ & -x_9x_6^2 + x_8^2x_7 \end{aligned}$$

3. Pseudo remainder with p_{20} over variable x_{20}:
   Polynomial too big for output (text size is 4292 characters, number of terms is 134)

4. Pseudo remainder with p_{19} over variable x_{19}:
   Polynomial too big for output (number of terms is 2612)

5. Pseudo remainder with p_{18} over variable x_{18}:

5 Prover results

Status: Proving failed - Space limit has been reached.

Space Complexity: The biggest polynomial obtained during prover execution contains 180808 terms.

Time Complexity: Time spent by the prover is 69.864 seconds.