# OpenGeoProver Output for conjecture "Chou 082"

Wu's method used

February 21, 2012

## 1 Validation of Construction Protocol

### Construction steps:

- Free point A
- Free point B
- Free point C
- Line c through two points A and B
- Line a through two points B and C
- Line b through two points C and A
- Midpoint F of segment BC
- Line footPointPerpLine487 through point C perpendicular to line c
- Intersection point D of point sets footPointPerpLine487 and c
- Line footPointPerpLine421 through point B perpendicular to line b
- Intersection point E of point sets footPointPerpLine421 and b
- Line de through two points D and E
- Line footPointPerpLine25 through point F perpendicular to line de
- Intersection point G of point sets footPointPerpLine25 and de

### Theorem statement:

- Ratio of oriented segments DG/GE equals 1.0

**Validation result:** Construction protocol is valid.

# 2 Transformation of Construction Protocol to algebraic form

### Transformation of Construction steps

## 2.1 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

## 2.2 Transformation of point B:

- Point B has been assigned following coordinates: (0, $u_1$)

## 2.3 Transformation of point C:

- Point C has been assigned following coordinates: ($u_2$, $u_3$)

## 2.4 Transformation of point F:

- Point F has been assigned following coordinates: ($x_1$, $x_2$)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_1 - 0.5u_2$$

**Info:** Polynomial

$$p = x_1 - 0.5u_2$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_1 - 0.5u_2$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_2 + (-0.5u_3 - 0.5u_1)$$

**Info:** Polynomial

$$p = x_2 + (-0.5u_3 - 0.5u_1)$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_2 + (-0.5u_3 - 0.5u_1)$$

is added to polynomial system

## 2.5 Transformation of point D:

- Point D has been assigned following coordinates: ($x_3$, $x_4$)
- Polynomial that point D has to satisfy is:

$$p = x_4 - u_3$$

- Processing of polynomial

$$p = x_4 - u_3$$

**Info:** Will try to rename Y coordinate of point D

**Info:** Y coordinate of point D renamed by independent variable $u_3$

- Point D has been renamed. Point D has been assigned following coordinates: ($x_3$, $u_3$)
- Polynomial that point D has to satisfy is:

$$p = x_3$$

- Processing of polynomial

$$p = x_3$$

**Info:** Will try to rename X coordinate of point D

**Info:** X coordinate of point D renamed by zero

- Point D has been renamed. Point D has been assigned following coordinates: (0, $u_3$)

## 2.6 Transformation of point E:

- Point E has been assigned following coordinates: ($x_3$, $x_4$)
- Polynomial that point E has to satisfy is:

$$p = u_3x_4 + u_2x_3 - u_3u_1$$

- Processing of polynomial

$$p = u_3x_4 + u_2x_3 - u_3u_1$$

**Info:** Polynomial

$$p = u_3x_4 + u_2x_3 - u_3u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point E has to satisfy is:

$$p = u_2x_4 - u_3x_3$$

- Processing of polynomial

$$p = u_2x_4 - u_3x_3$$

**Info:** Polynomial

$$p = u_2x_4 - u_3x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.7 Transformation of point G:

- Point G has been assigned following coordinates: ($x_5$, $x_6$)
- Polynomial that point G has to satisfy is:

$$p = x_6x_4 - u_3x_6 + x_5x_3 - x_4x_2 - x_3x_1 + u_3x_2$$

- Processing of polynomial

$$p = x_6x_4 - u_3x_6 + x_5x_3 - x_4x_2 - x_3x_1 + u_3x_2$$

**Info:** Polynomial

$$p = x_6x_4 - u_3x_6 + x_5x_3 - x_4x_2 - x_3x_1 + u_3x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point G has to satisfy is:

$$p = x_6x_3 - x_5x_4 + u_3x_5 - u_3x_3$$

- Processing of polynomial

$$p = x_6x_3 - x_5x_4 + u_3x_5 - u_3x_3$$

**Info:** Polynomial

$$p = x_6x_3 - x_5x_4 + u_3x_5 - u_3x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_5 - 0.5x_3$$

### Time spent for transformation of Construction Protocol to algebraic form

- 0.096 seconds

## 3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= x_1 - 0.5u_2 \\
p_2 &= x_2 + (-0.5u_3 - 0.5u_1) \\
p_3 &= u_3x_4 + u_2x_3 - u_3u_1 \\
p_4 &= u_2x_4 - u_3x_3 \\
p_5 &= x_6x_4 - u_3x_6 + x_5x_3 - x_4x_2 - x_3x_1 + u_3x_2 \\
p_6 &= x_6x_3 - x_5x_4 + u_3x_5 - u_3x_3
\end{aligned}$$

### 3.1 Triangulation, step 1

**Choosing variable:** Trying the variable with index 6.

**Variable $x_6$ selected:** The number of polynomials with this variable, with indexes from 1 to 6, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable $x_6$ from all other polynomials by reducing them with polynomial $p_5$ from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= x_1 - 0.5u_2 \\
p_2 &= x_2 + (-0.5u_3 - 0.5u_1) \\
p_3 &= u_3x_4 + u_2x_3 - u_3u_1 \\
p_4 &= u_2x_4 - u_3x_3 \\
p_5 &= -x_5x_4^2 + 2u_3x_5x_4 - x_5x_3^2 - u_3^2x_5 + x_4x_3x_2 \\
&\quad -u_3x_4x_3 + x_3^2x_1 - u_3x_3x_2 + u_3^2x_3 \\
p_6 &= x_6x_4 - u_3x_6 + x_5x_3 - x_4x_2 - x_3x_1 + u_3x_2
\end{aligned}$$

### 3.2 Triangulation, step 2

**Choosing variable:** Trying the variable with index 5.

**Variable $x_5$ selected:** The number of polynomials with this variable, with indexes from 1 to 5, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_5$. No reduction needed.

The triangular system has not been changed.

### 3.3 Triangulation, step 3

**Choosing variable:** Trying the variable with index 4.

**Variable $x_4$ selected:** The number of polynomials with this variable, with indexes from 1 to 4, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable $x_4$ from all other polynomials by reducing them with polynomial $p_3$ from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= x_1 - 0.5u_2 \\
p_2 &= x_2 + (-0.5u_3 - 0.5u_1) \\
p_3 &= (-u_3^2 - u_2^2)x_3 + u_3u_2u_1 \\
p_4 &= u_3x_4 + u_2x_3 - u_3u_1 \\
p_5 &= -x_5x_4^2 + 2u_3x_5x_4 - x_5x_3^2 - u_3^2x_5 + x_4x_3x_2 \\
&\quad -u_3x_4x_3 + x_3^2x_1 - u_3x_3x_2 + u_3^2x_3 \\
p_6 &= x_6x_4 - u_3x_6 + x_5x_3 - x_4x_2 - x_3x_1 + u_3x_2
\end{aligned}$$

### 3.4 Triangulation, step 4

**Choosing variable:** Trying the variable with index 3.

**Variable $x_3$ selected:** The number of polynomials with this variable, with indexes from 1 to 3, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_3$. No reduction needed.

The triangular system has not been changed.

### 3.5 Triangulation, step 5

**Choosing variable:** Trying the variable with index 2.

**Variable $x_2$ selected:** The number of polynomials with this variable, with indexes from 1 to 2, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_2$. No reduction needed.

The triangular system has not been changed.

### 3.6 Triangulation, step 6

**Choosing variable:** Trying the variable with index 1.

**Variable $x_1$ selected:** The number of polynomials with this variable, with indexes from 1 to 1, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_1$. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_1 &= x_1 - 0.5u_2 \\
p_2 &= x_2 + (-0.5u_3 - 0.5u_1) \\
p_3 &= (-u_3^2 - u_2^2)x_3 + u_3u_2u_1 \\
p_4 &= u_3x_4 + u_2x_3 - u_3u_1 \\
p_5 &= -x_5x_4^2 + 2u_3x_5x_4 - x_5x_3^2 - u_3^2x_5 + x_4x_3x_2 \\
&\quad -u_3x_4x_3 + x_3^2x_1 - u_3x_3x_2 + u_3^2x_3 \\
p_6 &= x_6x_4 - u_3x_6 + x_5x_3 - x_4x_2 - x_3x_1 + u_3x_2
\end{aligned}$$

# 4 Final Remainder

## 4.1 Final remainder for conjecture Chou 082

Calculating final remainder of the conclusion:

$$g = x_5 - 0.5x_3$$

with respect to the triangular system.

1. Pseudo remainder with $p_6$ over variable $x_6$:

$$g = x_5 - 0.5x_3$$

2. Pseudo remainder with $p_5$ over variable $x_5$:

$$\begin{aligned}
g &= 0.5x_4^2x_3 - x_4x_3x_2 + 0.5x_3^3 - x_3^2x_1 + u_3x_3x_2 \\
&\quad -0.5u_3^2x_3
\end{aligned}$$

3. Pseudo remainder with $p_4$ over variable $x_4$:

$$\begin{aligned}
g &= (0.5u_3^2 + 0.5u_2^2)x_3^3 + u_3u_2x_3^2x_2 - u_3^2x_3^2x_1 \\
&\quad -u_3u_2u_1x_3^2 + (u_3^3 - u_3^2u_1)x_3x_2 + \\
&\quad (-0.5u_3^4 + 0.5u_3^2u_1^2)x_3
\end{aligned}$$

4. Pseudo remainder with $p_3$ over variable $x_3$:

$$\begin{aligned} g &= (-u_3^8 u_2 u_1 + u_3^7 u_2 u_1^2 - 2u_3^6 u_2^3 u_1 + \\ &\quad u_3^5 u_2^3 u_1^2 - u_3^4 u_2^5 u_1) \\ &\quad x_2 \\ &\quad +(u_3^6 u_2^2 u_1^2 + u_3^4 u_2^4 u_1^2) x_1 + \\ &\quad (0.5u_3^9 u_2 u_1 + u_3^7 u_2^3 u_1 - 0.5u_3^7 u_2 u_1^3 + \\ &\quad 0.5u_3^5 u_2^5 u_1 - 0.5u_3^5 u_2^3 u_1^3) \end{aligned}$$

5. Pseudo remainder with $p_2$ over variable $x_2$:

$$\begin{aligned} g &= (u_3^6 u_2^2 u_1^2 + u_3^4 u_2^4 u_1^2) x_1 + \\ &\quad (-0.5u_3^6 u_2^3 u_1^2 - 0.5u_3^4 u_2^5 u_1^2) \end{aligned}$$

6. Pseudo remainder with $p_1$ over variable $x_1$:

$$g = 0$$

# 5 Prover results

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.089 seconds.

# 6 NDG Conditions

## NDG Conditions in readable form

- Points D, A, B and C are not collinear
- Points D, A, B and C are not collinear
- Points D and E are not identical
- Points D, E and C are not collinear

## Time spent for processing NDG Conditions

- 3.647 seconds