OpenGeoProver Output for conjecture "Chou 321 (Converse of Menelaus' Theorem)"

Wu's method used

February 20, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Free point C
- Line BC through two points B and C
- Random point D from line BC
- Line CA through two points C and A
- Random point E from line CA
- Generalized segment division point F of segment AB with respect to ratio product (CD/DB)*(AE/EC) and coefficient -1.0

Theorem statement:

- Points D, E, F are collinear

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

2.2 Transformation of point B:

- Point B has been assigned following coordinates: (0, u_1)

2.3 Transformation of point C:

- Point C has been assigned following coordinates: (u_2, u_3)

2.4 Transformation of point D:

- Point D has been assigned following coordinates: (u_4, x_1)
- Polynomial that point D has to satisfy is:

$$p = u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1)$$

- Processing of polynomial

$$p = u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1)$$

Info: Polynomial

$$p = u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of point E:

- Point E has been assigned following coordinates: (u_5, x_2)
- Polynomial that point E has to satisfy is:

$$p = u_2x_2 - u_5u_3$$

- Processing of polynomial

$$p = u_2x_2 - u_5u_3$$

Info: Polynomial

$$p = u_2x_2 - u_5u_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point F:

- Point F has been assigned following coordinates: (x_3, x_4)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_3$$

Info: Will try to rename X coordinate of point F

Info: Y coordinate of point F will be replaced by X coordinate

Info: X coordinate of point F renamed by zero

- Point F has been renamed. Point F has been assigned following coordinates: (0, x_3)
- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = (u_5u_2 - u_4u_2)x_3 + (u_5u_4u_1 - u_5u_2u_1)$$

Info: Polynomial

$$p = (u_5u_2 - u_4u_2)x_3 + (u_5u_4u_1 - u_5u_2u_1)$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = (u_5u_2 - u_4u_2)x_3 + (u_5u_4u_1 - u_5u_2u_1)$$

is added to polynomial system

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = (u_5 - u_4)x_3 + u_4x_2 - u_5x_1$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.04 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned} p_1 &= u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1) \\ p_2 &= u_2x_2 - u_5u_3 \\ p_3 &= (u_5u_2 - u_4u_2)x_3 + (u_5u_4u_1 - u_5u_2u_1) \end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned} p_1 &= u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1) \\ p_2 &= u_2x_2 - u_5u_3 \\ p_3 &= (u_5u_2 - u_4u_2)x_3 + (u_5u_4u_1 - u_5u_2u_1) \end{aligned}$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 321 (Converse of Menelaus' Theorem)

Calculating final remainder of the conclusion:

$$g = (u_5 - u_4)x_3 + u_4x_2 - u_5x_1$$

with respect to the triangular system.

1. Pseudo remainder with p_3 over variable x_3:

$$\begin{aligned} g = {} & (u_5u_4u_2 - u_4^2u_2)x_2 + (-u_5^2u_2 + u_5u_4u_2)x_1 + \\ & (-u_5^2u_4u_1 + u_5^2u_2u_1 + u_5u_4^2u_1 - u_5u_4u_2u_1) \end{aligned}$$

2. Pseudo remainder with p_2 over variable x_2:

$$\begin{aligned} g &= (-u_5^2u_2^2 + u_5u_4u_2^2)x_1 + \\ &\quad (u_5^2u_4u_3u_2 - u_5^2u_4u_2u_1 + u_5^2u_2^2u_1 \\ &\quad -u_5u_4^2u_3u_2 + u_5u_4^2u_2u_1 - u_5u_4u_2^2u_1) \end{aligned}$$

3. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.02 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points F, A, B and C are not collinear
- Points D, E and A are not collinear

Time spent for processing NDG Conditions

- 0.624 seconds