

OpenGeoProver Output for RC-Constructibility problem “Wernick 124 (Ha, Hb, Hc — A)”

Used algebraic method (with triangulation)

July 14, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point Ha
- Free point Hb
- Free point Hc
- Angle bisector hb of angle HaHbHc
- Angle bisector hc of angle HbHcHa
- Line b through point Hb perpendicular to line hb
- Line c through point Hc perpendicular to line hc
- Intersection point A of point sets b and c

Free points:

- Ha
- Hb
- Hc

Points to be constructed:

- A

Info: Attempting to add the construction of new random point tempPoint-779hb necessary for completion of construction of line b

Warning: Generated new random point tempPoint-779hb on line hb in order to complete the construction of perpendicular line b

Info: Attempting to add the construction of new random point tempPoint-203hc necessary for completion of construction of line c

Warning: Generated new random point tempPoint-203hc on line hc in order to complete the construction of perpendicular line c

Validation result: Construction protocol is valid.

2 Instantiation of points with symbolic variables

- Point A has been assigned following coordinates: (x_1, x_2)
- Point Ha has been assigned following coordinates: (u_1, u_2)
- Point Hb has been assigned following coordinates: (u_3, u_4)
- Point Hc has been assigned following coordinates: (u_5, u_6)
- Point tempPoint-779hb has been assigned following coordinates: (u_7, x_3)
- Point tempPoint-203hc has been assigned following coordinates: (u_8, x_4)

3 Transformation of geometry conditions for points to polynomial form

3.1 Transformation, step 1

Point to transform: Ha

Polynomial condition(s): N/A - free point

3.2 Transformation, step 2

Point to transform: Hb

Polynomial condition(s): N/A - free point

3.3 Transformation, step 3

Point to transform: Hc

Polynomial condition(s): N/A - free point

3.4 Transformation, step 4

Point to transform: tempPoint-779hb

Polynomial condition(s): One polynomial

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$$\begin{aligned}
p = & (-u_6u_3 + u_6u_1 - u_5u_4 + u_5u_2 + 2u_4u_3 - u_4u_1 - u_3u_2)x_3^2 + \\
& (2u_7u_6u_4 - 2u_7u_6u_2 - 2u_7u_5u_3 + 2u_7u_5u_1 - 2u_7u_4^2 + \\
& 2u_7u_4u_2 + 2u_7u_3^2 - 2u_7u_3u_1 - 2u_6u_4u_1 + 2u_6u_3u_2 + \\
& 2u_5u_4^2 - 2u_5u_4u_2 + 2u_5u_3^2 - 2u_5u_3u_1 - 2u_4^2u_3 + \\
& 2u_4^2u_1 - 2u_3^3 + 2u_3^2u_1) \\
& x_3 \\
& + \\
& (u_7^2u_6u_3 - u_7^2u_6u_1 + u_7^2u_5u_4 - u_7^2u_5u_2 \\
& - 2u_7^2u_4u_3 + u_7^2u_4u_1 + u_7^2u_3u_2 - 2u_7u_6u_4^2 + \\
& 2u_7u_6u_4u_2 - 2u_7u_6u_3^2 + 2u_7u_6u_3u_1 - 2u_7u_5u_4u_1 + \\
& 2u_7u_5u_3u_2 + 2u_7u_4^3 - 2u_7u_4^2u_2 + 2u_7u_4u_3^2 \\
& - 2u_7u_3^2u_2 + u_6u_4^2u_3 + u_6u_4^2u_1 - 2u_6u_4u_3u_2 + \\
& u_6u_3^3 - u_6u_3^2u_1 - u_5u_4^3 + u_5u_4^2u_2 - u_5u_4u_3^2 + \\
& 2u_5u_4u_3u_1 - u_5u_3^2u_2 - u_4^3u_1 + u_4^2u_3u_2 \\
& - u_4u_3^2u_1 + u_3^3u_2)
\end{aligned}$$

3.5 Transformation, step 5

Point to transform: tempPoint-203hc

Polynomial condition(s): One polynomial

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$$\begin{aligned}
p = & (2u_6u_5 - u_6u_3 - u_6u_1 - u_5u_4 - u_5u_2 + u_4u_1 + u_3u_2)x_4^2 + \\
& (-2u_8u_6^2 + 2u_8u_6u_4 + 2u_8u_6u_2 + 2u_8u_5^2 - 2u_8u_5u_3 \\
& - 2u_8u_5u_1 - 2u_8u_4u_2 + 2u_8u_3u_1 - 2u_6^2u_5 + 2u_6^2u_3 + \\
& 2u_6^2u_1 - 2u_6u_4u_1 - 2u_6u_3u_2 - 2u_5^3 + 2u_5^2u_3 + 2u_5^2u_1 + \\
& 2u_5u_4u_2 - 2u_5u_3u_1) \\
& x_4 \\
& + \\
& (-2u_8^2u_6u_5 + u_8^2u_6u_3 + u_8^2u_6u_1 + u_8^2u_5u_4 + \\
& u_8^2u_5u_2 - u_8^2u_4u_1 - u_8^2u_3u_2 + 2u_8u_6^3 \\
& - 2u_8u_6^2u_4 - 2u_8u_6^2u_2 + 2u_8u_6u_5^2 + 2u_8u_6u_4u_2 \\
& - 2u_8u_6u_3u_1 - 2u_8u_5^2u_4 - 2u_8u_5^2u_2 + 2u_8u_5u_4u_1 + \\
& 2u_8u_5u_3u_2 - u_6^3u_3 - u_6^3u_1 + u_6^2u_5u_4 + u_6^2u_5u_2 + \\
& u_6^2u_4u_1 + u_6^2u_3u_2 - u_6u_5^2u_3 - u_6u_5^2u_1 \\
& - 2u_6u_5u_4u_2 + 2u_6u_5u_3u_1 + u_5^3u_4 + u_5^3u_2 \\
& - u_5^2u_4u_1 - u_5^2u_3u_2)
\end{aligned}$$

3.6 Transformation, step 6

Point to transform: A

Polynomial condition(s): Two polynomials

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$$p = x_3x_2 - u_4x_3 - u_4x_2 + (u_7 - u_3)x_1 + (-u_7u_3 + u_4^2 + u_3^2)$$

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$$p = x_4x_2 - u_6x_4 - u_6x_2 + (u_8 - u_5)x_1 + (-u_8u_5 + u_6^2 + u_5^2)$$

4 Triangulation of polynomial system

The input system is:

$$\begin{aligned} p_1 = & (-u_6u_3 + u_6u_1 - u_5u_4 + u_5u_2 + 2u_4u_3 - u_4u_1 - u_3u_2)x_3^2 + \\ & (2u_7u_6u_4 - 2u_7u_6u_2 - 2u_7u_5u_3 + 2u_7u_5u_1 - 2u_7u_4^2 + \\ & 2u_7u_4u_2 + 2u_7u_3^2 - 2u_7u_3u_1 - 2u_6u_4u_1 + 2u_6u_3u_2 + \\ & 2u_5u_4^2 - 2u_5u_4u_2 + 2u_5u_3^2 - 2u_5u_3u_1 - 2u_4^2u_3 + \\ & 2u_4^2u_1 - 2u_3^3 + 2u_3^2u_1) \\ & x_3 \\ & + \\ & (u_7^2u_6u_3 - u_7^2u_6u_1 + u_7^2u_5u_4 - u_7^2u_5u_2 \\ & - 2u_7^2u_4u_3 + u_7^2u_4u_1 + u_7^2u_3u_2 - 2u_7u_6u_4^2 + \\ & 2u_7u_6u_4u_2 - 2u_7u_6u_3^2 + 2u_7u_6u_3u_1 - 2u_7u_5u_4u_1 + \\ & 2u_7u_5u_3u_2 + 2u_7u_4^3 - 2u_7u_4^2u_2 + 2u_7u_4u_3^2 \\ & - 2u_7u_3^2u_2 + u_6u_4^2u_3 + u_6u_4^2u_1 - 2u_6u_4u_3u_2 + \\ & u_6u_3^3 - u_6u_3^2u_1 - u_5u_4^3 + u_5u_4^2u_2 - u_5u_4u_3^2 + \\ & 2u_5u_4u_3u_1 - u_5u_3^2u_2 - u_4^3u_1 + u_4^2u_3u_2 \\ & - u_4u_3^2u_1 + u_3^3u_2) \\ p_2 = & (2u_6u_5 - u_6u_3 - u_6u_1 - u_5u_4 - u_5u_2 + u_4u_1 + u_3u_2)x_4^2 + \\ & (-2u_8u_6^2 + 2u_8u_6u_4 + 2u_8u_6u_2 + 2u_8u_5^2 - 2u_8u_5u_3 \\ & - 2u_8u_5u_1 - 2u_8u_4u_2 + 2u_8u_3u_1 - 2u_6^2u_5 + 2u_6^2u_3 + \\ & 2u_6^2u_1 - 2u_6u_4u_1 - 2u_6u_3u_2 - 2u_5^3 + 2u_5^2u_3 + 2u_5^2u_1 + \\ & 2u_5u_4u_2 - 2u_5u_3u_1) \\ & x_4 \\ & + \\ & (-2u_8^2u_6u_5 + u_8^2u_6u_3 + u_8^2u_6u_1 + u_8^2u_5u_4 + \\ & u_8^2u_5u_2 - u_8^2u_4u_1 - u_8^2u_3u_2 + 2u_8u_6^3 \\ & - 2u_8u_6^2u_4 - 2u_8u_6^2u_2 + 2u_8u_6u_5^2 + 2u_8u_6u_4u_2 \end{aligned}$$

$$\begin{aligned}
& -2u_8u_6u_3u_1 - 2u_8u_5^2u_4 - 2u_8u_5^2u_2 + 2u_8u_5u_4u_1 + \\
& 2u_8u_5u_3u_2 - u_6^3u_3 - u_6^3u_1 + u_6^2u_5u_4 + u_6^2u_5u_2 + \\
& u_6^2u_4u_1 + u_6^2u_3u_2 - u_6u_5^2u_3 - u_6u_5^2u_1 \\
& -2u_6u_5u_4u_2 + 2u_6u_5u_3u_1 + u_5^3u_4 + u_5^3u_2 \\
& -u_5^2u_4u_1 - u_5^2u_3u_2)
\end{aligned}$$

$$\begin{aligned}
p_3 &= x_3x_2 - u_4x_3 - u_4x_2 + (u_7 - u_3)x_1 + (-u_7u_3 + u_4^2 + u_3^2) \\
p_4 &= x_4x_2 - u_6x_4 - u_6x_2 + (u_8 - u_5)x_1 + (-u_8u_5 + u_6^2 + u_5^2)
\end{aligned}$$

4.1 Triangulation, step 1

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 2.

Minimal degrees: 1 polynomial(s) with degree 1 and 1 polynomial(s) with degree 2.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_4 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (-u_6u_3 + u_6u_1 - u_5u_4 + u_5u_2 + 2u_4u_3 - u_4u_1 - u_3u_2)x_3^2 + \\
& (2u_7u_6u_4 - 2u_7u_6u_2 - 2u_7u_5u_3 + 2u_7u_5u_1 - 2u_7u_4^2 + \\
& 2u_7u_4u_2 + 2u_7u_3^2 - 2u_7u_3u_1 - 2u_6u_4u_1 + 2u_6u_3u_2 + \\
& 2u_5u_4^2 - 2u_5u_4u_2 + 2u_5u_3^2 - 2u_5u_3u_1 - 2u_4^2u_3 + \\
& 2u_4^2u_1 - 2u_3^3 + 2u_3^2u_1) \\
& x_3 \\
& + \\
& (u_7^2u_6u_3 - u_7^2u_6u_1 + u_7^2u_5u_4 - u_7^2u_5u_2 \\
& - 2u_7^2u_4u_3 + u_7^2u_4u_1 + u_7^2u_3u_2 - 2u_7u_6u_4^2 + \\
& 2u_7u_6u_4u_2 - 2u_7u_6u_3^2 + 2u_7u_6u_3u_1 - 2u_7u_5u_4u_1 + \\
& 2u_7u_5u_3u_2 + 2u_7u_4^3 - 2u_7u_4^2u_2 + 2u_7u_4u_3^2 \\
& - 2u_7u_3^2u_2 + u_6u_4^2u_3 + u_6u_4^2u_1 - 2u_6u_4u_3u_2 + \\
& u_6u_3^3 - u_6u_3^2u_1 - u_5u_4^3 + u_5u_4^2u_2 - u_5u_4u_3^2 + \\
& 2u_5u_4u_3u_1 - u_5u_3^2u_2 - u_4^3u_1 + u_4^2u_3u_2 \\
& - u_4u_3^2u_1 + u_3^3u_2) \\
p_2 &= x_3x_2 - u_4x_3 - u_4x_2 + (u_7 - u_3)x_1 + (-u_7u_3 + u_4^2 + u_3^2) \\
p_3 &= \dots \\
p_4 &= x_4x_2 - u_6x_4 - u_6x_2 + (u_8 - u_5)x_1 + (-u_8u_5 + u_6^2 + u_5^2)
\end{aligned}$$

4.2 Triangulation, step 2

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 2.

Minimal degrees: 1 polynomial(s) with degree 1 and 1 polynomial(s) with degree 2.

Polynomial with linear degree: Removing variable x_3 from all other polynomials by reducing them with polynomial p_2 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_1 &= \dots \\ p_2 &= \dots \\ p_3 &= x_3x_2 - u_4x_3 - u_4x_2 + (u_7 - u_3)x_1 + (-u_7u_3 + u_4^2 + u_3^2) \\ p_4 &= x_4x_2 - u_6x_4 - u_6x_2 + (u_8 - u_5)x_1 + (-u_8u_5 + u_6^2 + u_5^2) \end{aligned}$$

4.3 Triangulation, step 3

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 2.

No linear degree polynomials: Reducing polynomial p_2 (of degree 2) with p_1 (of degree 2).

Finished a triangulation step, the current system is:

$$\begin{aligned} p_1 &= \dots \\ p_2 &= \dots \\ p_3 &= x_3x_2 - u_4x_3 - u_4x_2 + (u_7 - u_3)x_1 + (-u_7u_3 + u_4^2 + u_3^2) \\ p_4 &= x_4x_2 - u_6x_4 - u_6x_2 + (u_8 - u_5)x_1 + (-u_8u_5 + u_6^2 + u_5^2) \end{aligned}$$

4.4 Triangulation, step 4

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1 . No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_1 &= \dots \\
p_2 &= \dots \\
p_3 &= x_3x_2 - u_4x_3 - u_4x_2 + (u_7 - u_3)x_1 + (-u_7u_3 + u_4^2 + u_3^2) \\
p_4 &= x_4x_2 - u_6x_4 - u_6x_2 + (u_8 - u_5)x_1 + (-u_8u_5 + u_6^2 + u_5^2)
\end{aligned}$$

List of leading monic terms from triangular polynomial system

$$p_1 = x_1^4$$

$$p_2 = x_2x_1$$

$$p_3 = x_3x_2$$

$$p_4 = x_4x_2$$

5 Instantiation of triangular polynomial system

5.1 Instances for points' coordinates

- Ha: $x = 0.0, y = 0.0$
- Hb: $x = 0.0, y = 1.0$
- Hc: $x = 1.0, y = 0.0$

5.2 Instantiated triangular polynomial system

$$\begin{aligned}
p_1 &= (-16u_8^6u_7^4 + 96u_8^5u_7^4 - 240u_8^4u_7^4 + 320u_8^3u_7^4 \\
&\quad - 240u_8^2u_7^4 + 96u_8u_7^4 - 16u_7^4) \\
&\quad x_1^4 \\
&\quad + \\
&\quad (32u_8^6u_7^4 - 192u_8^5u_7^4 + 480u_8^4u_7^4 - 640u_8^3u_7^4 + \\
&\quad 480u_8^2u_7^4 - 192u_8u_7^4 + 32u_7^4) \\
&\quad x_1^3 \\
&\quad +
\end{aligned}$$

$$\begin{aligned}
& (-16u_8^6u_7^4 + 96u_8^5u_7^4 - 240u_8^4u_7^4 + 320u_8^3u_7^4 \\
& -240u_8^2u_7^4 + 96u_8u_7^4 - 16u_7^4) \\
& x_1 \\
& + \\
& (4u_8^6u_7^4 - 24u_8^5u_7^4 + 60u_8^4u_7^4 - 80u_8^3u_7^4 + \\
& 60u_8^2u_7^4 - 24u_8u_7^4 + 4u_7^4) \\
\\
p_2 &= (4u_8^2u_7^2 - 8u_8u_7^2 + 4u_7^2)x_2x_1 + \\
& (-4u_8^2u_7^2 + 8u_8u_7^2 - 4u_7^2)x_2 + \\
& (-4u_8^2u_7^2 + 8u_8u_7^2 - 4u_7^2)x_1 + \\
& (2u_8^2u_7^2 - 4u_8u_7^2 + 2u_7^2) \\
p_3 &= x_3x_2 - x_3 - x_2 + u_7x_1 + 1 \\
p_4 &= x_4x_2 + (u_8 - 1)x_1 + (-u_8 + 1)
\end{aligned}$$

6 Result of transformation of RC-constructibility problem to polynomial form

Success Message: Successful completion.

Space Complexity: The biggest polynomial obtained during application execution contains 7 terms.

Time Complexity: Time spent in execution is 205.276 seconds.