OpenGeoProver Output for conjecture "Chou 001 (Pappus' Theorem)"

Wu's method used

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1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Line AB through two points A and B
- Random point C from line AB
- Free point A1
- Free point B1
- Line A1B1 through two points A1 and B1
- Random point C1 from line A1B1
- Line AB1 through two points A and B1
- Line A1B through two points A1 and B
- Intersection point P of point sets AB1 and A1B
- Line AC1 through two points A and C1
- Line A1C through two points A1 and C
- Intersection point Q of point sets AC1 and A1C
- Line BC1 through two points B and C1
- Line B1C through two points B1 and C
- Intersection point R of point sets BC1 and B1C

Theorem statement:

- Points P, Q, R are collinear

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

2.2 Transformation of point B:

- Point B has been assigned following coordinates: $(0,\ u_1)$

2.3 Transformation of point C:

- Point C has been assigned following coordinates: $(u_2,\ x_1)$
- Polynomial that point C has to satisfy is:

$$p = x_1$$

- Processing of polynomial

$$p = x_1$$

Info: Will try to rename X coordinate of point C

Info: X coordinate of point C renamed by zero

- Point C has been renamed. Point C has been assigned following coordinates: $(0,\ u_2)$

2.4 Transformation of point A1:

- Point A1 has been assigned following coordinates: $(u_3,\ u_4)$

2.5 Transformation of point B1:

- Point B1 has been assigned following coordinates: $(u_5,\ u_6)$

2.6 Transformation of point C1:

- Point C1 has been assigned following coordinates: $(u_7,\ x_1)$
- Polynomial that point C1 has to satisfy is:

$$p = (u_5 - u_3)x_1 + (-u_7u_6 + u_7u_4 + u_6u_3 - u_5u_4)$$

- Processing of polynomial

$$p = (u_5 - u_3)x_1 + (-u_7u_6 + u_7u_4 + u_6u_3 - u_5u_4)$$

Info: Polynomial

$$p = (u_5 - u_3)x_1 + (-u_7u_6 + u_7u_4 + u_6u_3 - u_5u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point P:

- Point P has been assigned following coordinates: (x_2, x_3)
- Polynomial that point P has to satisfy is:

$$p = u_5x_3 - u_6x_2$$

- Processing of polynomial

$$p = u_5x_3 - u_6x_2$$

Info: Polynomial

$$p = u_5x_3 - u_6x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point P has to satisfy is:

$$p = u_3x_3 + (-u_4 + u_1)x_2 - u_3u_1$$

- Processing of polynomial

$$p = u_3x_3 + (-u_4 + u_1)x_2 - u_3u_1$$

Info: Polynomial

$$p = u_3x_3 + (-u_4 + u_1)x_2 - u_3u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.8 Transformation of point Q:

- Point Q has been assigned following coordinates: (x_4, x_5)
- Polynomial that point Q has to satisfy is:

$$p = u_7x_5 - x_4x_1$$

- Processing of polynomial

$$p = u_7x_5 - x_4x_1$$

Info: Polynomial

$$p = u_7x_5 - x_4x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point Q has to satisfy is:

$$p = u_3x_5 + (-u_4 + u_2)x_4 - u_3u_2$$

- Processing of polynomial

$$p = u_3x_5 + (-u_4 + u_2)x_4 - u_3u_2$$

Info: Polynomial

$$p = u_3x_5 + (-u_4 + u_2)x_4 - u_3u_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.9 Transformation of point R:

- Point R has been assigned following coordinates: (x_6, x_7)
- Polynomial that point R has to satisfy is:

$$p = u_7x_7 - x_6x_1 + u_1x_6 - u_7u_1$$

- Processing of polynomial

$$p = u_7x_7 - x_6x_1 + u_1x_6 - u_7u_1$$

Info: Polynomial

$$p = u_7x_7 - x_6x_1 + u_1x_6 - u_7u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point R has to satisfy is:

$$p = u_5x_7 + (-u_6 + u_2)x_6 - u_5u_2$$

- Processing of polynomial

$$p = u_5x_7 + (-u_6 + u_2)x_6 - u_5u_2$$

Info: Polynomial

$$p = u_5x_7 + (-u_6 + u_2)x_6 - u_5u_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_7x_4 - x_7x_2 - x_6x_5 + x_6x_3 + x_5x_2 - x_4x_3$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.078 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$
\begin{aligned}
p_1 &= (u_5 - u_3)x_1 + (-u_7u_6 + u_7u_4 + u_6u_3 - u_5u_4) \\
p_2 &= u_5x_3 - u_6x_2 \\
p_3 &= u_3x_3 + (-u_4 + u_1)x_2 - u_3u_1 \\
p_4 &= u_7x_5 - x_4x_1 \\
p_5 &= u_3x_5 + (-u_4 + u_2)x_4 - u_3u_2 \\
p_6 &= u_7x_7 - x_6x_1 + u_1x_6 - u_7u_1 \\
p_7 &= u_5x_7 + (-u_6 + u_2)x_6 - u_5u_2
\end{aligned}
$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_7 from all other polynomials by reducing them with polynomial p_6 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (u_5 - u_3)x_1 + (-u_7u_6 + u_7u_4 + u_6u_3 - u_5u_4) \\
p_2 &= u_5x_3 - u_6x_2 \\
p_3 &= u_3x_3 + (-u_4 + u_1)x_2 - u_3u_1 \\
p_4 &= u_7x_5 - x_4x_1 \\
p_5 &= u_3x_5 + (-u_4 + u_2)x_4 - u_3u_2 \\
p_6 &= u_5x_6x_1 + (-u_7u_6 + u_7u_2 - u_5u_1)x_6 + (-u_7u_5u_2 + u_7u_5u_1) \\
p_7 &= u_7x_7 - x_6x_1 + u_1x_6 - u_7u_1
\end{aligned}
$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_5 from all other polynomials by reducing them with polynomial p_4 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_5 - u_3)x_1 + (-u_7u_6 + u_7u_4 + u_6u_3 - u_5u_4)\\
p_2 &= u_5x_3 - u_6x_2\\
p_3 &= u_3x_3 + (-u_4 + u_1)x_2 - u_3u_1\\
p_4 &= u_3x_4x_1 + (-u_7u_4 + u_7u_2)x_4 - u_7u_3u_2\\
p_5 &= u_7x_5 - x_4x_1\\
p_6 &= u_5x_6x_1 + (-u_7u_6 + u_7u_2 - u_5u_1)x_6 + (-u_7u_5u_2 + u_7u_5u_1)\\
p_7 &= u_7x_7 - x_6x_1 + u_1x_6 - u_7u_1
\end{aligned}$$

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_3 from all other polynomials by reducing them with polynomial p_2 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (u_5 - u_3)x_1 + (-u_7u_6 + u_7u_4 + u_6u_3 - u_5u_4) \\
p_2 &= (u_6u_3 - u_5u_4 + u_5u_1)x_2 - u_5u_3u_1 \\
p_3 &= u_5x_3 - u_6x_2 \\
p_4 &= u_3x_4x_1 + (-u_7u_4 + u_7u_2)x_4 - u_7u_3u_2 \\
p_5 &= u_7x_5 - x_4x_1 \\
p_6 &= u_5x_6x_1 + (-u_7u_6 + u_7u_2 - u_5u_1)x_6 + (-u_7u_5u_2 + u_7u_5u_1) \\
p_7 &= u_7x_7 - x_6x_1 + u_1x_6 - u_7u_1
\end{aligned}
$$

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= (u_5 - u_3)x_1 + (-u_7u_6 + u_7u_4 + u_6u_3 - u_5u_4) \\
p_2 &= (u_6u_3 - u_5u_4 + u_5u_1)x_2 - u_5u_3u_1 \\
p_3 &= u_5x_3 - u_6x_2 \\
p_4 &= u_3x_4x_1 + (-u_7u_4 + u_7u_2)x_4 - u_7u_3u_2 \\
p_5 &= u_7x_5 - x_4x_1 \\
p_6 &= u_5x_6x_1 + (-u_7u_6 + u_7u_2 - u_5u_1)x_6 + (-u_7u_5u_2 + u_7u_5u_1) \\
p_7 &= u_7x_7 - x_6x_1 + u_1x_6 - u_7u_1
\end{aligned}
$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 001 (Pappus' Theorem)

Calculating final remainder of the conclusion:

$$g = x_7x_4 - x_7x_2 - x_6x_5 + x_6x_3 + x_5x_2 - x_4x_3$$

with respect to the triangular system.

1. Pseudo remainder with p_7 over variable x_7:

$$\begin{aligned} g = & -u_7x_6x_5 + x_6x_4x_1 - u_1x_6x_4 + u_7x_6x_3 - x_6x_2x_1 + \\ & u_1x_6x_2 + u_7x_5x_2 - u_7x_4x_3 + u_7u_1x_4 - u_7u_1x_2 \end{aligned}$$

2. Pseudo remainder with p_6 over variable x_6:

$$\begin{aligned} g = & u_7u_5x_5x_2x_1 + (-u_7^2u_6 + u_7^2u_2 - u_7u_5u_1)x_5x_2 + \\ & (-u_7^2u_5u_2 + u_7^2u_5u_1)x_5 - u_7u_5x_4x_3x_1 + \\ & (u_7^2u_6 - u_7^2u_2 + u_7u_5u_1)x_4x_3 + u_7u_5u_2x_4x_1 + \\ & (-u_7^2u_6u_1 + u_7^2u_2u_1 - u_7u_5u_2u_1)x_4 + \\ & (u_7^2u_5u_2 - u_7^2u_5u_1)x_3 - u_7u_5u_2x_2x_1 + \\ & (u_7^2u_6u_1 - u_7^2u_2u_1 + u_7u_5u_2u_1)x_2 \end{aligned}$$

3. Pseudo remainder with p_5 over variable x_5:

$$\begin{aligned} g = & -u_7^2u_5x_4x_3x_1 + \\ & (u_7^3u_6 - u_7^3u_2 + u_7^2u_5u_1)x_4x_3 + u_7u_5x_4x_2x_1^2 + \\ & (-u_7^2u_6 + u_7^2u_2 - u_7u_5u_1)x_4x_2x_1 + \\ & u_7^2u_5u_1x_4x_1 + \\ & (-u_7^3u_6u_1 + u_7^3u_2u_1 - u_7^2u_5u_2u_1)x_4 + \\ & (u_7^3u_5u_2 - u_7^3u_5u_1)x_3 - u_7^2u_5u_2x_2x_1 + \\ & (u_7^3u_6u_1 - u_7^3u_2u_1 + u_7^2u_5u_2u_1)x_2 \end{aligned}$$

4. Pseudo remainder with p_4 over variable x_4:

$$\begin{aligned} g = & -u_7^3u_5u_3u_1x_3x_1 + \\ & (u_7^4u_6u_3u_2 - u_7^4u_5u_4u_2 + u_7^4u_5u_4u_1 + \\ & u_7^4u_5u_2^2 - u_7^4u_5u_2u_1 - u_7^4u_3u_2^2 + \\ & u_7^3u_5u_3u_2u_1) \end{aligned}$$

$$
\begin{aligned}
& x_3 \\
& + \\
& (-u_7^3u_6u_3u_2 + u_7^3u_6u_3u_1 + u_7^3u_5u_4u_2 \\
& -u_7^3u_5u_2^2 + u_7^3u_3u_2^2 - u_7^3u_3u_2u_1) \\
& x_2x_1 \\
& + \\
& (-u_7^4u_6u_4u_1 + u_7^4u_6u_2u_1 + u_7^4u_4u_2u_1 \\
& -u_7^4u_2^2u_1 - u_7^3u_5u_4u_2u_1 + u_7^3u_5u_2^2u_1) \\
& x_2 \\
& +u_7^3u_5u_3u_2u_1x_1 + \\
& (-u_7^4u_6u_3u_2u_1 + u_7^4u_3u_2^2u_1 \\
& -u_7^3u_5u_3u_2^2u_1)
\end{aligned}
$$

5. Pseudo remainder with p_3 over variable x_3:

$$
\begin{aligned}
g \quad = \quad & (-u_7^3u_6u_5u_3u_2 + u_7^3u_5^2u_4u_2 - u_7^3u_5^2u_2^2 + \\
& u_7^3u_5u_3u_2^2 - u_7^3u_5u_3u_2u_1) \\
& x_2x_1 \\
& + \\
& (u_7^4u_6^2u_3u_2 - u_7^4u_6u_5u_4u_2 + u_7^4u_6u_5u_2^2 \\
& -u_7^4u_6u_3u_2^2 + u_7^4u_5u_4u_2u_1 - u_7^4u_5u_2^2u_1 + \\
& u_7^3u_6u_5u_3u_2u_1 - u_7^3u_5^2u_4u_2u_1 + \\
& u_7^3u_5^2u_2^2u_1) \\
& x_2 \\
& +u_7^3u_5^2u_3u_2u_1x_1 + \\
& (-u_7^4u_6u_5u_3u_2u_1 + u_7^4u_5u_3u_2^2u_1 \\
& -u_7^3u_5^2u_3u_2^2u_1)
\end{aligned}
$$

6. Pseudo remainder with p_2 over variable x_2:

$$
\begin{aligned}
g \quad = \quad & (-u_7^3u_5^3u_3u_2^2u_1 + u_7^3u_5^3u_3u_2u_1^2 + \\
& u_7^3u_5^2u_3^2u_2^2u_1 - u_7^3u_5^2u_3^2u_2u_1^2) \\
& x_1 \\
& + \\
& (u_7^4u_6u_5^2u_3u_2^2u_1 - u_7^4u_6u_5^2u_3u_2u_1^2 \\
& -u_7^4u_5^2u_4u_3u_2^2u_1 + u_7^4u_5^2u_4u_3u_2u_1^2 \\
& -u_7^3u_6u_5^2u_3^2u_2^2u_1 + \\
& u_7^3u_6u_5^2u_3^2u_2u_1^2 + u_7^3u_5^3u_4u_3u_2^2u_1
\end{aligned}
$$

$$-u_7^3u_5^3u_4u_3u_2u_1^2)$$

7. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 10 terms.

Time Complexity: Time spent by the prover is 0.125 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points A1, A, C and B1 are not collinear
- Points A1, A, B and B1 are not collinear
- Points C and B1 are not identical
- Line through points A1 and C is not parallel with line through points A and C1
- Points A and C1 are not identical
- Line through points B and C1 is not parallel with line through points C and B1

Time spent for processing NDG Conditions

- 1.248 seconds