OpenGeoProver Output for conjecture "Polar 002"

Wu's method used

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1 Validation of Construction Protocol

Construction steps:

- General Conic Section c
- Free point P
- Random point A from general conic c
- Random point C from general conic c
- Random point E from general conic c
- Line s1 through two points P and A
- Line s2 through two points P and C
- Line s3 through two points P and E
- Intersection point B of point sets s1 and c
- Intersection point D of point sets s2 and c
- Intersection point F of point sets s3 and c
- Fourth harmonic conjugate point G of tripple of points A, B and P
- Fourth harmonic conjugate point H of tripple of points C, D and P
- Fourth harmonic conjugate point I of tripple of points E, F and P

Theorem statement:

- Points G, H, I are collinear

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of general conic section c:

List of parametric points

- Point Ac has been assigned following coordinates: $(u_1, 0)$
- Point Bc has been assigned following coordinates: $(u_2, 0)$
- Point Cc has been assigned following coordinates: $(u_3, 0)$
- Point Dc has been assigned following coordinates: $(u_4, 0)$
- Point Ec has been assigned following coordinates: $(u_5, 0)$
- Condition for point X(x_1, x_2) to belong to this conic section is following equation:

$$p = u_3x_2^2 + u_2x_2x_1 + u_5x_2 + u_1x_1^2 + u_4x_1$$

2.2 Transformation of point P:

- Point P has been assigned following coordinates: (u_6, u_7)

2.3 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

2.4 Transformation of point C:

- Point C has been assigned following coordinates: $(0, x_1)$
- Polynomial that point C has to satisfy is:

$$p = u_3x_1^2 + u_5x_1$$

- Processing of polynomial

$$p = u_3x_1^2 + u_5x_1$$

Info: Polynomial

$$p = u_3x_1^2 + u_5x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of point E:

- Point E has been assigned following coordinates: (u_8, x_2)
- Polynomial that point E has to satisfy is:

$$p = u_3x_2^2 + (u_8u_2 + u_5)x_2 + (u_8^2u_1 + u_8u_4)$$

- Processing of polynomial

$$p = u_3x_2^2 + (u_8u_2 + u_5)x_2 + (u_8^2u_1 + u_8u_4)$$

Info: Polynomial

$$p = u_3x_2^2 + (u_8u_2 + u_5)x_2 + (u_8^2u_1 + u_8u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point B:

- Point B has been assigned following coordinates: (x_3, x_4)
- Polynomial that point B has to satisfy is:

$$p = u_6x_4 - u_7x_3$$

- Processing of polynomial

$$p = u_6x_4 - u_7x_3$$

Info: Polynomial

$$p = u_6x_4 - u_7x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point B has to satisfy is:

$$p = u_3x_4^2 + u_2x_4x_3 + u_5x_4 + u_1x_3^2 + u_4x_3$$

- Processing of polynomial

$$p = u_3x_4^2 + u_2x_4x_3 + u_5x_4 + u_1x_3^2 + u_4x_3$$

Info: Polynomial

$$p = u_3x_4^2 + u_2x_4x_3 + u_5x_4 + u_1x_3^2 + u_4x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point D:

- Point D has been assigned following coordinates: (x_5, x_6)
- Polynomial that point D has to satisfy is:

$$p = u_6x_6 + x_5x_1 - u_7x_5 - u_6x_1$$

- Processing of polynomial

$$p = u_6x_6 + x_5x_1 - u_7x_5 - u_6x_1$$

Info: Polynomial

$$p = u_6x_6 + x_5x_1 - u_7x_5 - u_6x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point D has to satisfy is:

$$p = u_3x_6^2 + u_2x_6x_5 + u_5x_6 + u_1x_5^2 + u_4x_5$$

- Processing of polynomial

$$p = u_3x_6^2 + u_2x_6x_5 + u_5x_6 + u_1x_5^2 + u_4x_5$$

Info: Polynomial

$$p = u_3x_6^2 + u_2x_6x_5 + u_5x_6 + u_1x_5^2 + u_4x_5$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.8 Transformation of point F:

- Point F has been assigned following coordinates: (x_7, x_8)
- Polynomial that point F has to satisfy is:

$$p = (u_8 - u_6)x_8 - x_7x_2 + u_7x_7 + u_6x_2 - u_8u_7$$

- Processing of polynomial

$$p = (u_8 - u_6)x_8 - x_7x_2 + u_7x_7 + u_6x_2 - u_8u_7$$

Info: Polynomial

$$p = (u_8 - u_6)x_8 - x_7x_2 + u_7x_7 + u_6x_2 - u_8u_7$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point F has to satisfy is:

$$p \quad = \quad u_3x_8^2 + u_2x_8x_7 + u_5x_8 + u_1x_7^2 + u_4x_7$$

- Processing of polynomial

$$p \quad = \quad u_3x_8^2 + u_2x_8x_7 + u_5x_8 + u_1x_7^2 + u_4x_7$$

Info: Polynomial

$$p \quad = \quad u_3x_8^2 + u_2x_8x_7 + u_5x_8 + u_1x_7^2 + u_4x_7$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.9 Transformation of point G:

- Point G has been assigned following coordinates: (x_9, x_{10})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p \quad = \quad x_9x_3 - 2u_6x_9 + u_6x_3$$

Info: Polynomial

$$p \quad = \quad x_9x_3 - 2u_6x_9 + u_6x_3$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p \quad = \quad x_9x_3 - 2u_6x_9 + u_6x_3$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p \quad = \quad x_{10}x_4 - 2u_7x_{10} + u_7x_4$$

Info: Polynomial

$$p \quad = \quad x_{10}x_4 - 2u_7x_{10} + u_7x_4$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p \quad = \quad x_{10}x_4 - 2u_7x_{10} + u_7x_4$$

is added to polynomial system

2.10 Transformation of point H:

- Point H has been assigned following coordinates: (x_{11}, x_{12})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{11}x_5 - 2u_6x_{11} + u_6x_5$$

Info: Polynomial

$$p = x_{11}x_5 - 2u_6x_{11} + u_6x_5$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{11}x_5 - 2u_6x_{11} + u_6x_5$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{12}x_6 + x_{12}x_1 - 2u_7x_{12} - 2x_6x_1 + u_7x_6 + u_7x_1$$

Info: Polynomial

$$p = x_{12}x_6 + x_{12}x_1 - 2u_7x_{12} - 2x_6x_1 + u_7x_6 + u_7x_1$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{12}x_6 + x_{12}x_1 - 2u_7x_{12} - 2x_6x_1 + u_7x_6 + u_7x_1$$

is added to polynomial system

2.11 Transformation of point I:

- Point I has been assigned following coordinates: (x_{13}, x_{14})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{13}x_7 + (u_8 - 2u_6)x_{13} + (-2u_8 + u_6)x_7 + u_8u_6$$

Info: Polynomial

$$p = x_{13}x_7 + (u_8 - 2u_6)x_{13} + (-2u_8 + u_6)x_7 + u_8u_6$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{13}x_7 + (u_8 - 2u_6)x_{13} + (-2u_8 + u_6)x_7 + u_8u_6$$

  is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{14}x_8 + x_{14}x_2 - 2u_7x_{14} - 2x_8x_2 + u_7x_8 + u_7x_2$$

  Info: Polynomial

$$p = x_{14}x_8 + x_{14}x_2 - 2u_7x_{14} - 2x_8x_2 + u_7x_8 + u_7x_2$$

  added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{14}x_8 + x_{14}x_2 - 2u_7x_{14} - 2x_8x_2 + u_7x_8 + u_7x_2$$

  is added to polynomial system

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_{14}x_{11} - x_{14}x_9 - x_{13}x_{12} + x_{13}x_{10} + x_{12}x_9 - x_{11}x_{10}$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.126 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_8u_2 + u_5)x_2 + (u_8^2u_1 + u_8u_4) \\
p_3 &= u_6x_4 - u_7x_3 \\
p_4 &= u_3x_4^2 + u_2x_4x_3 + u_5x_4 + u_1x_3^2 + u_4x_3 \\
p_5 &= u_6x_6 + x_5x_1 - u_7x_5 - u_6x_1 \\
p_6 &= u_3x_6^2 + u_2x_6x_5 + u_5x_6 + u_1x_5^2 + u_4x_5 \\
p_7 &= (u_8 - u_6)x_8 - x_7x_2 + u_7x_7 + u_6x_2 - u_8u_7 \\
p_8 &= u_3x_8^2 + u_2x_8x_7 + u_5x_8 + u_1x_7^2 + u_4x_7 \\
p_9 &= x_9x_3 - 2u_6x_9 + u_6x_3
\end{aligned}$$

$$
\begin{aligned}
p_{10} &= x_{10}x_4 - 2u_7x_{10} + u_7x_4 \\
p_{11} &= x_{11}x_5 - 2u_6x_{11} + u_6x_5 \\
p_{12} &= x_{12}x_6 + x_{12}x_1 - 2u_7x_{12} - 2x_6x_1 + u_7x_6 + u_7x_1 \\
p_{13} &= x_{13}x_7 + (u_8 - 2u_6)x_{13} + (-2u_8 + u_6)x_7 + u_8u_6 \\
p_{14} &= x_{14}x_8 + x_{14}x_2 - 2u_7x_{14} - 2x_8x_2 + u_7x_8 + u_7x_2
\end{aligned}
$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14}. No reduction needed.

The triangular system has not been changed.

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{13}. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{12}. No reduction needed.

The triangular system has not been changed.

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{11}. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10}. No reduction needed.

The triangular system has not been changed.

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_9. No reduction needed.

The triangular system has not been changed.

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 2.

Minimal degrees: 1 polynomial(s) with degree 1 and 1 polynomial(s) with degree 2.

Polynomial with linear degree: Removing variable x_8 from all other polynomials by reducing them with polynomial p_7 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_8u_2 + u_5)x_2 + (u_8^2u_1 + u_8u_4) \\
p_3 &= u_6x_4 - u_7x_3 \\
p_4 &= u_3x_4^2 + u_2x_4x_3 + u_5x_4 + u_1x_3^2 + u_4x_3 \\
p_5 &= u_6x_6 + x_5x_1 - u_7x_5 - u_6x_1 \\
p_6 &= u_3x_6^2 + u_2x_6x_5 + u_5x_6 + u_1x_5^2 + u_4x_5 \\
p_7 &= u_3x_7^2x_2^2 + (u_8u_2 - 2u_7u_3 - u_6u_2)x_7^2x_2 + \\
&\quad (u_8^2u_1 - u_8u_7u_2 - 2u_8u_6u_1 + u_7^2u_3 + u_7u_6u_2 + u_6^2u_1) \\
&\quad x_7^2 \\
&\quad -2u_6u_3x_7x_2^2 + \\
&\quad (2u_8u_7u_3 - u_8u_6u_2 + u_8u_5 + 2u_7u_6u_3 + u_6^2u_2 - u_6u_5)
\end{aligned}
$$

$$
\begin{aligned}
& x_7x_2 \\
& + \\
& (u_8^2u_7u_2 + u_8^2u_4 - 2u_8u_7^2u_3 - u_8u_7u_6u_2 \\
& -u_8u_7u_5 - 2u_8u_6u_4 + u_7u_6u_5 + u_6^2u_4) \\
& x_7 \\
& +u_6^2u_3x_2^2 + (-2u_8u_7u_6u_3 - u_8u_6u_5 + u_6^2u_5)x_2 + \\
& (u_8^2u_7^2u_3 + u_8^2u_7u_5 - u_8u_7u_6u_5)
\end{aligned}
$$

$$
\begin{aligned}
p_8 &= (u_8 - u_6)x_8 - x_7x_2 + u_7x_7 + u_6x_2 - u_8u_7 \\
p_9 &= x_9x_3 - 2u_6x_9 + u_6x_3 \\
p_{10} &= x_{10}x_4 - 2u_7x_{10} + u_7x_4 \\
p_{11} &= x_{11}x_5 - 2u_6x_{11} + u_6x_5 \\
p_{12} &= x_{12}x_6 + x_{12}x_1 - 2u_7x_{12} - 2x_6x_1 + u_7x_6 + u_7x_1 \\
p_{13} &= x_{13}x_7 + (u_8 - 2u_6)x_{13} + (-2u_8 + u_6)x_7 + u_8u_6 \\
p_{14} &= x_{14}x_8 + x_{14}x_2 - 2u_7x_{14} - 2x_8x_2 + u_7x_8 + u_7x_2
\end{aligned}
$$

3.8 Triangulation, step 8

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7. No reduction needed.

The triangular system has not been changed.

3.9 Triangulation, step 9

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 2.

Minimal degrees: 1 polynomial(s) with degree 1 and 1 polynomial(s) with degree 2.

Polynomial with linear degree: Removing variable x_6 from all other polynomials by reducing them with polynomial p_5 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_8u_2 + u_5)x_2 + (u_8^2u_1 + u_8u_4) \\
p_3 &= u_6x_4 - u_7x_3 \\
p_4 &= u_3x_4^2 + u_2x_4x_3 + u_5x_4 + u_1x_3^2 + u_4x_3 \\
p_5 &= u_3x_5^2x_1^2 + (-2u_7u_3 - u_6u_2)x_5^2x_1 +
\end{aligned}
$$

$$
\begin{aligned}
& (u_7^2u_3 + u_7u_6u_2 + u_6^2u_1)x_5^2 - 2u_6u_3x_5x_1^2 + \\
& (2u_7u_6u_3 + u_6^2u_2 - u_6u_5)x_5x_1 + \\
& (u_7u_6u_5 + u_6^2u_4)x_5 + u_6^2u_3x_1^2 + u_6^2u_5x_1 \\
p_6 &= u_6x_6 + x_5x_1 - u_7x_5 - u_6x_1 \\
p_7 &= u_3x_7^2x_2^2 + (u_8u_2 - 2u_7u_3 - u_6u_2)x_7^2x_2 + \\
& (u_8^2u_1 - u_8u_7u_2 - 2u_8u_6u_1 + u_7^2u_3 + u_7u_6u_2 + u_6^2u_1) \\
& x_7^2 \\
& -2u_6u_3x_7x_2^2 + \\
& (2u_8u_7u_3 - u_8u_6u_2 + u_8u_5 + 2u_7u_6u_3 + u_6^2u_2 - u_6u_5) \\
& x_7x_2 \\
& + \\
& (u_8^2u_7u_2 + u_8^2u_4 - 2u_8u_7^2u_3 - u_8u_7u_6u_2 \\
& -u_8u_7u_5 - 2u_8u_6u_4 + u_7u_6u_5 + u_6^2u_4) \\
& x_7 \\
& +u_6^2u_3x_2^2 + (-2u_8u_7u_6u_3 - u_8u_6u_5 + u_6^2u_5)x_2 + \\
& (u_8^2u_7^2u_3 + u_8^2u_7u_5 - u_8u_7u_6u_5) \\
p_8 &= (u_8 - u_6)x_8 - x_7x_2 + u_7x_7 + u_6x_2 - u_8u_7 \\
p_9 &= x_9x_3 - 2u_6x_9 + u_6x_3 \\
p_{10} &= x_{10}x_4 - 2u_7x_{10} + u_7x_4 \\
p_{11} &= x_{11}x_5 - 2u_6x_{11} + u_6x_5 \\
p_{12} &= x_{12}x_6 + x_{12}x_1 - 2u_7x_{12} - 2x_6x_1 + u_7x_6 + u_7x_1 \\
p_{13} &= x_{13}x_7 + (u_8 - 2u_6)x_{13} + (-2u_8 + u_6)x_7 + u_8u_6 \\
p_{14} &= x_{14}x_8 + x_{14}x_2 - 2u_7x_{14} - 2x_8x_2 + u_7x_8 + u_7x_2
\end{aligned}
$$

3.10 Triangulation, step 10

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

3.11 Triangulation, step 11

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 2.

Minimal degrees: 1 polynomial(s) with degree 1 and 1 polynomial(s) with degree 2.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_3 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_8u_2 + u_5)x_2 + (u_8^2u_1 + u_8u_4) \\
p_3 &= (u_7^2u_3 + u_7u_6u_2 + u_6^2u_1)x_3^2 + \\
&\quad (u_7u_6u_5 + u_6^2u_4)x_3 \\
p_4 &= u_6x_4 - u_7x_3 \\
p_5 &= u_3x_5^2x_1^2 + (-2u_7u_3 - u_6u_2)x_5^2x_1 + \\
&\quad (u_7^2u_3 + u_7u_6u_2 + u_6^2u_1)x_5^2 - 2u_6u_3x_5x_1^2 + \\
&\quad (2u_7u_6u_3 + u_6^2u_2 - u_6u_5)x_5x_1 + \\
&\quad (u_7u_6u_5 + u_6^2u_4)x_5 + u_6^2u_3x_1^2 + u_6^2u_5x_1 \\
p_6 &= u_6x_6 + x_5x_1 - u_7x_5 - u_6x_1 \\
p_7 &= u_3x_7^2x_2^2 + (u_8u_2 - 2u_7u_3 - u_6u_2)x_7^2x_2 + \\
&\quad (u_8^2u_1 - u_8u_7u_2 - 2u_8u_6u_1 + u_7^2u_3 + u_7u_6u_2 + u_6^2u_1) \\
&\quad x_7^2 \\
&\quad -2u_6u_3x_7x_2^2 + \\
&\quad (2u_8u_7u_3 - u_8u_6u_2 + u_8u_5 + 2u_7u_6u_3 + u_6^2u_2 - u_6u_5) \\
&\quad x_7x_2 \\
&\quad + \\
&\quad (u_8^2u_7u_2 + u_8^2u_4 - 2u_8u_7^2u_3 - u_8u_7u_6u_2 \\
&\quad -u_8u_7u_5 - 2u_8u_6u_4 + u_7u_6u_5 + u_6^2u_4) \\
&\quad x_7 \\
&\quad +u_6^2u_3x_2^2 + (-2u_8u_7u_6u_3 - u_8u_6u_5 + u_6^2u_5)x_2 + \\
&\quad (u_8^2u_7^2u_3 + u_8^2u_7u_5 - u_8u_7u_6u_5) \\
p_8 &= (u_8 - u_6)x_8 - x_7x_2 + u_7x_7 + u_6x_2 - u_8u_7 \\
p_9 &= x_9x_3 - 2u_6x_9 + u_6x_3 \\
p_{10} &= x_{10}x_4 - 2u_7x_{10} + u_7x_4 \\
p_{11} &= x_{11}x_5 - 2u_6x_{11} + u_6x_5 \\
p_{12} &= x_{12}x_6 + x_{12}x_1 - 2u_7x_{12} - 2x_6x_1 + u_7x_6 + u_7x_1 \\
p_{13} &= x_{13}x_7 + (u_8 - 2u_6)x_{13} + (-2u_8 + u_6)x_7 + u_8u_6 \\
p_{14} &= x_{14}x_8 + x_{14}x_2 - 2u_7x_{14} - 2x_8x_2 + u_7x_8 + u_7x_2
\end{aligned}
$$

3.12 Triangulation, step 12

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.13 Triangulation, step 13

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.14 Triangulation, step 14

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_8u_2 + u_5)x_2 + (u_8^2u_1 + u_8u_4) \\
p_3 &= (u_7^2u_3 + u_7u_6u_2 + u_6^2u_1)x_3^2 + \\
&\quad (u_7u_6u_5 + u_6^2u_4)x_3 \\
p_4 &= u_6x_4 - u_7x_3 \\
p_5 &= u_3x_5^2x_1^2 + (-2u_7u_3 - u_6u_2)x_5^2x_1 + \\
&\quad (u_7^2u_3 + u_7u_6u_2 + u_6^2u_1)x_5^2 - 2u_6u_3x_5x_1^2 + \\
&\quad (2u_7u_6u_3 + u_6^2u_2 - u_6u_5)x_5x_1 + \\
&\quad (u_7u_6u_5 + u_6^2u_4)x_5 + u_6^2u_3x_1^2 + u_6^2u_5x_1 \\
p_6 &= u_6x_6 + x_5x_1 - u_7x_5 - u_6x_1 \\
p_7 &= u_3x_7^2x_2^2 + (u_8u_2 - 2u_7u_3 - u_6u_2)x_7^2x_2 + \\
&\quad (u_8^2u_1 - u_8u_7u_2 - 2u_8u_6u_1 + u_7^2u_3 + u_7u_6u_2 + u_6^2u_1) \\
&\quad x_7^2 \\
&\quad -2u_6u_3x_7x_2^2 + \\
&\quad (2u_8u_7u_3 - u_8u_6u_2 + u_8u_5 + 2u_7u_6u_3 + u_6^2u_2 - u_6u_5) \\
&\quad x_7x_2 \\
&\quad + \\
&\quad (u_8^2u_7u_2 + u_8^2u_4 - 2u_8u_7^2u_3 - u_8u_7u_6u_2 \\
&\quad -u_8u_7u_5 - 2u_8u_6u_4 + u_7u_6u_5 + u_6^2u_4)
\end{aligned}
$$

$$
\begin{aligned}
& x_7 \\
& +u_6^2u_3x_2^2 + (-2u_8u_7u_6u_3 - u_8u_6u_5 + u_6^2u_5)x_2 + \\
& (u_8^2u_7^2u_3 + u_8^2u_7u_5 - u_8u_7u_6u_5) \\
p_8 &= (u_8 - u_6)x_8 - x_7x_2 + u_7x_7 + u_6x_2 - u_8u_7 \\
p_9 &= x_9x_3 - 2u_6x_9 + u_6x_3 \\
p_{10} &= x_{10}x_4 - 2u_7x_{10} + u_7x_4 \\
p_{11} &= x_{11}x_5 - 2u_6x_{11} + u_6x_5 \\
p_{12} &= x_{12}x_6 + x_{12}x_1 - 2u_7x_{12} - 2x_6x_1 + u_7x_6 + u_7x_1 \\
p_{13} &= x_{13}x_7 + (u_8 - 2u_6)x_{13} + (-2u_8 + u_6)x_7 + u_8u_6 \\
p_{14} &= x_{14}x_8 + x_{14}x_2 - 2u_7x_{14} - 2x_8x_2 + u_7x_8 + u_7x_2
\end{aligned}
$$

4 Final Remainder

4.1 Final remainder for conjecture Polar 002

Calculating final remainder of the conclusion:

$$g = x_{14}x_{11} - x_{14}x_9 - x_{13}x_{12} + x_{13}x_{10} + x_{12}x_9 - x_{11}x_{10}$$

with respect to the triangular system.

1. Pseudo remainder with p_{14} over variable x_{14}:

$$
\begin{aligned}
g = & -x_{13}x_{12}x_8 - x_{13}x_{12}x_2 + 2u_7x_{13}x_{12} + x_{13}x_{10}x_8 + x_{13}x_{10}x_2 \\
& -2u_7x_{13}x_{10} + x_{12}x_9x_8 + x_{12}x_9x_2 - 2u_7x_{12}x_9 - x_{11}x_{10}x_8 \\
& -x_{11}x_{10}x_2 + 2u_7x_{11}x_{10} + 2x_{11}x_8x_2 - u_7x_{11}x_8 - u_7x_{11}x_2 \\
& -2x_9x_8x_2 + u_7x_9x_8 + u_7x_9x_2
\end{aligned}
$$

2. Pseudo remainder with p_{13} over variable x_{13}:

$$
\begin{aligned}
g = & \; x_{12}x_9x_8x_7 + (u_8 - 2u_6)x_{12}x_9x_8 + x_{12}x_9x_7x_2 \\
& -2u_7x_{12}x_9x_7 + (u_8 - 2u_6)x_{12}x_9x_2 + (-2u_8u_7 + 4u_7u_6)x_{12}x_9 + \\
& (-2u_8 + u_6)x_{12}x_8x_7 + u_8u_6x_{12}x_8 + (-2u_8 + u_6)x_{12}x_7x_2 + \\
& (4u_8u_7 - 2u_7u_6)x_{12}x_7 + u_8u_6x_{12}x_2 - 2u_8u_7u_6x_{12} \\
& -x_{11}x_{10}x_8x_7 + (-u_8 + 2u_6)x_{11}x_{10}x_8 - x_{11}x_{10}x_7x_2 + \\
& 2u_7x_{11}x_{10}x_7 + (-u_8 + 2u_6)x_{11}x_{10}x_2 + \\
& (2u_8u_7 - 4u_7u_6)x_{11}x_{10} + 2x_{11}x_8x_7x_2 - u_7x_{11}x_8x_7 + \\
& (2u_8 - 4u_6)x_{11}x_8x_2 + (-u_8u_7 + 2u_7u_6)x_{11}x_8 - u_7x_{11}x_7x_2 + \\
& (-u_8u_7 + 2u_7u_6)x_{11}x_2 + (2u_8 - u_6)x_{10}x_8x_7 - u_8u_6x_{10}x_8 + \\
& (2u_8 - u_6)x_{10}x_7x_2 + (-4u_8u_7 + 2u_7u_6)x_{10}x_7 - u_8u_6x_{10}x_2 + \\
& 2u_8u_7u_6x_{10} - 2x_9x_8x_7x_2 + u_7x_9x_8x_7 + \\
& (-2u_8 + 4u_6)x_9x_8x_2 + (u_8u_7 - 2u_7u_6)x_9x_8 + u_7x_9x_7x_2 + \\
& (u_8u_7 - 2u_7u_6)x_9x_2
\end{aligned}
$$

3. Pseudo remainder with p_{12} over variable x_{12}:

   Polynomial too big for output (text size is 3341 *characters, number of terms is* 92*)*

4. Pseudo remainder with p_{11} over variable x_{11}:

   Polynomial too big for output (text size is 4992 *characters, number of terms is* 122*)*

5. Pseudo remainder with p_{10} over variable x_{10}:

   Polynomial too big for output (text size is 7484 *characters, number of terms is* 160*)*

6. Pseudo remainder with p_9 over variable x_9:

   Polynomial too big for output (text size is 7702 *characters, number of terms is* 160*)*

7. Pseudo remainder with p_8 over variable x_8:

   Polynomial too big for output (text size is 12985 *characters, number of terms is* 188*)*

8. Pseudo remainder with p_7 over variable x_7:

   Polynomial too big for output (text size is 77208 *characters, number of terms is* 232*)*

9. Pseudo remainder with p_6 over variable x_6:

   Polynomial too big for output (number of terms is 296*)*

10. Pseudo remainder with p_5 over variable x_5:

    Polynomial too big for output (number of terms is 368*)*

11. Pseudo remainder with p_4 over variable x_4:

    Polynomial too big for output (number of terms is 276*)*

12. Pseudo remainder with p_3 over variable x_3:

    Polynomial too big for output (text size is 470541 *characters, number of terms is* 170*)*

13. Pseudo remainder with p_2 over variable x_2:

    Polynomial too big for output (text size is greater than 2000 *characters, number of terms is* 72*)*

14. Pseudo remainder with p_1 over variable x_1:

    Polynomial too big for output (text size is greater than 2000 *characters, number of terms is* 24*)*

5 Prover results

Status: Theorem can't be neither proved nor disproved.

Space Complexity: The biggest polynomial obtained during prover execution contains 368 terms.

Time Complexity: Time spent by the prover is 3.34 seconds.