

OpenGeoProver Output for conjecture “Chou 040”

Wu’s method used

February 18, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Free point D
- Line vA through two points A and D
- Line uB through two points B and D
- Angle ray AC of angle with vertex A and point B from first ray, which is three times greater than angle BAD
- Angle ray BC of angle with vertex B and point A from first ray, which is three times greater than angle ABD
- Intersection point C of point sets AC and BC
- Angle ray uA of angle with vertex A and point D from first ray, which is congruent to angle BAD
- Angle alpha of 60 degrees
- Angle ray vC of angle with vertex C and point A from first ray, which is third angle to 60 degrees for angles BAD and DBA
- Intersection point F of point sets vC and uA
- Angle ray vB of angle with vertex B and point D from first ray, which is congruent to angle ABD
- Angle ray uC of angle with vertex C and point B from first ray, which is congruent to angle FCA
- Intersection point E of point sets vB and uC
- Intersection point U of point sets vB and uA
- Intersection point V of point sets vC and uB

- Intersection point W of point sets vA and uC
- Line UD through two points U and D
- Line VE through two points V and E
- Line WF through two points W and F

Theorem statement:

- Lines UD, VE, WF are concurrent

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point A:

- Point A has been assigned following coordinates: $(0, 0)$

2.2 Transformation of point B:

- Point B has been assigned following coordinates: $(0, u_1)$

2.3 Transformation of point D:

- Point D has been assigned following coordinates: (u_2, u_3)

2.4 Transformation of point C:

- Point C has been assigned following coordinates: (x_1, x_2)
- Polynomial that point C has to satisfy is:

$$p = (u_3^2 u_2 - 0.333333 u_2^3) x_2 + (-0.333333 u_3^3 + u_3 u_2^2) x_1$$

- Processing of polynomial

$$p = (u_3^2 u_2 - 0.333333 u_2^3) x_2 + (-0.333333 u_3^3 + u_3 u_2^2) x_1$$

Info: Polynomial

$$p = (u_3^2 u_2 - 0.333333 u_2^3) x_2 + (-0.333333 u_3^3 + u_3 u_2^2) x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point C has to satisfy is:

$$\begin{aligned}
p = & (u_3^2 u_2 - 2u_3 u_2 u_1 - 0.333333 u_2^3 + u_2 u_1^2) x_2 + \\
& (-0.333333 u_3^3 + u_3^2 u_1 + u_3 u_2^2 - u_3 u_1^2 - u_2^2 u_1 + \\
& 0.333333 u_1^3) \\
& x_1 \\
& + (-u_3^2 u_2 u_1 + 2u_3 u_2 u_1^2 + 0.333333 u_2^3 u_1 - u_2 u_1^3)
\end{aligned}$$

- Processing of polynomial

$$\begin{aligned}
p = & (u_3^2 u_2 - 2u_3 u_2 u_1 - 0.333333 u_2^3 + u_2 u_1^2) x_2 + \\
& (-0.333333 u_3^3 + u_3^2 u_1 + u_3 u_2^2 - u_3 u_1^2 - u_2^2 u_1 + \\
& 0.333333 u_1^3) \\
& x_1 \\
& + (-u_3^2 u_2 u_1 + 2u_3 u_2 u_1^2 + 0.333333 u_2^3 u_1 - u_2 u_1^3)
\end{aligned}$$

Info: Polynomial

$$\begin{aligned}
p = & (u_3^2 u_2 - 2u_3 u_2 u_1 - 0.333333 u_2^3 + u_2 u_1^2) x_2 + \\
& (-0.333333 u_3^3 + u_3^2 u_1 + u_3 u_2^2 - u_3 u_1^2 - u_2^2 u_1 + \\
& 0.333333 u_1^3) \\
& x_1 \\
& + (-u_3^2 u_2 u_1 + 2u_3 u_2 u_1^2 + 0.333333 u_2^3 u_1 - u_2 u_1^3)
\end{aligned}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of angle alpha of 60 degrees:

- Parametric pointPoint Aalpha has been assigned following coordinates: $(x_3, 0)$
- Polynomial

$$p = x_3^2 - 3$$

added to system of hypotheses.

2.6 Transformation of point F:

- Point F has been assigned following coordinates: (x_4, x_5)
- Polynomial that point F has to satisfy is:

$$\begin{aligned}
p = & (u_3^2 - u_3 u_1 + u_2^2) x_5 x_3 x_2 - u_2 u_1 x_5 x_3 x_1 \\
& - u_2 u_1 x_5 x_2 + (-u_3^2 + u_3 u_1 - u_2^2) x_5 x_1 + u_2 u_1 x_4 x_3 x_2 + \\
& (u_3^2 - u_3 u_1 + u_2^2) x_4 x_3 x_1 + (u_3^2 - u_3 u_1 + u_2^2) x_4 x_2 \\
& - u_2 u_1 x_4 x_1 + (-u_3^2 + u_3 u_1 - u_2^2) x_3 x_2^2 + \\
& (-u_3^2 + u_3 u_1 - u_2^2) x_3 x_1^2 + u_2 u_1 x_2^2 + u_2 u_1 x_1^2
\end{aligned}$$

- Processing of polynomial

$$\begin{aligned}
p = & (u_3^2 - u_3u_1 + u_2^2)x_5x_3x_2 - u_2u_1x_5x_3x_1 \\
& - u_2u_1x_5x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_5x_1 + u_2u_1x_4x_3x_2 + \\
& (u_3^2 - u_3u_1 + u_2^2)x_4x_3x_1 + (u_3^2 - u_3u_1 + u_2^2)x_4x_2 \\
& - u_2u_1x_4x_1 + (-u_3^2 + u_3u_1 - u_2^2)x_3x_2^2 + \\
& (-u_3^2 + u_3u_1 - u_2^2)x_3x_1^2 + u_2u_1x_2^2 + u_2u_1x_1^2
\end{aligned}$$

Info: Polynomial

$$\begin{aligned}
p = & (u_3^2 - u_3u_1 + u_2^2)x_5x_3x_2 - u_2u_1x_5x_3x_1 \\
& - u_2u_1x_5x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_5x_1 + u_2u_1x_4x_3x_2 + \\
& (u_3^2 - u_3u_1 + u_2^2)x_4x_3x_1 + (u_3^2 - u_3u_1 + u_2^2)x_4x_2 \\
& - u_2u_1x_4x_1 + (-u_3^2 + u_3u_1 - u_2^2)x_3x_2^2 + \\
& (-u_3^2 + u_3u_1 - u_2^2)x_3x_1^2 + u_2u_1x_2^2 + u_2u_1x_1^2
\end{aligned}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point F has to satisfy is:

$$p = u_3u_2x_5 + (-0.5u_3^2 + 0.5u_2^2)x_4$$

- Processing of polynomial

$$p = u_3u_2x_5 + (-0.5u_3^2 + 0.5u_2^2)x_4$$

Info: Polynomial

$$p = u_3u_2x_5 + (-0.5u_3^2 + 0.5u_2^2)x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point E:

- Point E has been assigned following coordinates: (x_6, x_7)
- Polynomial that point E has to satisfy is:

$$\begin{aligned}
p = & (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 + \\
& (-u_3u_2u_1 + u_2u_1^2)
\end{aligned}$$

- Processing of polynomial

$$\begin{aligned}
p = & (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 + \\
& (-u_3u_2u_1 + u_2u_1^2)
\end{aligned}$$

Info: Polynomial

$$p = (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 + (-u_3u_2u_1 + u_2u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point E has to satisfy is:

$$p = x_7x_5x_2x_1 - 0.5u_1x_7x_5x_1 - 0.5x_7x_4x_2^2 + 0.5u_1x_7x_4x_2 + 0.5x_7x_4x_1^2 - 0.5x_7x_2^2x_1 - 0.5x_7x_1^3 - 0.5x_6x_5x_2^2 + 0.5u_1x_6x_5x_2 + 0.5x_6x_5x_1^2 - x_6x_4x_2x_1 + 0.5u_1x_6x_4x_1 + 0.5x_6x_2^3 - 0.5u_1x_6x_2^2 + 0.5x_6x_2x_1^2 - 0.5u_1x_6x_1^2 - 0.5x_5x_2^2x_1 - 0.5x_5x_1^3 + 0.5x_4x_2^3 - 0.5u_1x_4x_2^2 + 0.5x_4x_2x_1^2 - 0.5u_1x_4x_1^2 + 0.5u_1x_2^2x_1 + 0.5u_1x_1^3$$

- Processing of polynomial

$$p = x_7x_5x_2x_1 - 0.5u_1x_7x_5x_1 - 0.5x_7x_4x_2^2 + 0.5u_1x_7x_4x_2 + 0.5x_7x_4x_1^2 - 0.5x_7x_2^2x_1 - 0.5x_7x_1^3 - 0.5x_6x_5x_2^2 + 0.5u_1x_6x_5x_2 + 0.5x_6x_5x_1^2 - x_6x_4x_2x_1 + 0.5u_1x_6x_4x_1 + 0.5x_6x_2^3 - 0.5u_1x_6x_2^2 + 0.5x_6x_2x_1^2 - 0.5u_1x_6x_1^2 - 0.5x_5x_2^2x_1 - 0.5x_5x_1^3 + 0.5x_4x_2^3 - 0.5u_1x_4x_2^2 + 0.5x_4x_2x_1^2 - 0.5u_1x_4x_1^2 + 0.5u_1x_2^2x_1 + 0.5u_1x_1^3$$

Info: Polynomial

$$p = x_7x_5x_2x_1 - 0.5u_1x_7x_5x_1 - 0.5x_7x_4x_2^2 + 0.5u_1x_7x_4x_2 + 0.5x_7x_4x_1^2 - 0.5x_7x_2^2x_1 - 0.5x_7x_1^3 - 0.5x_6x_5x_2^2 + 0.5u_1x_6x_5x_2 + 0.5x_6x_5x_1^2 - x_6x_4x_2x_1 + 0.5u_1x_6x_4x_1 + 0.5x_6x_2^3 - 0.5u_1x_6x_2^2 + 0.5x_6x_2x_1^2 - 0.5u_1x_6x_1^2 - 0.5x_5x_2^2x_1 - 0.5x_5x_1^3 + 0.5x_4x_2^3 - 0.5u_1x_4x_2^2 + 0.5x_4x_2x_1^2 - 0.5u_1x_4x_1^2 + 0.5u_1x_2^2x_1 + 0.5u_1x_1^3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.8 Transformation of point U:

- Point U has been assigned following coordinates: (x_8, x_9)
- Polynomial that point U has to satisfy is:

$$p = (u_3u_2 - u_2u_1)x_9 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_8 + (-u_3u_2u_1 + u_2u_1^2)$$

- Processing of polynomial

$$p = (u_3u_2 - u_2u_1)x_9 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_8 + (-u_3u_2u_1 + u_2u_1^2)$$

Info: Polynomial

$$p = (u_3u_2 - u_2u_1)x_9 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_8 + (-u_3u_2u_1 + u_2u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point U has to satisfy is:

$$p = u_3u_2x_9 + (-0.5u_3^2 + 0.5u_2^2)x_8$$

- Processing of polynomial

$$p = u_3u_2x_9 + (-0.5u_3^2 + 0.5u_2^2)x_8$$

Info: Polynomial

$$p = u_3u_2x_9 + (-0.5u_3^2 + 0.5u_2^2)x_8$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.9 Transformation of point V:

- Point V has been assigned following coordinates: (x_{10}, x_{11})
- Polynomial that point V has to satisfy is:

$$p = x_{11}x_4 - x_{11}x_1 - x_{10}x_5 + x_{10}x_2 + x_5x_1 - x_4x_2$$

- Processing of polynomial

$$p = x_{11}x_4 - x_{11}x_1 - x_{10}x_5 + x_{10}x_2 + x_5x_1 - x_4x_2$$

Info: Polynomial

$$p = x_{11}x_4 - x_{11}x_1 - x_{10}x_5 + x_{10}x_2 + x_5x_1 - x_4x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point V has to satisfy is:

$$p = u_2x_{11} + (-u_3 + u_1)x_{10} - u_2u_1$$

- Processing of polynomial

$$p = u_2x_{11} + (-u_3 + u_1)x_{10} - u_2u_1$$

Info: Polynomial

$$p = u_2x_{11} + (-u_3 + u_1)x_{10} - u_2u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.10 Transformation of point W:

- Point W has been assigned following coordinates: (x_{12}, x_{13})
- Polynomial that point W has to satisfy is:

$$p = u_2x_{13} - u_3x_{12}$$

- Processing of polynomial

$$p = u_2x_{13} - u_3x_{12}$$

Info: Polynomial

$$p = u_2x_{13} - u_3x_{12}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point W has to satisfy is:

$$p = x_{13}x_6 - x_{13}x_1 - x_{12}x_7 + x_{12}x_2 + x_7x_1 - x_6x_2$$

- Processing of polynomial

$$p = x_{13}x_6 - x_{13}x_1 - x_{12}x_7 + x_{12}x_2 + x_7x_1 - x_6x_2$$

Info: Polynomial

$$p = x_{13}x_6 - x_{13}x_1 - x_{12}x_7 + x_{12}x_2 + x_7x_1 - x_6x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

2.11 Transformation of point intersectPoint-UD.VE:

- Point intersectPoint-UD.VE has been assigned following coordinates: (x_{14}, x_{15})
- Polynomial that point intersectPoint-UD.VE has to satisfy is:

$$p = x_{15}x_8 - u_2x_{15} - x_{14}x_9 + u_3x_{14} + u_2x_9 - u_3x_8$$

- Processing of polynomial

$$p = x_{15}x_8 - u_2x_{15} - x_{14}x_9 + u_3x_{14} + u_2x_9 - u_3x_8$$

Info: Polynomial

$$p = x_{15}x_8 - u_2x_{15} - x_{14}x_9 + u_3x_{14} + u_2x_9 - u_3x_8$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point intersectPoint-UD.VE has to satisfy is:

$$p = x_{15}x_{10} - x_{15}x_6 - x_{14}x_{11} + x_{14}x_7 + x_{11}x_6 - x_{10}x_7$$

- Processing of polynomial

$$p = x_{15}x_{10} - x_{15}x_6 - x_{14}x_{11} + x_{14}x_7 + x_{11}x_6 - x_{10}x_7$$

Info: Polynomial

$$p = x_{15}x_{10} - x_{15}x_6 - x_{14}x_{11} + x_{14}x_7 + x_{11}x_6 - x_{10}x_7$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial for theorem statement:

$$p = x_{15}x_{12} - x_{15}x_4 - x_{14}x_{13} + x_{14}x_5 + x_{13}x_4 - x_{12}x_5$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.78 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= (u_3^2u_2 - 0.333333u_2^3)x_2 + (-0.333333u_3^3 + u_3u_2^2)x_1 \\
p_2 &= (u_3^2u_2 - 2u_3u_2u_1 - 0.333333u_2^3 + u_2u_1^2)x_2 + \\
&\quad (-0.333333u_3^3 + u_3^2u_1 + u_3u_2^2 - u_3u_1^2 - u_2^2u_1 + \\
&\quad 0.333333u_1^3) \\
&\quad x_1 \\
&\quad + (-u_3^2u_2u_1 + 2u_3u_2u_1^2 + 0.333333u_2^3u_1 - u_2u_1^3) \\
p_3 &= x_3^2 - 3 \\
p_4 &= (u_3^2 - u_3u_1 + u_2^2)x_5x_3x_2 - u_2u_1x_5x_3x_1 \\
&\quad - u_2u_1x_5x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_5x_1 + u_2u_1x_4x_3x_2 + \\
&\quad (u_3^2 - u_3u_1 + u_2^2)x_4x_3x_1 + (u_3^2 - u_3u_1 + u_2^2)x_4x_2 \\
&\quad - u_2u_1x_4x_1 + (-u_3^2 + u_3u_1 - u_2^2)x_3x_2^2 + \\
&\quad (-u_3^2 + u_3u_1 - u_2^2)x_3x_1^2 + u_2u_1x_2^2 + u_2u_1x_1^2 \\
p_5 &= u_3u_2x_5 + (-0.5u_3^2 + 0.5u_2^2)x_4 \\
p_6 &= (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 + \\
&\quad (-u_3u_2u_1 + u_2u_1^2)
\end{aligned}$$

$$\begin{aligned}
p_7 &= x_7x_5x_2x_1 - 0.5u_1x_7x_5x_1 - 0.5x_7x_4x_2^2 + 0.5u_1x_7x_4x_2 + \\
&\quad 0.5x_7x_4x_1^2 - 0.5x_7x_2^2x_1 - 0.5x_7x_1^3 - 0.5x_6x_5x_2^2 + \\
&\quad 0.5u_1x_6x_5x_2 + 0.5x_6x_5x_1^2 - x_6x_4x_2x_1 + 0.5u_1x_6x_4x_1 + \\
&\quad 0.5x_6x_2^3 - 0.5u_1x_6x_2^2 + 0.5x_6x_2x_1^2 - 0.5u_1x_6x_1^2 \\
&\quad - 0.5x_5x_2^2x_1 - 0.5x_5x_1^3 + 0.5x_4x_2^3 - 0.5u_1x_4x_2^2 + \\
&\quad 0.5x_4x_2x_1^2 - 0.5u_1x_4x_1^2 + 0.5u_1x_2^2x_1 + 0.5u_1x_1^3 \\
p_8 &= (u_3u_2 - u_2u_1)x_9 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_8 + \\
&\quad (-u_3u_2u_1 + u_2u_1^2) \\
p_9 &= u_3u_2x_9 + (-0.5u_3^2 + 0.5u_2^2)x_8 \\
p_{10} &= x_{11}x_4 - x_{11}x_1 - x_{10}x_5 + x_{10}x_2 + x_5x_1 - x_4x_2 \\
p_{11} &= u_2x_{11} + (-u_3 + u_1)x_{10} - u_2u_1 \\
p_{12} &= u_2x_{13} - u_3x_{12} \\
p_{13} &= x_{13}x_6 - x_{13}x_1 - x_{12}x_7 + x_{12}x_2 + x_7x_1 - x_6x_2 \\
p_{14} &= x_{15}x_8 - u_2x_{15} - x_{14}x_9 + u_3x_{14} + u_2x_9 - u_3x_8 \\
p_{15} &= x_{15}x_{10} - x_{15}x_6 - x_{14}x_{11} + x_{14}x_7 + x_{11}x_6 - x_{10}x_7
\end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{15} from all other polynomials by reducing them with polynomial p_{14} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_3^2u_2 - 0.333333u_2^3)x_2 + (-0.333333u_3^3 + u_3u_2^2)x_1 \\
p_2 &= (u_3^2u_2 - 2u_3u_2u_1 - 0.333333u_2^3 + u_2u_1^2)x_2 + \\
&\quad (-0.333333u_3^3 + u_3^2u_1 + u_3u_2^2 - u_3u_1^2 - u_2^2u_1 + \\
&\quad 0.333333u_1^3) \\
&\quad x_1 \\
&\quad + (-u_3^2u_2u_1 + 2u_3u_2u_1^2 + 0.333333u_2^3u_1 - u_2u_1^3) \\
p_3 &= x_3^2 - 3 \\
p_4 &= (u_3^2 - u_3u_1 + u_2^2)x_5x_3x_2 - u_2u_1x_5x_3x_1 \\
&\quad - u_2u_1x_5x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_5x_1 + u_2u_1x_4x_3x_2 + \\
&\quad (u_3^2 - u_3u_1 + u_2^2)x_4x_3x_1 + (u_3^2 - u_3u_1 + u_2^2)x_4x_2 \\
&\quad - u_2u_1x_4x_1 + (-u_3^2 + u_3u_1 - u_2^2)x_3x_2^2 + \\
&\quad (-u_3^2 + u_3u_1 - u_2^2)x_3x_1^2 + u_2u_1x_2^2 + u_2u_1x_1^2 \\
p_5 &= u_3u_2x_5 + (-0.5u_3^2 + 0.5u_2^2)x_4 \\
p_6 &= (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 +
\end{aligned}$$

$$\begin{aligned}
& (-u_3u_2u_1 + u_2u_1^2) \\
p_7 &= x_7x_5x_2x_1 - 0.5u_1x_7x_5x_1 - 0.5x_7x_4x_2^2 + 0.5u_1x_7x_4x_2 + \\
& 0.5x_7x_4x_1^2 - 0.5x_7x_2^2x_1 - 0.5x_7x_1^3 - 0.5x_6x_5x_2^2 + \\
& 0.5u_1x_6x_5x_2 + 0.5x_6x_5x_1^2 - x_6x_4x_2x_1 + 0.5u_1x_6x_4x_1 + \\
& 0.5x_6x_2^3 - 0.5u_1x_6x_2^2 + 0.5x_6x_2x_1^2 - 0.5u_1x_6x_1^2 \\
& - 0.5x_5x_2^2x_1 - 0.5x_5x_1^3 + 0.5x_4x_2^3 - 0.5u_1x_4x_2^2 + \\
& 0.5x_4x_2x_1^2 - 0.5u_1x_4x_1^2 + 0.5u_1x_2^2x_1 + 0.5u_1x_1^3 \\
p_8 &= (u_3u_2 - u_2u_1)x_9 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_8 + \\
& (-u_3u_2u_1 + u_2u_1^2) \\
p_9 &= u_3u_2x_9 + (-0.5u_3^2 + 0.5u_2^2)x_8 \\
p_{10} &= x_{11}x_4 - x_{11}x_1 - x_{10}x_5 + x_{10}x_2 + x_5x_1 - x_4x_2 \\
p_{11} &= u_2x_{11} + (-u_3 + u_1)x_{10} - u_2u_1 \\
p_{12} &= u_2x_{13} - u_3x_{12} \\
p_{13} &= x_{13}x_6 - x_{13}x_1 - x_{12}x_7 + x_{12}x_2 + x_7x_1 - x_6x_2 \\
p_{14} &= -x_{14}x_{11}x_8 + u_2x_{14}x_{11} + x_{14}x_{10}x_9 - u_3x_{14}x_{10} - x_{14}x_9x_6 + \\
& x_{14}x_8x_7 - u_2x_{14}x_7 + u_3x_{14}x_6 + x_{11}x_8x_6 - u_2x_{11}x_6 \\
& - u_2x_{10}x_9 - x_{10}x_8x_7 + u_3x_{10}x_8 + u_2x_{10}x_7 + u_2x_9x_6 \\
& - u_3x_8x_6 \\
p_{15} &= x_{15}x_8 - u_2x_{15} - x_{14}x_9 + u_3x_{14} + u_2x_9 - u_3x_8
\end{aligned}$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14} . No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{13} from all other polynomials by reducing them with polynomial p_{12} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_3^2 u_2 - 0.333333 u_2^3) x_2 + (-0.333333 u_3^3 + u_3 u_2^2) x_1 \\
p_2 &= (u_3^2 u_2 - 2 u_3 u_2 u_1 - 0.333333 u_2^3 + u_2 u_1^2) x_2 + \\
&\quad (-0.333333 u_3^3 + u_3^2 u_1 + u_3 u_2^2 - u_3 u_1^2 - u_2^2 u_1 + \\
&\quad 0.333333 u_1^3) \\
&\quad x_1 \\
&\quad + (-u_3^2 u_2 u_1 + 2 u_3 u_2 u_1^2 + 0.333333 u_2^3 u_1 - u_2 u_1^3) \\
p_3 &= x_3^2 - 3 \\
p_4 &= (u_3^2 - u_3 u_1 + u_2^2) x_5 x_3 x_2 - u_2 u_1 x_5 x_3 x_1 \\
&\quad - u_2 u_1 x_5 x_2 + (-u_3^2 + u_3 u_1 - u_2^2) x_5 x_1 + u_2 u_1 x_4 x_3 x_2 + \\
&\quad (u_3^2 - u_3 u_1 + u_2^2) x_4 x_3 x_1 + (u_3^2 - u_3 u_1 + u_2^2) x_4 x_2 \\
&\quad - u_2 u_1 x_4 x_1 + (-u_3^2 + u_3 u_1 - u_2^2) x_3 x_2^2 + \\
&\quad (-u_3^2 + u_3 u_1 - u_2^2) x_3 x_1^2 + u_2 u_1 x_2^2 + u_2 u_1 x_1^2 \\
p_5 &= u_3 u_2 x_5 + (-0.5 u_3^2 + 0.5 u_2^2) x_4 \\
p_6 &= (u_3 u_2 - u_2 u_1) x_7 + (-0.5 u_3^2 + u_3 u_1 + 0.5 u_2^2 - 0.5 u_1^2) x_6 + \\
&\quad (-u_3 u_2 u_1 + u_2 u_1^2) \\
p_7 &= x_7 x_5 x_2 x_1 - 0.5 u_1 x_7 x_5 x_1 - 0.5 x_7 x_4 x_2^2 + 0.5 u_1 x_7 x_4 x_2 + \\
&\quad 0.5 x_7 x_4 x_1^2 - 0.5 x_7 x_2^2 x_1 - 0.5 x_7 x_1^3 - 0.5 x_6 x_5 x_2^2 + \\
&\quad 0.5 u_1 x_6 x_5 x_2 + 0.5 x_6 x_5 x_1^2 - x_6 x_4 x_2 x_1 + 0.5 u_1 x_6 x_4 x_1 + \\
&\quad 0.5 x_6 x_2^3 - 0.5 u_1 x_6 x_2^2 + 0.5 x_6 x_2 x_1^2 - 0.5 u_1 x_6 x_1^2 \\
&\quad - 0.5 x_5 x_2^2 x_1 - 0.5 x_5 x_1^3 + 0.5 x_4 x_2^3 - 0.5 u_1 x_4 x_2^2 + \\
&\quad 0.5 x_4 x_2 x_1^2 - 0.5 u_1 x_4 x_1^2 + 0.5 u_1 x_2^2 x_1 + 0.5 u_1 x_1^3 \\
p_8 &= (u_3 u_2 - u_2 u_1) x_9 + (-0.5 u_3^2 + u_3 u_1 + 0.5 u_2^2 - 0.5 u_1^2) x_8 + \\
&\quad (-u_3 u_2 u_1 + u_2 u_1^2) \\
p_9 &= u_3 u_2 x_9 + (-0.5 u_3^2 + 0.5 u_2^2) x_8 \\
p_{10} &= x_{11} x_4 - x_{11} x_1 - x_{10} x_5 + x_{10} x_2 + x_5 x_1 - x_4 x_2 \\
p_{11} &= u_2 x_{11} + (-u_3 + u_1) x_{10} - u_2 u_1 \\
p_{12} &= -u_2 x_{12} x_7 + u_3 x_{12} x_6 + u_2 x_{12} x_2 - u_3 x_{12} x_1 + u_2 x_7 x_1 \\
&\quad - u_2 x_6 x_2 \\
p_{13} &= u_2 x_{13} - u_3 x_{12} \\
p_{14} &= -x_{14} x_{11} x_8 + u_2 x_{14} x_{11} + x_{14} x_{10} x_9 - u_3 x_{14} x_{10} - x_{14} x_9 x_6 + \\
&\quad x_{14} x_8 x_7 - u_2 x_{14} x_7 + u_3 x_{14} x_6 + x_{11} x_8 x_6 - u_2 x_{11} x_6 \\
&\quad - u_2 x_{10} x_9 - x_{10} x_8 x_7 + u_3 x_{10} x_8 + u_2 x_{10} x_7 + u_2 x_9 x_6 \\
&\quad - u_3 x_8 x_6 \\
p_{15} &= x_{15} x_8 - u_2 x_{15} - x_{14} x_9 + u_3 x_{14} + u_2 x_9 - u_3 x_8
\end{aligned}$$

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{12} . No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{11} from all other polynomials by reducing them with polynomial p_{10} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_3^2 u_2 - 0.333333 u_2^3) x_2 + (-0.333333 u_3^3 + u_3 u_2^2) x_1 \\
p_2 &= (u_3^2 u_2 - 2 u_3 u_2 u_1 - 0.333333 u_2^3 + u_2 u_1^2) x_2 + \\
&\quad (-0.333333 u_3^3 + u_3^2 u_1 + u_3 u_2^2 - u_3 u_1^2 - u_2^2 u_1 + \\
&\quad 0.333333 u_1^3) \\
&\quad x_1 \\
&\quad + (-u_3^2 u_2 u_1 + 2 u_3 u_2 u_1^2 + 0.333333 u_2^3 u_1 - u_2 u_1^3) \\
p_3 &= x_3^2 - 3 \\
p_4 &= (u_3^2 - u_3 u_1 + u_2^2) x_5 x_3 x_2 - u_2 u_1 x_5 x_3 x_1 \\
&\quad - u_2 u_1 x_5 x_2 + (-u_3^2 + u_3 u_1 - u_2^2) x_5 x_1 + u_2 u_1 x_4 x_3 x_2 + \\
&\quad (u_3^2 - u_3 u_1 + u_2^2) x_4 x_3 x_1 + (u_3^2 - u_3 u_1 + u_2^2) x_4 x_2 \\
&\quad - u_2 u_1 x_4 x_1 + (-u_3^2 + u_3 u_1 - u_2^2) x_3 x_2^2 + \\
&\quad (-u_3^2 + u_3 u_1 - u_2^2) x_3 x_1^2 + u_2 u_1 x_2^2 + u_2 u_1 x_1^2 \\
p_5 &= u_3 u_2 x_5 + (-0.5 u_3^2 + 0.5 u_2^2) x_4 \\
p_6 &= (u_3 u_2 - u_2 u_1) x_7 + (-0.5 u_3^2 + u_3 u_1 + 0.5 u_2^2 - 0.5 u_1^2) x_6 + \\
&\quad (-u_3 u_2 u_1 + u_2 u_1^2) \\
p_7 &= x_7 x_5 x_2 x_1 - 0.5 u_1 x_7 x_5 x_1 - 0.5 x_7 x_4 x_2^2 + 0.5 u_1 x_7 x_4 x_2 + \\
&\quad 0.5 x_7 x_4 x_1^2 - 0.5 x_7 x_2^2 x_1 - 0.5 x_7 x_1^3 - 0.5 x_6 x_5 x_2^2 + \\
&\quad 0.5 u_1 x_6 x_5 x_2 + 0.5 x_6 x_5 x_1^2 - x_6 x_4 x_2 x_1 + 0.5 u_1 x_6 x_4 x_1 + \\
&\quad 0.5 x_6 x_2^2 - 0.5 u_1 x_6 x_2^2 + 0.5 x_6 x_2 x_1^2 - 0.5 u_1 x_6 x_1^2 \\
&\quad - 0.5 x_5 x_2^2 x_1 - 0.5 x_5 x_1^3 + 0.5 x_4 x_2^3 - 0.5 u_1 x_4 x_2^2 + \\
&\quad 0.5 x_4 x_2 x_1^2 - 0.5 u_1 x_4 x_1^2 + 0.5 u_1 x_2^2 x_1 + 0.5 u_1 x_1^3 \\
p_8 &= (u_3 u_2 - u_2 u_1) x_9 + (-0.5 u_3^2 + u_3 u_1 + 0.5 u_2^2 - 0.5 u_1^2) x_8 + \\
&\quad (-u_3 u_2 u_1 + u_2 u_1^2) \\
p_9 &= u_3 u_2 x_9 + (-0.5 u_3^2 + 0.5 u_2^2) x_8 \\
p_{10} &= u_2 x_{10} x_5 + (-u_3 + u_1) x_{10} x_4 - u_2 x_{10} x_2 + (u_3 - u_1) x_{10} x_1 \\
&\quad - u_2 x_5 x_1 + u_2 x_4 x_2 - u_2 u_1 x_4 + u_2 u_1 x_1
\end{aligned}$$

$$\begin{aligned}
p_{11} &= x_{11}x_4 - x_{11}x_1 - x_{10}x_5 + x_{10}x_2 + x_5x_1 - x_4x_2 \\
p_{12} &= -u_2x_{12}x_7 + u_3x_{12}x_6 + u_2x_{12}x_2 - u_3x_{12}x_1 + u_2x_7x_1 \\
&\quad - u_2x_6x_2 \\
p_{13} &= u_2x_{13} - u_3x_{12} \\
p_{14} &= -x_{14}x_{11}x_8 + u_2x_{14}x_{11} + x_{14}x_{10}x_9 - u_3x_{14}x_{10} - x_{14}x_9x_6 + \\
&\quad x_{14}x_8x_7 - u_2x_{14}x_7 + u_3x_{14}x_6 + x_{11}x_8x_6 - u_2x_{11}x_6 \\
&\quad - u_2x_{10}x_9 - x_{10}x_8x_7 + u_3x_{10}x_8 + u_2x_{10}x_7 + u_2x_9x_6 \\
&\quad - u_3x_8x_6 \\
p_{15} &= x_{15}x_8 - u_2x_{15} - x_{14}x_9 + u_3x_{14} + u_2x_9 - u_3x_8
\end{aligned}$$

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10} . No reduction needed.

The triangular system has not been changed.

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_9 from all other polynomials by reducing them with polynomial p_8 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_3^2u_2 - 0.333333u_2^3)x_2 + (-0.333333u_3^3 + u_3u_2^2)x_1 \\
p_2 &= (u_3^2u_2 - 2u_3u_2u_1 - 0.333333u_2^3 + u_2u_1^2)x_2 + \\
&\quad (-0.333333u_3^3 + u_3^2u_1 + u_3u_2^2 - u_3u_1^2 - u_2^2u_1 + \\
&\quad 0.333333u_1^3) \\
&\quad x_1 \\
&\quad + (-u_3^2u_2u_1 + 2u_3u_2u_1^2 + 0.333333u_2^3u_1 - u_2u_1^3) \\
p_3 &= x_3^2 - 3 \\
p_4 &= (u_3^2 - u_3u_1 + u_2^2)x_5x_3x_2 - u_2u_1x_5x_3x_1 \\
&\quad - u_2u_1x_5x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_5x_1 + u_2u_1x_4x_3x_2 + \\
&\quad (u_3^2 - u_3u_1 + u_2^2)x_4x_3x_1 + (u_3^2 - u_3u_1 + u_2^2)x_4x_2 \\
&\quad - u_2u_1x_4x_1 + (-u_3^2 + u_3u_1 - u_2^2)x_3x_2^2 +
\end{aligned}$$

$$\begin{aligned}
& (-u_3^2 + u_3u_1 - u_2^2)x_3x_1^2 + u_2u_1x_2^2 + u_2u_1x_1^2 \\
p_5 &= u_3u_2x_5 + (-0.5u_3^2 + 0.5u_2^2)x_4 \\
p_6 &= (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 + \\
& \quad (-u_3u_2u_1 + u_2u_1^2) \\
p_7 &= x_7x_5x_2x_1 - 0.5u_1x_7x_5x_1 - 0.5x_7x_4x_2^2 + 0.5u_1x_7x_4x_2 + \\
& \quad 0.5x_7x_4x_1^2 - 0.5x_7x_2^2x_1 - 0.5x_7x_1^3 - 0.5x_6x_5x_2^2 + \\
& \quad 0.5u_1x_6x_5x_2 + 0.5x_6x_5x_1^2 - x_6x_4x_2x_1 + 0.5u_1x_6x_4x_1 + \\
& \quad 0.5x_6x_2^3 - 0.5u_1x_6x_2^2 + 0.5x_6x_2x_1^2 - 0.5u_1x_6x_1^2 \\
& \quad - 0.5x_5x_2^2x_1 - 0.5x_5x_1^3 + 0.5x_4x_2^3 - 0.5u_1x_4x_2^2 + \\
& \quad 0.5x_4x_2x_1^2 - 0.5u_1x_4x_1^2 + 0.5u_1x_2^2x_1 + 0.5u_1x_1^3 \\
p_8 &= (-0.5u_3^2u_2u_1 + 0.5u_3u_2u_1^2 - 0.5u_2^3u_1)x_8 + \\
& \quad (u_3^2u_2^2u_1 - u_3u_2^2u_1^2) \\
p_9 &= (u_3u_2 - u_2u_1)x_9 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_8 + \\
& \quad (-u_3u_2u_1 + u_2u_1^2) \\
p_{10} &= u_2x_{10}x_5 + (-u_3 + u_1)x_{10}x_4 - u_2x_{10}x_2 + (u_3 - u_1)x_{10}x_1 \\
& \quad - u_2x_5x_1 + u_2x_4x_2 - u_2u_1x_4 + u_2u_1x_1 \\
p_{11} &= x_{11}x_4 - x_{11}x_1 - x_{10}x_5 + x_{10}x_2 + x_5x_1 - x_4x_2 \\
p_{12} &= -u_2x_{12}x_7 + u_3x_{12}x_6 + u_2x_{12}x_2 - u_3x_{12}x_1 + u_2x_7x_1 \\
& \quad - u_2x_6x_2 \\
p_{13} &= u_2x_{13} - u_3x_{12} \\
p_{14} &= -x_{14}x_{11}x_8 + u_2x_{14}x_{11} + x_{14}x_{10}x_9 - u_3x_{14}x_{10} - x_{14}x_9x_6 + \\
& \quad x_{14}x_8x_7 - u_2x_{14}x_7 + u_3x_{14}x_6 + x_{11}x_8x_6 - u_2x_{11}x_6 \\
& \quad - u_2x_{10}x_9 - x_{10}x_8x_7 + u_3x_{10}x_8 + u_2x_{10}x_7 + u_2x_9x_6 \\
& \quad - u_3x_8x_6 \\
p_{15} &= x_{15}x_8 - u_2x_{15} - x_{14}x_9 + u_3x_{14} + u_2x_9 - u_3x_8
\end{aligned}$$

3.8 Triangulation, step 8

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8 . No reduction needed.

The triangular system has not been changed.

3.9 Triangulation, step 9

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_7 from all other polynomials by reducing them with polynomial p_6 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_3^2 u_2 - 0.333333 u_2^3) x_2 + (-0.333333 u_3^3 + u_3 u_2^2) x_1 \\
p_2 &= (u_3^2 u_2 - 2 u_3 u_2 u_1 - 0.333333 u_2^3 + u_2 u_1^2) x_2 + \\
&\quad (-0.333333 u_3^3 + u_3^2 u_1 + u_3 u_2^2 - u_3 u_1^2 - u_2^2 u_1 + \\
&\quad 0.333333 u_1^3) \\
&\quad x_1 \\
&\quad + (-u_3^2 u_2 u_1 + 2 u_3 u_2 u_1^2 + 0.333333 u_2^3 u_1 - u_2 u_1^3) \\
p_3 &= x_3^2 - 3 \\
p_4 &= (u_3^2 - u_3 u_1 + u_2^2) x_5 x_3 x_2 - u_2 u_1 x_5 x_3 x_1 \\
&\quad - u_2 u_1 x_5 x_2 + (-u_3^2 + u_3 u_1 - u_2^2) x_5 x_1 + u_2 u_1 x_4 x_3 x_2 + \\
&\quad (u_3^2 - u_3 u_1 + u_2^2) x_4 x_3 x_1 + (u_3^2 - u_3 u_1 + u_2^2) x_4 x_2 \\
&\quad - u_2 u_1 x_4 x_1 + (-u_3^2 + u_3 u_1 - u_2^2) x_3 x_2^2 + \\
&\quad (-u_3^2 + u_3 u_1 - u_2^2) x_3 x_1^2 + u_2 u_1 x_2^2 + u_2 u_1 x_1^2 \\
p_5 &= u_3 u_2 x_5 + (-0.5 u_3^2 + 0.5 u_2^2) x_4 \\
p_6 &= (-0.5 u_3 u_2 + 0.5 u_2 u_1) x_6 x_5 x_2^2 + \\
&\quad (0.5 u_3^2 - u_3 u_1 - 0.5 u_2^2 + 0.5 u_1^2) x_6 x_5 x_2 x_1 + \\
&\quad (0.5 u_3 u_2 u_1 - 0.5 u_2 u_1^2) x_6 x_5 x_2 + \\
&\quad (0.5 u_3 u_2 - 0.5 u_2 u_1) x_6 x_5 x_1^2 + \\
&\quad (-0.25 u_3^2 u_1 + 0.5 u_3 u_1^2 + 0.25 u_2^2 u_1 - 0.25 u_1^3) x_6 x_5 x_1 + \\
&\quad (-0.25 u_3^2 + 0.5 u_3 u_1 + 0.25 u_2^2 - 0.25 u_1^2) x_6 x_4 x_2^2 + \\
&\quad (-u_3 u_2 + u_2 u_1) x_6 x_4 x_2 x_1 + \\
&\quad (0.25 u_3^2 u_1 - 0.5 u_3 u_1^2 - 0.25 u_2^2 u_1 + 0.25 u_1^3) x_6 x_4 x_2 + \\
&\quad (0.25 u_3^2 - 0.5 u_3 u_1 - 0.25 u_2^2 + 0.25 u_1^2) x_6 x_4 x_1^2 + \\
&\quad (0.5 u_3 u_2 u_1 - 0.5 u_2 u_1^2) x_6 x_4 x_1 + \\
&\quad (0.5 u_3 u_2 - 0.5 u_2 u_1) x_6 x_2^3 + \\
&\quad (-0.25 u_3^2 + 0.5 u_3 u_1 + 0.25 u_2^2 - 0.25 u_1^2) x_6 x_2^2 x_1 + \\
&\quad (-0.5 u_3 u_2 u_1 + 0.5 u_2 u_1^2) x_6 x_2^2 + \\
&\quad (0.5 u_3 u_2 - 0.5 u_2 u_1) x_6 x_2 x_1^2 + \\
&\quad (-0.25 u_3^2 + 0.5 u_3 u_1 + 0.25 u_2^2 - 0.25 u_1^2) x_6 x_1^3 + \\
&\quad (-0.5 u_3 u_2 u_1 + 0.5 u_2 u_1^2) x_6 x_1^2 + \\
&\quad (-0.5 u_3 u_2 + 0.5 u_2 u_1) x_5 x_2^2 x_1 + \\
&\quad (u_3 u_2 u_1 - u_2 u_1^2) x_5 x_2 x_1 + \\
&\quad (-0.5 u_3 u_2 + 0.5 u_2 u_1) x_5 x_1^3 + \\
&\quad (-0.5 u_3 u_2 u_1^2 + 0.5 u_2 u_1^3) x_5 x_1 + \\
&\quad (0.5 u_3 u_2 - 0.5 u_2 u_1) x_4 x_2^3 + (-u_3 u_2 u_1 + u_2 u_1^2) x_4 x_2^2 + \\
&\quad (0.5 u_3 u_2 - 0.5 u_2 u_1) x_4 x_2 x_1^2 + \\
&\quad (0.5 u_3 u_2 u_1^2 - 0.5 u_2 u_1^3) x_4 x_2
\end{aligned}$$

$$\begin{aligned}
p_7 &= (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 + \\
&\quad (-u_3u_2u_1 + u_2u_1^2) \\
p_8 &= (-0.5u_3^2u_2u_1 + 0.5u_3u_2u_1^2 - 0.5u_2^3u_1)x_8 + \\
&\quad (u_3^2u_2^2u_1 - u_3u_2^2u_1^2) \\
p_9 &= (u_3u_2 - u_2u_1)x_9 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_8 + \\
&\quad (-u_3u_2u_1 + u_2u_1^2) \\
p_{10} &= u_2x_{10}x_5 + (-u_3 + u_1)x_{10}x_4 - u_2x_{10}x_2 + (u_3 - u_1)x_{10}x_1 \\
&\quad - u_2x_5x_1 + u_2x_4x_2 - u_2u_1x_4 + u_2u_1x_1 \\
p_{11} &= x_{11}x_4 - x_{11}x_1 - x_{10}x_5 + x_{10}x_2 + x_5x_1 - x_4x_2 \\
p_{12} &= -u_2x_{12}x_7 + u_3x_{12}x_6 + u_2x_{12}x_2 - u_3x_{12}x_1 + u_2x_7x_1 \\
&\quad - u_2x_6x_2 \\
p_{13} &= u_2x_{13} - u_3x_{12} \\
p_{14} &= -x_{14}x_{11}x_8 + u_2x_{14}x_{11} + x_{14}x_{10}x_9 - u_3x_{14}x_{10} - x_{14}x_9x_6 + \\
&\quad x_{14}x_8x_7 - u_2x_{14}x_7 + u_3x_{14}x_6 + x_{11}x_8x_6 - u_2x_{11}x_6 \\
&\quad - u_2x_{10}x_9 - x_{10}x_8x_7 + u_3x_{10}x_8 + u_2x_{10}x_7 + u_2x_9x_6 \\
&\quad - u_3x_8x_6 \\
p_{15} &= x_{15}x_8 - u_2x_{15} - x_{14}x_9 + u_3x_{14} + u_2x_9 - u_3x_8
\end{aligned}$$

3.10 Triangulation, step 10

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6 . No reduction needed.

The triangular system has not been changed.

3.11 Triangulation, step 11

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_5 from all other polynomials by reducing them with polynomial p_4 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_3^2u_2 - 0.333333u_2^3)x_2 + (-0.333333u_3^3 + u_3u_2^2)x_1 \\
p_2 &= (u_3^2u_2 - 2u_3u_2u_1 - 0.333333u_2^3 + u_2u_1^2)x_2 + \\
&\quad (-0.333333u_3^3 + u_3^2u_1 + u_3u_2^2 - u_3u_1^2 - u_2^2u_1 +
\end{aligned}$$

$$\begin{aligned}
& 0.333333u_1^3) \\
& x_1 \\
& +(-u_3^2u_2u_1 + 2u_3u_2u_1^2 + 0.333333u_2^3u_1 - u_2u_1^3) \\
p_3 = & x_3^2 - 3 \\
p_4 = & (-0.5u_3^4 + 0.5u_3^3u_1 - 1.5u_3u_2^2u_1 + 0.5u_2^4)x_4x_3x_2 + \\
& (-u_3^3u_2 + 1.5u_3^2u_2u_1 - u_3u_2^3 - 0.5u_2^3u_1)x_4x_3x_1 + \\
& (-u_3^3u_2 + 1.5u_3^2u_2u_1 - u_3u_2^3 - 0.5u_2^3u_1)x_4x_2 + \\
& (0.5u_3^4 - 0.5u_3^3u_1 + 1.5u_3u_2^2u_1 - 0.5u_2^4)x_4x_1 + \\
& (u_3^3u_2 - u_3^2u_2u_1 + u_3u_2^3)x_3x_2^2 + \\
& (u_3^3u_2 - u_3^2u_2u_1 + u_3u_2^3)x_3x_1^2 \\
& -u_3u_2^2u_1x_2^2 - u_3u_2^2u_1x_1^2 \\
p_5 = & (u_3^2 - u_3u_1 + u_2^2)x_5x_3x_2 - u_2u_1x_5x_3x_1 \\
& -u_2u_1x_5x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_5x_1 + u_2u_1x_4x_3x_2 + \\
& (u_3^2 - u_3u_1 + u_2^2)x_4x_3x_1 + (u_3^2 - u_3u_1 + u_2^2)x_4x_2 \\
& -u_2u_1x_4x_1 + (-u_3^2 + u_3u_1 - u_2^2)x_3x_2^2 + \\
& (-u_3^2 + u_3u_1 - u_2^2)x_3x_1^2 + u_2u_1x_2^2 + u_2u_1x_1^2 \\
p_6 = & (-0.5u_3u_2 + 0.5u_2u_1)x_6x_5x_2^2 + \\
& (0.5u_3^2 - u_3u_1 - 0.5u_2^2 + 0.5u_1^2)x_6x_5x_2x_1 + \\
& (0.5u_3u_2u_1 - 0.5u_2u_1^2)x_6x_5x_2 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_6x_5x_1^2 + \\
& (-0.25u_3^2u_1 + 0.5u_3u_1^2 + 0.25u_2^2u_1 - 0.25u_1^3)x_6x_5x_1 + \\
& (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_4x_2^2 + \\
& (-u_3u_2 + u_2u_1)x_6x_4x_2x_1 + \\
& (0.25u_3^2u_1 - 0.5u_3u_1^2 - 0.25u_2^2u_1 + 0.25u_1^3)x_6x_4x_2 + \\
& (0.25u_3^2 - 0.5u_3u_1 - 0.25u_2^2 + 0.25u_1^2)x_6x_4x_1^2 + \\
& (0.5u_3u_2u_1 - 0.5u_2u_1^2)x_6x_4x_1 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_6x_2^3 + \\
& (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_2^2x_1 + \\
& (-0.5u_3u_2u_1 + 0.5u_2u_1^2)x_6x_2^2 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_6x_2x_1^2 + \\
& (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_1^3 + \\
& (-0.5u_3u_2u_1 + 0.5u_2u_1^2)x_6x_1^2 + \\
& (-0.5u_3u_2 + 0.5u_2u_1)x_5x_2^2x_1 + \\
& (u_3u_2u_1 - u_2u_1^2)x_5x_2x_1 + \\
& (-0.5u_3u_2 + 0.5u_2u_1)x_5x_1^3 + \\
& (-0.5u_3u_2u_1^2 + 0.5u_2u_1^3)x_5x_1 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_4x_2^3 + (-u_3u_2u_1 + u_2u_1^2)x_4x_2^2 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_4x_2x_1^2 + \\
& (0.5u_3u_2u_1^2 - 0.5u_2u_1^3)x_4x_2 \\
p_7 = & (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 +
\end{aligned}$$

$$\begin{aligned}
& (-u_3u_2u_1 + u_2u_1^2) \\
p_8 &= (-0.5u_3^2u_2u_1 + 0.5u_3u_2u_1^2 - 0.5u_2^3u_1)x_8 + \\
& (u_3^2u_2^2u_1 - u_3u_2^2u_1^2) \\
p_9 &= (u_3u_2 - u_2u_1)x_9 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_8 + \\
& (-u_3u_2u_1 + u_2u_1^2) \\
p_{10} &= u_2x_{10}x_5 + (-u_3 + u_1)x_{10}x_4 - u_2x_{10}x_2 + (u_3 - u_1)x_{10}x_1 \\
& -u_2x_5x_1 + u_2x_4x_2 - u_2u_1x_4 + u_2u_1x_1 \\
p_{11} &= x_{11}x_4 - x_{11}x_1 - x_{10}x_5 + x_{10}x_2 + x_5x_1 - x_4x_2 \\
p_{12} &= -u_2x_{12}x_7 + u_3x_{12}x_6 + u_2x_{12}x_2 - u_3x_{12}x_1 + u_2x_7x_1 \\
& -u_2x_6x_2 \\
p_{13} &= u_2x_{13} - u_3x_{12} \\
p_{14} &= -x_{14}x_{11}x_8 + u_2x_{14}x_{11} + x_{14}x_{10}x_9 - u_3x_{14}x_{10} - x_{14}x_9x_6 + \\
& x_{14}x_8x_7 - u_2x_{14}x_7 + u_3x_{14}x_6 + x_{11}x_8x_6 - u_2x_{11}x_6 \\
& -u_2x_{10}x_9 - x_{10}x_8x_7 + u_3x_{10}x_8 + u_2x_{10}x_7 + u_2x_9x_6 \\
& -u_3x_8x_6 \\
p_{15} &= x_{15}x_8 - u_2x_{15} - x_{14}x_9 + u_3x_{14} + u_2x_9 - u_3x_8
\end{aligned}$$

3.12 Triangulation, step 12

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4 . No reduction needed.

The triangular system has not been changed.

3.13 Triangulation, step 13

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3 . No reduction needed.

The triangular system has not been changed.

3.14 Triangulation, step 14

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_2 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (0.333333u_3^4u_2u_1 - 0.666667u_3^3u_2u_1^2 + 0.666667u_3^2u_2^3u_1 + \\
&\quad 0.333333u_3^2u_2u_1^3 - 0.666667u_3u_2^3u_1^2 + 0.333333u_2^5u_1 \\
&\quad - 0.111111u_2^3u_1^3) \\
&\quad x_1 \\
&\quad + \\
&\quad (-u_3^4u_2^2u_1 + 2u_3^3u_2^2u_1^2 + 0.666667u_3^2u_2^4u_1 \\
&\quad - u_3^2u_2^2u_1^3 - 0.666667u_3u_2^4u_1^2 - 0.111111u_2^6u_1 + \\
&\quad 0.333333u_2^4u_1^3) \\
p_2 &= (u_3^2u_2 - 0.333333u_2^3)x_2 + (-0.333333u_3^3 + u_3u_2^2)x_1 \\
p_3 &= x_3^2 - 3 \\
p_4 &= (-0.5u_3^4 + 0.5u_3^3u_1 - 1.5u_3u_2^2u_1 + 0.5u_2^4)x_4x_3x_2 + \\
&\quad (-u_3^3u_2 + 1.5u_3^2u_2u_1 - u_3u_2^3 - 0.5u_2^3u_1)x_4x_3x_1 + \\
&\quad (-u_3^3u_2 + 1.5u_3^2u_2u_1 - u_3u_2^3 - 0.5u_2^3u_1)x_4x_2 + \\
&\quad (0.5u_3^4 - 0.5u_3^3u_1 + 1.5u_3u_2^2u_1 - 0.5u_2^4)x_4x_1 + \\
&\quad (u_3^3u_2 - u_3^2u_2u_1 + u_3u_2^3)x_3x_2^2 + \\
&\quad (u_3^3u_2 - u_3^2u_2u_1 + u_3u_2^3)x_3x_1^2 \\
&\quad - u_3u_2^2u_1x_2^2 - u_3u_2^2u_1x_1^2 \\
p_5 &= (u_3^2 - u_3u_1 + u_2^2)x_5x_3x_2 - u_2u_1x_5x_3x_1 \\
&\quad - u_2u_1x_5x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_5x_1 + u_2u_1x_4x_3x_2 + \\
&\quad (u_3^2 - u_3u_1 + u_2^2)x_4x_3x_1 + (u_3^2 - u_3u_1 + u_2^2)x_4x_2 \\
&\quad - u_2u_1x_4x_1 + (-u_3^2 + u_3u_1 - u_2^2)x_3x_2^2 + \\
&\quad (-u_3^2 + u_3u_1 - u_2^2)x_3x_1^2 + u_2u_1x_2^2 + u_2u_1x_1^2 \\
p_6 &= (-0.5u_3u_2 + 0.5u_2u_1)x_6x_5x_2^2 + \\
&\quad (0.5u_3^2 - u_3u_1 - 0.5u_2^2 + 0.5u_1^2)x_6x_5x_2x_1 + \\
&\quad (0.5u_3u_2u_1 - 0.5u_2u_1^2)x_6x_5x_2 + \\
&\quad (0.5u_3u_2 - 0.5u_2u_1)x_6x_5x_1^2 + \\
&\quad (-0.25u_3^2u_1 + 0.5u_3u_1^2 + 0.25u_2^2u_1 - 0.25u_1^3)x_6x_5x_1 + \\
&\quad (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_4x_2^2 + \\
&\quad (-u_3u_2 + u_2u_1)x_6x_4x_2x_1 + \\
&\quad (0.25u_3^2u_1 - 0.5u_3u_1^2 - 0.25u_2^2u_1 + 0.25u_1^3)x_6x_4x_2 + \\
&\quad (0.25u_3^2 - 0.5u_3u_1 - 0.25u_2^2 + 0.25u_1^2)x_6x_4x_1^2 + \\
&\quad (0.5u_3u_2u_1 - 0.5u_2u_1^2)x_6x_4x_1 + \\
&\quad (0.5u_3u_2 - 0.5u_2u_1)x_6x_2^3 + \\
&\quad (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_2^2x_1 + \\
&\quad (-0.5u_3u_2u_1 + 0.5u_2u_1^2)x_6x_2^2 +
\end{aligned}$$

$$\begin{aligned}
& (0.5u_3u_2 - 0.5u_2u_1)x_6x_2x_1^2 + \\
& (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_1^3 + \\
& (-0.5u_3u_2u_1 + 0.5u_2u_1^2)x_6x_1^2 + \\
& (-0.5u_3u_2 + 0.5u_2u_1)x_5x_2^2x_1 + \\
& (u_3u_2u_1 - u_2u_1^2)x_5x_2x_1 + \\
& (-0.5u_3u_2 + 0.5u_2u_1)x_5x_1^3 + \\
& (-0.5u_3u_2u_1^2 + 0.5u_2u_1^3)x_5x_1 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_4x_2^3 + (-u_3u_2u_1 + u_2u_1^2)x_4x_2^2 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_4x_2x_1^2 + \\
& (0.5u_3u_2u_1^2 - 0.5u_2u_1^3)x_4x_2 \\
p_7 = & (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 + \\
& (-u_3u_2u_1 + u_2u_1^2) \\
p_8 = & (-0.5u_3^2u_2u_1 + 0.5u_3u_2u_1^2 - 0.5u_2^3u_1)x_8 + \\
& (u_3^2u_2^2u_1 - u_3u_2^2u_1^2) \\
p_9 = & (u_3u_2 - u_2u_1)x_9 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_8 + \\
& (-u_3u_2u_1 + u_2u_1^2) \\
p_{10} = & u_2x_{10}x_5 + (-u_3 + u_1)x_{10}x_4 - u_2x_{10}x_2 + (u_3 - u_1)x_{10}x_1 \\
& - u_2x_5x_1 + u_2x_4x_2 - u_2u_1x_4 + u_2u_1x_1 \\
p_{11} = & x_{11}x_4 - x_{11}x_1 - x_{10}x_5 + x_{10}x_2 + x_5x_1 - x_4x_2 \\
p_{12} = & -u_2x_{12}x_7 + u_3x_{12}x_6 + u_2x_{12}x_2 - u_3x_{12}x_1 + u_2x_7x_1 \\
& - u_2x_6x_2 \\
p_{13} = & u_2x_{13} - u_3x_{12} \\
p_{14} = & -x_{14}x_{11}x_8 + u_2x_{14}x_{11} + x_{14}x_{10}x_9 - u_3x_{14}x_{10} - x_{14}x_9x_6 + \\
& x_{14}x_8x_7 - u_2x_{14}x_7 + u_3x_{14}x_6 + x_{11}x_8x_6 - u_2x_{11}x_6 \\
& - u_2x_{10}x_9 - x_{10}x_8x_7 + u_3x_{10}x_8 + u_2x_{10}x_7 + u_2x_9x_6 \\
& - u_3x_8x_6 \\
p_{15} = & x_{15}x_8 - u_2x_{15} - x_{14}x_9 + u_3x_{14} + u_2x_9 - u_3x_8
\end{aligned}$$

3.15 Triangulation, step 15

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1 . No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_1 = & (0.333333u_3^4u_2u_1 - 0.666667u_3^3u_2u_1^2 + 0.666667u_3^2u_2^3u_1 + \\
& 0.333333u_3^2u_2u_1^3 - 0.666667u_3u_2^3u_1^2 + 0.333333u_2^5u_1
\end{aligned}$$

$$\begin{aligned}
& -0.111111u_2^3u_1^3) \\
& x_1 \\
& + \\
& (-u_3^4u_2^2u_1 + 2u_3^3u_2^2u_1^2 + 0.666667u_3^2u_2^4u_1 \\
& -u_3^2u_2^2u_1^3 - 0.666667u_3u_2^4u_1^2 - 0.111111u_2^6u_1 + \\
& 0.333333u_2^4u_1^3) \\
p_2 &= (u_3^2u_2 - 0.333333u_2^3)x_2 + (-0.333333u_3^3 + u_3u_2^2)x_1 \\
p_3 &= x_3^2 - 3 \\
p_4 &= (-0.5u_3^4 + 0.5u_3^3u_1 - 1.5u_3u_2^2u_1 + 0.5u_2^4)x_4x_3x_2 + \\
& (-u_3^3u_2 + 1.5u_3^2u_2u_1 - u_3u_2^3 - 0.5u_2^3u_1)x_4x_3x_1 + \\
& (-u_3^3u_2 + 1.5u_3^2u_2u_1 - u_3u_2^3 - 0.5u_2^3u_1)x_4x_2 + \\
& (0.5u_3^4 - 0.5u_3^3u_1 + 1.5u_3u_2^2u_1 - 0.5u_2^4)x_4x_1 + \\
& (u_3^3u_2 - u_3^2u_2u_1 + u_3u_2^3)x_3x_2^2 + \\
& (u_3^3u_2 - u_3^2u_2u_1 + u_3u_2^3)x_3x_1^2 \\
& -u_3u_2^2u_1x_2^2 - u_3u_2^2u_1x_1^2 \\
p_5 &= (u_3^2 - u_3u_1 + u_2^2)x_5x_3x_2 - u_2u_1x_5x_3x_1 \\
& -u_2u_1x_5x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_5x_1 + u_2u_1x_4x_3x_2 + \\
& (u_3^2 - u_3u_1 + u_2^2)x_4x_3x_1 + (u_3^2 - u_3u_1 + u_2^2)x_4x_2 \\
& -u_2u_1x_4x_1 + (-u_3^2 + u_3u_1 - u_2^2)x_3x_2^2 + \\
& (-u_3^2 + u_3u_1 - u_2^2)x_3x_1^2 + u_2u_1x_2^2 + u_2u_1x_1^2 \\
p_6 &= (-0.5u_3u_2 + 0.5u_2u_1)x_6x_5x_2^2 + \\
& (0.5u_3^2 - u_3u_1 - 0.5u_2^2 + 0.5u_1^2)x_6x_5x_2x_1 + \\
& (0.5u_3u_2u_1 - 0.5u_2u_1^2)x_6x_5x_2 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_6x_5x_1^2 + \\
& (-0.25u_3^2u_1 + 0.5u_3u_1^2 + 0.25u_2^2u_1 - 0.25u_1^3)x_6x_5x_1 + \\
& (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_4x_2^2 + \\
& (-u_3u_2 + u_2u_1)x_6x_4x_2x_1 + \\
& (0.25u_3^2u_1 - 0.5u_3u_1^2 - 0.25u_2^2u_1 + 0.25u_1^3)x_6x_4x_2 + \\
& (0.25u_3^2 - 0.5u_3u_1 - 0.25u_2^2 + 0.25u_1^2)x_6x_4x_1^2 + \\
& (0.5u_3u_2u_1 - 0.5u_2u_1^2)x_6x_4x_1 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_6x_2^3 + \\
& (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_2^2x_1 + \\
& (-0.5u_3u_2u_1 + 0.5u_2u_1^2)x_6x_2^2 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_6x_2x_1^2 + \\
& (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_1^3 + \\
& (-0.5u_3u_2u_1 + 0.5u_2u_1^2)x_6x_1^2 + \\
& (-0.5u_3u_2 + 0.5u_2u_1)x_5x_2^2x_1 + \\
& (u_3u_2u_1 - u_2u_1^2)x_5x_2x_1 + \\
& (-0.5u_3u_2 + 0.5u_2u_1)x_5x_1^3 +
\end{aligned}$$

$$\begin{aligned}
& (-0.5u_3u_2u_1^2 + 0.5u_2u_1^3)x_5x_1 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_4x_2^3 + (-u_3u_2u_1 + u_2u_1^2)x_4x_2^2 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_4x_2x_1^2 + \\
& (0.5u_3u_2u_1^2 - 0.5u_2u_1^3)x_4x_2 \\
p_7 = & (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 + \\
& (-u_3u_2u_1 + u_2u_1^2) \\
p_8 = & (-0.5u_3^2u_2u_1 + 0.5u_3u_2u_1^2 - 0.5u_2^3u_1)x_8 + \\
& (u_3^2u_2^2u_1 - u_3u_2^2u_1^2) \\
p_9 = & (u_3u_2 - u_2u_1)x_9 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_8 + \\
& (-u_3u_2u_1 + u_2u_1^2) \\
p_{10} = & u_2x_{10}x_5 + (-u_3 + u_1)x_{10}x_4 - u_2x_{10}x_2 + (u_3 - u_1)x_{10}x_1 \\
& - u_2x_5x_1 + u_2x_4x_2 - u_2u_1x_4 + u_2u_1x_1 \\
p_{11} = & x_{11}x_4 - x_{11}x_1 - x_{10}x_5 + x_{10}x_2 + x_5x_1 - x_4x_2 \\
p_{12} = & -u_2x_{12}x_7 + u_3x_{12}x_6 + u_2x_{12}x_2 - u_3x_{12}x_1 + u_2x_7x_1 \\
& - u_2x_6x_2 \\
p_{13} = & u_2x_{13} - u_3x_{12} \\
p_{14} = & -x_{14}x_{11}x_8 + u_2x_{14}x_{11} + x_{14}x_{10}x_9 - u_3x_{14}x_{10} - x_{14}x_9x_6 + \\
& x_{14}x_8x_7 - u_2x_{14}x_7 + u_3x_{14}x_6 + x_{11}x_8x_6 - u_2x_{11}x_6 \\
& - u_2x_{10}x_9 - x_{10}x_8x_7 + u_3x_{10}x_8 + u_2x_{10}x_7 + u_2x_9x_6 \\
& - u_3x_8x_6 \\
p_{15} = & x_{15}x_8 - u_2x_{15} - x_{14}x_9 + u_3x_{14} + u_2x_9 - u_3x_8
\end{aligned}$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 040

Calculating final remainder of the conclusion:

$$g = x_{15}x_{12} - x_{15}x_4 - x_{14}x_{13} + x_{14}x_5 + x_{13}x_4 - x_{12}x_5$$

with respect to the triangular system.

1. Pseudo remainder with p_{15} over variable x_{15} :

$$\begin{aligned}
g = & -x_{14}x_{13}x_8 + u_2x_{14}x_{13} + x_{14}x_{12}x_9 - u_3x_{14}x_{12} - x_{14}x_9x_4 + \\
& x_{14}x_8x_5 - u_2x_{14}x_5 + u_3x_{14}x_4 + x_{13}x_8x_4 - u_2x_{13}x_4 \\
& - u_2x_{12}x_9 - x_{12}x_8x_5 + u_3x_{12}x_8 + u_2x_{12}x_5 + u_2x_9x_4 \\
& - u_3x_8x_4
\end{aligned}$$

2. Pseudo remainder with p_{14} over variable x_{14} :

Polynomial too big for output (text size is 2523 characters, number of terms is 88)

3. Pseudo remainder with p_{13} over variable x_{13} :
Polynomial too big for output (text size is 2688 characters, number of terms is 82)
4. Pseudo remainder with p_{12} over variable x_{12} :
Polynomial too big for output (text size is 8104 characters, number of terms is 204)
5. Pseudo remainder with p_{11} over variable x_{11} :
Polynomial too big for output (number of terms is 369)
6. Pseudo remainder with p_{10} over variable x_{10} :
Polynomial too big for output (number of terms is 722)
7. Pseudo remainder with p_9 over variable x_9 :
Polynomial too big for output (number of terms is 507)
8. Pseudo remainder with p_8 over variable x_8 :
Polynomial too big for output (text size is 61594 characters, number of terms is 174)
9. Pseudo remainder with p_7 over variable x_7 :
Polynomial too big for output (text size is 86709 characters, number of terms is 118)
10. Pseudo remainder with p_6 over variable x_6 :
Polynomial too big for output (number of terms is 618)
11. Pseudo remainder with p_5 over variable x_5 :
Polynomial too big for output (number of terms is 1860)
12. Pseudo remainder with p_4 over variable x_4 :
Polynomial too big for output (number of terms is 1170)
13. Pseudo remainder with p_3 over variable x_3 :
Polynomial too big for output (text size is greater than 2000 characters, number of terms is 234)
14. Pseudo remainder with p_2 over variable x_2 :
Polynomial too big for output (text size is greater than 2000 characters, number of terms is 12)
15. Pseudo remainder with p_1 over variable x_1 :
Polynomial too big for output (text size is greater than 2000 characters, number of terms is 2)

5 Prover results

Status: Theorem can't be neither proved nor disproved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2142 terms.

Time Complexity: Time spent by the prover is 237.183 seconds.