# OpenGeoProver Output for conjecture “Three squares theorem”

Wu’s method used

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## 1 Validation of Construction Protocol

### Construction steps:

- Free point A
- Free point B
- Rotated point C of point A around point B for angle of -90.0 degrees
- Translated point D of point C for vector BA
- Translated point E of point B for vector AB
- Translated point F of point C for vector DC
- Translated point G of point E for vector BE
- Translated point H of point F for vector CF

### Theorem statement:

- Algebraic sum of angles EDF, GDH and BDC is zero

**Validation result:** Construction protocol is valid.

## 2 Transformation of Construction Protocol to algebraic form

### Transformation of Construction steps

### 2.1 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

### 2.2 Transformation of point B:

- Point B has been assigned following coordinates: (0, $u_1$)

## 2.3 Transformation of point C:

- Point C has been assigned following coordinates: $(x_1, x_2)$
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_1 + u_1$$

**Info:** Polynomial

$$p = x_1 + u_1$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_1 + u_1$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_2 - u_1$$

**Info:** Will try to rename Y coordinate of point C

**Info:** Y coordinate of point C renamed by independent variable $u_1$

- Point C has been renamed. Point C has been assigned following coordinates: $(x_1, u_1)$
- Repeating instantiation of condition for X-coordinate of this point, after it has been renamed
- Processing of polynomial

$$p = x_1 + u_1$$

**Info:** Polynomial

$$p = x_1 + u_1$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_1 + u_1$$

is added to polynomial system

### 2.4 Transformation of point D:

- Point D has been assigned following coordinates: ($x_2$, $x_3$)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_2 - x_1$$

**Info:** Will try to rename X coordinate of point D

**Info:** Y coordinate of point D will be replaced by X coordinate

**Info:** X coordinate of point D renamed by dependent variable $x_1$

- Point D has been renamed. Point D has been assigned following coordinates: ($x_1$, $x_2$)
- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_2$$

**Info:** Will try to rename Y coordinate of point D

**Info:** Y coordinate of point D renamed by zero

- Point D has been renamed. Point D has been assigned following coordinates: ($x_1$, 0)

### 2.5 Transformation of point E:

- Point E has been assigned following coordinates: ($x_2$, $x_3$)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_2$$

**Info:** Will try to rename X coordinate of point E

**Info:** Y coordinate of point E will be replaced by X coordinate

**Info:** X coordinate of point E renamed by zero

- Point E has been renamed. Point E has been assigned following coordinates: (0, $x_2$)
- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_2 - 2u_1$$

**Info:** Polynomial

$$p = x_2 - 2u_1$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_2 - 2u_1$$

is added to polynomial system

## 2.6 Transformation of point F:

- Point F has been assigned following coordinates: $(x_3, x_4)$
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_3 - x_1$$

**Info:** Will try to rename X coordinate of point F

**Info:** Y coordinate of point F will be replaced by X coordinate

**Info:** X coordinate of point F renamed by dependent variable $x_1$

- Point F has been renamed. Point F has been assigned following coordinates: $(x_1, x_3)$
- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_3 - 2u_1$$

**Info:** Polynomial

$$p = x_3 - 2u_1$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_3 - 2u_1$$

is added to polynomial system

## 2.7 Transformation of point G:

- Point G has been assigned following coordinates: $(x_4, x_5)$
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_4$$

**Info:** Will try to rename X coordinate of point G

**Info:** Y coordinate of point G will be replaced by X coordinate

**Info:** X coordinate of point G renamed by zero

- Point G has been renamed. Point G has been assigned following coordinates: $(0, x_4)$
- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_4 - 2x_2 + u_1$$

**Info:** Polynomial

$$p = x_4 - 2x_2 + u_1$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_4 - 2x_2 + u_1$$

is added to polynomial system

### 2.8 Transformation of point H:

- Point H has been assigned following coordinates: $(x_5, x_6)$
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_5 - x_1$$

**Info:** Will try to rename X coordinate of point H

**Info:** Y coordinate of point H will be replaced by X coordinate

**Info:** X coordinate of point H renamed by dependent variable $x_1$

- Point H has been renamed. Point H has been assigned following coordinates: $(x_1, x_5)$
- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_5 - 2x_3 + u_1$$

**Info:** Polynomial

$$p = x_5 - 2x_3 + u_1$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_5 - 2x_3 + u_1$$

is added to polynomial system

### Transformation of Theorem statement

- Polynomial for theorem statement:

$$\begin{array}{rcl} p & = & x_5x_4x_3x_2x_1 - u_1x_5x_4x_3x_1 - u_1x_5x_3x_2x_1 \\ & & -x_5x_3x_1^3 \end{array}$$

### Time spent for transformation of Construction Protocol to algebraic form

- 0.254 seconds

# 3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{array}{rcl} p_1 & = & x_1 + u_1 \\ p_2 & = & x_2 - 2u_1 \\ p_3 & = & x_3 - 2u_1 \\ p_4 & = & x_4 - 2x_2 + u_1 \\ p_5 & = & x_5 - 2x_3 + u_1 \end{array}$$

## 3.1 Triangulation, step 1

**Choosing variable:** Trying the variable with index 5.

**Variable $x_5$ selected:** The number of polynomials with this variable, with indexes from 1 to 5, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_5$. No reduction needed.

The triangular system has not been changed.

## 3.2 Triangulation, step 2

**Choosing variable:** Trying the variable with index 4.

**Variable $x_4$ selected:** The number of polynomials with this variable, with indexes from 1 to 4, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_4$. No reduction needed.

The triangular system has not been changed.

### 3.3 Triangulation, step 3

**Choosing variable:** Trying the variable with index 3.

**Variable $x_3$ selected:** The number of polynomials with this variable, with indexes from 1 to 3, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_3$. No reduction needed.

The triangular system has not been changed.

### 3.4 Triangulation, step 4

**Choosing variable:** Trying the variable with index 2.

**Variable $x_2$ selected:** The number of polynomials with this variable, with indexes from 1 to 2, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_2$. No reduction needed.

The triangular system has not been changed.

### 3.5 Triangulation, step 5

**Choosing variable:** Trying the variable with index 1.

**Variable $x_1$ selected:** The number of polynomials with this variable, with indexes from 1 to 1, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_1$. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{array}{rcl} p_1 & = & x_1 + u_1 \\ p_2 & = & x_2 - 2u_1 \\ p_3 & = & x_3 - 2u_1 \\ p_4 & = & x_4 - 2x_2 + u_1 \\ p_5 & = & x_5 - 2x_3 + u_1 \end{array}$$

## 4 Final Remainder

### 4.1 Final remainder for conjecture Three squares theorem

Calculating final remainder of the conclusion:

$$\begin{array}{rcl} g & = & x_5x_4x_3x_2x_1 - u_1x_5x_4x_3x_1 - u_1x_5x_3x_2x_1 \\ & & -x_5x_3x_1^3 \end{array}$$

with respect to the triangular system.

1. Pseudo remainder with $p_5$ over variable $x_5$:

$$\begin{aligned} g &= 2x_4x_3^2x_2x_1 - 2u_1x_4x_3^2x_1 - u_1x_4x_3x_2x_1 + \\ &\quad u_1^2x_4x_3x_1 - 2u_1x_3^2x_2x_1 - 2x_3^2x_1^3 + \\ &\quad u_1^2x_3x_2x_1 + u_1x_3x_1^3 \end{aligned}$$

2. Pseudo remainder with $p_4$ over variable $x_4$:

$$\begin{aligned} g &= 4x_3^2x_2^2x_1 - 8u_1x_3^2x_2x_1 - 2x_3^2x_1^3 + \\ &\quad 2u_1^2x_3^2x_1 - 2u_1x_3x_2^2x_1 + 4u_1^2x_3x_2x_1 + \\ &\quad u_1x_3x_1^3 - u_1^3x_3x_1 \end{aligned}$$

3. Pseudo remainder with $p_3$ over variable $x_3$:

$$g = 12u_1^2x_2^2x_1 - 24u_1^3x_2x_1 - 6u_1^2x_1^3 + 6u_1^4x_1$$

4. Pseudo remainder with $p_2$ over variable $x_2$:

$$g = -6u_1^2x_1^3 + 6u_1^4x_1$$

5. Pseudo remainder with $p_1$ over variable $x_1$:

$$g = 0$$

# 5 Prover results

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 8 terms.

**Time Complexity:** Time spent by the prover is 0.077 seconds.

# 6 NDG Conditions

## NDG Conditions in readable form

- There are no NDG conditions for this theorem