OpenGeoProver Output for conjecture "Golden section in pentagon"

Wu's method used

February 23, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point O
- Free point A
- Rotated point B of point A around point O for angle of 72.0 degrees
- Rotated point C of point B around point O for angle of 72.0 degrees

Theorem statement:

- Algebraic sum of segments AB*AB, AB*AC and AC*AC is zero

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point O:

- Point O has been assigned following coordinates: (0, 0)

2.2 Transformation of point A:

- Point A has been assigned following coordinates: (0, u_1)

2.3 Transformation of point B:

- Point B has been assigned following coordinates: (x_1, x_2)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p \quad = \quad x_1 + 0.951057u_1$$

Info: Polynomial

$$p \quad = \quad x_1 + 0.951057u_1$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p \quad = \quad x_1 + 0.951057u_1$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p \quad = \quad x_2 - 0.309017u_1$$

Info: Polynomial

$$p \quad = \quad x_2 - 0.309017u_1$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p \quad = \quad x_2 - 0.309017u_1$$

is added to polynomial system

2.4 Transformation of point C:

- Point C has been assigned following coordinates: (x_3, x_4)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p \quad = \quad x_3 + 0.951057x_2 - 0.309017x_1$$

Info: Polynomial

$$p \quad = \quad x_3 + 0.951057x_2 - 0.309017x_1$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p \quad = \quad x_3 + 0.951057x_2 - 0.309017x_1$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p \quad = \quad x_4 - 0.309017x_2 - 0.951057x_1$$

Info: Polynomial

$$p = x_4 - 0.309017x_2 - 0.951057x_1$$

added to system of polynomials that represents the constructions

- Instantiated condition

  $$p = x_4 - 0.309017x_2 - 0.951057x_1$$

  is added to polynomial system

Transformation of Theorem statement

- Polynomial for theorem statement: *Polynomial too big for output (text size is* 4384 *characters, number of terms is* 155*)*

Time spent for transformation of Construction Protocol to algebraic form

- 0.227 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned} p_1 &= x_1 + 0.951057u_1 \\ p_2 &= x_2 - 0.309017u_1 \\ p_3 &= x_3 + 0.951057x_2 - 0.309017x_1 \\ p_4 &= x_4 - 0.309017x_2 - 0.951057x_1 \end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned} p_1 &= x_1 + 0.951057u_1 \\ p_2 &= x_2 - 0.309017u_1 \\ p_3 &= x_3 + 0.951057x_2 - 0.309017x_1 \\ p_4 &= x_4 - 0.309017x_2 - 0.951057x_1 \end{aligned}$$

4 Final Remainder

4.1 Final remainder for conjecture Golden section in pentagon

Calculating final remainder of the conclusion:

Polynomial too big for output (text size is 4384 *characters, number of terms is* 155*)*

with respect to the triangular system.

1. Pseudo remainder with p_4 over variable x_4:

   Polynomial too big for output (text size is 3125 *characters, number of terms is* 95*)*

2. Pseudo remainder with p_3 over variable x_3:

$$\begin{aligned} g &= -3x_2^8 + 15.708204u_1x_2^7 - 12x_2^6x_1^2 + 11.412678u_1x_2^6x_1 \\ &\quad -36.159054u_1^2x_2^6 + 47.124612u_1x_2^5x_1^2 \\ &\quad -63.218758u_1^2x_2^5x_1 + 56.54102u_1^3x_2^5 - 18x_2^4x_1^4 + \end{aligned}$$

$$
\begin{aligned}
&34.238035u_1x_2^4x_1^3 - 87.936141u_1^2x_2^4x_1^2 + \\
&128.364653u_1^3x_2^4x_1 - 62.532889u_1^4x_2^4 + \\
&47.124612u_1x_2^3x_1^4 - 126.437516u_1^2x_2^3x_1^3 + \\
&148.137767u_1^3x_2^3x_1^2 - 173.198287u_1^4x_2^3x_1 + \\
&56.54102u_1^5x_2^3 - 12x_2^2x_1^6 + 34.238035u_1x_2^2x_1^5 \\
&-67.395122u_1^2x_2^2x_1^4 + 148.838868u_1^3x_2^2x_1^3 \\
&-124.567331u_1^4x_2^2x_1^2 + 128.364653u_1^5x_2^2x_1 \\
&-36.159054u_1^6x_2^2 + 15.708204u_1x_2x_1^6 - 63.218758u_1^2x_2x_1^5 + \\
&91.596748u_1^3x_2x_1^4 - 136.952138u_1^4x_2x_1^3 + \\
&91.596748u_1^5x_2x_1^2 - 63.218758u_1^6x_2x_1 + 15.708204u_1^7x_2 \\
&-3x_1^8 + 11.412678u_1x_1^7 - 15.618034u_1^2x_1^6 + 20.474215u_1^3x_1^5 \\
&-12.145898u_1^4x_1^4 + 20.474215u_1^5x_1^3 - 15.618034u_1^6x_1^2 + \\
&11.412678u_1^7x_1 - 3u_1^8
\end{aligned}
$$

3. Pseudo remainder with p_2 over variable x_2:

$$
\begin{aligned}
g \quad = \quad & -3x_1^8 + 11.412678u_1x_1^7 - 11.90983u_1^2x_1^6 + 4.207986u_1^3x_1^5 + \\
& 10.949833u_1^4x_1^4 - 11.052253u_1^5x_1^3 + 4.483601u_1^6x_1^2 + \\
& 0.026229u_1^7x_1 - 0.36874u_1^8
\end{aligned}
$$

4. Pseudo remainder with p_1 over variable x_1:

$$g \quad = \quad 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 156 terms.

Time Complexity: Time spent by the prover is 0.186 seconds.

6 NDG Conditions

NDG Conditions in readable form

- There are no NDG conditions for this theorem