OpenGeoProver Output for conjecture "Chou 039 (Morley's Theorem)"

Wu's method used

February 18, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Free point D
- Line vA through two points A and D
- Line uB through two points B and D
- Angle ray AC of angle with vertex A and point B from first ray, which is three times greater than angle BAD
- Angle ray BC of angle with vertex B and point A from first ray, which is three times greater than angle ABD
- Intersection point C of point sets AC and BC
- Angle ray uA of angle with vertex A and point D from first ray, which is congruent to angle BAD
- Angle alpha of 60 degrees
- Angle ray vC of angle with vertex C and point A from first ray, which is third angle to 60 degrees for angles BAD and DBA
- Intersection point F of point sets vC and uA
- Angle ray vB of angle with vertex B and point D from first ray, which is congruent to angle ABD
- Angle ray uC of angle with vertex C and point B from first ray, which is congruent to angle FCA
- Intersection point E of point sets vB and uC

Theorem statement:

- Angle DEF is equal to angle alpha

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

2.2 Transformation of point B:

- Point B has been assigned following coordinates: (0, u_1)

2.3 Transformation of point D:

- Point D has been assigned following coordinates: (u_2, u_3)

2.4 Transformation of point C:

- Point C has been assigned following coordinates: (x_1, x_2)
- Polynomial that point C has to satisfy is:

$$p = (u_3^2 u_2 - 0.333333 u_2^3) x_2 + (-0.333333 u_3^3 + u_3 u_2^2) x_1$$

- Processing of polynomial

$$p = (u_3^2 u_2 - 0.333333 u_2^3) x_2 + (-0.333333 u_3^3 + u_3 u_2^2) x_1$$

Info: Polynomial

$$p = (u_3^2 u_2 - 0.333333 u_2^3) x_2 + (-0.333333 u_3^3 + u_3 u_2^2) x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point C has to satisfy is:

$$\begin{aligned} p = {} & (u_3^2 u_2 - 2 u_3 u_2 u_1 - 0.333333 u_2^3 + u_2 u_1^2) x_2 + \\ & (-0.333333 u_3^3 + u_3^2 u_1 + u_3 u_2^2 - u_3 u_1^2 - u_2^2 u_1 + \\ & 0.333333 u_1^3) \\ & x_1 \\ & + (-u_3^2 u_2 u_1 + 2 u_3 u_2 u_1^2 + 0.333333 u_2^3 u_1 - u_2 u_1^3) \end{aligned}$$

- Processing of polynomial

$$\begin{aligned} p &= (u_3^2u_2 - 2u_3u_2u_1 - 0.333333u_2^3 + u_2u_1^2)x_2 + \\ &\quad (-0.333333u_3^3 + u_3^2u_1 + u_3u_2^2 - u_3u_1^2 - u_2^2u_1 + \\ &\quad 0.333333u_1^3) \\ &\quad x_1 \\ &\quad +(-u_3^2u_2u_1 + 2u_3u_2u_1^2 + 0.333333u_2^3u_1 - u_2u_1^3) \end{aligned}$$

Info: Polynomial

$$\begin{aligned} p &= (u_3^2u_2 - 2u_3u_2u_1 - 0.333333u_2^3 + u_2u_1^2)x_2 + \\ &\quad (-0.333333u_3^3 + u_3^2u_1 + u_3u_2^2 - u_3u_1^2 - u_2^2u_1 + \\ &\quad 0.333333u_1^3) \\ &\quad x_1 \\ &\quad +(-u_3^2u_2u_1 + 2u_3u_2u_1^2 + 0.333333u_2^3u_1 - u_2u_1^3) \end{aligned}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of angle alpha of 60 degrees:

- Parametric pointPoint Aalpha has been assigned following coordinates: (x_3, 0)
- Polynomial

$$p = x_3^2 - 3$$

added to system of hypotheses.

2.6 Transformation of point F:

- Point F has been assigned following coordinates: (x_4, x_5)
- Polynomial that point F has to satisfy is:

$$\begin{aligned} p &= (u_3^2 - u_3u_1 + u_2^2)x_5x_3x_2 - u_2u_1x_5x_3x_1 \\ &\quad -u_2u_1x_5x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_5x_1 + u_2u_1x_4x_3x_2 + \\ &\quad (u_3^2 - u_3u_1 + u_2^2)x_4x_3x_1 + (u_3^2 - u_3u_1 + u_2^2)x_4x_2 \\ &\quad -u_2u_1x_4x_1 + (-u_3^2 + u_3u_1 - u_2^2)x_3x_2^2 + \\ &\quad (-u_3^2 + u_3u_1 - u_2^2)x_3x_1^2 + u_2u_1x_2^2 + u_2u_1x_1^2 \end{aligned}$$

- Processing of polynomial

$$\begin{aligned} p &= (u_3^2 - u_3u_1 + u_2^2)x_5x_3x_2 - u_2u_1x_5x_3x_1 \\ &\quad -u_2u_1x_5x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_5x_1 + u_2u_1x_4x_3x_2 + \\ &\quad (u_3^2 - u_3u_1 + u_2^2)x_4x_3x_1 + (u_3^2 - u_3u_1 + u_2^2)x_4x_2 \\ &\quad -u_2u_1x_4x_1 + (-u_3^2 + u_3u_1 - u_2^2)x_3x_2^2 + \\ &\quad (-u_3^2 + u_3u_1 - u_2^2)x_3x_1^2 + u_2u_1x_2^2 + u_2u_1x_1^2 \end{aligned}$$

Info: Polynomial

$$\begin{aligned} p &= (u_3^2 - u_3u_1 + u_2^2)x_5x_3x_2 - u_2u_1x_5x_3x_1 \\ &\quad -u_2u_1x_5x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_5x_1 + u_2u_1x_4x_3x_2 + \\ &\quad (u_3^2 - u_3u_1 + u_2^2)x_4x_3x_1 + (u_3^2 - u_3u_1 + u_2^2)x_4x_2 \\ &\quad -u_2u_1x_4x_1 + (-u_3^2 + u_3u_1 - u_2^2)x_3x_2^2 + \\ &\quad (-u_3^2 + u_3u_1 - u_2^2)x_3x_1^2 + u_2u_1x_2^2 + u_2u_1x_1^2 \end{aligned}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point F has to satisfy is:

$$p = u_3u_2x_5 + (-0.5u_3^2 + 0.5u_2^2)x_4$$

- Processing of polynomial

$$p = u_3u_2x_5 + (-0.5u_3^2 + 0.5u_2^2)x_4$$

Info: Polynomial

$$p = u_3u_2x_5 + (-0.5u_3^2 + 0.5u_2^2)x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point E:

- Point E has been assigned following coordinates: (x_6, x_7)
- Polynomial that point E has to satisfy is:

$$\begin{aligned} p &= (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 + \\ &\quad (-u_3u_2u_1 + u_2u_1^2) \end{aligned}$$

- Processing of polynomial

$$\begin{aligned} p &= (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 + \\ &\quad (-u_3u_2u_1 + u_2u_1^2) \end{aligned}$$

Info: Polynomial

$$\begin{aligned} p &= (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 + \\ &\quad (-u_3u_2u_1 + u_2u_1^2) \end{aligned}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point E has to satisfy is:

$$\begin{aligned} p &= x_7x_5x_2x_1 - 0.5u_1x_7x_5x_1 - 0.5x_7x_4x_2^2 + 0.5u_1x_7x_4x_2 + \\ &\quad 0.5x_7x_4x_1^2 - 0.5x_7x_2^2x_1 - 0.5x_7x_1^3 - 0.5x_6x_5x_2^2 + \\ &\quad 0.5u_1x_6x_5x_2 + 0.5x_6x_5x_1^2 - x_6x_4x_2x_1 + 0.5u_1x_6x_4x_1 + \\ &\quad 0.5x_6x_2^3 - 0.5u_1x_6x_2^2 + 0.5x_6x_2x_1^2 - 0.5u_1x_6x_1^2 \\ &\quad -0.5x_5x_2^2x_1 - 0.5x_5x_1^3 + 0.5x_4x_2^3 - 0.5u_1x_4x_2^2 + \\ &\quad 0.5x_4x_2x_1^2 - 0.5u_1x_4x_1^2 + 0.5u_1x_2^2x_1 + 0.5u_1x_1^3 \end{aligned}$$

- Processing of polynomial

$$\begin{aligned} p &= x_7x_5x_2x_1 - 0.5u_1x_7x_5x_1 - 0.5x_7x_4x_2^2 + 0.5u_1x_7x_4x_2 + \\ &\quad 0.5x_7x_4x_1^2 - 0.5x_7x_2^2x_1 - 0.5x_7x_1^3 - 0.5x_6x_5x_2^2 + \\ &\quad 0.5u_1x_6x_5x_2 + 0.5x_6x_5x_1^2 - x_6x_4x_2x_1 + 0.5u_1x_6x_4x_1 + \\ &\quad 0.5x_6x_2^3 - 0.5u_1x_6x_2^2 + 0.5x_6x_2x_1^2 - 0.5u_1x_6x_1^2 \\ &\quad -0.5x_5x_2^2x_1 - 0.5x_5x_1^3 + 0.5x_4x_2^3 - 0.5u_1x_4x_2^2 + \\ &\quad 0.5x_4x_2x_1^2 - 0.5u_1x_4x_1^2 + 0.5u_1x_2^2x_1 + 0.5u_1x_1^3 \end{aligned}$$

  Info: Polynomial

$$\begin{aligned} p &= x_7x_5x_2x_1 - 0.5u_1x_7x_5x_1 - 0.5x_7x_4x_2^2 + 0.5u_1x_7x_4x_2 + \\ &\quad 0.5x_7x_4x_1^2 - 0.5x_7x_2^2x_1 - 0.5x_7x_1^3 - 0.5x_6x_5x_2^2 + \\ &\quad 0.5u_1x_6x_5x_2 + 0.5x_6x_5x_1^2 - x_6x_4x_2x_1 + 0.5u_1x_6x_4x_1 + \\ &\quad 0.5x_6x_2^3 - 0.5u_1x_6x_2^2 + 0.5x_6x_2x_1^2 - 0.5u_1x_6x_1^2 \\ &\quad -0.5x_5x_2^2x_1 - 0.5x_5x_1^3 + 0.5x_4x_2^3 - 0.5u_1x_4x_2^2 + \\ &\quad 0.5x_4x_2x_1^2 - 0.5u_1x_4x_1^2 + 0.5u_1x_2^2x_1 + 0.5u_1x_1^3 \end{aligned}$$

  added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

- Polynomial for theorem statement:

$$\begin{aligned} p &= x_7^2x_3 - x_7x_5x_3 - x_7x_4 - u_3x_7x_3 + u_2x_7 + x_6^2x_3 + \\ &\quad x_6x_5 - x_6x_4x_3 - u_2x_6x_3 - u_3x_6 + u_3x_5x_3 - u_2x_5 + \\ &\quad u_2x_4x_3 + u_3x_4 \end{aligned}$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.515 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$
\begin{aligned}
p_1 &= (u_3^2u_2 - 0.333333u_2^3)x_2 + (-0.333333u_3^3 + u_3u_2^2)x_1 \\
p_2 &= (u_3^2u_2 - 2u_3u_2u_1 - 0.333333u_2^3 + u_2u_1^2)x_2 + \\
&\quad (-0.333333u_3^3 + u_3^2u_1 + u_3u_2^2 - u_3u_1^2 - u_2^2u_1 + \\
&\quad 0.333333u_1^3) \\
&\quad x_1 \\
&\quad +(-u_3^2u_2u_1 + 2u_3u_2u_1^2 + 0.333333u_2^3u_1 - u_2u_1^3) \\
p_3 &= x_3^2 - 3 \\
p_4 &= (u_3^2 - u_3u_1 + u_2^2)x_5x_3x_2 - u_2u_1x_5x_3x_1 \\
&\quad -u_2u_1x_5x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_5x_1 + u_2u_1x_4x_3x_2 + \\
&\quad (u_3^2 - u_3u_1 + u_2^2)x_4x_3x_1 + (u_3^2 - u_3u_1 + u_2^2)x_4x_2 \\
&\quad -u_2u_1x_4x_1 + (-u_3^2 + u_3u_1 - u_2^2)x_3x_2^2 + \\
&\quad (-u_3^2 + u_3u_1 - u_2^2)x_3x_1^2 + u_2u_1x_2^2 + u_2u_1x_1^2 \\
p_5 &= u_3u_2x_5 + (-0.5u_3^2 + 0.5u_2^2)x_4 \\
p_6 &= (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 + \\
&\quad (-u_3u_2u_1 + u_2u_1^2) \\
p_7 &= x_7x_5x_2x_1 - 0.5u_1x_7x_5x_1 - 0.5x_7x_4x_2^2 + 0.5u_1x_7x_4x_2 + \\
&\quad 0.5x_7x_4x_1^2 - 0.5x_7x_2^2x_1 - 0.5x_7x_1^3 - 0.5x_6x_5x_2^2 + \\
&\quad 0.5u_1x_6x_5x_2 + 0.5x_6x_5x_1^2 - x_6x_4x_2x_1 + 0.5u_1x_6x_4x_1 + \\
&\quad 0.5x_6x_2^3 - 0.5u_1x_6x_2^2 + 0.5x_6x_2x_1^2 - 0.5u_1x_6x_1^2 \\
&\quad -0.5x_5x_2^2x_1 - 0.5x_5x_1^3 + 0.5x_4x_2^3 - 0.5u_1x_4x_2^2 + \\
&\quad 0.5x_4x_2x_1^2 - 0.5u_1x_4x_1^2 + 0.5u_1x_2^2x_1 + 0.5u_1x_1^3
\end{aligned}
$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_7 from all other polynomials by reducing them with polynomial p_6 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (u_3^2u_2 - 0.333333u_2^3)x_2 + (-0.333333u_3^3 + u_3u_2^2)x_1 \\
p_2 &= (u_3^2u_2 - 2u_3u_2u_1 - 0.333333u_2^3 + u_2u_1^2)x_2 + \\
&\quad (-0.333333u_3^3 + u_3^2u_1 + u_3u_2^2 - u_3u_1^2 - u_2^2u_1 + \\
&\quad 0.333333u_1^3) \\
&\quad x_1
\end{aligned}
$$

$$
\begin{aligned}
& +(-u_3^2u_2u_1 + 2u_3u_2u_1^2 + 0.333333u_2^3u_1 - u_2u_1^3) \\
p_3 &= x_3^2 - 3 \\
p_4 &= (u_3^2 - u_3u_1 + u_2^2)x_5x_3x_2 - u_2u_1x_5x_3x_1 \\
& -u_2u_1x_5x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_5x_1 + u_2u_1x_4x_3x_2 + \\
& (u_3^2 - u_3u_1 + u_2^2)x_4x_3x_1 + (u_3^2 - u_3u_1 + u_2^2)x_4x_2 \\
& -u_2u_1x_4x_1 + (-u_3^2 + u_3u_1 - u_2^2)x_3x_2^2 + \\
& (-u_3^2 + u_3u_1 - u_2^2)x_3x_1^2 + u_2u_1x_2^2 + u_2u_1x_1^2 \\
p_5 &= u_3u_2x_5 + (-0.5u_3^2 + 0.5u_2^2)x_4 \\
p_6 &= (-0.5u_3u_2 + 0.5u_2u_1)x_6x_5x_2^2 + \\
& (0.5u_3^2 - u_3u_1 - 0.5u_2^2 + 0.5u_1^2)x_6x_5x_2x_1 + \\
& (0.5u_3u_2u_1 - 0.5u_2u_1^2)x_6x_5x_2 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_6x_5x_1^2 + \\
& (-0.25u_3^2u_1 + 0.5u_3u_1^2 + 0.25u_2^2u_1 - 0.25u_1^3)x_6x_5x_1 + \\
& (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_4x_2^2 + \\
& (-u_3u_2 + u_2u_1)x_6x_4x_2x_1 + \\
& (0.25u_3^2u_1 - 0.5u_3u_1^2 - 0.25u_2^2u_1 + 0.25u_1^3)x_6x_4x_2 + \\
& (0.25u_3^2 - 0.5u_3u_1 - 0.25u_2^2 + 0.25u_1^2)x_6x_4x_1^2 + \\
& (0.5u_3u_2u_1 - 0.5u_2u_1^2)x_6x_4x_1 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_6x_2^3 + \\
& (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_2^2x_1 + \\
& (-0.5u_3u_2u_1 + 0.5u_2u_1^2)x_6x_2^2 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_6x_2x_1^2 + \\
& (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_1^3 + \\
& (-0.5u_3u_2u_1 + 0.5u_2u_1^2)x_6x_1^2 + \\
& (-0.5u_3u_2 + 0.5u_2u_1)x_5x_2^2x_1 + \\
& (u_3u_2u_1 - u_2u_1^2)x_5x_2x_1 + \\
& (-0.5u_3u_2 + 0.5u_2u_1)x_5x_1^3 + \\
& (-0.5u_3u_2u_1^2 + 0.5u_2u_1^3)x_5x_1 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_4x_2^3 + (-u_3u_2u_1 + u_2u_1^2)x_4x_2^2 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_4x_2x_1^2 + \\
& (0.5u_3u_2u_1^2 - 0.5u_2u_1^3)x_4x_2 \\
p_7 &= (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 + \\
& (-u_3u_2u_1 + u_2u_1^2)
\end{aligned}
$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_5 from all other polynomials by reducing them with polynomial p_4 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (u_3^2u_2 - 0.333333u_2^3)x_2 + (-0.333333u_3^3 + u_3u_2^2)x_1 \\
p_2 &= (u_3^2u_2 - 2u_3u_2u_1 - 0.333333u_2^3 + u_2u_1^2)x_2 + \\
&\quad (-0.333333u_3^3 + u_3^2u_1 + u_3u_2^2 - u_3u_1^2 - u_2^2u_1 + \\
&\quad 0.333333u_1^3) \\
&\quad x_1 \\
&\quad +(-u_3^2u_2u_1 + 2u_3u_2u_1^2 + 0.333333u_2^3u_1 - u_2u_1^3) \\
p_3 &= x_3^2 - 3 \\
p_4 &= (-0.5u_3^4 + 0.5u_3^3u_1 - 1.5u_3u_2^2u_1 + 0.5u_2^4)x_4x_3x_2 + \\
&\quad (-u_3^3u_2 + 1.5u_3^2u_2u_1 - u_3u_2^3 - 0.5u_2^3u_1)x_4x_3x_1 + \\
&\quad (-u_3^3u_2 + 1.5u_3^2u_2u_1 - u_3u_2^3 - 0.5u_2^3u_1)x_4x_2 + \\
&\quad (0.5u_3^4 - 0.5u_3^3u_1 + 1.5u_3u_2^2u_1 - 0.5u_2^4)x_4x_1 + \\
&\quad (u_3^3u_2 - u_3^2u_2u_1 + u_3u_2^3)x_3x_2^2 + \\
&\quad (u_3^3u_2 - u_3^2u_2u_1 + u_3u_2^3)x_3x_1^2 \\
&\quad -u_3u_2^2u_1x_2^2 - u_3u_2^2u_1x_1^2 \\
p_5 &= (u_3^2 - u_3u_1 + u_2^2)x_5x_3x_2 - u_2u_1x_5x_3x_1 \\
&\quad -u_2u_1x_5x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_5x_1 + u_2u_1x_4x_3x_2 + \\
&\quad (u_3^2 - u_3u_1 + u_2^2)x_4x_3x_1 + (u_3^2 - u_3u_1 + u_2^2)x_4x_2 \\
&\quad -u_2u_1x_4x_1 + (-u_3^2 + u_3u_1 - u_2^2)x_3x_2^2 + \\
&\quad (-u_3^2 + u_3u_1 - u_2^2)x_3x_1^2 + u_2u_1x_2^2 + u_2u_1x_1^2 \\
p_6 &= (-0.5u_3u_2 + 0.5u_2u_1)x_6x_5x_2^2 + \\
&\quad (0.5u_3^2 - u_3u_1 - 0.5u_2^2 + 0.5u_1^2)x_6x_5x_2x_1 + \\
&\quad (0.5u_3u_2u_1 - 0.5u_2u_1^2)x_6x_5x_2 + \\
&\quad (0.5u_3u_2 - 0.5u_2u_1)x_6x_5x_1^2 + \\
&\quad (-0.25u_3^2u_1 + 0.5u_3u_1^2 + 0.25u_2^2u_1 - 0.25u_1^3)x_6x_5x_1 + \\
&\quad (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_4x_2^2 + \\
&\quad (-u_3u_2 + u_2u_1)x_6x_4x_2x_1 + \\
&\quad (0.25u_3^2u_1 - 0.5u_3u_1^2 - 0.25u_2^2u_1 + 0.25u_1^3)x_6x_4x_2 + \\
&\quad (0.25u_3^2 - 0.5u_3u_1 - 0.25u_2^2 + 0.25u_1^2)x_6x_4x_1^2 +
\end{aligned}
$$

$$
\begin{aligned}
& (0.5u_3u_2u_1 - 0.5u_2u_1^2)x_6x_4x_1 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_6x_2^3 + \\
& (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_2^2x_1 + \\
& (-0.5u_3u_2u_1 + 0.5u_2u_1^2)x_6x_2^2 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_6x_2x_1^2 + \\
& (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_1^3 + \\
& (-0.5u_3u_2u_1 + 0.5u_2u_1^2)x_6x_1^2 + \\
& (-0.5u_3u_2 + 0.5u_2u_1)x_5x_2^2x_1 + \\
& (u_3u_2u_1 - u_2u_1^2)x_5x_2x_1 + \\
& (-0.5u_3u_2 + 0.5u_2u_1)x_5x_1^3 + \\
& (-0.5u_3u_2u_1^2 + 0.5u_2u_1^3)x_5x_1 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_4x_2^3 + (-u_3u_2u_1 + u_2u_1^2)x_4x_2^2 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_4x_2x_1^2 + \\
& (0.5u_3u_2u_1^2 - 0.5u_2u_1^3)x_4x_2 \\
p_7 \quad = \quad & (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 + \\
& (-u_3u_2u_1 + u_2u_1^2)
\end{aligned}
$$

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_2 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (0.333333u_3^4u_2u_1 - 0.666667u_3^3u_2u_1^2 + 0.666667u_3^2u_2^3u_1 + \\
&\quad 0.333333u_3^2u_2u_1^3 - 0.666667u_3u_2^3u_1^2 + 0.333333u_2^5u_1 \\
&\quad -0.111111u_2^3u_1^3) \\
&\quad x_1 \\
&\quad + \\
&\quad (-u_3^4u_2^2u_1 + 2u_3^3u_2^2u_1^2 + 0.666667u_3^2u_2^4u_1 \\
&\quad -u_3^2u_2^2u_1^3 - 0.666667u_3u_2^4u_1^2 - 0.111111u_2^6u_1 + \\
&\quad 0.333333u_2^4u_1^3) \\
\\
p_2 &= (u_3^2u_2 - 0.333333u_2^3)x_2 + (-0.333333u_3^3 + u_3u_2^2)x_1 \\
p_3 &= x_3^2 - 3 \\
p_4 &= (-0.5u_3^4 + 0.5u_3^3u_1 - 1.5u_3u_2^2u_1 + 0.5u_2^4)x_4x_3x_2 + \\
&\quad (-u_3^3u_2 + 1.5u_3^2u_2u_1 - u_3u_2^3 - 0.5u_2^3u_1)x_4x_3x_1 + \\
&\quad (-u_3^3u_2 + 1.5u_3^2u_2u_1 - u_3u_2^3 - 0.5u_2^3u_1)x_4x_2 + \\
&\quad (0.5u_3^4 - 0.5u_3^3u_1 + 1.5u_3u_2^2u_1 - 0.5u_2^4)x_4x_1 + \\
&\quad (u_3^3u_2 - u_3^2u_2u_1 + u_3u_2^3)x_3x_2^2 + \\
&\quad (u_3^3u_2 - u_3^2u_2u_1 + u_3u_2^3)x_3x_1^2 \\
&\quad -u_3u_2^2u_1x_2^2 - u_3u_2^2u_1x_1^2 \\
p_5 &= (u_3^2 - u_3u_1 + u_2^2)x_5x_3x_2 - u_2u_1x_5x_3x_1 \\
&\quad -u_2u_1x_5x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_5x_1 + u_2u_1x_4x_3x_2 + \\
&\quad (u_3^2 - u_3u_1 + u_2^2)x_4x_3x_1 + (u_3^2 - u_3u_1 + u_2^2)x_4x_2 \\
&\quad -u_2u_1x_4x_1 + (-u_3^2 + u_3u_1 - u_2^2)x_3x_2^2 + \\
&\quad (-u_3^2 + u_3u_1 - u_2^2)x_3x_1^2 + u_2u_1x_2^2 + u_2u_1x_1^2 \\
p_6 &= (-0.5u_3u_2 + 0.5u_2u_1)x_6x_5x_2^2 + \\
&\quad (0.5u_3^2 - u_3u_1 - 0.5u_2^2 + 0.5u_1^2)x_6x_5x_2x_1 + \\
&\quad (0.5u_3u_2u_1 - 0.5u_2u_1^2)x_6x_5x_2 + \\
&\quad (0.5u_3u_2 - 0.5u_2u_1)x_6x_5x_1^2 + \\
&\quad (-0.25u_3^2u_1 + 0.5u_3u_1^2 + 0.25u_2^2u_1 - 0.25u_1^3)x_6x_5x_1 + \\
&\quad (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_4x_2^2 + \\
&\quad (-u_3u_2 + u_2u_1)x_6x_4x_2x_1 + \\
&\quad (0.25u_3^2u_1 - 0.5u_3u_1^2 - 0.25u_2^2u_1 + 0.25u_1^3)x_6x_4x_2 + \\
&\quad (0.25u_3^2 - 0.5u_3u_1 - 0.25u_2^2 + 0.25u_1^2)x_6x_4x_1^2 + \\
&\quad (0.5u_3u_2u_1 - 0.5u_2u_1^2)x_6x_4x_1 + \\
&\quad (0.5u_3u_2 - 0.5u_2u_1)x_6x_2^3 + \\
&\quad (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_2^2x_1 + \\
&\quad (-0.5u_3u_2u_1 + 0.5u_2u_1^2)x_6x_2^2 +
\end{aligned}
$$

$$
\begin{aligned}
& (0.5u_3u_2 - 0.5u_2u_1)x_6x_2x_1^2 + \\
& (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_1^3 + \\
& (-0.5u_3u_2u_1 + 0.5u_2u_1^2)x_6x_1^2 + \\
& (-0.5u_3u_2 + 0.5u_2u_1)x_5x_2^2x_1 + \\
& (u_3u_2u_1 - u_2u_1^2)x_5x_2x_1 + \\
& (-0.5u_3u_2 + 0.5u_2u_1)x_5x_1^3 + \\
& (-0.5u_3u_2u_1^2 + 0.5u_2u_1^3)x_5x_1 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_4x_2^3 + (-u_3u_2u_1 + u_2u_1^2)x_4x_2^2 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_4x_2x_1^2 + \\
& (0.5u_3u_2u_1^2 - 0.5u_2u_1^3)x_4x_2 \\
p_7 \;=\; & (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 + \\
& (-u_3u_2u_1 + u_2u_1^2)
\end{aligned}
$$

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 \;=\; & (0.333333u_3^4u_2u_1 - 0.666667u_3^3u_2u_1^2 + 0.666667u_3^2u_2^3u_1 + \\
& 0.333333u_3^2u_2u_1^3 - 0.666667u_3u_2^3u_1^2 + 0.333333u_2^5u_1 \\
& -0.111111u_2^3u_1^3) \\
& x_1 \\
& + \\
& (-u_3^4u_2^2u_1 + 2u_3^3u_2^2u_1^2 + 0.666667u_3^2u_2^4u_1 \\
& -u_3^2u_2^2u_1^3 - 0.666667u_3u_2^4u_1^2 - 0.111111u_2^6u_1 + \\
& 0.333333u_2^4u_1^3)
\end{aligned}
$$

$$
\begin{aligned}
p_2 \;=\; & (u_3^2u_2 - 0.333333u_2^3)x_2 + (-0.333333u_3^3 + u_3u_2^2)x_1 \\
p_3 \;=\; & x_3^2 - 3 \\
p_4 \;=\; & (-0.5u_3^4 + 0.5u_3^3u_1 - 1.5u_3u_2^2u_1 + 0.5u_2^4)x_4x_3x_2 + \\
& (-u_3^3u_2 + 1.5u_3^2u_2u_1 - u_3u_2^3 - 0.5u_2^3u_1)x_4x_3x_1 + \\
& (-u_3^3u_2 + 1.5u_3^2u_2u_1 - u_3u_2^3 - 0.5u_2^3u_1)x_4x_2 + \\
& (0.5u_3^4 - 0.5u_3^3u_1 + 1.5u_3u_2^2u_1 - 0.5u_2^4)x_4x_1 + \\
& (u_3^3u_2 - u_3^2u_2u_1 + u_3u_2^3)x_3x_2^2 + \\
& (u_3^3u_2 - u_3^2u_2u_1 + u_3u_2^3)x_3x_1^2
\end{aligned}
$$

$$
\begin{aligned}
& -u_3u_2^2u_1x_2^2 - u_3u_2^2u_1x_1^2 \\
p_5 \;=\; & (u_3^2 - u_3u_1 + u_2^2)x_5x_3x_2 - u_2u_1x_5x_3x_1 \\
& -u_2u_1x_5x_2 + (-u_3^2 + u_3u_1 - u_2^2)x_5x_1 + u_2u_1x_4x_3x_2 + \\
& (u_3^2 - u_3u_1 + u_2^2)x_4x_3x_1 + (u_3^2 - u_3u_1 + u_2^2)x_4x_2 \\
& -u_2u_1x_4x_1 + (-u_3^2 + u_3u_1 - u_2^2)x_3x_2^2 + \\
& (-u_3^2 + u_3u_1 - u_2^2)x_3x_1^2 + u_2u_1x_2^2 + u_2u_1x_1^2 \\
p_6 \;=\; & (-0.5u_3u_2 + 0.5u_2u_1)x_6x_5x_2^2 + \\
& (0.5u_3^2 - u_3u_1 - 0.5u_2^2 + 0.5u_1^2)x_6x_5x_2x_1 + \\
& (0.5u_3u_2u_1 - 0.5u_2u_1^2)x_6x_5x_2 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_6x_5x_1^2 + \\
& (-0.25u_3^2u_1 + 0.5u_3u_1^2 + 0.25u_2^2u_1 - 0.25u_1^3)x_6x_5x_1 + \\
& (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_4x_2^2 + \\
& (-u_3u_2 + u_2u_1)x_6x_4x_2x_1 + \\
& (0.25u_3^2u_1 - 0.5u_3u_1^2 - 0.25u_2^2u_1 + 0.25u_1^3)x_6x_4x_2 + \\
& (0.25u_3^2 - 0.5u_3u_1 - 0.25u_2^2 + 0.25u_1^2)x_6x_4x_1^2 + \\
& (0.5u_3u_2u_1 - 0.5u_2u_1^2)x_6x_4x_1 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_6x_2^3 + \\
& (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_2^2x_1 + \\
& (-0.5u_3u_2u_1 + 0.5u_2u_1^2)x_6x_2^2 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_6x_2x_1^2 + \\
& (-0.25u_3^2 + 0.5u_3u_1 + 0.25u_2^2 - 0.25u_1^2)x_6x_1^3 + \\
& (-0.5u_3u_2u_1 + 0.5u_2u_1^2)x_6x_1^2 + \\
& (-0.5u_3u_2 + 0.5u_2u_1)x_5x_2^2x_1 + \\
& (u_3u_2u_1 - u_2u_1^2)x_5x_2x_1 + \\
& (-0.5u_3u_2 + 0.5u_2u_1)x_5x_1^3 + \\
& (-0.5u_3u_2u_1^2 + 0.5u_2u_1^3)x_5x_1 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_4x_2^3 + (-u_3u_2u_1 + u_2u_1^2)x_4x_2^2 + \\
& (0.5u_3u_2 - 0.5u_2u_1)x_4x_2x_1^2 + \\
& (0.5u_3u_2u_1^2 - 0.5u_2u_1^3)x_4x_2 \\
p_7 \;=\; & (u_3u_2 - u_2u_1)x_7 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_6 + \\
& (-u_3u_2u_1 + u_2u_1^2)
\end{aligned}
$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 039 (Morley's Theorem)

Calculating final remainder of the conclusion:

$$
\begin{aligned}
g \;=\; & x_7^2x_3 - x_7x_5x_3 - x_7x_4 - u_3x_7x_3 + u_2x_7 + x_6^2x_3 + \\
& x_6x_5 - x_6x_4x_3 - u_2x_6x_3 - u_3x_6 + u_3x_5x_3 - u_2x_5 +
\end{aligned}
$$

$$u_2x_4x_3 + u_3x_4$$

with respect to the triangular system.

1. Pseudo remainder with p_7 over variable x_7:

$$\begin{aligned} g \;=\; & (0.25u_3^4 - u_3^3u_1 + 0.5u_3^2u_2^2 + 1.5u_3^2u_1^2 - u_3u_2^2u_1 \\ & -u_3u_1^3 + 0.25u_2^4 + 0.5u_2^2u_1^2 + 0.25u_1^4) \\ & x_6^2x_3 \\ & + \\ & (-0.5u_3^3u_2 + 1.5u_3^2u_2u_1 + 0.5u_3u_2^3 - 1.5u_3u_2u_1^2 \\ & -0.5u_2^3u_1 + 0.5u_2u_1^3) \\ & x_6x_5x_3 \\ & +(u_3^2u_2^2 - 2u_3u_2^2u_1 + u_2^2u_1^2)x_6x_5 + \\ & (-u_3^2u_2^2 + 2u_3u_2^2u_1 - u_2^2u_1^2)x_6x_4x_3 + \\ & (-0.5u_3^3u_2 + 1.5u_3^2u_2u_1 + 0.5u_3u_2^3 - 1.5u_3u_2u_1^2 \\ & -0.5u_2^3u_1 + 0.5u_2u_1^3) \\ & x_6x_4 \\ & + \\ & (-0.5u_3^4u_2 + 2.5u_3^3u_2u_1 - 0.5u_3^2u_2^3 - 4.5u_3^2u_2u_1^2 + \\ & 0.5u_3u_2^3u_1 + 3.5u_3u_2u_1^3 - u_2u_1^4) \\ & x_6x_3 \\ & + \\ & (-0.5u_3^3u_2^2 + 0.5u_3^2u_2^2u_1 - 0.5u_3u_2^4 + \\ & 0.5u_3u_2^2u_1^2 + 0.5u_2^4u_1 - 0.5u_2^2u_1^3) \\ & x_6 \\ & + \\ & (u_3^3u_2^2 - 3u_3^2u_2^2u_1 + 3u_3u_2^2u_1^2 - u_2^2u_1^3) \\ & x_5x_3 \\ & +(-u_3^2u_2^3 + 2u_3u_2^3u_1 - u_2^3u_1^2)x_5 + \\ & (u_3^2u_2^3 - 2u_3u_2^3u_1 + u_2^3u_1^2)x_4x_3 + \\ & (u_3^3u_2^2 - 3u_3^2u_2^2u_1 + 3u_3u_2^2u_1^2 - u_2^2u_1^3) \\ & x_4 \\ & + \\ & (-u_3^3u_2^2u_1 + 3u_3^2u_2^2u_1^2 - 3u_3u_2^2u_1^3 + \\ & u_2^2u_1^4) \\ & x_3 \\ & +(u_3^2u_2^3u_1 - 2u_3u_2^3u_1^2 + u_2^3u_1^3) \end{aligned}$$

2. Pseudo remainder with p_6 over variable x_6:

   Polynomial too big for output (number of terms is 391)

3. Pseudo remainder with p_5 over variable x_5:

   Polynomial too big for output (number of terms is 750*)*

4. Pseudo remainder with p_4 over variable x_4:

   Polynomial too big for output (number of terms is 520*)*

5. Pseudo remainder with p_3 over variable x_3:

   Polynomial too big for output (text size is 579876 *characters, number of terms is* 130*)*

6. Pseudo remainder with p_2 over variable x_2:

   Polynomial too big for output (text size is greater than 2000 *characters, number of terms is* 10*)*

7. Pseudo remainder with p_1 over variable x_1:

   Polynomial too big for output (text size is greater than 2000 *characters, number of terms is* 2*)*

5 Prover results

Status: Theorem can't be neither proved nor disproved.

Space Complexity: The biggest polynomial obtained during prover execution contains 836 terms.

Time Complexity: Time spent by the prover is 23.166 seconds.