

OpenGeoProver Output for conjecture “Chou 013 (Steiner’s Theorem)”

Wu’s method used

February 12, 2012

1 Validation of Construction Protocol

Construction steps:

- General Conic Section c
- Random point A from general conic c
- Random point B from general conic c
- Random point C from general conic c
- Random point D from general conic c
- Random point E from general conic c
- Random point F from general conic c
- Line AB through two points A and B
- Line DC through two points D and C
- Intersection point $P1$ of point sets AB and DC
- Line BE through two points B and E
- Line CF through two points C and F
- Intersection point $Q1$ of point sets BE and CF
- Line $P1Q1$ through two points $P1$ and $Q1$
- Line FE through two points F and E
- Intersection point $P2$ of point sets DC and FE
- Line DA through two points D and A
- Intersection point $Q2$ of point sets DA and BE
- Line $P2Q2$ through two points $P2$ and $Q2$
- Intersection point $P3$ of point sets FE and AB
- Intersection point $Q3$ of point sets CF and DA
- Line $P3Q3$ through two points $P3$ and $Q3$

Theorem statement:

- Lines P1Q1, P2Q2, P3Q3 are concurrent

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of general conic section c:

List of parametric points

- Point Ac has been assigned following coordinates: $(u_1, 0)$
- Point Bc has been assigned following coordinates: $(u_2, 0)$
- Point Cc has been assigned following coordinates: $(u_3, 0)$
- Point Dc has been assigned following coordinates: $(u_4, 0)$
- Point Ec has been assigned following coordinates: $(u_5, 0)$
- Condition for point $X(x_1, x_2)$ to belong to this conic section is following equation:

$$p = u_3x_2^2 + u_2x_2x_1 + u_5x_2 + u_1x_1^2 + u_4x_1$$

2.2 Transformation of point A:

- Point A has been assigned following coordinates: $(0, 0)$

2.3 Transformation of point B:

- Point B has been assigned following coordinates: $(0, x_1)$
- Polynomial that point B has to satisfy is:

$$p = u_3x_1^2 + u_5x_1$$

- Processing of polynomial

$$p = u_3x_1^2 + u_5x_1$$

Info: Polynomial

$$p = u_3x_1^2 + u_5x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.4 Transformation of point C:

- Point C has been assigned following coordinates: (u_6, x_2)
- Polynomial that point C has to satisfy is:

$$p = u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)$$

- Processing of polynomial

$$p = u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)$$

Info: Polynomial

$$p = u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of point D:

- Point D has been assigned following coordinates: (u_7, x_3)
- Polynomial that point D has to satisfy is:

$$p = u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4)$$

- Processing of polynomial

$$p = u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4)$$

Info: Polynomial

$$p = u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point E:

- Point E has been assigned following coordinates: (u_8, x_4)
- Polynomial that point E has to satisfy is:

$$p = u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)$$

- Processing of polynomial

$$p = u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)$$

Info: Polynomial

$$p = u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point F:

- Point F has been assigned following coordinates: (u_9, x_5)
- Polynomial that point F has to satisfy is:

$$p = u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4)$$

- Processing of polynomial

$$p = u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4)$$

Info: Polynomial

$$p = u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.8 Transformation of point P1:

- Point P1 has been assigned following coordinates: (x_6, x_7)
- Polynomial that point P1 has to satisfy is:

$$p = x_6x_1$$

- Processing of polynomial

$$p = x_6x_1$$

Info: Polynomial

$$p = x_6x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point P1 has to satisfy is:

$$p = (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2$$

- Processing of polynomial

$$p = (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2$$

Info: Polynomial

$$p = (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.9 Transformation of point Q1:

- Point Q1 has been assigned following coordinates: (x_8, x_9)
- Polynomial that point Q1 has to satisfy is:

$$p = u_8x_9 - x_8x_4 + x_8x_1 - u_8x_1$$

- Processing of polynomial

$$p = u_8x_9 - x_8x_4 + x_8x_1 - u_8x_1$$

Info: Polynomial

$$p = u_8x_9 - x_8x_4 + x_8x_1 - u_8x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point Q1 has to satisfy is:

$$p = (u_9 - u_6)x_9 - x_8x_5 + x_8x_2 + u_6x_5 - u_9x_2$$

- Processing of polynomial

$$p = (u_9 - u_6)x_9 - x_8x_5 + x_8x_2 + u_6x_5 - u_9x_2$$

Info: Polynomial

$$p = (u_9 - u_6)x_9 - x_8x_5 + x_8x_2 + u_6x_5 - u_9x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.10 Transformation of point P2:

- Point P2 has been assigned following coordinates: (x_{10}, x_{11})
- Polynomial that point P2 has to satisfy is:

$$p = (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2$$

- Processing of polynomial

$$p = (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2$$

Info: Polynomial

$$p = (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point P2 has to satisfy is:

$$p = (u_9 - u_8)x_{11} - x_{10}x_5 + x_{10}x_4 + u_8x_5 - u_9x_4$$

- Processing of polynomial

$$p = (u_9 - u_8)x_{11} - x_{10}x_5 + x_{10}x_4 + u_8x_5 - u_9x_4$$

Info: Polynomial

$$p = (u_9 - u_8)x_{11} - x_{10}x_5 + x_{10}x_4 + u_8x_5 - u_9x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.11 Transformation of point Q2:

- Point Q2 has been assigned following coordinates: (x_{12}, x_{13})
- Polynomial that point Q2 has to satisfy is:

$$p = u_7x_{13} - x_{12}x_3$$

- Processing of polynomial

$$p = u_7x_{13} - x_{12}x_3$$

Info: Polynomial

$$p = u_7x_{13} - x_{12}x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point Q2 has to satisfy is:

$$p = u_8x_{13} - x_{12}x_4 + x_{12}x_1 - u_8x_1$$

- Processing of polynomial

$$p = u_8x_{13} - x_{12}x_4 + x_{12}x_1 - u_8x_1$$

Info: Polynomial

$$p = u_8x_{13} - x_{12}x_4 + x_{12}x_1 - u_8x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.12 Transformation of point P3:

- Point P3 has been assigned following coordinates: (x_{14}, x_{15})
- Polynomial that point P3 has to satisfy is:

$$p = (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4$$

- Processing of polynomial

$$p = (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4$$

Info: Polynomial

$$p = (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point P3 has to satisfy is:

$$p = x_{14}x_1$$

- Processing of polynomial

$$p = x_{14}x_1$$

Info: Polynomial

$$p = x_{14}x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.13 Transformation of point Q3:

- Point Q3 has been assigned following coordinates: (x_{16}, x_{17})
- Polynomial that point Q3 has to satisfy is:

$$p = (u_9 - u_6)x_{17} - x_{16}x_5 + x_{16}x_2 + u_6x_5 - u_9x_2$$

- Processing of polynomial

$$p = (u_9 - u_6)x_{17} - x_{16}x_5 + x_{16}x_2 + u_6x_5 - u_9x_2$$

Info: Polynomial

$$p = (u_9 - u_6)x_{17} - x_{16}x_5 + x_{16}x_2 + u_6x_5 - u_9x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point Q3 has to satisfy is:

$$p = u_7x_{17} - x_{16}x_3$$

- Processing of polynomial

$$p = u_7x_{17} - x_{16}x_3$$

Info: Polynomial

$$p = u_7x_{17} - x_{16}x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

2.14 Transformation of point intersectPoint-P1Q1.P2Q2:

- Point intersectPoint-P1Q1.P2Q2 has been assigned following coordinates:
(x_{18}, x_{19})

- Polynomial that point intersectPoint-P1Q1.P2Q2 has to satisfy is:

$$p = x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7$$

- Processing of polynomial

$$p = x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7$$

Info: Polynomial

$$p = x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point intersectPoint-P1Q1.P2Q2 has to satisfy is:

$$p = x_{19}x_{12} - x_{19}x_{10} - x_{18}x_{13} + x_{18}x_{11} + x_{13}x_{10} - x_{12}x_{11}$$

- Processing of polynomial

$$p = x_{19}x_{12} - x_{19}x_{10} - x_{18}x_{13} + x_{18}x_{11} + x_{13}x_{10} - x_{12}x_{11}$$

Info: Polynomial

$$p = x_{19}x_{12} - x_{19}x_{10} - x_{18}x_{13} + x_{18}x_{11} + x_{13}x_{10} - x_{12}x_{11}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial for theorem statement:

$$p = x_{19}x_{16} - x_{19}x_{14} - x_{18}x_{17} + x_{18}x_{15} + x_{17}x_{14} - x_{16}x_{15}$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.23 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned} p_1 &= u_3x_1^2 + u_5x_1 \\ p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\ p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\ p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\ p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\ p_6 &= x_6x_1 \\ p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\ p_8 &= u_8x_9 - x_8x_4 + x_8x_1 - u_8x_1 \\ p_9 &= (u_9 - u_6)x_9 - x_8x_5 + x_8x_2 + u_6x_5 - u_9x_2 \\ p_{10} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2 \\ p_{11} &= (u_9 - u_8)x_{11} - x_{10}x_5 + x_{10}x_4 + u_8x_5 - u_9x_4 \\ p_{12} &= u_7x_{13} - x_{12}x_3 \\ p_{13} &= u_8x_{13} - x_{12}x_4 + x_{12}x_1 - u_8x_1 \\ p_{14} &= (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4 \\ p_{15} &= x_{14}x_1 \\ p_{16} &= (u_9 - u_6)x_{17} - x_{16}x_5 + x_{16}x_2 + u_6x_5 - u_9x_2 \\ p_{17} &= u_7x_{17} - x_{16}x_3 \\ p_{18} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7 \\ p_{19} &= x_{19}x_{12} - x_{19}x_{10} - x_{18}x_{13} + x_{18}x_{11} + x_{13}x_{10} - x_{12}x_{11} \end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 19.

Variable x_{19} selected: The number of polynomials with this variable, with indexes from 1 to 19, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{19} from all other polynomials by reducing them with polynomial p_{18} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= u_8x_9 - x_8x_4 + x_8x_1 - u_8x_1 \\
p_9 &= (u_9 - u_6)x_9 - x_8x_5 + x_8x_2 + u_6x_5 - u_9x_2 \\
p_{10} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2 \\
p_{11} &= (u_9 - u_8)x_{11} - x_{10}x_5 + x_{10}x_4 + u_8x_5 - u_9x_4 \\
p_{12} &= u_7x_{13} - x_{12}x_3 \\
p_{13} &= u_8x_{13} - x_{12}x_4 + x_{12}x_1 - u_8x_1 \\
p_{14} &= (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4 \\
p_{15} &= x_{14}x_1 \\
p_{16} &= (u_9 - u_6)x_{17} - x_{16}x_5 + x_{16}x_2 + u_6x_5 - u_9x_2 \\
p_{17} &= u_7x_{17} - x_{16}x_3 \\
p_{18} &= -x_{18}x_{13}x_8 + x_{18}x_{13}x_6 + x_{18}x_{12}x_9 - x_{18}x_{12}x_7 + x_{18}x_{11}x_8 \\
&\quad - x_{18}x_{11}x_6 - x_{18}x_{10}x_9 + x_{18}x_{10}x_7 + x_{13}x_{10}x_8 - x_{13}x_{10}x_6 \\
&\quad - x_{12}x_{11}x_8 + x_{12}x_{11}x_6 - x_{12}x_9x_6 + x_{12}x_8x_7 + x_{10}x_9x_6 \\
&\quad - x_{10}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7
\end{aligned}$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{18} . No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{17} from all other polynomials by reducing them with polynomial p_{16} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= u_8x_9 - x_8x_4 + x_8x_1 - u_8x_1 \\
p_9 &= (u_9 - u_6)x_9 - x_8x_5 + x_8x_2 + u_6x_5 - u_9x_2 \\
p_{10} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2 \\
p_{11} &= (u_9 - u_8)x_{11} - x_{10}x_5 + x_{10}x_4 + u_8x_5 - u_9x_4 \\
p_{12} &= u_7x_{13} - x_{12}x_3 \\
p_{13} &= u_8x_{13} - x_{12}x_4 + x_{12}x_1 - u_8x_1 \\
p_{14} &= (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4 \\
p_{15} &= x_{14}x_1 \\
p_{16} &= u_7x_{16}x_5 + (-u_9 + u_6)x_{16}x_3 - u_7x_{16}x_2 - u_7u_6x_5 + u_9u_7x_2 \\
p_{17} &= (u_9 - u_6)x_{17} - x_{16}x_5 + x_{16}x_2 + u_6x_5 - u_9x_2 \\
p_{18} &= -x_{18}x_{13}x_8 + x_{18}x_{13}x_6 + x_{18}x_{12}x_9 - x_{18}x_{12}x_7 + x_{18}x_{11}x_8 \\
&\quad - x_{18}x_{11}x_6 - x_{18}x_{10}x_9 + x_{18}x_{10}x_7 + x_{13}x_{10}x_8 - x_{13}x_{10}x_6 \\
&\quad - x_{12}x_{11}x_8 + x_{12}x_{11}x_6 - x_{12}x_9x_6 + x_{12}x_8x_7 + x_{10}x_9x_6 \\
&\quad - x_{10}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7
\end{aligned}$$

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{16} . No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14} . No reduction needed.

The triangular system has not been changed.

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14} . No reduction needed.

The triangular system has not been changed.

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{13} from all other polynomials by reducing them with polynomial p_{12} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= u_8x_9 - x_8x_4 + x_8x_1 - u_8x_1 \\
p_9 &= (u_9 - u_6)x_9 - x_8x_5 + x_8x_2 + u_6x_5 - u_9x_2 \\
p_{10} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2 \\
p_{11} &= (u_9 - u_8)x_{11} - x_{10}x_5 + x_{10}x_4 + u_8x_5 - u_9x_4 \\
p_{12} &= -u_7x_{12}x_4 + u_8x_{12}x_3 + u_7x_{12}x_1 - u_8u_7x_1 \\
p_{13} &= u_7x_{13} - x_{12}x_3 \\
p_{14} &= x_{14}x_1 \\
p_{15} &= (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4 \\
p_{16} &= u_7x_{16}x_5 + (-u_9 + u_6)x_{16}x_3 - u_7x_{16}x_2 - u_7u_6x_5 + u_9u_7x_2 \\
p_{17} &= (u_9 - u_6)x_{17} - x_{16}x_5 + x_{16}x_2 + u_6x_5 - u_9x_2 \\
p_{18} &= -x_{18}x_{13}x_8 + x_{18}x_{13}x_6 + x_{18}x_{12}x_9 - x_{18}x_{12}x_7 + x_{18}x_{11}x_8 \\
&\quad - x_{18}x_{11}x_6 - x_{18}x_{10}x_9 + x_{18}x_{10}x_7 + x_{13}x_{10}x_8 - x_{13}x_{10}x_6
\end{aligned}$$

$$\begin{aligned}
& -x_{12}x_{11}x_8 + x_{12}x_{11}x_6 - x_{12}x_9x_6 + x_{12}x_8x_7 + x_{10}x_9x_6 \\
& -x_{10}x_8x_7 \\
p_{19} = & x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7
\end{aligned}$$

3.8 Triangulation, step 8

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{12} . No reduction needed.

The triangular system has not been changed.

3.9 Triangulation, step 9

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{11} from all other polynomials by reducing them with polynomial p_{10} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= u_8x_9 - x_8x_4 + x_8x_1 - u_8x_1 \\
p_9 &= (u_9 - u_6)x_9 - x_8x_5 + x_8x_2 + u_6x_5 - u_9x_2 \\
p_{10} &= (-u_7 + u_6)x_{10}x_5 + (u_7 - u_6)x_{10}x_4 + (u_9 - u_8)x_{10}x_3 + \\
& (-u_9 + u_8)x_{10}x_2 + (u_8u_7 - u_8u_6)x_5 + (-u_9u_7 + u_9u_6)x_4 + \\
& (-u_9u_6 + u_8u_6)x_3 + (u_9u_7 - u_8u_7)x_2 \\
p_{11} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2 \\
p_{12} &= -u_7x_{12}x_4 + u_8x_{12}x_3 + u_7x_{12}x_1 - u_8u_7x_1 \\
p_{13} &= u_7x_{13} - x_{12}x_3 \\
p_{14} &= x_{14}x_1 \\
p_{15} &= (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4
\end{aligned}$$

$$\begin{aligned}
p_{16} &= u_7x_{16}x_5 + (-u_9 + u_6)x_{16}x_3 - u_7x_{16}x_2 - u_7u_6x_5 + u_9u_7x_2 \\
p_{17} &= (u_9 - u_6)x_{17} - x_{16}x_5 + x_{16}x_2 + u_6x_5 - u_9x_2 \\
p_{18} &= -x_{18}x_{13}x_8 + x_{18}x_{13}x_6 + x_{18}x_{12}x_9 - x_{18}x_{12}x_7 + x_{18}x_{11}x_8 \\
&\quad - x_{18}x_{11}x_6 - x_{18}x_{10}x_9 + x_{18}x_{10}x_7 + x_{13}x_{10}x_8 - x_{13}x_{10}x_6 \\
&\quad - x_{12}x_{11}x_8 + x_{12}x_{11}x_6 - x_{12}x_9x_6 + x_{12}x_8x_7 + x_{10}x_9x_6 \\
&\quad - x_{10}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7
\end{aligned}$$

3.10 Triangulation, step 10

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10} . No reduction needed.

The triangular system has not been changed.

3.11 Triangulation, step 11

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_9 from all other polynomials by reducing them with polynomial p_8 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= -u_8x_8x_5 + (u_9 - u_6)x_8x_4 + u_8x_8x_2 + (-u_9 + u_6)x_8x_1 + \\
&\quad u_8u_6x_5 - u_9u_8x_2 + (u_9u_8 - u_8u_6)x_1 \\
p_9 &= u_8x_9 - x_8x_4 + x_8x_1 - u_8x_1 \\
p_{10} &= (-u_7 + u_6)x_{10}x_5 + (u_7 - u_6)x_{10}x_4 + (u_9 - u_8)x_{10}x_3 + \\
&\quad (-u_9 + u_8)x_{10}x_2 + (u_8u_7 - u_8u_6)x_5 + (-u_9u_7 + u_9u_6)x_4 + \\
&\quad (-u_9u_6 + u_8u_6)x_3 + (u_9u_7 - u_8u_7)x_2
\end{aligned}$$

$$\begin{aligned}
p_{11} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2 \\
p_{12} &= -u_7x_{12}x_4 + u_8x_{12}x_3 + u_7x_{12}x_1 - u_8u_7x_1 \\
p_{13} &= u_7x_{13} - x_{12}x_3 \\
p_{14} &= x_{14}x_1 \\
p_{15} &= (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4 \\
p_{16} &= u_7x_{16}x_5 + (-u_9 + u_6)x_{16}x_3 - u_7x_{16}x_2 - u_7u_6x_5 + u_9u_7x_2 \\
p_{17} &= (u_9 - u_6)x_{17} - x_{16}x_5 + x_{16}x_2 + u_6x_5 - u_9x_2 \\
p_{18} &= -x_{18}x_{13}x_8 + x_{18}x_{13}x_6 + x_{18}x_{12}x_9 - x_{18}x_{12}x_7 + x_{18}x_{11}x_8 \\
&\quad - x_{18}x_{11}x_6 - x_{18}x_{10}x_9 + x_{18}x_{10}x_7 + x_{13}x_{10}x_8 - x_{13}x_{10}x_6 \\
&\quad - x_{12}x_{11}x_8 + x_{12}x_{11}x_6 - x_{12}x_9x_6 + x_{12}x_8x_7 + x_{10}x_9x_6 \\
&\quad - x_{10}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7
\end{aligned}$$

3.12 Triangulation, step 12

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8 . No reduction needed.

The triangular system has not been changed.

3.13 Triangulation, step 13

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7 . No reduction needed.

The triangular system has not been changed.

3.14 Triangulation, step 14

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6 . No reduction needed.

The triangular system has not been changed.

3.15 Triangulation, step 15

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5 . No reduction needed.

The triangular system has not been changed.

3.16 Triangulation, step 16

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4 . No reduction needed.

The triangular system has not been changed.

3.17 Triangulation, step 17

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3 . No reduction needed.

The triangular system has not been changed.

3.18 Triangulation, step 18

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2 . No reduction needed.

The triangular system has not been changed.

3.19 Triangulation, step 19

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1 . No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= -u_8x_8x_5 + (u_9 - u_6)x_8x_4 + u_8x_8x_2 + (-u_9 + u_6)x_8x_1 + \\
&\quad u_8u_6x_5 - u_9u_8x_2 + (u_9u_8 - u_8u_6)x_1 \\
p_9 &= u_8x_9 - x_8x_4 + x_8x_1 - u_8x_1 \\
p_{10} &= (-u_7 + u_6)x_{10}x_5 + (u_7 - u_6)x_{10}x_4 + (u_9 - u_8)x_{10}x_3 + \\
&\quad (-u_9 + u_8)x_{10}x_2 + (u_8u_7 - u_8u_6)x_5 + (-u_9u_7 + u_9u_6)x_4 + \\
&\quad (-u_9u_6 + u_8u_6)x_3 + (u_9u_7 - u_8u_7)x_2 \\
p_{11} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2 \\
p_{12} &= -u_7x_{12}x_4 + u_8x_{12}x_3 + u_7x_{12}x_1 - u_8u_7x_1 \\
p_{13} &= u_7x_{13} - x_{12}x_3 \\
p_{14} &= x_{14}x_1 \\
p_{15} &= (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4 \\
p_{16} &= u_7x_{16}x_5 + (-u_9 + u_6)x_{16}x_3 - u_7x_{16}x_2 - u_7u_6x_5 + u_9u_7x_2 \\
p_{17} &= (u_9 - u_6)x_{17} - x_{16}x_5 + x_{16}x_2 + u_6x_5 - u_9x_2 \\
p_{18} &= -x_{18}x_{13}x_8 + x_{18}x_{13}x_6 + x_{18}x_{12}x_9 - x_{18}x_{12}x_7 + x_{18}x_{11}x_8 \\
&\quad -x_{18}x_{11}x_6 - x_{18}x_{10}x_9 + x_{18}x_{10}x_7 + x_{13}x_{10}x_8 - x_{13}x_{10}x_6 \\
&\quad -x_{12}x_{11}x_8 + x_{12}x_{11}x_6 - x_{12}x_9x_6 + x_{12}x_8x_7 + x_{10}x_9x_6 \\
&\quad -x_{10}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7
\end{aligned}$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 013 (Steiner's Theorem)

Calculating final remainder of the conclusion:

$$g = x_{19}x_{16} - x_{19}x_{14} - x_{18}x_{17} + x_{18}x_{15} + x_{17}x_{14} - x_{16}x_{15}$$

with respect to the triangular system.

1. Pseudo remainder with p_{19} over variable x_{19} :

$$\begin{aligned} g = & -x_{18}x_{17}x_8 + x_{18}x_{17}x_6 + x_{18}x_{16}x_9 - x_{18}x_{16}x_7 + x_{18}x_{15}x_8 \\ & -x_{18}x_{15}x_6 - x_{18}x_{14}x_9 + x_{18}x_{14}x_7 + x_{17}x_{14}x_8 - x_{17}x_{14}x_6 \\ & -x_{16}x_{15}x_8 + x_{16}x_{15}x_6 - x_{16}x_9x_6 + x_{16}x_8x_7 + x_{14}x_9x_6 \\ & -x_{14}x_8x_7 \end{aligned}$$

2. Pseudo remainder with p_{18} over variable x_{18} :

Polynomial too big for output (text size is 2642 characters, number of terms is 88)

3. Pseudo remainder with p_{17} over variable x_{17} :

Polynomial too big for output (text size is 6601 characters, number of terms is 172)

4. Pseudo remainder with p_{16} over variable x_{16} :

Polynomial too big for output (text size is 12912 characters, number of terms is 206)

5. Pseudo remainder with p_{15} over variable x_{15} :

Polynomial too big for output (number of terms is 402)

6. Pseudo remainder with p_{14} over variable x_{14} :

Polynomial too big for output (text size is 16322 characters, number of terms is 186)

7. Pseudo remainder with p_{13} over variable x_{13} :

Polynomial too big for output (text size is 16613 characters, number of terms is 174)

8. Pseudo remainder with p_{12} over variable x_{12} :

Polynomial too big for output (number of terms is 282)

9. Pseudo remainder with p_{11} over variable x_{11} :

Polynomial too big for output (number of terms is 402)

10. Pseudo remainder with p_{10} over variable x_{10} :
Polynomial too big for output (number of terms is 428)
11. Pseudo remainder with p_9 over variable x_9 :
Polynomial too big for output (number of terms is 543)
12. Pseudo remainder with p_8 over variable x_8 :
Polynomial too big for output (number of terms is 881)
13. Pseudo remainder with p_7 over variable x_7 :
Polynomial too big for output (number of terms is 712)
14. Pseudo remainder with p_6 over variable x_6 :
Polynomial too big for output (text size is 246434 characters, number of terms is 228)
15. Pseudo remainder with p_5 over variable x_5 :
Polynomial too big for output (number of terms is 336)
16. Pseudo remainder with p_4 over variable x_4 :
Polynomial too big for output (text size is greater than 2000 characters, number of terms is 251)
17. Pseudo remainder with p_3 over variable x_3 :
Polynomial too big for output (text size is greater than 2000 characters, number of terms is 148)
18. Pseudo remainder with p_2 over variable x_2 :
Polynomial too big for output (text size is greater than 2000 characters, number of terms is 64)
19. Pseudo remainder with p_1 over variable x_1 :

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 881 terms.

Time Complexity: Time spent by the prover is 8.271 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Conic Section is not in degenerate form
- Points A and B are not identical
- Points D and C are not identical
- Line through points E and B is not parallel with line through points F and C
- Points E and A are not identical
- Line through points D and C is not parallel with line through points E and F
- Line through points D and A is not parallel with line through points E and B
- Points D and A are not identical
- Points E and F are not identical
- Line through points D and A is not parallel with line through points F and C
- Points F and C are not identical
- Line through points Q1 and P1 is not parallel with line through points Q2 and P2
- Segment with endpoints Q1 and P1 is not collinear and congruent with segment with endpoints A and B

Time spent for processing NDG Conditions

- 1.739 seconds