OpenGeoProver Output for conjecture "Chou 063"

Wu's method used

February 21, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point B
- Free point C
- Free point A
- Line c through two points A and B
- Line a through two points B and C
- Line b through two points C and A
- Free point O
- Line footPointPerpLine96 through point O perpendicular to line a
- Intersection point A' of point sets footPointPerpLine96 and a
- Line footPointPerpLine793 through point O perpendicular to line b
- Intersection point B' of point sets footPointPerpLine793 and b
- Line footPointPerpLine70 through point O perpendicular to line c
- Intersection point C' of point sets footPointPerpLine70 and c
- Line n through two points O and A'
- Random point P from line n
- Circle k with center O and one point P
- Inverse point A1 of point A' with respect to circle k
- Inverse point B1 of point B' with respect to circle k
- Inverse point C1 of point C' with respect to circle k
- Line a1 through two points A and A1
- Line b1 through two points B and B1
- Intersection point I of point sets a1 and b1

Theorem statement:

- Points C, C1, I are collinear

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point B:

- Point B has been assigned following coordinates: $(0, 0)$

2.2 Transformation of point C:

- Point C has been assigned following coordinates: $(0, u_1)$

2.3 Transformation of point A:

- Point A has been assigned following coordinates: (u_2, u_3)

2.4 Transformation of point O:

- Point O has been assigned following coordinates: (u_4, u_5)

2.5 Transformation of point A':

- Point A' has been assigned following coordinates: (x_1, x_2)

- Polynomial that point A' has to satisfy is:

$$p \quad = \quad x_2 - u_5$$

- Processing of polynomial

$$p \quad = \quad x_2 - u_5$$

  Info: Will try to rename Y coordinate of point A'

  Info: Y coordinate of point A' renamed by independent variable u_5

- Point A' has been renamed. Point A' has been assigned following coordinates: (x_1, u_5)

- Polynomial that point A' has to satisfy is:

$$p \quad = \quad x_1$$

- Processing of polynomial

$$p \quad = \quad x_1$$

Info: Will try to rename X coordinate of point A'

Info: X coordinate of point A' renamed by zero

- Point A' has been renamed. Point A' has been assigned following coordinates: $(0, u_5)$

2.6 Transformation of point B':

- Point B' has been assigned following coordinates: (x_1, x_2)

- Polynomial that point B' has to satisfy is:

$$p \quad = \quad (u_3 - u_1)x_2 + u_2x_1 + (-u_5u_3 + u_5u_1 - u_4u_2)$$

- Processing of polynomial

$$p \quad = \quad (u_3 - u_1)x_2 + u_2x_1 + (-u_5u_3 + u_5u_1 - u_4u_2)$$

Info: Polynomial

$$p \quad = \quad (u_3 - u_1)x_2 + u_2x_1 + (-u_5u_3 + u_5u_1 - u_4u_2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point B' has to satisfy is:

$$p \quad = \quad u_2x_2 + (-u_3 + u_1)x_1 - u_2u_1$$

- Processing of polynomial

$$p \quad = \quad u_2x_2 + (-u_3 + u_1)x_1 - u_2u_1$$

Info: Polynomial

$$p \quad = \quad u_2x_2 + (-u_3 + u_1)x_1 - u_2u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point C':

- Point C' has been assigned following coordinates: (x_3, x_4)

- Polynomial that point C' has to satisfy is:

$$p \quad = \quad u_3x_4 + u_2x_3 + (-u_5u_3 - u_4u_2)$$

- Processing of polynomial

$$p \quad = \quad u_3x_4 + u_2x_3 + (-u_5u_3 - u_4u_2)$$

Info: Polynomial

$$p \quad = \quad u_3 x_4 + u_2 x_3 + (-u_5 u_3 - u_4 u_2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point C' has to satisfy is:

$$p \quad = \quad u_2 x_4 - u_3 x_3$$

- Processing of polynomial

$$p \quad = \quad u_2 x_4 - u_3 x_3$$

Info: Polynomial

$$p \quad = \quad u_2 x_4 - u_3 x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.8 Transformation of point P:

- Point P has been assigned following coordinates: $(u_6,\, x_5)$

- Polynomial that point P has to satisfy is:

$$p \quad = \quad x_5 - u_5$$

- Processing of polynomial

$$p \quad = \quad x_5 - u_5$$

Info: Will try to rename Y coordinate of point P

Info: Y coordinate of point P renamed by independent variable u_5

- Point P has been renamed. Point P has been assigned following coordinates: $(u_6,\, u_5)$

2.9 Transformation of point secantPoint840:

- Point secantPoint840 has been assigned following coordinates: $(x_5,\, x_6)$

- Polynomial that point secantPoint840 has to satisfy is:

$$p \quad = \quad x_6^2 - 2u_5 x_6 + x_5^2 - 2u_4 x_5 + (-u_6^2 + 2u_6 u_4 + u_5^2)$$

- Processing of polynomial

$$p \quad = \quad x_6^2 - 2u_5 x_6 + x_5^2 - 2u_4 x_5 + (-u_6^2 + 2u_6 u_4 + u_5^2)$$

Info: Polynomial

$$p \quad = \quad x_6^2 - 2u_5x_6 + x_5^2 - 2u_4x_5 + (-u_6^2 + 2u_6u_4 + u_5^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point secantPoint840 has to satisfy is:

$$p \quad = \quad x_6 - u_5$$

- Processing of polynomial

$$p \quad = \quad x_6 - u_5$$

Info: Will try to rename Y coordinate of point secantPoint840

Info: Y coordinate of point secantPoint840 renamed by independent variable u_5

- Point secantPoint840 has been renamed. Point secantPoint840 has been assigned following coordinates: $(x_5,\ u_5)$

- Repeating instantiation of first condition of this point, after its coordinate has been renamed

- Polynomial that point secantPoint840 has to satisfy is:

$$p \quad = \quad x_5^2 - 2u_4x_5 + (-u_6^2 + 2u_6u_4)$$

- Processing of polynomial

$$p \quad = \quad x_5^2 - 2u_4x_5 + (-u_6^2 + 2u_6u_4)$$

Info: Polynomial

$$p \quad = \quad x_5^2 - 2u_4x_5 + (-u_6^2 + 2u_6u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.10 Transformation of point A1:

- Point A1 has been assigned following coordinates: $(x_6,\ x_7)$

- Instantiating condition for X-coordinate of this point

- Processing of polynomial

$$p \quad = \quad u_4x_6 + x_5^2 - 2u_4x_5$$

Info: Polynomial

$$p \quad = \quad u_4x_6 + x_5^2 - 2u_4x_5$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p \quad = \quad u_4 x_6 + x_5^2 - 2u_4 x_5$$

  is added to polynomial system

- Instantiating condition for Y-coordinate of this point

- Processing of polynomial

$$p \quad = \quad x_7 - u_5$$

  Info: Will try to rename Y coordinate of point A1

  Info: Y coordinate of point A1 renamed by independent variable u_5

- Point A1 has been renamed. Point A1 has been assigned following coordinates: (x_6, u_5)

- Repeating instantiation of condition for X-coordinate of this point, after it has been renamed

- Processing of polynomial

$$p \quad = \quad u_4 x_6 + x_5^2 - 2u_4 x_5$$

  Info: Polynomial

$$p \quad = \quad u_4 x_6 + x_5^2 - 2u_4 x_5$$

  added to system of polynomials that represents the constructions

- Instantiated condition

$$p \quad = \quad u_4 x_6 + x_5^2 - 2u_4 x_5$$

  is added to polynomial system

2.11   Transformation of point secantPoint371:

- Point secantPoint371 has been assigned following coordinates: (x_7, x_8)

- Polynomial that point secantPoint371 has to satisfy is:

$$p \quad = \quad x_8^2 - 2u_5 x_8 + x_7^2 - 2u_4 x_7 + (-u_6^2 + 2u_6 u_4 + u_5^2)$$

- Processing of polynomial

$$p \quad = \quad x_8^2 - 2u_5 x_8 + x_7^2 - 2u_4 x_7 + (-u_6^2 + 2u_6 u_4 + u_5^2)$$

  Info: Polynomial

$$p \quad = \quad x_8^2 - 2u_5 x_8 + x_7^2 - 2u_4 x_7 + (-u_6^2 + 2u_6 u_4 + u_5^2)$$

  added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point secantPoint371 has to satisfy is:

$$p \;\; = \;\; x_8 x_1 - u_4 x_8 - x_7 x_2 + u_5 x_7 + u_4 x_2 - u_5 x_1$$

- Processing of polynomial

$$p \;\; = \;\; x_8 x_1 - u_4 x_8 - x_7 x_2 + u_5 x_7 + u_4 x_2 - u_5 x_1$$

Info: Polynomial

$$p \;\; = \;\; x_8 x_1 - u_4 x_8 - x_7 x_2 + u_5 x_7 + u_4 x_2 - u_5 x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.12   Transformation of point B1:

- Point B1 has been assigned following coordinates: $(x_9,\, x_{10})$
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p \;\; = \;\; x_9 x_1 - u_4 x_9 - x_7^2 + 2 u_4 x_7 - u_4 x_1$$

Info: Polynomial

$$p \;\; = \;\; x_9 x_1 - u_4 x_9 - x_7^2 + 2 u_4 x_7 - u_4 x_1$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p \;\; = \;\; x_9 x_1 - u_4 x_9 - x_7^2 + 2 u_4 x_7 - u_4 x_1$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p \;\; = \;\; x_{10} x_2 - u_5 x_{10} - x_8^2 + 2 u_5 x_8 - u_5 x_2$$

Info: Polynomial

$$p \;\; = \;\; x_{10} x_2 - u_5 x_{10} - x_8^2 + 2 u_5 x_8 - u_5 x_2$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p \;\; = \;\; x_{10} x_2 - u_5 x_{10} - x_8^2 + 2 u_5 x_8 - u_5 x_2$$

is added to polynomial system

2.13 Transformation of point secantPoint356:

- Point secantPoint356 has been assigned following coordinates: (x_{11}, x_{12})
- Polynomial that point secantPoint356 has to satisfy is:

$$p \quad = \quad x_{12}^2 - 2u_5 x_{12} + x_{11}^2 - 2u_4 x_{11} + (-u_6^2 + 2u_6 u_4 + u_5^2)$$

- Processing of polynomial

$$p \quad = \quad x_{12}^2 - 2u_5 x_{12} + x_{11}^2 - 2u_4 x_{11} + (-u_6^2 + 2u_6 u_4 + u_5^2)$$

  Info: Polynomial

$$p \quad = \quad x_{12}^2 - 2u_5 x_{12} + x_{11}^2 - 2u_4 x_{11} + (-u_6^2 + 2u_6 u_4 + u_5^2)$$

  added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point secantPoint356 has to satisfy is:

$$p \quad = \quad x_{12} x_3 - u_4 x_{12} - x_{11} x_4 + u_5 x_{11} + u_4 x_4 - u_5 x_3$$

- Processing of polynomial

$$p \quad = \quad x_{12} x_3 - u_4 x_{12} - x_{11} x_4 + u_5 x_{11} + u_4 x_4 - u_5 x_3$$

  Info: Polynomial

$$p \quad = \quad x_{12} x_3 - u_4 x_{12} - x_{11} x_4 + u_5 x_{11} + u_4 x_4 - u_5 x_3$$

  added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.14 Transformation of point C1:

- Point C1 has been assigned following coordinates: (x_{13}, x_{14})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p \quad = \quad x_{13} x_3 - u_4 x_{13} - x_{11}^2 + 2u_4 x_{11} - u_4 x_3$$

  Info: Polynomial

$$p \quad = \quad x_{13} x_3 - u_4 x_{13} - x_{11}^2 + 2u_4 x_{11} - u_4 x_3$$

  added to system of polynomials that represents the constructions

- Instantiated condition

$$p \quad = \quad x_{13} x_3 - u_4 x_{13} - x_{11}^2 + 2u_4 x_{11} - u_4 x_3$$

  is added to polynomial system

- Instantiating condition for Y-coordinate of this point

- Processing of polynomial

$$p \quad = \quad x_{14}x_4 - u_5 x_{14} - x_{12}^2 + 2u_5 x_{12} - u_5 x_4$$

Info: Polynomial

$$p \quad = \quad x_{14}x_4 - u_5 x_{14} - x_{12}^2 + 2u_5 x_{12} - u_5 x_4$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p \quad = \quad x_{14}x_4 - u_5 x_{14} - x_{12}^2 + 2u_5 x_{12} - u_5 x_4$$

is added to polynomial system

2.15 Transformation of point I:

- Point I has been assigned following coordinates: $(x_{15},\ x_{16})$

- Polynomial that point I has to satisfy is:

$$p \quad = \quad x_{16}x_6 - u_2 x_{16} + (-u_5 + u_3)x_{15} - u_3 x_6 + u_5 u_2$$

- Processing of polynomial

$$p \quad = \quad x_{16}x_6 - u_2 x_{16} + (-u_5 + u_3)x_{15} - u_3 x_6 + u_5 u_2$$

Info: Polynomial

$$p \quad = \quad x_{16}x_6 - u_2 x_{16} + (-u_5 + u_3)x_{15} - u_3 x_6 + u_5 u_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point I has to satisfy is:

$$p \quad = \quad x_{16}x_9 - x_{15}x_{10}$$

- Processing of polynomial

$$p \quad = \quad x_{16}x_9 - x_{15}x_{10}$$

Info: Polynomial

$$p \quad = \quad x_{16}x_9 - x_{15}x_{10}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p \quad = \quad x_{16}x_{13} - x_{15}x_{14} + u_1 x_{15} - u_1 x_{13}$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.268 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$
\begin{aligned}
p_1 &= (u_3 - u_1)x_2 + u_2 x_1 + (-u_5 u_3 + u_5 u_1 - u_4 u_2) \\
p_2 &= u_2 x_2 + (-u_3 + u_1)x_1 - u_2 u_1 \\
p_3 &= u_3 x_4 + u_2 x_3 + (-u_5 u_3 - u_4 u_2) \\
p_4 &= u_2 x_4 - u_3 x_3 \\
p_5 &= x_5^2 - 2u_4 x_5 + (-u_6^2 + 2u_6 u_4) \\
p_6 &= u_4 x_6 + x_5^2 - 2u_4 x_5 \\
p_7 &= x_8^2 - 2u_5 x_8 + x_7^2 - 2u_4 x_7 + (-u_6^2 + 2u_6 u_4 + u_5^2) \\
p_8 &= x_8 x_1 - u_4 x_8 - x_7 x_2 + u_5 x_7 + u_4 x_2 - u_5 x_1 \\
p_9 &= x_9 x_1 - u_4 x_9 - x_7^2 + 2u_4 x_7 - u_4 x_1 \\
p_{10} &= x_{10} x_2 - u_5 x_{10} - x_8^2 + 2u_5 x_8 - u_5 x_2 \\
p_{11} &= x_{12}^2 - 2u_5 x_{12} + x_{11}^2 - 2u_4 x_{11} + (-u_6^2 + 2u_6 u_4 + u_5^2) \\
p_{12} &= x_{12} x_3 - u_4 x_{12} - x_{11} x_4 + u_5 x_{11} + u_4 x_4 - u_5 x_3 \\
p_{13} &= x_{13} x_3 - u_4 x_{13} - x_{11}^2 + 2u_4 x_{11} - u_4 x_3 \\
p_{14} &= x_{14} x_4 - u_5 x_{14} - x_{12}^2 + 2u_5 x_{12} - u_5 x_4 \\
p_{15} &= x_{16} x_6 - u_2 x_{16} + (-u_5 + u_3)x_{15} - u_3 x_6 + u_5 u_2 \\
p_{16} &= x_{16} x_9 - x_{15} x_{10}
\end{aligned}
$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{16} from all other polynomials by reducing them with polynomial p_{15} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (u_3 - u_1)x_2 + u_2x_1 + (-u_5u_3 + u_5u_1 - u_4u_2) \\
p_2 &= u_2x_2 + (-u_3 + u_1)x_1 - u_2u_1 \\
p_3 &= u_3x_4 + u_2x_3 + (-u_5u_3 - u_4u_2) \\
p_4 &= u_2x_4 - u_3x_3 \\
p_5 &= x_5^2 - 2u_4x_5 + (-u_6^2 + 2u_6u_4) \\
p_6 &= u_4x_6 + x_5^2 - 2u_4x_5 \\
p_7 &= x_8^2 - 2u_5x_8 + x_7^2 - 2u_4x_7 + (-u_6^2 + 2u_6u_4 + u_5^2) \\
p_8 &= x_8x_1 - u_4x_8 - x_7x_2 + u_5x_7 + u_4x_2 - u_5x_1 \\
p_9 &= x_9x_1 - u_4x_9 - x_7^2 + 2u_4x_7 - u_4x_1 \\
p_{10} &= x_{10}x_2 - u_5x_{10} - x_8^2 + 2u_5x_8 - u_5x_2 \\
p_{11} &= x_{12}^2 - 2u_5x_{12} + x_{11}^2 - 2u_4x_{11} + (-u_6^2 + 2u_6u_4 + u_5^2) \\
p_{12} &= x_{12}x_3 - u_4x_{12} - x_{11}x_4 + u_5x_{11} + u_4x_4 - u_5x_3 \\
p_{13} &= x_{13}x_3 - u_4x_{13} - x_{11}^2 + 2u_4x_{11} - u_4x_3 \\
p_{14} &= x_{14}x_4 - u_5x_{14} - x_{12}^2 + 2u_5x_{12} - u_5x_4 \\
p_{15} &= -x_{15}x_{10}x_6 + u_2x_{15}x_{10} + (u_5 - u_3)x_{15}x_9 + u_3x_9x_6 - u_5u_2x_9 \\
p_{16} &= x_{16}x_6 - u_2x_{16} + (-u_5 + u_3)x_{15} - u_3x_6 + u_5u_2
\end{aligned}
$$

3.2   Triangulation, step 2

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{15}. No reduction needed.

The triangular system has not been changed.

3.3   Triangulation, step 3

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14}. No reduction needed.

The triangular system has not been changed.

3.4   Triangulation, step 4

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{13}. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 2.

Minimal degrees: 1 polynomial(s) with degree 1 and 1 polynomial(s) with degree 2.

Polynomial with linear degree: Removing variable x_{12} from all other polynomials by reducing them with polynomial p_{12} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (u_3 - u_1)x_2 + u_2x_1 + (-u_5u_3 + u_5u_1 - u_4u_2) \\
p_2 &= u_2x_2 + (-u_3 + u_1)x_1 - u_2u_1 \\
p_3 &= u_3x_4 + u_2x_3 + (-u_5u_3 - u_4u_2) \\
p_4 &= u_2x_4 - u_3x_3 \\
p_5 &= x_5^2 - 2u_4x_5 + (-u_6^2 + 2u_6u_4) \\
p_6 &= u_4x_6 + x_5^2 - 2u_4x_5 \\
p_7 &= x_8^2 - 2u_5x_8 + x_7^2 - 2u_4x_7 + (-u_6^2 + 2u_6u_4 + u_5^2) \\
p_8 &= x_8x_1 - u_4x_8 - x_7x_2 + u_5x_7 + u_4x_2 - u_5x_1 \\
p_9 &= x_9x_1 - u_4x_9 - x_7^2 + 2u_4x_7 - u_4x_1 \\
p_{10} &= x_{10}x_2 - u_5x_{10} - x_8^2 + 2u_5x_8 - u_5x_2 \\
p_{11} &= x_{11}^2x_4^2 - 2u_5x_{11}^2x_4 + x_{11}^2x_3^2 - 2u_4x_{11}^2x_3 + \\
&\quad (u_5^2 + u_4^2)x_{11}^2 - 2u_4x_{11}x_4^2 + 4u_5u_4x_{11}x_4 \\
&\quad -2u_4x_{11}x_3^2 + 4u_4^2x_{11}x_3 + (-2u_5^2u_4 - 2u_4^3)x_{11} + \\
&\quad u_4^2x_4^2 - 2u_5u_4^2x_4 + (-u_6^2 + 2u_6u_4)x_3^2 + \\
&\quad (2u_6^2u_4 - 4u_6u_4^2)x_3 + \\
&\quad (-u_6^2u_4^2 + 2u_6u_4^3 + u_5^2u_4^2) \\
p_{12} &= x_{12}x_3 - u_4x_{12} - x_{11}x_4 + u_5x_{11} + u_4x_4 - u_5x_3 \\
p_{13} &= x_{13}x_3 - u_4x_{13} - x_{11}^2 + 2u_4x_{11} - u_4x_3 \\
p_{14} &= x_{14}x_4 - u_5x_{14} - x_{12}^2 + 2u_5x_{12} - u_5x_4 \\
p_{15} &= -x_{15}x_{10}x_6 + u_2x_{15}x_{10} + (u_5 - u_3)x_{15}x_9 + u_3x_9x_6 - u_5u_2x_9 \\
p_{16} &= x_{16}x_6 - u_2x_{16} + (-u_5 + u_3)x_{15} - u_3x_6 + u_5u_2
\end{aligned}
$$

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{11}. No reduction needed.

The triangular system has not been changed.

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10}. No reduction needed.

The triangular system has not been changed.

3.8 Triangulation, step 8

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_9. No reduction needed.

The triangular system has not been changed.

3.9 Triangulation, step 9

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 2.

Minimal degrees: 1 polynomial(s) with degree 1 and 1 polynomial(s) with degree 2.

Polynomial with linear degree: Removing variable x_8 from all other polynomials by reducing them with polynomial p_8 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (u_3 - u_1)x_2 + u_2 x_1 + (-u_5 u_3 + u_5 u_1 - u_4 u_2) \\
p_2 &= u_2 x_2 + (-u_3 + u_1)x_1 - u_2 u_1 \\
p_3 &= u_3 x_4 + u_2 x_3 + (-u_5 u_3 - u_4 u_2) \\
p_4 &= u_2 x_4 - u_3 x_3 \\
p_5 &= x_5^2 - 2u_4 x_5 + (-u_6^2 + 2u_6 u_4) \\
p_6 &= u_4 x_6 + x_5^2 - 2u_4 x_5 \\
p_7 &= x_7^2 x_2^2 - 2u_5 x_7^2 x_2 + x_7^2 x_1^2 - 2u_4 x_7^2 x_1 + \\
&\quad (u_5^2 + u_4^2)x_7^2 - 2u_4 x_7 x_2^2 + 4u_5 u_4 x_7 x_2 \\
&\quad -2u_4 x_7 x_1^2 + 4u_4^2 x_7 x_1 + (-2u_5^2 u_4 - 2u_4^3)x_7 + \\
&\quad u_4^2 x_2^2 - 2u_5 u_4^2 x_2 + (-u_6^2 + 2u_6 u_4)x_1^2 + \\
&\quad (2u_6^2 u_4 - 4u_6 u_4^2)x_1 + \\
&\quad (-u_6^2 u_4^2 + 2u_6 u_4^3 + u_5^2 u_4^2) \\
p_8 &= x_8 x_1 - u_4 x_8 - x_7 x_2 + u_5 x_7 + u_4 x_2 - u_5 x_1 \\
p_9 &= x_9 x_1 - u_4 x_9 - x_7^2 + 2u_4 x_7 - u_4 x_1 \\
p_{10} &= x_{10} x_2 - u_5 x_{10} - x_8^2 + 2u_5 x_8 - u_5 x_2 \\
p_{11} &= x_{11}^2 x_4^2 - 2u_5 x_{11}^2 x_4 + x_{11}^2 x_3^2 - 2u_4 x_{11}^2 x_3 + \\
&\quad (u_5^2 + u_4^2)x_{11}^2 - 2u_4 x_{11} x_4^2 + 4u_5 u_4 x_{11} x_4 \\
&\quad -2u_4 x_{11} x_3^2 + 4u_4^2 x_{11} x_3 + (-2u_5^2 u_4 - 2u_4^3)x_{11} + \\
&\quad u_4^2 x_4^2 - 2u_5 u_4^2 x_4 + (-u_6^2 + 2u_6 u_4)x_3^2 + \\
&\quad (2u_6^2 u_4 - 4u_6 u_4^2)x_3 + \\
&\quad (-u_6^2 u_4^2 + 2u_6 u_4^3 + u_5^2 u_4^2) \\
p_{12} &= x_{12} x_3 - u_4 x_{12} - x_{11} x_4 + u_5 x_{11} + u_4 x_4 - u_5 x_3 \\
p_{13} &= x_{13} x_3 - u_4 x_{13} - x_{11}^2 + 2u_4 x_{11} - u_4 x_3 \\
p_{14} &= x_{14} x_4 - u_5 x_{14} - x_{12}^2 + 2u_5 x_{12} - u_5 x_4 \\
p_{15} &= -x_{15} x_{10} x_6 + u_2 x_{15} x_{10} + (u_5 - u_3)x_{15} x_9 + u_3 x_9 x_6 - u_5 u_2 x_9 \\
p_{16} &= x_{16} x_6 - u_2 x_{16} + (-u_5 + u_3)x_{15} - u_3 x_6 + u_5 u_2
\end{aligned}
$$

3.10 Triangulation, step 10

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7. No reduction needed.

The triangular system has not been changed.

3.11 Triangulation, step 11

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

3.12   Triangulation, step 12

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

3.13   Triangulation, step 13

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_3 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (u_3 - u_1)x_2 + u_2 x_1 + (-u_5 u_3 + u_5 u_1 - u_4 u_2) \\
p_2 &= u_2 x_2 + (-u_3 + u_1)x_1 - u_2 u_1 \\
p_3 &= (-u_3^2 - u_2^2)x_3 + (u_5 u_3 u_2 + u_4 u_2^2) \\
p_4 &= u_3 x_4 + u_2 x_3 + (-u_5 u_3 - u_4 u_2) \\
p_5 &= x_5^2 - 2u_4 x_5 + (-u_6^2 + 2u_6 u_4) \\
p_6 &= u_4 x_6 + x_5^2 - 2u_4 x_5 \\
p_7 &= x_7^2 x_2^2 - 2u_5 x_7^2 x_2 + x_7^2 x_1^2 - 2u_4 x_7^2 x_1 + \\
&\quad (u_5^2 + u_4^2)x_7^2 - 2u_4 x_7 x_2^2 + 4u_5 u_4 x_7 x_2 \\
&\quad -2u_4 x_7 x_1^2 + 4u_4^2 x_7 x_1 + (-2u_5^2 u_4 - 2u_4^3)x_7 + \\
&\quad u_4^2 x_2^2 - 2u_5 u_4^2 x_2 + (-u_6^2 + 2u_6 u_4)x_1^2 + \\
&\quad (2u_6^2 u_4 - 4u_6 u_4^2)x_1 + \\
&\quad (-u_6^2 u_4^2 + 2u_6 u_4^3 + u_5^2 u_4^2) \\
p_8 &= x_8 x_1 - u_4 x_8 - x_7 x_2 + u_5 x_7 + u_4 x_2 - u_5 x_1 \\
p_9 &= x_9 x_1 - u_4 x_9 - x_7^2 + 2u_4 x_7 - u_4 x_1 \\
p_{10} &= x_{10} x_2 - u_5 x_{10} - x_8^2 + 2u_5 x_8 - u_5 x_2
\end{aligned}
$$

$$
\begin{aligned}
p_{11} &= x_{11}^2 x_4^2 - 2u_5 x_{11}^2 x_4 + x_{11}^2 x_3^2 - 2u_4 x_{11}^2 x_3 + \\
&\quad (u_5^2 + u_4^2) x_{11}^2 - 2u_4 x_{11} x_4^2 + 4u_5 u_4 x_{11} x_4 \\
&\quad -2u_4 x_{11} x_3^2 + 4u_4^2 x_{11} x_3 + (-2u_5^2 u_4 - 2u_4^3) x_{11} + \\
&\quad u_4^2 x_4^2 - 2u_5 u_4^2 x_4 + (-u_6^2 + 2u_6 u_4) x_3^2 + \\
&\quad (2u_6^2 u_4 - 4u_6 u_4^2) x_3 + \\
&\quad (-u_6^2 u_4^2 + 2u_6 u_4^3 + u_5^2 u_4^2) \\
p_{12} &= x_{12} x_3 - u_4 x_{12} - x_{11} x_4 + u_5 x_{11} + u_4 x_4 - u_5 x_3 \\
p_{13} &= x_{13} x_3 - u_4 x_{13} - x_{11}^2 + 2u_4 x_{11} - u_4 x_3 \\
p_{14} &= x_{14} x_4 - u_5 x_{14} - x_{12}^2 + 2u_5 x_{12} - u_5 x_4 \\
p_{15} &= -x_{15} x_{10} x_6 + u_2 x_{15} x_{10} + (u_5 - u_3) x_{15} x_9 + u_3 x_9 x_6 - u_5 u_2 x_9 \\
p_{16} &= x_{16} x_6 - u_2 x_{16} + (-u_5 + u_3) x_{15} - u_3 x_6 + u_5 u_2
\end{aligned}
$$

3.14   Triangulation, step 14

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.15   Triangulation, step 15

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_2 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (-u_3^2 + 2u_3 u_1 - u_2^2 - u_1^2) x_1 + \\
&\quad (u_5 u_3 u_2 - u_5 u_2 u_1 + u_4 u_2^2 - u_3 u_2 u_1 + u_2 u_1^2) \\
p_2 &= (u_3 - u_1) x_2 + u_2 x_1 + (-u_5 u_3 + u_5 u_1 - u_4 u_2) \\
p_3 &= (-u_3^2 - u_2^2) x_3 + (u_5 u_3 u_2 + u_4 u_2^2) \\
p_4 &= u_3 x_4 + u_2 x_3 + (-u_5 u_3 - u_4 u_2) \\
p_5 &= x_5^2 - 2u_4 x_5 + (-u_6^2 + 2u_6 u_4) \\
p_6 &= u_4 x_6 + x_5^2 - 2u_4 x_5 \\
p_7 &= x_7^2 x_2^2 - 2u_5 x_7^2 x_2 + x_7^2 x_1^2 - 2u_4 x_7^2 x_1 + \\
&\quad (u_5^2 + u_4^2) x_7^2 - 2u_4 x_7 x_2^2 + 4u_5 u_4 x_7 x_2
\end{aligned}
$$

$$-2u_4x_7x_1^2 + 4u_4^2x_7x_1 + (-2u_5^2u_4 - 2u_4^3)x_7 +$$
$$u_4^2x_2^2 - 2u_5u_4^2x_2 + (-u_6^2 + 2u_6u_4)x_1^2 +$$
$$(2u_6^2u_4 - 4u_6u_4^2)x_1 +$$
$$(-u_6^2u_4^2 + 2u_6u_4^3 + u_5^2u_4^2)$$

$$p_8 = x_8x_1 - u_4x_8 - x_7x_2 + u_5x_7 + u_4x_2 - u_5x_1$$

$$p_9 = x_9x_1 - u_4x_9 - x_7^2 + 2u_4x_7 - u_4x_1$$

$$p_{10} = x_{10}x_2 - u_5x_{10} - x_8^2 + 2u_5x_8 - u_5x_2$$

$$p_{11} = x_{11}^2x_4^2 - 2u_5x_{11}^2x_4 + x_{11}^2x_3^2 - 2u_4x_{11}^2x_3 +$$
$$(u_5^2 + u_4^2)x_{11}^2 - 2u_4x_{11}x_4^2 + 4u_5u_4x_{11}x_4$$
$$-2u_4x_{11}x_3^2 + 4u_4^2x_{11}x_3 + (-2u_5^2u_4 - 2u_4^3)x_{11} +$$
$$u_4^2x_4^2 - 2u_5u_4^2x_4 + (-u_6^2 + 2u_6u_4)x_3^2 +$$
$$(2u_6^2u_4 - 4u_6u_4^2)x_3 +$$
$$(-u_6^2u_4^2 + 2u_6u_4^3 + u_5^2u_4^2)$$

$$p_{12} = x_{12}x_3 - u_4x_{12} - x_{11}x_4 + u_5x_{11} + u_4x_4 - u_5x_3$$

$$p_{13} = x_{13}x_3 - u_4x_{13} - x_{11}^2 + 2u_4x_{11} - u_4x_3$$

$$p_{14} = x_{14}x_4 - u_5x_{14} - x_{12}^2 + 2u_5x_{12} - u_5x_4$$

$$p_{15} = -x_{15}x_{10}x_6 + u_2x_{15}x_{10} + (u_5 - u_3)x_{15}x_9 + u_3x_9x_6 - u_5u_2x_9$$

$$p_{16} = x_{16}x_6 - u_2x_{16} + (-u_5 + u_3)x_{15} - u_3x_6 + u_5u_2$$

3.16 Triangulation, step 16

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$p_1 = (-u_3^2 + 2u_3u_1 - u_2^2 - u_1^2)x_1 +$$
$$(u_5u_3u_2 - u_5u_2u_1 + u_4u_2^2 - u_3u_2u_1 + u_2u_1^2)$$

$$p_2 = (u_3 - u_1)x_2 + u_2x_1 + (-u_5u_3 + u_5u_1 - u_4u_2)$$

$$p_3 = (-u_3^2 - u_2^2)x_3 + (u_5u_3u_2 + u_4u_2^2)$$

$$p_4 = u_3x_4 + u_2x_3 + (-u_5u_3 - u_4u_2)$$

$$p_5 = x_5^2 - 2u_4x_5 + (-u_6^2 + 2u_6u_4)$$

$$p_6 = u_4x_6 + x_5^2 - 2u_4x_5$$

$$p_7 = x_7^2x_2^2 - 2u_5x_7^2x_2 + x_7^2x_1^2 - 2u_4x_7^2x_1 +$$
$$(u_5^2 + u_4^2)x_7^2 - 2u_4x_7x_2^2 + 4u_5u_4x_7x_2$$
$$-2u_4x_7x_1^2 + 4u_4^2x_7x_1 + (-2u_5^2u_4 - 2u_4^3)x_7 +$$
$$u_4^2x_2^2 - 2u_5u_4^2x_2 + (-u_6^2 + 2u_6u_4)x_1^2 +$$

$$(2u_6^2u_4 - 4u_6u_4^2)x_1 +$$
$$(-u_6^2u_4^2 + 2u_6u_4^3 + u_5^2u_4^2)$$

$$p_8 = x_8x_1 - u_4x_8 - x_7x_2 + u_5x_7 + u_4x_2 - u_5x_1$$

$$p_9 = x_9x_1 - u_4x_9 - x_7^2 + 2u_4x_7 - u_4x_1$$

$$p_{10} = x_{10}x_2 - u_5x_{10} - x_8^2 + 2u_5x_8 - u_5x_2$$

$$p_{11} = x_{11}^2x_4^2 - 2u_5x_{11}^2x_4 + x_{11}^2x_3^2 - 2u_4x_{11}^2x_3 +$$
$$(u_5^2 + u_4^2)x_{11}^2 - 2u_4x_{11}x_4^2 + 4u_5u_4x_{11}x_4$$
$$-2u_4x_{11}x_3^2 + 4u_4^2x_{11}x_3 + (-2u_5^2u_4 - 2u_4^3)x_{11} +$$
$$u_4^2x_4^2 - 2u_5u_4^2x_4 + (-u_6^2 + 2u_6u_4)x_3^2 +$$
$$(2u_6^2u_4 - 4u_6u_4^2)x_3 +$$
$$(-u_6^2u_4^2 + 2u_6u_4^3 + u_5^2u_4^2)$$

$$p_{12} = x_{12}x_3 - u_4x_{12} - x_{11}x_4 + u_5x_{11} + u_4x_4 - u_5x_3$$

$$p_{13} = x_{13}x_3 - u_4x_{13} - x_{11}^2 + 2u_4x_{11} - u_4x_3$$

$$p_{14} = x_{14}x_4 - u_5x_{14} - x_{12}^2 + 2u_5x_{12} - u_5x_4$$

$$p_{15} = -x_{15}x_{10}x_6 + u_2x_{15}x_{10} + (u_5 - u_3)x_{15}x_9 + u_3x_9x_6 - u_5u_2x_9$$

$$p_{16} = x_{16}x_6 - u_2x_{16} + (-u_5 + u_3)x_{15} - u_3x_6 + u_5u_2$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 063

Calculating final remainder of the conclusion:

$$g = x_{16}x_{13} - x_{15}x_{14} + u_1x_{15} - u_1x_{13}$$

with respect to the triangular system.

1. Pseudo remainder with p_{16} over variable x_{16}:

$$g = -x_{15}x_{14}x_6 + u_2x_{15}x_{14} + (u_5 - u_3)x_{15}x_{13} + u_1x_{15}x_6$$
$$-u_2u_1x_{15} + (u_3 - u_1)x_{13}x_6 + (-u_5u_2 + u_2u_1)x_{13}$$

2. Pseudo remainder with p_{15} over variable x_{15}:

$$g = u_3x_{14}x_9x_6^2 + (-u_5u_2 - u_3u_2)x_{14}x_9x_6 + u_5u_2^2x_{14}x_9 +$$
$$(-u_3 + u_1)x_{13}x_{10}x_6^2 + (u_5u_2 + u_3u_2 - 2u_2u_1)x_{13}x_{10}x_6 +$$
$$(-u_5u_2^2 + u_2^2u_1)x_{13}x_{10} + (-u_5u_1 + u_3u_1)x_{13}x_9x_6 +$$
$$(u_5u_2u_1 - u_3u_2u_1)x_{13}x_9 - u_3u_1x_9x_6^2 +$$
$$(u_5u_2u_1 + u_3u_2u_1)x_9x_6 - u_5u_2^2u_1x_9$$

3. Pseudo remainder with p_{14} over variable x_{14}:

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$$
\begin{aligned}
g \;=\; & (-u_3 + u_1)x_{13}x_{10}x_6^2 x_4 + (u_5 u_3 - u_5 u_1)x_{13}x_{10}x_6^2 + \\
& (u_5 u_2 + u_3 u_2 - 2u_2 u_1)x_{13}x_{10}x_6 x_4 + \\
& (-u_5^2 u_2 - u_5 u_3 u_2 + 2u_5 u_2 u_1)x_{13}x_{10}x_6 + \\
& (-u_5 u_2^2 + u_2^2 u_1)x_{13}x_{10}x_4 + \\
& (u_5^2 u_2^2 - u_5 u_2^2 u_1)x_{13}x_{10} + \\
& (-u_5 u_1 + u_3 u_1)x_{13}x_9 x_6 x_4 + \\
& (u_5^2 u_1 - u_5 u_3 u_1)x_{13}x_9 x_6 + \\
& (u_5 u_2 u_1 - u_3 u_2 u_1)x_{13}x_9 x_4 + \\
& (-u_5^2 u_2 u_1 + u_5 u_3 u_2 u_1)x_{13}x_9 + u_3 x_{12}^2 x_9 x_6^2 + \\
& (-u_5 u_2 - u_3 u_2)x_{12}^2 x_9 x_6 + u_5 u_2^2 x_{12}^2 x_9 \\
& -2u_5 u_3 x_{12}x_9 x_6^2 + (2u_5^2 u_2 + 2u_5 u_3 u_2)x_{12}x_9 x_6 \\
& -2u_5^2 u_2^2 x_{12}x_9 + (u_5 u_3 - u_3 u_1)x_9 x_6^2 x_4 + \\
& u_5 u_3 u_1 x_9 x_6^2 + \\
& (-u_5^2 u_2 - u_5 u_3 u_2 + u_5 u_2 u_1 + u_3 u_2 u_1)x_9 x_6 x_4 + \\
& (-u_5^2 u_2 u_1 - u_5 u_3 u_2 u_1)x_9 x_6 + \\
& (u_5^2 u_2^2 - u_5 u_2^2 u_1)x_9 x_4 + u_5^2 u_2^2 u_1 x_9
\end{aligned}
$$

4. Pseudo remainder with p_{13} over variable x_{13}:

   Polynomial too big for output (text size is 3108 characters, number of terms is 50)

5. Pseudo remainder with p_{12} over variable x_{12}:

   Polynomial too big for output (text size is 10567 characters, number of terms is 128)

6. Pseudo remainder with p_{11} over variable x_{11}:

   Polynomial too big for output (text size is 26113 characters, number of terms is 120)

7. Pseudo remainder with p_{10} over variable x_{10}:

   Polynomial too big for output (number of terms is 300)

8. Pseudo remainder with p_9 over variable x_9:

   Polynomial too big for output (number of terms is 660)

9. Pseudo remainder with p_8 over variable x_8:

   Polynomial too big for output (number of terms is 1500)

10. Pseudo remainder with p_7 over variable x_7:

    Polynomial too big for output (number of terms is 1200)

11. Pseudo remainder with p_6 over variable x_6:

   Polynomial too big for output (number of terms is 2000)

12. Pseudo remainder with p_5 over variable x_5:

   Polynomial too big for output (number of terms is 400)

13. Pseudo remainder with p_4 over variable x_4:

   Polynomial too big for output (text size is 676908 characters, number of terms is 140)

14. Pseudo remainder with p_3 over variable x_3:

   Polynomial too big for output (text size is greater than 2000 characters, number of terms is 20)

15. Pseudo remainder with p_2 over variable x_2:

   Polynomial too big for output (text size is greater than 2000 characters, number of terms is 7)

16. Pseudo remainder with p_1 over variable x_1:

$$g \quad = \quad 0$$

5   Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2000 terms.

Time Complexity: Time spent by the prover is 13.926 seconds.

6   NDG Conditions

NDG Conditions in readable form

- Points A, B and C are not collinear
- Points A, B and C are not collinear
- Points A and B are not identical
- Points A and B are not identical
- Points B, C and O are not collinear
- Points B' and O are not identical
- Line through points A' and O is not perpendicular to line through points O and B'

- Line through points B' and A' is not perpendicular to line through points A' and B

- Points C' and O are not identical

- Line through points C' and O is not parallel with line through points A' and B

- Point O is not the midpoint of segment with endpoints C' and P

- Line through points A1 and A is not parallel with line through points B and B1

- Line through points A1 and A is not parallel with line through points B and C

Time spent for processing NDG Conditions

- 7.159 seconds