OpenGeoProver Output for conjecture "Chou 142 (Centroid Theorem)"

Wu's method used

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1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Free point C
- Midpoint D of segment BC
- Midpoint E of segment CA
- Midpoint F of segment AB
- Line ta through two points A and D
- Line tb through two points B and E
- Intersection point T of point sets ta and tb

Theorem statement:

- Points C, F, T are collinear

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

2.2 Transformation of point B:

- Point B has been assigned following coordinates: (0, u_1)

2.3 Transformation of point C:

- Point C has been assigned following coordinates: (u_2, u_3)

2.4 Transformation of point D:

- Point D has been assigned following coordinates: (x_1, x_2)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_1 - 0.5u_2$$

Info: Polynomial

$$p = x_1 - 0.5u_2$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_1 - 0.5u_2$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_2 + (-0.5u_3 - 0.5u_1)$$

Info: Polynomial

$$p = x_2 + (-0.5u_3 - 0.5u_1)$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_2 + (-0.5u_3 - 0.5u_1)$$

is added to polynomial system

2.5 Transformation of point E:

- Point E has been assigned following coordinates: (x_3, x_4)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_3 - 0.5u_2$$

Info: Polynomial

$$p = x_3 - 0.5u_2$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p \quad = \quad x_3 - 0.5u_2$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p \quad = \quad x_4 - 0.5u_3$$

Info: Polynomial

$$p \quad = \quad x_4 - 0.5u_3$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p \quad = \quad x_4 - 0.5u_3$$

is added to polynomial system

2.6 Transformation of point F:

- Point F has been assigned following coordinates: (x_5, x_6)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p \quad = \quad x_5$$

Info: Will try to rename X coordinate of point F

Info: Y coordinate of point F will be replaced by X coordinate

Info: X coordinate of point F renamed by zero

- Point F has been renamed. Point F has been assigned following coordinates: (0, x_5)
- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p \quad = \quad x_5 - 0.5u_1$$

Info: Polynomial

$$p \quad = \quad x_5 - 0.5u_1$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p \quad = \quad x_5 - 0.5u_1$$

is added to polynomial system

2.7 Transformation of point T:

- Point T has been assigned following coordinates: (x_6, x_7)
- Polynomial that point T has to satisfy is:

$$p = x_7x_1 - x_6x_2$$

- Processing of polynomial

$$p = x_7x_1 - x_6x_2$$

Info: Polynomial

$$p = x_7x_1 - x_6x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point T has to satisfy is:

$$p = x_7x_3 - x_6x_4 + u_1x_6 - u_1x_3$$

- Processing of polynomial

$$p = x_7x_3 - x_6x_4 + u_1x_6 - u_1x_3$$

Info: Polynomial

$$p = x_7x_3 - x_6x_4 + u_1x_6 - u_1x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = u_2x_7 + x_6x_5 - u_3x_6 - u_2x_5$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.124 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$
\begin{aligned}
p_1 &= x_1 - 0.5u_2 \\
p_2 &= x_2 + (-0.5u_3 - 0.5u_1) \\
p_3 &= x_3 - 0.5u_2 \\
p_4 &= x_4 - 0.5u_3 \\
p_5 &= x_5 - 0.5u_1 \\
p_6 &= x_7x_1 - x_6x_2 \\
p_7 &= x_7x_3 - x_6x_4 + u_1x_6 - u_1x_3
\end{aligned}
$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_7 from all other polynomials by reducing them with polynomial p_6 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= x_1 - 0.5u_2 \\
p_2 &= x_2 + (-0.5u_3 - 0.5u_1) \\
p_3 &= x_3 - 0.5u_2 \\
p_4 &= x_4 - 0.5u_3 \\
p_5 &= x_5 - 0.5u_1 \\
p_6 &= -x_6x_4x_1 + x_6x_3x_2 + u_1x_6x_1 - u_1x_3x_1 \\
p_7 &= x_7x_1 - x_6x_2
\end{aligned}
$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_1 &= x_1 - 0.5u_2 \\
p_2 &= x_2 + (-0.5u_3 - 0.5u_1) \\
p_3 &= x_3 - 0.5u_2 \\
p_4 &= x_4 - 0.5u_3 \\
p_5 &= x_5 - 0.5u_1 \\
p_6 &= -x_6x_4x_1 + x_6x_3x_2 + u_1x_6x_1 - u_1x_3x_1 \\
p_7 &= x_7x_1 - x_6x_2
\end{aligned}$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 142 (Centroid Theorem)

Calculating final remainder of the conclusion:

$$g = u_2x_7 + x_6x_5 - u_3x_6 - u_2x_5$$

with respect to the triangular system.

1. Pseudo remainder with p_7 over variable x_7:

$$g = x_6x_5x_1 + u_2x_6x_2 - u_3x_6x_1 - u_2x_5x_1$$

2. Pseudo remainder with p_6 over variable x_6:

$$\begin{aligned}
g = {} & u_2x_5x_4x_1^2 - u_2x_5x_3x_2x_1 + u_1x_5x_3x_1^2 \\
& -u_2u_1x_5x_1^2 + u_2u_1x_3x_2x_1 - u_3u_1x_3x_1^2
\end{aligned}$$

3. Pseudo remainder with p_5 over variable x_5:

$$\begin{aligned}
g = {} & 0.5u_2u_1x_4x_1^2 + 0.5u_2u_1x_3x_2x_1 + \\
& (-u_3u_1 + 0.5u_1^2)x_3x_1^2 - 0.5u_2u_1^2x_1^2
\end{aligned}$$

4. Pseudo remainder with p_4 over variable x_4:

$$\begin{aligned}
g = {} & 0.5u_2u_1x_3x_2x_1 + (-u_3u_1 + 0.5u_1^2)x_3x_1^2 + \\
& (0.25u_3u_2u_1 - 0.5u_2u_1^2)x_1^2
\end{aligned}$$

5. Pseudo remainder with p_3 over variable x_3:

$$g = 0.25u_2^2u_1x_2x_1 + (-0.25u_3u_2u_1 - 0.25u_2u_1^2)x_1^2$$

6. Pseudo remainder with p_2 over variable x_2:

$$\begin{aligned} g &= (-0.25u_3u_2u_1 - 0.25u_2u_1^2)x_1^2 + \\ &\quad (0.125u_3u_2^2u_1 + 0.125u_2^2u_1^2)x_1 \end{aligned}$$

7. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

Time Complexity: Time spent by the prover is 0.151 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Line through points D and A is not parallel with line through points E and B
- Points D, F and A are not collinear

Time spent for processing NDG Conditions

- 1.514 seconds