OpenGeoProver Output for RC-Constructibility problem “Wernick 004 (A,B,G —– C)”

Used algebraic method (with triangulation)

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1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Free point C
- Midpoint Ma of segment BC
- Midpoint Mb of segment CA
- Line ta through two points A and Ma
- Line tb through two points B and Mb
- Intersection point G of point sets ta and tb

Free points:

- A
- B
- G

Points to be constructed:

- C

Validation result: Construction protocol is valid.

2 Instantiation of points with symbolic variables

- Point C has been assigned following coordinates: (x_1, x_2)
- Point A has been assigned following coordinates: (u_1, u_2)
- Point B has been assigned following coordinates: (u_3, u_4)
- Point G has been assigned following coordinates: (u_5, u_6)
- Point Ma has been assigned following coordinates: (x_3, x_4)
- Point Mb has been assigned following coordinates: (x_5, x_6)

3 Transformation of geometry conditions for points to polynomial form

3.1 Transformation, step 1

Point to transform: A

Polynomial condition(s): N/A - free point

3.2 Transformation, step 2

Point to transform: B

Polynomial condition(s): N/A - free point

3.3 Transformation, step 3

Point to transform: C

Polynomial condition(s): N/A - free point

3.4 Transformation, step 4

Point to transform: Ma

Polynomial condition(s): Two polynomials

- $$p = 2x_3 - x_1 - u_3$$
- $$p = 2x_4 - x_2 - u_4$$

3.5 Transformation, step 5

Point to transform: Mb

Polynomial condition(s): Two polynomials

- $$p = 2x_5 - x_1 - u_1$$

- $$p = 2x_6 - x_2 - u_2$$

3.6 Transformation, step 6

Point to transform: G

Polynomial condition(s): Two polynomials

- $$p = (u_5 - u_1)x_4 + (-u_6 + u_2)x_3 + (u_6u_1 - u_5u_2)$$

- $$p = (u_5 - u_3)x_6 + (-u_6 + u_4)x_5 + (u_6u_3 - u_5u_4)$$

4 Triangulation of polynomial system

The input system is:

$$\begin{aligned}
p_1 &= 2x_3 - x_1 - u_3\\
p_2 &= 2x_4 - x_2 - u_4\\
p_3 &= 2x_5 - x_1 - u_1\\
p_4 &= 2x_6 - x_2 - u_2\\
p_5 &= (u_5 - u_1)x_4 + (-u_6 + u_2)x_3 + (u_6u_1 - u_5u_2)\\
p_6 &= (u_5 - u_3)x_6 + (-u_6 + u_4)x_5 + (u_6u_3 - u_5u_4)
\end{aligned}$$

4.1 Triangulation, step 1

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_6 from all other polynomials by reducing them with polynomial p_4 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_3 - x_1 - u_3 \\
p_2 &= 2x_4 - x_2 - u_4 \\
p_3 &= 2x_5 - x_1 - u_1 \\
p_4 &= (u_5 - u_1)x_4 + (-u_6 + u_2)x_3 + (u_6u_1 - u_5u_2) \\
p_5 &= (-2u_6 + 2u_4)x_5 + (u_5 - u_3)x_2 + (2u_6u_3 - 2u_5u_4 + u_5u_2 - u_3u_2) \\
p_6 &= 2x_6 - x_2 - u_2
\end{aligned}
$$

4.2 Triangulation, step 2

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_5 from all other polynomials by reducing them with polynomial p_3 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_3 - x_1 - u_3 \\
p_2 &= 2x_4 - x_2 - u_4 \\
p_3 &= (u_5 - u_1)x_4 + (-u_6 + u_2)x_3 + (u_6u_1 - u_5u_2) \\
p_4 &= (2u_5 - 2u_3)x_2 + (-2u_6 + 2u_4)x_1 + \\
&\quad (4u_6u_3 - 2u_6u_1 - 4u_5u_4 + 2u_5u_2 + 2u_4u_1 - 2u_3u_2) \\
p_5 &= 2x_5 - x_1 - u_1 \\
p_6 &= 2x_6 - x_2 - u_2
\end{aligned}
$$

4.3 Triangulation, step 3

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_2 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_3 - x_1 - u_3 \\
p_2 &= (2u_5 - 2u_3)x_2 + (-2u_6 + 2u_4)x_1 + \\
&\quad (4u_6u_3 - 2u_6u_1 - 4u_5u_4 + 2u_5u_2 + 2u_4u_1 - 2u_3u_2) \\
p_3 &= (-2u_6 + 2u_2)x_3 + (u_5 - u_1)x_2 + (2u_6u_1 + u_5u_4 - 2u_5u_2 - u_4u_1) \\
p_4 &= 2x_4 - x_2 - u_4 \\
p_5 &= 2x_5 - x_1 - u_1 \\
p_6 &= 2x_6 - x_2 - u_2
\end{aligned}
$$

4.4 Triangulation, step 4

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_3 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (2u_5 - 2u_3)x_2 + (-2u_6 + 2u_4)x_1 + \\
&\quad (4u_6u_3 - 2u_6u_1 - 4u_5u_4 + 2u_5u_2 + 2u_4u_1 - 2u_3u_2) \\
p_2 &= (2u_5 - 2u_1)x_2 + (-2u_6 + 2u_2)x_1 + \\
&\quad (-2u_6u_3 + 4u_6u_1 + 2u_5u_4 - 4u_5u_2 - 2u_4u_1 + 2u_3u_2) \\
p_3 &= 2x_3 - x_1 - u_3 \\
p_4 &= 2x_4 - x_2 - u_4 \\
p_5 &= 2x_5 - x_1 - u_1 \\
p_6 &= 2x_6 - x_2 - u_2
\end{aligned}
$$

4.5 Triangulation, step 5

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_2 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (4u_6u_3 - 4u_6u_1 - 4u_5u_4 + 4u_5u_2 + 4u_4u_1 - 4u_3u_2)x_1 + \\
&\quad (-12u_6u_5u_3 + 12u_6u_5u_1 + 4u_6u_3^2 - 4u_6u_1^2 + 12u_5^2u_4 \\
&\quad -12u_5^2u_2 - 4u_5u_4u_3 - 16u_5u_4u_1 + 16u_5u_3u_2 + 4u_5u_2u_1 + \\
&\quad 4u_4u_3u_1 + 4u_4u_1^2 - 4u_3^2u_2 - 4u_3u_2u_1) \\
\\
p_2 &= (2u_5 - 2u_3)x_2 + (-2u_6 + 2u_4)x_1 + \\
&\quad (4u_6u_3 - 2u_6u_1 - 4u_5u_4 + 2u_5u_2 + 2u_4u_1 - 2u_3u_2) \\
p_3 &= 2x_3 - x_1 - u_3 \\
p_4 &= 2x_4 - x_2 - u_4 \\
p_5 &= 2x_5 - x_1 - u_1 \\
p_6 &= 2x_6 - x_2 - u_2
\end{aligned}
$$

4.6 Triangulation, step 6

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= (4u_6u_3 - 4u_6u_1 - 4u_5u_4 + 4u_5u_2 + 4u_4u_1 - 4u_3u_2)x_1 + \\
&\quad (-12u_6u_5u_3 + 12u_6u_5u_1 + 4u_6u_3^2 - 4u_6u_1^2 + 12u_5^2u_4 \\
&\quad -12u_5^2u_2 - 4u_5u_4u_3 - 16u_5u_4u_1 + 16u_5u_3u_2 + 4u_5u_2u_1 + \\
&\quad 4u_4u_3u_1 + 4u_4u_1^2 - 4u_3^2u_2 - 4u_3u_2u_1) \\
\\
p_2 &= (2u_5 - 2u_3)x_2 + (-2u_6 + 2u_4)x_1 + \\
&\quad (4u_6u_3 - 2u_6u_1 - 4u_5u_4 + 2u_5u_2 + 2u_4u_1 - 2u_3u_2) \\
p_3 &= 2x_3 - x_1 - u_3 \\
p_4 &= 2x_4 - x_2 - u_4 \\
p_5 &= 2x_5 - x_1 - u_1 \\
p_6 &= 2x_6 - x_2 - u_2
\end{aligned}
$$

List of leading monic terms from triangular polynomial system

$$
p_1 = x_1
$$

$$p_2 = x_2$$

$$p_3 = x_3$$

$$p_4 = x_4$$

$$p_5 = x_5$$

$$p_6 = x_6$$

5 Result of transformation of RC-constructibility problem to polynomial form

Success Message: Successful completion.

Space Complexity: The biggest polynomial obtained during application execution contains 3 terms.

Time Complexity: Time spent in execution is 0.11 seconds.