OpenGeoProver Output for conjecture "Chou 042"

Wu's method used

February 19, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Free point C
- Free point D
- Line AD through two points A and D
- Line BC through two points B and C
- Intersection point W of point sets AD and BC
- Midpoint X of segment AC
- Midpoint Y of segment BD

Theorem statement:

- Linear combination of double signed polygon areas: 4.0*WXY-1.0*ABCD equals zero

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

2.2 Transformation of point B:

- Point B has been assigned following coordinates: $(0,\ u_1)$

2.3 Transformation of point C:

- Point C has been assigned following coordinates: $(u_2,\ u_3)$

2.4 Transformation of point D:

- Point D has been assigned following coordinates: $(u_4,\ u_5)$

2.5 Transformation of point W:

- Point W has been assigned following coordinates: $(x_1,\ x_2)$
- Polynomial that point W has to satisfy is:

$$p \quad = \quad u_4x_2 - u_5x_1$$

- Processing of polynomial

$$p \quad = \quad u_4x_2 - u_5x_1$$

Info: Polynomial

$$p \quad = \quad u_4x_2 - u_5x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point W has to satisfy is:

$$p \quad = \quad u_2x_2 + (-u_3 + u_1)x_1 - u_2u_1$$

- Processing of polynomial

$$p \quad = \quad u_2x_2 + (-u_3 + u_1)x_1 - u_2u_1$$

Info: Polynomial

$$p \quad = \quad u_2x_2 + (-u_3 + u_1)x_1 - u_2u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point X:

- Point X has been assigned following coordinates: (x_3, x_4)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_3 - 0.5u_2$$

Info: Polynomial

$$p = x_3 - 0.5u_2$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_3 - 0.5u_2$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_4 - 0.5u_3$$

Info: Polynomial

$$p = x_4 - 0.5u_3$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_4 - 0.5u_3$$

is added to polynomial system

2.7 Transformation of point Y:

- Point Y has been assigned following coordinates: (x_5, x_6)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_5 - 0.5u_4$$

Info: Polynomial

$$p = x_5 - 0.5u_4$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_5 - 0.5u_4$$

  is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_6 + (-0.5u_5 - 0.5u_1)$$

  Info: Polynomial

$$p = x_6 + (-0.5u_5 - 0.5u_1)$$

  added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_6 + (-0.5u_5 - 0.5u_1)$$

  is added to polynomial system

Transformation of Theorem statement

- Polynomial for theorem statement:

$$\begin{aligned} p = {} & x_6x_3 - x_6x_1 - x_5x_4 + x_5x_2 + x_4x_1 - x_3x_2 + \\ & (-0.25u_5u_2 + 0.25u_4u_3 + 0.25u_2u_1) \end{aligned}$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.063 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned} p_1 &= u_4x_2 - u_5x_1 \\ p_2 &= u_2x_2 + (-u_3 + u_1)x_1 - u_2u_1 \\ p_3 &= x_3 - 0.5u_2 \\ p_4 &= x_4 - 0.5u_3 \\ p_5 &= x_5 - 0.5u_4 \\ p_6 &= x_6 + (-0.5u_5 - 0.5u_1) \end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_2 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_1 &= (u_5u_2 - u_4u_3 + u_4u_1)x_1 - u_4u_2u_1 \\ p_2 &= u_4x_2 - u_5x_1 \\ p_3 &= x_3 - 0.5u_2 \\ p_4 &= x_4 - 0.5u_3 \\ p_5 &= x_5 - 0.5u_4 \\ p_6 &= x_6 + (-0.5u_5 - 0.5u_1) \end{aligned}$$

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned} p_1 &= (u_5u_2 - u_4u_3 + u_4u_1)x_1 - u_4u_2u_1 \\ p_2 &= u_4x_2 - u_5x_1 \\ p_3 &= x_3 - 0.5u_2 \\ p_4 &= x_4 - 0.5u_3 \\ p_5 &= x_5 - 0.5u_4 \\ p_6 &= x_6 + (-0.5u_5 - 0.5u_1) \end{aligned}$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 042

Calculating final remainder of the conclusion:

$$\begin{aligned} g &= x_6x_3 - x_6x_1 - x_5x_4 + x_5x_2 + x_4x_1 - x_3x_2 + \\ &\quad (-0.25u_5u_2 + 0.25u_4u_3 + 0.25u_2u_1) \end{aligned}$$

with respect to the triangular system.

1. Pseudo remainder with p_6 over variable x_6:

$$\begin{aligned} g &= -x_5x_4 + x_5x_2 + x_4x_1 - x_3x_2 + (0.5u_5 + 0.5u_1)x_3 + \\ &\quad (-0.5u_5 - 0.5u_1)x_1 + (-0.25u_5u_2 + 0.25u_4u_3 + 0.25u_2u_1) \end{aligned}$$

2. Pseudo remainder with p_5 over variable x_5:

$$\begin{aligned} g &= x_4x_1 - 0.5u_4x_4 - x_3x_2 + (0.5u_5 + 0.5u_1)x_3 + 0.5u_4x_2 + \\ &\quad (-0.5u_5 - 0.5u_1)x_1 + (-0.25u_5u_2 + 0.25u_4u_3 + 0.25u_2u_1) \end{aligned}$$

3. Pseudo remainder with p_4 over variable x_4:

$$\begin{aligned} g &= -x_3x_2 + (0.5u_5 + 0.5u_1)x_3 + 0.5u_4x_2 + (-0.5u_5 + 0.5u_3 - 0.5u_1)x_1 + \\ &\quad (-0.25u_5u_2 + 0.25u_2u_1) \end{aligned}$$

4. Pseudo remainder with p_3 over variable x_3:

$$g = (0.5u_4 - 0.5u_2)x_2 + (-0.5u_5 + 0.5u_3 - 0.5u_1)x_1 + 0.5u_2u_1$$

5. Pseudo remainder with p_2 over variable x_2:

$$g = (-0.5u_5u_2 + 0.5u_4u_3 - 0.5u_4u_1)x_1 + 0.5u_4u_2u_1$$

6. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 7 terms.

Time Complexity: Time spent by the prover is 0.078 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points D, A, B and C are not collinear
- Points D and A are not identical

Time spent for processing NDG Conditions

- 0.203 seconds