OpenGeoProver Output for RC-Constructibility problem "Wernick 080 (O,H,I —— A)"

Used algebraic method (with triangulation)

July 14, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Free point I
- Angle ray b of angle with vertex A and point I from first ray, which is congruent to angle BAI
- Angle ray a of angle with vertex B and point I from first ray, which is congruent to angle ABI
- Intersection point C of point sets a and b
- Line ha through point A perpendicular to line a
- Line hb through point B perpendicular to line b
- Intersection point H of point sets ha and hb
- Perpendicular bisector ma of segment BC
- Perpendicular bisector mb of segment AC
- Intersection point O of point sets ma and mb

Free points:

- I
- H
- O

Points to be constructed:

- A

Validation result: Construction protocol is valid.

2 Instantiation of points with symbolic variables

- Point A has been assigned following coordinates: (x_1, x_2)
- Point I has been assigned following coordinates: (u_1, u_2)
- Point H has been assigned following coordinates: (u_3, u_4)
- Point O has been assigned following coordinates: (u_5, u_6)
- Point B has been assigned following coordinates: (x_3, x_4)
- Point C has been assigned following coordinates: (x_5, x_6)

3 Transformation of geometry conditions for points to polynomial form

3.1 Transformation, step 1

Point to transform: A

Polynomial condition(s): N/A - free point

3.2 Transformation, step 2

Point to transform: B

Polynomial condition(s): N/A - free point

3.3 Transformation, step 3

Point to transform: I

Polynomial condition(s): N/A - free point

3.4 Transformation, step 4

Point to transform: C

Polynomial condition(s): Two polynomials

- $$\begin{aligned} p \;=\; & x_6x_4^2x_3 + x_6x_4^2x_1 - 2u_1x_6x_4^2 - 2x_6x_4x_3x_2 + \\ & 2u_1x_6x_4x_2 - 2u_2x_6x_4x_1 + 2u_2u_1x_6x_4 + x_6x_3^3 \\ & -x_6x_3^2x_1 - 2u_1x_6x_3^2 + 2u_2x_6x_3x_2 + 2u_1x_6x_3x_1 + \\ & (-u_2^2 + u_1^2)x_6x_3 - 2u_2u_1x_6x_2 + (u_2^2 - u_1^2)x_6x_1 \\ & -x_5x_4^3 + x_5x_4^2x_2 + 2u_2x_5x_4^2 - x_5x_4x_3^2 + \\ & 2x_5x_4x_3x_1 - 2u_2x_5x_4x_2 - 2u_1x_5x_4x_1 + \\ & (-u_2^2 + u_1^2)x_5x_4 - x_5x_3^2x_2 + 2u_2x_5x_3^2 + \\ & 2u_1x_5x_3x_2 - 2u_2x_5x_3x_1 - 2u_2u_1x_5x_3 + \end{aligned}$$

$$
\begin{aligned}
&(u_2^2 - u_1^2)x_5x_2 + 2u_2u_1x_5x_1 - x_4^3x_1 + 2u_1x_4^3 + \\
&x_4^2x_3x_2 - 2u_2x_4^2x_3 - 2u_1x_4^2x_2 + 2u_2x_4^2x_1 \\
&-2u_2u_1x_4^2 - x_4x_3^2x_1 + 2u_1x_4x_3^2 + \\
&(2u_2^2 - 2u_1^2)x_4x_3 + 2u_2u_1x_4x_2 + (-u_2^2 + u_1^2)x_4x_1 + \\
&x_3^3x_2 - 2u_2x_3^3 - 2u_1x_3^2x_2 + 2u_2x_3^2x_1 + \\
&2u_2u_1x_3^2 + (-u_2^2 + u_1^2)x_3x_2 - 2u_2u_1x_3x_1
\end{aligned}
$$

- $$
\begin{aligned}
p \;=\; & -2x_6x_4x_2x_1 + 2u_1x_6x_4x_2 + 2u_2x_6x_4x_1 - 2u_2u_1x_6x_4 + \\
& x_6x_3x_2^2 - 2u_2x_6x_3x_2 - x_6x_3x_1^2 + 2u_1x_6x_3x_1 + \\
& (u_2^2 - u_1^2)x_6x_3 + x_6x_2^2x_1 - 2u_1x_6x_2^2 + \\
& 2u_2u_1x_6x_2 + x_6x_1^3 - 2u_1x_6x_1^2 + (-u_2^2 + u_1^2)x_6x_1 + \\
& x_5x_4x_2^2 - 2u_2x_5x_4x_2 - x_5x_4x_1^2 + 2u_1x_5x_4x_1 + \\
& (u_2^2 - u_1^2)x_5x_4 + 2x_5x_3x_2x_1 - 2u_1x_5x_3x_2 \\
& -2u_2x_5x_3x_1 + 2u_2u_1x_5x_3 - x_5x_2^3 + 2u_2x_5x_2^2 \\
& -x_5x_2x_1^2 + (-u_2^2 + u_1^2)x_5x_2 + 2u_2x_5x_1^2 \\
& -2u_2u_1x_5x_1 + x_4x_2^2x_1 - 2u_1x_4x_2^2 + 2u_2u_1x_4x_2 + \\
& x_4x_1^3 - 2u_1x_4x_1^2 + (-u_2^2 + u_1^2)x_4x_1 - x_3x_2^3 + \\
& 2u_2x_3x_2^2 - x_3x_2x_1^2 + (-u_2^2 + u_1^2)x_3x_2 + \\
& 2u_2x_3x_1^2 - 2u_2u_1x_3x_1 + 2u_1x_2^3 - 2u_2x_2^2x_1 \\
& -2u_2u_1x_2^2 + 2u_1x_2x_1^2 + (2u_2^2 - 2u_1^2)x_2x_1 - 2u_2x_1^3 + \\
& 2u_2u_1x_1^2
\end{aligned}
$$

3.5 Transformation, step 5

Point to transform: H

Polynomial condition(s): Two polynomials

- $$p \;=\; -x_6x_2 + u_4x_6 - x_5x_1 + u_3x_5 + x_4x_2 - u_4x_4 + x_3x_1 - u_3x_3$$
- $$p \;=\; -x_6x_4 + u_4x_6 - x_5x_3 + u_3x_5 + x_4x_2 + x_3x_1 - u_4x_2 - u_3x_1$$

3.6 Transformation, step 6

Point to transform: O

Polynomial condition(s): Two polynomials

- $$p \;=\; -x_6^2 + 2u_6x_6 - x_5^2 + 2u_5x_5 + x_4^2 - 2u_6x_4 + x_3^2 - 2u_5x_3$$
- $$p \;=\; -x_6^2 + 2u_6x_6 - x_5^2 + 2u_5x_5 + x_2^2 - 2u_6x_2 + x_1^2 - 2u_5x_1$$

4 Instantiation of polynomial system

4.1 Instances for points' coordinates

- I: x = 1.0, y = 0.0
- H: x = 0.0, y = 1.0
- O: x = 0.0, y = 0.0

4.2 Instantiated polynomial system

$$
\begin{aligned}
p_1 &= x_6x_4^2x_3 + x_6x_4^2x_1 - 2x_6x_4^2 - 2x_6x_4x_3x_2 + \\
&\quad 2x_6x_4x_2 + x_6x_3^3 - x_6x_3^2x_1 - 2x_6x_3^2 + 2x_6x_3x_1 + \\
&\quad x_6x_3 - x_6x_1 - x_5x_4^3 + x_5x_4^2x_2 - x_5x_4x_3^2 + \\
&\quad 2x_5x_4x_3x_1 - 2x_5x_4x_1 + x_5x_4 - x_5x_3^2x_2 + 2x_5x_3x_2 \\
&\quad -x_5x_2 - x_4^3x_1 + 2x_4^3 + x_4^2x_3x_2 - 2x_4^2x_2 \\
&\quad -x_4x_3^2x_1 + 2x_4x_3^2 - 2x_4x_3 + x_4x_1 + x_3^3x_2 - 2x_3^2x_2 + \\
&\quad x_3x_2 \\
p_2 &= -2x_6x_4x_2x_1 + 2x_6x_4x_2 + x_6x_3x_2^2 - x_6x_3x_1^2 + \\
&\quad 2x_6x_3x_1 - x_6x_3 + x_6x_2^2x_1 - 2x_6x_2^2 + x_6x_1^3 \\
&\quad -2x_6x_1^2 + x_6x_1 + x_5x_4x_2^2 - x_5x_4x_1^2 + 2x_5x_4x_1 \\
&\quad -x_5x_4 + 2x_5x_3x_2x_1 - 2x_5x_3x_2 - x_5x_2^3 - x_5x_2x_1^2 + \\
&\quad x_5x_2 + x_4x_2^2x_1 - 2x_4x_2^2 + x_4x_1^3 - 2x_4x_1^2 + x_4x_1 \\
&\quad -x_3x_2^3 - x_3x_2x_1^2 + x_3x_2 + 2x_2^3 + 2x_2x_1^2 - 2x_2x_1 \\
p_3 &= -x_6x_2 + x_6 - x_5x_1 + x_4x_2 - x_4 + x_3x_1 \\
p_4 &= -x_6x_4 + x_6 - x_5x_3 + x_4x_2 + x_3x_1 - x_2 \\
p_5 &= -x_6^2 - x_5^2 + x_4^2 + x_3^2 \\
p_6 &= -x_6^2 - x_5^2 + x_2^2 + x_1^2
\end{aligned}
$$

5 Triangulation of polynomial system

The input system is:

$$
\begin{aligned}
p_1 &= x_6x_4^2x_3 + x_6x_4^2x_1 - 2x_6x_4^2 - 2x_6x_4x_3x_2 + \\
&\quad 2x_6x_4x_2 + x_6x_3^3 - x_6x_3^2x_1 - 2x_6x_3^2 + 2x_6x_3x_1 + \\
&\quad x_6x_3 - x_6x_1 - x_5x_4^3 + x_5x_4^2x_2 - x_5x_4x_3^2 + \\
&\quad 2x_5x_4x_3x_1 - 2x_5x_4x_1 + x_5x_4 - x_5x_3^2x_2 + 2x_5x_3x_2 \\
&\quad -x_5x_2 - x_4^3x_1 + 2x_4^3 + x_4^2x_3x_2 - 2x_4^2x_2 \\
&\quad -x_4x_3^2x_1 + 2x_4x_3^2 - 2x_4x_3 + x_4x_1 + x_3^3x_2 - 2x_3^2x_2 + \\
&\quad x_3x_2 \\
p_2 &= -2x_6x_4x_2x_1 + 2x_6x_4x_2 + x_6x_3x_2^2 - x_6x_3x_1^2 +
\end{aligned}
$$

$$
\begin{aligned}
& 2x_6x_3x_1 - x_6x_3 + x_6x_2^2x_1 - 2x_6x_2^2 + x_6x_1^3 \\
& -2x_6x_1^2 + x_6x_1 + x_5x_4x_2^2 - x_5x_4x_1^2 + 2x_5x_4x_1 \\
& -x_5x_4 + 2x_5x_3x_2x_1 - 2x_5x_3x_2 - x_5x_2^3 - x_5x_2x_1^2 + \\
& x_5x_2 + x_4x_2^2x_1 - 2x_4x_2^2 + x_4x_1^3 - 2x_4x_1^2 + x_4x_1 \\
& -x_3x_2^3 - x_3x_2x_1^2 + x_3x_2 + 2x_2^3 + 2x_2x_1^2 - 2x_2x_1 \\
p_3 \;=\; & -x_6x_2 + x_6 - x_5x_1 + x_4x_2 - x_4 + x_3x_1 \\
p_4 \;=\; & -x_6x_4 + x_6 - x_5x_3 + x_4x_2 + x_3x_1 - x_2 \\
p_5 \;=\; & -x_6^2 - x_5^2 + x_4^2 + x_3^2 \\
p_6 \;=\; & -x_6^2 - x_5^2 + x_2^2 + x_1^2
\end{aligned}
$$

5.1 Triangulation, step 1

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 6.

Minimal degrees: 4 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_6 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 \;=\; & \ldots \\
p_2 \;=\; & -x_5x_4^3x_2 + x_5x_4^3 - x_5x_4^2x_3x_1 + x_5x_4^2x_2^2 \\
& -x_5x_4^2x_2 - x_5x_4^2x_1^2 + 2x_5x_4^2x_1 - x_5x_4x_3^2x_2 + \\
& x_5x_4x_3^2 + 4x_5x_4x_3x_2x_1 - 2x_5x_4x_3x_1 - 4x_5x_4x_2x_1 + \\
& x_5x_4x_2 + 2x_5x_4x_1 - x_5x_4 - x_5x_3^3x_1 - x_5x_3^2x_2^2 + \\
& x_5x_3^2x_2 + x_5x_3^2x_1^2 + 2x_5x_3^2x_1 + 2x_5x_3x_2^2 \\
& -2x_5x_3x_2 - 2x_5x_3x_1^2 - x_5x_3x_1 - x_5x_2^2 + x_5x_2 + \\
& x_5x_1^2 + x_4^3x_3x_2 - x_4^3x_3 + x_4^2x_3^2x_1 \\
& -x_4^2x_3x_2^2 + x_4^2x_3x_2 + x_4^2x_3x_1^2 - 2x_4^2x_3x_1 + \\
& x_4x_3^3x_2 - x_4x_3^3 - 4x_4x_3^2x_2x_1 + 2x_4x_3^2x_1 + \\
& 4x_4x_3x_2x_1 - x_4x_3x_2 - 2x_4x_3x_1 + x_4x_3 + x_3^4x_1 + \\
& x_3^3x_2^2 - x_3^3x_2 - x_3^3x_1^2 - 2x_3^3x_1 - 2x_3^2x_2^2 + \\
& 2x_3^2x_2 + 2x_3^2x_1^2 + x_3^2x_1 + x_3x_2^2 - x_3x_2 - x_3x_1^2 \\
p_3 \;=\; & -x_5x_4^4 + x_5x_4^3x_2 + x_5x_4^3 - 2x_5x_4^2x_3^2 + \\
& x_5x_4^2x_3x_1 + 2x_5x_4^2x_3 - x_5x_4^2x_2 - 2x_5x_4^2x_1 + \\
& x_5x_4^2 + x_5x_4x_3^2x_2 + x_5x_4x_3^2 - 2x_5x_4x_3x_1 \\
& -x_5x_4x_2 + 2x_5x_4x_1 - x_5x_4 - x_5x_3^4 + x_5x_3^3x_1 + \\
& 2x_5x_3^3 + x_5x_3^2x_2 - 2x_5x_3^2x_1 - x_5x_3^2 - 2x_5x_3x_2 + \\
& x_5x_3x_1 + x_5x_2 - x_4^4x_1 + 2x_4^4 + 2x_4^3x_3x_2 + x_4^3x_2x_1 \\
& -4x_4^3x_2 + x_4^3x_1 - 2x_4^3 + 2x_4^2x_3^2 - 2x_4^2x_3x_2^2
\end{aligned}
$$

$$
\begin{aligned}
& -2x_4^2x_3x_2 + x_4^2x_3x_1^2 - 2x_4^2x_3x_1 - 2x_4^2x_3 + \\
& 2x_4^2x_2^2 - x_4^2x_2x_1 + 4x_4^2x_2 + x_4^2x_1 + 2x_4x_3^3x_2 \\
& -3x_4x_3^2x_2x_1 - 4x_4x_3^2x_2 + x_4x_3^2x_1 - 2x_4x_3^2 + \\
& 2x_4x_3x_2^2 + 4x_4x_3x_2x_1 + 2x_4x_3x_2 + 2x_4x_3 - 2x_4x_2^2 \\
& -x_4x_2x_1 - x_4x_1 + x_3^4x_1 - 2x_3^3x_2 - x_3^3x_1^2 \\
& -2x_3^3x_1 + x_3^2x_2x_1 + 4x_3^2x_2 + 2x_3^2x_1^2 + x_3^2x_1 \\
& -2x_3x_2x_1 - 2x_3x_2 - x_3x_1^2 + x_2x_1
\end{aligned}
$$

$$
\begin{aligned}
p_4 &= \ldots \\
p_5 &= \ldots \\
p_6 &= x_6x_4^2x_3 + x_6x_4^2x_1 - 2x_6x_4^2 - 2x_6x_4x_3x_2 + \\
&\quad 2x_6x_4x_2 + x_6x_3^3 - x_6x_3^2x_1 - 2x_6x_3^2 + 2x_6x_3x_1 + \\
&\quad x_6x_3 - x_6x_1 - x_5x_4^3 + x_5x_4^2x_2 - x_5x_4x_3^2 + \\
&\quad 2x_5x_4x_3x_1 - 2x_5x_4x_1 + x_5x_4 - x_5x_3^2x_2 + 2x_5x_3x_2 \\
&\quad -x_5x_2 - x_4^3x_1 + 2x_4^3 + x_4^2x_3x_2 - 2x_4^2x_2 \\
&\quad -x_4x_3^2x_1 + 2x_4x_3^2 - 2x_4x_3 + x_4x_1 + x_3^3x_2 - 2x_3^2x_2 + \\
&\quad x_3x_2
\end{aligned}
$$

5.2 Triangulation, step 2

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 5.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_5 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= \ldots \\
p_2 &= \ldots \\
p_3 &= \ldots \\
p_4 &= \ldots \\
p_5 &= \ldots \\
p_6 &= x_6x_4^2x_3 + x_6x_4^2x_1 - 2x_6x_4^2 - 2x_6x_4x_3x_2 + \\
&\quad 2x_6x_4x_2 + x_6x_3^3 - x_6x_3^2x_1 - 2x_6x_3^2 + 2x_6x_3x_1 + \\
&\quad x_6x_3 - x_6x_1 - x_5x_4^3 + x_5x_4^2x_2 - x_5x_4x_3^2 + \\
&\quad 2x_5x_4x_3x_1 - 2x_5x_4x_1 + x_5x_4 - x_5x_3^2x_2 + 2x_5x_3x_2 \\
&\quad -x_5x_2 - x_4^3x_1 + 2x_4^3 + x_4^2x_3x_2 - 2x_4^2x_2 \\
&\quad -x_4x_3^2x_1 + 2x_4x_3^2 - 2x_4x_3 + x_4x_1 + x_3^3x_2 - 2x_3^2x_2 + \\
&\quad x_3x_2
\end{aligned}
$$

5.3 Triangulation, step 3

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 4.

Minimal degrees: 2 polynomial(s) with degree 7.

No linear degree polynomials: Reducing polynomial p_2 (of degree 7) with p_1 (of degree 7).

Failed to triangulate system of polynomials

6 Result of transformation of RC-constructibility problem to polynomial form

Success Message: Transformation failed since space limit of 100000 polynomial terms has been reached - the biggest polynomial obtained during execution contains 114214 terms.

Space Complexity: The biggest polynomial obtained during application execution contains 114214 terms.

Time Complexity: Time spent in execution is 609.869 seconds.