OpenGeoProver Output for conjecture "Chou 334 (Ceva's Theorem)"

Wu's method used

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1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Free point C
- Line BC through two points B and C
- Random point D from line BC
- Line CA through two points C and A
- Random point E from line CA
- Line AD through two points A and D
- Line BE through two points B and E
- Intersection point S of point sets AD and BE
- Line CS through two points C and S
- Line AB through two points A and B
- Intersection point F of point sets AB and CS

Theorem statement:

- Ratio product (AF/FB)*(BD/DC)*(CE/EA) is equal to 1.0

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

2.2 Transformation of point B:

- Point B has been assigned following coordinates: (0, u_1)

2.3 Transformation of point C:

- Point C has been assigned following coordinates: (u_2, u_3)

2.4 Transformation of point D:

- Point D has been assigned following coordinates: (u_4, x_1)
- Polynomial that point D has to satisfy is:

$$p = u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1)$$

- Processing of polynomial

$$p = u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1)$$

Info: Polynomial

$$p = u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of point E:

- Point E has been assigned following coordinates: (u_5, x_2)
- Polynomial that point E has to satisfy is:

$$p = u_2x_2 - u_5u_3$$

- Processing of polynomial

$$p = u_2x_2 - u_5u_3$$

Info: Polynomial

$$p = u_2x_2 - u_5u_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point S:

- Point S has been assigned following coordinates: (x_3, x_4)
- Polynomial that point S has to satisfy is:

$$p = u_4x_4 - x_3x_1$$

- Processing of polynomial

$$p = u_4x_4 - x_3x_1$$

Info: Polynomial

$$p = u_4x_4 - x_3x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point S has to satisfy is:

$$p = u_5x_4 - x_3x_2 + u_1x_3 - u_5u_1$$

- Processing of polynomial

$$p = u_5x_4 - x_3x_2 + u_1x_3 - u_5u_1$$

Info: Polynomial

$$p = u_5x_4 - x_3x_2 + u_1x_3 - u_5u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point F:

- Point F has been assigned following coordinates: (x_5, x_6)
- Polynomial that point F has to satisfy is:

$$p = x_5$$

- Processing of polynomial

$$p = x_5$$

Info: Will try to rename X coordinate of point F

Info: Y coordinate of point F will be replaced by X coordinate

Info: X coordinate of point F renamed by zero

- Point F has been renamed. Point F has been assigned following coordinates: (0, x_5)

- Polynomial that point F has to satisfy is:

$$p = x_5x_3 - u_2x_5 + u_2x_4 - u_3x_3$$

- Processing of polynomial

$$p = x_5x_3 - u_2x_5 + u_2x_4 - u_3x_3$$

Info: Polynomial

$$p = x_5x_3 - u_2x_5 + u_2x_4 - u_3x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = (u_5u_4 - 0.5u_5u_2 - 0.5u_4u_2)x_5 + (-0.5u_5u_4u_1 + 0.5u_5u_2u_1)$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.06 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned} p_1 &= u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1) \\ p_2 &= u_2x_2 - u_5u_3 \\ p_3 &= u_4x_4 - x_3x_1 \\ p_4 &= u_5x_4 - x_3x_2 + u_1x_3 - u_5u_1 \\ p_5 &= x_5x_3 - u_2x_5 + u_2x_4 - u_3x_3 \end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_3 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_1 &= u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1) \\ p_2 &= u_2x_2 - u_5u_3 \\ p_3 &= -u_4x_3x_2 + u_5x_3x_1 + u_4u_1x_3 - u_5u_4u_1 \\ p_4 &= u_4x_4 - x_3x_1 \\ p_5 &= x_5x_3 - u_2x_5 + u_2x_4 - u_3x_3 \end{aligned}$$

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_1 &= u_2x_1 + (-u_4u_3 + u_4u_1 - u_2u_1) \\
p_2 &= u_2x_2 - u_5u_3 \\
p_3 &= -u_4x_3x_2 + u_5x_3x_1 + u_4u_1x_3 - u_5u_4u_1 \\
p_4 &= u_4x_4 - x_3x_1 \\
p_5 &= x_5x_3 - u_2x_5 + u_2x_4 - u_3x_3
\end{aligned}$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 334 (Ceva's Theorem)

Calculating final remainder of the conclusion:

$$g = (u_5u_4 - 0.5u_5u_2 - 0.5u_4u_2)x_5 + (-0.5u_5u_4u_1 + 0.5u_5u_2u_1)$$

with respect to the triangular system.

1. Pseudo remainder with p_5 over variable x_5:

$$\begin{aligned}
g = {} & (-u_5u_4u_2 + 0.5u_5u_2^2 + 0.5u_4u_2^2)x_4 + \\
& (u_5u_4u_3 - 0.5u_5u_4u_1 - 0.5u_5u_3u_2 + 0.5u_5u_2u_1 - 0.5u_4u_3u_2) \\
& x_3 \\
& +(0.5u_5u_4u_2u_1 - 0.5u_5u_2^2u_1)
\end{aligned}$$

2. Pseudo remainder with p_4 over variable x_4:

$$\begin{aligned}
g = {} & (-u_5u_4u_2 + 0.5u_5u_2^2 + 0.5u_4u_2^2)x_3x_1 + \\
& (u_5u_4^2u_3 - 0.5u_5u_4^2u_1 - 0.5u_5u_4u_3u_2 + 0.5u_5u_4u_2u_1 \\
& -0.5u_4^2u_3u_2) \\
& x_3 \\
& +(0.5u_5u_4^2u_2u_1 - 0.5u_5u_4u_2^2u_1)
\end{aligned}$$

3. Pseudo remainder with p_3 over variable x_3:

$$\begin{aligned}
g = {} & (-0.5u_5u_4^3u_2u_1 + 0.5u_5u_4^2u_2^2u_1)x_2 + \\
& (-0.5u_5^2u_4^2u_2u_1 + 0.5u_5u_4^2u_2^2u_1)x_1 + \\
& (u_5^2u_4^3u_3u_1 - 0.5u_5^2u_4^3u_1^2
\end{aligned}$$

$$
\begin{aligned}
&-0.5u_5^2u_4^2u_3u_2u_1 + 0.5u_5^2u_4^2u_2u_1^2 \\
&-0.5u_5u_4^3u_3u_2u_1 + 0.5u_5u_4^3u_2u_1^2 \\
&-0.5u_5u_4^2u_2^2u_1^2)
\end{aligned}
$$

4. Pseudo remainder with p_2 over variable x_2:

$$
\begin{aligned}
g \;=\; & (-0.5u_5^2u_4^2u_2^2u_1 + 0.5u_5u_4^2u_2^3u_1)x_1 + \\
& (0.5u_5^2u_4^3u_3u_2u_1 - 0.5u_5^2u_4^3u_2u_1^2 + \\
& 0.5u_5^2u_4^2u_2^2u_1^2 - 0.5u_5u_4^3u_3u_2^2u_1 + \\
& 0.5u_5u_4^3u_2^2u_1^2 - 0.5u_5u_4^2u_2^3u_1^2)
\end{aligned}
$$

5. Pseudo remainder with p_1 over variable x_1:

$$
g \;=\; 0
$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 4 terms.

Time Complexity: Time spent by the prover is 0.07 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points F, A, B and C are not collinear
- Line through points D and A is not parallel with line through points E and B
- Points D, F and A are not collinear
- Line through points A and B is not parallel with line through points S and C

Time spent for processing NDG Conditions

- 1.947 seconds