OpenGeoProver Output for conjecture “Converse of Thales’ theorem”

Wu’s method used

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1 Validation of Construction Protocol

Construction steps:

- Free point S
- Free point A
- Free point B
- Line SA through two points S and A
- Random point C from line SA
- Line AB through two points A and B
- Line CD through point C parallel with line AB
- Line SB through two points S and B
- Intersection point D of point sets CD and SB

Theorem statement:

- Ratio product (SA/SB) is equal to 1.0*(SC/SD)

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point S:

- Point S has been assigned following coordinates: (0, 0)

2.2 Transformation of point A:

- Point A has been assigned following coordinates: $(0, u_1)$

2.3 Transformation of point B:

- Point B has been assigned following coordinates: (u_2, u_3)

2.4 Transformation of point C:

- Point C has been assigned following coordinates: (u_4, x_1)
- Polynomial that point C has to satisfy is:

$$p = x_1$$

- Processing of polynomial

$$p = x_1$$

Info: Will try to rename X coordinate of point C

Info: X coordinate of point C renamed by zero

- Point C has been renamed. Point C has been assigned following coordinates: $(0, u_4)$

2.5 Transformation of point D:

- Point D has been assigned following coordinates: (x_1, x_2)
- Polynomial that point D has to satisfy is:

$$p = u_2x_2 + (-u_3 + u_1)x_1 - u_4u_2$$

- Processing of polynomial

$$p = u_2x_2 + (-u_3 + u_1)x_1 - u_4u_2$$

Info: Polynomial

$$p = u_2x_2 + (-u_3 + u_1)x_1 - u_4u_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point D has to satisfy is:

$$p = u_2x_2 - u_3x_1$$

- Processing of polynomial

$$p = u_2x_2 - u_3x_1$$

Info: Polynomial

$$p = u_2x_2 - u_3x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = u_1x_2 - u_4u_3$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.032 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned} p_1 &= u_2x_2 + (-u_3 + u_1)x_1 - u_4u_2 \\ p_2 &= u_2x_2 - u_3x_1 \end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_2 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_1 &= -u_2u_1x_1 + u_4u_2^2 \\ p_2 &= u_2x_2 + (-u_3 + u_1)x_1 - u_4u_2 \end{aligned}$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned} p_1 &= -u_2u_1x_1 + u_4u_2^2 \\ p_2 &= u_2x_2 + (-u_3 + u_1)x_1 - u_4u_2 \end{aligned}$$

4 Final Remainder

4.1 Final remainder for conjecture Converse of Thales' theorem

Calculating final remainder of the conclusion:

$$g = u_1x_2 - u_4u_3$$

with respect to the triangular system.

1. Pseudo remainder with p_2 over variable x_2:

$$g = (u_3u_1 - u_1^2)x_1 + (-u_4u_3u_2 + u_4u_2u_1)$$

2. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.025 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points A, S, B and C are not collinear
- Points A and S are not identical

Time spent for processing NDG Conditions

- 0.15 seconds