OpenGeoProver Output for conjecture "Chou 031"

Wu's method used

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1 Validation of Construction Protocol

Construction steps:

- Free point O
- Free point A
- Circle k with center O and one point A
- Random point B from circle k
- Random point C from circle k
- Line c through two points A and B
- Line hc through point C perpendicular to line c
- Line b through two points A and C
- Line hb through point B perpendicular to line b
- Intersection point H of point sets hb and hc

Theorem statement:

- Linear combination of squares of segments: 1.0*sqr(OH) -9.0*sqr(OA) +1.0*sqr(BC) +1.0*sqr(CA) +1.0*sqr(AB) equals zero

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point O:

- Point O has been assigned following coordinates: (0, 0)

2.2 Transformation of point A:

- Point A has been assigned following coordinates: $(0,\ u_1)$

2.3 Transformation of point B:

- Point B has been assigned following coordinates: $(u_2,\ x_1)$
- Polynomial that point B has to satisfy is:

$$p \quad = \quad x_1^2 + (u_2^2 - u_1^2)$$

- Processing of polynomial

$$p \quad = \quad x_1^2 + (u_2^2 - u_1^2)$$

Info: Polynomial

$$p \quad = \quad x_1^2 + (u_2^2 - u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.4 Transformation of point C:

- Point C has been assigned following coordinates: $(u_3,\ x_2)$
- Polynomial that point C has to satisfy is:

$$p \quad = \quad x_2^2 + (u_3^2 - u_1^2)$$

- Processing of polynomial

$$p \quad = \quad x_2^2 + (u_3^2 - u_1^2)$$

Info: Polynomial

$$p \quad = \quad x_2^2 + (u_3^2 - u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of point H:

- Point H has been assigned following coordinates: $(x_3,\ x_4)$
- Polynomial that point H has to satisfy is:

$$p \quad = \quad x_4x_2 - u_1x_4 + u_3x_3 - x_2x_1 + u_1x_1 - u_3u_2$$

- Processing of polynomial

$$p \quad = \quad x_4x_2 - u_1x_4 + u_3x_3 - x_2x_1 + u_1x_1 - u_3u_2$$

Info: Polynomial

$$p = x_4x_2 - u_1x_4 + u_3x_3 - x_2x_1 + u_1x_1 - u_3u_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point H has to satisfy is:

$$p = x_4x_1 - u_1x_4 + u_2x_3 - x_2x_1 + u_1x_2 - u_3u_2$$

- Processing of polynomial

$$p = x_4x_1 - u_1x_4 + u_2x_3 - x_2x_1 + u_1x_2 - u_3u_2$$

Info: Polynomial

$$p = x_4x_1 - u_1x_4 + u_2x_3 - x_2x_1 + u_1x_2 - u_3u_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

- Polynomial for theorem statement:

$$\begin{aligned} p = {} & x_4^2 + x_3^2 + 2x_2^2 - 2x_2x_1 - 2u_1x_2 + 2x_1^2 - 2u_1x_1 + \\ & (2u_3^2 - 2u_3u_2 + 2u_2^2 - 7u_1^2) \end{aligned}$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.047 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned} p_1 &= x_1^2 + (u_2^2 - u_1^2) \\ p_2 &= x_2^2 + (u_3^2 - u_1^2) \\ p_3 &= x_4x_2 - u_1x_4 + u_3x_3 - x_2x_1 + u_1x_1 - u_3u_2 \\ p_4 &= x_4x_1 - u_1x_4 + u_2x_3 - x_2x_1 + u_1x_2 - u_3u_2 \end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_3 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= x_1^2 + (u_2^2 - u_1^2) \\
p_2 &= x_2^2 + (u_3^2 - u_1^2) \\
p_3 &= u_2x_3x_2 - u_3x_3x_1 + (u_3u_1 - u_2u_1)x_3 - x_2^2x_1 + u_1x_2^2 + \\
&\quad x_2x_1^2 + (-u_3u_2 - u_1^2)x_2 - u_1x_1^2 + (u_3u_2 + u_1^2)x_1 \\
p_4 &= x_4x_2 - u_1x_4 + u_3x_3 - x_2x_1 + u_1x_1 - u_3u_2
\end{aligned}$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_1 &= x_1^2 + (u_2^2 - u_1^2) \\
p_2 &= x_2^2 + (u_3^2 - u_1^2) \\
p_3 &= u_2x_3x_2 - u_3x_3x_1 + (u_3u_1 - u_2u_1)x_3 - x_2^2x_1 + u_1x_2^2 + \\
&\quad x_2x_1^2 + (-u_3u_2 - u_1^2)x_2 - u_1x_1^2 + (u_3u_2 + u_1^2)x_1 \\
p_4 &= x_4x_2 - u_1x_4 + u_3x_3 - x_2x_1 + u_1x_1 - u_3u_2
\end{aligned}$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 031

Calculating final remainder of the conclusion:

$$\begin{aligned}
g &= x_4^2 + x_3^2 + 2x_2^2 - 2x_2x_1 - 2u_1x_2 + 2x_1^2 - 2u_1x_1 + \\
&\quad (2u_3^2 - 2u_3u_2 + 2u_2^2 - 7u_1^2)
\end{aligned}$$

with respect to the triangular system.

1. Pseudo remainder with p_4 over variable x_4:

$$\begin{aligned}
g &= x_3^2x_2^2 - 2u_1x_3^2x_2 + (u_3^2 + u_1^2)x_3^2 \\
&\quad -2u_3x_3x_2x_1 + 2u_3u_1x_3x_1 - 2u_3^2u_2x_3 + 2x_2^4 - 2x_2^3x_1 \\
&\quad -6u_1x_2^3 + 3x_2^2x_1^2 + 2u_1x_2^2x_1 + \\
&\quad (2u_3^2 - 2u_3u_2 + 2u_2^2 - u_1^2)x_2^2 - 6u_1x_2x_1^2 + \\
&\quad (2u_3u_2 + 2u_1^2)x_2x_1 + \\
&\quad (-4u_3^2u_1 + 4u_3u_2u_1 - 4u_2^2u_1 + 12u_1^3)x_2 + 3u_1^2x_1^2 + \\
&\quad (-2u_3u_2u_1 - 2u_1^3)x_1 + \\
&\quad (u_3^2u_2^2 + 2u_3^2u_1^2 - 2u_3u_2u_1^2 + 2u_2^2u_1^2 - 7u_1^4)
\end{aligned}$$

2. Pseudo remainder with p_3 over variable x_3:

   Polynomial too big for output (text size is 3246 *characters, number of terms is* 31*)*

3. Pseudo remainder with p_2 over variable x_2:

$$\begin{aligned}
g &= -8u_1^3x_2x_1^4 + \\
&\quad (2u_3^3u_2 + 16u_3^2u_1^2 - 8u_3u_2u_1^2)x_2x_1^3 + \\
&\quad (-8u_3^3u_2u_1 + 6u_3^2u_2^2u_1 - 16u_3^2u_1^3 + 32u_3u_2u_1^3
\end{aligned}$$

$$
\begin{aligned}
&-24u_2^2u_1^3 + 16u_1^5) \\
&x_2x_1^2 \\
&+ \\
&(2u_3^3u_2^3 - 2u_3^3u_2u_1^2 + 16u_3^2u_2^2u_1^2 \\
&-16u_3^2u_1^4 - 8u_3u_2^3u_1^2 + 8u_3u_2u_1^4) \\
&x_2x_1 \\
&+ \\
&(-8u_3^3u_2^3u_1 + 8u_3^3u_2u_1^3 + 6u_3^2u_2^4u_1 \\
&-22u_3^2u_2^2u_1^3 + 16u_3^2u_1^5 + 32u_3u_2^3u_1^3 \\
&-32u_3u_2u_1^5 - 16u_2^4u_1^3 + 24u_2^2u_1^5 - 8u_1^7) \\
&x_2 \\
&+(-u_3^4 - 4u_3^2u_1^2 + 8u_1^4)x_1^4 + \\
&(8u_3^4u_1 - 6u_3^3u_2u_1 - 16u_3^2u_1^3 + 8u_3u_2u_1^3)x_1^3 + \\
&(-6u_3^4u_1^2 + 24u_3^3u_2u_1^2 - 18u_3^2u_2^2u_1^2 + \\
&24u_3^2u_1^4 - 32u_3u_2u_1^4 + 24u_2^2u_1^4 - 16u_1^6) \\
&x_1^2 \\
&+ \\
&(8u_3^4u_2^2u_1 - 8u_3^4u_1^3 - 6u_3^3u_2^3u_1 + \\
&6u_3^3u_2u_1^3 - 16u_3^2u_2^2u_1^3 + 16u_3^2u_1^5 + \\
&8u_3u_2^3u_1^3 - 8u_3u_2u_1^5) \\
&x_1 \\
&+ \\
&(u_3^4u_2^4 - 8u_3^4u_2^2u_1^2 + 7u_3^4u_1^4 + \\
&24u_3^3u_2^3u_1^2 - 24u_3^3u_2u_1^4 - 14u_3^2u_2^4u_1^2 + \\
&34u_3^2u_2^2u_1^4 - 20u_3^2u_1^6 - 32u_3u_2^3u_1^4 + \\
&32u_3u_2u_1^6 + 16u_2^4u_1^4 - 24u_2^2u_1^6 + 8u_1^8)
\end{aligned}
$$

4. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 32 terms.

Time Complexity: Time spent by the prover is 0.219 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points A, B and C are not collinear
- Line through points C and A is not perpendicular to line through points A and O

Time spent for processing NDG Conditions

- 0.374 seconds