OpenGeoProver Output for conjecture "Chou 346 (Desargues' Theorem)"

Wu's method used

February 20, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point S
- Free point A
- Free point B
- Free point C
- Line SA through two points S and A
- Line SB through two points S and B
- Line SC through two points S and C
- Random point A1 from line SA
- Random point B1 from line SB
- Random point C1 from line SC
- Line BC through two points B and C
- Line B1C1 through two points B1 and C1
- Intersection point P of point sets BC and B1C1
- Line CA through two points C and A
- Line C1A1 through two points C1 and A1
- Intersection point Q of point sets CA and C1A1
- Line AB through two points A and B
- Line A1B1 through two points A1 and B1
- Intersection point R of point sets AB and A1B1

Theorem statement:

- Points P, Q, R are collinear

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point S:

- Point S has been assigned following coordinates: (0, 0)

2.2 Transformation of point A:

- Point A has been assigned following coordinates: (0, u_1)

2.3 Transformation of point B:

- Point B has been assigned following coordinates: (u_2, u_3)

2.4 Transformation of point C:

- Point C has been assigned following coordinates: (u_4, u_5)

2.5 Transformation of point A1:

- Point A1 has been assigned following coordinates: (u_6, x_1)
- Polynomial that point A1 has to satisfy is:

$$p = x_1$$

- Processing of polynomial

$$p = x_1$$

  Info: Will try to rename X coordinate of point A1

  Info: X coordinate of point A1 renamed by zero

- Point A1 has been renamed. Point A1 has been assigned following coordinates: (0, u_6)

2.6 Transformation of point B1:

- Point B1 has been assigned following coordinates: (u_7, x_1)
- Polynomial that point B1 has to satisfy is:

$$p = u_2x_1 - u_7u_3$$

- Processing of polynomial

$$p = u_2x_1 - u_7u_3$$

Info: Polynomial

$$p = u_2x_1 - u_7u_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point C1:

- Point C1 has been assigned following coordinates: (u_8, x_2)
- Polynomial that point C1 has to satisfy is:

$$p = u_4x_2 - u_8u_5$$

- Processing of polynomial

$$p = u_4x_2 - u_8u_5$$

Info: Polynomial

$$p = u_4x_2 - u_8u_5$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.8 Transformation of point P:

- Point P has been assigned following coordinates: (x_3, x_4)
- Polynomial that point P has to satisfy is:

$$p = (u_4 - u_2)x_4 + (-u_5 + u_3)x_3 + (u_5u_2 - u_4u_3)$$

- Processing of polynomial

$$p = (u_4 - u_2)x_4 + (-u_5 + u_3)x_3 + (u_5u_2 - u_4u_3)$$

Info: Polynomial

$$p = (u_4 - u_2)x_4 + (-u_5 + u_3)x_3 + (u_5u_2 - u_4u_3)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point P has to satisfy is:

$$p = (u_8 - u_7)x_4 - x_3x_2 + x_3x_1 + u_7x_2 - u_8x_1$$

- Processing of polynomial

$$p = (u_8 - u_7)x_4 - x_3x_2 + x_3x_1 + u_7x_2 - u_8x_1$$

Info: Polynomial

$$p = (u_8 - u_7)x_4 - x_3x_2 + x_3x_1 + u_7x_2 - u_8x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.9 Transformation of point Q:

- Point Q has been assigned following coordinates: (x_5, x_6)
- Polynomial that point Q has to satisfy is:

$$p = u_4x_6 + (-u_5 + u_1)x_5 - u_4u_1$$

- Processing of polynomial

$$p = u_4x_6 + (-u_5 + u_1)x_5 - u_4u_1$$

Info: Polynomial

$$p = u_4x_6 + (-u_5 + u_1)x_5 - u_4u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point Q has to satisfy is:

$$p = u_8x_6 - x_5x_2 + u_6x_5 - u_8u_6$$

- Processing of polynomial

$$p = u_8x_6 - x_5x_2 + u_6x_5 - u_8u_6$$

Info: Polynomial

$$p = u_8x_6 - x_5x_2 + u_6x_5 - u_8u_6$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.10 Transformation of point R:

- Point R has been assigned following coordinates: (x_7, x_8)
- Polynomial that point R has to satisfy is:

$$p = u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1$$

- Processing of polynomial

$$p = u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1$$

Info: Polynomial

$$p = u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point R has to satisfy is:

$$p = u_7x_8 - x_7x_1 + u_6x_7 - u_7u_6$$

- Processing of polynomial

$$p = u_7x_8 - x_7x_1 + u_6x_7 - u_7u_6$$

Info: Polynomial

$$p = u_7x_8 - x_7x_1 + u_6x_7 - u_7u_6$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_8x_5 - x_8x_3 - x_7x_6 + x_7x_4 + x_6x_3 - x_5x_4$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.088 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$
\begin{aligned}
p_1 &= u_2x_1 - u_7u_3 \\
p_2 &= u_4x_2 - u_8u_5 \\
p_3 &= (u_4 - u_2)x_4 + (-u_5 + u_3)x_3 + (u_5u_2 - u_4u_3) \\
p_4 &= (u_8 - u_7)x_4 - x_3x_2 + x_3x_1 + u_7x_2 - u_8x_1 \\
p_5 &= u_4x_6 + (-u_5 + u_1)x_5 - u_4u_1 \\
p_6 &= u_8x_6 - x_5x_2 + u_6x_5 - u_8u_6 \\
p_7 &= u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1 \\
p_8 &= u_7x_8 - x_7x_1 + u_6x_7 - u_7u_6
\end{aligned}
$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_8 from all other polynomials by reducing them with polynomial p_7 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_2x_1 - u_7u_3 \\
p_2 &= u_4x_2 - u_8u_5 \\
p_3 &= (u_4 - u_2)x_4 + (-u_5 + u_3)x_3 + (u_5u_2 - u_4u_3) \\
p_4 &= (u_8 - u_7)x_4 - x_3x_2 + x_3x_1 + u_7x_2 - u_8x_1 \\
p_5 &= u_4x_6 + (-u_5 + u_1)x_5 - u_4u_1 \\
p_6 &= u_8x_6 - x_5x_2 + u_6x_5 - u_8u_6 \\
p_7 &= -u_2x_7x_1 + (u_7u_3 - u_7u_1 + u_6u_2)x_7 + (-u_7u_6u_2 + u_7u_2u_1) \\
p_8 &= u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1
\end{aligned}
$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_6 from all other polynomials by reducing them with polynomial p_5 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_2x_1 - u_7u_3 \\
p_2 &= u_4x_2 - u_8u_5 \\
p_3 &= (u_4 - u_2)x_4 + (-u_5 + u_3)x_3 + (u_5u_2 - u_4u_3) \\
p_4 &= (u_8 - u_7)x_4 - x_3x_2 + x_3x_1 + u_7x_2 - u_8x_1 \\
p_5 &= -u_4x_5x_2 + (u_8u_5 - u_8u_1 + u_6u_4)x_5 + (-u_8u_6u_4 + u_8u_4u_1) \\
p_6 &= u_4x_6 + (-u_5 + u_1)x_5 - u_4u_1 \\
p_7 &= -u_2x_7x_1 + (u_7u_3 - u_7u_1 + u_6u_2)x_7 + (-u_7u_6u_2 + u_7u_2u_1) \\
p_8 &= u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1
\end{aligned}
$$

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_3 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_2x_1 - u_7u_3 \\
p_2 &= u_4x_2 - u_8u_5 \\
p_3 &= (-u_4 + u_2)x_3x_2 + (u_4 - u_2)x_3x_1 + \\
&\quad (u_8u_5 - u_8u_3 - u_7u_5 + u_7u_3)x_3 + (u_7u_4 - u_7u_2)x_2 + \\
&\quad (-u_8u_4 + u_8u_2)x_1 + \\
&\quad (-u_8u_5u_2 + u_8u_4u_3 + u_7u_5u_2 - u_7u_4u_3) \\
p_4 &= (u_4 - u_2)x_4 + (-u_5 + u_3)x_3 + (u_5u_2 - u_4u_3) \\
p_5 &= -u_4x_5x_2 + (u_8u_5 - u_8u_1 + u_6u_4)x_5 + (-u_8u_6u_4 + u_8u_4u_1) \\
p_6 &= u_4x_6 + (-u_5 + u_1)x_5 - u_4u_1 \\
p_7 &= -u_2x_7x_1 + (u_7u_3 - u_7u_1 + u_6u_2)x_7 + (-u_7u_6u_2 + u_7u_2u_1) \\
p_8 &= u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1
\end{aligned}
$$

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.8 Triangulation, step 8

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= u_2x_1 - u_7u_3 \\
p_2 &= u_4x_2 - u_8u_5 \\
p_3 &= (-u_4 + u_2)x_3x_2 + (u_4 - u_2)x_3x_1 + \\
&\quad (u_8u_5 - u_8u_3 - u_7u_5 + u_7u_3)x_3 + (u_7u_4 - u_7u_2)x_2 + \\
&\quad (-u_8u_4 + u_8u_2)x_1 + \\
&\quad (-u_8u_5u_2 + u_8u_4u_3 + u_7u_5u_2 - u_7u_4u_3) \\
p_4 &= (u_4 - u_2)x_4 + (-u_5 + u_3)x_3 + (u_5u_2 - u_4u_3) \\
p_5 &= -u_4x_5x_2 + (u_8u_5 - u_8u_1 + u_6u_4)x_5 + (-u_8u_6u_4 + u_8u_4u_1) \\
p_6 &= u_4x_6 + (-u_5 + u_1)x_5 - u_4u_1 \\
p_7 &= -u_2x_7x_1 + (u_7u_3 - u_7u_1 + u_6u_2)x_7 + (-u_7u_6u_2 + u_7u_2u_1) \\
p_8 &= u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1
\end{aligned}
$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 346 (Desargues' Theorem)

Calculating final remainder of the conclusion:

$$g = x_8x_5 - x_8x_3 - x_7x_6 + x_7x_4 + x_6x_3 - x_5x_4$$

with respect to the triangular system.

1. Pseudo remainder with p_8 over variable x_8:

$$
\begin{aligned}
g &= -u_2x_7x_6 + (u_3 - u_1)x_7x_5 + u_2x_7x_4 + (-u_3 + u_1)x_7x_3 + \\
&\quad u_2x_6x_3 - u_2x_5x_4 + u_2u_1x_5 - u_2u_1x_3
\end{aligned}
$$

2. Pseudo remainder with p_7 over variable x_7:

$$
\begin{aligned}
g &= -u_2^2x_6x_3x_1 + (u_7u_3u_2 - u_7u_2u_1 + u_6u_2^2)x_6x_3 + \\
&\quad (-u_7u_6u_2^2 + u_7u_2^2u_1)x_6 + u_2^2x_5x_4x_1 + \\
&\quad (-u_7u_3u_2 + u_7u_2u_1 - u_6u_2^2)x_5x_4 - u_2^2u_1x_5x_1 + \\
&\quad (u_7u_6u_3u_2 - u_7u_6u_2u_1 + u_6u_2^2u_1)x_5 + \\
&\quad (u_7u_6u_2^2 - u_7u_2^2u_1)x_4 + u_2^2u_1x_3x_1 + \\
&\quad (-u_7u_6u_3u_2 + u_7u_6u_2u_1 - u_6u_2^2u_1)x_3
\end{aligned}
$$

3. Pseudo remainder with p_6 over variable x_6:

$$
\begin{aligned}
g \;=\;& u_4u_2^2x_5x_4x_1 + \\
& (-u_7u_4u_3u_2 + u_7u_4u_2u_1 - u_6u_4u_2^2)x_5x_4 + \\
& (-u_5u_2^2 + u_2^2u_1)x_5x_3x_1 + \\
& (u_7u_5u_3u_2 - u_7u_5u_2u_1 - u_7u_3u_2u_1 + u_7u_2u_1^2 + \\
& u_6u_5u_2^2 - u_6u_2^2u_1) \\
& x_5x_3 \\
& -u_4u_2^2u_1x_5x_1 + \\
& (-u_7u_6u_5u_2^2 + u_7u_6u_4u_3u_2 - u_7u_6u_4u_2u_1 + \\
& u_7u_6u_2^2u_1 + u_7u_5u_2^2u_1 - u_7u_2^2u_1^2 + \\
& u_6u_4u_2^2u_1) \\
& x_5 \\
& +(u_7u_6u_4u_2^2 - u_7u_4u_2^2u_1)x_4 + \\
& (-u_7u_6u_4u_3u_2 + u_7u_6u_4u_2u_1 + u_7u_4u_3u_2u_1 \\
& -u_7u_4u_2u_1^2) \\
& x_3 \\
& +(-u_7u_6u_4u_2^2u_1 + u_7u_4u_2^2u_1^2)
\end{aligned}
$$

4. Pseudo remainder with p_5 over variable x_5:

$$
\begin{aligned}
g \;=\;& (-u_7u_6u_4^2u_2^2 + u_7u_4^2u_2^2u_1)x_4x_2 + \\
& (u_8u_6u_4^2u_2^2 - u_8u_4^2u_2^2u_1)x_4x_1 + \\
& (u_8u_7u_6u_5u_4u_2^2 - u_8u_7u_6u_4^2u_3u_2 + \\
& u_8u_7u_6u_4^2u_2u_1 - u_8u_7u_6u_4u_2^2u_1 \\
& -u_8u_7u_5u_4u_2^2u_1 + u_8u_7u_4^2u_3u_2u_1 \\
& -u_8u_7u_4^2u_2u_1^2 + u_8u_7u_4u_2^2u_1^2 \\
& -u_8u_6^2u_4^2u_2^2 + u_8u_6u_4^2u_2^2u_1 + \\
& u_7u_6^2u_4^2u_2^2 - u_7u_6u_4^2u_2^2u_1) \\
& x_4 \\
& + \\
& (u_7u_6u_4^2u_3u_2 - u_7u_6u_4^2u_2u_1 - u_7u_4^2u_3u_2u_1 + \\
& u_7u_4^2u_2u_1^2) \\
& x_3x_2 \\
& + \\
& (-u_8u_6u_5u_4u_2^2 + u_8u_6u_4u_2^2u_1 + u_8u_5u_4u_2^2u_1 \\
& -u_8u_4u_2^2u_1^2) \\
& x_3x_1 \\
& + \\
& (u_8u_6^2u_5u_4u_2^2 - u_8u_6^2u_4u_2^2u_1 \\
& -u_8u_6u_5u_4u_2^2u_1 + u_8u_6u_4u_2^2u_1^2
\end{aligned}
$$

$$
\begin{aligned}
&-u_7u_6^2u_4^2u_3u_2 + u_7u_6^2u_4^2u_2u_1 + \\
&u_7u_6u_4^2u_3u_2u_1 - u_7u_6u_4^2u_2u_1^2) \\
&x_3 \\
&+(u_7u_6u_4^2u_2^2u_1 - u_7u_4^2u_2^2u_1^2)x_2 + \\
&(-u_8u_6u_4^2u_2^2u_1 + u_8u_4^2u_2^2u_1^2)x_1 + \\
&(-u_8u_7u_6^2u_5u_4u_2^2 + u_8u_7u_6^2u_4^2u_3u_2 \\
&-u_8u_7u_6^2u_4^2u_2u_1 + u_8u_7u_6^2u_4u_2^2u_1 + \\
&u_8u_7u_6u_5u_4u_2^2u_1 - u_8u_7u_6u_4^2u_3u_2u_1 + \\
&u_8u_7u_6u_4^2u_2u_1^2 - u_8u_7u_6u_4u_2^2u_1^2 + \\
&u_8u_6^2u_4^2u_2^2u_1 - u_8u_6u_4^2u_2^2u_1^2 \\
&-u_7u_6^2u_4^2u_2^2u_1 + u_7u_6u_4^2u_2^2u_1^2)
\end{aligned}
$$

5. Pseudo remainder with p_4 over variable x_4:

   Polynomial too big for output (text size is 3871 *characters, number of terms is* 6*)*

6. Pseudo remainder with p_3 over variable x_3:

   Polynomial too big for output (text size is 17248 *characters, number of terms is* 6*)*

7. Pseudo remainder with p_2 over variable x_2:

   Polynomial too big for output (text size is 9285 *characters, number of terms is* 3*)*

8. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 10 terms.

Time Complexity: Time spent by the prover is 0.284 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points A and B are not identical
- Points S and C are not identical
- Line through points B and C is not parallel with line through points C1 and B1

- Points A, S, B and C are not collinear
- Line through points A1 and C1 is not parallel with line through points A and C
- Line through points A1 and B1 is not parallel with line through points A and B

Time spent for processing NDG Conditions

- 1.099 seconds