OpenGeoProver Output for conjecture "Chou 079 (Ptolemy's Theorem)"

Wu's method used

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1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Free point C
- Circumscribed circle k around triangle ABC
- Random point D from circle k

Theorem statement:

- Algebraic sum of segments AC*BD, AB*CD and BC*DA is zero

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

2.2 Transformation of point B:

- Point B has been assigned following coordinates: (0, u_1)

2.3 Transformation of point C:

- Point C has been assigned following coordinates: (u_2, u_3)

2.4 Transformation of point D:

- Point D has been assigned following coordinates: (u_4, x_1)
- Polynomial that point D has to satisfy is:

$$\begin{aligned} p &= u_2x_1^2 - u_2u_1x_1 + \\ &\quad (u_4^2u_2 - u_4u_3^2 + u_4u_3u_1 - u_4u_2^2) \end{aligned}$$

- Processing of polynomial

$$\begin{aligned} p &= u_2x_1^2 - u_2u_1x_1 + \\ &\quad (u_4^2u_2 - u_4u_3^2 + u_4u_3u_1 - u_4u_2^2) \end{aligned}$$

  Info: Polynomial

$$\begin{aligned} p &= u_2x_1^2 - u_2u_1x_1 + \\ &\quad (u_4^2u_2 - u_4u_3^2 + u_4u_3u_1 - u_4u_2^2) \end{aligned}$$

  added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

- Polynomial for theorem statement:

$$\begin{aligned} p &= u_2^2x_1^4 - 2u_2^2u_1x_1^3 + \\ &\quad (2u_4^2u_2^2 - 2u_4u_3^2u_2 + 2u_4u_3u_2u_1 - 2u_4u_2^3 + \\ &\quad u_2^2u_1^2) \\ &\quad x_1^2 \\ &\quad + \\ &\quad (-2u_4^2u_2^2u_1 + 2u_4u_3^2u_2u_1 - 2u_4u_3u_2u_1^2 + \\ &\quad 2u_4u_2^3u_1) \\ &\quad x_1 \\ &\quad + \\ &\quad (u_4^4u_2^2 - 2u_4^3u_3^2u_2 + 2u_4^3u_3u_2u_1 - 2u_4^3u_2^3 + \\ &\quad u_4^2u_3^4 - 2u_4^2u_3^3u_1 + 2u_4^2u_3^2u_2^2 + \\ &\quad u_4^2u_3^2u_1^2 - 2u_4^2u_3u_2^2u_1 + u_4^2u_2^4) \end{aligned}$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.097 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The system is already triangular.

$$\begin{aligned} p_1 &= u_2 x_1^2 - u_2 u_1 x_1 + \\ &\quad (u_4^2 u_2 - u_4 u_3^2 + u_4 u_3 u_1 - u_4 u_2^2) \end{aligned}$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 079 (Ptolemy's Theorem)

Calculating final remainder of the conclusion:

$$\begin{aligned} g &= u_2^2 x_1^4 - 2u_2^2 u_1 x_1^3 + \\ &\quad (2u_4^2 u_2^2 - 2u_4 u_3^2 u_2 + 2u_4 u_3 u_2 u_1 - 2u_4 u_2^3 + \\ &\quad u_2^2 u_1^2) \\ &\quad x_1^2 \\ &\quad + \\ &\quad (-2u_4^2 u_2^2 u_1 + 2u_4 u_3^2 u_2 u_1 - 2u_4 u_3 u_2 u_1^2 + \\ &\quad 2u_4 u_2^3 u_1) \\ &\quad x_1 \\ &\quad + \\ &\quad (u_4^4 u_2^2 - 2u_4^3 u_3^2 u_2 + 2u_4^3 u_3 u_2 u_1 - 2u_4^3 u_2^3 + \\ &\quad u_4^2 u_3^4 - 2u_4^2 u_3^3 u_1 + 2u_4^2 u_3^2 u_2^2 + \\ &\quad u_4^2 u_3^2 u_1^2 - 2u_4^2 u_3 u_2^2 u_1 + u_4^2 u_2^4) \end{aligned}$$

with respect to the triangular system.

1. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 5 terms.

Time Complexity: Time spent by the prover is 0.019 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points B and C are not identical

Time spent for processing NDG Conditions

- 0.026 seconds