

# OpenGeoProver Output for conjecture “Chou 012”

Wu’s method used

February 12, 2012

## 1 Validation of Construction Protocol

### Construction steps:

- Free point A0
- Free point A1
- Perpendicular bisector m of segment A0A1
- Random point O from line m
- Circle c with center O and one point A0
- Random point A2 from circle c
- Random point A3 from circle c
- Random point A4 from circle c
- Random point A5 from circle c
- Line A0A1 through two points A0 and A1
- Line A2A3 through two points A2 and A3
- Intersection point A of point sets A0A1 and A2A3
- Line A4A5 through two points A4 and A5
- Intersection point B of point sets A0A1 and A4A5
- Line A0A2 through two points A0 and A2
- Line A3A4 through two points A3 and A4
- Intersection point C of point sets A0A2 and A3A4
- Line A0A3 through two points A0 and A3
- Line A2A5 through two points A2 and A5
- Intersection point D of point sets A0A3 and A2A5

- Line A1A4 through two points A1 and A4
- Intersection point E of point sets A1A4 and A2A5
- Line A1A5 through two points A1 and A5
- Intersection point F of point sets A1A5 and A3A4
- Line AB through two points A and B
- Line DE through two points D and E
- Line BC through two points B and C
- Line EF through two points E and F
- Line CD through two points C and D
- Line FA through two points F and A
- Intersection point P of point sets AB and DE
- Intersection point Q of point sets BC and EF
- Intersection point R of point sets CD and FA

**Theorem statement:**

- Points P, Q, R are collinear

**Validation result:** Construction protocol is valid.

## 2 Transformation of Construction Protocol to algebraic form

### Transformation of Construction steps

#### 2.1 Transformation of point A0:

- Point A0 has been assigned following coordinates:  $(0, 0)$

#### 2.2 Transformation of point A1:

- Point A1 has been assigned following coordinates:  $(0, u_1)$

#### 2.3 Transformation of point O:

- Point O has been assigned following coordinates:  $(u_2, x_1)$
- Polynomial that point O has to satisfy is:

$$p = x_1 - 0.5u_1$$

- Processing of polynomial

$$p = x_1 - 0.5u_1$$

**Info:** Polynomial

$$p = x_1 - 0.5u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.4 Transformation of point A2:

- Point A2 has been assigned following coordinates:  $(u_3, x_2)$
- Polynomial that point A2 has to satisfy is:

$$p = x_2^2 - 2x_2x_1 + (u_3^2 - 2u_3u_2)$$

- Processing of polynomial

$$p = x_2^2 - 2x_2x_1 + (u_3^2 - 2u_3u_2)$$

**Info:** Polynomial

$$p = x_2^2 - 2x_2x_1 + (u_3^2 - 2u_3u_2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.5 Transformation of point A3:

- Point A3 has been assigned following coordinates:  $(u_4, x_3)$
- Polynomial that point A3 has to satisfy is:

$$p = x_3^2 - 2x_3x_1 + (u_4^2 - 2u_4u_2)$$

- Processing of polynomial

$$p = x_3^2 - 2x_3x_1 + (u_4^2 - 2u_4u_2)$$

**Info:** Polynomial

$$p = x_3^2 - 2x_3x_1 + (u_4^2 - 2u_4u_2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.6 Transformation of point A4:

- Point A4 has been assigned following coordinates:  $(u_5, x_4)$
- Polynomial that point A4 has to satisfy is:

$$p = x_4^2 - 2x_4x_1 + (u_5^2 - 2u_5u_2)$$

- Processing of polynomial

$$p = x_4^2 - 2x_4x_1 + (u_5^2 - 2u_5u_2)$$

**Info:** Polynomial

$$p = x_4^2 - 2x_4x_1 + (u_5^2 - 2u_5u_2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.7 Transformation of point A5:

- Point A5 has been assigned following coordinates:  $(u_6, x_5)$
- Polynomial that point A5 has to satisfy is:

$$p = x_5^2 - 2x_5x_1 + (u_6^2 - 2u_6u_2)$$

- Processing of polynomial

$$p = x_5^2 - 2x_5x_1 + (u_6^2 - 2u_6u_2)$$

**Info:** Polynomial

$$p = x_5^2 - 2x_5x_1 + (u_6^2 - 2u_6u_2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.8 Transformation of point A:

- Point A has been assigned following coordinates:  $(x_6, x_7)$
- Polynomial that point A has to satisfy is:

$$p = x_6$$

- Processing of polynomial

$$p = x_6$$

**Info:** Will try to rename X coordinate of point A

**Info:** Y coordinate of point A will be replaced by X coordinate

**Info:** X coordinate of point A renamed by zero

- Point A has been renamed. Point A has been assigned following coordinates:  $(0, x_6)$
- Polynomial that point A has to satisfy is:

$$p = (u_4 - u_3)x_6 + u_3x_3 - u_4x_2$$

- Processing of polynomial

$$p = (u_4 - u_3)x_6 + u_3x_3 - u_4x_2$$

**Info:** Polynomial

$$p = (u_4 - u_3)x_6 + u_3x_3 - u_4x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.9 Transformation of point B:

- Point B has been assigned following coordinates:  $(x_7, x_8)$
- Polynomial that point B has to satisfy is:

$$p = x_7$$

- Processing of polynomial

$$p = x_7$$

**Info:** Will try to rename X coordinate of point B

**Info:** Y coordinate of point B will be replaced by X coordinate

**Info:** X coordinate of point B renamed by zero

- Point B has been renamed. Point B has been assigned following coordinates:  $(0, x_7)$
- Polynomial that point B has to satisfy is:

$$p = (u_6 - u_5)x_7 + u_5x_5 - u_6x_4$$

- Processing of polynomial

$$p = (u_6 - u_5)x_7 + u_5x_5 - u_6x_4$$

**Info:** Polynomial

$$p = (u_6 - u_5)x_7 + u_5x_5 - u_6x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.10 Transformation of point C:

- Point C has been assigned following coordinates:  $(x_8, x_9)$
- Polynomial that point C has to satisfy is:

$$p = u_3x_9 - x_8x_2$$

- Processing of polynomial

$$p = u_3x_9 - x_8x_2$$

**Info:** Polynomial

$$p = u_3x_9 - x_8x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point C has to satisfy is:

$$p = (u_5 - u_4)x_9 - x_8x_4 + x_8x_3 + u_4x_4 - u_5x_3$$

- Processing of polynomial

$$p = (u_5 - u_4)x_9 - x_8x_4 + x_8x_3 + u_4x_4 - u_5x_3$$

**Info:** Polynomial

$$p = (u_5 - u_4)x_9 - x_8x_4 + x_8x_3 + u_4x_4 - u_5x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.11 Transformation of point D:

- Point D has been assigned following coordinates:  $(x_{10}, x_{11})$
- Polynomial that point D has to satisfy is:

$$p = u_4x_{11} - x_{10}x_3$$

- Processing of polynomial

$$p = u_4x_{11} - x_{10}x_3$$

**Info:** Polynomial

$$p = u_4x_{11} - x_{10}x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point D has to satisfy is:

$$p = (u_6 - u_3)x_{11} - x_{10}x_5 + x_{10}x_2 + u_3x_5 - u_6x_2$$

- Processing of polynomial

$$p = (u_6 - u_3)x_{11} - x_{10}x_5 + x_{10}x_2 + u_3x_5 - u_6x_2$$

**Info:** Polynomial

$$p = (u_6 - u_3)x_{11} - x_{10}x_5 + x_{10}x_2 + u_3x_5 - u_6x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.12 Transformation of point E:

- Point E has been assigned following coordinates:  $(x_{12}, x_{13})$
- Polynomial that point E has to satisfy is:

$$p = u_5x_{13} - x_{12}x_4 + u_1x_{12} - u_5u_1$$

- Processing of polynomial

$$p = u_5x_{13} - x_{12}x_4 + u_1x_{12} - u_5u_1$$

**Info:** Polynomial

$$p = u_5x_{13} - x_{12}x_4 + u_1x_{12} - u_5u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point E has to satisfy is:

$$p = (u_6 - u_3)x_{13} - x_{12}x_5 + x_{12}x_2 + u_3x_5 - u_6x_2$$

- Processing of polynomial

$$p = (u_6 - u_3)x_{13} - x_{12}x_5 + x_{12}x_2 + u_3x_5 - u_6x_2$$

**Info:** Polynomial

$$p = (u_6 - u_3)x_{13} - x_{12}x_5 + x_{12}x_2 + u_3x_5 - u_6x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

### 2.13 Transformation of point F:

- Point F has been assigned following coordinates:  $(x_{14}, x_{15})$
- Polynomial that point F has to satisfy is:

$$p = u_6x_{15} - x_{14}x_5 + u_1x_{14} - u_6u_1$$

- Processing of polynomial

$$p = u_6x_{15} - x_{14}x_5 + u_1x_{14} - u_6u_1$$

**Info:** Polynomial

$$p = u_6x_{15} - x_{14}x_5 + u_1x_{14} - u_6u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point F has to satisfy is:

$$p = (u_5 - u_4)x_{15} - x_{14}x_4 + x_{14}x_3 + u_4x_4 - u_5x_3$$

- Processing of polynomial

$$p = (u_5 - u_4)x_{15} - x_{14}x_4 + x_{14}x_3 + u_4x_4 - u_5x_3$$

**Info:** Polynomial

$$p = (u_5 - u_4)x_{15} - x_{14}x_4 + x_{14}x_3 + u_4x_4 - u_5x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

### 2.14 Transformation of point P:

- Point P has been assigned following coordinates:  $(x_{16}, x_{17})$
- Polynomial that point P has to satisfy is:

$$p = x_{16}x_7 - x_{16}x_6$$

- Processing of polynomial

$$p = x_{16}x_7 - x_{16}x_6$$

**Info:** Polynomial

$$p = x_{16}x_7 - x_{16}x_6$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses



- Polynomial that point P has to satisfy is:

$$p = x_{17}x_{12} - x_{17}x_{10} - x_{16}x_{13} + x_{16}x_{11} + x_{13}x_{10} - x_{12}x_{11}$$

- Processing of polynomial

$$p = x_{17}x_{12} - x_{17}x_{10} - x_{16}x_{13} + x_{16}x_{11} + x_{13}x_{10} - x_{12}x_{11}$$

**Info:** Polynomial

$$p = x_{17}x_{12} - x_{17}x_{10} - x_{16}x_{13} + x_{16}x_{11} + x_{13}x_{10} - x_{12}x_{11}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.15 Transformation of point Q:

- Point Q has been assigned following coordinates:  $(x_{18}, x_{19})$
- Polynomial that point Q has to satisfy is:

$$p = x_{19}x_8 - x_{18}x_9 + x_{18}x_7 - x_8x_7$$

- Processing of polynomial

$$p = x_{19}x_8 - x_{18}x_9 + x_{18}x_7 - x_8x_7$$

**Info:** Polynomial

$$p = x_{19}x_8 - x_{18}x_9 + x_{18}x_7 - x_8x_7$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point Q has to satisfy is:

$$p = x_{19}x_{14} - x_{19}x_{12} - x_{18}x_{15} + x_{18}x_{13} + x_{15}x_{12} - x_{14}x_{13}$$

- Processing of polynomial

$$p = x_{19}x_{14} - x_{19}x_{12} - x_{18}x_{15} + x_{18}x_{13} + x_{15}x_{12} - x_{14}x_{13}$$

**Info:** Polynomial

$$p = x_{19}x_{14} - x_{19}x_{12} - x_{18}x_{15} + x_{18}x_{13} + x_{15}x_{12} - x_{14}x_{13}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

### 2.16 Transformation of point R:

- Point R has been assigned following coordinates:  $(x_{20}, x_{21})$
- Polynomial that point R has to satisfy is:

$$p = x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9$$

- Processing of polynomial

$$p = x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9$$

**Info:** Polynomial

$$p = x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point R has to satisfy is:

$$p = x_{21}x_{14} - x_{20}x_{15} + x_{20}x_6 - x_{14}x_6$$

- Processing of polynomial

$$p = x_{21}x_{14} - x_{20}x_{15} + x_{20}x_6 - x_{14}x_6$$

**Info:** Polynomial

$$p = x_{21}x_{14} - x_{20}x_{15} + x_{20}x_6 - x_{14}x_6$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

### Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_{21}x_{18} - x_{21}x_{16} - x_{20}x_{19} + x_{20}x_{17} + x_{19}x_{16} - x_{18}x_{17}$$

### Time spent for transformation of Construction Protocol to algebraic form

- 0.442 seconds

## 3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= x_1 - 0.5u_1 \\
p_2 &= x_2^2 - 2x_2x_1 + (u_3^2 - 2u_3u_2) \\
p_3 &= x_3^2 - 2x_3x_1 + (u_4^2 - 2u_4u_2) \\
p_4 &= x_4^2 - 2x_4x_1 + (u_5^2 - 2u_5u_2) \\
p_5 &= x_5^2 - 2x_5x_1 + (u_6^2 - 2u_6u_2) \\
p_6 &= (u_4 - u_3)x_6 + u_3x_3 - u_4x_2 \\
p_7 &= (u_6 - u_5)x_7 + u_5x_5 - u_6x_4 \\
p_8 &= u_3x_9 - x_8x_2 \\
p_9 &= (u_5 - u_4)x_9 - x_8x_4 + x_8x_3 + u_4x_4 - u_5x_3 \\
p_{10} &= u_4x_{11} - x_{10}x_3 \\
p_{11} &= (u_6 - u_3)x_{11} - x_{10}x_5 + x_{10}x_2 + u_3x_5 - u_6x_2 \\
p_{12} &= u_5x_{13} - x_{12}x_4 + u_1x_{12} - u_5u_1 \\
p_{13} &= (u_6 - u_3)x_{13} - x_{12}x_5 + x_{12}x_2 + u_3x_5 - u_6x_2 \\
p_{14} &= u_6x_{15} - x_{14}x_5 + u_1x_{14} - u_6u_1 \\
p_{15} &= (u_5 - u_4)x_{15} - x_{14}x_4 + x_{14}x_3 + u_4x_4 - u_5x_3 \\
p_{16} &= x_{16}x_7 - x_{16}x_6 \\
p_{17} &= x_{17}x_{12} - x_{17}x_{10} - x_{16}x_{13} + x_{16}x_{11} + x_{13}x_{10} - x_{12}x_{11} \\
p_{18} &= x_{19}x_8 - x_{18}x_9 + x_{18}x_7 - x_8x_7 \\
p_{19} &= x_{19}x_{14} - x_{19}x_{12} - x_{18}x_{15} + x_{18}x_{13} + x_{15}x_{12} - x_{14}x_{13} \\
p_{20} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9 \\
p_{21} &= x_{21}x_{14} - x_{20}x_{15} + x_{20}x_6 - x_{14}x_6
\end{aligned}$$

### 3.1 Triangulation, step 1

**Choosing variable:** Trying the variable with index 21.

**Variable  $x_{21}$  selected:** The number of polynomials with this variable, with indexes from 1 to 21, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{21}$  from all other polynomials by reducing them with polynomial  $p_{20}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= x_1 - 0.5u_1 \\
p_2 &= x_2^2 - 2x_2x_1 + (u_3^2 - 2u_3u_2) \\
p_3 &= x_3^2 - 2x_3x_1 + (u_4^2 - 2u_4u_2) \\
p_4 &= x_4^2 - 2x_4x_1 + (u_5^2 - 2u_5u_2) \\
p_5 &= x_5^2 - 2x_5x_1 + (u_6^2 - 2u_6u_2) \\
p_6 &= (u_4 - u_3)x_6 + u_3x_3 - u_4x_2
\end{aligned}$$

$$\begin{aligned}
p_7 &= (u_6 - u_5)x_7 + u_5x_5 - u_6x_4 \\
p_8 &= u_3x_9 - x_8x_2 \\
p_9 &= (u_5 - u_4)x_9 - x_8x_4 + x_8x_3 + u_4x_4 - u_5x_3 \\
p_{10} &= u_4x_{11} - x_{10}x_3 \\
p_{11} &= (u_6 - u_3)x_{11} - x_{10}x_5 + x_{10}x_2 + u_3x_5 - u_6x_2 \\
p_{12} &= u_5x_{13} - x_{12}x_4 + u_1x_{12} - u_5u_1 \\
p_{13} &= (u_6 - u_3)x_{13} - x_{12}x_5 + x_{12}x_2 + u_3x_5 - u_6x_2 \\
p_{14} &= u_6x_{15} - x_{14}x_5 + u_1x_{14} - u_6u_1 \\
p_{15} &= (u_5 - u_4)x_{15} - x_{14}x_4 + x_{14}x_3 + u_4x_4 - u_5x_3 \\
p_{16} &= x_{16}x_7 - x_{16}x_6 \\
p_{17} &= x_{17}x_{12} - x_{17}x_{10} - x_{16}x_{13} + x_{16}x_{11} + x_{13}x_{10} - x_{12}x_{11} \\
p_{18} &= x_{19}x_8 - x_{18}x_9 + x_{18}x_7 - x_8x_7 \\
p_{19} &= x_{19}x_{14} - x_{19}x_{12} - x_{18}x_{15} + x_{18}x_{13} + x_{15}x_{12} - x_{14}x_{13} \\
p_{20} &= -x_{20}x_{15}x_{10} + x_{20}x_{15}x_8 + x_{20}x_{14}x_{11} - x_{20}x_{14}x_9 + x_{20}x_{10}x_6 \\
&\quad - x_{20}x_8x_6 - x_{14}x_{11}x_8 + x_{14}x_{10}x_9 - x_{14}x_{10}x_6 + x_{14}x_8x_6 \\
p_{21} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9
\end{aligned}$$

### 3.2 Triangulation, step 2

**Choosing variable:** Trying the variable with index 20.

**Variable  $x_{20}$  selected:** The number of polynomials with this variable, with indexes from 1 to 20, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{20}$ . No reduction needed.

The triangular system has not been changed.

### 3.3 Triangulation, step 3

**Choosing variable:** Trying the variable with index 19.

**Variable  $x_{19}$  selected:** The number of polynomials with this variable, with indexes from 1 to 19, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{19}$  from all other polynomials by reducing them with polynomial  $p_{18}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= x_1 - 0.5u_1 \\
p_2 &= x_2^2 - 2x_2x_1 + (u_3^2 - 2u_3u_2) \\
p_3 &= x_3^2 - 2x_3x_1 + (u_4^2 - 2u_4u_2) \\
p_4 &= x_4^2 - 2x_4x_1 + (u_5^2 - 2u_5u_2)
\end{aligned}$$

$$\begin{aligned}
p_5 &= x_5^2 - 2x_5x_1 + (u_6^2 - 2u_6u_2) \\
p_6 &= (u_4 - u_3)x_6 + u_3x_3 - u_4x_2 \\
p_7 &= (u_6 - u_5)x_7 + u_5x_5 - u_6x_4 \\
p_8 &= u_3x_9 - x_8x_2 \\
p_9 &= (u_5 - u_4)x_9 - x_8x_4 + x_8x_3 + u_4x_4 - u_5x_3 \\
p_{10} &= u_4x_{11} - x_{10}x_3 \\
p_{11} &= (u_6 - u_3)x_{11} - x_{10}x_5 + x_{10}x_2 + u_3x_5 - u_6x_2 \\
p_{12} &= u_5x_{13} - x_{12}x_4 + u_1x_{12} - u_5u_1 \\
p_{13} &= (u_6 - u_3)x_{13} - x_{12}x_5 + x_{12}x_2 + u_3x_5 - u_6x_2 \\
p_{14} &= u_6x_{15} - x_{14}x_5 + u_1x_{14} - u_6u_1 \\
p_{15} &= (u_5 - u_4)x_{15} - x_{14}x_4 + x_{14}x_3 + u_4x_4 - u_5x_3 \\
p_{16} &= x_{16}x_7 - x_{16}x_6 \\
p_{17} &= x_{17}x_{12} - x_{17}x_{10} - x_{16}x_{13} + x_{16}x_{11} + x_{13}x_{10} - x_{12}x_{11} \\
p_{18} &= -x_{18}x_{15}x_8 + x_{18}x_{14}x_9 - x_{18}x_{14}x_7 + x_{18}x_{13}x_8 - x_{18}x_{12}x_9 + \\
&\quad x_{18}x_{12}x_7 + x_{15}x_{12}x_8 - x_{14}x_{13}x_8 + x_{14}x_8x_7 - x_{12}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{18}x_9 + x_{18}x_7 - x_8x_7 \\
p_{20} &= -x_{20}x_{15}x_{10} + x_{20}x_{15}x_8 + x_{20}x_{14}x_{11} - x_{20}x_{14}x_9 + x_{20}x_{10}x_6 \\
&\quad - x_{20}x_8x_6 - x_{14}x_{11}x_8 + x_{14}x_{10}x_9 - x_{14}x_{10}x_6 + x_{14}x_8x_6 \\
p_{21} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9
\end{aligned}$$

### 3.4 Triangulation, step 4

**Choosing variable:** Trying the variable with index 18.

**Variable  $x_{18}$  selected:** The number of polynomials with this variable, with indexes from 1 to 18, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{18}$ . No reduction needed.

The triangular system has not been changed.

### 3.5 Triangulation, step 5

**Choosing variable:** Trying the variable with index 17.

**Variable  $x_{17}$  selected:** The number of polynomials with this variable, with indexes from 1 to 17, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{17}$ . No reduction needed.

The triangular system has not been changed.

### 3.6 Triangulation, step 6

**Choosing variable:** Trying the variable with index 16.

**Variable  $x_{16}$  selected:** The number of polynomials with this variable, with indexes from 1 to 16, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{16}$ . No reduction needed.

The triangular system has not been changed.

### 3.7 Triangulation, step 7

**Choosing variable:** Trying the variable with index 15.

**Variable  $x_{15}$  selected:** The number of polynomials with this variable, with indexes from 1 to 15, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{15}$  from all other polynomials by reducing them with polynomial  $p_{14}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= x_1 - 0.5u_1 \\
p_2 &= x_2^2 - 2x_2x_1 + (u_3^2 - 2u_3u_2) \\
p_3 &= x_3^2 - 2x_3x_1 + (u_4^2 - 2u_4u_2) \\
p_4 &= x_4^2 - 2x_4x_1 + (u_5^2 - 2u_5u_2) \\
p_5 &= x_5^2 - 2x_5x_1 + (u_6^2 - 2u_6u_2) \\
p_6 &= (u_4 - u_3)x_6 + u_3x_3 - u_4x_2 \\
p_7 &= (u_6 - u_5)x_7 + u_5x_5 - u_6x_4 \\
p_8 &= u_3x_9 - x_8x_2 \\
p_9 &= (u_5 - u_4)x_9 - x_8x_4 + x_8x_3 + u_4x_4 - u_5x_3 \\
p_{10} &= u_4x_{11} - x_{10}x_3 \\
p_{11} &= (u_6 - u_3)x_{11} - x_{10}x_5 + x_{10}x_2 + u_3x_5 - u_6x_2 \\
p_{12} &= u_5x_{13} - x_{12}x_4 + u_1x_{12} - u_5u_1 \\
p_{13} &= (u_6 - u_3)x_{13} - x_{12}x_5 + x_{12}x_2 + u_3x_5 - u_6x_2 \\
p_{14} &= (u_5 - u_4)x_{14}x_5 - u_6x_{14}x_4 + u_6x_{14}x_3 + (-u_5u_1 + u_4u_1)x_{14} + \\
&\quad u_6u_4x_4 - u_6u_5x_3 + (u_6u_5u_1 - u_6u_4u_1) \\
p_{15} &= u_6x_{15} - x_{14}x_5 + u_1x_{14} - u_6u_1 \\
p_{16} &= x_{16}x_7 - x_{16}x_6 \\
p_{17} &= x_{17}x_{12} - x_{17}x_{10} - x_{16}x_{13} + x_{16}x_{11} + x_{13}x_{10} - x_{12}x_{11} \\
p_{18} &= -x_{18}x_{15}x_8 + x_{18}x_{14}x_9 - x_{18}x_{14}x_7 + x_{18}x_{13}x_8 - x_{18}x_{12}x_9 + \\
&\quad x_{18}x_{12}x_7 + x_{15}x_{12}x_8 - x_{14}x_{13}x_8 + x_{14}x_8x_7 - x_{12}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{18}x_9 + x_{18}x_7 - x_8x_7 \\
p_{20} &= -x_{20}x_{15}x_{10} + x_{20}x_{15}x_8 + x_{20}x_{14}x_{11} - x_{20}x_{14}x_9 + x_{20}x_{10}x_6 \\
&\quad - x_{20}x_8x_6 - x_{14}x_{11}x_8 + x_{14}x_{10}x_9 - x_{14}x_{10}x_6 + x_{14}x_8x_6 \\
p_{21} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9
\end{aligned}$$

### 3.8 Triangulation, step 8

**Choosing variable:** Trying the variable with index 14.

**Variable  $x_{14}$  selected:** The number of polynomials with this variable, with indexes from 1 to 14, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{14}$ . No reduction needed.

The triangular system has not been changed.

### 3.9 Triangulation, step 9

**Choosing variable:** Trying the variable with index 13.

**Variable  $x_{13}$  selected:** The number of polynomials with this variable, with indexes from 1 to 13, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{13}$  from all other polynomials by reducing them with polynomial  $p_{12}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_1 &= x_1 - 0.5u_1 \\ p_2 &= x_2^2 - 2x_2x_1 + (u_3^2 - 2u_3u_2) \\ p_3 &= x_3^2 - 2x_3x_1 + (u_4^2 - 2u_4u_2) \\ p_4 &= x_4^2 - 2x_4x_1 + (u_5^2 - 2u_5u_2) \\ p_5 &= x_5^2 - 2x_5x_1 + (u_6^2 - 2u_6u_2) \\ p_6 &= (u_4 - u_3)x_6 + u_3x_3 - u_4x_2 \\ p_7 &= (u_6 - u_5)x_7 + u_5x_5 - u_6x_4 \\ p_8 &= u_3x_9 - x_8x_2 \\ p_9 &= (u_5 - u_4)x_9 - x_8x_4 + x_8x_3 + u_4x_4 - u_5x_3 \\ p_{10} &= u_4x_{11} - x_{10}x_3 \\ p_{11} &= (u_6 - u_3)x_{11} - x_{10}x_5 + x_{10}x_2 + u_3x_5 - u_6x_2 \\ p_{12} &= -u_5x_{12}x_5 + (u_6 - u_3)x_{12}x_4 + u_5x_{12}x_2 + (-u_6u_1 + u_3u_1)x_{12} + \\ &\quad u_5u_3x_5 - u_6u_5x_2 + (u_6u_5u_1 - u_5u_3u_1) \\ p_{13} &= u_5x_{13} - x_{12}x_4 + u_1x_{12} - u_5u_1 \\ p_{14} &= (u_5 - u_4)x_{14}x_5 - u_6x_{14}x_4 + u_6x_{14}x_3 + (-u_5u_1 + u_4u_1)x_{14} + \\ &\quad u_6u_4x_4 - u_6u_5x_3 + (u_6u_5u_1 - u_6u_4u_1) \\ p_{15} &= u_6x_{15} - x_{14}x_5 + u_1x_{14} - u_6u_1 \\ p_{16} &= x_{16}x_7 - x_{16}x_6 \\ p_{17} &= x_{17}x_{12} - x_{17}x_{10} - x_{16}x_{13} + x_{16}x_{11} + x_{13}x_{10} - x_{12}x_{11} \\ p_{18} &= -x_{18}x_{15}x_8 + x_{18}x_{14}x_9 - x_{18}x_{14}x_7 + x_{18}x_{13}x_8 - x_{18}x_{12}x_9 + \\ &\quad x_{18}x_{12}x_7 + x_{15}x_{12}x_8 - x_{14}x_{13}x_8 + x_{14}x_8x_7 - x_{12}x_8x_7 \end{aligned}$$

$$\begin{aligned}
p_{19} &= x_{19}x_8 - x_{18}x_9 + x_{18}x_7 - x_8x_7 \\
p_{20} &= -x_{20}x_{15}x_{10} + x_{20}x_{15}x_8 + x_{20}x_{14}x_{11} - x_{20}x_{14}x_9 + x_{20}x_{10}x_6 \\
&\quad - x_{20}x_8x_6 - x_{14}x_{11}x_8 + x_{14}x_{10}x_9 - x_{14}x_{10}x_6 + x_{14}x_8x_6 \\
p_{21} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9
\end{aligned}$$

### 3.10 Triangulation, step 10

**Choosing variable:** Trying the variable with index 12.

**Variable  $x_{12}$  selected:** The number of polynomials with this variable, with indexes from 1 to 12, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{12}$ . No reduction needed.

The triangular system has not been changed.

### 3.11 Triangulation, step 11

**Choosing variable:** Trying the variable with index 11.

**Variable  $x_{11}$  selected:** The number of polynomials with this variable, with indexes from 1 to 11, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{11}$  from all other polynomials by reducing them with polynomial  $p_{10}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= x_1 - 0.5u_1 \\
p_2 &= x_2^2 - 2x_2x_1 + (u_3^2 - 2u_3u_2) \\
p_3 &= x_3^2 - 2x_3x_1 + (u_4^2 - 2u_4u_2) \\
p_4 &= x_4^2 - 2x_4x_1 + (u_5^2 - 2u_5u_2) \\
p_5 &= x_5^2 - 2x_5x_1 + (u_6^2 - 2u_6u_2) \\
p_6 &= (u_4 - u_3)x_6 + u_3x_3 - u_4x_2 \\
p_7 &= (u_6 - u_5)x_7 + u_5x_5 - u_6x_4 \\
p_8 &= u_3x_9 - x_8x_2 \\
p_9 &= (u_5 - u_4)x_9 - x_8x_4 + x_8x_3 + u_4x_4 - u_5x_3 \\
p_{10} &= -u_4x_{10}x_5 + (u_6 - u_3)x_{10}x_3 + u_4x_{10}x_2 + u_4u_3x_5 - u_6u_4x_2 \\
p_{11} &= u_4x_{11} - x_{10}x_3 \\
p_{12} &= -u_5x_{12}x_5 + (u_6 - u_3)x_{12}x_4 + u_5x_{12}x_2 + (-u_6u_1 + u_3u_1)x_{12} + \\
&\quad u_5u_3x_5 - u_6u_5x_2 + (u_6u_5u_1 - u_5u_3u_1) \\
p_{13} &= u_5x_{13} - x_{12}x_4 + u_1x_{12} - u_5u_1 \\
p_{14} &= (u_5 - u_4)x_{14}x_5 - u_6x_{14}x_4 + u_6x_{14}x_3 + (-u_5u_1 + u_4u_1)x_{14} + \\
&\quad u_6u_4x_4 - u_6u_5x_3 + (u_6u_5u_1 - u_6u_4u_1)
\end{aligned}$$



$$\begin{aligned}
p_{15} &= u_6x_{15} - x_{14}x_5 + u_1x_{14} - u_6u_1 \\
p_{16} &= x_{16}x_7 - x_{16}x_6 \\
p_{17} &= x_{17}x_{12} - x_{17}x_{10} - x_{16}x_{13} + x_{16}x_{11} + x_{13}x_{10} - x_{12}x_{11} \\
p_{18} &= -x_{18}x_{15}x_8 + x_{18}x_{14}x_9 - x_{18}x_{14}x_7 + x_{18}x_{13}x_8 - x_{18}x_{12}x_9 + \\
&\quad x_{18}x_{12}x_7 + x_{15}x_{12}x_8 - x_{14}x_{13}x_8 + x_{14}x_8x_7 - x_{12}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{18}x_9 + x_{18}x_7 - x_8x_7 \\
p_{20} &= -x_{20}x_{15}x_{10} + x_{20}x_{15}x_8 + x_{20}x_{14}x_{11} - x_{20}x_{14}x_9 + x_{20}x_{10}x_6 \\
&\quad - x_{20}x_8x_6 - x_{14}x_{11}x_8 + x_{14}x_{10}x_9 - x_{14}x_{10}x_6 + x_{14}x_8x_6 \\
p_{21} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9
\end{aligned}$$

### 3.12 Triangulation, step 12

**Choosing variable:** Trying the variable with index 10.

**Variable  $x_{10}$  selected:** The number of polynomials with this variable, with indexes from 1 to 10, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{10}$ . No reduction needed.

The triangular system has not been changed.

### 3.13 Triangulation, step 13

**Choosing variable:** Trying the variable with index 9.

**Variable  $x_9$  selected:** The number of polynomials with this variable, with indexes from 1 to 9, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_9$  from all other polynomials by reducing them with polynomial  $p_8$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= x_1 - 0.5u_1 \\
p_2 &= x_2^2 - 2x_2x_1 + (u_3^2 - 2u_3u_2) \\
p_3 &= x_3^2 - 2x_3x_1 + (u_4^2 - 2u_4u_2) \\
p_4 &= x_4^2 - 2x_4x_1 + (u_5^2 - 2u_5u_2) \\
p_5 &= x_5^2 - 2x_5x_1 + (u_6^2 - 2u_6u_2) \\
p_6 &= (u_4 - u_3)x_6 + u_3x_3 - u_4x_2 \\
p_7 &= (u_6 - u_5)x_7 + u_5x_5 - u_6x_4 \\
p_8 &= -u_3x_8x_4 + u_3x_8x_3 + (u_5 - u_4)x_8x_2 + u_4u_3x_4 - u_5u_3x_3 \\
p_9 &= u_3x_9 - x_8x_2 \\
p_{10} &= -u_4x_{10}x_5 + (u_6 - u_3)x_{10}x_3 + u_4x_{10}x_2 + u_4u_3x_5 - u_6u_4x_2 \\
p_{11} &= u_4x_{11} - x_{10}x_3
\end{aligned}$$

$$\begin{aligned}
p_{12} &= -u_5x_{12}x_5 + (u_6 - u_3)x_{12}x_4 + u_5x_{12}x_2 + (-u_6u_1 + u_3u_1)x_{12} + \\
&\quad u_5u_3x_5 - u_6u_5x_2 + (u_6u_5u_1 - u_5u_3u_1) \\
p_{13} &= u_5x_{13} - x_{12}x_4 + u_1x_{12} - u_5u_1 \\
p_{14} &= (u_5 - u_4)x_{14}x_5 - u_6x_{14}x_4 + u_6x_{14}x_3 + (-u_5u_1 + u_4u_1)x_{14} + \\
&\quad u_6u_4x_4 - u_6u_5x_3 + (u_6u_5u_1 - u_6u_4u_1) \\
p_{15} &= u_6x_{15} - x_{14}x_5 + u_1x_{14} - u_6u_1 \\
p_{16} &= x_{16}x_7 - x_{16}x_6 \\
p_{17} &= x_{17}x_{12} - x_{17}x_{10} - x_{16}x_{13} + x_{16}x_{11} + x_{13}x_{10} - x_{12}x_{11} \\
p_{18} &= -x_{18}x_{15}x_8 + x_{18}x_{14}x_9 - x_{18}x_{14}x_7 + x_{18}x_{13}x_8 - x_{18}x_{12}x_9 + \\
&\quad x_{18}x_{12}x_7 + x_{15}x_{12}x_8 - x_{14}x_{13}x_8 + x_{14}x_8x_7 - x_{12}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{18}x_9 + x_{18}x_7 - x_8x_7 \\
p_{20} &= -x_{20}x_{15}x_{10} + x_{20}x_{15}x_8 + x_{20}x_{14}x_{11} - x_{20}x_{14}x_9 + x_{20}x_{10}x_6 \\
&\quad -x_{20}x_8x_6 - x_{14}x_{11}x_8 + x_{14}x_{10}x_9 - x_{14}x_{10}x_6 + x_{14}x_8x_6 \\
p_{21} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9
\end{aligned}$$

### 3.14 Triangulation, step 14

**Choosing variable:** Trying the variable with index 8.

**Variable  $x_8$  selected:** The number of polynomials with this variable, with indexes from 1 to 8, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_8$ . No reduction needed.

The triangular system has not been changed.

### 3.15 Triangulation, step 15

**Choosing variable:** Trying the variable with index 7.

**Variable  $x_7$  selected:** The number of polynomials with this variable, with indexes from 1 to 7, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_7$ . No reduction needed.

The triangular system has not been changed.

### 3.16 Triangulation, step 16

**Choosing variable:** Trying the variable with index 6.

**Variable  $x_6$  selected:** The number of polynomials with this variable, with indexes from 1 to 6, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_6$ . No reduction needed.

The triangular system has not been changed.

### 3.17 Triangulation, step 17

**Choosing variable:** Trying the variable with index 5.

**Variable  $x_5$  selected:** The number of polynomials with this variable, with indexes from 1 to 5, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_5$ . No reduction needed.

The triangular system has not been changed.

### 3.18 Triangulation, step 18

**Choosing variable:** Trying the variable with index 4.

**Variable  $x_4$  selected:** The number of polynomials with this variable, with indexes from 1 to 4, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_4$ . No reduction needed.

The triangular system has not been changed.

### 3.19 Triangulation, step 19

**Choosing variable:** Trying the variable with index 3.

**Variable  $x_3$  selected:** The number of polynomials with this variable, with indexes from 1 to 3, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_3$ . No reduction needed.

The triangular system has not been changed.

### 3.20 Triangulation, step 20

**Choosing variable:** Trying the variable with index 2.

**Variable  $x_2$  selected:** The number of polynomials with this variable, with indexes from 1 to 2, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_2$ . No reduction needed.

The triangular system has not been changed.

### 3.21 Triangulation, step 21

**Choosing variable:** Trying the variable with index 1.

**Variable  $x_1$  selected:** The number of polynomials with this variable, with indexes from 1 to 1, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_1$ . No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned} p_1 &= x_1 - 0.5u_1 \\ p_2 &= x_2^2 - 2x_2x_1 + (u_3^2 - 2u_3u_2) \\ p_3 &= x_3^2 - 2x_3x_1 + (u_4^2 - 2u_4u_2) \\ p_4 &= x_4^2 - 2x_4x_1 + (u_5^2 - 2u_5u_2) \\ p_5 &= x_5^2 - 2x_5x_1 + (u_6^2 - 2u_6u_2) \\ p_6 &= (u_4 - u_3)x_6 + u_3x_3 - u_4x_2 \\ p_7 &= (u_6 - u_5)x_7 + u_5x_5 - u_6x_4 \\ p_8 &= -u_3x_8x_4 + u_3x_8x_3 + (u_5 - u_4)x_8x_2 + u_4u_3x_4 - u_5u_3x_3 \\ p_9 &= u_3x_9 - x_8x_2 \\ p_{10} &= -u_4x_{10}x_5 + (u_6 - u_3)x_{10}x_3 + u_4x_{10}x_2 + u_4u_3x_5 - u_6u_4x_2 \\ p_{11} &= u_4x_{11} - x_{10}x_3 \\ p_{12} &= -u_5x_{12}x_5 + (u_6 - u_3)x_{12}x_4 + u_5x_{12}x_2 + (-u_6u_1 + u_3u_1)x_{12} + \\ &\quad u_5u_3x_5 - u_6u_5x_2 + (u_6u_5u_1 - u_5u_3u_1) \\ p_{13} &= u_5x_{13} - x_{12}x_4 + u_1x_{12} - u_5u_1 \\ p_{14} &= (u_5 - u_4)x_{14}x_5 - u_6x_{14}x_4 + u_6x_{14}x_3 + (-u_5u_1 + u_4u_1)x_{14} + \\ &\quad u_6u_4x_4 - u_6u_5x_3 + (u_6u_5u_1 - u_6u_4u_1) \\ p_{15} &= u_6x_{15} - x_{14}x_5 + u_1x_{14} - u_6u_1 \\ p_{16} &= x_{16}x_7 - x_{16}x_6 \\ p_{17} &= x_{17}x_{12} - x_{17}x_{10} - x_{16}x_{13} + x_{16}x_{11} + x_{13}x_{10} - x_{12}x_{11} \\ p_{18} &= -x_{18}x_{15}x_8 + x_{18}x_{14}x_9 - x_{18}x_{14}x_7 + x_{18}x_{13}x_8 - x_{18}x_{12}x_9 + \\ &\quad x_{18}x_{12}x_7 + x_{15}x_{12}x_8 - x_{14}x_{13}x_8 + x_{14}x_8x_7 - x_{12}x_8x_7 \\ p_{19} &= x_{19}x_8 - x_{18}x_9 + x_{18}x_7 - x_8x_7 \\ p_{20} &= -x_{20}x_{15}x_{10} + x_{20}x_{15}x_8 + x_{20}x_{14}x_{11} - x_{20}x_{14}x_9 + x_{20}x_{10}x_6 \\ &\quad - x_{20}x_8x_6 - x_{14}x_{11}x_8 + x_{14}x_{10}x_9 - x_{14}x_{10}x_6 + x_{14}x_8x_6 \\ p_{21} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9 \end{aligned}$$

## 4 Final Remainder

### 4.1 Final remainder for conjecture Chou 012

Calculating final remainder of the conclusion:

$$g = x_{21}x_{18} - x_{21}x_{16} - x_{20}x_{19} + x_{20}x_{17} + x_{19}x_{16} - x_{18}x_{17}$$

with respect to the triangular system.

1. Pseudo remainder with  $p_{21}$  over variable  $x_{21}$ :

$$\begin{aligned} g = & -x_{20}x_{19}x_{10} + x_{20}x_{19}x_8 + x_{20}x_{18}x_{11} - x_{20}x_{18}x_9 + x_{20}x_{17}x_{10} \\ & -x_{20}x_{17}x_8 - x_{20}x_{16}x_{11} + x_{20}x_{16}x_9 + x_{19}x_{16}x_{10} - x_{19}x_{16}x_8 \\ & -x_{18}x_{17}x_{10} + x_{18}x_{17}x_8 - x_{18}x_{11}x_8 + x_{18}x_{10}x_9 + x_{16}x_{11}x_8 \\ & -x_{16}x_{10}x_9 \end{aligned}$$

2. Pseudo remainder with  $p_{20}$  over variable  $x_{20}$ :

$$\begin{aligned} g = & -x_{19}x_{16}x_{15}x_{10}^2 + 2x_{19}x_{16}x_{15}x_{10}x_8 - x_{19}x_{16}x_{15}x_8^2 + \\ & x_{19}x_{16}x_{14}x_{11}x_{10} - x_{19}x_{16}x_{14}x_{11}x_8 - x_{19}x_{16}x_{14}x_{10}x_9 + \\ & x_{19}x_{16}x_{14}x_9x_8 + x_{19}x_{16}x_{10}^2x_6 - 2x_{19}x_{16}x_{10}x_8x_6 + \\ & x_{19}x_{16}x_8^2x_6 - x_{19}x_{14}x_{11}x_{10}x_8 + x_{19}x_{14}x_{11}x_8^2 + \\ & x_{19}x_{14}x_{10}^2x_9 - x_{19}x_{14}x_{10}^2x_6 - x_{19}x_{14}x_{10}x_9x_8 + \\ & 2x_{19}x_{14}x_{10}x_8x_6 - x_{19}x_{14}x_8^2x_6 + x_{18}x_{17}x_{15}x_{10}^2 \\ & - 2x_{18}x_{17}x_{15}x_{10}x_8 + x_{18}x_{17}x_{15}x_8^2 - x_{18}x_{17}x_{14}x_{11}x_{10} + \\ & x_{18}x_{17}x_{14}x_{11}x_8 + x_{18}x_{17}x_{14}x_{10}x_9 - x_{18}x_{17}x_{14}x_9x_8 \\ & - x_{18}x_{17}x_{10}^2x_6 + 2x_{18}x_{17}x_{10}x_8x_6 - x_{18}x_{17}x_8^2x_6 + \\ & x_{18}x_{15}x_{11}x_{10}x_8 - x_{18}x_{15}x_{11}x_8^2 - x_{18}x_{15}x_{10}^2x_9 + \\ & x_{18}x_{15}x_{10}x_9x_8 + x_{18}x_{14}x_{11}x_{10}x_6 - x_{18}x_{14}x_{11}x_8x_6 \\ & - x_{18}x_{14}x_{10}x_9x_6 + x_{18}x_{14}x_9x_8x_6 - x_{18}x_{11}x_{10}x_8x_6 + \\ & x_{18}x_{11}x_8^2x_6 + x_{18}x_{10}^2x_9x_6 - x_{18}x_{10}x_9x_8x_6 + \\ & x_{17}x_{14}x_{11}x_{10}x_8 - x_{17}x_{14}x_{11}x_8^2 - x_{17}x_{14}x_{10}^2x_9 + \\ & x_{17}x_{14}x_{10}^2x_6 + x_{17}x_{14}x_{10}x_9x_8 - 2x_{17}x_{14}x_{10}x_8x_6 + \\ & x_{17}x_{14}x_8^2x_6 - x_{16}x_{15}x_{11}x_{10}x_8 + x_{16}x_{15}x_{11}x_8^2 + \\ & x_{16}x_{15}x_{10}^2x_9 - x_{16}x_{15}x_{10}x_9x_8 - x_{16}x_{14}x_{11}x_{10}x_6 + \\ & x_{16}x_{14}x_{11}x_8x_6 + x_{16}x_{14}x_{10}x_9x_6 - x_{16}x_{14}x_9x_8x_6 + \\ & x_{16}x_{11}x_{10}x_8x_6 - x_{16}x_{11}x_8^2x_6 - x_{16}x_{10}^2x_9x_6 + \\ & x_{16}x_{10}x_9x_8x_6 \end{aligned}$$

3. Pseudo remainder with  $p_{19}$  over variable  $x_{19}$ :

*Polynomial too big for output (text size is 3079 characters, number of terms is 89)*

4. Pseudo remainder with  $p_{18}$  over variable  $x_{18}$ :

*Polynomial too big for output (number of terms is 266)*

5. Pseudo remainder with  $p_{17}$  over variable  $x_{17}$ :  
*Polynomial too big for output (number of terms is 439)*
6. Pseudo remainder with  $p_{16}$  over variable  $x_{16}$ :  
*Polynomial too big for output (number of terms is 268)*
7. Pseudo remainder with  $p_{15}$  over variable  $x_{15}$ :  
*Polynomial too big for output (number of terms is 550)*
8. Pseudo remainder with  $p_{14}$  over variable  $x_{14}$ :  
*Polynomial too big for output (number of terms is 2360)*
9. Pseudo remainder with  $p_{13}$  over variable  $x_{13}$ :  
*Polynomial too big for output (number of terms is 3516)*
10. Pseudo remainder with  $p_{12}$  over variable  $x_{12}$ :  
*Polynomial too big for output (number of terms is 8950)*
11. Pseudo remainder with  $p_{11}$  over variable  $x_{11}$ :  
*Polynomial too big for output (number of terms is 7424)*
12. Pseudo remainder with  $p_{10}$  over variable  $x_{10}$ :  
*Polynomial too big for output (number of terms is 11430)*
13. Pseudo remainder with  $p_9$  over variable  $x_9$ :  
*Polynomial too big for output (number of terms is 7388)*
14. Pseudo remainder with  $p_8$  over variable  $x_8$ :  
*Polynomial too big for output (number of terms is 7532)*
15. Pseudo remainder with  $p_7$  over variable  $x_7$ :  
*Polynomial too big for output (number of terms is 5323)*
16. Pseudo remainder with  $p_6$  over variable  $x_6$ :  
*Polynomial too big for output (number of terms is 2333)*
17. Pseudo remainder with  $p_5$  over variable  $x_5$ :  
*Polynomial too big for output (number of terms is 4454)*

18. Pseudo remainder with  $p_4$  over variable  $x_4$ :  
*Polynomial too big for output (number of terms is 2264)*
19. Pseudo remainder with  $p_3$  over variable  $x_3$ :  
*Polynomial too big for output (number of terms is 766)*
20. Pseudo remainder with  $p_2$  over variable  $x_2$ :  
*Polynomial too big for output (text size is greater than 2000 characters, number of terms is 208)*
21. Pseudo remainder with  $p_1$  over variable  $x_1$ :

$$g = 0$$

## 5 Prover results

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 11430 terms.

**Time Complexity:** Time spent by the prover is 3277.26 seconds.

## 6 NDG Conditions

### NDG Conditions in readable form

- Points A2, A3, B and A0 are not collinear
- Points A4, A and A5 are not collinear
- Line through points A2 and A0 is not parallel with line through points A4 and A3
- Points A2, A and A0 are not collinear
- Line through points A2 and A5 is not parallel with line through points A3 and A0
- Points A3, A and B are not collinear
- Line through points A2 and A5 is not parallel with line through points A1 and A4
- Points A4, A, B and A0 are not collinear
- Line through points A1 and A5 is not parallel with line through points A4 and A3
- Points A, A5 and A0 are not collinear

- Line through points A1 and A0 is not perpendicular to line through points A and B
- Line through points D and E is not parallel with line through points A1 and A0
- Line through points E and F is not parallel with line through points B and C
- Points A1, C and A0 are not collinear
- Line through points D and C is not parallel with line through points F and A
- Line through points D and C is not parallel with line through points A1 and A0

### **Time spent for processing NDG Conditions**

- 7.503 seconds