# OpenGeoProver Output for conjecture "Thales' theorem"

Wu's method used

February 21, 2012

## 1 Validation of Construction Protocol

**Construction steps:**

- Free point S

- Free point A

- Free point D

- Line SA through two points S and A

- Random point B from line SA

- Generalized segment division point C of segment SD with respect to ratio product (SA/AB) and coefficient 1.0

- Line CA through two points C and A

- Line DB through two points D and B

**Theorem statement:**

- Line CA is parallel with line DB

**Validation result:** Construction protocol is valid.

## 2 Transformation of Construction Protocol to algebraic form

**Transformation of Construction steps**

### 2.1 Transformation of point S:

- Point S has been assigned following coordinates: $(0, 0)$

### 2.2 Transformation of point A:

- Point A has been assigned following coordinates: $(0, u_1)$

## 2.3 Transformation of point D:

- Point D has been assigned following coordinates: $(u_2,\, u_3)$

## 2.4 Transformation of point B:

- Point B has been assigned following coordinates: $(u_4,\, x_1)$
- Polynomial that point B has to satisfy is:

$$p \;=\; x_1$$

- Processing of polynomial

$$p \;=\; x_1$$

  **Info:** Will try to rename X coordinate of point B

  **Info:** X coordinate of point B renamed by zero

- Point B has been renamed. Point B has been assigned following coordinates: $(0,\, u_4)$

## 2.5 Transformation of point C:

- Point C has been assigned following coordinates: $(x_1,\, x_2)$
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p \;=\; u_4 x_1 - u_2 u_1$$

  **Info:** Polynomial

$$p \;=\; u_4 x_1 - u_2 u_1$$

  added to system of polynomials that represents the constructions

- Instantiated condition

$$p \;=\; u_4 x_1 - u_2 u_1$$

  is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p \;=\; u_4 x_2 - u_3 u_1$$

  **Info:** Polynomial

$$p \;=\; u_4 x_2 - u_3 u_1$$

  added to system of polynomials that represents the constructions

- Instantiated condition

$$p \;=\; u_4 x_2 - u_3 u_1$$

  is added to polynomial system

**Transformation of Theorem statement**

- Polynomial for theorem statement:

$$p = u_2 x_2 + (u_4 - u_3)x_1 - u_2 u_1$$

**Time spent for transformation of Construction Protocol to algebraic form**

- 0.032 seconds

# 3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$p_1 = u_4 x_1 - u_2 u_1$$
$$p_2 = u_4 x_2 - u_3 u_1$$

## 3.1 Triangulation, step 1

**Choosing variable:** Trying the variable with index 2.

**Variable $x_2$ selected:** The number of polynomials with this variable, with indexes from 1 to 2, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_2$. No reduction needed.

The triangular system has not been changed.

## 3.2 Triangulation, step 2

**Choosing variable:** Trying the variable with index 1.

**Variable $x_1$ selected:** The number of polynomials with this variable, with indexes from 1 to 1, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_1$. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$p_1 = u_4 x_1 - u_2 u_1$$
$$p_2 = u_4 x_2 - u_3 u_1$$

# 4  Final Remainder

## 4.1  Final remainder for conjecture Thales' theorem

Calculating final remainder of the conclusion:

$$g \quad = \quad u_2 x_2 + (u_4 - u_3)x_1 - u_2 u_1$$

with respect to the triangular system.

1. Pseudo remainder with $p_2$ over variable $x_2$:

$$g \quad = \quad (u_4^2 - u_4 u_3)x_1 + (-u_4 u_2 u_1 + u_3 u_2 u_1)$$

2. Pseudo remainder with $p_1$ over variable $x_1$:

$$g \quad = \quad 0$$

# 5  Prover results

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.019 seconds.

# 6  NDG Conditions

## NDG Conditions in readable form

- Points A and B are not identical

## Time spent for processing NDG Conditions

- 0.05 seconds