

OpenGeoProver Output for conjecture “Chou 011”

Wu’s method used

February 12, 2012

1 Validation of Construction Protocol

Construction steps:

- General Conic Section c
- Random point A_0 from general conic c
- Random point A_1 from general conic c
- Random point A_2 from general conic c
- Random point A_3 from general conic c
- Random point A_4 from general conic c
- Random point A_5 from general conic c
- Line A_0A_1 through two points A_0 and A_1
- Line A_2A_3 through two points A_2 and A_3
- Intersection point A of point sets A_0A_1 and A_2A_3
- Line A_4A_5 through two points A_4 and A_5
- Intersection point B of point sets A_0A_1 and A_4A_5
- Line A_0A_2 through two points A_0 and A_2
- Line A_1A_4 through two points A_1 and A_4
- Intersection point C of point sets A_0A_2 and A_1A_4
- Line A_0A_3 through two points A_0 and A_3
- Line A_1A_5 through two points A_1 and A_5
- Intersection point D of point sets A_0A_3 and A_1A_5
- Line A_0A_4 through two points A_0 and A_4
- Line A_1A_2 through two points A_1 and A_2

- Intersection point E of point sets A0A4 and A1A2
- Line A0A5 through two points A0 and A5
- Line A1A3 through two points A1 and A3
- Intersection point F of point sets A0A5 and A1A3
- Line AB through two points A and B
- Line DE through two points D and E
- Line BC through two points B and C
- Line EF through two points E and F
- Line CD through two points C and D
- Line FA through two points F and A
- Intersection point P of point sets AB and DE
- Intersection point Q of point sets BC and EF
- Intersection point R of point sets CD and FA

Theorem statement:

- Points P, Q, R are collinear

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of general conic section c:

List of parametric points

- Point Ac has been assigned following coordinates: $(u_1, 0)$
- Point Bc has been assigned following coordinates: $(u_2, 0)$
- Point Cc has been assigned following coordinates: $(u_3, 0)$
- Point Dc has been assigned following coordinates: $(u_4, 0)$
- Point Ec has been assigned following coordinates: $(u_5, 0)$
- Condition for point $X(x_1, x_2)$ to belong to this conic section is following equation:

$$p = u_3x_2^2 + u_2x_2x_1 + u_5x_2 + u_1x_1^2 + u_4x_1$$

2.2 Transformation of point A0:

- Point A0 has been assigned following coordinates: $(0, 0)$

2.3 Transformation of point A1:

- Point A1 has been assigned following coordinates: $(0, x_1)$
- Polynomial that point A1 has to satisfy is:

$$p = u_3x_1^2 + u_5x_1$$

- Processing of polynomial

$$p = u_3x_1^2 + u_5x_1$$

Info: Polynomial

$$p = u_3x_1^2 + u_5x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.4 Transformation of point A2:

- Point A2 has been assigned following coordinates: (u_6, x_2)
- Polynomial that point A2 has to satisfy is:

$$p = u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)$$

- Processing of polynomial

$$p = u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)$$

Info: Polynomial

$$p = u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of point A3:

- Point A3 has been assigned following coordinates: (u_7, x_3)
- Polynomial that point A3 has to satisfy is:

$$p = u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4)$$

- Processing of polynomial

$$p = u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4)$$

Info: Polynomial

$$p = u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point A4:

- Point A4 has been assigned following coordinates: (u_8, x_4)
- Polynomial that point A4 has to satisfy is:

$$p = u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)$$

- Processing of polynomial

$$p = u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)$$

Info: Polynomial

$$p = u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point A5:

- Point A5 has been assigned following coordinates: (u_9, x_5)
- Polynomial that point A5 has to satisfy is:

$$p = u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4)$$

- Processing of polynomial

$$p = u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4)$$

Info: Polynomial

$$p = u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.8 Transformation of point A:

- Point A has been assigned following coordinates: (x_6, x_7)
- Polynomial that point A has to satisfy is:

$$p = x_6x_1$$

- Processing of polynomial

$$p = x_6x_1$$

Info: Polynomial

$$p = x_6x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point A has to satisfy is:

$$p = (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2$$

- Processing of polynomial

$$p = (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2$$

Info: Polynomial

$$p = (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.9 Transformation of point B:

- Point B has been assigned following coordinates: (x_8, x_9)
- Polynomial that point B has to satisfy is:

$$p = x_8x_1$$

- Processing of polynomial

$$p = x_8x_1$$

Info: Polynomial

$$p = x_8x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point B has to satisfy is:

$$p = (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4$$

- Processing of polynomial

$$p = (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4$$

Info: Polynomial

$$p = (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.10 Transformation of point C:

- Point C has been assigned following coordinates: (x_{10}, x_{11})
- Polynomial that point C has to satisfy is:

$$p = u_6x_{11} - x_{10}x_2$$

- Processing of polynomial

$$p = u_6x_{11} - x_{10}x_2$$

Info: Polynomial

$$p = u_6x_{11} - x_{10}x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point C has to satisfy is:

$$p = u_8x_{11} - x_{10}x_4 + x_{10}x_1 - u_8x_1$$

- Processing of polynomial

$$p = u_8x_{11} - x_{10}x_4 + x_{10}x_1 - u_8x_1$$

Info: Polynomial

$$p = u_8x_{11} - x_{10}x_4 + x_{10}x_1 - u_8x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.11 Transformation of point D:

- Point D has been assigned following coordinates: (x_{12}, x_{13})
- Polynomial that point D has to satisfy is:

$$p = u_7x_{13} - x_{12}x_3$$

- Processing of polynomial

$$p = u_7x_{13} - x_{12}x_3$$

Info: Polynomial

$$p = u_7x_{13} - x_{12}x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point D has to satisfy is:

$$p = u_9x_{13} - x_{12}x_5 + x_{12}x_1 - u_9x_1$$

- Processing of polynomial

$$p = u_9x_{13} - x_{12}x_5 + x_{12}x_1 - u_9x_1$$

Info: Polynomial

$$p = u_9x_{13} - x_{12}x_5 + x_{12}x_1 - u_9x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.12 Transformation of point E:

- Point E has been assigned following coordinates: (x_{14}, x_{15})
- Polynomial that point E has to satisfy is:

$$p = u_8x_{15} - x_{14}x_4$$

- Processing of polynomial

$$p = u_8x_{15} - x_{14}x_4$$

Info: Polynomial

$$p = u_8x_{15} - x_{14}x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point E has to satisfy is:

$$p = u_6x_{15} - x_{14}x_2 + x_{14}x_1 - u_6x_1$$

- Processing of polynomial

$$p = u_6x_{15} - x_{14}x_2 + x_{14}x_1 - u_6x_1$$

Info: Polynomial

$$p = u_6x_{15} - x_{14}x_2 + x_{14}x_1 - u_6x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.13 Transformation of point F:

- Point F has been assigned following coordinates: (x_{16}, x_{17})
- Polynomial that point F has to satisfy is:

$$p = u_9x_{17} - x_{16}x_5$$

- Processing of polynomial

$$p = u_9x_{17} - x_{16}x_5$$

Info: Polynomial

$$p = u_9x_{17} - x_{16}x_5$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point F has to satisfy is:

$$p = u_7x_{17} - x_{16}x_3 + x_{16}x_1 - u_7x_1$$

- Processing of polynomial

$$p = u_7x_{17} - x_{16}x_3 + x_{16}x_1 - u_7x_1$$

Info: Polynomial

$$p = u_7x_{17} - x_{16}x_3 + x_{16}x_1 - u_7x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.14 Transformation of point P:

- Point P has been assigned following coordinates: (x_{18}, x_{19})
- Polynomial that point P has to satisfy is:

$$p = x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7$$

- Processing of polynomial

$$p = x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7$$

Info: Polynomial

$$p = x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point P has to satisfy is:

$$p = x_{19}x_{14} - x_{19}x_{12} - x_{18}x_{15} + x_{18}x_{13} + x_{15}x_{12} - x_{14}x_{13}$$

- Processing of polynomial

$$p = x_{19}x_{14} - x_{19}x_{12} - x_{18}x_{15} + x_{18}x_{13} + x_{15}x_{12} - x_{14}x_{13}$$

Info: Polynomial

$$p = x_{19}x_{14} - x_{19}x_{12} - x_{18}x_{15} + x_{18}x_{13} + x_{15}x_{12} - x_{14}x_{13}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.15 Transformation of point Q:

- Point Q has been assigned following coordinates: (x_{20}, x_{21})
- Polynomial that point Q has to satisfy is:

$$p = x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9$$

- Processing of polynomial

$$p = x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9$$

Info: Polynomial

$$p = x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point Q has to satisfy is:

$$p = x_{21}x_{16} - x_{21}x_{14} - x_{20}x_{17} + x_{20}x_{15} + x_{17}x_{14} - x_{16}x_{15}$$

- Processing of polynomial

$$p = x_{21}x_{16} - x_{21}x_{14} - x_{20}x_{17} + x_{20}x_{15} + x_{17}x_{14} - x_{16}x_{15}$$

Info: Polynomial

$$p = x_{21}x_{16} - x_{21}x_{14} - x_{20}x_{17} + x_{20}x_{15} + x_{17}x_{14} - x_{16}x_{15}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.16 Transformation of point R:

- Point R has been assigned following coordinates: (x_{22}, x_{23})
- Polynomial that point R has to satisfy is:

$$p = x_{23}x_{12} - x_{23}x_{10} - x_{22}x_{13} + x_{22}x_{11} + x_{13}x_{10} - x_{12}x_{11}$$

- Processing of polynomial

$$p = x_{23}x_{12} - x_{23}x_{10} - x_{22}x_{13} + x_{22}x_{11} + x_{13}x_{10} - x_{12}x_{11}$$

Info: Polynomial

$$p = x_{23}x_{12} - x_{23}x_{10} - x_{22}x_{13} + x_{22}x_{11} + x_{13}x_{10} - x_{12}x_{11}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point R has to satisfy is:

$$p = x_{23}x_{16} - x_{23}x_6 - x_{22}x_{17} + x_{22}x_7 + x_{17}x_6 - x_{16}x_7$$

- Processing of polynomial

$$p = x_{23}x_{16} - x_{23}x_6 - x_{22}x_{17} + x_{22}x_7 + x_{17}x_6 - x_{16}x_7$$

Info: Polynomial

$$p = x_{23}x_{16} - x_{23}x_6 - x_{22}x_{17} + x_{22}x_7 + x_{17}x_6 - x_{16}x_7$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_{23}x_{20} - x_{23}x_{18} - x_{22}x_{21} + x_{22}x_{19} + x_{21}x_{18} - x_{20}x_{19}$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.395 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= x_8x_1 \\
p_9 &= (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4 \\
p_{10} &= u_6x_{11} - x_{10}x_2 \\
p_{11} &= u_8x_{11} - x_{10}x_4 + x_{10}x_1 - u_8x_1 \\
p_{12} &= u_7x_{13} - x_{12}x_3 \\
p_{13} &= u_9x_{13} - x_{12}x_5 + x_{12}x_1 - u_9x_1 \\
p_{14} &= u_8x_{15} - x_{14}x_4 \\
p_{15} &= u_6x_{15} - x_{14}x_2 + x_{14}x_1 - u_6x_1 \\
p_{16} &= u_9x_{17} - x_{16}x_5 \\
p_{17} &= u_7x_{17} - x_{16}x_3 + x_{16}x_1 - u_7x_1 \\
p_{18} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7 \\
p_{19} &= x_{19}x_{14} - x_{19}x_{12} - x_{18}x_{15} + x_{18}x_{13} + x_{15}x_{12} - x_{14}x_{13} \\
p_{20} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9 \\
p_{21} &= x_{21}x_{16} - x_{21}x_{14} - x_{20}x_{17} + x_{20}x_{15} + x_{17}x_{14} - x_{16}x_{15} \\
p_{22} &= x_{23}x_{12} - x_{23}x_{10} - x_{22}x_{13} + x_{22}x_{11} + x_{13}x_{10} - x_{12}x_{11} \\
p_{23} &= x_{23}x_{16} - x_{23}x_6 - x_{22}x_{17} + x_{22}x_7 + x_{17}x_6 - x_{16}x_7
\end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 23.

Variable x_{23} selected: The number of polynomials with this variable, with indexes from 1 to 23, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{23} from all other polynomials by reducing them with polynomial p_{22} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)
\end{aligned}$$

$$\begin{aligned}
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= x_8x_1 \\
p_9 &= (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4 \\
p_{10} &= u_6x_{11} - x_{10}x_2 \\
p_{11} &= u_8x_{11} - x_{10}x_4 + x_{10}x_1 - u_8x_1 \\
p_{12} &= u_7x_{13} - x_{12}x_3 \\
p_{13} &= u_9x_{13} - x_{12}x_5 + x_{12}x_1 - u_9x_1 \\
p_{14} &= u_8x_{15} - x_{14}x_4 \\
p_{15} &= u_6x_{15} - x_{14}x_2 + x_{14}x_1 - u_6x_1 \\
p_{16} &= u_9x_{17} - x_{16}x_5 \\
p_{17} &= u_7x_{17} - x_{16}x_3 + x_{16}x_1 - u_7x_1 \\
p_{18} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7 \\
p_{19} &= x_{19}x_{14} - x_{19}x_{12} - x_{18}x_{15} + x_{18}x_{13} + x_{15}x_{12} - x_{14}x_{13} \\
p_{20} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9 \\
p_{21} &= x_{21}x_{16} - x_{21}x_{14} - x_{20}x_{17} + x_{20}x_{15} + x_{17}x_{14} - x_{16}x_{15} \\
p_{22} &= -x_{22}x_{17}x_{12} + x_{22}x_{17}x_{10} + x_{22}x_{16}x_{13} - x_{22}x_{16}x_{11} \\
&\quad -x_{22}x_{13}x_6 + x_{22}x_{12}x_7 + x_{22}x_{11}x_6 - x_{22}x_{10}x_7 + x_{17}x_{12}x_6 \\
&\quad -x_{17}x_{10}x_6 - x_{16}x_{13}x_{10} + x_{16}x_{12}x_{11} - x_{16}x_{12}x_7 + x_{16}x_{10}x_7 + \\
&\quad x_{13}x_{10}x_6 - x_{12}x_{11}x_6 \\
p_{23} &= x_{23}x_{12} - x_{23}x_{10} - x_{22}x_{13} + x_{22}x_{11} + x_{13}x_{10} - x_{12}x_{11}
\end{aligned}$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 22.

Variable x_{22} selected: The number of polynomials with this variable, with indexes from 1 to 22, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{22} . No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 21.

Variable x_{21} selected: The number of polynomials with this variable, with indexes from 1 to 21, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{21} from all other polynomials by reducing them with polynomial p_{20} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= x_8x_1 \\
p_9 &= (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4 \\
p_{10} &= u_6x_{11} - x_{10}x_2 \\
p_{11} &= u_8x_{11} - x_{10}x_4 + x_{10}x_1 - u_8x_1 \\
p_{12} &= u_7x_{13} - x_{12}x_3 \\
p_{13} &= u_9x_{13} - x_{12}x_5 + x_{12}x_1 - u_9x_1 \\
p_{14} &= u_8x_{15} - x_{14}x_4 \\
p_{15} &= u_6x_{15} - x_{14}x_2 + x_{14}x_1 - u_6x_1 \\
p_{16} &= u_9x_{17} - x_{16}x_5 \\
p_{17} &= u_7x_{17} - x_{16}x_3 + x_{16}x_1 - u_7x_1 \\
p_{18} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7 \\
p_{19} &= x_{19}x_{14} - x_{19}x_{12} - x_{18}x_{15} + x_{18}x_{13} + x_{15}x_{12} - x_{14}x_{13} \\
p_{20} &= -x_{20}x_{17}x_{10} + x_{20}x_{17}x_8 + x_{20}x_{16}x_{11} - x_{20}x_{16}x_9 + x_{20}x_{15}x_{10} \\
&\quad -x_{20}x_{15}x_8 - x_{20}x_{14}x_{11} + x_{20}x_{14}x_9 + x_{17}x_{14}x_{10} - x_{17}x_{14}x_8 \\
&\quad -x_{16}x_{15}x_{10} + x_{16}x_{15}x_8 - x_{16}x_{11}x_8 + x_{16}x_{10}x_9 + x_{14}x_{11}x_8 \\
&\quad -x_{14}x_{10}x_9 \\
p_{21} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9 \\
p_{22} &= -x_{22}x_{17}x_{12} + x_{22}x_{17}x_{10} + x_{22}x_{16}x_{13} - x_{22}x_{16}x_{11} \\
&\quad -x_{22}x_{13}x_6 + x_{22}x_{12}x_7 + x_{22}x_{11}x_6 - x_{22}x_{10}x_7 + x_{17}x_{12}x_6 \\
&\quad -x_{17}x_{10}x_6 - x_{16}x_{13}x_{10} + x_{16}x_{12}x_{11} - x_{16}x_{12}x_7 + x_{16}x_{10}x_7 + \\
&\quad x_{13}x_{10}x_6 - x_{12}x_{11}x_6 \\
p_{23} &= x_{23}x_{12} - x_{23}x_{10} - x_{22}x_{13} + x_{22}x_{11} + x_{13}x_{10} - x_{12}x_{11}
\end{aligned}$$

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 20.

Variable x_{20} selected: The number of polynomials with this variable, with indexes from 1 to 20, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{20} . No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 19.

Variable x_{19} selected: The number of polynomials with this variable, with indexes from 1 to 19, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{19} from all other polynomials by reducing them with polynomial p_{18} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= x_8x_1 \\
p_9 &= (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4 \\
p_{10} &= u_6x_{11} - x_{10}x_2 \\
p_{11} &= u_8x_{11} - x_{10}x_4 + x_{10}x_1 - u_8x_1 \\
p_{12} &= u_7x_{13} - x_{12}x_3 \\
p_{13} &= u_9x_{13} - x_{12}x_5 + x_{12}x_1 - u_9x_1 \\
p_{14} &= u_8x_{15} - x_{14}x_4 \\
p_{15} &= u_6x_{15} - x_{14}x_2 + x_{14}x_1 - u_6x_1 \\
p_{16} &= u_9x_{17} - x_{16}x_5 \\
p_{17} &= u_7x_{17} - x_{16}x_3 + x_{16}x_1 - u_7x_1 \\
p_{18} &= -x_{18}x_{15}x_8 + x_{18}x_{15}x_6 + x_{18}x_{14}x_9 - x_{18}x_{14}x_7 + x_{18}x_{13}x_8 \\
&\quad -x_{18}x_{13}x_6 - x_{18}x_{12}x_9 + x_{18}x_{12}x_7 + x_{15}x_{12}x_8 - x_{15}x_{12}x_6 \\
&\quad -x_{14}x_{13}x_8 + x_{14}x_{13}x_6 - x_{14}x_9x_6 + x_{14}x_8x_7 + x_{12}x_9x_6 \\
&\quad -x_{12}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7 \\
p_{20} &= -x_{20}x_{17}x_{10} + x_{20}x_{17}x_8 + x_{20}x_{16}x_{11} - x_{20}x_{16}x_9 + x_{20}x_{15}x_{10} \\
&\quad -x_{20}x_{15}x_8 - x_{20}x_{14}x_{11} + x_{20}x_{14}x_9 + x_{17}x_{14}x_{10} - x_{17}x_{14}x_8 \\
&\quad -x_{16}x_{15}x_{10} + x_{16}x_{15}x_8 - x_{16}x_{11}x_8 + x_{16}x_{10}x_9 + x_{14}x_{11}x_8 \\
&\quad -x_{14}x_{10}x_9 \\
p_{21} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9 \\
p_{22} &= -x_{22}x_{17}x_{12} + x_{22}x_{17}x_{10} + x_{22}x_{16}x_{13} - x_{22}x_{16}x_{11} \\
&\quad -x_{22}x_{13}x_6 + x_{22}x_{12}x_7 + x_{22}x_{11}x_6 - x_{22}x_{10}x_7 + x_{17}x_{12}x_6 \\
&\quad -x_{17}x_{10}x_6 - x_{16}x_{13}x_{10} + x_{16}x_{12}x_{11} - x_{16}x_{12}x_7 + x_{16}x_{10}x_7 +
\end{aligned}$$

$$\begin{aligned}
& x_{13}x_{10}x_6 - x_{12}x_{11}x_6 \\
p_{23} = & x_{23}x_{12} - x_{23}x_{10} - x_{22}x_{13} + x_{22}x_{11} + x_{13}x_{10} - x_{12}x_{11}
\end{aligned}$$

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{18} . No reduction needed.

The triangular system has not been changed.

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{17} from all other polynomials by reducing them with polynomial p_{16} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= x_8x_1 \\
p_9 &= (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4 \\
p_{10} &= u_6x_{11} - x_{10}x_2 \\
p_{11} &= u_8x_{11} - x_{10}x_4 + x_{10}x_1 - u_8x_1 \\
p_{12} &= u_7x_{13} - x_{12}x_3 \\
p_{13} &= u_9x_{13} - x_{12}x_5 + x_{12}x_1 - u_9x_1 \\
p_{14} &= u_8x_{15} - x_{14}x_4 \\
p_{15} &= u_6x_{15} - x_{14}x_2 + x_{14}x_1 - u_6x_1 \\
p_{16} &= u_7x_{16}x_5 - u_9x_{16}x_3 + u_9x_{16}x_1 - u_9u_7x_1 \\
p_{17} &= u_9x_{17} - x_{16}x_5 \\
p_{18} &= -x_{18}x_{15}x_8 + x_{18}x_{15}x_6 + x_{18}x_{14}x_9 - x_{18}x_{14}x_7 + x_{18}x_{13}x_8
\end{aligned}$$

$$\begin{aligned}
& -x_{18}x_{13}x_6 - x_{18}x_{12}x_9 + x_{18}x_{12}x_7 + x_{15}x_{12}x_8 - x_{15}x_{12}x_6 \\
& -x_{14}x_{13}x_8 + x_{14}x_{13}x_6 - x_{14}x_9x_6 + x_{14}x_8x_7 + x_{12}x_9x_6 \\
& -x_{12}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7 \\
p_{20} &= -x_{20}x_{17}x_{10} + x_{20}x_{17}x_8 + x_{20}x_{16}x_{11} - x_{20}x_{16}x_9 + x_{20}x_{15}x_{10} \\
& -x_{20}x_{15}x_8 - x_{20}x_{14}x_{11} + x_{20}x_{14}x_9 + x_{17}x_{14}x_{10} - x_{17}x_{14}x_8 \\
& -x_{16}x_{15}x_{10} + x_{16}x_{15}x_8 - x_{16}x_{11}x_8 + x_{16}x_{10}x_9 + x_{14}x_{11}x_8 \\
& -x_{14}x_{10}x_9 \\
p_{21} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9 \\
p_{22} &= -x_{22}x_{17}x_{12} + x_{22}x_{17}x_{10} + x_{22}x_{16}x_{13} - x_{22}x_{16}x_{11} \\
& -x_{22}x_{13}x_6 + x_{22}x_{12}x_7 + x_{22}x_{11}x_6 - x_{22}x_{10}x_7 + x_{17}x_{12}x_6 \\
& -x_{17}x_{10}x_6 - x_{16}x_{13}x_{10} + x_{16}x_{12}x_{11} - x_{16}x_{12}x_7 + x_{16}x_{10}x_7 + \\
& x_{13}x_{10}x_6 - x_{12}x_{11}x_6 \\
p_{23} &= x_{23}x_{12} - x_{23}x_{10} - x_{22}x_{13} + x_{22}x_{11} + x_{13}x_{10} - x_{12}x_{11}
\end{aligned}$$

3.8 Triangulation, step 8

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{16} . No reduction needed.

The triangular system has not been changed.

3.9 Triangulation, step 9

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{15} from all other polynomials by reducing them with polynomial p_{14} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1
\end{aligned}$$

$$\begin{aligned}
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= x_8x_1 \\
p_9 &= (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4 \\
p_{10} &= u_6x_{11} - x_{10}x_2 \\
p_{11} &= u_8x_{11} - x_{10}x_4 + x_{10}x_1 - u_8x_1 \\
p_{12} &= u_7x_{13} - x_{12}x_3 \\
p_{13} &= u_9x_{13} - x_{12}x_5 + x_{12}x_1 - u_9x_1 \\
p_{14} &= u_6x_{14}x_4 - u_8x_{14}x_2 + u_8x_{14}x_1 - u_8u_6x_1 \\
p_{15} &= u_8x_{15} - x_{14}x_4 \\
p_{16} &= u_7x_{16}x_5 - u_9x_{16}x_3 + u_9x_{16}x_1 - u_9u_7x_1 \\
p_{17} &= u_9x_{17} - x_{16}x_5 \\
p_{18} &= -x_{18}x_{15}x_8 + x_{18}x_{15}x_6 + x_{18}x_{14}x_9 - x_{18}x_{14}x_7 + x_{18}x_{13}x_8 \\
&\quad -x_{18}x_{13}x_6 - x_{18}x_{12}x_9 + x_{18}x_{12}x_7 + x_{15}x_{12}x_8 - x_{15}x_{12}x_6 \\
&\quad -x_{14}x_{13}x_8 + x_{14}x_{13}x_6 - x_{14}x_9x_6 + x_{14}x_8x_7 + x_{12}x_9x_6 \\
&\quad -x_{12}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7 \\
p_{20} &= -x_{20}x_{17}x_{10} + x_{20}x_{17}x_8 + x_{20}x_{16}x_{11} - x_{20}x_{16}x_9 + x_{20}x_{15}x_{10} \\
&\quad -x_{20}x_{15}x_8 - x_{20}x_{14}x_{11} + x_{20}x_{14}x_9 + x_{17}x_{14}x_{10} - x_{17}x_{14}x_8 \\
&\quad -x_{16}x_{15}x_{10} + x_{16}x_{15}x_8 - x_{16}x_{11}x_8 + x_{16}x_{10}x_9 + x_{14}x_{11}x_8 \\
&\quad -x_{14}x_{10}x_9 \\
p_{21} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9 \\
p_{22} &= -x_{22}x_{17}x_{12} + x_{22}x_{17}x_{10} + x_{22}x_{16}x_{13} - x_{22}x_{16}x_{11} \\
&\quad -x_{22}x_{13}x_6 + x_{22}x_{12}x_7 + x_{22}x_{11}x_6 - x_{22}x_{10}x_7 + x_{17}x_{12}x_6 \\
&\quad -x_{17}x_{10}x_6 - x_{16}x_{13}x_{10} + x_{16}x_{12}x_{11} - x_{16}x_{12}x_7 + x_{16}x_{10}x_7 + \\
&\quad x_{13}x_{10}x_6 - x_{12}x_{11}x_6 \\
p_{23} &= x_{23}x_{12} - x_{23}x_{10} - x_{22}x_{13} + x_{22}x_{11} + x_{13}x_{10} - x_{12}x_{11}
\end{aligned}$$

3.10 Triangulation, step 10

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14} . No reduction needed.

The triangular system has not been changed.

3.11 Triangulation, step 11

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{13} from all other polynomials by reducing them with polynomial p_{12} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= x_8x_1 \\
p_9 &= (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4 \\
p_{10} &= u_6x_{11} - x_{10}x_2 \\
p_{11} &= u_8x_{11} - x_{10}x_4 + x_{10}x_1 - u_8x_1 \\
p_{12} &= -u_7x_{12}x_5 + u_9x_{12}x_3 + u_7x_{12}x_1 - u_9u_7x_1 \\
p_{13} &= u_7x_{13} - x_{12}x_3 \\
p_{14} &= u_6x_{14}x_4 - u_8x_{14}x_2 + u_8x_{14}x_1 - u_8u_6x_1 \\
p_{15} &= u_8x_{15} - x_{14}x_4 \\
p_{16} &= u_7x_{16}x_5 - u_9x_{16}x_3 + u_9x_{16}x_1 - u_9u_7x_1 \\
p_{17} &= u_9x_{17} - x_{16}x_5 \\
p_{18} &= -x_{18}x_{15}x_8 + x_{18}x_{15}x_6 + x_{18}x_{14}x_9 - x_{18}x_{14}x_7 + x_{18}x_{13}x_8 \\
&\quad - x_{18}x_{13}x_6 - x_{18}x_{12}x_9 + x_{18}x_{12}x_7 + x_{15}x_{12}x_8 - x_{15}x_{12}x_6 \\
&\quad - x_{14}x_{13}x_8 + x_{14}x_{13}x_6 - x_{14}x_9x_6 + x_{14}x_8x_7 + x_{12}x_9x_6 \\
&\quad - x_{12}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7 \\
p_{20} &= -x_{20}x_{17}x_{10} + x_{20}x_{17}x_8 + x_{20}x_{16}x_{11} - x_{20}x_{16}x_9 + x_{20}x_{15}x_{10} \\
&\quad - x_{20}x_{15}x_8 - x_{20}x_{14}x_{11} + x_{20}x_{14}x_9 + x_{17}x_{14}x_{10} - x_{17}x_{14}x_8 \\
&\quad - x_{16}x_{15}x_{10} + x_{16}x_{15}x_8 - x_{16}x_{11}x_8 + x_{16}x_{10}x_9 + x_{14}x_{11}x_8 \\
&\quad - x_{14}x_{10}x_9 \\
p_{21} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9 \\
p_{22} &= -x_{22}x_{17}x_{12} + x_{22}x_{17}x_{10} + x_{22}x_{16}x_{13} - x_{22}x_{16}x_{11} \\
&\quad - x_{22}x_{13}x_6 + x_{22}x_{12}x_7 + x_{22}x_{11}x_6 - x_{22}x_{10}x_7 + x_{17}x_{12}x_6 \\
&\quad - x_{17}x_{10}x_6 - x_{16}x_{13}x_{10} + x_{16}x_{12}x_{11} - x_{16}x_{12}x_7 + x_{16}x_{10}x_7 + \\
&\quad x_{13}x_{10}x_6 - x_{12}x_{11}x_6 \\
p_{23} &= x_{23}x_{12} - x_{23}x_{10} - x_{22}x_{13} + x_{22}x_{11} + x_{13}x_{10} - x_{12}x_{11}
\end{aligned}$$

3.12 Triangulation, step 12

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{12} . No reduction needed.

The triangular system has not been changed.

3.13 Triangulation, step 13

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{11} from all other polynomials by reducing them with polynomial p_{10} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= x_8x_1 \\
p_9 &= (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4 \\
p_{10} &= -u_6x_{10}x_4 + u_8x_{10}x_2 + u_6x_{10}x_1 - u_8u_6x_1 \\
p_{11} &= u_6x_{11} - x_{10}x_2 \\
p_{12} &= -u_7x_{12}x_5 + u_9x_{12}x_3 + u_7x_{12}x_1 - u_9u_7x_1 \\
p_{13} &= u_7x_{13} - x_{12}x_3 \\
p_{14} &= u_6x_{14}x_4 - u_8x_{14}x_2 + u_8x_{14}x_1 - u_8u_6x_1 \\
p_{15} &= u_8x_{15} - x_{14}x_4 \\
p_{16} &= u_7x_{16}x_5 - u_9x_{16}x_3 + u_9x_{16}x_1 - u_9u_7x_1 \\
p_{17} &= u_9x_{17} - x_{16}x_5 \\
p_{18} &= -x_{18}x_{15}x_8 + x_{18}x_{15}x_6 + x_{18}x_{14}x_9 - x_{18}x_{14}x_7 + x_{18}x_{13}x_8 \\
&\quad -x_{18}x_{13}x_6 - x_{18}x_{12}x_9 + x_{18}x_{12}x_7 + x_{15}x_{12}x_8 - x_{15}x_{12}x_6 \\
&\quad -x_{14}x_{13}x_8 + x_{14}x_{13}x_6 - x_{14}x_9x_6 + x_{14}x_8x_7 + x_{12}x_9x_6 \\
&\quad -x_{12}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7 \\
p_{20} &= -x_{20}x_{17}x_{10} + x_{20}x_{17}x_8 + x_{20}x_{16}x_{11} - x_{20}x_{16}x_9 + x_{20}x_{15}x_{10}
\end{aligned}$$

$$\begin{aligned}
& -x_{20}x_{15}x_8 - x_{20}x_{14}x_{11} + x_{20}x_{14}x_9 + x_{17}x_{14}x_{10} - x_{17}x_{14}x_8 \\
& -x_{16}x_{15}x_{10} + x_{16}x_{15}x_8 - x_{16}x_{11}x_8 + x_{16}x_{10}x_9 + x_{14}x_{11}x_8 \\
& -x_{14}x_{10}x_9 \\
p_{21} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9 \\
p_{22} &= -x_{22}x_{17}x_{12} + x_{22}x_{17}x_{10} + x_{22}x_{16}x_{13} - x_{22}x_{16}x_{11} \\
& -x_{22}x_{13}x_6 + x_{22}x_{12}x_7 + x_{22}x_{11}x_6 - x_{22}x_{10}x_7 + x_{17}x_{12}x_6 \\
& -x_{17}x_{10}x_6 - x_{16}x_{13}x_{10} + x_{16}x_{12}x_{11} - x_{16}x_{12}x_7 + x_{16}x_{10}x_7 + \\
& x_{13}x_{10}x_6 - x_{12}x_{11}x_6 \\
p_{23} &= x_{23}x_{12} - x_{23}x_{10} - x_{22}x_{13} + x_{22}x_{11} + x_{13}x_{10} - x_{12}x_{11}
\end{aligned}$$

3.14 Triangulation, step 14

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10} . No reduction needed.

The triangular system has not been changed.

3.15 Triangulation, step 15

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_9 . No reduction needed.

The triangular system has not been changed.

3.16 Triangulation, step 16

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8 . No reduction needed.

The triangular system has not been changed.

3.17 Triangulation, step 17

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7 . No reduction needed.

The triangular system has not been changed.

3.18 Triangulation, step 18

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6 . No reduction needed.

The triangular system has not been changed.

3.19 Triangulation, step 19

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5 . No reduction needed.

The triangular system has not been changed.

3.20 Triangulation, step 20

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4 . No reduction needed.

The triangular system has not been changed.

3.21 Triangulation, step 21

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3 . No reduction needed.

The triangular system has not been changed.

3.22 Triangulation, step 22

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2 . No reduction needed.

The triangular system has not been changed.

3.23 Triangulation, step 23

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1 . No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned} p_1 &= u_3x_1^2 + u_5x_1 \\ p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\ p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\ p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\ p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\ p_6 &= x_6x_1 \\ p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\ p_8 &= x_8x_1 \\ p_9 &= (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4 \\ p_{10} &= -u_6x_{10}x_4 + u_8x_{10}x_2 + u_6x_{10}x_1 - u_8u_6x_1 \\ p_{11} &= u_6x_{11} - x_{10}x_2 \\ p_{12} &= -u_7x_{12}x_5 + u_9x_{12}x_3 + u_7x_{12}x_1 - u_9u_7x_1 \end{aligned}$$

$$\begin{aligned}
p_{13} &= u_7x_{13} - x_{12}x_3 \\
p_{14} &= u_6x_{14}x_4 - u_8x_{14}x_2 + u_8x_{14}x_1 - u_8u_6x_1 \\
p_{15} &= u_8x_{15} - x_{14}x_4 \\
p_{16} &= u_7x_{16}x_5 - u_9x_{16}x_3 + u_9x_{16}x_1 - u_9u_7x_1 \\
p_{17} &= u_9x_{17} - x_{16}x_5 \\
p_{18} &= -x_{18}x_{15}x_8 + x_{18}x_{15}x_6 + x_{18}x_{14}x_9 - x_{18}x_{14}x_7 + x_{18}x_{13}x_8 \\
&\quad -x_{18}x_{13}x_6 - x_{18}x_{12}x_9 + x_{18}x_{12}x_7 + x_{15}x_{12}x_8 - x_{15}x_{12}x_6 \\
&\quad -x_{14}x_{13}x_8 + x_{14}x_{13}x_6 - x_{14}x_9x_6 + x_{14}x_8x_7 + x_{12}x_9x_6 \\
&\quad -x_{12}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7 \\
p_{20} &= -x_{20}x_{17}x_{10} + x_{20}x_{17}x_8 + x_{20}x_{16}x_{11} - x_{20}x_{16}x_9 + x_{20}x_{15}x_{10} \\
&\quad -x_{20}x_{15}x_8 - x_{20}x_{14}x_{11} + x_{20}x_{14}x_9 + x_{17}x_{14}x_{10} - x_{17}x_{14}x_8 \\
&\quad -x_{16}x_{15}x_{10} + x_{16}x_{15}x_8 - x_{16}x_{11}x_8 + x_{16}x_{10}x_9 + x_{14}x_{11}x_8 \\
&\quad -x_{14}x_{10}x_9 \\
p_{21} &= x_{21}x_{10} - x_{21}x_8 - x_{20}x_{11} + x_{20}x_9 + x_{11}x_8 - x_{10}x_9 \\
p_{22} &= -x_{22}x_{17}x_{12} + x_{22}x_{17}x_{10} + x_{22}x_{16}x_{13} - x_{22}x_{16}x_{11} \\
&\quad -x_{22}x_{13}x_6 + x_{22}x_{12}x_7 + x_{22}x_{11}x_6 - x_{22}x_{10}x_7 + x_{17}x_{12}x_6 \\
&\quad -x_{17}x_{10}x_6 - x_{16}x_{13}x_{10} + x_{16}x_{12}x_{11} - x_{16}x_{12}x_7 + x_{16}x_{10}x_7 + \\
&\quad x_{13}x_{10}x_6 - x_{12}x_{11}x_6 \\
p_{23} &= x_{23}x_{12} - x_{23}x_{10} - x_{22}x_{13} + x_{22}x_{11} + x_{13}x_{10} - x_{12}x_{11}
\end{aligned}$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 011

Calculating final remainder of the conclusion:

$$g = x_{23}x_{20} - x_{23}x_{18} - x_{22}x_{21} + x_{22}x_{19} + x_{21}x_{18} - x_{20}x_{19}$$

with respect to the triangular system.

1. Pseudo remainder with p_{23} over variable x_{23} :

$$\begin{aligned}
g &= -x_{22}x_{21}x_{12} + x_{22}x_{21}x_{10} + x_{22}x_{20}x_{13} - x_{22}x_{20}x_{11} + \\
&\quad x_{22}x_{19}x_{12} - x_{22}x_{19}x_{10} - x_{22}x_{18}x_{13} + x_{22}x_{18}x_{11} + \\
&\quad x_{21}x_{18}x_{12} - x_{21}x_{18}x_{10} - x_{20}x_{19}x_{12} + x_{20}x_{19}x_{10} \\
&\quad -x_{20}x_{13}x_{10} + x_{20}x_{12}x_{11} + x_{18}x_{13}x_{10} - x_{18}x_{12}x_{11}
\end{aligned}$$

2. Pseudo remainder with p_{22} over variable x_{22} :

Polynomial too big for output (text size is 2762 characters, number of terms is 88)

3. Pseudo remainder with p_{21} over variable x_{21} :
Polynomial too big for output (text size is 7948 characters, number of terms is 220)
4. Pseudo remainder with p_{20} over variable x_{20} :
Polynomial too big for output (number of terms is 1050)
5. Pseudo remainder with p_{19} over variable x_{19} :
Polynomial too big for output (number of terms is 1864)
6. Pseudo remainder with p_{18} over variable x_{18} :
Polynomial too big for output (number of terms is 4316)
7. Pseudo remainder with p_{17} over variable x_{17} :
Polynomial too big for output (number of terms is 4316)
8. Pseudo remainder with p_{16} over variable x_{16} :
Polynomial too big for output (number of terms is 11012)
9. Pseudo remainder with p_{15} over variable x_{15} :
Polynomial too big for output (number of terms is 11012)
10. Pseudo remainder with p_{14} over variable x_{14} :
Polynomial too big for output (number of terms is 19438)
11. Pseudo remainder with p_{13} over variable x_{13} :
Polynomial too big for output (number of terms is 18221)
12. Pseudo remainder with p_{12} over variable x_{12} :
Polynomial too big for output (number of terms is 22631)
13. Pseudo remainder with p_{11} over variable x_{11} :
Polynomial too big for output (number of terms is 19225)
14. Pseudo remainder with p_{10} over variable x_{10} :
Polynomial too big for output (number of terms is 16224)
15. Pseudo remainder with p_9 over variable x_9 :
Polynomial too big for output (number of terms is 17036)

16. Pseudo remainder with p_8 over variable x_8 :
Polynomial too big for output (number of terms is 1712)
17. Pseudo remainder with p_7 over variable x_7 :
Polynomial too big for output (number of terms is 1456)
18. Pseudo remainder with p_6 over variable x_6 :

$$g = 0$$

19. Pseudo remainder with p_5 over variable x_5 :

$$g = 0$$

20. Pseudo remainder with p_4 over variable x_4 :

$$g = 0$$

21. Pseudo remainder with p_3 over variable x_3 :

$$g = 0$$

22. Pseudo remainder with p_2 over variable x_2 :

$$g = 0$$

23. Pseudo remainder with p_1 over variable x_1 :

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 27515 terms.

Time Complexity: Time spent by the prover is 48.5 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Conic Section is not in degenerate form
- Points A1 and A0 are not identical
- Points A2 and A3 are not identical
- Points A4 and A5 are not identical
- Line through points A2 and A0 is not parallel with line through points A1 and A4
- Points A2 and A1 are not identical
- Line through points A1 and A5 is not parallel with line through points A3 and A0
- Points A1 and A3 are not identical
- Line through points A2 and A1 is not parallel with line through points A4 and A0
- Points A1 and A4 are not identical
- Line through points A1 and A3 is not parallel with line through points A5 and A0
- Points A1 and A5 are not identical
- Line through points D and E is not parallel with line through points A and B
-

$$p = x_8 - x_6$$

- Line through points E and F is not parallel with line through points B and C
-

$$p = x_{10} - x_8$$

- Line through points D and C is not parallel with line through points F and A
- Segment with endpoints D and A1 is not collinear and congruent with segment with endpoints C and A0

Time spent for processing NDG Conditions

- 2.444 seconds