OpenGeoProver Output for conjecture "Chou 014 (Kirkman's Theorem)"

Wu's method used

February 14, 2012

1 Validation of Construction Protocol

Construction steps:

- General Conic Section c
- Random point A from general conic c
- Random point B from general conic c
- Random point C from general conic c
- Random point D from general conic c
- Random point E from general conic c
- Random point F from general conic c
- Line AB through two points A and B
- Line DC through two points D and C
- Intersection point P1 of point sets AB and DC
- Line AE through two points A and E
- Line DF through two points D and F
- Intersection point Q1 of point sets AE and DF
- Line P1Q1 through two points P1 and Q1
- Line FE through two points F and E
- Intersection point P2 of point sets DC and FE
- Line DB through two points D and B
- Intersection point Q2 of point sets DB and AE
- Line P2Q2 through two points P2 and Q2
- Intersection point P3 of point sets FE and AB

- Line EC through two points E and C
- Line BD through two points B and D
- Intersection point Q3 of point sets EC and BD
- Line P3Q3 through two points P3 and Q3

Theorem statement:

- Lines P1Q1, P2Q2, P3Q3 are concurrent

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of general conic section c:

List of parametric points

- Point Ac has been assigned following coordinates: $(u_1, 0)$
- Point Bc has been assigned following coordinates: $(u_2, 0)$
- Point Cc has been assigned following coordinates: $(u_3, 0)$
- Point Dc has been assigned following coordinates: $(u_4, 0)$
- Point Ec has been assigned following coordinates: $(u_5, 0)$
- Condition for point X(x_1, x_2) to belong to this conic section is following equation:

$$p = u_3x_2^2 + u_2x_2x_1 + u_5x_2 + u_1x_1^2 + u_4x_1$$

2.2 Transformation of point A:

- Point A has been assigned following coordinates: $(0, 0)$

2.3 Transformation of point B:

- Point B has been assigned following coordinates: $(0, x_1)$
- Polynomial that point B has to satisfy is:

$$p = u_3x_1^2 + u_5x_1$$

- Processing of polynomial

$$p = u_3x_1^2 + u_5x_1$$

Info: Polynomial

$$p = u_3x_1^2 + u_5x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.4 Transformation of point C:

- Point C has been assigned following coordinates: (u_6, x_2)
- Polynomial that point C has to satisfy is:

$$p = u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)$$

- Processing of polynomial

$$p = u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)$$

Info: Polynomial

$$p = u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of point D:

- Point D has been assigned following coordinates: (u_7, x_3)
- Polynomial that point D has to satisfy is:

$$p = u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4)$$

- Processing of polynomial

$$p = u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4)$$

Info: Polynomial

$$p = u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point E:

- Point E has been assigned following coordinates: (u_8, x_4)
- Polynomial that point E has to satisfy is:

$$p = u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)$$

- Processing of polynomial

$$p = u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)$$

Info: Polynomial

$$p = u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point F:

- Point F has been assigned following coordinates: (u_9, x_5)
- Polynomial that point F has to satisfy is:

$$p = u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4)$$

- Processing of polynomial

$$p = u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4)$$

Info: Polynomial

$$p = u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.8 Transformation of point P1:

- Point P1 has been assigned following coordinates: (x_6, x_7)
- Polynomial that point P1 has to satisfy is:

$$p = x_6x_1$$

- Processing of polynomial

$$p = x_6x_1$$

Info: Polynomial

$$p = x_6x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point P1 has to satisfy is:

$$p = (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2$$

- Processing of polynomial

$$p = (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2$$

Info: Polynomial

$$p = (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.9 Transformation of point Q1:

- Point Q1 has been assigned following coordinates: (x_8, x_9)
- Polynomial that point Q1 has to satisfy is:

$$p = u_8x_9 - x_8x_4$$

- Processing of polynomial

$$p = u_8x_9 - x_8x_4$$

Info: Polynomial

$$p = u_8x_9 - x_8x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point Q1 has to satisfy is:

$$p = (u_9 - u_7)x_9 - x_8x_5 + x_8x_3 + u_7x_5 - u_9x_3$$

- Processing of polynomial

$$p = (u_9 - u_7)x_9 - x_8x_5 + x_8x_3 + u_7x_5 - u_9x_3$$

Info: Polynomial

$$p = (u_9 - u_7)x_9 - x_8x_5 + x_8x_3 + u_7x_5 - u_9x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.10 Transformation of point P2:

- Point P2 has been assigned following coordinates: (x_{10}, x_{11})
- Polynomial that point P2 has to satisfy is:

$$p = (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2$$

- Processing of polynomial

$$p = (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2$$

Info: Polynomial

$$p = (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point P2 has to satisfy is:

$$p = (u_9 - u_8)x_{11} - x_{10}x_5 + x_{10}x_4 + u_8x_5 - u_9x_4$$

- Processing of polynomial

$$p = (u_9 - u_8)x_{11} - x_{10}x_5 + x_{10}x_4 + u_8x_5 - u_9x_4$$

Info: Polynomial

$$p = (u_9 - u_8)x_{11} - x_{10}x_5 + x_{10}x_4 + u_8x_5 - u_9x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.11 Transformation of point Q2:

- Point Q2 has been assigned following coordinates: (x_{12}, x_{13})
- Polynomial that point Q2 has to satisfy is:

$$p = u_7x_{13} - x_{12}x_3 + x_{12}x_1 - u_7x_1$$

- Processing of polynomial

$$p = u_7x_{13} - x_{12}x_3 + x_{12}x_1 - u_7x_1$$

Info: Polynomial

$$p = u_7x_{13} - x_{12}x_3 + x_{12}x_1 - u_7x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point Q2 has to satisfy is:

$$p = u_8 x_{13} - x_{12} x_4$$

- Processing of polynomial

$$p = u_8 x_{13} - x_{12} x_4$$

Info: Polynomial

$$p = u_8 x_{13} - x_{12} x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.12 Transformation of point P3:

- Point P3 has been assigned following coordinates: (x_{14}, x_{15})
- Polynomial that point P3 has to satisfy is:

$$p = (u_9 - u_8) x_{15} - x_{14} x_5 + x_{14} x_4 + u_8 x_5 - u_9 x_4$$

- Processing of polynomial

$$p = (u_9 - u_8) x_{15} - x_{14} x_5 + x_{14} x_4 + u_8 x_5 - u_9 x_4$$

Info: Polynomial

$$p = (u_9 - u_8) x_{15} - x_{14} x_5 + x_{14} x_4 + u_8 x_5 - u_9 x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point P3 has to satisfy is:

$$p = x_{14} x_1$$

- Processing of polynomial

$$p = x_{14} x_1$$

Info: Polynomial

$$p = x_{14} x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.13 Transformation of point Q3:

- Point Q3 has been assigned following coordinates: (x_{16}, x_{17})
- Polynomial that point Q3 has to satisfy is:

$$p = (u_8 - u_6)x_{17} - x_{16}x_4 + x_{16}x_2 + u_6x_4 - u_8x_2$$

- Processing of polynomial

$$p = (u_8 - u_6)x_{17} - x_{16}x_4 + x_{16}x_2 + u_6x_4 - u_8x_2$$

Info: Polynomial

$$p = (u_8 - u_6)x_{17} - x_{16}x_4 + x_{16}x_2 + u_6x_4 - u_8x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point Q3 has to satisfy is:

$$p = u_7x_{17} - x_{16}x_3 + x_{16}x_1 - u_7x_1$$

- Processing of polynomial

$$p = u_7x_{17} - x_{16}x_3 + x_{16}x_1 - u_7x_1$$

Info: Polynomial

$$p = u_7x_{17} - x_{16}x_3 + x_{16}x_1 - u_7x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

2.14 Transformation of point intersectPoint-P1Q1.P2Q2:

- Point intersectPoint-P1Q1.P2Q2 has been assigned following coordinates: (x_{18}, x_{19})
- Polynomial that point intersectPoint-P1Q1.P2Q2 has to satisfy is:

$$p = x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7$$

- Processing of polynomial

$$p = x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7$$

Info: Polynomial

$$p = x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point intersectPoint-P1Q1.P2Q2 has to satisfy is:

$$p = x_{19}x_{12} - x_{19}x_{10} - x_{18}x_{13} + x_{18}x_{11} + x_{13}x_{10} - x_{12}x_{11}$$

- Processing of polynomial

$$p = x_{19}x_{12} - x_{19}x_{10} - x_{18}x_{13} + x_{18}x_{11} + x_{13}x_{10} - x_{12}x_{11}$$

Info: Polynomial

$$p = x_{19}x_{12} - x_{19}x_{10} - x_{18}x_{13} + x_{18}x_{11} + x_{13}x_{10} - x_{12}x_{11}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial for theorem statement:

$$p = x_{19}x_{16} - x_{19}x_{14} - x_{18}x_{17} + x_{18}x_{15} + x_{17}x_{14} - x_{16}x_{15}$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.234 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= u_8x_9 - x_8x_4 \\
p_9 &= (u_9 - u_7)x_9 - x_8x_5 + x_8x_3 + u_7x_5 - u_9x_3 \\
p_{10} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2 \\
p_{11} &= (u_9 - u_8)x_{11} - x_{10}x_5 + x_{10}x_4 + u_8x_5 - u_9x_4 \\
p_{12} &= u_7x_{13} - x_{12}x_3 + x_{12}x_1 - u_7x_1 \\
p_{13} &= u_8x_{13} - x_{12}x_4 \\
p_{14} &= (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4 \\
p_{15} &= x_{14}x_1
\end{aligned}$$

$$
\begin{aligned}
p_{16} &= (u_8 - u_6)x_{17} - x_{16}x_4 + x_{16}x_2 + u_6x_4 - u_8x_2 \\
p_{17} &= u_7x_{17} - x_{16}x_3 + x_{16}x_1 - u_7x_1 \\
p_{18} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7 \\
p_{19} &= x_{19}x_{12} - x_{19}x_{10} - x_{18}x_{13} + x_{18}x_{11} + x_{13}x_{10} - x_{12}x_{11}
\end{aligned}
$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 19.

Variable x_{19} selected: The number of polynomials with this variable, with indexes from 1 to 19, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{19} from all other polynomials by reducing them with polynomial p_{18} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= u_8x_9 - x_8x_4 \\
p_9 &= (u_9 - u_7)x_9 - x_8x_5 + x_8x_3 + u_7x_5 - u_9x_3 \\
p_{10} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2 \\
p_{11} &= (u_9 - u_8)x_{11} - x_{10}x_5 + x_{10}x_4 + u_8x_5 - u_9x_4 \\
p_{12} &= u_7x_{13} - x_{12}x_3 + x_{12}x_1 - u_7x_1 \\
p_{13} &= u_8x_{13} - x_{12}x_4 \\
p_{14} &= (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4 \\
p_{15} &= x_{14}x_1 \\
p_{16} &= (u_8 - u_6)x_{17} - x_{16}x_4 + x_{16}x_2 + u_6x_4 - u_8x_2 \\
p_{17} &= u_7x_{17} - x_{16}x_3 + x_{16}x_1 - u_7x_1 \\
p_{18} &= -x_{18}x_{13}x_8 + x_{18}x_{13}x_6 + x_{18}x_{12}x_9 - x_{18}x_{12}x_7 + x_{18}x_{11}x_8 \\
&\quad -x_{18}x_{11}x_6 - x_{18}x_{10}x_9 + x_{18}x_{10}x_7 + x_{13}x_{10}x_8 - x_{13}x_{10}x_6 \\
&\quad -x_{12}x_{11}x_8 + x_{12}x_{11}x_6 - x_{12}x_9x_6 + x_{12}x_8x_7 + x_{10}x_9x_6 \\
&\quad -x_{10}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7
\end{aligned}
$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{18}. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{17} from all other polynomials by reducing them with polynomial p_{16} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= u_8x_9 - x_8x_4 \\
p_9 &= (u_9 - u_7)x_9 - x_8x_5 + x_8x_3 + u_7x_5 - u_9x_3 \\
p_{10} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2 \\
p_{11} &= (u_9 - u_8)x_{11} - x_{10}x_5 + x_{10}x_4 + u_8x_5 - u_9x_4 \\
p_{12} &= u_7x_{13} - x_{12}x_3 + x_{12}x_1 - u_7x_1 \\
p_{13} &= u_8x_{13} - x_{12}x_4 \\
p_{14} &= (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4 \\
p_{15} &= x_{14}x_1 \\
p_{16} &= u_7x_{16}x_4 + (-u_8 + u_6)x_{16}x_3 - u_7x_{16}x_2 + (u_8 - u_6)x_{16}x_1 \\
&\quad -u_7u_6x_4 + u_8u_7x_2 + (-u_8u_7 + u_7u_6)x_1 \\
p_{17} &= (u_8 - u_6)x_{17} - x_{16}x_4 + x_{16}x_2 + u_6x_4 - u_8x_2 \\
p_{18} &= -x_{18}x_{13}x_8 + x_{18}x_{13}x_6 + x_{18}x_{12}x_9 - x_{18}x_{12}x_7 + x_{18}x_{11}x_8 \\
&\quad -x_{18}x_{11}x_6 - x_{18}x_{10}x_9 + x_{18}x_{10}x_7 + x_{13}x_{10}x_8 - x_{13}x_{10}x_6 \\
&\quad -x_{12}x_{11}x_8 + x_{12}x_{11}x_6 - x_{12}x_9x_6 + x_{12}x_8x_7 + x_{10}x_9x_6 \\
&\quad -x_{10}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7
\end{aligned}
$$

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{16}. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14}. No reduction needed.

The triangular system has not been changed.

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14}. No reduction needed.

The triangular system has not been changed.

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{13} from all other polynomials by reducing them with polynomial p_{12} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= u_8x_9 - x_8x_4 \\
p_9 &= (u_9 - u_7)x_9 - x_8x_5 + x_8x_3 + u_7x_5 - u_9x_3 \\
p_{10} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2 \\
p_{11} &= (u_9 - u_8)x_{11} - x_{10}x_5 + x_{10}x_4 + u_8x_5 - u_9x_4 \\
p_{12} &= -u_7x_{12}x_4 + u_8x_{12}x_3 - u_8x_{12}x_1 + u_8u_7x_1 \\
p_{13} &= u_7x_{13} - x_{12}x_3 + x_{12}x_1 - u_7x_1 \\
p_{14} &= x_{14}x_1 \\
p_{15} &= (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4 \\
p_{16} &= u_7x_{16}x_4 + (-u_8 + u_6)x_{16}x_3 - u_7x_{16}x_2 + (u_8 - u_6)x_{16}x_1 \\
&\quad -u_7u_6x_4 + u_8u_7x_2 + (-u_8u_7 + u_7u_6)x_1 \\
p_{17} &= (u_8 - u_6)x_{17} - x_{16}x_4 + x_{16}x_2 + u_6x_4 - u_8x_2 \\
p_{18} &= -x_{18}x_{13}x_8 + x_{18}x_{13}x_6 + x_{18}x_{12}x_9 - x_{18}x_{12}x_7 + x_{18}x_{11}x_8 \\
&\quad -x_{18}x_{11}x_6 - x_{18}x_{10}x_9 + x_{18}x_{10}x_7 + x_{13}x_{10}x_8 - x_{13}x_{10}x_6 \\
&\quad -x_{12}x_{11}x_8 + x_{12}x_{11}x_6 - x_{12}x_9x_6 + x_{12}x_8x_7 + x_{10}x_9x_6 \\
&\quad -x_{10}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7
\end{aligned}
$$

3.8 Triangulation, step 8

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{12}. No reduction needed.

The triangular system has not been changed.

3.9 Triangulation, step 9

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{11} from all other polynomials by reducing them with polynomial p_{10} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= u_8x_9 - x_8x_4 \\
p_9 &= (u_9 - u_7)x_9 - x_8x_5 + x_8x_3 + u_7x_5 - u_9x_3 \\
p_{10} &= (-u_7 + u_6)x_{10}x_5 + (u_7 - u_6)x_{10}x_4 + (u_9 - u_8)x_{10}x_3 + \\
&\quad (-u_9 + u_8)x_{10}x_2 + (u_8u_7 - u_8u_6)x_5 + (-u_9u_7 + u_9u_6)x_4 + \\
&\quad (-u_9u_6 + u_8u_6)x_3 + (u_9u_7 - u_8u_7)x_2 \\
p_{11} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2 \\
p_{12} &= -u_7x_{12}x_4 + u_8x_{12}x_3 - u_8x_{12}x_1 + u_8u_7x_1 \\
p_{13} &= u_7x_{13} - x_{12}x_3 + x_{12}x_1 - u_7x_1 \\
p_{14} &= x_{14}x_1 \\
p_{15} &= (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4 \\
p_{16} &= u_7x_{16}x_4 + (-u_8 + u_6)x_{16}x_3 - u_7x_{16}x_2 + (u_8 - u_6)x_{16}x_1 \\
&\quad -u_7u_6x_4 + u_8u_7x_2 + (-u_8u_7 + u_7u_6)x_1 \\
p_{17} &= (u_8 - u_6)x_{17} - x_{16}x_4 + x_{16}x_2 + u_6x_4 - u_8x_2 \\
p_{18} &= -x_{18}x_{13}x_8 + x_{18}x_{13}x_6 + x_{18}x_{12}x_9 - x_{18}x_{12}x_7 + x_{18}x_{11}x_8 \\
&\quad -x_{18}x_{11}x_6 - x_{18}x_{10}x_9 + x_{18}x_{10}x_7 + x_{13}x_{10}x_8 - x_{13}x_{10}x_6 \\
&\quad -x_{12}x_{11}x_8 + x_{12}x_{11}x_6 - x_{12}x_9x_6 + x_{12}x_8x_7 + x_{10}x_9x_6 \\
&\quad -x_{10}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7
\end{aligned}
$$

3.10 Triangulation, step 10

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10}. No reduction needed.

The triangular system has not been changed.

3.11 Triangulation, step 11

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_9 from all other polynomials by reducing them with polynomial p_8 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= -u_8x_8x_5 + (u_9 - u_7)x_8x_4 + u_8x_8x_3 + u_8u_7x_5 - u_9u_8x_3 \\
p_9 &= u_8x_9 - x_8x_4 \\
p_{10} &= (-u_7 + u_6)x_{10}x_5 + (u_7 - u_6)x_{10}x_4 + (u_9 - u_8)x_{10}x_3 + \\
&\quad (-u_9 + u_8)x_{10}x_2 + (u_8u_7 - u_8u_6)x_5 + (-u_9u_7 + u_9u_6)x_4 + \\
&\quad (-u_9u_6 + u_8u_6)x_3 + (u_9u_7 - u_8u_7)x_2 \\
p_{11} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2 \\
p_{12} &= -u_7x_{12}x_4 + u_8x_{12}x_3 - u_8x_{12}x_1 + u_8u_7x_1 \\
p_{13} &= u_7x_{13} - x_{12}x_3 + x_{12}x_1 - u_7x_1 \\
p_{14} &= x_{14}x_1 \\
p_{15} &= (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4 \\
p_{16} &= u_7x_{16}x_4 + (-u_8 + u_6)x_{16}x_3 - u_7x_{16}x_2 + (u_8 - u_6)x_{16}x_1 \\
&\quad -u_7u_6x_4 + u_8u_7x_2 + (-u_8u_7 + u_7u_6)x_1 \\
p_{17} &= (u_8 - u_6)x_{17} - x_{16}x_4 + x_{16}x_2 + u_6x_4 - u_8x_2 \\
p_{18} &= -x_{18}x_{13}x_8 + x_{18}x_{13}x_6 + x_{18}x_{12}x_9 - x_{18}x_{12}x_7 + x_{18}x_{11}x_8 \\
&\quad -x_{18}x_{11}x_6 - x_{18}x_{10}x_9 + x_{18}x_{10}x_7 + x_{13}x_{10}x_8 - x_{13}x_{10}x_6 \\
&\quad -x_{12}x_{11}x_8 + x_{12}x_{11}x_6 - x_{12}x_9x_6 + x_{12}x_8x_7 + x_{10}x_9x_6 \\
&\quad -x_{10}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7
\end{aligned}
$$

3.12 Triangulation, step 12

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8. No reduction needed.

The triangular system has not been changed.

3.13 Triangulation, step 13

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7. No reduction needed.

The triangular system has not been changed.

3.14 Triangulation, step 14

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

3.15 Triangulation, step 15

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

3.16 Triangulation, step 16

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

3.17 Triangulation, step 17

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.18 Triangulation, step 18

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.19 Triangulation, step 19

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_7 - u_6)x_7 - x_6x_3 + x_6x_2 + u_6x_3 - u_7x_2 \\
p_8 &= -u_8x_8x_5 + (u_9 - u_7)x_8x_4 + u_8x_8x_3 + u_8u_7x_5 - u_9u_8x_3 \\
p_9 &= u_8x_9 - x_8x_4 \\
p_{10} &= (-u_7 + u_6)x_{10}x_5 + (u_7 - u_6)x_{10}x_4 + (u_9 - u_8)x_{10}x_3 + \\
&\quad (-u_9 + u_8)x_{10}x_2 + (u_8u_7 - u_8u_6)x_5 + (-u_9u_7 + u_9u_6)x_4 + \\
&\quad (-u_9u_6 + u_8u_6)x_3 + (u_9u_7 - u_8u_7)x_2
\end{aligned}
$$

$$
\begin{aligned}
p_{11} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2 \\
p_{12} &= -u_7x_{12}x_4 + u_8x_{12}x_3 - u_8x_{12}x_1 + u_8u_7x_1 \\
p_{13} &= u_7x_{13} - x_{12}x_3 + x_{12}x_1 - u_7x_1 \\
p_{14} &= x_{14}x_1 \\
p_{15} &= (u_9 - u_8)x_{15} - x_{14}x_5 + x_{14}x_4 + u_8x_5 - u_9x_4 \\
p_{16} &= u_7x_{16}x_4 + (-u_8 + u_6)x_{16}x_3 - u_7x_{16}x_2 + (u_8 - u_6)x_{16}x_1 \\
&\quad -u_7u_6x_4 + u_8u_7x_2 + (-u_8u_7 + u_7u_6)x_1 \\
p_{17} &= (u_8 - u_6)x_{17} - x_{16}x_4 + x_{16}x_2 + u_6x_4 - u_8x_2 \\
p_{18} &= -x_{18}x_{13}x_8 + x_{18}x_{13}x_6 + x_{18}x_{12}x_9 - x_{18}x_{12}x_7 + x_{18}x_{11}x_8 \\
&\quad -x_{18}x_{11}x_6 - x_{18}x_{10}x_9 + x_{18}x_{10}x_7 + x_{13}x_{10}x_8 - x_{13}x_{10}x_6 \\
&\quad -x_{12}x_{11}x_8 + x_{12}x_{11}x_6 - x_{12}x_9x_6 + x_{12}x_8x_7 + x_{10}x_9x_6 \\
&\quad -x_{10}x_8x_7 \\
p_{19} &= x_{19}x_8 - x_{19}x_6 - x_{18}x_9 + x_{18}x_7 + x_9x_6 - x_8x_7
\end{aligned}
$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 014 (Kirkman's Theorem)

Calculating final remainder of the conclusion:

$$
g = x_{19}x_{16} - x_{19}x_{14} - x_{18}x_{17} + x_{18}x_{15} + x_{17}x_{14} - x_{16}x_{15}
$$

with respect to the triangular system.

1. Pseudo remainder with p_{19} over variable x_{19}:

$$
\begin{aligned}
g &= -x_{18}x_{17}x_8 + x_{18}x_{17}x_6 + x_{18}x_{16}x_9 - x_{18}x_{16}x_7 + x_{18}x_{15}x_8 \\
&\quad -x_{18}x_{15}x_6 - x_{18}x_{14}x_9 + x_{18}x_{14}x_7 + x_{17}x_{14}x_8 - x_{17}x_{14}x_6 \\
&\quad -x_{16}x_{15}x_8 + x_{16}x_{15}x_6 - x_{16}x_9x_6 + x_{16}x_8x_7 + x_{14}x_9x_6 \\
&\quad -x_{14}x_8x_7
\end{aligned}
$$

2. Pseudo remainder with p_{18} over variable x_{18}:

   Polynomial too big for output (text size is 2642 *characters, number of terms is* 88*)*

3. Pseudo remainder with p_{17} over variable x_{17}:

   Polynomial too big for output (text size is 6601 *characters, number of terms is* 172*)*

4. Pseudo remainder with p_{16} over variable x_{16}:

   Polynomial too big for output (number of terms is 322*)*

5. Pseudo remainder with p_{15} over variable x_{15}:

   Polynomial too big for output (number of terms is 574*)*

6. Pseudo remainder with p_{14} over variable x_{14}:

   Polynomial too big for output (number of terms is 272*)*

7. Pseudo remainder with p_{13} over variable x_{13}:

   Polynomial too big for output (number of terms is 383*)*

8. Pseudo remainder with p_{12} over variable x_{12}:

   Polynomial too big for output (number of terms is 352*)*

9. Pseudo remainder with p_{11} over variable x_{11}:

   Polynomial too big for output (number of terms is 496*)*

10. Pseudo remainder with p_{10} over variable x_{10}:

    Polynomial too big for output (number of terms is 515*)*

11. Pseudo remainder with p_9 over variable x_9:

    Polynomial too big for output (number of terms is 444*)*

12. Pseudo remainder with p_8 over variable x_8:

    Polynomial too big for output (number of terms is 649*)*

13. Pseudo remainder with p_7 over variable x_7:

    Polynomial too big for output (number of terms is 552*)*

14. Pseudo remainder with p_6 over variable x_6:

    Polynomial too big for output (text size is 147868 *characters, number of terms is* 174*)*

15. Pseudo remainder with p_5 over variable x_5:

    Polynomial too big for output (text size is 693336 *characters, number of terms is* 253*)*

16. Pseudo remainder with p_4 over variable x_4:

    Polynomial too big for output (text size is greater than 2000 *characters, number of terms is* 192*)*

17. Pseudo remainder with p_3 over variable x_3:

    Polynomial too big for output (text size is greater than 2000 *characters, number of terms is* 88*)*

18. Pseudo remainder with p_2 over variable x_2:

    Polynomial too big for output (text size is greater than 2000 *characters, number of terms is* 48*)*

19. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 649 terms.

Time Complexity: Time spent by the prover is 6.583 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Conic Section is not in degenerate form
- Points A and B are not identical
- Points D and C are not identical
- Line through points D and F is not parallel with line through points E and A
- Points E and A are not identical
- Line through points D and C is not parallel with line through points E and F
- Line through points D and B is not parallel with line through points E and A
- Points D and A are not identical
- Points E and F are not identical
- Line through points D and B is not parallel with line through points E and C
- Points E and C are not identical
- Line through points Q1 and P1 is not parallel with line through points Q2 and P2
- Segment with endpoints Q1 and P1 is not collinear and congruent with segment with endpoints A and B

Time spent for processing NDG Conditions

- 1.295 seconds