

OpenGeoProver Output for conjecture “Chou 017 (Gauss conic)”

Wu’s method used

February 14, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point A0
- Free point A1
- Free point A2
- Free point A3
- Free point A4
- Free point A5
- Line A0A1 through two points A0 and A1
- Line A3A4 through two points A3 and A4
- Intersection point X of point sets A0A1 and A3A4
- Line A1A2 through two points A1 and A2
- Line A4A5 through two points A4 and A5
- Intersection point Y of point sets A1A2 and A4A5
- Line A2A3 through two points A2 and A3
- Line A5A0 through two points A5 and A0
- Intersection point Z of point sets A2A3 and A5A0
- Intersection point B1 of point sets A2A3 and A0A1
- Intersection point B2 of point sets A4A5 and A0A1
- Intersection point B3 of point sets A1A2 and A5A0
- Intersection point B4 of point sets A3A4 and A5A0
- Midpoint M1 of segment A1A3

- Midpoint M2 of segment A2X
- Midpoint M3 of segment A1A5
- Midpoint M4 of segment A0Y
- Midpoint M5 of segment B1A4
- Midpoint M6 of segment B2A3
- Midpoint M7 of segment A2A5
- Midpoint M8 of segment ZY
- Midpoint M9 of segment A0A4
- Midpoint N1 of segment A5X
- Midpoint N2 of segment A3B3
- Midpoint N3 of segment A2B4
- Line M1M2 through two points M1 and M2
- Line M3M4 through two points M3 and M4
- Intersection point P of point sets M1M2 and M3M4
- Line M5M6 through two points M5 and M6
- Line M7M8 through two points M7 and M8
- Intersection point Q of point sets M5M6 and M7M8
- Line N2N3 through two points N2 and N3
- Line M9N1 through two points M9 and N1
- Intersection point R of point sets N2N3 and M9N1

Theorem statement:

- Points P, Q, R are collinear

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point A0:

- Point A0 has been assigned following coordinates: $(0, 0)$

2.2 Transformation of point A1:

- Point A1 has been assigned following coordinates: $(0, u_1)$

2.3 Transformation of point A2:

- Point A2 has been assigned following coordinates: (u_2, u_3)

2.4 Transformation of point A3:

- Point A3 has been assigned following coordinates: (u_4, u_5)

2.5 Transformation of point A4:

- Point A4 has been assigned following coordinates: (u_6, u_7)

2.6 Transformation of point A5:

- Point A5 has been assigned following coordinates: (u_8, u_9)

2.7 Transformation of point X:

- Point X has been assigned following coordinates: (x_1, x_2)
- Polynomial that point X has to satisfy is:

$$p = x_1$$

- Processing of polynomial

$$p = x_1$$

Info: Will try to rename X coordinate of point X

Info: Y coordinate of point X will be replaced by X coordinate

Info: X coordinate of point X renamed by zero

- Point X has been renamed. Point X has been assigned following coordinates: $(0, x_1)$
- Polynomial that point X has to satisfy is:

$$p = (u_6 - u_4)x_1 + (u_7u_4 - u_6u_5)$$

- Processing of polynomial

$$p = (u_6 - u_4)x_1 + (u_7u_4 - u_6u_5)$$

Info: Polynomial

$$p = (u_6 - u_4)x_1 + (u_7u_4 - u_6u_5)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.8 Transformation of point Y:

- Point Y has been assigned following coordinates: (x_2, x_3)
- Polynomial that point Y has to satisfy is:

$$p = u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1$$

- Processing of polynomial

$$p = u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1$$

Info: Polynomial

$$p = u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point Y has to satisfy is:

$$p = (u_8 - u_6)x_3 + (-u_9 + u_7)x_2 + (u_9u_6 - u_8u_7)$$

- Processing of polynomial

$$p = (u_8 - u_6)x_3 + (-u_9 + u_7)x_2 + (u_9u_6 - u_8u_7)$$

Info: Polynomial

$$p = (u_8 - u_6)x_3 + (-u_9 + u_7)x_2 + (u_9u_6 - u_8u_7)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.9 Transformation of point Z:

- Point Z has been assigned following coordinates: (x_4, x_5)
- Polynomial that point Z has to satisfy is:

$$p = (u_4 - u_2)x_5 + (-u_5 + u_3)x_4 + (u_5u_2 - u_4u_3)$$

- Processing of polynomial

$$p = (u_4 - u_2)x_5 + (-u_5 + u_3)x_4 + (u_5u_2 - u_4u_3)$$

Info: Polynomial

$$p = (u_4 - u_2)x_5 + (-u_5 + u_3)x_4 + (u_5u_2 - u_4u_3)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point Z has to satisfy is:

$$p = u_8x_5 - u_9x_4$$

- Processing of polynomial

$$p = u_8x_5 - u_9x_4$$

Info: Polynomial

$$p = u_8x_5 - u_9x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.10 Transformation of point B1:

- Point B1 has been assigned following coordinates: (x_6, x_7)
- Polynomial that point B1 has to satisfy is:

$$p = (u_4 - u_2)x_7 + (-u_5 + u_3)x_6 + (u_5u_2 - u_4u_3)$$

- Processing of polynomial

$$p = (u_4 - u_2)x_7 + (-u_5 + u_3)x_6 + (u_5u_2 - u_4u_3)$$

Info: Polynomial

$$p = (u_4 - u_2)x_7 + (-u_5 + u_3)x_6 + (u_5u_2 - u_4u_3)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point B1 has to satisfy is:

$$p = x_6$$

- Processing of polynomial

$$p = x_6$$

Info: Will try to rename X coordinate of point B1

Info: Y coordinate of point B1 will be replaced by X coordinate

Info: X coordinate of point B1 renamed by zero

- Point B1 has been renamed. Point B1 has been assigned following coordinates: $(0, x_6)$
- Repeating instantiation of first condition of this point, after its coordinate has been renamed

- Polynomial that point B1 has to satisfy is:

$$p = (u_4 - u_2)x_6 + (u_5u_2 - u_4u_3)$$

- Processing of polynomial

$$p = (u_4 - u_2)x_6 + (u_5u_2 - u_4u_3)$$

Info: Polynomial

$$p = (u_4 - u_2)x_6 + (u_5u_2 - u_4u_3)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.11 Transformation of point B2:

- Point B2 has been assigned following coordinates: (x_7, x_8)
- Polynomial that point B2 has to satisfy is:

$$p = (u_8 - u_6)x_8 + (-u_9 + u_7)x_7 + (u_9u_6 - u_8u_7)$$

- Processing of polynomial

$$p = (u_8 - u_6)x_8 + (-u_9 + u_7)x_7 + (u_9u_6 - u_8u_7)$$

Info: Polynomial

$$p = (u_8 - u_6)x_8 + (-u_9 + u_7)x_7 + (u_9u_6 - u_8u_7)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point B2 has to satisfy is:

$$p = x_7$$

- Processing of polynomial

$$p = x_7$$

Info: Will try to rename X coordinate of point B2

Info: Y coordinate of point B2 will be replaced by X coordinate

Info: X coordinate of point B2 renamed by zero

- Point B2 has been renamed. Point B2 has been assigned following coordinates: $(0, x_7)$
- Repeating instantiation of first condition of this point, after its coordinate has been renamed

- Polynomial that point B2 has to satisfy is:

$$p = (u_8 - u_6)x_7 + (u_9u_6 - u_8u_7)$$

- Processing of polynomial

$$p = (u_8 - u_6)x_7 + (u_9u_6 - u_8u_7)$$

Info: Polynomial

$$p = (u_8 - u_6)x_7 + (u_9u_6 - u_8u_7)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.12 Transformation of point B3:

- Point B3 has been assigned following coordinates: (x_8, x_9)
- Polynomial that point B3 has to satisfy is:

$$p = u_2x_9 + (-u_3 + u_1)x_8 - u_2u_1$$

- Processing of polynomial

$$p = u_2x_9 + (-u_3 + u_1)x_8 - u_2u_1$$

Info: Polynomial

$$p = u_2x_9 + (-u_3 + u_1)x_8 - u_2u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point B3 has to satisfy is:

$$p = u_8x_9 - u_9x_8$$

- Processing of polynomial

$$p = u_8x_9 - u_9x_8$$

Info: Polynomial

$$p = u_8x_9 - u_9x_8$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.13 Transformation of point B4:

- Point B4 has been assigned following coordinates: (x_{10}, x_{11})
- Polynomial that point B4 has to satisfy is:

$$p = (u_6 - u_4)x_{11} + (-u_7 + u_5)x_{10} + (u_7u_4 - u_6u_5)$$

- Processing of polynomial

$$p = (u_6 - u_4)x_{11} + (-u_7 + u_5)x_{10} + (u_7u_4 - u_6u_5)$$

Info: Polynomial

$$p = (u_6 - u_4)x_{11} + (-u_7 + u_5)x_{10} + (u_7u_4 - u_6u_5)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point B4 has to satisfy is:

$$p = u_8x_{11} - u_9x_{10}$$

- Processing of polynomial

$$p = u_8x_{11} - u_9x_{10}$$

Info: Polynomial

$$p = u_8x_{11} - u_9x_{10}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.14 Transformation of point M1:

- Point M1 has been assigned following coordinates: (x_{12}, x_{13})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{12} - 0.5u_4$$

Info: Polynomial

$$p = x_{12} - 0.5u_4$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{12} - 0.5u_4$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{13} + (-0.5u_5 - 0.5u_1)$$

Info: Polynomial

$$p = x_{13} + (-0.5u_5 - 0.5u_1)$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{13} + (-0.5u_5 - 0.5u_1)$$

is added to polynomial system

2.15 Transformation of point M2:

- Point M2 has been assigned following coordinates: (x_{14}, x_{15})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{14} - 0.5u_2$$

Info: Polynomial

$$p = x_{14} - 0.5u_2$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{14} - 0.5u_2$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{15} - 0.5x_1 - 0.5u_3$$

Info: Polynomial

$$p = x_{15} - 0.5x_1 - 0.5u_3$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{15} - 0.5x_1 - 0.5u_3$$

is added to polynomial system

2.16 Transformation of point M3:

- Point M3 has been assigned following coordinates: (x_{16}, x_{17})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{16} - 0.5u_8$$

Info: Polynomial

$$p = x_{16} - 0.5u_8$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{16} - 0.5u_8$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{17} + (-0.5u_9 - 0.5u_1)$$

Info: Polynomial

$$p = x_{17} + (-0.5u_9 - 0.5u_1)$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{17} + (-0.5u_9 - 0.5u_1)$$

is added to polynomial system

2.17 Transformation of point M4:

- Point M4 has been assigned following coordinates: (x_{18}, x_{19})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{18} - 0.5x_2$$

Info: Polynomial

$$p = x_{18} - 0.5x_2$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{18} - 0.5x_2$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{19} - 0.5x_3$$

Info: Polynomial

$$p = x_{19} - 0.5x_3$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{19} - 0.5x_3$$

is added to polynomial system

2.18 Transformation of point M5:

- Point M5 has been assigned following coordinates: (x_{20}, x_{21})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{20} - 0.5u_6$$

Info: Polynomial

$$p = x_{20} - 0.5u_6$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{20} - 0.5u_6$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{21} - 0.5x_6 - 0.5u_7$$

Info: Polynomial

$$p = x_{21} - 0.5x_6 - 0.5u_7$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{21} - 0.5x_6 - 0.5u_7$$

is added to polynomial system

2.19 Transformation of point M6:

- Point M6 has been assigned following coordinates: (x_{22}, x_{23})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{22} - 0.5u_4$$

Info: Polynomial

$$p = x_{22} - 0.5u_4$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{22} - 0.5u_4$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{23} - 0.5x_7 - 0.5u_5$$

Info: Polynomial

$$p = x_{23} - 0.5x_7 - 0.5u_5$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{23} - 0.5x_7 - 0.5u_5$$

is added to polynomial system

2.20 Transformation of point M7:

- Point M7 has been assigned following coordinates: (x_{24}, x_{25})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{24} + (-0.5u_8 - 0.5u_2)$$

Info: Polynomial

$$p = x_{24} + (-0.5u_8 - 0.5u_2)$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{24} + (-0.5u_8 - 0.5u_2)$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{25} + (-0.5u_9 - 0.5u_3)$$

Info: Polynomial

$$p = x_{25} + (-0.5u_9 - 0.5u_3)$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{25} + (-0.5u_9 - 0.5u_3)$$

is added to polynomial system

2.21 Transformation of point M8:

- Point M8 has been assigned following coordinates: (x_{26}, x_{27})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{26} - 0.5x_4 - 0.5x_2$$

Info: Polynomial

$$p = x_{26} - 0.5x_4 - 0.5x_2$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{26} - 0.5x_4 - 0.5x_2$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{27} - 0.5x_5 - 0.5x_3$$

Info: Polynomial

$$p = x_{27} - 0.5x_5 - 0.5x_3$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{27} - 0.5x_5 - 0.5x_3$$

is added to polynomial system

2.22 Transformation of point M9:

- Point M9 has been assigned following coordinates: (x_{28}, x_{29})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{28} - 0.5u_6$$

Info: Polynomial

$$p = x_{28} - 0.5u_6$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{28} - 0.5u_6$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{29} - 0.5u_7$$

Info: Polynomial

$$p = x_{29} - 0.5u_7$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{29} - 0.5u_7$$

is added to polynomial system

2.23 Transformation of point N1:

- Point N1 has been assigned following coordinates: (x_{30}, x_{31})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{30} - 0.5u_8$$

Info: Polynomial

$$p = x_{30} - 0.5u_8$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{30} - 0.5u_8$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{31} - 0.5x_1 - 0.5u_9$$

Info: Polynomial

$$p = x_{31} - 0.5x_1 - 0.5u_9$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{31} - 0.5x_1 - 0.5u_9$$

is added to polynomial system

2.24 Transformation of point N2:

- Point N2 has been assigned following coordinates: (x_{32}, x_{33})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{32} - 0.5x_8 - 0.5u_4$$

Info: Polynomial

$$p = x_{32} - 0.5x_8 - 0.5u_4$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{32} - 0.5x_8 - 0.5u_4$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{33} - 0.5x_9 - 0.5u_5$$

Info: Polynomial

$$p = x_{33} - 0.5x_9 - 0.5u_5$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{33} - 0.5x_9 - 0.5u_5$$

is added to polynomial system

2.25 Transformation of point N3:

- Point N3 has been assigned following coordinates: (x_{34}, x_{35})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{34} - 0.5x_{10} - 0.5u_2$$

Info: Polynomial

$$p = x_{34} - 0.5x_{10} - 0.5u_2$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{34} - 0.5x_{10} - 0.5u_2$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{35} - 0.5x_{11} - 0.5u_3$$

Info: Polynomial

$$p = x_{35} - 0.5x_{11} - 0.5u_3$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{35} - 0.5x_{11} - 0.5u_3$$

is added to polynomial system

2.26 Transformation of point P:

- Point P has been assigned following coordinates: (x_{36}, x_{37})
- Polynomial that point P has to satisfy is:

$$p = x_{37}x_{14} - x_{37}x_{12} - x_{36}x_{15} + x_{36}x_{13} + x_{15}x_{12} - x_{14}x_{13}$$

- Processing of polynomial

$$p = x_{37}x_{14} - x_{37}x_{12} - x_{36}x_{15} + x_{36}x_{13} + x_{15}x_{12} - x_{14}x_{13}$$

Info: Polynomial

$$p = x_{37}x_{14} - x_{37}x_{12} - x_{36}x_{15} + x_{36}x_{13} + x_{15}x_{12} - x_{14}x_{13}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point P has to satisfy is:

$$p = x_{37}x_{18} - x_{37}x_{16} - x_{36}x_{19} + x_{36}x_{17} + x_{19}x_{16} - x_{18}x_{17}$$

- Processing of polynomial

$$p = x_{37}x_{18} - x_{37}x_{16} - x_{36}x_{19} + x_{36}x_{17} + x_{19}x_{16} - x_{18}x_{17}$$

Info: Polynomial

$$p = x_{37}x_{18} - x_{37}x_{16} - x_{36}x_{19} + x_{36}x_{17} + x_{19}x_{16} - x_{18}x_{17}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.27 Transformation of point Q:

- Point Q has been assigned following coordinates: (x_{38}, x_{39})
- Polynomial that point Q has to satisfy is:

$$p = x_{39}x_{22} - x_{39}x_{20} - x_{38}x_{23} + x_{38}x_{21} + x_{23}x_{20} - x_{22}x_{21}$$

- Processing of polynomial

$$p = x_{39}x_{22} - x_{39}x_{20} - x_{38}x_{23} + x_{38}x_{21} + x_{23}x_{20} - x_{22}x_{21}$$

Info: Polynomial

$$p = x_{39}x_{22} - x_{39}x_{20} - x_{38}x_{23} + x_{38}x_{21} + x_{23}x_{20} - x_{22}x_{21}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point Q has to satisfy is:

$$p = x_{39}x_{26} - x_{39}x_{24} - x_{38}x_{27} + x_{38}x_{25} + x_{27}x_{24} - x_{26}x_{25}$$

- Processing of polynomial

$$p = x_{39}x_{26} - x_{39}x_{24} - x_{38}x_{27} + x_{38}x_{25} + x_{27}x_{24} - x_{26}x_{25}$$

Info: Polynomial

$$p = x_{39}x_{26} - x_{39}x_{24} - x_{38}x_{27} + x_{38}x_{25} + x_{27}x_{24} - x_{26}x_{25}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.28 Transformation of point R:

- Point R has been assigned following coordinates: (x_{40}, x_{41})
- Polynomial that point R has to satisfy is:

$$p = x_{41}x_{34} - x_{41}x_{32} - x_{40}x_{35} + x_{40}x_{33} + x_{35}x_{32} - x_{34}x_{33}$$

- Processing of polynomial

$$p = x_{41}x_{34} - x_{41}x_{32} - x_{40}x_{35} + x_{40}x_{33} + x_{35}x_{32} - x_{34}x_{33}$$

Info: Polynomial

$$p = x_{41}x_{34} - x_{41}x_{32} - x_{40}x_{35} + x_{40}x_{33} + x_{35}x_{32} - x_{34}x_{33}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point R has to satisfy is:

$$p = x_{41}x_{30} - x_{41}x_{28} - x_{40}x_{31} + x_{40}x_{29} + x_{31}x_{28} - x_{30}x_{29}$$

- Processing of polynomial

$$p = x_{41}x_{30} - x_{41}x_{28} - x_{40}x_{31} + x_{40}x_{29} + x_{31}x_{28} - x_{30}x_{29}$$

Info: Polynomial

$$p = x_{41}x_{30} - x_{41}x_{28} - x_{40}x_{31} + x_{40}x_{29} + x_{31}x_{28} - x_{30}x_{29}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_{41}x_{38} - x_{41}x_{36} - x_{40}x_{39} + x_{40}x_{37} + x_{39}x_{36} - x_{38}x_{37}$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.359 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= (u_6 - u_4)x_1 + (u_7u_4 - u_6u_5) \\
p_2 &= u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1 \\
p_3 &= (u_8 - u_6)x_3 + (-u_9 + u_7)x_2 + (u_9u_6 - u_8u_7) \\
p_4 &= (u_4 - u_2)x_5 + (-u_5 + u_3)x_4 + (u_5u_2 - u_4u_3) \\
p_5 &= u_8x_5 - u_9x_4 \\
p_6 &= (u_4 - u_2)x_6 + (u_5u_2 - u_4u_3) \\
p_7 &= (u_8 - u_6)x_7 + (u_9u_6 - u_8u_7) \\
p_8 &= u_2x_9 + (-u_3 + u_1)x_8 - u_2u_1 \\
p_9 &= u_8x_9 - u_9x_8 \\
p_{10} &= (u_6 - u_4)x_{11} + (-u_7 + u_5)x_{10} + (u_7u_4 - u_6u_5) \\
p_{11} &= u_8x_{11} - u_9x_{10} \\
p_{12} &= x_{12} - 0.5u_4 \\
p_{13} &= x_{13} + (-0.5u_5 - 0.5u_1) \\
p_{14} &= x_{14} - 0.5u_2 \\
p_{15} &= x_{15} - 0.5x_1 - 0.5u_3 \\
p_{16} &= x_{16} - 0.5u_8 \\
p_{17} &= x_{17} + (-0.5u_9 - 0.5u_1) \\
p_{18} &= x_{18} - 0.5x_2 \\
p_{19} &= x_{19} - 0.5x_3 \\
p_{20} &= x_{20} - 0.5u_6 \\
p_{21} &= x_{21} - 0.5x_6 - 0.5u_7 \\
p_{22} &= x_{22} - 0.5u_4 \\
p_{23} &= x_{23} - 0.5x_7 - 0.5u_5 \\
p_{24} &= x_{24} + (-0.5u_8 - 0.5u_2) \\
p_{25} &= x_{25} + (-0.5u_9 - 0.5u_3) \\
p_{26} &= x_{26} - 0.5x_4 - 0.5x_2 \\
p_{27} &= x_{27} - 0.5x_5 - 0.5x_3 \\
p_{28} &= x_{28} - 0.5u_6 \\
p_{29} &= x_{29} - 0.5u_7 \\
p_{30} &= x_{30} - 0.5u_8 \\
p_{31} &= x_{31} - 0.5x_1 - 0.5u_9 \\
p_{32} &= x_{32} - 0.5x_8 - 0.5u_4 \\
p_{33} &= x_{33} - 0.5x_9 - 0.5u_5 \\
p_{34} &= x_{34} - 0.5x_{10} - 0.5u_2 \\
p_{35} &= x_{35} - 0.5x_{11} - 0.5u_3 \\
p_{36} &= x_{37}x_{14} - x_{37}x_{12} - x_{36}x_{15} + x_{36}x_{13} + x_{15}x_{12} - x_{14}x_{13} \\
p_{37} &= x_{37}x_{18} - x_{37}x_{16} - x_{36}x_{19} + x_{36}x_{17} + x_{19}x_{16} - x_{18}x_{17} \\
p_{38} &= x_{39}x_{22} - x_{39}x_{20} - x_{38}x_{23} + x_{38}x_{21} + x_{23}x_{20} - x_{22}x_{21} \\
p_{39} &= x_{39}x_{26} - x_{39}x_{24} - x_{38}x_{27} + x_{38}x_{25} + x_{27}x_{24} - x_{26}x_{25}
\end{aligned}$$

$$\begin{aligned}
p_{40} &= x_{41}x_{34} - x_{41}x_{32} - x_{40}x_{35} + x_{40}x_{33} + x_{35}x_{32} - x_{34}x_{33} \\
p_{41} &= x_{41}x_{30} - x_{41}x_{28} - x_{40}x_{31} + x_{40}x_{29} + x_{31}x_{28} - x_{30}x_{29}
\end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 41.

Variable x_{41} selected: The number of polynomials with this variable, with indexes from 1 to 41, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{41} from all other polynomials by reducing them with polynomial p_{40} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_6 - u_4)x_1 + (u_7u_4 - u_6u_5) \\
p_2 &= u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1 \\
p_3 &= (u_8 - u_6)x_3 + (-u_9 + u_7)x_2 + (u_9u_6 - u_8u_7) \\
p_4 &= (u_4 - u_2)x_5 + (-u_5 + u_3)x_4 + (u_5u_2 - u_4u_3) \\
p_5 &= u_8x_5 - u_9x_4 \\
p_6 &= (u_4 - u_2)x_6 + (u_5u_2 - u_4u_3) \\
p_7 &= (u_8 - u_6)x_7 + (u_9u_6 - u_8u_7) \\
p_8 &= u_2x_9 + (-u_3 + u_1)x_8 - u_2u_1 \\
p_9 &= u_8x_9 - u_9x_8 \\
p_{10} &= (u_6 - u_4)x_{11} + (-u_7 + u_5)x_{10} + (u_7u_4 - u_6u_5) \\
p_{11} &= u_8x_{11} - u_9x_{10} \\
p_{12} &= x_{12} - 0.5u_4 \\
p_{13} &= x_{13} + (-0.5u_5 - 0.5u_1) \\
p_{14} &= x_{14} - 0.5u_2 \\
p_{15} &= x_{15} - 0.5x_1 - 0.5u_3 \\
p_{16} &= x_{16} - 0.5u_8 \\
p_{17} &= x_{17} + (-0.5u_9 - 0.5u_1) \\
p_{18} &= x_{18} - 0.5x_2 \\
p_{19} &= x_{19} - 0.5x_3 \\
p_{20} &= x_{20} - 0.5u_6 \\
p_{21} &= x_{21} - 0.5x_6 - 0.5u_7 \\
p_{22} &= x_{22} - 0.5u_4 \\
p_{23} &= x_{23} - 0.5x_7 - 0.5u_5 \\
p_{24} &= x_{24} + (-0.5u_8 - 0.5u_2) \\
p_{25} &= x_{25} + (-0.5u_9 - 0.5u_3) \\
p_{26} &= x_{26} - 0.5x_4 - 0.5x_2 \\
p_{27} &= x_{27} - 0.5x_5 - 0.5x_3
\end{aligned}$$

$$\begin{aligned}
p_{28} &= x_{28} - 0.5u_6 \\
p_{29} &= x_{29} - 0.5u_7 \\
p_{30} &= x_{30} - 0.5u_8 \\
p_{31} &= x_{31} - 0.5x_1 - 0.5u_9 \\
p_{32} &= x_{32} - 0.5x_8 - 0.5u_4 \\
p_{33} &= x_{33} - 0.5x_9 - 0.5u_5 \\
p_{34} &= x_{34} - 0.5x_{10} - 0.5u_2 \\
p_{35} &= x_{35} - 0.5x_{11} - 0.5u_3 \\
p_{36} &= x_{37}x_{14} - x_{37}x_{12} - x_{36}x_{15} + x_{36}x_{13} + x_{15}x_{12} - x_{14}x_{13} \\
p_{37} &= x_{37}x_{18} - x_{37}x_{16} - x_{36}x_{19} + x_{36}x_{17} + x_{19}x_{16} - x_{18}x_{17} \\
p_{38} &= x_{39}x_{22} - x_{39}x_{20} - x_{38}x_{23} + x_{38}x_{21} + x_{23}x_{20} - x_{22}x_{21} \\
p_{39} &= x_{39}x_{26} - x_{39}x_{24} - x_{38}x_{27} + x_{38}x_{25} + x_{27}x_{24} - x_{26}x_{25} \\
p_{40} &= x_{40}x_{35}x_{30} - x_{40}x_{35}x_{28} - x_{40}x_{34}x_{31} + x_{40}x_{34}x_{29} \\
&\quad - x_{40}x_{33}x_{30} + x_{40}x_{33}x_{28} + x_{40}x_{32}x_{31} - x_{40}x_{32}x_{29} \\
&\quad - x_{35}x_{32}x_{30} + x_{35}x_{32}x_{28} + x_{34}x_{33}x_{30} - x_{34}x_{33}x_{28} + \\
&\quad x_{34}x_{31}x_{28} - x_{34}x_{30}x_{29} - x_{32}x_{31}x_{28} + x_{32}x_{30}x_{29} \\
p_{41} &= x_{41}x_{34} - x_{41}x_{32} - x_{40}x_{35} + x_{40}x_{33} + x_{35}x_{32} - x_{34}x_{33}
\end{aligned}$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 40.

Variable x_{40} selected: The number of polynomials with this variable, with indexes from 1 to 40, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{40} . No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 39.

Variable x_{39} selected: The number of polynomials with this variable, with indexes from 1 to 39, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{39} from all other polynomials by reducing them with polynomial p_{38} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_6 - u_4)x_1 + (u_7u_4 - u_6u_5) \\
p_2 &= u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1 \\
p_3 &= (u_8 - u_6)x_3 + (-u_9 + u_7)x_2 + (u_9u_6 - u_8u_7)
\end{aligned}$$

$$\begin{aligned}
p_4 &= (u_4 - u_2)x_5 + (-u_5 + u_3)x_4 + (u_5u_2 - u_4u_3) \\
p_5 &= u_8x_5 - u_9x_4 \\
p_6 &= (u_4 - u_2)x_6 + (u_5u_2 - u_4u_3) \\
p_7 &= (u_8 - u_6)x_7 + (u_9u_6 - u_8u_7) \\
p_8 &= u_2x_9 + (-u_3 + u_1)x_8 - u_2u_1 \\
p_9 &= u_8x_9 - u_9x_8 \\
p_{10} &= (u_6 - u_4)x_{11} + (-u_7 + u_5)x_{10} + (u_7u_4 - u_6u_5) \\
p_{11} &= u_8x_{11} - u_9x_{10} \\
p_{12} &= x_{12} - 0.5u_4 \\
p_{13} &= x_{13} + (-0.5u_5 - 0.5u_1) \\
p_{14} &= x_{14} - 0.5u_2 \\
p_{15} &= x_{15} - 0.5x_1 - 0.5u_3 \\
p_{16} &= x_{16} - 0.5u_8 \\
p_{17} &= x_{17} + (-0.5u_9 - 0.5u_1) \\
p_{18} &= x_{18} - 0.5x_2 \\
p_{19} &= x_{19} - 0.5x_3 \\
p_{20} &= x_{20} - 0.5u_6 \\
p_{21} &= x_{21} - 0.5x_6 - 0.5u_7 \\
p_{22} &= x_{22} - 0.5u_4 \\
p_{23} &= x_{23} - 0.5x_7 - 0.5u_5 \\
p_{24} &= x_{24} + (-0.5u_8 - 0.5u_2) \\
p_{25} &= x_{25} + (-0.5u_9 - 0.5u_3) \\
p_{26} &= x_{26} - 0.5x_4 - 0.5x_2 \\
p_{27} &= x_{27} - 0.5x_5 - 0.5x_3 \\
p_{28} &= x_{28} - 0.5u_6 \\
p_{29} &= x_{29} - 0.5u_7 \\
p_{30} &= x_{30} - 0.5u_8 \\
p_{31} &= x_{31} - 0.5x_1 - 0.5u_9 \\
p_{32} &= x_{32} - 0.5x_8 - 0.5u_4 \\
p_{33} &= x_{33} - 0.5x_9 - 0.5u_5 \\
p_{34} &= x_{34} - 0.5x_{10} - 0.5u_2 \\
p_{35} &= x_{35} - 0.5x_{11} - 0.5u_3 \\
p_{36} &= x_{37}x_{14} - x_{37}x_{12} - x_{36}x_{15} + x_{36}x_{13} + x_{15}x_{12} - x_{14}x_{13} \\
p_{37} &= x_{37}x_{18} - x_{37}x_{16} - x_{36}x_{19} + x_{36}x_{17} + x_{19}x_{16} - x_{18}x_{17} \\
p_{38} &= -x_{38}x_{27}x_{22} + x_{38}x_{27}x_{20} + x_{38}x_{26}x_{23} - x_{38}x_{26}x_{21} + \\
&\quad x_{38}x_{25}x_{22} - x_{38}x_{25}x_{20} - x_{38}x_{24}x_{23} + x_{38}x_{24}x_{21} + \\
&\quad x_{27}x_{24}x_{22} - x_{27}x_{24}x_{20} - x_{26}x_{25}x_{22} + x_{26}x_{25}x_{20} \\
&\quad - x_{26}x_{23}x_{20} + x_{26}x_{22}x_{21} + x_{24}x_{23}x_{20} - x_{24}x_{22}x_{21} \\
p_{39} &= x_{39}x_{22} - x_{39}x_{20} - x_{38}x_{23} + x_{38}x_{21} + x_{23}x_{20} - x_{22}x_{21} \\
p_{40} &= x_{40}x_{35}x_{30} - x_{40}x_{35}x_{28} - x_{40}x_{34}x_{31} + x_{40}x_{34}x_{29}
\end{aligned}$$

$$\begin{aligned}
& -x_{40}x_{33}x_{30} + x_{40}x_{33}x_{28} + x_{40}x_{32}x_{31} - x_{40}x_{32}x_{29} \\
& -x_{35}x_{32}x_{30} + x_{35}x_{32}x_{28} + x_{34}x_{33}x_{30} - x_{34}x_{33}x_{28} + \\
& x_{34}x_{31}x_{28} - x_{34}x_{30}x_{29} - x_{32}x_{31}x_{28} + x_{32}x_{30}x_{29} \\
p_{41} = & x_{41}x_{34} - x_{41}x_{32} - x_{40}x_{35} + x_{40}x_{33} + x_{35}x_{32} - x_{34}x_{33}
\end{aligned}$$

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 38.

Variable x_{38} selected: The number of polynomials with this variable, with indexes from 1 to 38, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{38} . No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 37.

Variable x_{37} selected: The number of polynomials with this variable, with indexes from 1 to 37, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{37} from all other polynomials by reducing them with polynomial p_{36} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_6 - u_4)x_1 + (u_7u_4 - u_6u_5) \\
p_2 &= u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1 \\
p_3 &= (u_8 - u_6)x_3 + (-u_9 + u_7)x_2 + (u_9u_6 - u_8u_7) \\
p_4 &= (u_4 - u_2)x_5 + (-u_5 + u_3)x_4 + (u_5u_2 - u_4u_3) \\
p_5 &= u_8x_5 - u_9x_4 \\
p_6 &= (u_4 - u_2)x_6 + (u_5u_2 - u_4u_3) \\
p_7 &= (u_8 - u_6)x_7 + (u_9u_6 - u_8u_7) \\
p_8 &= u_2x_9 + (-u_3 + u_1)x_8 - u_2u_1 \\
p_9 &= u_8x_9 - u_9x_8 \\
p_{10} &= (u_6 - u_4)x_{11} + (-u_7 + u_5)x_{10} + (u_7u_4 - u_6u_5) \\
p_{11} &= u_8x_{11} - u_9x_{10} \\
p_{12} &= x_{12} - 0.5u_4 \\
p_{13} &= x_{13} + (-0.5u_5 - 0.5u_1) \\
p_{14} &= x_{14} - 0.5u_2 \\
p_{15} &= x_{15} - 0.5x_1 - 0.5u_3 \\
p_{16} &= x_{16} - 0.5u_8
\end{aligned}$$

$$\begin{aligned}
p_{17} &= x_{17} + (-0.5u_9 - 0.5u_1) \\
p_{18} &= x_{18} - 0.5x_2 \\
p_{19} &= x_{19} - 0.5x_3 \\
p_{20} &= x_{20} - 0.5u_6 \\
p_{21} &= x_{21} - 0.5x_6 - 0.5u_7 \\
p_{22} &= x_{22} - 0.5u_4 \\
p_{23} &= x_{23} - 0.5x_7 - 0.5u_5 \\
p_{24} &= x_{24} + (-0.5u_8 - 0.5u_2) \\
p_{25} &= x_{25} + (-0.5u_9 - 0.5u_3) \\
p_{26} &= x_{26} - 0.5x_4 - 0.5x_2 \\
p_{27} &= x_{27} - 0.5x_5 - 0.5x_3 \\
p_{28} &= x_{28} - 0.5u_6 \\
p_{29} &= x_{29} - 0.5u_7 \\
p_{30} &= x_{30} - 0.5u_8 \\
p_{31} &= x_{31} - 0.5x_1 - 0.5u_9 \\
p_{32} &= x_{32} - 0.5x_8 - 0.5u_4 \\
p_{33} &= x_{33} - 0.5x_9 - 0.5u_5 \\
p_{34} &= x_{34} - 0.5x_{10} - 0.5u_2 \\
p_{35} &= x_{35} - 0.5x_{11} - 0.5u_3 \\
p_{36} &= -x_{36}x_{19}x_{14} + x_{36}x_{19}x_{12} + x_{36}x_{18}x_{15} - x_{36}x_{18}x_{13} + \\
&\quad x_{36}x_{17}x_{14} - x_{36}x_{17}x_{12} - x_{36}x_{16}x_{15} + x_{36}x_{16}x_{13} + \\
&\quad x_{19}x_{16}x_{14} - x_{19}x_{16}x_{12} - x_{18}x_{17}x_{14} + x_{18}x_{17}x_{12} \\
&\quad - x_{18}x_{15}x_{12} + x_{18}x_{14}x_{13} + x_{16}x_{15}x_{12} - x_{16}x_{14}x_{13} \\
p_{37} &= x_{37}x_{14} - x_{37}x_{12} - x_{36}x_{15} + x_{36}x_{13} + x_{15}x_{12} - x_{14}x_{13} \\
p_{38} &= -x_{38}x_{27}x_{22} + x_{38}x_{27}x_{20} + x_{38}x_{26}x_{23} - x_{38}x_{26}x_{21} + \\
&\quad x_{38}x_{25}x_{22} - x_{38}x_{25}x_{20} - x_{38}x_{24}x_{23} + x_{38}x_{24}x_{21} + \\
&\quad x_{27}x_{24}x_{22} - x_{27}x_{24}x_{20} - x_{26}x_{25}x_{22} + x_{26}x_{25}x_{20} \\
&\quad - x_{26}x_{23}x_{20} + x_{26}x_{22}x_{21} + x_{24}x_{23}x_{20} - x_{24}x_{22}x_{21} \\
p_{39} &= x_{39}x_{22} - x_{39}x_{20} - x_{38}x_{23} + x_{38}x_{21} + x_{23}x_{20} - x_{22}x_{21} \\
p_{40} &= x_{40}x_{35}x_{30} - x_{40}x_{35}x_{28} - x_{40}x_{34}x_{31} + x_{40}x_{34}x_{29} \\
&\quad - x_{40}x_{33}x_{30} + x_{40}x_{33}x_{28} + x_{40}x_{32}x_{31} - x_{40}x_{32}x_{29} \\
&\quad - x_{35}x_{32}x_{30} + x_{35}x_{32}x_{28} + x_{34}x_{33}x_{30} - x_{34}x_{33}x_{28} + \\
&\quad x_{34}x_{31}x_{28} - x_{34}x_{30}x_{29} - x_{32}x_{31}x_{28} + x_{32}x_{30}x_{29} \\
p_{41} &= x_{41}x_{34} - x_{41}x_{32} - x_{40}x_{35} + x_{40}x_{33} + x_{35}x_{32} - x_{34}x_{33}
\end{aligned}$$

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 36.

Variable x_{36} selected: The number of polynomials with this variable, with indexes from 1 to 36, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{36} . No reduction needed.

The triangular system has not been changed.

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 35.

Variable x_{35} selected: The number of polynomials with this variable, with indexes from 1 to 35, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{35} . No reduction needed.

The triangular system has not been changed.

3.8 Triangulation, step 8

Choosing variable: Trying the variable with index 34.

Variable x_{34} selected: The number of polynomials with this variable, with indexes from 1 to 34, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{34} . No reduction needed.

The triangular system has not been changed.

3.9 Triangulation, step 9

Choosing variable: Trying the variable with index 33.

Variable x_{33} selected: The number of polynomials with this variable, with indexes from 1 to 33, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{33} . No reduction needed.

The triangular system has not been changed.

3.10 Triangulation, step 10

Choosing variable: Trying the variable with index 32.

Variable x_{32} selected: The number of polynomials with this variable, with indexes from 1 to 32, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{32} . No reduction needed.

The triangular system has not been changed.

3.11 Triangulation, step 11

Choosing variable: Trying the variable with index 31.

Variable x_{31} selected: The number of polynomials with this variable, with indexes from 1 to 31, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{31} . No reduction needed.

The triangular system has not been changed.

3.12 Triangulation, step 12

Choosing variable: Trying the variable with index 30.

Variable x_{30} selected: The number of polynomials with this variable, with indexes from 1 to 30, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{30} . No reduction needed.

The triangular system has not been changed.

3.13 Triangulation, step 13

Choosing variable: Trying the variable with index 29.

Variable x_{29} selected: The number of polynomials with this variable, with indexes from 1 to 29, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{29} . No reduction needed.

The triangular system has not been changed.

3.14 Triangulation, step 14

Choosing variable: Trying the variable with index 28.

Variable x_{28} selected: The number of polynomials with this variable, with indexes from 1 to 28, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{28} . No reduction needed.

The triangular system has not been changed.

3.15 Triangulation, step 15

Choosing variable: Trying the variable with index 27.

Variable x_{27} selected: The number of polynomials with this variable, with indexes from 1 to 27, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{27} . No reduction needed.

The triangular system has not been changed.

3.16 Triangulation, step 16

Choosing variable: Trying the variable with index 26.

Variable x_{26} selected: The number of polynomials with this variable, with indexes from 1 to 26, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{26} . No reduction needed.

The triangular system has not been changed.

3.17 Triangulation, step 17

Choosing variable: Trying the variable with index 25.

Variable x_{25} selected: The number of polynomials with this variable, with indexes from 1 to 25, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{25} . No reduction needed.

The triangular system has not been changed.

3.18 Triangulation, step 18

Choosing variable: Trying the variable with index 24.

Variable x_{24} selected: The number of polynomials with this variable, with indexes from 1 to 24, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{24} . No reduction needed.

The triangular system has not been changed.

3.19 Triangulation, step 19

Choosing variable: Trying the variable with index 23.

Variable x_{23} selected: The number of polynomials with this variable, with indexes from 1 to 23, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{23} . No reduction needed.

The triangular system has not been changed.

3.20 Triangulation, step 20

Choosing variable: Trying the variable with index 22.

Variable x_{22} selected: The number of polynomials with this variable, with indexes from 1 to 22, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{22} . No reduction needed.

The triangular system has not been changed.

3.21 Triangulation, step 21

Choosing variable: Trying the variable with index 21.

Variable x_{21} selected: The number of polynomials with this variable, with indexes from 1 to 21, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{21} . No reduction needed.

The triangular system has not been changed.

3.22 Triangulation, step 22

Choosing variable: Trying the variable with index 20.

Variable x_{20} selected: The number of polynomials with this variable, with indexes from 1 to 20, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{20} . No reduction needed.

The triangular system has not been changed.

3.23 Triangulation, step 23

Choosing variable: Trying the variable with index 19.

Variable x_{19} selected: The number of polynomials with this variable, with indexes from 1 to 19, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{19} . No reduction needed.

The triangular system has not been changed.

3.24 Triangulation, step 24

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{18} . No reduction needed.

The triangular system has not been changed.

3.25 Triangulation, step 25

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{17} . No reduction needed.

The triangular system has not been changed.

3.26 Triangulation, step 26

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{16} . No reduction needed.

The triangular system has not been changed.

3.27 Triangulation, step 27

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{15} . No reduction needed.

The triangular system has not been changed.

3.28 Triangulation, step 28

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14} . No reduction needed.

The triangular system has not been changed.

3.29 Triangulation, step 29

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{13} . No reduction needed.

The triangular system has not been changed.

3.30 Triangulation, step 30

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{12} . No reduction needed.

The triangular system has not been changed.

3.31 Triangulation, step 31

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{11} from all other polynomials by reducing them with polynomial p_{10} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_1 &= (u_6 - u_4)x_1 + (u_7u_4 - u_6u_5) \\ p_2 &= u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1 \\ p_3 &= (u_8 - u_6)x_3 + (-u_9 + u_7)x_2 + (u_9u_6 - u_8u_7) \\ p_4 &= (u_4 - u_2)x_5 + (-u_5 + u_3)x_4 + (u_5u_2 - u_4u_3) \\ p_5 &= u_8x_5 - u_9x_4 \\ p_6 &= (u_4 - u_2)x_6 + (u_5u_2 - u_4u_3) \\ p_7 &= (u_8 - u_6)x_7 + (u_9u_6 - u_8u_7) \\ p_8 &= u_2x_9 + (-u_3 + u_1)x_8 - u_2u_1 \\ p_9 &= u_8x_9 - u_9x_8 \\ p_{10} &= (-u_9u_6 + u_9u_4 + u_8u_7 - u_8u_5)x_{10} + (-u_8u_7u_4 + u_8u_6u_5) \\ p_{11} &= (u_6 - u_4)x_{11} + (-u_7 + u_5)x_{10} + (u_7u_4 - u_6u_5) \\ p_{12} &= x_{12} - 0.5u_4 \\ p_{13} &= x_{13} + (-0.5u_5 - 0.5u_1) \\ p_{14} &= x_{14} - 0.5u_2 \\ p_{15} &= x_{15} - 0.5x_1 - 0.5u_3 \\ p_{16} &= x_{16} - 0.5u_8 \\ p_{17} &= x_{17} + (-0.5u_9 - 0.5u_1) \\ p_{18} &= x_{18} - 0.5x_2 \\ p_{19} &= x_{19} - 0.5x_3 \\ p_{20} &= x_{20} - 0.5u_6 \\ p_{21} &= x_{21} - 0.5x_6 - 0.5u_7 \\ p_{22} &= x_{22} - 0.5u_4 \\ p_{23} &= x_{23} - 0.5x_7 - 0.5u_5 \\ p_{24} &= x_{24} + (-0.5u_8 - 0.5u_2) \\ p_{25} &= x_{25} + (-0.5u_9 - 0.5u_3) \\ p_{26} &= x_{26} - 0.5x_4 - 0.5x_2 \\ p_{27} &= x_{27} - 0.5x_5 - 0.5x_3 \\ p_{28} &= x_{28} - 0.5u_6 \\ p_{29} &= x_{29} - 0.5u_7 \\ p_{30} &= x_{30} - 0.5u_8 \end{aligned}$$

$$\begin{aligned}
p_{31} &= x_{31} - 0.5x_1 - 0.5u_9 \\
p_{32} &= x_{32} - 0.5x_8 - 0.5u_4 \\
p_{33} &= x_{33} - 0.5x_9 - 0.5u_5 \\
p_{34} &= x_{34} - 0.5x_{10} - 0.5u_2 \\
p_{35} &= x_{35} - 0.5x_{11} - 0.5u_3 \\
p_{36} &= -x_{36}x_{19}x_{14} + x_{36}x_{19}x_{12} + x_{36}x_{18}x_{15} - x_{36}x_{18}x_{13} + \\
&\quad x_{36}x_{17}x_{14} - x_{36}x_{17}x_{12} - x_{36}x_{16}x_{15} + x_{36}x_{16}x_{13} + \\
&\quad x_{19}x_{16}x_{14} - x_{19}x_{16}x_{12} - x_{18}x_{17}x_{14} + x_{18}x_{17}x_{12} \\
&\quad - x_{18}x_{15}x_{12} + x_{18}x_{14}x_{13} + x_{16}x_{15}x_{12} - x_{16}x_{14}x_{13} \\
p_{37} &= x_{37}x_{14} - x_{37}x_{12} - x_{36}x_{15} + x_{36}x_{13} + x_{15}x_{12} - x_{14}x_{13} \\
p_{38} &= -x_{38}x_{27}x_{22} + x_{38}x_{27}x_{20} + x_{38}x_{26}x_{23} - x_{38}x_{26}x_{21} + \\
&\quad x_{38}x_{25}x_{22} - x_{38}x_{25}x_{20} - x_{38}x_{24}x_{23} + x_{38}x_{24}x_{21} + \\
&\quad x_{27}x_{24}x_{22} - x_{27}x_{24}x_{20} - x_{26}x_{25}x_{22} + x_{26}x_{25}x_{20} \\
&\quad - x_{26}x_{23}x_{20} + x_{26}x_{22}x_{21} + x_{24}x_{23}x_{20} - x_{24}x_{22}x_{21} \\
p_{39} &= x_{39}x_{22} - x_{39}x_{20} - x_{38}x_{23} + x_{38}x_{21} + x_{23}x_{20} - x_{22}x_{21} \\
p_{40} &= x_{40}x_{35}x_{30} - x_{40}x_{35}x_{28} - x_{40}x_{34}x_{31} + x_{40}x_{34}x_{29} \\
&\quad - x_{40}x_{33}x_{30} + x_{40}x_{33}x_{28} + x_{40}x_{32}x_{31} - x_{40}x_{32}x_{29} \\
&\quad - x_{35}x_{32}x_{30} + x_{35}x_{32}x_{28} + x_{34}x_{33}x_{30} - x_{34}x_{33}x_{28} + \\
&\quad x_{34}x_{31}x_{28} - x_{34}x_{30}x_{29} - x_{32}x_{31}x_{28} + x_{32}x_{30}x_{29} \\
p_{41} &= x_{41}x_{34} - x_{41}x_{32} - x_{40}x_{35} + x_{40}x_{33} + x_{35}x_{32} - x_{34}x_{33}
\end{aligned}$$

3.32 Triangulation, step 32

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10} . No reduction needed.

The triangular system has not been changed.

3.33 Triangulation, step 33

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_9 from all other polynomials by reducing them with polynomial p_8 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_6 - u_4)x_1 + (u_7u_4 - u_6u_5) \\
p_2 &= u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1 \\
p_3 &= (u_8 - u_6)x_3 + (-u_9 + u_7)x_2 + (u_9u_6 - u_8u_7) \\
p_4 &= (u_4 - u_2)x_5 + (-u_5 + u_3)x_4 + (u_5u_2 - u_4u_3) \\
p_5 &= u_8x_5 - u_9x_4 \\
p_6 &= (u_4 - u_2)x_6 + (u_5u_2 - u_4u_3) \\
p_7 &= (u_8 - u_6)x_7 + (u_9u_6 - u_8u_7) \\
p_8 &= (-u_9u_2 + u_8u_3 - u_8u_1)x_8 + u_8u_2u_1 \\
p_9 &= u_2x_9 + (-u_3 + u_1)x_8 - u_2u_1 \\
p_{10} &= (-u_9u_6 + u_9u_4 + u_8u_7 - u_8u_5)x_{10} + (-u_8u_7u_4 + u_8u_6u_5) \\
p_{11} &= (u_6 - u_4)x_{11} + (-u_7 + u_5)x_{10} + (u_7u_4 - u_6u_5) \\
p_{12} &= x_{12} - 0.5u_4 \\
p_{13} &= x_{13} + (-0.5u_5 - 0.5u_1) \\
p_{14} &= x_{14} - 0.5u_2 \\
p_{15} &= x_{15} - 0.5x_1 - 0.5u_3 \\
p_{16} &= x_{16} - 0.5u_8 \\
p_{17} &= x_{17} + (-0.5u_9 - 0.5u_1) \\
p_{18} &= x_{18} - 0.5x_2 \\
p_{19} &= x_{19} - 0.5x_3 \\
p_{20} &= x_{20} - 0.5u_6 \\
p_{21} &= x_{21} - 0.5x_6 - 0.5u_7 \\
p_{22} &= x_{22} - 0.5u_4 \\
p_{23} &= x_{23} - 0.5x_7 - 0.5u_5 \\
p_{24} &= x_{24} + (-0.5u_8 - 0.5u_2) \\
p_{25} &= x_{25} + (-0.5u_9 - 0.5u_3) \\
p_{26} &= x_{26} - 0.5x_4 - 0.5x_2 \\
p_{27} &= x_{27} - 0.5x_5 - 0.5x_3 \\
p_{28} &= x_{28} - 0.5u_6 \\
p_{29} &= x_{29} - 0.5u_7 \\
p_{30} &= x_{30} - 0.5u_8 \\
p_{31} &= x_{31} - 0.5x_1 - 0.5u_9 \\
p_{32} &= x_{32} - 0.5x_8 - 0.5u_4 \\
p_{33} &= x_{33} - 0.5x_9 - 0.5u_5 \\
p_{34} &= x_{34} - 0.5x_{10} - 0.5u_2 \\
p_{35} &= x_{35} - 0.5x_{11} - 0.5u_3 \\
p_{36} &= -x_{36}x_{19}x_{14} + x_{36}x_{19}x_{12} + x_{36}x_{18}x_{15} - x_{36}x_{18}x_{13} + \\
&\quad x_{36}x_{17}x_{14} - x_{36}x_{17}x_{12} - x_{36}x_{16}x_{15} + x_{36}x_{16}x_{13} + \\
&\quad x_{19}x_{16}x_{14} - x_{19}x_{16}x_{12} - x_{18}x_{17}x_{14} + x_{18}x_{17}x_{12} \\
&\quad - x_{18}x_{15}x_{12} + x_{18}x_{14}x_{13} + x_{16}x_{15}x_{12} - x_{16}x_{14}x_{13}
\end{aligned}$$

$$\begin{aligned}
p_{37} &= x_{37}x_{14} - x_{37}x_{12} - x_{36}x_{15} + x_{36}x_{13} + x_{15}x_{12} - x_{14}x_{13} \\
p_{38} &= -x_{38}x_{27}x_{22} + x_{38}x_{27}x_{20} + x_{38}x_{26}x_{23} - x_{38}x_{26}x_{21} + \\
&\quad x_{38}x_{25}x_{22} - x_{38}x_{25}x_{20} - x_{38}x_{24}x_{23} + x_{38}x_{24}x_{21} + \\
&\quad x_{27}x_{24}x_{22} - x_{27}x_{24}x_{20} - x_{26}x_{25}x_{22} + x_{26}x_{25}x_{20} \\
&\quad - x_{26}x_{23}x_{20} + x_{26}x_{22}x_{21} + x_{24}x_{23}x_{20} - x_{24}x_{22}x_{21} \\
p_{39} &= x_{39}x_{22} - x_{39}x_{20} - x_{38}x_{23} + x_{38}x_{21} + x_{23}x_{20} - x_{22}x_{21} \\
p_{40} &= x_{40}x_{35}x_{30} - x_{40}x_{35}x_{28} - x_{40}x_{34}x_{31} + x_{40}x_{34}x_{29} \\
&\quad - x_{40}x_{33}x_{30} + x_{40}x_{33}x_{28} + x_{40}x_{32}x_{31} - x_{40}x_{32}x_{29} \\
&\quad - x_{35}x_{32}x_{30} + x_{35}x_{32}x_{28} + x_{34}x_{33}x_{30} - x_{34}x_{33}x_{28} + \\
&\quad x_{34}x_{31}x_{28} - x_{34}x_{30}x_{29} - x_{32}x_{31}x_{28} + x_{32}x_{30}x_{29} \\
p_{41} &= x_{41}x_{34} - x_{41}x_{32} - x_{40}x_{35} + x_{40}x_{33} + x_{35}x_{32} - x_{34}x_{33}
\end{aligned}$$

3.34 Triangulation, step 34

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8 . No reduction needed.

The triangular system has not been changed.

3.35 Triangulation, step 35

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7 . No reduction needed.

The triangular system has not been changed.

3.36 Triangulation, step 36

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6 . No reduction needed.

The triangular system has not been changed.

3.37 Triangulation, step 37

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_5 from all other polynomials by reducing them with polynomial p_4 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_1 &= (u_6 - u_4)x_1 + (u_7u_4 - u_6u_5) \\ p_2 &= u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1 \\ p_3 &= (u_8 - u_6)x_3 + (-u_9 + u_7)x_2 + (u_9u_6 - u_8u_7) \\ p_4 &= (-u_9u_4 + u_9u_2 + u_8u_5 - u_8u_3)x_4 + (-u_8u_5u_2 + u_8u_4u_3) \\ p_5 &= (u_4 - u_2)x_5 + (-u_5 + u_3)x_4 + (u_5u_2 - u_4u_3) \\ p_6 &= (u_4 - u_2)x_6 + (u_5u_2 - u_4u_3) \\ p_7 &= (u_8 - u_6)x_7 + (u_9u_6 - u_8u_7) \\ p_8 &= (-u_9u_2 + u_8u_3 - u_8u_1)x_8 + u_8u_2u_1 \\ p_9 &= u_2x_9 + (-u_3 + u_1)x_8 - u_2u_1 \\ p_{10} &= (-u_9u_6 + u_9u_4 + u_8u_7 - u_8u_5)x_{10} + (-u_8u_7u_4 + u_8u_6u_5) \\ p_{11} &= (u_6 - u_4)x_{11} + (-u_7 + u_5)x_{10} + (u_7u_4 - u_6u_5) \\ p_{12} &= x_{12} - 0.5u_4 \\ p_{13} &= x_{13} + (-0.5u_5 - 0.5u_1) \\ p_{14} &= x_{14} - 0.5u_2 \\ p_{15} &= x_{15} - 0.5x_1 - 0.5u_3 \\ p_{16} &= x_{16} - 0.5u_8 \\ p_{17} &= x_{17} + (-0.5u_9 - 0.5u_1) \\ p_{18} &= x_{18} - 0.5x_2 \\ p_{19} &= x_{19} - 0.5x_3 \\ p_{20} &= x_{20} - 0.5u_6 \\ p_{21} &= x_{21} - 0.5x_6 - 0.5u_7 \\ p_{22} &= x_{22} - 0.5u_4 \\ p_{23} &= x_{23} - 0.5x_7 - 0.5u_5 \\ p_{24} &= x_{24} + (-0.5u_8 - 0.5u_2) \\ p_{25} &= x_{25} + (-0.5u_9 - 0.5u_3) \\ p_{26} &= x_{26} - 0.5x_4 - 0.5x_2 \\ p_{27} &= x_{27} - 0.5x_5 - 0.5x_3 \\ p_{28} &= x_{28} - 0.5u_6 \\ p_{29} &= x_{29} - 0.5u_7 \\ p_{30} &= x_{30} - 0.5u_8 \end{aligned}$$

$$\begin{aligned}
p_{31} &= x_{31} - 0.5x_1 - 0.5u_9 \\
p_{32} &= x_{32} - 0.5x_8 - 0.5u_4 \\
p_{33} &= x_{33} - 0.5x_9 - 0.5u_5 \\
p_{34} &= x_{34} - 0.5x_{10} - 0.5u_2 \\
p_{35} &= x_{35} - 0.5x_{11} - 0.5u_3 \\
p_{36} &= -x_{36}x_{19}x_{14} + x_{36}x_{19}x_{12} + x_{36}x_{18}x_{15} - x_{36}x_{18}x_{13} + \\
&\quad x_{36}x_{17}x_{14} - x_{36}x_{17}x_{12} - x_{36}x_{16}x_{15} + x_{36}x_{16}x_{13} + \\
&\quad x_{19}x_{16}x_{14} - x_{19}x_{16}x_{12} - x_{18}x_{17}x_{14} + x_{18}x_{17}x_{12} \\
&\quad - x_{18}x_{15}x_{12} + x_{18}x_{14}x_{13} + x_{16}x_{15}x_{12} - x_{16}x_{14}x_{13} \\
p_{37} &= x_{37}x_{14} - x_{37}x_{12} - x_{36}x_{15} + x_{36}x_{13} + x_{15}x_{12} - x_{14}x_{13} \\
p_{38} &= -x_{38}x_{27}x_{22} + x_{38}x_{27}x_{20} + x_{38}x_{26}x_{23} - x_{38}x_{26}x_{21} + \\
&\quad x_{38}x_{25}x_{22} - x_{38}x_{25}x_{20} - x_{38}x_{24}x_{23} + x_{38}x_{24}x_{21} + \\
&\quad x_{27}x_{24}x_{22} - x_{27}x_{24}x_{20} - x_{26}x_{25}x_{22} + x_{26}x_{25}x_{20} \\
&\quad - x_{26}x_{23}x_{20} + x_{26}x_{22}x_{21} + x_{24}x_{23}x_{20} - x_{24}x_{22}x_{21} \\
p_{39} &= x_{39}x_{22} - x_{39}x_{20} - x_{38}x_{23} + x_{38}x_{21} + x_{23}x_{20} - x_{22}x_{21} \\
p_{40} &= x_{40}x_{35}x_{30} - x_{40}x_{35}x_{28} - x_{40}x_{34}x_{31} + x_{40}x_{34}x_{29} \\
&\quad - x_{40}x_{33}x_{30} + x_{40}x_{33}x_{28} + x_{40}x_{32}x_{31} - x_{40}x_{32}x_{29} \\
&\quad - x_{35}x_{32}x_{30} + x_{35}x_{32}x_{28} + x_{34}x_{33}x_{30} - x_{34}x_{33}x_{28} + \\
&\quad x_{34}x_{31}x_{28} - x_{34}x_{30}x_{29} - x_{32}x_{31}x_{28} + x_{32}x_{30}x_{29} \\
p_{41} &= x_{41}x_{34} - x_{41}x_{32} - x_{40}x_{35} + x_{40}x_{33} + x_{35}x_{32} - x_{34}x_{33}
\end{aligned}$$

3.38 Triangulation, step 38

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4 . No reduction needed.

The triangular system has not been changed.

3.39 Triangulation, step 39

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_3 from all other polynomials by reducing them with polynomial p_2 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_6 - u_4)x_1 + (u_7u_4 - u_6u_5) \\
p_2 &= (-u_9u_2 + u_8u_3 - u_8u_1 + u_7u_2 - u_6u_3 + u_6u_1)x_2 + \\
&\quad (u_9u_6u_2 - u_8u_7u_2 + u_8u_2u_1 - u_6u_2u_1) \\
p_3 &= u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1 \\
p_4 &= (-u_9u_4 + u_9u_2 + u_8u_5 - u_8u_3)x_4 + (-u_8u_5u_2 + u_8u_4u_3) \\
p_5 &= (u_4 - u_2)x_5 + (-u_5 + u_3)x_4 + (u_5u_2 - u_4u_3) \\
p_6 &= (u_4 - u_2)x_6 + (u_5u_2 - u_4u_3) \\
p_7 &= (u_8 - u_6)x_7 + (u_9u_6 - u_8u_7) \\
p_8 &= (-u_9u_2 + u_8u_3 - u_8u_1)x_8 + u_8u_2u_1 \\
p_9 &= u_2x_9 + (-u_3 + u_1)x_8 - u_2u_1 \\
p_{10} &= (-u_9u_6 + u_9u_4 + u_8u_7 - u_8u_5)x_{10} + (-u_8u_7u_4 + u_8u_6u_5) \\
p_{11} &= (u_6 - u_4)x_{11} + (-u_7 + u_5)x_{10} + (u_7u_4 - u_6u_5) \\
p_{12} &= x_{12} - 0.5u_4 \\
p_{13} &= x_{13} + (-0.5u_5 - 0.5u_1) \\
p_{14} &= x_{14} - 0.5u_2 \\
p_{15} &= x_{15} - 0.5x_1 - 0.5u_3 \\
p_{16} &= x_{16} - 0.5u_8 \\
p_{17} &= x_{17} + (-0.5u_9 - 0.5u_1) \\
p_{18} &= x_{18} - 0.5x_2 \\
p_{19} &= x_{19} - 0.5x_3 \\
p_{20} &= x_{20} - 0.5u_6 \\
p_{21} &= x_{21} - 0.5x_6 - 0.5u_7 \\
p_{22} &= x_{22} - 0.5u_4 \\
p_{23} &= x_{23} - 0.5x_7 - 0.5u_5 \\
p_{24} &= x_{24} + (-0.5u_8 - 0.5u_2) \\
p_{25} &= x_{25} + (-0.5u_9 - 0.5u_3) \\
p_{26} &= x_{26} - 0.5x_4 - 0.5x_2 \\
p_{27} &= x_{27} - 0.5x_5 - 0.5x_3 \\
p_{28} &= x_{28} - 0.5u_6 \\
p_{29} &= x_{29} - 0.5u_7 \\
p_{30} &= x_{30} - 0.5u_8 \\
p_{31} &= x_{31} - 0.5x_1 - 0.5u_9 \\
p_{32} &= x_{32} - 0.5x_8 - 0.5u_4 \\
p_{33} &= x_{33} - 0.5x_9 - 0.5u_5 \\
p_{34} &= x_{34} - 0.5x_{10} - 0.5u_2 \\
p_{35} &= x_{35} - 0.5x_{11} - 0.5u_3 \\
p_{36} &= -x_{36}x_{19}x_{14} + x_{36}x_{19}x_{12} + x_{36}x_{18}x_{15} - x_{36}x_{18}x_{13} + \\
&\quad x_{36}x_{17}x_{14} - x_{36}x_{17}x_{12} - x_{36}x_{16}x_{15} + x_{36}x_{16}x_{13} + \\
&\quad x_{19}x_{16}x_{14} - x_{19}x_{16}x_{12} - x_{18}x_{17}x_{14} + x_{18}x_{17}x_{12}
\end{aligned}$$

$$\begin{aligned}
& -x_{18}x_{15}x_{12} + x_{18}x_{14}x_{13} + x_{16}x_{15}x_{12} - x_{16}x_{14}x_{13} \\
p_{37} &= x_{37}x_{14} - x_{37}x_{12} - x_{36}x_{15} + x_{36}x_{13} + x_{15}x_{12} - x_{14}x_{13} \\
p_{38} &= -x_{38}x_{27}x_{22} + x_{38}x_{27}x_{20} + x_{38}x_{26}x_{23} - x_{38}x_{26}x_{21} + \\
& x_{38}x_{25}x_{22} - x_{38}x_{25}x_{20} - x_{38}x_{24}x_{23} + x_{38}x_{24}x_{21} + \\
& x_{27}x_{24}x_{22} - x_{27}x_{24}x_{20} - x_{26}x_{25}x_{22} + x_{26}x_{25}x_{20} \\
& -x_{26}x_{23}x_{20} + x_{26}x_{22}x_{21} + x_{24}x_{23}x_{20} - x_{24}x_{22}x_{21} \\
p_{39} &= x_{39}x_{22} - x_{39}x_{20} - x_{38}x_{23} + x_{38}x_{21} + x_{23}x_{20} - x_{22}x_{21} \\
p_{40} &= x_{40}x_{35}x_{30} - x_{40}x_{35}x_{28} - x_{40}x_{34}x_{31} + x_{40}x_{34}x_{29} \\
& -x_{40}x_{33}x_{30} + x_{40}x_{33}x_{28} + x_{40}x_{32}x_{31} - x_{40}x_{32}x_{29} \\
& -x_{35}x_{32}x_{30} + x_{35}x_{32}x_{28} + x_{34}x_{33}x_{30} - x_{34}x_{33}x_{28} + \\
& x_{34}x_{31}x_{28} - x_{34}x_{30}x_{29} - x_{32}x_{31}x_{28} + x_{32}x_{30}x_{29} \\
p_{41} &= x_{41}x_{34} - x_{41}x_{32} - x_{40}x_{35} + x_{40}x_{33} + x_{35}x_{32} - x_{34}x_{33}
\end{aligned}$$

3.40 Triangulation, step 40

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2 . No reduction needed.

The triangular system has not been changed.

3.41 Triangulation, step 41

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1 . No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_1 &= (u_6 - u_4)x_1 + (u_7u_4 - u_6u_5) \\
p_2 &= (-u_9u_2 + u_8u_3 - u_8u_1 + u_7u_2 - u_6u_3 + u_6u_1)x_2 + \\
& (u_9u_6u_2 - u_8u_7u_2 + u_8u_2u_1 - u_6u_2u_1) \\
p_3 &= u_2x_3 + (-u_3 + u_1)x_2 - u_2u_1 \\
p_4 &= (-u_9u_4 + u_9u_2 + u_8u_5 - u_8u_3)x_4 + (-u_8u_5u_2 + u_8u_4u_3) \\
p_5 &= (u_4 - u_2)x_5 + (-u_5 + u_3)x_4 + (u_5u_2 - u_4u_3) \\
p_6 &= (u_4 - u_2)x_6 + (u_5u_2 - u_4u_3) \\
p_7 &= (u_8 - u_6)x_7 + (u_9u_6 - u_8u_7)
\end{aligned}$$

$$\begin{aligned}
p_8 &= (-u_9u_2 + u_8u_3 - u_8u_1)x_8 + u_8u_2u_1 \\
p_9 &= u_2x_9 + (-u_3 + u_1)x_8 - u_2u_1 \\
p_{10} &= (-u_9u_6 + u_9u_4 + u_8u_7 - u_8u_5)x_{10} + (-u_8u_7u_4 + u_8u_6u_5) \\
p_{11} &= (u_6 - u_4)x_{11} + (-u_7 + u_5)x_{10} + (u_7u_4 - u_6u_5) \\
p_{12} &= x_{12} - 0.5u_4 \\
p_{13} &= x_{13} + (-0.5u_5 - 0.5u_1) \\
p_{14} &= x_{14} - 0.5u_2 \\
p_{15} &= x_{15} - 0.5x_1 - 0.5u_3 \\
p_{16} &= x_{16} - 0.5u_8 \\
p_{17} &= x_{17} + (-0.5u_9 - 0.5u_1) \\
p_{18} &= x_{18} - 0.5x_2 \\
p_{19} &= x_{19} - 0.5x_3 \\
p_{20} &= x_{20} - 0.5u_6 \\
p_{21} &= x_{21} - 0.5x_6 - 0.5u_7 \\
p_{22} &= x_{22} - 0.5u_4 \\
p_{23} &= x_{23} - 0.5x_7 - 0.5u_5 \\
p_{24} &= x_{24} + (-0.5u_8 - 0.5u_2) \\
p_{25} &= x_{25} + (-0.5u_9 - 0.5u_3) \\
p_{26} &= x_{26} - 0.5x_4 - 0.5x_2 \\
p_{27} &= x_{27} - 0.5x_5 - 0.5x_3 \\
p_{28} &= x_{28} - 0.5u_6 \\
p_{29} &= x_{29} - 0.5u_7 \\
p_{30} &= x_{30} - 0.5u_8 \\
p_{31} &= x_{31} - 0.5x_1 - 0.5u_9 \\
p_{32} &= x_{32} - 0.5x_8 - 0.5u_4 \\
p_{33} &= x_{33} - 0.5x_9 - 0.5u_5 \\
p_{34} &= x_{34} - 0.5x_{10} - 0.5u_2 \\
p_{35} &= x_{35} - 0.5x_{11} - 0.5u_3 \\
p_{36} &= -x_{36}x_{19}x_{14} + x_{36}x_{19}x_{12} + x_{36}x_{18}x_{15} - x_{36}x_{18}x_{13} + \\
&\quad x_{36}x_{17}x_{14} - x_{36}x_{17}x_{12} - x_{36}x_{16}x_{15} + x_{36}x_{16}x_{13} + \\
&\quad x_{19}x_{16}x_{14} - x_{19}x_{16}x_{12} - x_{18}x_{17}x_{14} + x_{18}x_{17}x_{12} \\
&\quad - x_{18}x_{15}x_{12} + x_{18}x_{14}x_{13} + x_{16}x_{15}x_{12} - x_{16}x_{14}x_{13} \\
p_{37} &= x_{37}x_{14} - x_{37}x_{12} - x_{36}x_{15} + x_{36}x_{13} + x_{15}x_{12} - x_{14}x_{13} \\
p_{38} &= -x_{38}x_{27}x_{22} + x_{38}x_{27}x_{20} + x_{38}x_{26}x_{23} - x_{38}x_{26}x_{21} + \\
&\quad x_{38}x_{25}x_{22} - x_{38}x_{25}x_{20} - x_{38}x_{24}x_{23} + x_{38}x_{24}x_{21} + \\
&\quad x_{27}x_{24}x_{22} - x_{27}x_{24}x_{20} - x_{26}x_{25}x_{22} + x_{26}x_{25}x_{20} \\
&\quad - x_{26}x_{23}x_{20} + x_{26}x_{22}x_{21} + x_{24}x_{23}x_{20} - x_{24}x_{22}x_{21} \\
p_{39} &= x_{39}x_{22} - x_{39}x_{20} - x_{38}x_{23} + x_{38}x_{21} + x_{23}x_{20} - x_{22}x_{21} \\
p_{40} &= x_{40}x_{35}x_{30} - x_{40}x_{35}x_{28} - x_{40}x_{34}x_{31} + x_{40}x_{34}x_{29} \\
&\quad - x_{40}x_{33}x_{30} + x_{40}x_{33}x_{28} + x_{40}x_{32}x_{31} - x_{40}x_{32}x_{29}
\end{aligned}$$

$$\begin{aligned}
& -x_{35}x_{32}x_{30} + x_{35}x_{32}x_{28} + x_{34}x_{33}x_{30} - x_{34}x_{33}x_{28} + \\
& x_{34}x_{31}x_{28} - x_{34}x_{30}x_{29} - x_{32}x_{31}x_{28} + x_{32}x_{30}x_{29} \\
p_{41} = & x_{41}x_{34} - x_{41}x_{32} - x_{40}x_{35} + x_{40}x_{33} + x_{35}x_{32} - x_{34}x_{33}
\end{aligned}$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 017 (Gauss conic)

Calculating final remainder of the conclusion:

$$g = x_{41}x_{38} - x_{41}x_{36} - x_{40}x_{39} + x_{40}x_{37} + x_{39}x_{36} - x_{38}x_{37}$$

with respect to the triangular system.

1. Pseudo remainder with p_{41} over variable x_{41} :

$$\begin{aligned}
g = & -x_{40}x_{39}x_{34} + x_{40}x_{39}x_{32} + x_{40}x_{38}x_{35} - x_{40}x_{38}x_{33} + \\
& x_{40}x_{37}x_{34} - x_{40}x_{37}x_{32} - x_{40}x_{36}x_{35} + x_{40}x_{36}x_{33} + \\
& x_{39}x_{36}x_{34} - x_{39}x_{36}x_{32} - x_{38}x_{37}x_{34} + x_{38}x_{37}x_{32} \\
& -x_{38}x_{35}x_{32} + x_{38}x_{34}x_{33} + x_{36}x_{35}x_{32} - x_{36}x_{34}x_{33}
\end{aligned}$$

2. Pseudo remainder with p_{40} over variable x_{40} :

Polynomial too big for output (text size is 2823 characters, number of terms is 88)

3. Pseudo remainder with p_{39} over variable x_{39} :

Polynomial too big for output (text size is 8894 characters, number of terms is 232)

4. Pseudo remainder with p_{38} over variable x_{38} :

Polynomial too big for output (number of terms is 1288)

5. Pseudo remainder with p_{37} over variable x_{37} :

Polynomial too big for output (number of terms is 3360)

6. Pseudo remainder with p_{36} over variable x_{36} :

Polynomial too big for output (number of terms is 18816)

7. Pseudo remainder with p_{35} over variable x_{35} :

Polynomial too big for output (number of terms is 25536)

8. Pseudo remainder with p_{34} over variable x_{34} :

Polynomial too big for output (number of terms is 44352)

9. Pseudo remainder with p_{33} over variable x_{33} :
Polynomial too big for output (number of terms is 49392)
10. Pseudo remainder with p_{32} over variable x_{32} :
Polynomial too big for output (number of terms is 52752)
11. Pseudo remainder with p_{31} over variable x_{31} :
Polynomial too big for output (number of terms is 65240)
12. Pseudo remainder with p_{30} over variable x_{30} :
Polynomial too big for output (number of terms is 60368)
13. Pseudo remainder with p_{29} over variable x_{29} :
Polynomial too big for output (number of terms is 48720)
14. Pseudo remainder with p_{28} over variable x_{28} :
Polynomial too big for output (number of terms is 30464)
15. Pseudo remainder with p_{27} over variable x_{27} :
Polynomial too big for output (number of terms is 39552)
16. Pseudo remainder with p_{26} over variable x_{26} :
Polynomial too big for output (number of terms is 50144)
17. Pseudo remainder with p_{25} over variable x_{25} :
Polynomial too big for output (number of terms is 48128)
18. Pseudo remainder with p_{24} over variable x_{24} :
Polynomial too big for output (number of terms is 45168)
19. Pseudo remainder with p_{23} over variable x_{23} :
Polynomial too big for output (number of terms is 54252)
20. Pseudo remainder with p_{22} over variable x_{22} :
Polynomial too big for output (number of terms is 39280)
21. Pseudo remainder with p_{21} over variable x_{21} :
Polynomial too big for output (number of terms is 39728)

22. Pseudo remainder with p_{20} over variable x_{20} :
Polynomial too big for output (number of terms is 15584)
23. Pseudo remainder with p_{19} over variable x_{19} :
Polynomial too big for output (number of terms is 15352)
24. Pseudo remainder with p_{18} over variable x_{18} :
Polynomial too big for output (number of terms is 14864)
25. Pseudo remainder with p_{17} over variable x_{17} :
Polynomial too big for output (number of terms is 13240)
26. Pseudo remainder with p_{16} over variable x_{16} :
Polynomial too big for output (number of terms is 10216)
27. Pseudo remainder with p_{15} over variable x_{15} :
Polynomial too big for output (number of terms is 10278)
28. Pseudo remainder with p_{14} over variable x_{14} :
Polynomial too big for output (number of terms is 6178)
29. Pseudo remainder with p_{13} over variable x_{13} :
Polynomial too big for output (number of terms is 4422)
30. Pseudo remainder with p_{12} over variable x_{12} :
Polynomial too big for output (number of terms is 1430)
31. Pseudo remainder with p_{11} over variable x_{11} :
Polynomial too big for output (number of terms is 1192)
32. Pseudo remainder with p_{10} over variable x_{10} :
Polynomial too big for output (number of terms is 473)
33. Pseudo remainder with p_9 over variable x_9 :
Polynomial too big for output (number of terms is 309)
34. Pseudo remainder with p_8 over variable x_8 :
Polynomial too big for output (text size is greater than 2000 characters, number of terms is 103)

35. Pseudo remainder with p_7 over variable x_7 :
Polynomial too big for output (text size is greater than 2000 characters, number of terms is 70)
36. Pseudo remainder with p_6 over variable x_6 :
Polynomial too big for output (text size is greater than 2000 characters, number of terms is 36)
37. Pseudo remainder with p_5 over variable x_5 :
Polynomial too big for output (text size is greater than 2000 characters, number of terms is 27)
38. Pseudo remainder with p_4 over variable x_4 :
Polynomial too big for output (text size is greater than 2000 characters, number of terms is 18)
39. Pseudo remainder with p_3 over variable x_3 :
Polynomial too big for output (text size is greater than 2000 characters, number of terms is 9)
40. Pseudo remainder with p_2 over variable x_2 :
Polynomial too big for output (text size is greater than 2000 characters, number of terms is 3)
41. Pseudo remainder with p_1 over variable x_1 :

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 65240 terms.

Time Complexity: Time spent by the prover is 7481.004 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points A4, A3 and A0 are not collinear
- Points A2, A1, A4 and A5 are not collinear
- Points A2 and X are not identical
- Points A2, A3, A5 and X are not collinear
- Points A2 and A3 are not identical
- Points A4, A5 and A0 are not collinear

- Points A2, A1, A5 and A0 are not collinear
- Points A1, A4, A3 and A5 are not collinear
- Line through points M4 and M3 is not parallel with line through points M2 and M1
- Line through points A1 and A0 is not parallel with line through points M2 and M1
- Line through points M8 and M7 is not parallel with line through points M6 and M5
- Line through points A1 and A0 is not parallel with line through points M6 and M5
- Line through points M9 and N1 is not parallel with line through points N3 and N2
- Line through points A1 and A0 is not parallel with line through points N3 and N2

Time spent for processing NDG Conditions

- 20.592 seconds