

# OpenGeoProver Output for conjecture “Chou 009”

Wu’s method used

February 12, 2012

## 1 Validation of Construction Protocol

**Construction steps:**

- Free point A0
- Free point A1
- Free point A2
- Free point A3
- Free point A4
- Line A0A1 through two points A0 and A1
- Line A2A3 through two points A2 and A3
- Intersection point A of point sets A0A1 and A2A3
- Line A2A4 through two points A2 and A4
- Intersection point B of point sets A0A1 and A2A4
- Line A0A2 through two points A0 and A2
- Line A1A3 through two points A1 and A3
- Intersection point C of point sets A0A2 and A1A3
- Line A1A4 through two points A1 and A4
- Intersection point D of point sets A0A2 and A1A4
- Line A0A3 through two points A0 and A3
- Line A1A2 through two points A1 and A2
- Intersection point E of point sets A0A3 and A1A2
- Line A0A4 through two points A0 and A4
- Intersection point F of point sets A0A4 and A1A2

- Line AB through two points A and B
- Line DE through two points D and E
- Line BC through two points B and C
- Line EF through two points E and F
- Line CD through two points C and D
- Line FA through two points F and A
- Intersection point P of point sets AB and DE
- Intersection point Q of point sets BC and EF
- Intersection point R of point sets CD and FA

**Theorem statement:**

- Points P, Q, R are collinear

**Validation result:** Construction protocol is valid.

## 2 Transformation of Construction Protocol to algebraic form

### Transformation of Construction steps

#### 2.1 Transformation of point A0:

- Point A0 has been assigned following coordinates:  $(0, 0)$

#### 2.2 Transformation of point A1:

- Point A1 has been assigned following coordinates:  $(0, u_1)$

#### 2.3 Transformation of point A2:

- Point A2 has been assigned following coordinates:  $(u_2, u_3)$

#### 2.4 Transformation of point A3:

- Point A3 has been assigned following coordinates:  $(u_4, u_5)$

#### 2.5 Transformation of point A4:

- Point A4 has been assigned following coordinates:  $(u_6, u_7)$

## 2.6 Transformation of point A:

- Point A has been assigned following coordinates:  $(x_1, x_2)$
- Polynomial that point A has to satisfy is:

$$p = x_1$$

- Processing of polynomial

$$p = x_1$$

**Info:** Will try to rename X coordinate of point A

**Info:** Y coordinate of point A will be replaced by X coordinate

**Info:** X coordinate of point A renamed by zero

- Point A has been renamed. Point A has been assigned following coordinates:  $(0, x_1)$
- Polynomial that point A has to satisfy is:

$$p = (u_4 - u_2)x_1 + (u_5u_2 - u_4u_3)$$

- Processing of polynomial

$$p = (u_4 - u_2)x_1 + (u_5u_2 - u_4u_3)$$

**Info:** Polynomial

$$p = (u_4 - u_2)x_1 + (u_5u_2 - u_4u_3)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.7 Transformation of point B:

- Point B has been assigned following coordinates:  $(x_2, x_3)$
- Polynomial that point B has to satisfy is:

$$p = x_2$$

- Processing of polynomial

$$p = x_2$$

**Info:** Will try to rename X coordinate of point B

**Info:** Y coordinate of point B will be replaced by X coordinate

**Info:** X coordinate of point B renamed by zero

- Point B has been renamed. Point B has been assigned following coordinates:  $(0, x_2)$

- Polynomial that point B has to satisfy is:

$$p = (u_6 - u_2)x_2 + (u_7u_2 - u_6u_3)$$

- Processing of polynomial

$$p = (u_6 - u_2)x_2 + (u_7u_2 - u_6u_3)$$

**Info:** Polynomial

$$p = (u_6 - u_2)x_2 + (u_7u_2 - u_6u_3)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.8 Transformation of point C:

- Point C has been assigned following coordinates:  $(x_3, x_4)$
- Polynomial that point C has to satisfy is:

$$p = u_2x_4 - u_3x_3$$

- Processing of polynomial

$$p = u_2x_4 - u_3x_3$$

**Info:** Polynomial

$$p = u_2x_4 - u_3x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point C has to satisfy is:

$$p = u_4x_4 + (-u_5 + u_1)x_3 - u_4u_1$$

- Processing of polynomial

$$p = u_4x_4 + (-u_5 + u_1)x_3 - u_4u_1$$

**Info:** Polynomial

$$p = u_4x_4 + (-u_5 + u_1)x_3 - u_4u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.9 Transformation of point D:

- Point D has been assigned following coordinates:  $(x_5, x_6)$
- Polynomial that point D has to satisfy is:

$$p = u_2x_6 - u_3x_5$$

- Processing of polynomial

$$p = u_2x_6 - u_3x_5$$

**Info:** Polynomial

$$p = u_2x_6 - u_3x_5$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point D has to satisfy is:

$$p = u_6x_6 + (-u_7 + u_1)x_5 - u_6u_1$$

- Processing of polynomial

$$p = u_6x_6 + (-u_7 + u_1)x_5 - u_6u_1$$

**Info:** Polynomial

$$p = u_6x_6 + (-u_7 + u_1)x_5 - u_6u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.10 Transformation of point E:

- Point E has been assigned following coordinates:  $(x_7, x_8)$
- Polynomial that point E has to satisfy is:

$$p = u_4x_8 - u_5x_7$$

- Processing of polynomial

$$p = u_4x_8 - u_5x_7$$

**Info:** Polynomial

$$p = u_4x_8 - u_5x_7$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point E has to satisfy is:

$$p = u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1$$

- Processing of polynomial

$$p = u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1$$

**Info:** Polynomial

$$p = u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.11 Transformation of point F:

- Point F has been assigned following coordinates:  $(x_9, x_{10})$
- Polynomial that point F has to satisfy is:

$$p = u_6x_{10} - u_7x_9$$

- Processing of polynomial

$$p = u_6x_{10} - u_7x_9$$

**Info:** Polynomial

$$p = u_6x_{10} - u_7x_9$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point F has to satisfy is:

$$p = u_2x_{10} + (-u_3 + u_1)x_9 - u_2u_1$$

- Processing of polynomial

$$p = u_2x_{10} + (-u_3 + u_1)x_9 - u_2u_1$$

**Info:** Polynomial

$$p = u_2x_{10} + (-u_3 + u_1)x_9 - u_2u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

### 2.12 Transformation of point P:

- Point P has been assigned following coordinates:  $(x_{11}, x_{12})$
- Polynomial that point P has to satisfy is:

$$p = x_{11}x_2 - x_{11}x_1$$

- Processing of polynomial

$$p = x_{11}x_2 - x_{11}x_1$$

**Info:** Polynomial

$$p = x_{11}x_2 - x_{11}x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point P has to satisfy is:

$$p = x_{12}x_7 - x_{12}x_5 - x_{11}x_8 + x_{11}x_6 + x_8x_5 - x_7x_6$$

- Processing of polynomial

$$p = x_{12}x_7 - x_{12}x_5 - x_{11}x_8 + x_{11}x_6 + x_8x_5 - x_7x_6$$

**Info:** Polynomial

$$p = x_{12}x_7 - x_{12}x_5 - x_{11}x_8 + x_{11}x_6 + x_8x_5 - x_7x_6$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

### 2.13 Transformation of point Q:

- Point Q has been assigned following coordinates:  $(x_{13}, x_{14})$
- Polynomial that point Q has to satisfy is:

$$p = x_{14}x_3 - x_{13}x_4 + x_{13}x_2 - x_3x_2$$

- Processing of polynomial

$$p = x_{14}x_3 - x_{13}x_4 + x_{13}x_2 - x_3x_2$$

**Info:** Polynomial

$$p = x_{14}x_3 - x_{13}x_4 + x_{13}x_2 - x_3x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point Q has to satisfy is:

$$p = x_{14}x_9 - x_{14}x_7 - x_{13}x_{10} + x_{13}x_8 + x_{10}x_7 - x_9x_8$$

- Processing of polynomial

$$p = x_{14}x_9 - x_{14}x_7 - x_{13}x_{10} + x_{13}x_8 + x_{10}x_7 - x_9x_8$$

**Info:** Polynomial

$$p = x_{14}x_9 - x_{14}x_7 - x_{13}x_{10} + x_{13}x_8 + x_{10}x_7 - x_9x_8$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## 2.14 Transformation of point R:

- Point R has been assigned following coordinates:  $(x_{15}, x_{16})$
- Polynomial that point R has to satisfy is:

$$p = x_{16}x_5 - x_{16}x_3 - x_{15}x_6 + x_{15}x_4 + x_6x_3 - x_5x_4$$

- Processing of polynomial

$$p = x_{16}x_5 - x_{16}x_3 - x_{15}x_6 + x_{15}x_4 + x_6x_3 - x_5x_4$$

**Info:** Polynomial

$$p = x_{16}x_5 - x_{16}x_3 - x_{15}x_6 + x_{15}x_4 + x_6x_3 - x_5x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point R has to satisfy is:

$$p = x_{16}x_9 - x_{15}x_{10} + x_{15}x_1 - x_9x_1$$

- Processing of polynomial

$$p = x_{16}x_9 - x_{15}x_{10} + x_{15}x_1 - x_9x_1$$

**Info:** Polynomial

$$p = x_{16}x_9 - x_{15}x_{10} + x_{15}x_1 - x_9x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

## Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_{16}x_{13} - x_{16}x_{11} - x_{15}x_{14} + x_{15}x_{12} + x_{14}x_{11} - x_{13}x_{12}$$



## Time spent for transformation of Construction Protocol to algebraic form

- 0.172 seconds

### 3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned} p_1 &= (u_4 - u_2)x_1 + (u_5u_2 - u_4u_3) \\ p_2 &= (u_6 - u_2)x_2 + (u_7u_2 - u_6u_3) \\ p_3 &= u_2x_4 - u_3x_3 \\ p_4 &= u_4x_4 + (-u_5 + u_1)x_3 - u_4u_1 \\ p_5 &= u_2x_6 - u_3x_5 \\ p_6 &= u_6x_6 + (-u_7 + u_1)x_5 - u_6u_1 \\ p_7 &= u_4x_8 - u_5x_7 \\ p_8 &= u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1 \\ p_9 &= u_6x_{10} - u_7x_9 \\ p_{10} &= u_2x_{10} + (-u_3 + u_1)x_9 - u_2u_1 \\ p_{11} &= x_{11}x_2 - x_{11}x_1 \\ p_{12} &= x_{12}x_7 - x_{12}x_5 - x_{11}x_8 + x_{11}x_6 + x_8x_5 - x_7x_6 \\ p_{13} &= x_{14}x_3 - x_{13}x_4 + x_{13}x_2 - x_3x_2 \\ p_{14} &= x_{14}x_9 - x_{14}x_7 - x_{13}x_{10} + x_{13}x_8 + x_{10}x_7 - x_9x_8 \\ p_{15} &= x_{16}x_5 - x_{16}x_3 - x_{15}x_6 + x_{15}x_4 + x_6x_3 - x_5x_4 \\ p_{16} &= x_{16}x_9 - x_{15}x_{10} + x_{15}x_1 - x_9x_1 \end{aligned}$$

#### 3.1 Triangulation, step 1

**Choosing variable:** Trying the variable with index 16.

**Variable  $x_{16}$  selected:** The number of polynomials with this variable, with indexes from 1 to 16, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{16}$  from all other polynomials by reducing them with polynomial  $p_{15}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_1 &= (u_4 - u_2)x_1 + (u_5u_2 - u_4u_3) \\ p_2 &= (u_6 - u_2)x_2 + (u_7u_2 - u_6u_3) \\ p_3 &= u_2x_4 - u_3x_3 \\ p_4 &= u_4x_4 + (-u_5 + u_1)x_3 - u_4u_1 \end{aligned}$$

$$\begin{aligned}
p_5 &= u_2x_6 - u_3x_5 \\
p_6 &= u_6x_6 + (-u_7 + u_1)x_5 - u_6u_1 \\
p_7 &= u_4x_8 - u_5x_7 \\
p_8 &= u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1 \\
p_9 &= u_6x_{10} - u_7x_9 \\
p_{10} &= u_2x_{10} + (-u_3 + u_1)x_9 - u_2u_1 \\
p_{11} &= x_{11}x_2 - x_{11}x_1 \\
p_{12} &= x_{12}x_7 - x_{12}x_5 - x_{11}x_8 + x_{11}x_6 + x_8x_5 - x_7x_6 \\
p_{13} &= x_{14}x_3 - x_{13}x_4 + x_{13}x_2 - x_3x_2 \\
p_{14} &= x_{14}x_9 - x_{14}x_7 - x_{13}x_{10} + x_{13}x_8 + x_{10}x_7 - x_9x_8 \\
p_{15} &= -x_{15}x_{10}x_5 + x_{15}x_{10}x_3 + x_{15}x_9x_6 - x_{15}x_9x_4 + x_{15}x_5x_1 \\
&\quad - x_{15}x_3x_1 - x_9x_6x_3 + x_9x_5x_4 - x_9x_5x_1 + x_9x_3x_1 \\
p_{16} &= x_{16}x_5 - x_{16}x_3 - x_{15}x_6 + x_{15}x_4 + x_6x_3 - x_5x_4
\end{aligned}$$

### 3.2 Triangulation, step 2

**Choosing variable:** Trying the variable with index 15.

**Variable  $x_{15}$  selected:** The number of polynomials with this variable, with indexes from 1 to 15, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{15}$ . No reduction needed.

The triangular system has not been changed.

### 3.3 Triangulation, step 3

**Choosing variable:** Trying the variable with index 14.

**Variable  $x_{14}$  selected:** The number of polynomials with this variable, with indexes from 1 to 14, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{14}$  from all other polynomials by reducing them with polynomial  $p_{13}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_4 - u_2)x_1 + (u_5u_2 - u_4u_3) \\
p_2 &= (u_6 - u_2)x_2 + (u_7u_2 - u_6u_3) \\
p_3 &= u_2x_4 - u_3x_3 \\
p_4 &= u_4x_4 + (-u_5 + u_1)x_3 - u_4u_1 \\
p_5 &= u_2x_6 - u_3x_5 \\
p_6 &= u_6x_6 + (-u_7 + u_1)x_5 - u_6u_1 \\
p_7 &= u_4x_8 - u_5x_7
\end{aligned}$$

$$\begin{aligned}
p_8 &= u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1 \\
p_9 &= u_6x_{10} - u_7x_9 \\
p_{10} &= u_2x_{10} + (-u_3 + u_1)x_9 - u_2u_1 \\
p_{11} &= x_{11}x_2 - x_{11}x_1 \\
p_{12} &= x_{12}x_7 - x_{12}x_5 - x_{11}x_8 + x_{11}x_6 + x_8x_5 - x_7x_6 \\
p_{13} &= -x_{13}x_{10}x_3 + x_{13}x_9x_4 - x_{13}x_9x_2 + x_{13}x_8x_3 - x_{13}x_7x_4 + \\
&\quad x_{13}x_7x_2 + x_{10}x_7x_3 - x_9x_8x_3 + x_9x_3x_2 - x_7x_3x_2 \\
p_{14} &= x_{14}x_3 - x_{13}x_4 + x_{13}x_2 - x_3x_2 \\
p_{15} &= -x_{15}x_{10}x_5 + x_{15}x_{10}x_3 + x_{15}x_9x_6 - x_{15}x_9x_4 + x_{15}x_5x_1 \\
&\quad - x_{15}x_3x_1 - x_9x_6x_3 + x_9x_5x_4 - x_9x_5x_1 + x_9x_3x_1 \\
p_{16} &= x_{16}x_5 - x_{16}x_3 - x_{15}x_6 + x_{15}x_4 + x_6x_3 - x_5x_4
\end{aligned}$$

### 3.4 Triangulation, step 4

**Choosing variable:** Trying the variable with index 13.

**Variable  $x_{13}$  selected:** The number of polynomials with this variable, with indexes from 1 to 13, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{13}$ . No reduction needed.

The triangular system has not been changed.

### 3.5 Triangulation, step 5

**Choosing variable:** Trying the variable with index 12.

**Variable  $x_{12}$  selected:** The number of polynomials with this variable, with indexes from 1 to 12, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{12}$ . No reduction needed.

The triangular system has not been changed.

### 3.6 Triangulation, step 6

**Choosing variable:** Trying the variable with index 11.

**Variable  $x_{11}$  selected:** The number of polynomials with this variable, with indexes from 1 to 11, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{11}$ . No reduction needed.

The triangular system has not been changed.

### 3.7 Triangulation, step 7

**Choosing variable:** Trying the variable with index 10.

**Variable  $x_{10}$  selected:** The number of polynomials with this variable, with indexes from 1 to 10, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{10}$  from all other polynomials by reducing them with polynomial  $p_9$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_4 - u_2)x_1 + (u_5u_2 - u_4u_3) \\
p_2 &= (u_6 - u_2)x_2 + (u_7u_2 - u_6u_3) \\
p_3 &= u_2x_4 - u_3x_3 \\
p_4 &= u_4x_4 + (-u_5 + u_1)x_3 - u_4u_1 \\
p_5 &= u_2x_6 - u_3x_5 \\
p_6 &= u_6x_6 + (-u_7 + u_1)x_5 - u_6u_1 \\
p_7 &= u_4x_8 - u_5x_7 \\
p_8 &= u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1 \\
p_9 &= (u_7u_2 - u_6u_3 + u_6u_1)x_9 - u_6u_2u_1 \\
p_{10} &= u_6x_{10} - u_7x_9 \\
p_{11} &= x_{11}x_2 - x_{11}x_1 \\
p_{12} &= x_{12}x_7 - x_{12}x_5 - x_{11}x_8 + x_{11}x_6 + x_8x_5 - x_7x_6 \\
p_{13} &= -x_{13}x_{10}x_3 + x_{13}x_9x_4 - x_{13}x_9x_2 + x_{13}x_8x_3 - x_{13}x_7x_4 + \\
&\quad x_{13}x_7x_2 + x_{10}x_7x_3 - x_9x_8x_3 + x_9x_3x_2 - x_7x_3x_2 \\
p_{14} &= x_{14}x_3 - x_{13}x_4 + x_{13}x_2 - x_3x_2 \\
p_{15} &= -x_{15}x_{10}x_5 + x_{15}x_{10}x_3 + x_{15}x_9x_6 - x_{15}x_9x_4 + x_{15}x_5x_1 \\
&\quad -x_{15}x_3x_1 - x_9x_6x_3 + x_9x_5x_4 - x_9x_5x_1 + x_9x_3x_1 \\
p_{16} &= x_{16}x_5 - x_{16}x_3 - x_{15}x_6 + x_{15}x_4 + x_6x_3 - x_5x_4
\end{aligned}$$

### 3.8 Triangulation, step 8

**Choosing variable:** Trying the variable with index 9.

**Variable  $x_9$  selected:** The number of polynomials with this variable, with indexes from 1 to 9, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_9$ . No reduction needed.

The triangular system has not been changed.

### 3.9 Triangulation, step 9

**Choosing variable:** Trying the variable with index 8.

**Variable  $x_8$  selected:** The number of polynomials with this variable, with indexes from 1 to 8, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_8$  from all other polynomials by reducing them with polynomial  $p_7$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_4 - u_2)x_1 + (u_5u_2 - u_4u_3) \\
p_2 &= (u_6 - u_2)x_2 + (u_7u_2 - u_6u_3) \\
p_3 &= u_2x_4 - u_3x_3 \\
p_4 &= u_4x_4 + (-u_5 + u_1)x_3 - u_4u_1 \\
p_5 &= u_2x_6 - u_3x_5 \\
p_6 &= u_6x_6 + (-u_7 + u_1)x_5 - u_6u_1 \\
p_7 &= (u_5u_2 - u_4u_3 + u_4u_1)x_7 - u_4u_2u_1 \\
p_8 &= u_4x_8 - u_5x_7 \\
p_9 &= (u_7u_2 - u_6u_3 + u_6u_1)x_9 - u_6u_2u_1 \\
p_{10} &= u_6x_{10} - u_7x_9 \\
p_{11} &= x_{11}x_2 - x_{11}x_1 \\
p_{12} &= x_{12}x_7 - x_{12}x_5 - x_{11}x_8 + x_{11}x_6 + x_8x_5 - x_7x_6 \\
p_{13} &= -x_{13}x_{10}x_3 + x_{13}x_9x_4 - x_{13}x_9x_2 + x_{13}x_8x_3 - x_{13}x_7x_4 + \\
&\quad x_{13}x_7x_2 + x_{10}x_7x_3 - x_9x_8x_3 + x_9x_3x_2 - x_7x_3x_2 \\
p_{14} &= x_{14}x_3 - x_{13}x_4 + x_{13}x_2 - x_3x_2 \\
p_{15} &= -x_{15}x_{10}x_5 + x_{15}x_{10}x_3 + x_{15}x_9x_6 - x_{15}x_9x_4 + x_{15}x_5x_1 \\
&\quad - x_{15}x_3x_1 - x_9x_6x_3 + x_9x_5x_4 - x_9x_5x_1 + x_9x_3x_1 \\
p_{16} &= x_{16}x_5 - x_{16}x_3 - x_{15}x_6 + x_{15}x_4 + x_6x_3 - x_5x_4
\end{aligned}$$

### 3.10 Triangulation, step 10

**Choosing variable:** Trying the variable with index 7.

**Variable  $x_7$  selected:** The number of polynomials with this variable, with indexes from 1 to 7, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_7$ . No reduction needed.

The triangular system has not been changed.

### 3.11 Triangulation, step 11

**Choosing variable:** Trying the variable with index 6.

**Variable  $x_6$  selected:** The number of polynomials with this variable, with indexes from 1 to 6, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_6$  from all other polynomials by reducing them with polynomial  $p_5$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_1 &= (u_4 - u_2)x_1 + (u_5u_2 - u_4u_3) \\ p_2 &= (u_6 - u_2)x_2 + (u_7u_2 - u_6u_3) \\ p_3 &= u_2x_4 - u_3x_3 \\ p_4 &= u_4x_4 + (-u_5 + u_1)x_3 - u_4u_1 \\ p_5 &= (-u_7u_2 + u_6u_3 + u_2u_1)x_5 - u_6u_2u_1 \\ p_6 &= u_2x_6 - u_3x_5 \\ p_7 &= (u_5u_2 - u_4u_3 + u_4u_1)x_7 - u_4u_2u_1 \\ p_8 &= u_4x_8 - u_5x_7 \\ p_9 &= (u_7u_2 - u_6u_3 + u_6u_1)x_9 - u_6u_2u_1 \\ p_{10} &= u_6x_{10} - u_7x_9 \\ p_{11} &= x_{11}x_2 - x_{11}x_1 \\ p_{12} &= x_{12}x_7 - x_{12}x_5 - x_{11}x_8 + x_{11}x_6 + x_8x_5 - x_7x_6 \\ p_{13} &= -x_{13}x_{10}x_3 + x_{13}x_9x_4 - x_{13}x_9x_2 + x_{13}x_8x_3 - x_{13}x_7x_4 + \\ &\quad x_{13}x_7x_2 + x_{10}x_7x_3 - x_9x_8x_3 + x_9x_3x_2 - x_7x_3x_2 \\ p_{14} &= x_{14}x_3 - x_{13}x_4 + x_{13}x_2 - x_3x_2 \\ p_{15} &= -x_{15}x_{10}x_5 + x_{15}x_{10}x_3 + x_{15}x_9x_6 - x_{15}x_9x_4 + x_{15}x_5x_1 \\ &\quad -x_{15}x_3x_1 - x_9x_6x_3 + x_9x_5x_4 - x_9x_5x_1 + x_9x_3x_1 \\ p_{16} &= x_{16}x_5 - x_{16}x_3 - x_{15}x_6 + x_{15}x_4 + x_6x_3 - x_5x_4 \end{aligned}$$

### 3.12 Triangulation, step 12

**Choosing variable:** Trying the variable with index 5.

**Variable  $x_5$  selected:** The number of polynomials with this variable, with indexes from 1 to 5, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_5$ . No reduction needed.

The triangular system has not been changed.

### 3.13 Triangulation, step 13

**Choosing variable:** Trying the variable with index 4.

**Variable  $x_4$  selected:** The number of polynomials with this variable, with indexes from 1 to 4, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_4$  from all other polynomials by reducing them with polynomial  $p_3$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_4 - u_2)x_1 + (u_5u_2 - u_4u_3) \\
p_2 &= (u_6 - u_2)x_2 + (u_7u_2 - u_6u_3) \\
p_3 &= (-u_5u_2 + u_4u_3 + u_2u_1)x_3 - u_4u_2u_1 \\
p_4 &= u_2x_4 - u_3x_3 \\
p_5 &= (-u_7u_2 + u_6u_3 + u_2u_1)x_5 - u_6u_2u_1 \\
p_6 &= u_2x_6 - u_3x_5 \\
p_7 &= (u_5u_2 - u_4u_3 + u_4u_1)x_7 - u_4u_2u_1 \\
p_8 &= u_4x_8 - u_5x_7 \\
p_9 &= (u_7u_2 - u_6u_3 + u_6u_1)x_9 - u_6u_2u_1 \\
p_{10} &= u_6x_{10} - u_7x_9 \\
p_{11} &= x_{11}x_2 - x_{11}x_1 \\
p_{12} &= x_{12}x_7 - x_{12}x_5 - x_{11}x_8 + x_{11}x_6 + x_8x_5 - x_7x_6 \\
p_{13} &= -x_{13}x_{10}x_3 + x_{13}x_9x_4 - x_{13}x_9x_2 + x_{13}x_8x_3 - x_{13}x_7x_4 + \\
&\quad x_{13}x_7x_2 + x_{10}x_7x_3 - x_9x_8x_3 + x_9x_3x_2 - x_7x_3x_2 \\
p_{14} &= x_{14}x_3 - x_{13}x_4 + x_{13}x_2 - x_3x_2 \\
p_{15} &= -x_{15}x_{10}x_5 + x_{15}x_{10}x_3 + x_{15}x_9x_6 - x_{15}x_9x_4 + x_{15}x_5x_1 \\
&\quad -x_{15}x_3x_1 - x_9x_6x_3 + x_9x_5x_4 - x_9x_5x_1 + x_9x_3x_1 \\
p_{16} &= x_{16}x_5 - x_{16}x_3 - x_{15}x_6 + x_{15}x_4 + x_6x_3 - x_5x_4
\end{aligned}$$

### 3.14 Triangulation, step 14

**Choosing variable:** Trying the variable with index 3.

**Variable  $x_3$  selected:** The number of polynomials with this variable, with indexes from 1 to 3, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_3$ . No reduction needed.

The triangular system has not been changed.

### 3.15 Triangulation, step 15

**Choosing variable:** Trying the variable with index 2.

**Variable  $x_2$  selected:** The number of polynomials with this variable, with indexes from 1 to 2, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_2$ . No reduction needed.

The triangular system has not been changed.

### 3.16 Triangulation, step 16

**Choosing variable:** Trying the variable with index 1.

**Variable  $x_1$  selected:** The number of polynomials with this variable, with indexes from 1 to 1, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_1$ . No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned} p_1 &= (u_4 - u_2)x_1 + (u_5u_2 - u_4u_3) \\ p_2 &= (u_6 - u_2)x_2 + (u_7u_2 - u_6u_3) \\ p_3 &= (-u_5u_2 + u_4u_3 + u_2u_1)x_3 - u_4u_2u_1 \\ p_4 &= u_2x_4 - u_3x_3 \\ p_5 &= (-u_7u_2 + u_6u_3 + u_2u_1)x_5 - u_6u_2u_1 \\ p_6 &= u_2x_6 - u_3x_5 \\ p_7 &= (u_5u_2 - u_4u_3 + u_4u_1)x_7 - u_4u_2u_1 \\ p_8 &= u_4x_8 - u_5x_7 \\ p_9 &= (u_7u_2 - u_6u_3 + u_6u_1)x_9 - u_6u_2u_1 \\ p_{10} &= u_6x_{10} - u_7x_9 \\ p_{11} &= x_{11}x_2 - x_{11}x_1 \\ p_{12} &= x_{12}x_7 - x_{12}x_5 - x_{11}x_8 + x_{11}x_6 + x_8x_5 - x_7x_6 \\ p_{13} &= -x_{13}x_{10}x_3 + x_{13}x_9x_4 - x_{13}x_9x_2 + x_{13}x_8x_3 - x_{13}x_7x_4 + \\ &\quad x_{13}x_7x_2 + x_{10}x_7x_3 - x_9x_8x_3 + x_9x_3x_2 - x_7x_3x_2 \\ p_{14} &= x_{14}x_3 - x_{13}x_4 + x_{13}x_2 - x_3x_2 \\ p_{15} &= -x_{15}x_{10}x_5 + x_{15}x_{10}x_3 + x_{15}x_9x_6 - x_{15}x_9x_4 + x_{15}x_5x_1 \\ &\quad -x_{15}x_3x_1 - x_9x_6x_3 + x_9x_5x_4 - x_9x_5x_1 + x_9x_3x_1 \\ p_{16} &= x_{16}x_5 - x_{16}x_3 - x_{15}x_6 + x_{15}x_4 + x_6x_3 - x_5x_4 \end{aligned}$$



## 4 Final Remainder

### 4.1 Final remainder for conjecture Chou 009

Calculating final remainder of the conclusion:

$$g = x_{16}x_{13} - x_{16}x_{11} - x_{15}x_{14} + x_{15}x_{12} + x_{14}x_{11} - x_{13}x_{12}$$

with respect to the triangular system.

1. Pseudo remainder with  $p_{16}$  over variable  $x_{16}$ :

$$\begin{aligned} g = & -x_{15}x_{14}x_5 + x_{15}x_{14}x_3 + x_{15}x_{13}x_6 - x_{15}x_{13}x_4 + x_{15}x_{12}x_5 \\ & -x_{15}x_{12}x_3 - x_{15}x_{11}x_6 + x_{15}x_{11}x_4 + x_{14}x_{11}x_5 - x_{14}x_{11}x_3 \\ & -x_{13}x_{12}x_5 + x_{13}x_{12}x_3 - x_{13}x_6x_3 + x_{13}x_5x_4 + x_{11}x_6x_3 \\ & -x_{11}x_5x_4 \end{aligned}$$

2. Pseudo remainder with  $p_{15}$  over variable  $x_{15}$ :

$$\begin{aligned} g = & -x_{14}x_{11}x_{10}x_5^2 + 2x_{14}x_{11}x_{10}x_5x_3 - x_{14}x_{11}x_{10}x_3^2 + \\ & x_{14}x_{11}x_9x_6x_5 - x_{14}x_{11}x_9x_6x_3 - x_{14}x_{11}x_9x_5x_4 + \\ & x_{14}x_{11}x_9x_4x_3 + x_{14}x_{11}x_5^2x_1 - 2x_{14}x_{11}x_5x_3x_1 + \\ & x_{14}x_{11}x_3^2x_1 - x_{14}x_9x_6x_5x_3 + x_{14}x_9x_6x_3^2 + \\ & x_{14}x_9x_5^2x_4 - x_{14}x_9x_5^2x_1 - x_{14}x_9x_5x_4x_3 + \\ & 2x_{14}x_9x_5x_3x_1 - x_{14}x_9x_3^2x_1 + x_{13}x_{12}x_{10}x_5^2 \\ & - 2x_{13}x_{12}x_{10}x_5x_3 + x_{13}x_{12}x_{10}x_3^2 - x_{13}x_{12}x_9x_6x_5 + \\ & x_{13}x_{12}x_9x_6x_3 + x_{13}x_{12}x_9x_5x_4 - x_{13}x_{12}x_9x_4x_3 \\ & - x_{13}x_{12}x_5^2x_1 + 2x_{13}x_{12}x_5x_3x_1 - x_{13}x_{12}x_3^2x_1 + \\ & x_{13}x_{10}x_6x_5x_3 - x_{13}x_{10}x_6x_3^2 - x_{13}x_{10}x_5^2x_4 + \\ & x_{13}x_{10}x_5x_4x_3 + x_{13}x_9x_6x_5x_1 - x_{13}x_9x_6x_3x_1 \\ & - x_{13}x_9x_5x_4x_1 + x_{13}x_9x_4x_3x_1 - x_{13}x_6x_5x_3x_1 + \\ & x_{13}x_6x_3^2x_1 + x_{13}x_5^2x_4x_1 - x_{13}x_5x_4x_3x_1 + \\ & x_{12}x_9x_6x_5x_3 - x_{12}x_9x_6x_3^2 - x_{12}x_9x_5^2x_4 + \\ & x_{12}x_9x_5^2x_1 + x_{12}x_9x_5x_4x_3 - 2x_{12}x_9x_5x_3x_1 + \\ & x_{12}x_9x_3^2x_1 - x_{11}x_{10}x_6x_5x_3 + x_{11}x_{10}x_6x_3^2 + \\ & x_{11}x_{10}x_5^2x_4 - x_{11}x_{10}x_5x_4x_3 - x_{11}x_9x_6x_5x_1 + \\ & x_{11}x_9x_6x_3x_1 + x_{11}x_9x_5x_4x_1 - x_{11}x_9x_4x_3x_1 + \\ & x_{11}x_6x_5x_3x_1 - x_{11}x_6x_3^2x_1 - x_{11}x_5^2x_4x_1 + \\ & x_{11}x_5x_4x_3x_1 \end{aligned}$$

3. Pseudo remainder with  $p_{14}$  over variable  $x_{14}$ :

*Polynomial too big for output (text size is 2939 characters, number of terms is 89)*

4. Pseudo remainder with  $p_{13}$  over variable  $x_{13}$ :  
*Polynomial too big for output (number of terms is 266)*
5. Pseudo remainder with  $p_{12}$  over variable  $x_{12}$ :  
*Polynomial too big for output (number of terms is 439)*
6. Pseudo remainder with  $p_{11}$  over variable  $x_{11}$ :  
*Polynomial too big for output (number of terms is 268)*
7. Pseudo remainder with  $p_{10}$  over variable  $x_{10}$ :  
*Polynomial too big for output (number of terms is 268)*
8. Pseudo remainder with  $p_9$  over variable  $x_9$ :  
*Polynomial too big for output (number of terms is 268)*
9. Pseudo remainder with  $p_8$  over variable  $x_8$ :  
*Polynomial too big for output (text size is 30410 characters, number of terms is 216)*
10. Pseudo remainder with  $p_7$  over variable  $x_7$ :  
*Polynomial too big for output (text size is 32336 characters, number of terms is 130)*
11. Pseudo remainder with  $p_6$  over variable  $x_6$ :  
*Polynomial too big for output (text size is 28702 characters, number of terms is 90)*
12. Pseudo remainder with  $p_5$  over variable  $x_5$ :  
*Polynomial too big for output (text size is 50326 characters, number of terms is 38)*
13. Pseudo remainder with  $p_4$  over variable  $x_4$ :  
*Polynomial too big for output (text size is 25635 characters, number of terms is 20)*
14. Pseudo remainder with  $p_3$  over variable  $x_3$ :  
*Polynomial too big for output (text size is 23540 characters, number of terms is 6)*
15. Pseudo remainder with  $p_2$  over variable  $x_2$ :  
*Polynomial too big for output (text size is greater than 2000 characters, number of terms is 3)*
16. Pseudo remainder with  $p_1$  over variable  $x_1$ :

$$g = 0$$

## 5 Prover results

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 439 terms.

**Time Complexity:** Time spent by the prover is 1.65 seconds.

## 6 NDG Conditions

### NDG Conditions in readable form

- Points A2, A3, B and A0 are not collinear
- Points A2 and A4 are not identical
- Points A2, A1 and A3 are not collinear
- Points A2, A and A0 are not collinear
- Points A2, A1, A4 and B are not collinear
- Points A2, A1 and A3 are not collinear
- Points A3, A and B are not collinear
- Points A2, A1, A4 and B are not collinear
- Points A4, A, B and A0 are not collinear
- Line through points A1 and A0 is not perpendicular to line through points A and B
- Line through points D and E is not parallel with line through points A1 and A0
- Line through points E and F is not parallel with line through points B and C
- Points A1, C and A0 are not collinear
- Line through points D and C is not parallel with line through points F and A
- Line through points D and C is not parallel with line through points A1 and A0

### Time spent for processing NDG Conditions

- 2.438 seconds