OpenGeoProver Output for conjecture "Polar 002"

Wu's method used

February 12, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point O
- Free point M
- Circle c with center O and one point M
- Free point P
- Random point A from circle c
- Random point C from circle c
- Random point E from circle c
- Line s1 through two points P and A
- Line s2 through two points P and C
- Line s3 through two points P and E
- Intersection point B of point sets s1 and c
- Intersection point D of point sets s2 and c
- Intersection point F of point sets s3 and c
- Fourth harmonic conjugate point G of tripple of points A, B and P
- Fourth harmonic conjugate point H of tripple of points C, D and P
- Fourth harmonic conjugate point I of tripple of points E, F and P

Theorem statement:

- Points G, H, I are collinear

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point O:

- Point O has been assigned following coordinates: (0, 0)

2.2 Transformation of point M:

- Point M has been assigned following coordinates: (0, u_1)

2.3 Transformation of point P:

- Point P has been assigned following coordinates: (u_2, u_3)

2.4 Transformation of point A:

- Point A has been assigned following coordinates: (u_4, x_1)
- Polynomial that point A has to satisfy is:

$$p = x_1^2 + (u_4^2 - u_1^2)$$

- Processing of polynomial

$$p = x_1^2 + (u_4^2 - u_1^2)$$

Info: Polynomial

$$p = x_1^2 + (u_4^2 - u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of point C:

- Point C has been assigned following coordinates: (u_5, x_2)
- Polynomial that point C has to satisfy is:

$$p = x_2^2 + (u_5^2 - u_1^2)$$

- Processing of polynomial

$$p = x_2^2 + (u_5^2 - u_1^2)$$

Info: Polynomial

$$p = x_2^2 + (u_5^2 - u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point E:

- Point E has been assigned following coordinates: (u_6, x_3)
- Polynomial that point E has to satisfy is:

$$p = x_3^2 + (u_6^2 - u_1^2)$$

- Processing of polynomial

$$p = x_3^2 + (u_6^2 - u_1^2)$$

Info: Polynomial

$$p = x_3^2 + (u_6^2 - u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point B:

- Point B has been assigned following coordinates: (x_4, x_5)
- Polynomial that point B has to satisfy is:

$$p = (u_4 - u_2)x_5 - x_4x_1 + u_3x_4 + u_2x_1 - u_4u_3$$

- Processing of polynomial

$$p = (u_4 - u_2)x_5 - x_4x_1 + u_3x_4 + u_2x_1 - u_4u_3$$

Info: Polynomial

$$p = (u_4 - u_2)x_5 - x_4x_1 + u_3x_4 + u_2x_1 - u_4u_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point B has to satisfy is:

$$p = x_5^2 + x_4^2 - u_1^2$$

- Processing of polynomial

$$p = x_5^2 + x_4^2 - u_1^2$$

Info: Polynomial

$$p = x_5^2 + x_4^2 - u_1^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.8 Transformation of point D:

- Point D has been assigned following coordinates: (x_6, x_7)
- Polynomial that point D has to satisfy is:

$$p = (u_5 - u_2)x_7 - x_6x_2 + u_3x_6 + u_2x_2 - u_5u_3$$

- Processing of polynomial

$$p = (u_5 - u_2)x_7 - x_6x_2 + u_3x_6 + u_2x_2 - u_5u_3$$

Info: Polynomial

$$p = (u_5 - u_2)x_7 - x_6x_2 + u_3x_6 + u_2x_2 - u_5u_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point D has to satisfy is:

$$p = x_7^2 + x_6^2 - u_1^2$$

- Processing of polynomial

$$p = x_7^2 + x_6^2 - u_1^2$$

Info: Polynomial

$$p = x_7^2 + x_6^2 - u_1^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.9 Transformation of point F:

- Point F has been assigned following coordinates: (x_8, x_9)
- Polynomial that point F has to satisfy is:

$$p = (u_6 - u_2)x_9 - x_8x_3 + u_3x_8 + u_2x_3 - u_6u_3$$

- Processing of polynomial

$$p = (u_6 - u_2)x_9 - x_8x_3 + u_3x_8 + u_2x_3 - u_6u_3$$

Info: Polynomial

$$p = (u_6 - u_2)x_9 - x_8x_3 + u_3x_8 + u_2x_3 - u_6u_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point F has to satisfy is:

$$p = x_9^2 + x_8^2 - u_1^2$$

- Processing of polynomial

$$p = x_9^2 + x_8^2 - u_1^2$$

Info: Polynomial

$$p = x_9^2 + x_8^2 - u_1^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.10 Transformation of point G:

- Point G has been assigned following coordinates: (x_{10}, x_{11})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{10}x_4 + (u_4 - 2u_2)x_{10} + (-2u_4 + u_2)x_4 + u_4u_2$$

Info: Polynomial

$$p = x_{10}x_4 + (u_4 - 2u_2)x_{10} + (-2u_4 + u_2)x_4 + u_4u_2$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{10}x_4 + (u_4 - 2u_2)x_{10} + (-2u_4 + u_2)x_4 + u_4u_2$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{11}x_5 + x_{11}x_1 - 2u_3x_{11} - 2x_5x_1 + u_3x_5 + u_3x_1$$

Info: Polynomial

$$p = x_{11}x_5 + x_{11}x_1 - 2u_3x_{11} - 2x_5x_1 + u_3x_5 + u_3x_1$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{11}x_5 + x_{11}x_1 - 2u_3x_{11} - 2x_5x_1 + u_3x_5 + u_3x_1$$

is added to polynomial system

2.11 Transformation of point H:

- Point H has been assigned following coordinates: (x_{12}, x_{13})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{12}x_6 + (u_5 - 2u_2)x_{12} + (-2u_5 + u_2)x_6 + u_5u_2$$

Info: Polynomial

$$p = x_{12}x_6 + (u_5 - 2u_2)x_{12} + (-2u_5 + u_2)x_6 + u_5u_2$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{12}x_6 + (u_5 - 2u_2)x_{12} + (-2u_5 + u_2)x_6 + u_5u_2$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{13}x_7 + x_{13}x_2 - 2u_3x_{13} - 2x_7x_2 + u_3x_7 + u_3x_2$$

Info: Polynomial

$$p = x_{13}x_7 + x_{13}x_2 - 2u_3x_{13} - 2x_7x_2 + u_3x_7 + u_3x_2$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{13}x_7 + x_{13}x_2 - 2u_3x_{13} - 2x_7x_2 + u_3x_7 + u_3x_2$$

is added to polynomial system

2.12 Transformation of point I:

- Point I has been assigned following coordinates: (x_{14}, x_{15})
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_{14}x_8 + (u_6 - 2u_2)x_{14} + (-2u_6 + u_2)x_8 + u_6u_2$$

Info: Polynomial

$$p = x_{14}x_8 + (u_6 - 2u_2)x_{14} + (-2u_6 + u_2)x_8 + u_6u_2$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{14}x_8 + (u_6 - 2u_2)x_{14} + (-2u_6 + u_2)x_8 + u_6u_2$$

  is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_{15}x_9 + x_{15}x_3 - 2u_3x_{15} - 2x_9x_3 + u_3x_9 + u_3x_3$$

  Info: Polynomial

$$p = x_{15}x_9 + x_{15}x_3 - 2u_3x_{15} - 2x_9x_3 + u_3x_9 + u_3x_3$$

  added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_{15}x_9 + x_{15}x_3 - 2u_3x_{15} - 2x_9x_3 + u_3x_9 + u_3x_3$$

  is added to polynomial system

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_{15}x_{12} - x_{15}x_{10} - x_{14}x_{13} + x_{14}x_{11} + x_{13}x_{10} - x_{12}x_{11}$$

Time spent for transformation of Construction Protocol to algebraic form

- 2.9 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= x_1^2 + (u_4^2 - u_1^2) \\
p_2 &= x_2^2 + (u_5^2 - u_1^2) \\
p_3 &= x_3^2 + (u_6^2 - u_1^2) \\
p_4 &= (u_4 - u_2)x_5 - x_4x_1 + u_3x_4 + u_2x_1 - u_4u_3 \\
p_5 &= x_5^2 + x_4^2 - u_1^2 \\
p_6 &= (u_5 - u_2)x_7 - x_6x_2 + u_3x_6 + u_2x_2 - u_5u_3 \\
p_7 &= x_7^2 + x_6^2 - u_1^2 \\
p_8 &= (u_6 - u_2)x_9 - x_8x_3 + u_3x_8 + u_2x_3 - u_6u_3 \\
p_9 &= x_9^2 + x_8^2 - u_1^2
\end{aligned}$$

$$
\begin{aligned}
p_{10} &= x_{10}x_4 + (u_4 - 2u_2)x_{10} + (-2u_4 + u_2)x_4 + u_4u_2 \\
p_{11} &= x_{11}x_5 + x_{11}x_1 - 2u_3x_{11} - 2x_5x_1 + u_3x_5 + u_3x_1 \\
p_{12} &= x_{12}x_6 + (u_5 - 2u_2)x_{12} + (-2u_5 + u_2)x_6 + u_5u_2 \\
p_{13} &= x_{13}x_7 + x_{13}x_2 - 2u_3x_{13} - 2x_7x_2 + u_3x_7 + u_3x_2 \\
p_{14} &= x_{14}x_8 + (u_6 - 2u_2)x_{14} + (-2u_6 + u_2)x_8 + u_6u_2 \\
p_{15} &= x_{15}x_9 + x_{15}x_3 - 2u_3x_{15} - 2x_9x_3 + u_3x_9 + u_3x_3
\end{aligned}
$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{15}. No reduction needed.

The triangular system has not been changed.

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14}. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{13}. No reduction needed.

The triangular system has not been changed.

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{12}. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{11}. No reduction needed.

The triangular system has not been changed.

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10}. No reduction needed.

The triangular system has not been changed.

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 2.

Minimal degrees: 1 polynomial(s) with degree 1 and 1 polynomial(s) with degree 2.

Polynomial with linear degree: Removing variable x_9 from all other polynomials by reducing them with polynomial p_8 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= x_1^2 + (u_4^2 - u_1^2) \\
p_2 &= x_2^2 + (u_5^2 - u_1^2) \\
p_3 &= x_3^2 + (u_6^2 - u_1^2) \\
p_4 &= (u_4 - u_2)x_5 - x_4x_1 + u_3x_4 + u_2x_1 - u_4u_3 \\
p_5 &= x_5^2 + x_4^2 - u_1^2 \\
p_6 &= (u_5 - u_2)x_7 - x_6x_2 + u_3x_6 + u_2x_2 - u_5u_3 \\
p_7 &= x_7^2 + x_6^2 - u_1^2 \\
p_8 &= x_8^2x_3^2 - 2u_3x_8^2x_3 + \\
&\quad (u_6^2 - 2u_6u_2 + u_3^2 + u_2^2)x_8^2 - 2u_2x_8x_3^2 + \\
&\quad (2u_6u_3 + 2u_3u_2)x_8x_3 - 2u_6u_3^2x_8 + u_2^2x_3^2 \\
&\quad -2u_6u_3u_2x_3 +
\end{aligned}
$$

$$
\begin{aligned}
& (u_6^2u_3^2 - u_6^2u_1^2 + 2u_6u_2u_1^2 - u_2^2u_1^2) \\
p_9 &= (u_6 - u_2)x_9 - x_8x_3 + u_3x_8 + u_2x_3 - u_6u_3 \\
p_{10} &= x_{10}x_4 + (u_4 - 2u_2)x_{10} + (-2u_4 + u_2)x_4 + u_4u_2 \\
p_{11} &= x_{11}x_5 + x_{11}x_1 - 2u_3x_{11} - 2x_5x_1 + u_3x_5 + u_3x_1 \\
p_{12} &= x_{12}x_6 + (u_5 - 2u_2)x_{12} + (-2u_5 + u_2)x_6 + u_5u_2 \\
p_{13} &= x_{13}x_7 + x_{13}x_2 - 2u_3x_{13} - 2x_7x_2 + u_3x_7 + u_3x_2 \\
p_{14} &= x_{14}x_8 + (u_6 - 2u_2)x_{14} + (-2u_6 + u_2)x_8 + u_6u_2 \\
p_{15} &= x_{15}x_9 + x_{15}x_3 - 2u_3x_{15} - 2x_9x_3 + u_3x_9 + u_3x_3
\end{aligned}
$$

3.8 Triangulation, step 8

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8. No reduction needed.

The triangular system has not been changed.

3.9 Triangulation, step 9

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 2.

Minimal degrees: 1 polynomial(s) with degree 1 and 1 polynomial(s) with degree 2.

Polynomial with linear degree: Removing variable x_7 from all other polynomials by reducing them with polynomial p_6 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= x_1^2 + (u_4^2 - u_1^2) \\
p_2 &= x_2^2 + (u_5^2 - u_1^2) \\
p_3 &= x_3^2 + (u_6^2 - u_1^2) \\
p_4 &= (u_4 - u_2)x_5 - x_4x_1 + u_3x_4 + u_2x_1 - u_4u_3 \\
p_5 &= x_5^2 + x_4^2 - u_1^2 \\
p_6 &= x_6^2x_2^2 - 2u_3x_6^2x_2 + \\
&\quad (u_5^2 - 2u_5u_2 + u_3^2 + u_2^2)x_6^2 - 2u_2x_6x_2^2 + \\
&\quad (2u_5u_3 + 2u_3u_2)x_6x_2 - 2u_5u_3^2x_6 + u_2^2x_2^2 \\
&\quad -2u_5u_3u_2x_2 + \\
&\quad (u_5^2u_3^2 - u_5^2u_1^2 + 2u_5u_2u_1^2 - u_2^2u_1^2) \\
p_7 &= (u_5 - u_2)x_7 - x_6x_2 + u_3x_6 + u_2x_2 - u_5u_3
\end{aligned}
$$

$$
\begin{aligned}
p_8 &= x_8^2x_3^2 - 2u_3x_8^2x_3 + \\
&\quad (u_6^2 - 2u_6u_2 + u_3^2 + u_2^2)x_8^2 - 2u_2x_8x_3^2 + \\
&\quad (2u_6u_3 + 2u_3u_2)x_8x_3 - 2u_6u_3^2x_8 + u_2^2x_3^2 \\
&\quad -2u_6u_3u_2x_3 + \\
&\quad (u_6^2u_3^2 - u_6^2u_1^2 + 2u_6u_2u_1^2 - u_2^2u_1^2) \\
p_9 &= (u_6 - u_2)x_9 - x_8x_3 + u_3x_8 + u_2x_3 - u_6u_3 \\
p_{10} &= x_{10}x_4 + (u_4 - 2u_2)x_{10} + (-2u_4 + u_2)x_4 + u_4u_2 \\
p_{11} &= x_{11}x_5 + x_{11}x_1 - 2u_3x_{11} - 2x_5x_1 + u_3x_5 + u_3x_1 \\
p_{12} &= x_{12}x_6 + (u_5 - 2u_2)x_{12} + (-2u_5 + u_2)x_6 + u_5u_2 \\
p_{13} &= x_{13}x_7 + x_{13}x_2 - 2u_3x_{13} - 2x_7x_2 + u_3x_7 + u_3x_2 \\
p_{14} &= x_{14}x_8 + (u_6 - 2u_2)x_{14} + (-2u_6 + u_2)x_8 + u_6u_2 \\
p_{15} &= x_{15}x_9 + x_{15}x_3 - 2u_3x_{15} - 2x_9x_3 + u_3x_9 + u_3x_3
\end{aligned}
$$

3.10 Triangulation, step 10

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

3.11 Triangulation, step 11

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 2.

Minimal degrees: 1 polynomial(s) with degree 1 and 1 polynomial(s) with degree 2.

Polynomial with linear degree: Removing variable x_5 from all other polynomials by reducing them with polynomial p_4 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= x_1^2 + (u_4^2 - u_1^2) \\
p_2 &= x_2^2 + (u_5^2 - u_1^2) \\
p_3 &= x_3^2 + (u_6^2 - u_1^2) \\
p_4 &= x_4^2x_1^2 - 2u_3x_4^2x_1 + \\
&\quad (u_4^2 - 2u_4u_2 + u_3^2 + u_2^2)x_4^2 - 2u_2x_4x_1^2 + \\
&\quad (2u_4u_3 + 2u_3u_2)x_4x_1 - 2u_4u_3^2x_4 + u_2^2x_1^2 \\
&\quad -2u_4u_3u_2x_1 +
\end{aligned}
$$

$$
\begin{aligned}
& (u_4^2u_3^2 - u_4^2u_1^2 + 2u_4u_2u_1^2 - u_2^2u_1^2) \\
p_5 &= (u_4 - u_2)x_5 - x_4x_1 + u_3x_4 + u_2x_1 - u_4u_3 \\
p_6 &= x_6^2x_2^2 - 2u_3x_6^2x_2 + \\
& (u_5^2 - 2u_5u_2 + u_3^2 + u_2^2)x_6^2 - 2u_2x_6x_2^2 + \\
& (2u_5u_3 + 2u_3u_2)x_6x_2 - 2u_5u_3^2x_6 + u_2^2x_2^2 \\
& -2u_5u_3u_2x_2 + \\
& (u_5^2u_3^2 - u_5^2u_1^2 + 2u_5u_2u_1^2 - u_2^2u_1^2) \\
p_7 &= (u_5 - u_2)x_7 - x_6x_2 + u_3x_6 + u_2x_2 - u_5u_3 \\
p_8 &= x_8^2x_3^2 - 2u_3x_8^2x_3 + \\
& (u_6^2 - 2u_6u_2 + u_3^2 + u_2^2)x_8^2 - 2u_2x_8x_3^2 + \\
& (2u_6u_3 + 2u_3u_2)x_8x_3 - 2u_6u_3^2x_8 + u_2^2x_3^2 \\
& -2u_6u_3u_2x_3 + \\
& (u_6^2u_3^2 - u_6^2u_1^2 + 2u_6u_2u_1^2 - u_2^2u_1^2) \\
p_9 &= (u_6 - u_2)x_9 - x_8x_3 + u_3x_8 + u_2x_3 - u_6u_3 \\
p_{10} &= x_{10}x_4 + (u_4 - 2u_2)x_{10} + (-2u_4 + u_2)x_4 + u_4u_2 \\
p_{11} &= x_{11}x_5 + x_{11}x_1 - 2u_3x_{11} - 2x_5x_1 + u_3x_5 + u_3x_1 \\
p_{12} &= x_{12}x_6 + (u_5 - 2u_2)x_{12} + (-2u_5 + u_2)x_6 + u_5u_2 \\
p_{13} &= x_{13}x_7 + x_{13}x_2 - 2u_3x_{13} - 2x_7x_2 + u_3x_7 + u_3x_2 \\
p_{14} &= x_{14}x_8 + (u_6 - 2u_2)x_{14} + (-2u_6 + u_2)x_8 + u_6u_2 \\
p_{15} &= x_{15}x_9 + x_{15}x_3 - 2u_3x_{15} - 2x_9x_3 + u_3x_9 + u_3x_3
\end{aligned}
$$

3.12 Triangulation, step 12

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

3.13 Triangulation, step 13

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.14 Triangulation, step 14

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.15 Triangulation, step 15

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= x_1^2 + (u_4^2 - u_1^2) \\
p_2 &= x_2^2 + (u_5^2 - u_1^2) \\
p_3 &= x_3^2 + (u_6^2 - u_1^2) \\
p_4 &= x_4^2 x_1^2 - 2u_3 x_4^2 x_1 + \\
&\quad (u_4^2 - 2u_4u_2 + u_3^2 + u_2^2)x_4^2 - 2u_2 x_4 x_1^2 + \\
&\quad (2u_4u_3 + 2u_3u_2)x_4x_1 - 2u_4u_3^2 x_4 + u_2^2 x_1^2 \\
&\quad -2u_4u_3u_2x_1 + \\
&\quad (u_4^2u_3^2 - u_4^2u_1^2 + 2u_4u_2u_1^2 - u_2^2u_1^2) \\
p_5 &= (u_4 - u_2)x_5 - x_4x_1 + u_3x_4 + u_2x_1 - u_4u_3 \\
p_6 &= x_6^2 x_2^2 - 2u_3 x_6^2 x_2 + \\
&\quad (u_5^2 - 2u_5u_2 + u_3^2 + u_2^2)x_6^2 - 2u_2 x_6 x_2^2 + \\
&\quad (2u_5u_3 + 2u_3u_2)x_6x_2 - 2u_5u_3^2 x_6 + u_2^2 x_2^2 \\
&\quad -2u_5u_3u_2x_2 + \\
&\quad (u_5^2u_3^2 - u_5^2u_1^2 + 2u_5u_2u_1^2 - u_2^2u_1^2) \\
p_7 &= (u_5 - u_2)x_7 - x_6x_2 + u_3x_6 + u_2x_2 - u_5u_3 \\
p_8 &= x_8^2 x_3^2 - 2u_3 x_8^2 x_3 + \\
&\quad (u_6^2 - 2u_6u_2 + u_3^2 + u_2^2)x_8^2 - 2u_2 x_8 x_3^2 + \\
&\quad (2u_6u_3 + 2u_3u_2)x_8x_3 - 2u_6u_3^2 x_8 + u_2^2 x_3^2 \\
&\quad -2u_6u_3u_2x_3 + \\
&\quad (u_6^2u_3^2 - u_6^2u_1^2 + 2u_6u_2u_1^2 - u_2^2u_1^2) \\
p_9 &= (u_6 - u_2)x_9 - x_8x_3 + u_3x_8 + u_2x_3 - u_6u_3
\end{aligned}
$$

$$
\begin{aligned}
p_{10} &= x_{10}x_4 + (u_4 - 2u_2)x_{10} + (-2u_4 + u_2)x_4 + u_4u_2 \\
p_{11} &= x_{11}x_5 + x_{11}x_1 - 2u_3x_{11} - 2x_5x_1 + u_3x_5 + u_3x_1 \\
p_{12} &= x_{12}x_6 + (u_5 - 2u_2)x_{12} + (-2u_5 + u_2)x_6 + u_5u_2 \\
p_{13} &= x_{13}x_7 + x_{13}x_2 - 2u_3x_{13} - 2x_7x_2 + u_3x_7 + u_3x_2 \\
p_{14} &= x_{14}x_8 + (u_6 - 2u_2)x_{14} + (-2u_6 + u_2)x_8 + u_6u_2 \\
p_{15} &= x_{15}x_9 + x_{15}x_3 - 2u_3x_{15} - 2x_9x_3 + u_3x_9 + u_3x_3
\end{aligned}
$$

4 Final Remainder

4.1 Final remainder for conjecture Polar 002

Calculating final remainder of the conclusion:

$$
g = x_{15}x_{12} - x_{15}x_{10} - x_{14}x_{13} + x_{14}x_{11} + x_{13}x_{10} - x_{12}x_{11}
$$

with respect to the triangular system.

1. Pseudo remainder with p_{15} over variable x_{15}:

$$
\begin{aligned}
g = {} & -x_{14}x_{13}x_9 - x_{14}x_{13}x_3 + 2u_3x_{14}x_{13} + x_{14}x_{11}x_9 + x_{14}x_{11}x_3 \\
& -2u_3x_{14}x_{11} + x_{13}x_{10}x_9 + x_{13}x_{10}x_3 - 2u_3x_{13}x_{10} - x_{12}x_{11}x_9 \\
& -x_{12}x_{11}x_3 + 2u_3x_{12}x_{11} + 2x_{12}x_9x_3 - u_3x_{12}x_9 - u_3x_{12}x_3 \\
& -2x_{10}x_9x_3 + u_3x_{10}x_9 + u_3x_{10}x_3
\end{aligned}
$$

2. Pseudo remainder with p_{14} over variable x_{14}:

$$
\begin{aligned}
g = {} & x_{13}x_{10}x_9x_8 + (u_6 - 2u_2)x_{13}x_{10}x_9 + x_{13}x_{10}x_8x_3 \\
& -2u_3x_{13}x_{10}x_8 + (u_6 - 2u_2)x_{13}x_{10}x_3 + \\
& (-2u_6u_3 + 4u_3u_2)x_{13}x_{10} + (-2u_6 + u_2)x_{13}x_9x_8 + u_6u_2x_{13}x_9 + \\
& (-2u_6 + u_2)x_{13}x_8x_3 + (4u_6u_3 - 2u_3u_2)x_{13}x_8 + u_6u_2x_{13}x_3 \\
& -2u_6u_3u_2x_{13} - x_{12}x_{11}x_9x_8 + (-u_6 + 2u_2)x_{12}x_{11}x_9 \\
& -x_{12}x_{11}x_8x_3 + 2u_3x_{12}x_{11}x_8 + (-u_6 + 2u_2)x_{12}x_{11}x_3 + \\
& (2u_6u_3 - 4u_3u_2)x_{12}x_{11} + 2x_{12}x_9x_8x_3 - u_3x_{12}x_9x_8 + \\
& (2u_6 - 4u_2)x_{12}x_9x_3 + (-u_6u_3 + 2u_3u_2)x_{12}x_9 - u_3x_{12}x_8x_3 + \\
& (-u_6u_3 + 2u_3u_2)x_{12}x_3 + (2u_6 - u_2)x_{11}x_9x_8 - u_6u_2x_{11}x_9 + \\
& (2u_6 - u_2)x_{11}x_8x_3 + (-4u_6u_3 + 2u_3u_2)x_{11}x_8 - u_6u_2x_{11}x_3 + \\
& 2u_6u_3u_2x_{11} - 2x_{10}x_9x_8x_3 + u_3x_{10}x_9x_8 + \\
& (-2u_6 + 4u_2)x_{10}x_9x_3 + (u_6u_3 - 2u_3u_2)x_{10}x_9 + u_3x_{10}x_8x_3 + \\
& (u_6u_3 - 2u_3u_2)x_{10}x_3
\end{aligned}
$$

3. Pseudo remainder with p_{13} over variable x_{13}:

   Polynomial too big for output (text size is 3363 *characters, number of terms is* 92*)*

4. Pseudo remainder with p_{12} over variable x_{12}:

   Polynomial too big for output (text size is 7730 *characters, number of terms is* 132*)*

5. Pseudo remainder with p_{11} over variable x_{11}:

   Polynomial too big for output (number of terms is 300*)*

6. Pseudo remainder with p_{10} over variable x_{10}:

   Polynomial too big for output (number of terms is 360*)*

7. Pseudo remainder with p_9 over variable x_9:

   Polynomial too big for output (number of terms is 452*)*

8. Pseudo remainder with p_8 over variable x_8:

   Polynomial too big for output (number of terms is 552*)*

9. Pseudo remainder with p_7 over variable x_7:

   Polynomial too big for output (number of terms is 600*)*

10. Pseudo remainder with p_6 over variable x_6:

    Polynomial too big for output (number of terms is 704*)*

11. Pseudo remainder with p_5 over variable x_5:

    Polynomial too big for output (number of terms is 768*)*

12. Pseudo remainder with p_4 over variable x_4:

    Polynomial too big for output (number of terms is 894*)*

13. Pseudo remainder with p_3 over variable x_3:

    Polynomial too big for output (number of terms is 384*)*

14. Pseudo remainder with p_2 over variable x_2:

    Polynomial too big for output (text size is greater than 2000 *characters, number of terms is* 160*)*

15. Pseudo remainder with p_1 over variable x_1:

    Polynomial too big for output (text size is greater than 2000 *characters, number of terms is* 64*)*

5 Prover results

Status: Theorem can't be neither proved nor disproved.

Space Complexity: The biggest polynomial obtained during prover execution contains 894 terms.

Time Complexity: Time spent by the prover is 20.218 seconds.