OpenGeoProver Output for conjecture “Chou 154 (Incenter Theorem)”

Wu’s method used

March 4, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Free point S
- Angle ray b of angle with vertex A and point S from first ray, which is congruent to angle BAS
- Angle ray a of angle with vertex B and point S from first ray, which is congruent to angle ABS
- Intersection point C of point sets a and b

Theorem statement:

- Angles ACS and SCB are equal

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

2.2 Transformation of point B:

- Point B has been assigned following coordinates: (0, u_1)

2.3 Transformation of point S:

- Point S has been assigned following coordinates: $(u_2,\, u_3)$

2.4 Transformation of point C:

- Point C has been assigned following coordinates: $(x_1,\, x_2)$
- Polynomial that point C has to satisfy is:

$$\begin{aligned} p &= (u_3u_2 - u_2u_1)x_2 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_1 + \\ &\quad (-u_3u_2u_1 + u_2u_1^2) \end{aligned}$$

- Processing of polynomial

$$\begin{aligned} p &= (u_3u_2 - u_2u_1)x_2 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_1 + \\ &\quad (-u_3u_2u_1 + u_2u_1^2) \end{aligned}$$

Info: Polynomial

$$\begin{aligned} p &= (u_3u_2 - u_2u_1)x_2 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_1 + \\ &\quad (-u_3u_2u_1 + u_2u_1^2) \end{aligned}$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point C has to satisfy is:

$$p = u_3u_2x_2 + (-0.5u_3^2 + 0.5u_2^2)x_1$$

- Processing of polynomial

$$p = u_3u_2x_2 + (-0.5u_3^2 + 0.5u_2^2)x_1$$

Info: Polynomial

$$p = u_3u_2x_2 + (-0.5u_3^2 + 0.5u_2^2)x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

- Polynomial for theorem statement:

$$\begin{aligned} p &= u_2x_2^3 + (-u_3 + 0.5u_1)x_2^2x_1 + (-u_3u_2 - u_2u_1)x_2^2 + \\ &\quad u_2x_2x_1^2 + (u_3^2 - u_2^2)x_2x_1 + u_3u_2u_1x_2 + \\ &\quad (-u_3 + 0.5u_1)x_1^3 + (u_3u_2 - u_2u_1)x_1^2 + \\ &\quad (-0.5u_3^2u_1 + 0.5u_2^2u_1)x_1 \end{aligned}$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.063 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$
\begin{aligned}
p_1 &= (u_3u_2 - u_2u_1)x_2 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_1 + \\
&\quad (-u_3u_2u_1 + u_2u_1^2) \\
p_2 &= u_3u_2x_2 + (-0.5u_3^2 + 0.5u_2^2)x_1
\end{aligned}
$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_2 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (-0.5u_3^2u_2u_1 + 0.5u_3u_2u_1^2 - 0.5u_2^3u_1)x_1 + \\
&\quad (u_3^2u_2^2u_1 - u_3u_2^2u_1^2) \\
p_2 &= (u_3u_2 - u_2u_1)x_2 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_1 + \\
&\quad (-u_3u_2u_1 + u_2u_1^2)
\end{aligned}
$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= (-0.5u_3^2u_2u_1 + 0.5u_3u_2u_1^2 - 0.5u_2^3u_1)x_1 + \\
&\quad (u_3^2u_2^2u_1 - u_3u_2^2u_1^2) \\
p_2 &= (u_3u_2 - u_2u_1)x_2 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_1 + \\
&\quad (-u_3u_2u_1 + u_2u_1^2)
\end{aligned}
$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 154 (Incenter Theorem)

Calculating final remainder of the conclusion:

$$
\begin{aligned}
g &= u_2x_2^3 + (-u_3 + 0.5u_1)x_2^2x_1 + (-u_3u_2 - u_2u_1)x_2^2 + \\
&\quad u_2x_2x_1^2 + (u_3^2 - u_2^2)x_2x_1 + u_3u_2u_1x_2 + \\
&\quad (-u_3 + 0.5u_1)x_1^3 + (u_3u_2 - u_2u_1)x_1^2 + \\
&\quad (-0.5u_3^2u_1 + 0.5u_2^2u_1)x_1
\end{aligned}
$$

with respect to the triangular system.

1. Pseudo remainder with p_2 over variable x_2:

$$
\begin{aligned}
g &= (-0.125u_3^6u_2 + 0.625u_3^5u_2u_1 - 0.375u_3^4u_2^3 \\
&\quad -1.25u_3^4u_2u_1^2 + 1.25u_3^3u_2^3u_1 + 1.25u_3^3u_2u_1^3 \\
&\quad -0.375u_3^2u_2^5 - 1.5u_3^2u_2^3u_1^2 - 0.625u_3^2u_2u_1^4 + \\
&\quad 0.625u_3u_2^5u_1 + 0.75u_3u_2^3u_1^3 + 0.125u_3u_2u_1^5 - 0.125u_2^7 \\
&\quad -0.25u_2^5u_1^2 - 0.125u_2^3u_1^4) \\
&\quad x_1^3 \\
&\quad + \\
&\quad (0.25u_3^6u_2^2 - 1.25u_3^5u_2^2u_1 + 0.5u_3^4u_2^4 + \\
&\quad 2.5u_3^4u_2^2u_1^2 - 1.5u_3^3u_2^4u_1 - 2.5u_3^3u_2^2u_1^3 + \\
&\quad 0.25u_3^2u_2^6 + 1.5u_3^2u_2^4u_1^2 + 1.25u_3^2u_2^2u_1^4 \\
&\quad -0.25u_3u_2^6u_1 - 0.5u_3u_2^4u_1^3 - 0.25u_3u_2^2u_1^5) \\
&\quad x_1^2
\end{aligned}
$$

2. Pseudo remainder with p_1 over variable x_1:

$$
g = 0
$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 9 terms.

Time Complexity: Time spent by the prover is 0.073 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points A and S are not identical
- Points A and B are not identical
- Points S and B are not identical
- Points S and B are not identical

Time spent for processing NDG Conditions

- 0.057 seconds