OpenGeoProver Output for conjecture "Chou 191 (Euler's Line Theorem)"

Wu's method used

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1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Free point C
- Line b through two points C and A
- Line c through two points A and B
- Line hb through point B perpendicular to line b
- Line hc through point C perpendicular to line c
- Intersection point H of point sets hb and hc
- Perpendicular bisector mb of segment CA
- Perpendicular bisector mc of segment AB
- Intersection point O of point sets mb and mc
- Midpoint B1 of segment CA
- Line tb through two points B and B1
- Midpoint C1 of segment AB
- Line tc through two points C and C1
- Intersection point T of point sets tb and tc

Theorem statement:

- Ratio of oriented segments HT/TO equals 2.0

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

2.2 Transformation of point B:

- Point B has been assigned following coordinates: (0, u_1)

2.3 Transformation of point C:

- Point C has been assigned following coordinates: (u_2, u_3)

2.4 Transformation of point H:

- Point H has been assigned following coordinates: (x_1, x_2)
- Polynomial that point H has to satisfy is:

$$p = u_3x_2 + u_2x_1 - u_3u_1$$

- Processing of polynomial

$$p = u_3x_2 + u_2x_1 - u_3u_1$$

Info: Polynomial

$$p = u_3x_2 + u_2x_1 - u_3u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point H has to satisfy is:

$$p = x_2 - u_3$$

- Processing of polynomial

$$p = x_2 - u_3$$

Info: Will try to rename Y coordinate of point H

Info: Y coordinate of point H renamed by independent variable u_3

- Point H has been renamed. Point H has been assigned following coordinates: (x_1, u_3)
- Repeating instantiation of first condition of this point, after its coordinate has been renamed

- Polynomial that point H has to satisfy is:

$$p = u_2x_1 + (u_3^2 - u_3u_1)$$

- Processing of polynomial

$$p = u_2x_1 + (u_3^2 - u_3u_1)$$

Info: Polynomial

$$p = u_2x_1 + (u_3^2 - u_3u_1)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of point O:

- Point O has been assigned following coordinates: (x_2, x_3)
- Polynomial that point O has to satisfy is:

$$p = u_3x_3 + u_2x_2 + (-0.5u_3^2 - 0.5u_2^2)$$

- Processing of polynomial

$$p = u_3x_3 + u_2x_2 + (-0.5u_3^2 - 0.5u_2^2)$$

Info: Polynomial

$$p = u_3x_3 + u_2x_2 + (-0.5u_3^2 - 0.5u_2^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point O has to satisfy is:

$$p = x_3 - 0.5u_1$$

- Processing of polynomial

$$p = x_3 - 0.5u_1$$

Info: Polynomial

$$p = x_3 - 0.5u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point B1:

- Point B1 has been assigned following coordinates: (x_4, x_5)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_4 - 0.5u_2$$

Info: Polynomial

$$p = x_4 - 0.5u_2$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_4 - 0.5u_2$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_5 - 0.5u_3$$

Info: Polynomial

$$p = x_5 - 0.5u_3$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_5 - 0.5u_3$$

is added to polynomial system

2.7 Transformation of point C1:

- Point C1 has been assigned following coordinates: (x_6, x_7)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_6$$

Info: Will try to rename X coordinate of point C1

Info: Y coordinate of point C1 will be replaced by X coordinate

Info: X coordinate of point C1 renamed by zero

- Point C1 has been renamed. Point C1 has been assigned following coordinates: $(0, x_6)$

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_6 - 0.5u_1$$

Info: Polynomial

$$p = x_6 - 0.5u_1$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_6 - 0.5u_1$$

is added to polynomial system

2.8 Transformation of point T:

- Point T has been assigned following coordinates: (x_7, x_8)
- Polynomial that point T has to satisfy is:

$$p = x_8x_4 - x_7x_5 + u_1x_7 - u_1x_4$$

- Processing of polynomial

$$p = x_8x_4 - x_7x_5 + u_1x_7 - u_1x_4$$

Info: Polynomial

$$p = x_8x_4 - x_7x_5 + u_1x_7 - u_1x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point T has to satisfy is:

$$p = u_2x_8 + x_7x_6 - u_3x_7 - u_2x_6$$

- Processing of polynomial

$$p = u_2x_8 + x_7x_6 - u_3x_7 - u_2x_6$$

Info: Polynomial

$$p = u_2x_8 + x_7x_6 - u_3x_7 - u_2x_6$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_7 - 0.666667x_2 - 0.333333x_1$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.093 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= u_2x_1 + (u_3^2 - u_3u_1) \\
p_2 &= u_3x_3 + u_2x_2 + (-0.5u_3^2 - 0.5u_2^2) \\
p_3 &= x_3 - 0.5u_1 \\
p_4 &= x_4 - 0.5u_2 \\
p_5 &= x_5 - 0.5u_3 \\
p_6 &= x_6 - 0.5u_1 \\
p_7 &= x_8x_4 - x_7x_5 + u_1x_7 - u_1x_4 \\
p_8 &= u_2x_8 + x_7x_6 - u_3x_7 - u_2x_6
\end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_8 from all other polynomials by reducing them with polynomial p_7 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= u_2x_1 + (u_3^2 - u_3u_1) \\
p_2 &= u_3x_3 + u_2x_2 + (-0.5u_3^2 - 0.5u_2^2) \\
p_3 &= x_3 - 0.5u_1 \\
p_4 &= x_4 - 0.5u_2 \\
p_5 &= x_5 - 0.5u_3 \\
p_6 &= x_6 - 0.5u_1 \\
p_7 &= x_7x_6x_4 + u_2x_7x_5 - u_3x_7x_4 - u_2u_1x_7 - u_2x_6x_4 + \\
&\quad u_2u_1x_4 \\
p_8 &= x_8x_4 - x_7x_5 + u_1x_7 - u_1x_4
\end{aligned}$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_3 from all other polynomials by reducing them with polynomial p_2 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_2x_1 + (u_3^2 - u_3u_1) \\
p_2 &= -u_2x_2 + (0.5u_3^2 - 0.5u_3u_1 + 0.5u_2^2) \\
p_3 &= u_3x_3 + u_2x_2 + (-0.5u_3^2 - 0.5u_2^2) \\
p_4 &= x_4 - 0.5u_2 \\
p_5 &= x_5 - 0.5u_3 \\
p_6 &= x_6 - 0.5u_1 \\
p_7 &= x_7x_6x_4 + u_2x_7x_5 - u_3x_7x_4 - u_2u_1x_7 - u_2x_6x_4 + \\
&\quad u_2u_1x_4 \\
p_8 &= x_8x_4 - x_7x_5 + u_1x_7 - u_1x_4
\end{aligned}
$$

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.8 Triangulation, step 8

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= u_2x_1 + (u_3^2 - u_3u_1)\\
p_2 &= -u_2x_2 + (0.5u_3^2 - 0.5u_3u_1 + 0.5u_2^2)\\
p_3 &= u_3x_3 + u_2x_2 + (-0.5u_3^2 - 0.5u_2^2)\\
p_4 &= x_4 - 0.5u_2\\
p_5 &= x_5 - 0.5u_3\\
p_6 &= x_6 - 0.5u_1\\
p_7 &= x_7x_6x_4 + u_2x_7x_5 - u_3x_7x_4 - u_2u_1x_7 - u_2x_6x_4 +\\
&\quad u_2u_1x_4\\
p_8 &= x_8x_4 - x_7x_5 + u_1x_7 - u_1x_4
\end{aligned}
$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 191 (Euler's Line Theorem)

Calculating final remainder of the conclusion:

$$g = x_7 - 0.666667x_2 - 0.333333x_1$$

with respect to the triangular system.

1. Pseudo remainder with p_8 over variable x_8:

$$g = x_7 - 0.666667x_2 - 0.333333x_1$$

2. Pseudo remainder with p_7 over variable x_7:

$$
\begin{aligned}
g = {} & -0.666667x_6x_4x_2 - 0.333333x_6x_4x_1 + u_2x_6x_4 - 0.666667u_2x_5x_2\\
& -0.333333u_2x_5x_1 + 0.666667u_3x_4x_2 + 0.333333u_3x_4x_1 - u_2u_1x_4 +\\
& 0.666667u_2u_1x_2 + 0.333333u_2u_1x_1
\end{aligned}
$$

3. Pseudo remainder with p_6 over variable x_6:

$$
\begin{aligned}
g = {} & -0.666667u_2x_5x_2 - 0.333333u_2x_5x_1 + (0.666667u_3 - 0.333333u_1)x_4x_2 +\\
& (0.333333u_3 - 0.166667u_1)x_4x_1 - 0.5u_2u_1x_4 + 0.666667u_2u_1x_2 +\\
& 0.333333u_2u_1x_1
\end{aligned}
$$

4. Pseudo remainder with p_5 over variable x_5:

$$
\begin{aligned}
g &= (0.666667u_3 - 0.333333u_1)x_4x_2 + (0.333333u_3 - 0.166667u_1)x_4x_1 \\
&\quad -0.5u_2u_1x_4 + (-0.333333u_3u_2 + 0.666667u_2u_1)x_2 + \\
&\quad (-0.166667u_3u_2 + 0.333333u_2u_1)x_1
\end{aligned}
$$

5. Pseudo remainder with p_4 over variable x_4:

$$g = 0.5u_2u_1x_2 + 0.25u_2u_1x_1 - 0.25u_2^2u_1$$

6. Pseudo remainder with p_3 over variable x_3:

$$g = 0.5u_2u_1x_2 + 0.25u_2u_1x_1 - 0.25u_2^2u_1$$

7. Pseudo remainder with p_2 over variable x_2:

$$g = -0.25u_2^2u_1x_1 + (-0.25u_3^2u_2u_1 + 0.25u_3u_2u_1^2)$$

8. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 10 terms.

Time Complexity: Time spent by the prover is 0.074 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points B, C and C1 are not collinear
- Points A, B, C and C1 are not collinear
- Line through points B and B1 is not parallel with line through points C and C1
- Points A, B and B1 are not collinear

Time spent for processing NDG Conditions

- 0.97 seconds