OpenGeoProver Output for conjecture "Chou 196 (Nine points circle Theorem)"

Wu's method used

February 26, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point A
- Free point B
- Free point C
- Line a through two points B and C
- Line b through two points C and A
- Line c through two points A and B
- Line footPointPerpLine440 through point A perpendicular to line a
- Intersection point D of point sets footPointPerpLine440 and a
- Line footPointPerpLine478 through point B perpendicular to line b
- Intersection point E of point sets footPointPerpLine478 and b
- Line footPointPerpLine308 through point C perpendicular to line c
- Intersection point F of point sets footPointPerpLine308 and c
- Midpoint C1 of segment AB
- Line ha through two points A and D
- Line hb through two points B and E
- Intersection point H of point sets ha and hb
- Midpoint A2 of segment AH

Theorem statement:

- Points D, E, F, C1, A2 are concyclic

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

2.2 Transformation of point B:

- Point B has been assigned following coordinates: (0, u_1)

2.3 Transformation of point C:

- Point C has been assigned following coordinates: (u_2, u_3)

2.4 Transformation of point D:

- Point D has been assigned following coordinates: (x_1, x_2)
- Polynomial that point D has to satisfy is:

$$p = (u_3 - u_1)x_2 + u_2x_1$$

- Processing of polynomial

$$p = (u_3 - u_1)x_2 + u_2x_1$$

Info: Polynomial

$$p = (u_3 - u_1)x_2 + u_2x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point D has to satisfy is:

$$p = u_2x_2 + (-u_3 + u_1)x_1 - u_2u_1$$

- Processing of polynomial

$$p = u_2x_2 + (-u_3 + u_1)x_1 - u_2u_1$$

Info: Polynomial

$$p = u_2x_2 + (-u_3 + u_1)x_1 - u_2u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of point E:

- Point E has been assigned following coordinates: (x_3, x_4)
- Polynomial that point E has to satisfy is:

$$p = u_3x_4 + u_2x_3 - u_3u_1$$

- Processing of polynomial

$$p = u_3x_4 + u_2x_3 - u_3u_1$$

Info: Polynomial

$$p = u_3x_4 + u_2x_3 - u_3u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point E has to satisfy is:

$$p = u_2x_4 - u_3x_3$$

- Processing of polynomial

$$p = u_2x_4 - u_3x_3$$

Info: Polynomial

$$p = u_2x_4 - u_3x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point F:

- Point F has been assigned following coordinates: (x_5, x_6)
- Polynomial that point F has to satisfy is:

$$p = x_6 - u_3$$

- Processing of polynomial

$$p = x_6 - u_3$$

Info: Will try to rename Y coordinate of point F

Info: Y coordinate of point F renamed by independent variable u_3

- Point F has been renamed. Point F has been assigned following coordinates: (x_5, u_3)

- Polynomial that point F has to satisfy is:

$$p = x_5$$

- Processing of polynomial

$$p = x_5$$

Info: Will try to rename X coordinate of point F

Info: X coordinate of point F renamed by zero

- Point F has been renamed. Point F has been assigned following coordinates: (0, u_3)

2.7 Transformation of point C1:

- Point C1 has been assigned following coordinates: (x_5, x_6)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_5$$

Info: Will try to rename X coordinate of point C1

Info: Y coordinate of point C1 will be replaced by X coordinate

Info: X coordinate of point C1 renamed by zero

- Point C1 has been renamed. Point C1 has been assigned following coordinates: (0, x_5)
- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_5 - 0.5u_1$$

Info: Polynomial

$$p = x_5 - 0.5u_1$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_5 - 0.5u_1$$

is added to polynomial system

2.8 Transformation of point H:

- Point H has been assigned following coordinates: (x_6, x_7)
- Polynomial that point H has to satisfy is:

$$p = x_7x_1 - x_6x_2$$

- Processing of polynomial

$$p = x_7x_1 - x_6x_2$$

Info: Polynomial

$$p = x_7x_1 - x_6x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point H has to satisfy is:

$$p = x_7x_3 - x_6x_4 + u_1x_6 - u_1x_3$$

- Processing of polynomial

$$p = x_7x_3 - x_6x_4 + u_1x_6 - u_1x_3$$

Info: Polynomial

$$p = x_7x_3 - x_6x_4 + u_1x_6 - u_1x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.9 Transformation of point A2:

- Point A2 has been assigned following coordinates: (x_8, x_9)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_8 - 0.5x_6$$

Info: Polynomial

$$p = x_8 - 0.5x_6$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_8 - 0.5x_6$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_9 - 0.5x_7$$

Info: Polynomial

$$p = x_9 - 0.5x_7$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_9 - 0.5x_7$$

is added to polynomial system

Transformation of Theorem statement

- Polynomial for theorem statement: *Polynomial too big for output (text size is* 7970 *characters, number of terms is* 227*)*

Time spent for transformation of Construction Protocol to algebraic form

- 2.313 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= (u_3 - u_1)x_2 + u_2x_1 \\
p_2 &= u_2x_2 + (-u_3 + u_1)x_1 - u_2u_1 \\
p_3 &= u_3x_4 + u_2x_3 - u_3u_1 \\
p_4 &= u_2x_4 - u_3x_3 \\
p_5 &= x_5 - 0.5u_1 \\
p_6 &= x_7x_1 - x_6x_2 \\
p_7 &= x_7x_3 - x_6x_4 + u_1x_6 - u_1x_3 \\
p_8 &= x_8 - 0.5x_6 \\
p_9 &= x_9 - 0.5x_7
\end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_9. No reduction needed.

The triangular system has not been changed.

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_7 from all other polynomials by reducing them with polynomial p_6 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (u_3 - u_1)x_2 + u_2x_1 \\
p_2 &= u_2x_2 + (-u_3 + u_1)x_1 - u_2u_1 \\
p_3 &= u_3x_4 + u_2x_3 - u_3u_1 \\
p_4 &= u_2x_4 - u_3x_3 \\
p_5 &= x_5 - 0.5u_1 \\
p_6 &= -x_6x_4x_1 + x_6x_3x_2 + u_1x_6x_1 - u_1x_3x_1 \\
p_7 &= x_7x_1 - x_6x_2 \\
p_8 &= x_8 - 0.5x_6 \\
p_9 &= x_9 - 0.5x_7
\end{aligned}
$$

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_3 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (u_3 - u_1)x_2 + u_2x_1 \\
p_2 &= u_2x_2 + (-u_3 + u_1)x_1 - u_2u_1 \\
p_3 &= (-u_3^2 - u_2^2)x_3 + u_3u_2u_1 \\
p_4 &= u_3x_4 + u_2x_3 - u_3u_1 \\
p_5 &= x_5 - 0.5u_1 \\
p_6 &= -x_6x_4x_1 + x_6x_3x_2 + u_1x_6x_1 - u_1x_3x_1 \\
p_7 &= x_7x_1 - x_6x_2 \\
p_8 &= x_8 - 0.5x_6 \\
p_9 &= x_9 - 0.5x_7
\end{aligned}
$$

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.8 Triangulation, step 8

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_2 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (-u_3^2 + 2u_3u_1 - u_2^2 - u_1^2)x_1 + (-u_3u_2u_1 + u_2u_1^2) \\
p_2 &= (u_3 - u_1)x_2 + u_2x_1 \\
p_3 &= (-u_3^2 - u_2^2)x_3 + u_3u_2u_1 \\
p_4 &= u_3x_4 + u_2x_3 - u_3u_1 \\
p_5 &= x_5 - 0.5u_1 \\
p_6 &= -x_6x_4x_1 + x_6x_3x_2 + u_1x_6x_1 - u_1x_3x_1 \\
p_7 &= x_7x_1 - x_6x_2 \\
p_8 &= x_8 - 0.5x_6 \\
p_9 &= x_9 - 0.5x_7
\end{aligned}$$

3.9 Triangulation, step 9

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_1 &= (-u_3^2 + 2u_3u_1 - u_2^2 - u_1^2)x_1 + (-u_3u_2u_1 + u_2u_1^2) \\
p_2 &= (u_3 - u_1)x_2 + u_2x_1 \\
p_3 &= (-u_3^2 - u_2^2)x_3 + u_3u_2u_1 \\
p_4 &= u_3x_4 + u_2x_3 - u_3u_1 \\
p_5 &= x_5 - 0.5u_1 \\
p_6 &= -x_6x_4x_1 + x_6x_3x_2 + u_1x_6x_1 - u_1x_3x_1 \\
p_7 &= x_7x_1 - x_6x_2 \\
p_8 &= x_8 - 0.5x_6 \\
p_9 &= x_9 - 0.5x_7
\end{aligned}$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 196 (Nine points circle Theorem)

Calculating final remainder of the conclusion:

Polynomial too big for output (text size is 7970 *characters, number of terms is* 227*)*

with respect to the triangular system.

1. Pseudo remainder with p_9 over variable x_9:

   Polynomial too big for output (text size is 8012 *characters, number of terms is* 227*)*

2. Pseudo remainder with p_8 over variable x_8:

   Polynomial too big for output (text size is 8213 *characters, number of terms is* 227*)*

3. Pseudo remainder with p_7 over variable x_7:

   Polynomial too big for output (text size is 6432 *characters, number of terms is* 166*)*

4. Pseudo remainder with p_6 over variable x_6:

   Polynomial too big for output (number of terms is 888*)*

5. Pseudo remainder with p_5 over variable x_5:

   Polynomial too big for output (text size is 24694 *characters, number of terms is* 250*)*

6. Pseudo remainder with p_4 over variable x_4:

   Polynomial too big for output (text size is 18811 *characters, number of terms is* 101*)*

7. Pseudo remainder with p_3 over variable x_3:

   Polynomial too big for output (text size is 22299 *characters, number of terms is* 35*)*

8. Pseudo remainder with p_2 over variable x_2:

   Polynomial too big for output (text size is 11165 *characters, number of terms is* 5*)*

9. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 888 terms.

Time Complexity: Time spent by the prover is 1.129 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points B, C and C1 are not collinear
- Points B, C and C1 are not collinear
- Points F, B, C and C1 are not collinear
- Points F, B, C and C1 are not collinear
- Line through points D and A is not parallel with line through points E and B
- Points D and A are not identical

Time spent for processing NDG Conditions

- 1.344 seconds