OpenGeoProver Output for conjecture "Chou 162 (Euler's Formula)"

Wu's method used

February 23, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point B
- Free point C
- Free point S
- Line a through two points B and C
- Line footPointPerpLine681 through point S perpendicular to line a
- Intersection point S1 of point sets footPointPerpLine681 and a
- Angle ray c of angle with vertex B and point S from first ray, which is congruent to angle CBS
- Angle ray b of angle with vertex C and point S from first ray, which is congruent to angle BCS
- Intersection point A of point sets b and c
- Perpendicular bisector ma of segment BC
- Perpendicular bisector mc of segment AB
- Intersection point O of point sets ma and mc
- Cental symmetric point B1 of point B with respect to center of symmetry O

Theorem statement:

- Algebraic sum of segments OB*OB, OS*OS and BB1*SS1 is zero

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point B:

- Point B has been assigned following coordinates: (0, 0)

2.2 Transformation of point C:

- Point C has been assigned following coordinates: (0, u_1)

2.3 Transformation of point S:

- Point S has been assigned following coordinates: (u_2, u_3)

2.4 Transformation of point S1:

- Point S1 has been assigned following coordinates: (x_1, x_2)
- Polynomial that point S1 has to satisfy is:

$$p = x_2 - u_3$$

- Processing of polynomial

$$p = x_2 - u_3$$

Info: Will try to rename Y coordinate of point S1

Info: Y coordinate of point S1 renamed by independent variable u_3

- Point S1 has been renamed. Point S1 has been assigned following coordinates: (x_1, u_3)
- Polynomial that point S1 has to satisfy is:

$$p = x_1$$

- Processing of polynomial

$$p = x_1$$

Info: Will try to rename X coordinate of point S1

Info: X coordinate of point S1 renamed by zero

- Point S1 has been renamed. Point S1 has been assigned following coordinates: (0, u_3)

2.5 Transformation of point A:

- Point A has been assigned following coordinates: (x_1, x_2)
- Polynomial that point A has to satisfy is:

$$p = (u_3u_2 - u_2u_1)x_2 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_1 + (-u_3u_2u_1 + u_2u_1^2)$$

- Processing of polynomial

$$p = (u_3u_2 - u_2u_1)x_2 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_1 + (-u_3u_2u_1 + u_2u_1^2)$$

Info: Polynomial

$$p = (u_3u_2 - u_2u_1)x_2 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_1 + (-u_3u_2u_1 + u_2u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point A has to satisfy is:

$$p = u_3u_2x_2 + (-0.5u_3^2 + 0.5u_2^2)x_1$$

- Processing of polynomial

$$p = u_3u_2x_2 + (-0.5u_3^2 + 0.5u_2^2)x_1$$

Info: Polynomial

$$p = u_3u_2x_2 + (-0.5u_3^2 + 0.5u_2^2)x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point O:

- Point O has been assigned following coordinates: (x_3, x_4)
- Polynomial that point O has to satisfy is:

$$p = x_4 - 0.5u_1$$

- Processing of polynomial

$$p = x_4 - 0.5u_1$$

Info: Polynomial

$$p = x_4 - 0.5u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point O has to satisfy is:

$$p = x_4x_2 + x_3x_1 - 0.5x_2^2 - 0.5x_1^2$$

- Processing of polynomial

$$p = x_4x_2 + x_3x_1 - 0.5x_2^2 - 0.5x_1^2$$

Info: Polynomial

$$p = x_4x_2 + x_3x_1 - 0.5x_2^2 - 0.5x_1^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point B1:

- Point B1 has been assigned following coordinates: (x_5, x_6)
- Instantiating condition for X-coordinate of this point
- Processing of polynomial

$$p = x_5 - 2x_3$$

Info: Polynomial

$$p = x_5 - 2x_3$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_5 - 2x_3$$

is added to polynomial system

- Instantiating condition for Y-coordinate of this point
- Processing of polynomial

$$p = x_6 - 2x_4$$

Info: Polynomial

$$p = x_6 - 2x_4$$

added to system of polynomials that represents the constructions

- Instantiated condition

$$p = x_6 - 2x_4$$

is added to polynomial system

Transformation of Theorem statement

- Polynomial for theorem statement: *Polynomial too big for output (text size is* 2796 *characters, number of terms is* 57*)*

Time spent for transformation of Construction Protocol to algebraic form

- 0.157 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$
\begin{aligned}
p_1 &= (u_3u_2 - u_2u_1)x_2 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_1 + \\
&\quad (-u_3u_2u_1 + u_2u_1^2) \\
p_2 &= u_3u_2x_2 + (-0.5u_3^2 + 0.5u_2^2)x_1 \\
p_3 &= x_4 - 0.5u_1 \\
p_4 &= x_4x_2 + x_3x_1 - 0.5x_2^2 - 0.5x_1^2 \\
p_5 &= x_5 - 2x_3 \\
p_6 &= x_6 - 2x_4
\end{aligned}
$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_3 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= (u_3u_2 - u_2u_1)x_2 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_1 + \\
&\quad (-u_3u_2u_1 + u_2u_1^2) \\
p_2 &= u_3u_2x_2 + (-0.5u_3^2 + 0.5u_2^2)x_1 \\
p_3 &= x_3x_1 - 0.5x_2^2 + 0.5u_1x_2 - 0.5x_1^2 \\
p_4 &= x_4 - 0.5u_1 \\
p_5 &= x_5 - 2x_3 \\
p_6 &= x_6 - 2x_4
\end{aligned}$$

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_2 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= (-0.5u_3^2u_2u_1 + 0.5u_3u_2u_1^2 - 0.5u_2^3u_1)x_1 + \\
&\quad (u_3^2u_2^2u_1 - u_3u_2^2u_1^2) \\
p_2 &= (u_3u_2 - u_2u_1)x_2 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_1 + \\
&\quad (-u_3u_2u_1 + u_2u_1^2) \\
p_3 &= x_3x_1 - 0.5x_2^2 + 0.5u_1x_2 - 0.5x_1^2 \\
p_4 &= x_4 - 0.5u_1 \\
p_5 &= x_5 - 2x_3 \\
p_6 &= x_6 - 2x_4
\end{aligned}
$$

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= (-0.5u_3^2u_2u_1 + 0.5u_3u_2u_1^2 - 0.5u_2^3u_1)x_1 + \\
&\quad (u_3^2u_2^2u_1 - u_3u_2^2u_1^2) \\
p_2 &= (u_3u_2 - u_2u_1)x_2 + (-0.5u_3^2 + u_3u_1 + 0.5u_2^2 - 0.5u_1^2)x_1 + \\
&\quad (-u_3u_2u_1 + u_2u_1^2) \\
p_3 &= x_3x_1 - 0.5x_2^2 + 0.5u_1x_2 - 0.5x_1^2 \\
p_4 &= x_4 - 0.5u_1 \\
p_5 &= x_5 - 2x_3 \\
p_6 &= x_6 - 2x_4
\end{aligned}
$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 162 (Euler's Formula)

Calculating final remainder of the conclusion:

Polynomial too big for output (text size is 2796 *characters, number of terms is* 57*)*

with respect to the triangular system.

1. Pseudo remainder with p_6 over variable x_6:

   Polynomial too big for output (text size is 2185 *characters, number of terms is* 42*)*

2. Pseudo remainder with p_5 over variable x_5:

$$
\begin{aligned}
g \;=\;& (16u_3^2 - 16u_2^2)x_4^6 + 32u_3u_2x_4^5x_3 + \\
& (-48u_3^3 + 16u_3u_2^2)x_4^5 + (32u_3^2 - 32u_2^2)x_4^4x_3^2 + \\
& (-112u_3^2u_2 + 16u_2^3)x_4^4x_3 + \\
& (68u_3^4 + 8u_3^2u_2^2 + 4u_2^4)x_4^4 + 64u_3u_2x_4^3x_3^3 + \\
& (-64u_3^3 - 64u_3u_2^2)x_4^3x_3^2 + \\
& (160u_3^3u_2 + 32u_3u_2^3)x_4^3x_3 + \\
& (-56u_3^5 - 48u_3^3u_2^2 + 8u_3u_2^4)x_4^3 + \\
& (16u_3^2 - 16u_2^2)x_4^2x_3^4 - 128u_3^2u_2x_4^2x_3^3 + \\
& (56u_3^4 + 144u_3^2u_2^2 + 24u_2^4)x_4^2x_3^2 + \\
& (-120u_3^4u_2 - 112u_3^2u_2^3 + 8u_2^5)x_4^2x_3 + \\
& (28u_3^6 + 52u_3^4u_2^2 + 20u_3^2u_2^4 - 4u_2^6)x_4^2 + \\
& 32u_3u_2x_4x_3^5 + (-16u_3^3 - 80u_3u_2^2)x_4x_3^4 + \\
& (96u_3^3u_2 + 96u_3u_2^3)x_4x_3^3 + \\
& (-24u_3^5 - 112u_3^3u_2^2 - 88u_3u_2^4)x_4x_3^2 + \\
& (48u_3^5u_2 + 96u_3^3u_2^3 + 48u_3u_2^5)x_4x_3 + \\
& (-8u_3^7 - 24u_3^5u_2^2 - 24u_3^3u_2^4 - 8u_3u_2^6)x_4 + \\
& (-16u_3^2u_2 - 16u_2^3)x_3^5 + \\
& (4u_3^4 + 40u_3^2u_2^2 + 36u_2^4)x_3^4 + \\
& (-24u_3^4u_2 - 48u_3^2u_2^3 - 24u_2^5)x_3^3 + \\
& (4u_3^6 + 28u_3^4u_2^2 + 44u_3^2u_2^4 + 20u_2^6)x_3^2 + \\
& (-8u_3^6u_2 - 24u_3^4u_2^3 - 24u_3^2u_2^5 - 8u_2^7)x_3 + \\
& (u_3^8 + 4u_3^6u_2^2 + 6u_3^4u_2^4 + 4u_3^2u_2^6 + u_2^8)
\end{aligned}
$$

3. Pseudo remainder with p_4 over variable x_4:

$$
\begin{aligned}
g \;=\;& (-16u_3^2u_2 + 16u_3u_2u_1 - 16u_2^3)x_3^5 + \\
& (4u_3^4 - 8u_3^3u_1 + 40u_3^2u_2^2 + 4u_3^2u_1^2 - 40u_3u_2^2u_1 + \\
& 36u_2^4 - 4u_2^2u_1^2) \\
& x_3^4 \\
& + \\
& (-24u_3^4u_2 + 48u_3^3u_2u_1 - 48u_3^2u_2^3 - 32u_3^2u_2u_1^2 + \\
& 48u_3u_2^3u_1 + 8u_3u_2u_1^3 - 24u_2^5) \\
& x_3^3 \\
& + \\
& (4u_3^6 - 12u_3^5u_1 + 28u_3^4u_2^2 + 14u_3^4u_1^2 \\
& -56u_3^3u_2^2u_1 - 8u_3^3u_1^3 + 44u_3^2u_2^4 + \\
& 36u_3^2u_2^2u_1^2 + 2u_3^2u_1^4 - 44u_3u_2^4u_1 \\
& -8u_3u_2^2u_1^3 + 20u_2^6 + 6u_2^4u_1^2 - 2u_2^2u_1^4)
\end{aligned}
$$

$$
\begin{aligned}
&x_3^2 \\
&+ \\
&(-8u_3^6u_2 + 24u_3^5u_2u_1 - 24u_3^4u_2^3 - 30u_3^4u_2u_1^2 + \\
&48u_3^3u_2^3u_1 + 20u_3^3u_2u_1^3 - 24u_3^2u_2^5 \\
&-28u_3^2u_2^3u_1^2 - 7u_3^2u_2u_1^4 + 24u_3u_2^5u_1 + \\
&4u_3u_2^3u_1^3 + u_3u_2u_1^5 - 8u_2^7 + 2u_2^5u_1^2 + \\
&u_2^3u_1^4) \\
&x_3 \\
&+ \\
&(u_3^8 - 4u_3^7u_1 + 4u_3^6u_2^2 + 7u_3^6u_1^2 - 12u_3^5u_2^2u_1 \\
&-7u_3^5u_1^3 + 6u_3^4u_2^4 + 13u_3^4u_2^2u_1^2 + 4.25u_3^4u_1^4 \\
&-12u_3^3u_2^4u_1 - 6u_3^3u_2^2u_1^3 - 1.5u_3^3u_1^5 + \\
&4u_3^2u_2^6 + 5u_3^2u_2^4u_1^2 + 0.5u_3^2u_2^2u_1^4 + \\
&0.25u_3^2u_1^6 - 4u_3u_2^6u_1 + u_3u_2^4u_1^3 + \\
&0.5u_3u_2^2u_1^5 + u_2^8 - u_2^6u_1^2 + 0.25u_2^4u_1^4 \\
&-0.25u_2^2u_1^6)
\end{aligned}
$$

4. Pseudo remainder with p_3 over variable x_3:

   Polynomial too big for output (text size is 8695 *characters, number of terms is* 46*)*

5. Pseudo remainder with p_2 over variable x_2:

   Polynomial too big for output (text size is 29175 *characters, number of terms is* 6*)*

6. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 57 terms.

Time Complexity: Time spent by the prover is 0.542 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points S, C and S1 are not collinear

- Points B and C are not identical
- Points S, C and S1 are not collinear
- Points S, C and S1 are not collinear
- Points A, B, C and S1 are not collinear

Time spent for processing NDG Conditions

- 0.284 seconds