OpenGeoProver Output for conjecture "Chou 036 (Butterfly Theorem)"

Wu's method used

February 23, 2012

1 Validation of Construction Protocol

Construction steps:

- Free point O
- Free point A
- Circle k with center O and one point A
- Random point B from circle k
- Random point C from circle k
- Random point D from circle k
- Line ac through two points A and C
- Line bd through two points B and D
- Intersection point E of point sets ac and bd
- Line oe through two points O and E
- Line ne through point E perpendicular to line oe
- Line ad through two points A and D
- Intersection point F of point sets ad and ne
- Line bc through two points B and C
- Intersection point G of point sets bc and ne

Theorem statement:

- Ratio of oriented segments FE/EG equals 1.0

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of point O:

- Point O has been assigned following coordinates: (0, 0)

2.2 Transformation of point A:

- Point A has been assigned following coordinates: (0, u_1)

2.3 Transformation of point B:

- Point B has been assigned following coordinates: (u_2, x_1)
- Polynomial that point B has to satisfy is:

$$p = x_1^2 + (u_2^2 - u_1^2)$$

- Processing of polynomial

$$p = x_1^2 + (u_2^2 - u_1^2)$$

Info: Polynomial

$$p = x_1^2 + (u_2^2 - u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.4 Transformation of point C:

- Point C has been assigned following coordinates: (u_3, x_2)
- Polynomial that point C has to satisfy is:

$$p = x_2^2 + (u_3^2 - u_1^2)$$

- Processing of polynomial

$$p = x_2^2 + (u_3^2 - u_1^2)$$

Info: Polynomial

$$p = x_2^2 + (u_3^2 - u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of point D:

- Point D has been assigned following coordinates: (u_4, x_3)
- Polynomial that point D has to satisfy is:

$$p = x_3^2 + (u_4^2 - u_1^2)$$

- Processing of polynomial

$$p = x_3^2 + (u_4^2 - u_1^2)$$

Info: Polynomial

$$p = x_3^2 + (u_4^2 - u_1^2)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point E:

- Point E has been assigned following coordinates: (x_4, x_5)
- Polynomial that point E has to satisfy is:

$$p = u_3x_5 - x_4x_2 + u_1x_4 - u_3u_1$$

- Processing of polynomial

$$p = u_3x_5 - x_4x_2 + u_1x_4 - u_3u_1$$

Info: Polynomial

$$p = u_3x_5 - x_4x_2 + u_1x_4 - u_3u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point E has to satisfy is:

$$p = (u_4 - u_2)x_5 - x_4x_3 + x_4x_1 + u_2x_3 - u_4x_1$$

- Processing of polynomial

$$p = (u_4 - u_2)x_5 - x_4x_3 + x_4x_1 + u_2x_3 - u_4x_1$$

Info: Polynomial

$$p = (u_4 - u_2)x_5 - x_4x_3 + x_4x_1 + u_2x_3 - u_4x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point F:

- Point F has been assigned following coordinates: (x_6, x_7)
- Polynomial that point F has to satisfy is:

$$p = u_4x_7 - x_6x_3 + u_1x_6 - u_4u_1$$

- Processing of polynomial

$$p = u_4x_7 - x_6x_3 + u_1x_6 - u_4u_1$$

Info: Polynomial

$$p = u_4x_7 - x_6x_3 + u_1x_6 - u_4u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point F has to satisfy is:

$$p = x_7x_5 + x_6x_4 - x_5^2 - x_4^2$$

- Processing of polynomial

$$p = x_7x_5 + x_6x_4 - x_5^2 - x_4^2$$

Info: Polynomial

$$p = x_7x_5 + x_6x_4 - x_5^2 - x_4^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.8 Transformation of point G:

- Point G has been assigned following coordinates: (x_8, x_9)
- Polynomial that point G has to satisfy is:

$$p = (u_3 - u_2)x_9 - x_8x_2 + x_8x_1 + u_2x_2 - u_3x_1$$

- Processing of polynomial

$$p = (u_3 - u_2)x_9 - x_8x_2 + x_8x_1 + u_2x_2 - u_3x_1$$

Info: Polynomial

$$p = (u_3 - u_2)x_9 - x_8x_2 + x_8x_1 + u_2x_2 - u_3x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point G has to satisfy is:

$$p = x_9x_5 + x_8x_4 - x_5^2 - x_4^2$$

- Processing of polynomial

$$p = x_9x_5 + x_8x_4 - x_5^2 - x_4^2$$

Info: Polynomial

$$p = x_9x_5 + x_8x_4 - x_5^2 - x_4^2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_8 + x_6 - 2x_4$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.15 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= x_1^2 + (u_2^2 - u_1^2) \\
p_2 &= x_2^2 + (u_3^2 - u_1^2) \\
p_3 &= x_3^2 + (u_4^2 - u_1^2) \\
p_4 &= u_3x_5 - x_4x_2 + u_1x_4 - u_3u_1 \\
p_5 &= (u_4 - u_2)x_5 - x_4x_3 + x_4x_1 + u_2x_3 - u_4x_1 \\
p_6 &= u_4x_7 - x_6x_3 + u_1x_6 - u_4u_1 \\
p_7 &= x_7x_5 + x_6x_4 - x_5^2 - x_4^2 \\
p_8 &= (u_3 - u_2)x_9 - x_8x_2 + x_8x_1 + u_2x_2 - u_3x_1 \\
p_9 &= x_9x_5 + x_8x_4 - x_5^2 - x_4^2
\end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_9 from all other polynomials by reducing them with polynomial p_8 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= x_1^2 + (u_2^2 - u_1^2) \\
p_2 &= x_2^2 + (u_3^2 - u_1^2) \\
p_3 &= x_3^2 + (u_4^2 - u_1^2) \\
p_4 &= u_3x_5 - x_4x_2 + u_1x_4 - u_3u_1 \\
p_5 &= (u_4 - u_2)x_5 - x_4x_3 + x_4x_1 + u_2x_3 - u_4x_1 \\
p_6 &= u_4x_7 - x_6x_3 + u_1x_6 - u_4u_1 \\
p_7 &= x_7x_5 + x_6x_4 - x_5^2 - x_4^2 \\
p_8 &= x_8x_5x_2 - x_8x_5x_1 + (u_3 - u_2)x_8x_4 + (-u_3 + u_2)x_5^2 \\
&\quad -u_2x_5x_2 + u_3x_5x_1 + (-u_3 + u_2)x_4^2 \\
p_9 &= (u_3 - u_2)x_9 - x_8x_2 + x_8x_1 + u_2x_2 - u_3x_1
\end{aligned}$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_7 from all other polynomials by reducing them with polynomial p_6 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= x_1^2 + (u_2^2 - u_1^2) \\
p_2 &= x_2^2 + (u_3^2 - u_1^2) \\
p_3 &= x_3^2 + (u_4^2 - u_1^2) \\
p_4 &= u_3x_5 - x_4x_2 + u_1x_4 - u_3u_1
\end{aligned}$$

$$
\begin{aligned}
p_5 &= (u_4 - u_2)x_5 - x_4x_3 + x_4x_1 + u_2x_3 - u_4x_1 \\
p_6 &= x_6x_5x_3 - u_1x_6x_5 + u_4x_6x_4 - u_4x_5^2 + u_4u_1x_5 \\
&\quad -u_4x_4^2 \\
p_7 &= u_4x_7 - x_6x_3 + u_1x_6 - u_4u_1 \\
p_8 &= x_8x_5x_2 - x_8x_5x_1 + (u_3 - u_2)x_8x_4 + (-u_3 + u_2)x_5^2 \\
&\quad -u_2x_5x_2 + u_3x_5x_1 + (-u_3 + u_2)x_4^2 \\
p_9 &= (u_3 - u_2)x_9 - x_8x_2 + x_8x_1 + u_2x_2 - u_3x_1
\end{aligned}
$$

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_5 from all other polynomials by reducing them with polynomial p_4 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= x_1^2 + (u_2^2 - u_1^2) \\
p_2 &= x_2^2 + (u_3^2 - u_1^2) \\
p_3 &= x_3^2 + (u_4^2 - u_1^2) \\
p_4 &= -u_3x_4x_3 + (u_4 - u_2)x_4x_2 + u_3x_4x_1 + (-u_4u_1 + u_2u_1)x_4 + \\
&\quad u_3u_2x_3 - u_4u_3x_1 + (u_4u_3u_1 - u_3u_2u_1) \\
p_5 &= u_3x_5 - x_4x_2 + u_1x_4 - u_3u_1 \\
p_6 &= x_6x_5x_3 - u_1x_6x_5 + u_4x_6x_4 - u_4x_5^2 + u_4u_1x_5 \\
&\quad -u_4x_4^2 \\
p_7 &= u_4x_7 - x_6x_3 + u_1x_6 - u_4u_1 \\
p_8 &= x_8x_5x_2 - x_8x_5x_1 + (u_3 - u_2)x_8x_4 + (-u_3 + u_2)x_5^2 \\
&\quad -u_2x_5x_2 + u_3x_5x_1 + (-u_3 + u_2)x_4^2 \\
p_9 &= (u_3 - u_2)x_9 - x_8x_2 + x_8x_1 + u_2x_2 - u_3x_1
\end{aligned}
$$

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.8 Triangulation, step 8

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.9 Triangulation, step 9

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= x_1^2 + (u_2^2 - u_1^2) \\
p_2 &= x_2^2 + (u_3^2 - u_1^2) \\
p_3 &= x_3^2 + (u_4^2 - u_1^2) \\
p_4 &= -u_3x_4x_3 + (u_4 - u_2)x_4x_2 + u_3x_4x_1 + (-u_4u_1 + u_2u_1)x_4 + \\
&\quad u_3u_2x_3 - u_4u_3x_1 + (u_4u_3u_1 - u_3u_2u_1) \\
p_5 &= u_3x_5 - x_4x_2 + u_1x_4 - u_3u_1 \\
p_6 &= x_6x_5x_3 - u_1x_6x_5 + u_4x_6x_4 - u_4x_5^2 + u_4u_1x_5 \\
&\quad -u_4x_4^2 \\
p_7 &= u_4x_7 - x_6x_3 + u_1x_6 - u_4u_1 \\
p_8 &= x_8x_5x_2 - x_8x_5x_1 + (u_3 - u_2)x_8x_4 + (-u_3 + u_2)x_5^2 \\
&\quad -u_2x_5x_2 + u_3x_5x_1 + (-u_3 + u_2)x_4^2 \\
p_9 &= (u_3 - u_2)x_9 - x_8x_2 + x_8x_1 + u_2x_2 - u_3x_1
\end{aligned}
$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 036 (Butterfly Theorem)

Calculating final remainder of the conclusion:

$$g = x_8 + x_6 - 2x_4$$

with respect to the triangular system.

1. Pseudo remainder with p_9 over variable x_9:

$$g = x_8 + x_6 - 2x_4$$

2. Pseudo remainder with p_8 over variable x_8:

$$
\begin{aligned}
g &= x_6x_5x_2 - x_6x_5x_1 + (u_3 - u_2)x_6x_4 + (u_3 - u_2)x_5^2 \\
&\quad -2x_5x_4x_2 + 2x_5x_4x_1 + u_2x_5x_2 - u_3x_5x_1 + (-u_3 + u_2)x_4^2
\end{aligned}
$$

3. Pseudo remainder with p_7 over variable x_7:

$$
\begin{aligned}
g &= x_6x_5x_2 - x_6x_5x_1 + (u_3 - u_2)x_6x_4 + (u_3 - u_2)x_5^2 \\
&\quad -2x_5x_4x_2 + 2x_5x_4x_1 + u_2x_5x_2 - u_3x_5x_1 + (-u_3 + u_2)x_4^2
\end{aligned}
$$

4. Pseudo remainder with p_6 over variable x_6:

$$
\begin{aligned}
g \;=\;& (u_3-u_2)x_5^3x_3 + u_4x_5^3x_2 - u_4x_5^3x_1 + \\
& (-u_3u_1+u_2u_1)x_5^3 - 2x_5^2x_4x_3x_2 + 2x_5^2x_4x_3x_1 + \\
& 2u_1x_5^2x_4x_2 - 2u_1x_5^2x_4x_1 + (2u_4u_3-2u_4u_2)x_5^2x_4 + \\
& u_2x_5^2x_3x_2 - u_3x_5^2x_3x_1 + (-u_4u_1-u_2u_1)x_5^2x_2 + \\
& (u_4u_1+u_3u_1)x_5^2x_1 + (-u_3+u_2)x_5x_4^2x_3 \\
& -u_4x_5x_4^2x_2 + u_4x_5x_4^2x_1 + (u_3u_1-u_2u_1)x_5x_4^2 + \\
& u_4u_2x_5x_4x_2 - u_4u_3x_5x_4x_1 + \\
& (-u_4u_3u_1+u_4u_2u_1)x_5x_4
\end{aligned}
$$

5. Pseudo remainder with p_5 over variable x_5:

   Polynomial too big for output (text size is 3063 *characters, number of terms is* 46*)*

6. Pseudo remainder with p_4 over variable x_4:

   Polynomial too big for output (text size is 17935 *characters, number of terms is* 75*)*

7. Pseudo remainder with p_3 over variable x_3:

   Polynomial too big for output (text size is 17423 *characters, number of terms is* 46*)*

8. Pseudo remainder with p_2 over variable x_2:

   Polynomial too big for output (text size is 11883 *characters, number of terms is* 20*)*

9. Pseudo remainder with p_1 over variable x_1:

$$g \;=\; 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 84 terms.

Time Complexity: Time spent by the prover is 0.494 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Line through points D and B is not parallel with line through points A and C
- Points A, C and O are not collinear

- Line through points D and A is not perpendicular to line through points E and O
- Points D and O are not identical
- Line through points E and O is not perpendicular to line through points B and C
- Points B, C and O are not collinear

Time spent for processing NDG Conditions

- 1.2 seconds