OpenGeoProver Output for conjecture "Chou 006 (Pascal's Theorem)"

Wu's method used

February 12, 2012

1 Validation of Construction Protocol

Construction steps:

- General Conic Section c
- Random point A from general conic c
- Random point B from general conic c
- Random point C from general conic c
- Random point D from general conic c
- Random point E from general conic c
- Random point F from general conic c
- Line AB through two points A and B
- Line DE through two points D and E
- Line BC through two points B and C
- Line EF through two points E and F
- Line CD through two points C and D
- Line FA through two points F and A
- Intersection point P of point sets AB and DE
- Intersection point Q of point sets BC and EF
- Intersection point R of point sets CD and FA

Theorem statement:

- Points P, Q, R are collinear

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

2.1 Transformation of general conic section c:

List of parametric points

- Point Ac has been assigned following coordinates: $(u_1, 0)$
- Point Bc has been assigned following coordinates: $(u_2, 0)$
- Point Cc has been assigned following coordinates: $(u_3, 0)$
- Point Dc has been assigned following coordinates: $(u_4, 0)$
- Point Ec has been assigned following coordinates: $(u_5, 0)$
- Condition for point X(x_1, x_2) to belong to this conic section is following equation:

$$p = u_3x_2^2 + u_2x_2x_1 + u_5x_2 + u_1x_1^2 + u_4x_1$$

2.2 Transformation of point A:

- Point A has been assigned following coordinates: (0, 0)

2.3 Transformation of point B:

- Point B has been assigned following coordinates: $(0, x_1)$
- Polynomial that point B has to satisfy is:

$$p = u_3x_1^2 + u_5x_1$$

- Processing of polynomial

$$p = u_3x_1^2 + u_5x_1$$

Info: Polynomial

$$p = u_3x_1^2 + u_5x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.4 Transformation of point C:

- Point C has been assigned following coordinates: (u_6, x_2)
- Polynomial that point C has to satisfy is:

$$p = u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)$$

- Processing of polynomial

$$p = u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)$$

Info: Polynomial

$$p = u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.5 Transformation of point D:

- Point D has been assigned following coordinates: (u_7, x_3)
- Polynomial that point D has to satisfy is:

$$p = u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4)$$

- Processing of polynomial

$$p = u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4)$$

Info: Polynomial

$$p = u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.6 Transformation of point E:

- Point E has been assigned following coordinates: (u_8, x_4)
- Polynomial that point E has to satisfy is:

$$p = u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)$$

- Processing of polynomial

$$p = u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)$$

Info: Polynomial

$$p = u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.7 Transformation of point F:

- Point F has been assigned following coordinates: (u_9, x_5)
- Polynomial that point F has to satisfy is:

$$p = u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4)$$

- Processing of polynomial

$$p = u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4)$$

Info: Polynomial

$$p = u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.8 Transformation of point P:

- Point P has been assigned following coordinates: (x_6, x_7)
- Polynomial that point P has to satisfy is:

$$p = x_6x_1$$

- Processing of polynomial

$$p = x_6x_1$$

Info: Polynomial

$$p = x_6x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point P has to satisfy is:

$$p = (u_8 - u_7)x_7 - x_6x_4 + x_6x_3 + u_7x_4 - u_8x_3$$

- Processing of polynomial

$$p = (u_8 - u_7)x_7 - x_6x_4 + x_6x_3 + u_7x_4 - u_8x_3$$

Info: Polynomial

$$p = (u_8 - u_7)x_7 - x_6x_4 + x_6x_3 + u_7x_4 - u_8x_3$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.9 Transformation of point Q:

- Point Q has been assigned following coordinates: (x_8, x_9)
- Polynomial that point Q has to satisfy is:

$$p = u_6x_9 - x_8x_2 + x_8x_1 - u_6x_1$$

- Processing of polynomial

$$p = u_6x_9 - x_8x_2 + x_8x_1 - u_6x_1$$

Info: Polynomial

$$p = u_6x_9 - x_8x_2 + x_8x_1 - u_6x_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point Q has to satisfy is:

$$p = (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4$$

- Processing of polynomial

$$p = (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4$$

Info: Polynomial

$$p = (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

2.10 Transformation of point R:

- Point R has been assigned following coordinates: (x_{10}, x_{11})
- Polynomial that point R has to satisfy is:

$$p = (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2$$

- Processing of polynomial

$$p = (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2$$

Info: Polynomial

$$p = (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

- Polynomial that point R has to satisfy is:

$$p = u_9x_{11} - x_{10}x_5$$

- Processing of polynomial

$$p = u_9x_{11} - x_{10}x_5$$

Info: Polynomial

$$p = u_9x_{11} - x_{10}x_5$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses

Transformation of Theorem statement

- Polynomial for theorem statement:

$$p = x_{11}x_8 - x_{11}x_6 - x_{10}x_9 + x_{10}x_7 + x_9x_6 - x_8x_7$$

Time spent for transformation of Construction Protocol to algebraic form

- 0.106 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_8 - u_7)x_7 - x_6x_4 + x_6x_3 + u_7x_4 - u_8x_3 \\
p_8 &= u_6x_9 - x_8x_2 + x_8x_1 - u_6x_1 \\
p_9 &= (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4 \\
p_{10} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2 \\
p_{11} &= u_9x_{11} - x_{10}x_5
\end{aligned}$$

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{11} from all other polynomials by reducing them with polynomial p_{10} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_8 - u_7)x_7 - x_6x_4 + x_6x_3 + u_7x_4 - u_8x_3 \\
p_8 &= u_6x_9 - x_8x_2 + x_8x_1 - u_6x_1 \\
p_9 &= (u_9 - u_8)x_9 - x_8x_5 + x_8x_4 + u_8x_5 - u_9x_4 \\
p_{10} &= (-u_7 + u_6)x_{10}x_5 + u_9x_{10}x_3 - u_9x_{10}x_2 - u_9u_6x_3 + u_9u_7x_2 \\
p_{11} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2
\end{aligned}
$$

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10}. No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_9 from all other polynomials by reducing them with polynomial p_8 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_8 - u_7)x_7 - x_6x_4 + x_6x_3 + u_7x_4 - u_8x_3 \\
p_8 &= -u_6x_8x_5 + u_6x_8x_4 + (u_9 - u_8)x_8x_2 + (-u_9 + u_8)x_8x_1 + \\
&\quad u_8u_6x_5 - u_9u_6x_4 + (u_9u_6 - u_8u_6)x_1 \\
p_9 &= u_6x_9 - x_8x_2 + x_8x_1 - u_6x_1 \\
p_{10} &= (-u_7 + u_6)x_{10}x_5 + u_9x_{10}x_3 - u_9x_{10}x_2 - u_9u_6x_3 + u_9u_7x_2 \\
p_{11} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2
\end{aligned}
$$

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8. No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7. No reduction needed.

The triangular system has not been changed.

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

3.8 Triangulation, step 8

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

3.9 Triangulation, step 9

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

3.10 Triangulation, step 10

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

3.11 Triangulation, step 11

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_1 &= u_3x_1^2 + u_5x_1 \\
p_2 &= u_3x_2^2 + (u_6u_2 + u_5)x_2 + (u_6^2u_1 + u_6u_4) \\
p_3 &= u_3x_3^2 + (u_7u_2 + u_5)x_3 + (u_7^2u_1 + u_7u_4) \\
p_4 &= u_3x_4^2 + (u_8u_2 + u_5)x_4 + (u_8^2u_1 + u_8u_4) \\
p_5 &= u_3x_5^2 + (u_9u_2 + u_5)x_5 + (u_9^2u_1 + u_9u_4) \\
p_6 &= x_6x_1 \\
p_7 &= (u_8 - u_7)x_7 - x_6x_4 + x_6x_3 + u_7x_4 - u_8x_3 \\
p_8 &= -u_6x_8x_5 + u_6x_8x_4 + (u_9 - u_8)x_8x_2 + (-u_9 + u_8)x_8x_1 + \\
&\quad u_8u_6x_5 - u_9u_6x_4 + (u_9u_6 - u_8u_6)x_1 \\
p_9 &= u_6x_9 - x_8x_2 + x_8x_1 - u_6x_1 \\
p_{10} &= (-u_7 + u_6)x_{10}x_5 + u_9x_{10}x_3 - u_9x_{10}x_2 - u_9u_6x_3 + u_9u_7x_2 \\
p_{11} &= (u_7 - u_6)x_{11} - x_{10}x_3 + x_{10}x_2 + u_6x_3 - u_7x_2
\end{aligned}$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 006 (Pascal's Theorem)

Calculating final remainder of the conclusion:

$$g = x_{11}x_8 - x_{11}x_6 - x_{10}x_9 + x_{10}x_7 + x_9x_6 - x_8x_7$$

with respect to the triangular system.

1. Pseudo remainder with p_{11} over variable x_{11}:

$$\begin{aligned}
g = {} & (-u_7 + u_6)x_{10}x_9 + x_{10}x_8x_3 - x_{10}x_8x_2 + (u_7 - u_6)x_{10}x_7 \\
& -x_{10}x_6x_3 + x_{10}x_6x_2 + (u_7 - u_6)x_9x_6 + (-u_7 + u_6)x_8x_7 \\
& -u_6x_8x_3 + u_7x_8x_2 + u_6x_6x_3 - u_7x_6x_2
\end{aligned}$$

2. Pseudo remainder with p_{10} over variable x_{10}:

$$\begin{aligned} g \;=\; & (-u_7^2 + 2u_7u_6 - u_6^2)x_9x_6x_5 + (u_9u_7 - u_9u_6)x_9x_6x_3 + \\ & (-u_9u_7 + u_9u_6)x_9x_6x_2 + (-u_9u_7u_6 + u_9u_6^2)x_9x_3 + \\ & (u_9u_7^2 - u_9u_7u_6)x_9x_2 + (u_7^2 - 2u_7u_6 + u_6^2)x_8x_7x_5 + \\ & (-u_9u_7 + u_9u_6)x_8x_7x_3 + (u_9u_7 - u_9u_6)x_8x_7x_2 + \\ & (u_7u_6 - u_6^2)x_8x_5x_3 + (-u_7^2 + u_7u_6)x_8x_5x_2 + \\ & (u_9u_7u_6 - u_9u_6^2)x_7x_3 + (-u_9u_7^2 + u_9u_7u_6)x_7x_2 + \\ & (-u_7u_6 + u_6^2)x_6x_5x_3 + (u_7^2 - u_7u_6)x_6x_5x_2 \end{aligned}$$

3. Pseudo remainder with p_9 over variable x_9:

$$\begin{aligned} g \;=\; & (u_7^2u_6 - 2u_7u_6^2 + u_6^3)x_8x_7x_5 + \\ & (-u_9u_7u_6 + u_9u_6^2)x_8x_7x_3 + \\ & (u_9u_7u_6 - u_9u_6^2)x_8x_7x_2 + \\ & (-u_7^2 + 2u_7u_6 - u_6^2)x_8x_6x_5x_2 + \\ & (u_7^2 - 2u_7u_6 + u_6^2)x_8x_6x_5x_1 + \\ & (u_9u_7 - u_9u_6)x_8x_6x_3x_2 + (-u_9u_7 + u_9u_6)x_8x_6x_3x_1 + \\ & (-u_9u_7 + u_9u_6)x_8x_6x_2^2 + (u_9u_7 - u_9u_6)x_8x_6x_2x_1 + \\ & (u_7u_6^2 - u_6^3)x_8x_5x_3 + (-u_7^2u_6 + u_7u_6^2)x_8x_5x_2 + \\ & (-u_9u_7u_6 + u_9u_6^2)x_8x_3x_2 + \\ & (u_9u_7u_6 - u_9u_6^2)x_8x_3x_1 + \\ & (u_9u_7^2 - u_9u_7u_6)x_8x_2^2 + \\ & (-u_9u_7^2 + u_9u_7u_6)x_8x_2x_1 + \\ & (u_9u_7u_6^2 - u_9u_6^3)x_7x_3 + \\ & (-u_9u_7^2u_6 + u_9u_7u_6^2)x_7x_2 + \\ & (-u_7u_6^2 + u_6^3)x_6x_5x_3 + (u_7^2u_6 - u_7u_6^2)x_6x_5x_2 + \\ & (-u_7^2u_6 + 2u_7u_6^2 - u_6^3)x_6x_5x_1 + \\ & (u_9u_7u_6 - u_9u_6^2)x_6x_3x_1 + \\ & (-u_9u_7u_6 + u_9u_6^2)x_6x_2x_1 + \\ & (-u_9u_7u_6^2 + u_9u_6^3)x_3x_1 + \\ & (u_9u_7^2u_6 - u_9u_7u_6^2)x_2x_1 \end{aligned}$$

4. Pseudo remainder with p_8 over variable x_8:

   Polynomial too big for output (text size is 3608 *characters, number of terms is* 36*)*

5. Pseudo remainder with p_7 over variable x_7:

   Polynomial too big for output (text size is 6736 *characters, number of terms is* 41*)*

6. Pseudo remainder with p_6 over variable x_6:

   Polynomial too big for output (text size is 2843 *characters, number of terms is* 18*)*

7. Pseudo remainder with p_5 over variable x_5:

   Polynomial too big for output (text size is 4559 *characters, number of terms is* 21*)*

8. Pseudo remainder with p_4 over variable x_4:

   Polynomial too big for output (text size is 5545 *characters, number of terms is* 21*)*

9. Pseudo remainder with p_3 over variable x_3:

   Polynomial too big for output (text size is 5435 *characters, number of terms is* 18*)*

10. Pseudo remainder with p_2 over variable x_2:

    Polynomial too big for output (text size is 3631 *characters, number of terms is* 12*)*

11. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 41 terms.

Time Complexity: Time spent by the prover is 0.343 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Conic Section is not in degenerate form
- Points A and B are not identical
- Points D and E are not identical
- Line through points E and F is not parallel with line through points B and C
- Points A and C are not identical
- Line through points D and C is not parallel with line through points F and A
- Points D and C are not identical

Time spent for processing NDG Conditions

- 0.904 seconds