Applied Multilevel Regression Modeling

Day 8: Complex Structures—Cross-classification and multiple membership

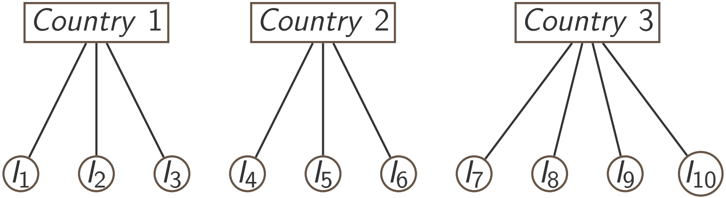
Constantin Manuel Bosancianu

WZB Berlin Social Science Center
Institutions and Political Inequality
bosancianu@icloud.com

August 7, 2019

Over the past 7 days...

So far, we have been focused on examples of strict hierarchical structure, wheres an observation could be a member of only one level-2 unit.



Standard examples: individuals in countries, firms in regions, multiple measurements in time for an individual, students in classrooms...

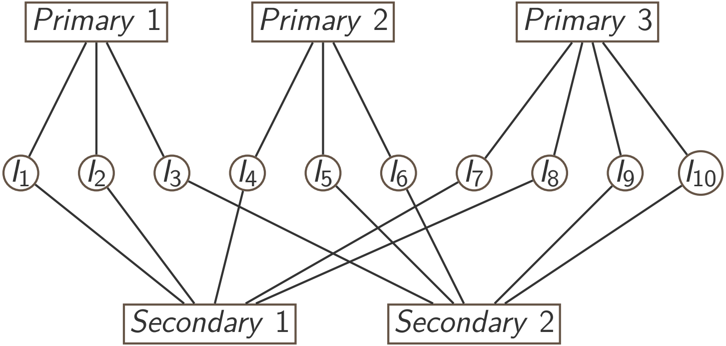
. . . but today. . .

What if you're interested in studying school achievement, and you have children in secondary schools?

Current school matters, but the primary school characteristics matter as well! (e.g., quality of the school, qualification of the professors in primary school)

This makes for a very odd structure—it's not hierarchical: students from the same primary school can end up in different secondary schools.

Cross-classified hierarchy



We have students nested in 2 structures that are not hierarchical, but cross-classified. It turns out that the MLM framework can handle such situations with minor additional hassle.

Model specifications

Cross-classified notation (1)

The notation is a bit harder, reflecting the complication in the structure.

Students are not technically nested in primary schools, or secondary schools, but in *combinations* of primary and secondary schools.

$$Y_{i(j_1,j_2)} = \beta_{0(j_1,j_2)} + \beta_{1(j_1,j_2)} X_{1(j_1,j_2)} + \epsilon_{i(j_1,j_2)} \tag{1}$$

Instead of j countries, we now have combinations of j_1 primary schools, and j_2 secondary schools.

Cross-classified notation (2)

At the second level, the notation reflects that there are two sources of error: one coming from primary schools, and one from secondary schools.

$$
\begin{cases}
\beta_{0(j_1,j_2)} = \gamma_{00} + \gamma_{01}Z_{j_1} + \gamma_{02}Z_{j_2} + \upsilon_{0(j_1)} + \upsilon_{0(j_2)} \\
\beta_{1(j_1,j_2)} = \gamma_{10} + \upsilon_{1(j_1)} + \upsilon_{1(j_2)}
\end{cases}
\tag{2}
$$

A further complication is that we have two "types" of variables at Level 2: referring to primary schools, and to secondary schools.

We can't ignore this and try to "simplify"—it would result in model misspecification and biased estimates for fixed effects.

Completely crossed vs. Partially crossed

Student scores nested in raters and measurement occasions.[1]

Each student is rated by the same team of raters (though some measurements might be randomly missing, due to rater unavailability)—completely crossed.

Groups of students are rated by separate teams of raters—partially crossed (some combinations of time and rater don't have an observation on them at all).

Additional complexity can yet be added! (Fielding & Goldstein, 2006)

[1] If you're not interested in rater effects, then maybe the approach from yesterday is better. If you want to account for both sources, then you may have to give up the longitudinal focus

The (dreaded) ICC

Easier to present it if we keep in mind that ICC can also be interpreted as the expected correlation between two individuals randomly sampled from same group.

Take the null model, where we have only $\epsilon_{i(j1,j2)}$, $\upsilon_{0(j1)}$, and $\upsilon_{0(j2)}$.

- Obs. from same $J1$ unit, but different $J2$ units: $\frac{\sigma^2_{\upsilon_{0(j1)}}}{\sigma^2_{\upsilon_{0(j1)}} + \sigma^2_{\upsilon_{0(j2)}} + \sigma^2_{\epsilon_{i(j1,j2)}}}$.
- Obs. from same $J2$ unit, but different $J1$ units: $\frac{\sigma^2_{\upsilon_{0(j2)}}}{\sigma^2_{\upsilon_{0(j1)}} + \sigma^2_{\upsilon_{0(j2)}} + \sigma^2_{\epsilon_{i(j1,j2)}}}$.
- Obs. from same combination of $J1$ and $J2$ units: $\frac{\sigma^2_{\upsilon_{0(j1)}} + \sigma^2_{\upsilon_{0(j2)}}}{\sigma^2_{\upsilon_{0(j1)}} + \sigma^2_{\upsilon_{0(j2)}} + \sigma^2_{\epsilon_{i(j1,j2)}}}$.

Multiple membership models

MM models

In this situation, the level 1 units (e.g., individuals) can be members of multiple upper-level units.

Say a researcher is interested in explaining a student's score on a national test, and gathers information about students' work habits between grade 5 and 8.

However, some students were exposed to effects of only 1 school, while others switched schools (e.g., family moved) and were exposed to effects from multiple schools.

MM models

In these instances, the researcher has to determine a weight which she allocated to each level 2 unit (under the $\sum w = 1$ constraint).

In some cases this involves very careful consideration, but other times it is easier. In my example with the schools, I could use as weight the time students spend in each school (e.g. 100%, 50% + 50%).

Then the analysis proceeds as with a standard MLM, but now level 2 variables are weighted based on the weights we supplied to the software.

Hierarchical APC models

An identification challenge (1)

- ✓ *Age* effect: changes which occur as people age
- ✓ *Period* effect: transformations experienced by all (the Depression in US, WWII in Europe);
- ✓ *Cohort* effect: changes experienced by a segment of society who shares a similar milestone, e.g birth (the students in 1968)

Knowing 2 of these uniquely identifies the 3rd (if added at the same level of analysis), which means it's impossible to add them all to a specification.

An identification challenge (2)

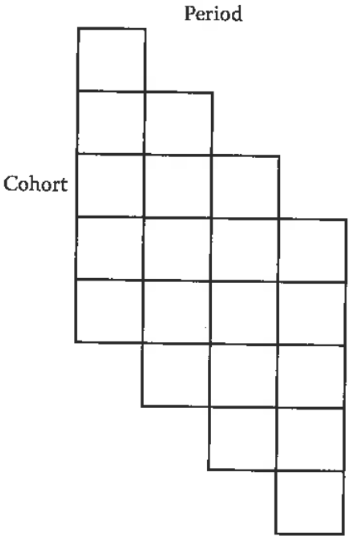
	p1980	p1990	p2000	p2010
a60	c1920	c1930	c1940	c1950
a70	c1910	c1920	c1930	c1940
a80	c1900	c1910	c1920	c1930
a90	c1890	c1900	c1910	c1920

Perfect collinearity in APC (Yang & Land, 2013, p. 10)

Longitudinal designs can't disentangle this—try moving across the diagonal...

Cross-sectional designs can't work either—try moving down columns...

Repeated cross-sections (1)



This can turn the problem (the authors claim. . .) into a cross-classified structure: individuals of varying ages are nested in *combinations* of periods and cohorts.

Repeated cross-sections (2)

Cohort by Period Cross-Classified Data Structure of GSS: Mean Verbal Test Score

Cohort	Year 1974	1976	1978	1982	1984	1987	1988	1989	1990	1991	1993	1994	1996	1998	2000	2004	2006	Mean
–1899	5.24	5.26	5.50	4.64	4.89	5.00	5.67											5.23
1900–1904	5.61	5.81	5.65	5.74	6.03	4.86	4.33	5.07	6.29	5.67	4.00							5.59
1905–1909	5.81	5.61	4.96	5.05	5.45	5.82	4.61	4.82	5.42	6.17	4.92	5.11	5.94	5.50				5.42
1910–1914	6.17	6.56	6.21	5.27	6.17	5.52	4.89	5.70	6.61	5.38	5.16	5.71	5.45	5.88	5.88			5.85
1915–1919	6.06	6.46	6.08	5.94	6.41	5.46	5.57	6.27	6.62	6.02	6.60	5.99	5.22	6.11	5.93	6.19	6.00	6.05
1920–1924	6.10	6.02	6.24	5.93	5.67	5.21	5.63	6.39	6.57	5.84	5.59	5.94	5.62	5.73	4.96	7.00	6.08	5.89
1925–1929	6.33	5.94	6.34	5.88	5.92	5.47	6.31	5.63	6.19	6.40	6.31	6.68	6.06	5.88	6.63	6.57	5.89	6.12
1930–1934	6.18	6.44	6.12	5.97	5.83	5.95	5.58	5.63	6.30	6.00	6.00	6.37	5.99	5.98	5.88	6.03	6.44	6.06
1935–1939	6.19	6.33	6.03	5.96	6.36	5.81	5.88	5.72	6.65	5.45	6.22	6.15	6.22	6.36	6.22	6.96	6.19	6.16
1940–1944	6.22	6.26	6.28	6.27	6.19	5.94	5.81	6.26	6.48	6.83	6.01	6.18	6.48	6.79	6.75	6.62	6.14	6.30
1945–1949	6.50	6.05	6.33	6.26	6.67	6.33	6.37	6.60	6.68	6.37	6.57	6.72	6.73	6.77	6.60	6.30	6.87	6.50
1950–1954	5.47	5.88	5.77	6.01	6.13	5.88	6.40	6.16	6.42	6.38	6.32	6.53	6.41	6.55	6.18	6.49	6.44	6.16
1955–1959	5.10	5.33	5.37	5.28	5.72	5.70	6.03	5.93	5.74	6.29	6.30	6.25	6.06	6.34	5.95	6.00	6.44	5.90
1960–1964			5.14	4.93	5.50	5.68	5.62	5.98	5.92	6.00	6.02	6.08	6.03	6.09	6.23	6.23	6.04	5.89
1965–1969					4.95	4.94	4.69	5.22	5.06	5.95	5.65	5.81	5.98	5.59	5.66	6.16	6.22	5.64
1970–1974							6.50	5.06	5.52	4.69	5.21	5.56	5.45	5.72	5.98	6.06	5.82	5.65
1975–1979											4.50	5.17	5.11	5.75	5.51	6.01	5.86	5.68
1980–1984														6.00	5.00	5.94	5.83	5.73
1985–1989																4.67	5.38	5.09
Mean	6.02	6.04	5.96	5.74	5.99	5.69	5.76	5.94	6.14	6.09	6.03	6.16	6.04	6.13	6.02	6.21	6.15	6.00

An example from the GSS on subjective happiness (Yang & Land, 2013, p. 29)

Hierarchical APC specification

Level 1 or "within-cell" model:

$$WORDSUM_{ijk} = \beta_{0jk} + \beta_1 AGE_{ijk} + \beta_2 AGE^2_{ijk} + \beta_3 EDUCATION_{ijk} + \beta_4 SEX_{ijk} + \beta_5 RACE_{ijk} + e_{ijk}$$

with $e_{ijk} \sim N(0, \sigma^2)$

Level 2 or "between-cell" model:

$$\beta_{0jk} = \gamma_0 + u_{0j} + v_{0k}, \text{ with } u_{0j} \sim N(0, \tau_u),\ v_{0k} \sim N(0, \tau_v)$$

Combined model:

$$WORDSUM_{ijk} = \gamma_0 + \beta_1 AGE_{ijk} + \beta_2 AGE^2_{ijk} + \beta_3 EDUCATION_{ijk} + \beta_4 SEX_{ijk} + \beta_5 RACE_{ijk} + u_{0j} + v_{0k} + e_{ijk}$$

for

$i = 1, 2, \ldots, n_{jk}$ individuals within cohort j and period k;
$j = 1, \ldots, 20$ birth cohorts;
$k = 1, \ldots, 17$ survey years;

Though the notation is a bit different, this is precisely a cross-classified specification—individuals nested simultaneously in a time period and a cohort.

Challenges (1)

The perspective is not without skeptics (particularly Andrew Bell and Kelvyn Jones).

[HTML] Clarifying **hierarchical** age–period–cohort models: A rejoinder to Bell and **Jones**
EN Reither, KC Land, SY Jeon, DA Powers… - Social science & …, 2015 - Elsevier
… (2015) demonstrated that **hierarchical** age–period … To contest this finding, Bell and **Jones** (2015) invent a data generating process (DGP) that borrows age, period and … When HAPC models applied to data simulated from this DGP fail to recover the patterning of **APC** effects, B&J …
Cited by 23 Related articles All 8 versions Web of Science: 13

Another'futile quest'? A simulation study of Yang and Land's **Hierarchical** Age-Period-Cohort model
AJ Bell, K **Jones** - Demographic Research, 2014 - JSTOR
BACKGROUND Whilst some argue that a solution to the age-period-cohort (APC)„identification problem" is impossible, numerous methodological solutions have been proposed, including Yang and Land" s Hierarchical-APC (HAPC) model: a multilevel model …
Cited by 118 Related articles All 20 versions Web of Science: 60

[HTML] Should age-period-cohort analysts accept innovation without scrutiny? A response to Reither, Masters, Yang, Powers, Zheng and Land
A Bell, K **Jones** - Social Science & Medicine, 2015 - Elsevier
This commentary clarifies our original commentary (Bell and Jones, 2014c) and illustrates some concerns we have regarding the response article in this issue (Reither et al., 2015). In particular, we argue that (a) linear effects do not have to be produced by exact linear …
Cited by 23 Related articles All 13 versions Web of Science: 13

[PDF] Age, period and cohort processes in longitudinal and life course analysis: A multilevel perspective
A Bell, K **Jones** - A life course perspective on health trajectories and …, 2015 - oapen.org
Age, period and cohort (APC) effects represent three distinct ways in which health can change over time, and researchers across the social and medical sciences have long been interested in how to differentiate and understand these changes. First, individuals can age …
Cited by 18 Related articles All 15 versions

[HTML] The **hierarchical** age–period–cohort model: Why does it find the results that it finds?
A Bell, K **Jones** - Quality & quantity, 2018 - Springer
It is claimed the hierarchical-age–period–cohort (HAPC) model solves the age–period–cohort (APC) identification problem. However, this is debateable; simulations show situations where the model produces incorrect results, countered by proponents of the …
Cited by 15 Related articles All 14 versions Web of Science: 8

Challenges (2)

The two main flaws that the HAPC framework is claimed to have is:

- ✓ it uncovers period or cohort effects even when the (simulated) DGP does not have them;[2]
- ✓ it tends to favor period effects, since in RCS designs there are typically considerably more cohorts than periods.

The second characteristic makes the findings from these models sensitive to the way in which cohorts are grouped, which is an undesirable aspect.

[2] The original authors don't dispute the findings, but claim that the DGP is unrealistic.

Thank you for the kind attention!

References

Fielding, A., & Goldstein, H. (2006). *Cross-classified and Multiple Membership Structures in Multilevel Models: An Introduction and Review* (Tech. Rep.). London: Institute of Education, University College. Retrieved from `http://dera.ioe.ac.uk/6469/1/RR791.pdf`

Yang, Y., & Land, K. C. (2013). *Age-Period-Cohort Analysis: New Models, Methods, and Empirical Applications*. Boca Raton, FL: CRC Press.