Multilevel Modeling: Principles and Applications in R

Day 3: Modeling Change over Time

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Up until now. . .

I have treated individuals as the L1 unit, with frequent examples where such individuals are nested in higher-order units:

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The variation we were interested in explaining occurred between groups.

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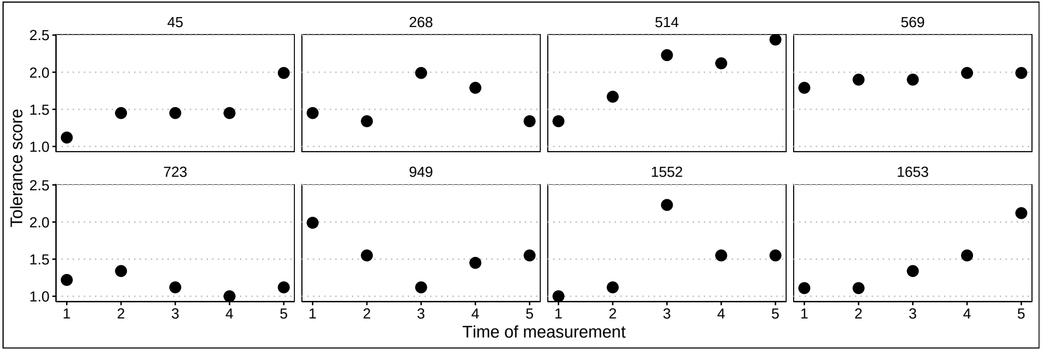
The L1 observations are measurements over time on these single units: performance on tests, share price, level of income inequality.

It takes some practice to adjust the interpretation of some of the measures from the past 2 days, e.g. ICC, to this new data configuration, but the MLM framework applies very well.

Change over time

2 kinds of questions



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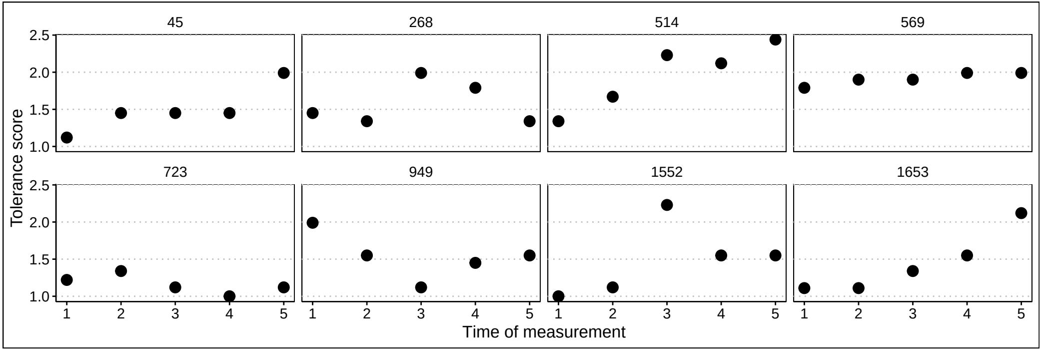

How is the pattern of change over time—linear, non-linear? (L1 model)

Do different kinds of units experience different types of change? (L2 model)

Different perspective

	Cross-sectional	Longitudinal
ICC	High value denotes group differences	High value denotes individual differences
Focus	L2 & L1*L2 effects	L1 & L1*L2 effects
Cross-level interactions	Added bonus	Vital component

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The approach should not be applied in every setting, though:

- ✓ 3 or more measurement time points;[1]

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- ✓ 3 or more measurement time points;[1]
- ✓ sufficient variance over time in outcome.

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Measuring time

You might opt for the natural metric (minutes, days, weeks, years), but you have more freedom than this: (1) number of sessions of treatment; (2) number of trips taken; (3) distance traveled; (4) school grade.

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Other choices: structured vs. unstructured schedule of measurement across individuals.

L1 trends

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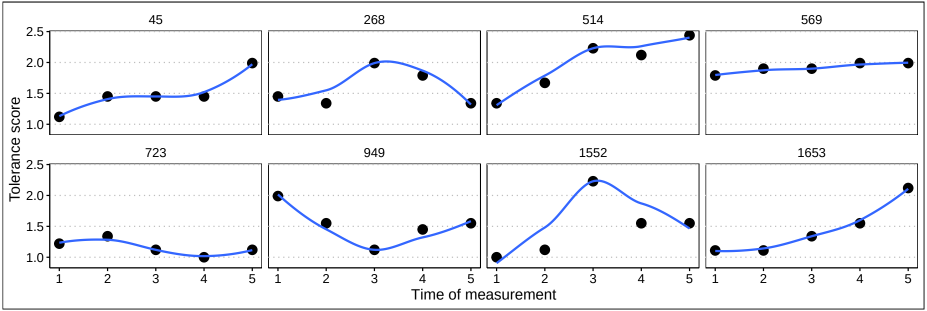

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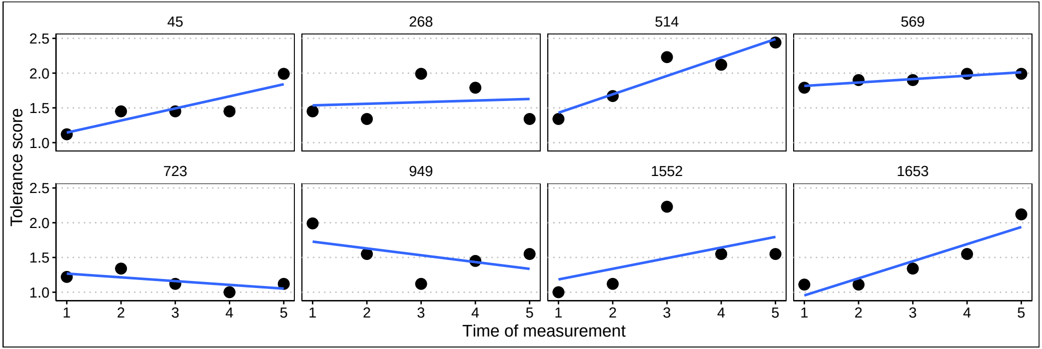

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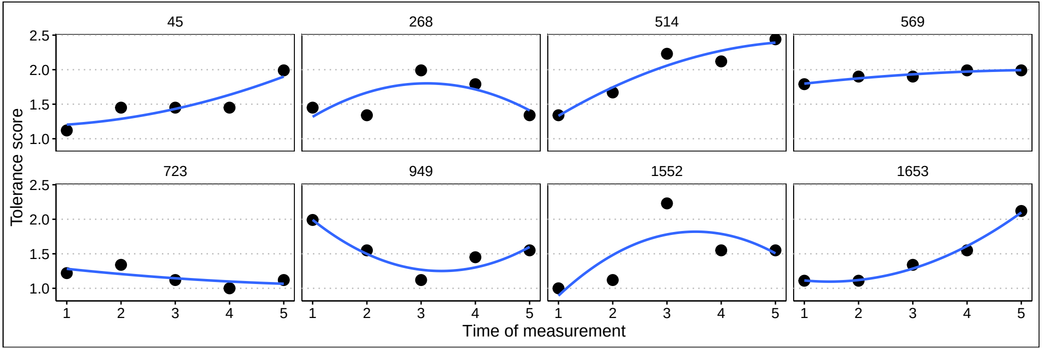

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More informative: split these trends up based on theoretically-relevant time-invariant (i.e. individual-level) predictors.

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ID	Intercept	Time	Time sq.
45	1.18	-0.01	0.03
268	0.75	0.68	-0.11
514	0.82	0.56	-0.05
569	1.70	0.10	-0.01
723	1.37	-0.10	0.01
949	2.75	-0.89	0.13
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These are preliminary exploratory phases—we haven't gotten yet to the *explanatory* one.

Model specifications

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Assume a very straightforward linear form:[2]

$$TOL_{ti} = \beta_{0i} + \beta_{1i} TIME_{ti} + \epsilon_{ti} \tag{1}$$

[2] I find Singer and Willett's notation convention a bit imprecise, e.g. the way that the position of the indicator for child, i, changes from one subscript to another. This is why I use a slightly altered notation.

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We're saying with that specification: each individual's level of tolerance has a linear growth pattern, described by individual growth parameters β_{0i} and β_{1i}.

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The L2 (individual) model links the variation in individual growth trajectories to time-invariant characteristics of individuals.

Composite model

$$\begin{cases} TOL_{ti} = \beta_{0i} + \beta_{1i} TIME_{ti} + \epsilon_{ti} \\ \beta_{0i} = \gamma_{00} + \gamma_{01} FEM_i + \upsilon_{0i} \\ \beta_{1i} = \gamma_{10} + \gamma_{11} FEM_i + \upsilon_{1i} \end{cases} \tag{3}$$

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$$\begin{aligned} TOL_{ti} = {} & (\gamma_{00} + \gamma_{01} * FEM_i + \upsilon_{0i}) + \\ & + (\gamma_{10} + \gamma_{11} * FEM_i + \upsilon_{1i}) * TIME_{ti} + \epsilon_{ti} = \\ = {} & \gamma_{00} + \gamma_{10} * TIME_{ti} + \gamma_{01} * FEM_i + \gamma_{11} * TIME_{ti} * FEM_i + \\ & + \upsilon_{1i} * TIME_{ti} + \upsilon_{0i} + \epsilon_{ti} \end{aligned} \tag{4}$$

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ϵ_{ti} likely have a more complex structure than before, because of auto-correlation.

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$$FEM_i = 0 \Rightarrow \begin{cases} Initial\ status = \gamma_{00} \\ Rate\ of\ change = \gamma_{10} \end{cases} \tag{6}$$

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υ_{0i} and υ_{1i} are all normally distributed, with constant variance, and we also get $Cov(\upsilon_{0i}; \upsilon_{1i})$—the association between baseline value and rate of growth.

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Additional predictors for individual intercepts or slopes can be added, slightly altering the interpretations above.

Unconditional means & unconditional growth

Unconditional means is what we called in past days the "null model".

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Building up models

We follow-up on these models by adding predictors at the individual-level (L2) that explain both initial status and rate of change, like in our earlier example with gender.

$$L2 = \begin{cases} \beta_{0i} = \gamma_{00} + \gamma_{01} * FEM_i + \upsilon_{0i} \\ \beta_{1i} = \gamma_{10} + \gamma_{11} * FEM_i + \upsilon_{1i} \end{cases} \tag{12}$$

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Time-varying predictors

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- ✓ γ_{11}: the change in the gap in TOL_{ti} between girls and boys for every passing unit of $TIME$

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Only linear and continuous change can be modeled with this specification.

Non-linear change

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Adding both models produces changes in both slope and elevation.

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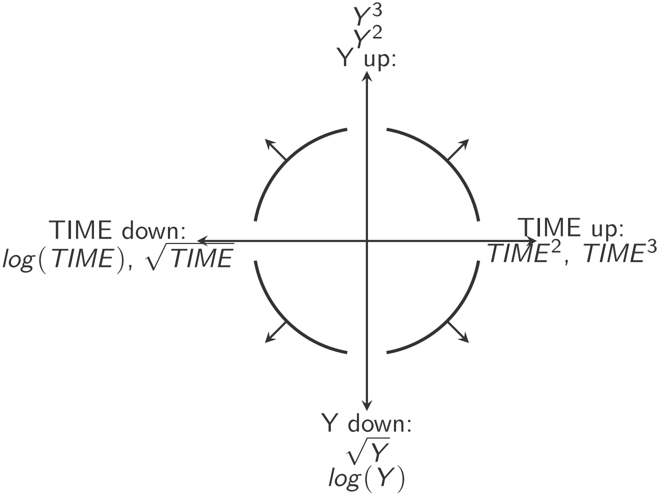

Mosteller and Tukey's (1977, p. 84) set of rules for transformations.

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Use GoF criteria to determine the moment at which you're overfitting the data.

I hope you enjoyed the workshop!

Let's keep in touch:

manuel.bosancianu@gmail.com

References

Mosteller, F., & Tukey, J. W. (1977). *Data Analysis and Regression. A Second Course in Statistics*. Reading, MA: Addison-Wesley.