Structural Equation Modeling with R and lavaan

Day 3: Multilevel Structural Equation Models

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Today's plan

Today we join some of the insights from the January workshop on MLM with what we covered in the past 2 days:

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- ✓ multilevel measurement models
- ✓ multilevel path models

From the software perspective, **lavaan** is still adding some more advanced capabilities, but the project is making constant advances in this.

The Multilevel Perspective

Value of MLM

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Many similar examples related to educational research, e.g. differences between schools in how much progress students make over a 4-year cycle.

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Trying to see the world like this trains your mind: how individual actions shape context, and how context, in turn, shapes individual action.

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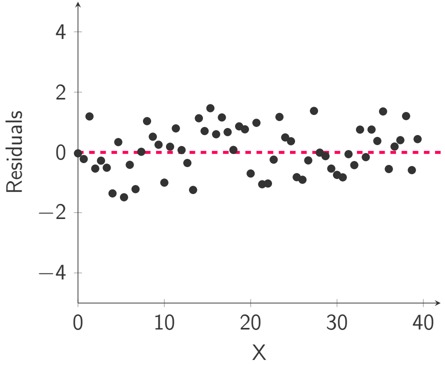
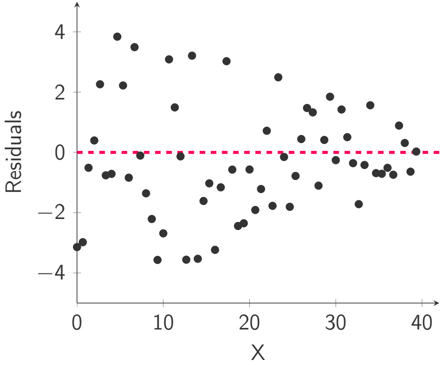
As SEs are incorporated into significance tests, we risk rejecting the null hypothesis more often than we should.

One OLS assumption

Homoskedasticity: $\epsilon \sim \mathcal{N}(0, \sigma^2)$.

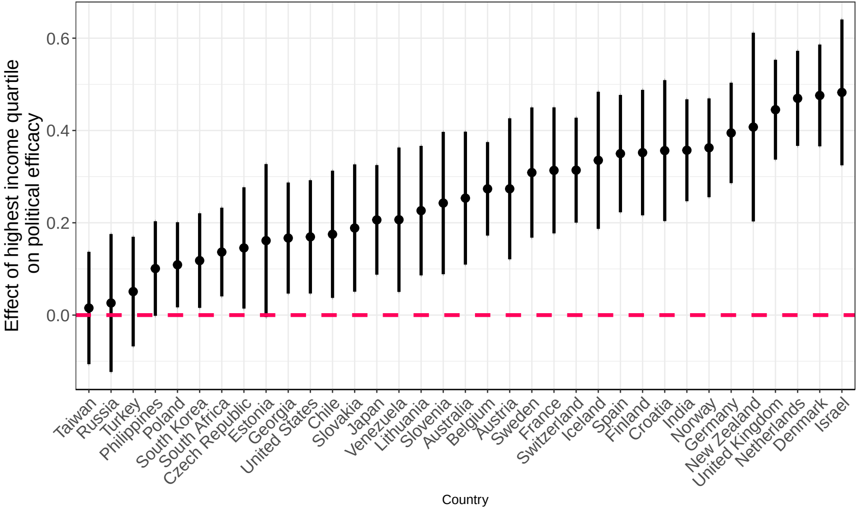


(a) Homoskedasticity



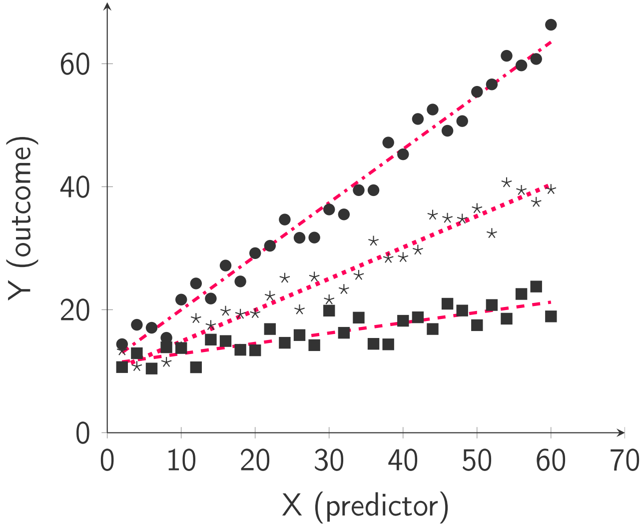
(b) Heteroskedasticity

The case of clustered data



Variation in the effect of income on political efficacy (ISSP Citizenship II, 2016)

Consequences of heterogeneity



In this instance, applying an overall slope ("naive pooling") to the data will generate heteroskedasticity.

This can be addressed with country dummies (*fixed effects*), but these won't explain *why* you're seeing a specific pattern.

MLM & SEM: substantive reasons

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These outcomes can be either observed variables or, even latent variables.

Tremendously helpful if we want to understand why and a measurement structure varies over (many) groups, or to explain variation in a structural component over the same groups.

Multilevel Path Models

Modifications to the SEM framework (I)

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The notation becomes cumbersome very quickly (there can now be multiple β_1s), so I focus on graphical representations.

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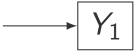
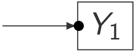


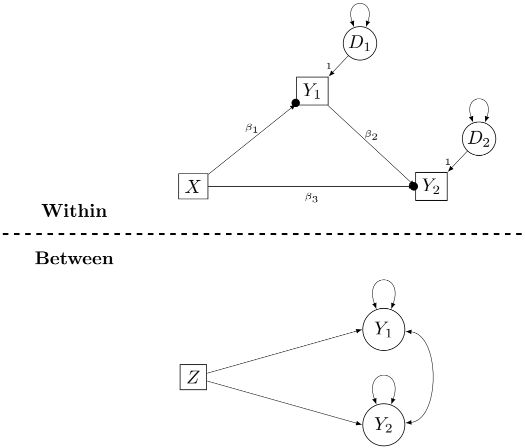
Fixed intercept



Varying intercept

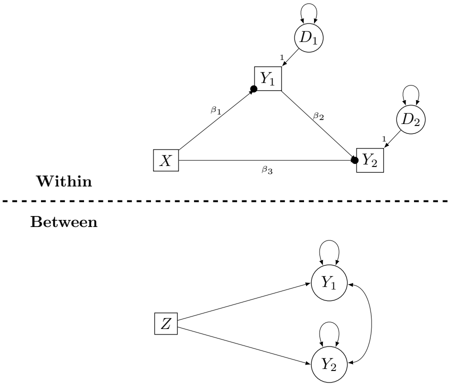
For modeling, all varying parameters at a lower level are considered latents at the higher level.

Multilevel path specification



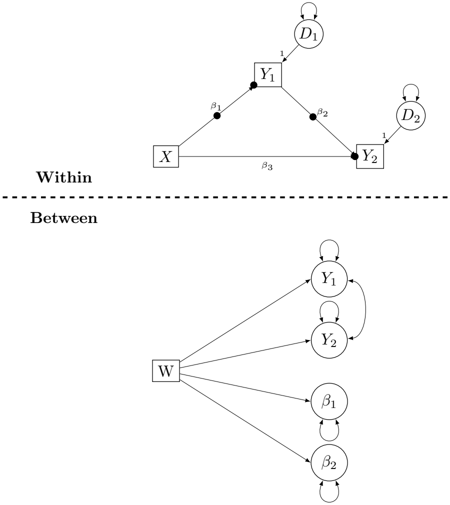
Level-1 intercepts are allowed to vary across level-2 groups.

Varying intercepts



Both intercepts (for Y_1 and Y_2) are assumed to follow a Gaussian distribution, with the means explained by Z_j (group-level factor).

Varying intercepts and slopes



Estimation and model fit

Similar to standard SEM, makes heavy use of ML and variations of it that are more robust to non-normality (robust ML, and the Bollen–Stine bootstrap).

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The SRMR is the only index that provides a model fit summary for the within-level and the between-level.

Partially-saturated model test

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Logic:

- ✓ model is specified with a saturated "between" part ⇒ misfit must come from "within" part
- ✓ model is then specified with a saturated "within" part ⇒ misfit must come from "between" part

Mediation in MSEM

Mediation poses problem in a standard multilevel setting, as it's hard to isolate cross-level dynamics ($2 \rightarrow 1 \rightarrow 1$): we can't disentangle within-group and between-group effects (Zhang, Zyphur, & Preacher, 2009).

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MSEM allows for tests of quite diverse linkages: $1 \rightarrow 1 \rightarrow 2$, $2 \rightarrow 1 \rightarrow 2$, or $1 \rightarrow 2 \rightarrow 2$ (level-2 constructs can be outcomes in SEM).

Multilevel Factor Models

Multiple-group CFA (MG-CFA)

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Invariance: individuals from two different populations with the same level on the latent construct have the same scores on the measured indicators.

This property is important if we want to apply the same measurement instrument (political efficacy, political trust, populist attitudes) across contexts.

Without it, cross-context differences might be just due to measurement error.

Multilevel CFA

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- ✓ it can test whether the within-group structure of measurement matches the between-group one
- ✓ level-2 covariates can be used to explain variance in level-2 constructs (becoming a full structural regression model)
- ✓ allows for testing of measurement invariance across many groups at the same time

2-level CFA: random intercepts (I)

The variance of an observed indicator is split into 2 components, within- and between-, which are additive and orthogonal:

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The "within" variance is explained by the level-1 latent factors. The "between" part is explained with the level-2 latent factors.

These two sets of specifications together comprise the 2-level measurement structure.

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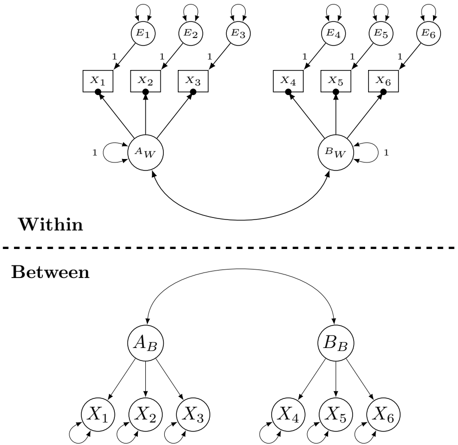

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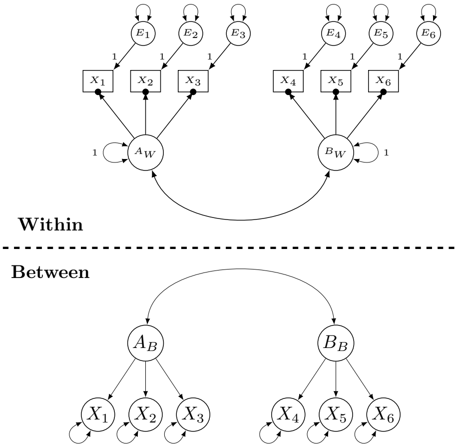

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We have to assume that the $g_1, g_2, \ldots g_j$ covariance matrices for the groups are identical $\Rightarrow$ measurement invariance.

Between-group variance

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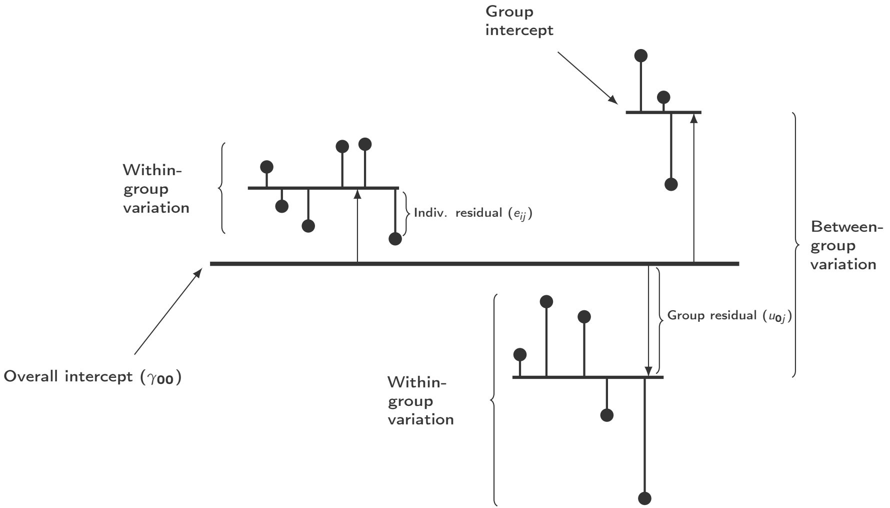
With insufficient variance, there is not much for group-level factors to explain.

ICC decomposition



Adapted from Merlo et al. (2005).

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Therefore, the maximum number of estimable parameters is $p(p+1)+k$, where k is the number of indicator intercepts.

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As before, we can add group-level indicators to explain the cross-group variance in loadings.

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For more advanced models, **Mplus** is, by and large, the most capable software. It also has a **R** package to send data for estimation: **MplusAutomation** (Hallquist & Wiley, 2018).

Thank you for the kind attention!

References I

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