Structural Equation Modeling with R and lavaan

Day 2: Full Structural Regression Models

Constantin Manuel Bosancianu

WZB Berlin Social Science Center
Institutions and Political Inequality
manuel.bosancianu@gmail.com

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Today's plan

Less theory than yesterday, to allow for more time in the lab session to explore **lavaan**:

- ✓ basic options for estimation of path models

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Best practices in analyzing and presenting results from these models to an audience.

Estimation

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- ✓ **simultaneous methods**: estimate all parameters in one go

Advantages and disadvantages to each, though the **simultaneous** approach tends to produce estimates with lower variance (is more *efficient*).

Single-equation methods (I)

Estimate one-by-one an equation for each endogenous variable in the *recursive* model using OLS.

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Main disadvantage: they don't provide a measure of *global* model fit.

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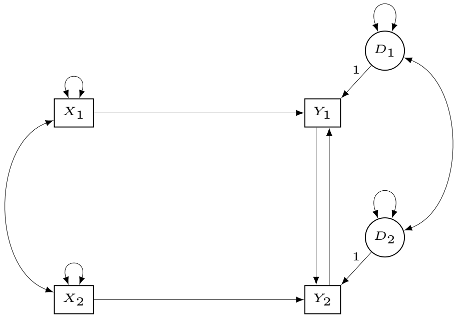

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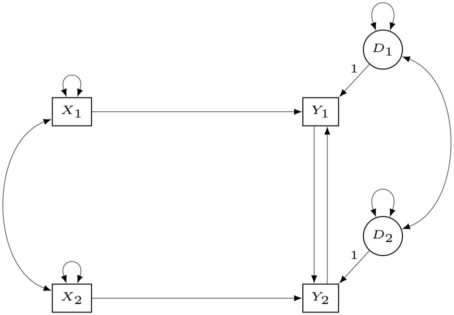

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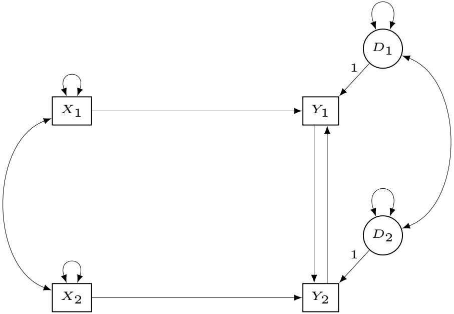

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- regress "problematic" variable (Y_1) on instrument (X_1), and save predicted values $\hat{Y}_1$
- regress Y_2 on predicted values $\hat{Y}_1$

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The most common such method is **maximum likelihood**, which comes in a variety of "flavors"; great advantage in that it works for latent variables as well.

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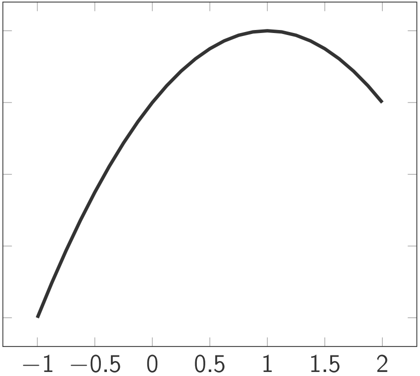
Problem if there are *multiple optima*: multiple sets of estimates that produce the same degree of fit between model and data.

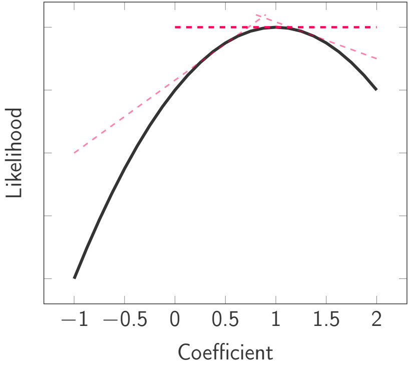
Maximum Likelihood (II)

Convergence of the algorithm is reached when the change in fit is extremely small.



Maximum Likelihood (III)

Setting the first derivative to the likelihood function to 0 gives you the coefficients.



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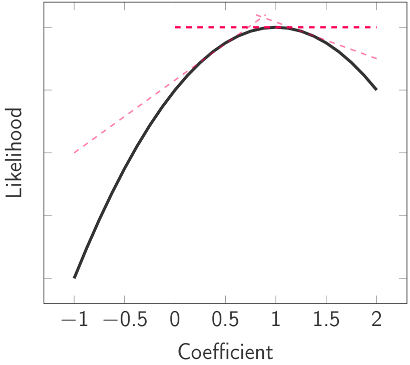

The second derivative to the likelihood function is used to determine if we found the minimum or maximum,[1] as well as to compute the standard errors.

[1] If it's negative, we found the maximum; if it's positive, it's a minimum.

Robust ML

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- ✓ the *p* values for estimates and model fit tests are also based on these replications

Categorical outcomes

With at least 6–7 categories, and distributions that approximate a Gaussian one, ML should provide reasonably accurate estimates (Rhemtulla, Brosseau-Liard, & Savalei, 2012).

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2. *robust WLS*
3. *full-information ML* based on numerical integration—complex in terms of calculations (read: **slow**), but implemented in software

Model fit

Local fit (I)

The goal is to understand for small subsets of a larger model, whether predicted covariances (under the model) fit observed covariances.

If not, where are the biggest gaps, and what can we learn from these patterns?

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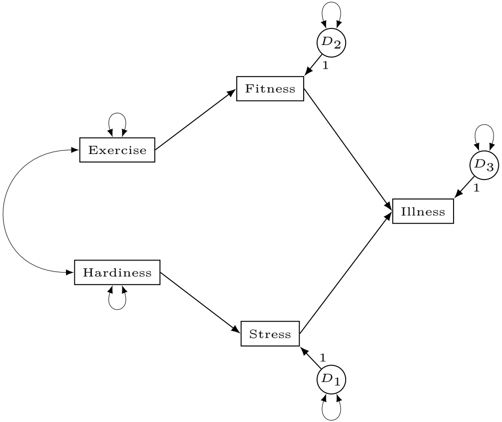
Roth et al. (1989) test an explanatory model of students' susceptibility to illness based on exercise, psychological hardiness, fitness, and stress.

Explaining illness



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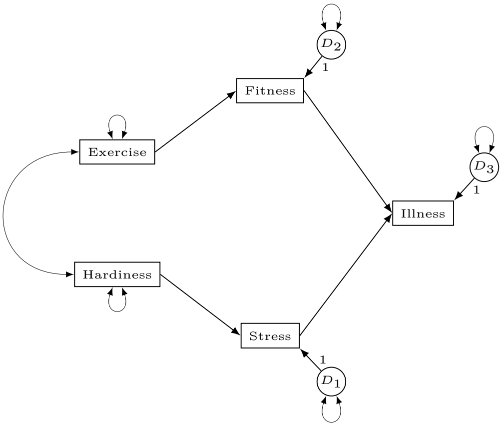

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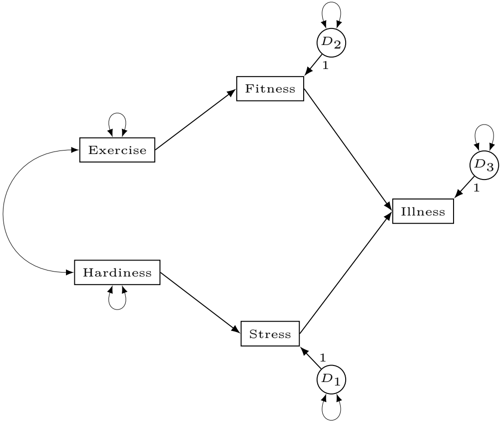

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Once we control for hardiness, exercise and stress should be independent of each other. 4 more such conditional independences can be evaluated.

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Independence	Controlling for	Partial correlation
Exercise $\perp\!\!\!\perp$ Stress	Hardiness	-0.058
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Hardiness $\perp\!\!\!\perp$ Fitness	Exercise	0.089
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It's possible we need to specify a causal link between fitness and stress (though the estimate is at the border).

After fit assessment

Estimation: Kline (2015) has an example for a *single-equation* approach (pp. 241–247), but here we jump directly to the *simultaneous* approach.

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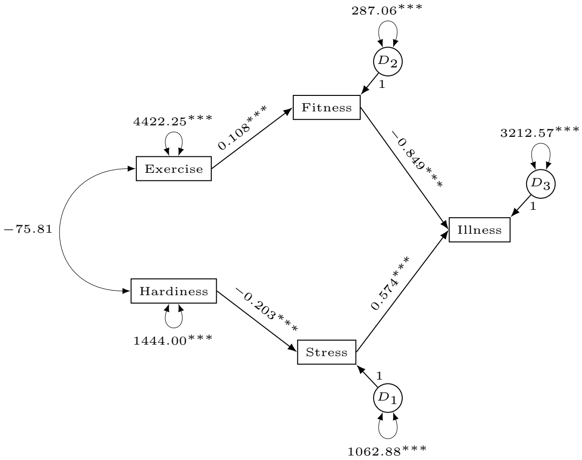
This also produces the measures of global fit we discuss in the next subsection, though these are not reported here.

Results simultaneous approach (I)



Unstandardized estimates (SEs not depicted)

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If doing this by hand (multiplying direct effects), there is a special approximate formula for the SE of the indirect effect (Sobel, 1982):

$$SE_{ab} = \sqrt{b^2 SE_a^2 + a^2 SE_b^2} \tag{1}$$

Residuals (I)

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	Fitness	Stress	Illness	Exercise	Hardiness
Fitness	0.000				
Stress	-0.133	0.000			
Illness	-0.038	0.030	0.000		
Exercise	0.000	-0.057	0.016	0.000	
Hardiness	0.082	0.000	-0.091	0.000	0.000

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As before, absolute values that are larger than 0.10 are problematic.

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Illness	-2.573	2.573	0.000		
Exercise	0.000	-1.128	0.358	0.000	
Hardiness	1.708	0.000	-1.921	0.000	0.000

Standardized residuals

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- ✓ good fit is not synonymous with theoretical soundness

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Never enough by itself; most valuable when used in conjunction with local fit testing.

Model χ^2

$$\chi^2 = (N - 1) * F_{ML} \tag{2}$$

N is sample size, and F_{ML} is the fit function minimized in ML estimation.[2]

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If $\chi^2 = 0$, then $F_{ML} = 0$, so there is no discrepancy between model and sample covariances. Higher values of χ^2 denote worse fit (a "badness-of-fit" indicator).

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χ^2 sensitivity

Easy to reduce by freeing parameters. Other influences on value:

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χ^2 variants

Various proposals for adjustments to χ^2.

With robust ML (MLR), you can obtain a **Satorra–Bentler scaled** χ^2, which applies a scaling correction factor (the average kurtosis in the data) to the model χ^2.

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There is also a **Satorra–Bentler adjusted** χ^2, though it's less used than the scaled version.

Approximate fit indices

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- **parsimony-adjusted**: incorporate a penalty for model complexity
- **predictive fit**: hypothetical fit in other samples from same population

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If $\hat{\Delta}_M = 0$, there is no departure from close fit. Otherwise:

$$\hat{\varepsilon} = \sqrt{\frac{\hat{\Delta}_M}{df_M(N-1)}} \tag{5}$$

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Using the Satorra–Bentler scaled χ^2 produces a more robust version of the RMSEA to departures from normality.

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Values greater than 0.10 indicate poor fit.

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CFI of 0.90 would indicate model fit that is about 90% better than the baseline model.

Usage recommendations

Report the model χ^2 and its degrees of freedom and p value. Irrespective of whether this test is passed or not, conduct local fit testing.

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Report values from approximate fit indices as well (RMSEA, CFI, SRMR), but keep in mind that thresholds for them depend considerably on (1) sample size, (2) distributional assumptions, (3) degree of mis-specification, and (4) estimation method (Xia & Yang, 2019).

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- **model building**: start with a simple model and free parameters—ideally, χ^2_M decreases considerably

Nested models

2 models are nested if one includes all the parameters of the other, plus at least one more. Example: one parameter is freed or constrained.

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Can be compared with the χ^2 difference statistic, with *df* the difference in the number of parameters between the two models.

Non-nested models (I)

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Models need to be estimated on the same sample to compare their AICs.

Non-nested models (II)

The Bayesian Information Criterion (BIC), however, does incorporate sample size differences:

$$BIC = \chi^2_M + k * ln(N) \tag{8}$$

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The Bayesian Information Criterion (BIC), however, does incorporate sample size differences:

$$BIC = \chi^2_M + k * ln(N) \tag{8}$$

where k is the number of free parameters and N is the sample size.

BIC should be used carefully, as with increasing sample size χ^2_M also may increase even though model fit wouldn't change.

Structural Regression Models

SR models (I)

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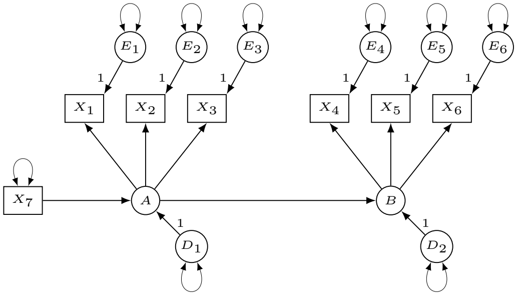
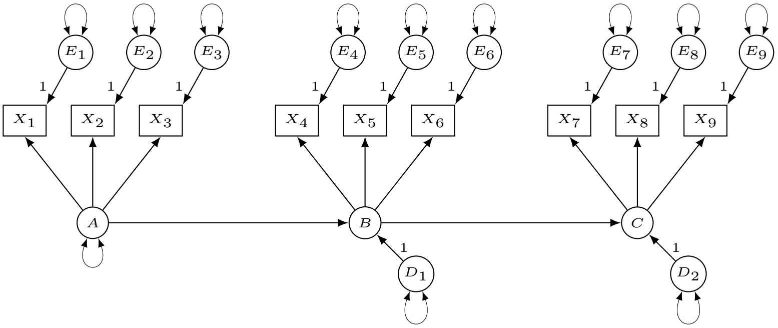


Partially latent SR model

SR models (II)



Fully latent SR model

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Both parts have to be identified (**two-step identification rule**).

2-step modeling (I)

First, specify a fully latent SR model as a CFA measurement model, to determine whether it fits the data.

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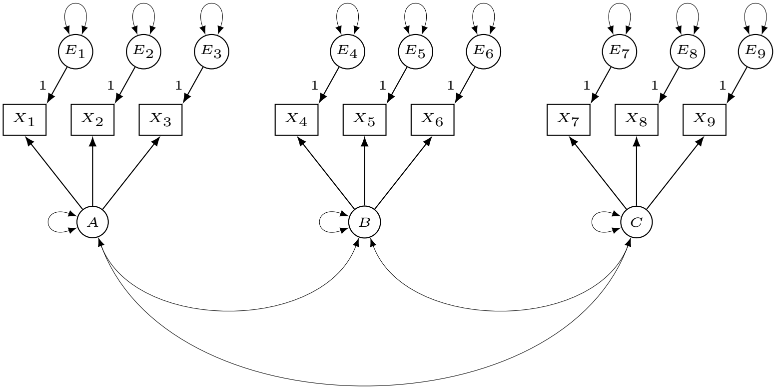

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A 4-step approach was also proposed, where 1st stage is an EFA measurement model, which can then be reduced to a CFA one in the 2nd stage.

Best practices

A few resources

All of these are currently uploaded on Moodle:

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5. Thompson (2000) lists the "10 commandments" of SEM

Selected suggestions: specification

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Don't specify reciprocal causation unless theory really suggests this is the case.

Keep in mind the importance of **parsimony**.

It's fine to constrain parameters, like forcing equality or proportionality, but make sure to defend this theoretically.

Selected suggestions: estimation

Don't rely on standard ML estimation for ordinal variables (6–7); go for full-information ML, or one of the least squares-based methods.

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For a fully latent SR model, don't do "single-shot" estimation. Instead:

1. ensure the measurement model is correctly specified
2. after this, proceed to adding causal links between latents

Selected suggestions: others

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Report the residual matrix (at a minimum in the appendix), and discuss it in the main text.

Report measures of model fit in the main text: model χ^2 with *df* and *p* value, RMSEA with 90% CI, CFI (or TLI), and the SRMR.

Thank you for the kind attention!

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