

# Structural Equation Modeling with R and lavaan

## Day 3: Multilevel Structural Equation Models

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# Today's plan

Today we join some of the insights from the January workshop on MLM with what we covered in the past 2 days:

- ✓ the need for multilevel specifications
- ✓ the logic of MSEM
- ✓ multilevel measurement models
- ✓ multilevel path models

From the software perspective, **lavaan** is still adding some more advanced capabilities, but the project is making constant advances in this.

# The Multilevel Perspective

# Value of MLM

MLMs are uniquely suited to capturing one type of social complexity: the way individuals/firms/NGOs act or think may be context-dependent.

An example which I focused on in January (and continue to do so in my research) are the cross-country differences in the likelihood that lower-income people participate in politics.

Many similar examples related to educational research, e.g. differences between schools in how much progress students make over a 4-year cycle.

# Cross-national variance

|                                | Micro              | Macro  |
|--------------------------------|--------------------|--|
| <i>Political participation</i> | income<br>efficacy | party polarization<br>welfare state institutions |
| <i>Trust</i>                   | education          | post-communist country                           |
| <i>Religiosity</i>             | age                | income inequality                                |

Trying to see the world like this trains your mind: how individual actions shape context, and how context, in turn, shapes individual action.

# MLM: statistical reasons

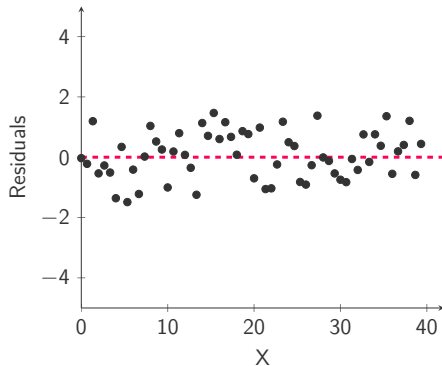
- ✓ obtain accurate SEs for estimates in instances of clustered data;
- ✓ *model* the heteroskedasticity (unobserved heterogeneity) in the data.

Because we're typically dealing with complex sample designs, formulas for standard errors are imprecise.

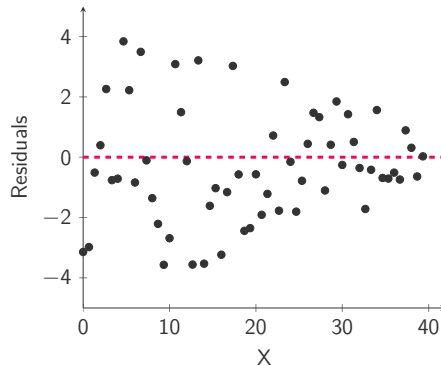
As SEs are incorporated into significance tests, we risk rejecting the null hypothesis more often than we should.

# One OLS assumption

**Homoskedasticity:**  $\epsilon \sim \mathcal{N}(0, \sigma^2)$ .

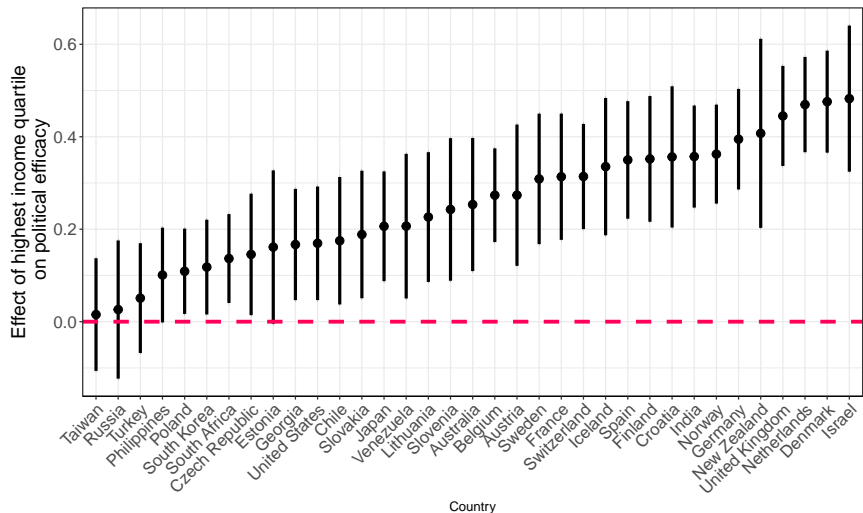


(a) Homoskedasticity



(b) Heteroskedasticity

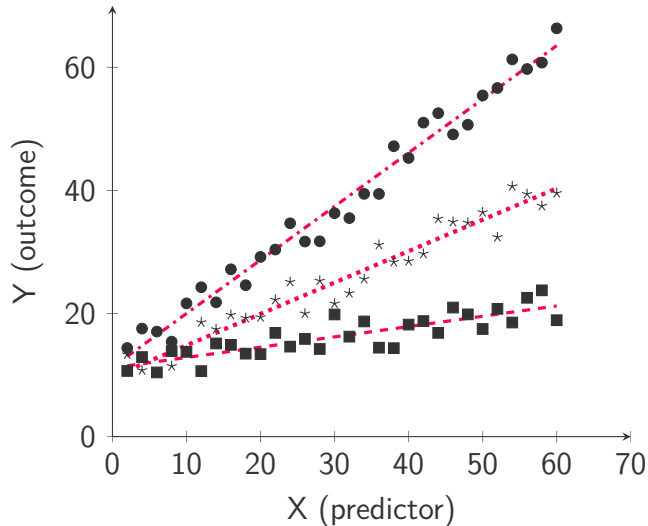
# The case of clustered data



Variation in the effect of income on political efficacy (ISSP Citizenship II, 2016)



# Consequences of heterogeneity



In this instance, applying an overall slope (“naive pooling”) to the data will generate heteroskedasticity.

This can be addressed with country dummies (*fixed effects*), but these won't explain *why* you're seeing a specific pattern.

# MLM & SEM: substantive reasons

Systematically account for how outcomes (or *effects*) vary across groups, beyond what can be explained by unit-level factors.

These outcomes can be either observed variables or, even latent variables.

Tremendously helpful if we want to understand why and a measurement structure varies over (many) groups, or to explain variation in a structural component over the same groups.

# Multilevel Path Models

# Modifications to the SEM framework (I)

The defining feature of the MLM framework was its ability to allow parameters to vary across groups:  $\beta_1 \rightarrow \beta_{1j}$ .

These varying parameters got a statistical model of their own, with upper-level variables:  $\beta_{1j} = \gamma_{10} + \gamma_{11} * Z_{1j} + \dots$

The same approach is used when moving from the SEM to the MSEM framework: allowing parameters, e.g. factor loadings, path coefficients, intercepts, to vary across groups.

The notation becomes cumbersome very quickly (there can now be multiple  $\beta_1$ s), so I focus on graphical representations.

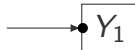
## Modifications to the SEM framework (II)

Estimated covariances can be either fixed ( $\longrightarrow$ ) or varying ( $\longrightarrow\bullet$ ) (different than *free* or *constrained*).

We haven't discussed yet about modeling a mean structure in a SEM, but the same can be done for intercepts



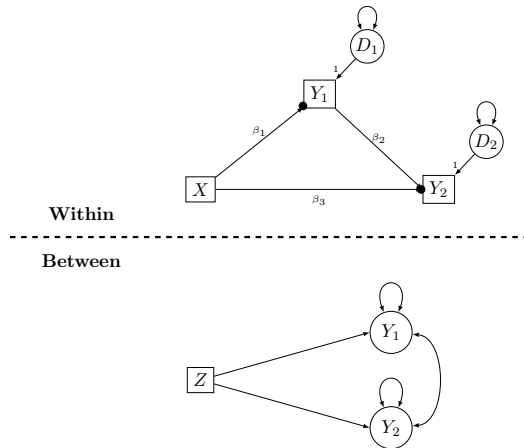
Fixed intercept



Varying intercept

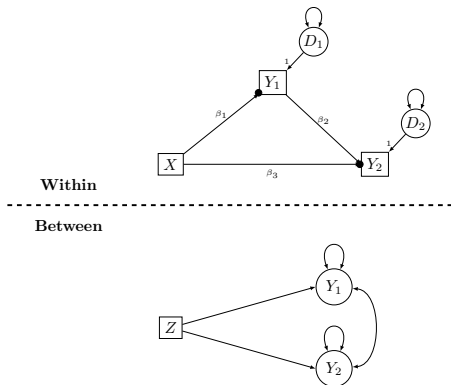
For modeling, all varying parameters at a lower level are considered latents at the higher level.

# Multilevel path specification



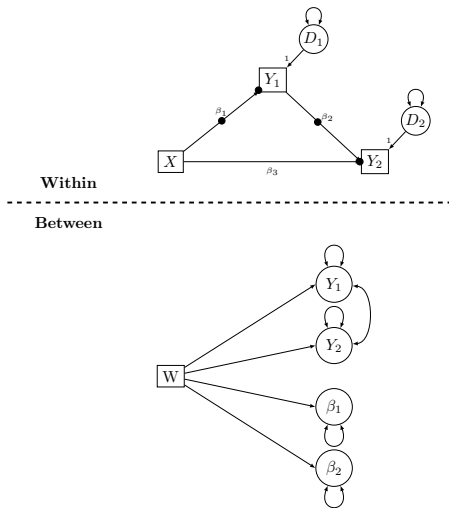
Level-1 intercepts are allowed to vary across level-2 groups.

# Varying intercepts



Both intercepts (for  $Y_1$  and  $Y_2$ ) are assumed to follow a Gaussian distribution, with the means explained by  $Z_j$  (group-level factor).

# Varying intercepts and slopes





# Estimation and model fit

Similar to standard SEM, makes heavy use of ML and variations of it that are more robust to non-normality (robust ML, and the Bollen–Stine bootstrap).

Model fit assessment is conducted using same indicators (model  $\chi^2$ , RMSEA, CFI, SRMR) as presented yesterday.

The first 3 provide a single number assessment; for multilevel models, that means sometimes the level-1 sample size dominates.

The SRMR is the only index that provides a model fit summary for the within-level and the between-level.

# Partially-saturated model test

A specific approach to MSEM: Ryu and West's (2009) *partially-saturated model test*; unfortunately, currently not implemented as a standard command.

Logic:

- ✓ model is specified with a saturated “between” part  $\Rightarrow$  misfit must come from “within” part
- ✓ model is then specified with a saturated “within” part  $\Rightarrow$  misfit must come from “between” part

# Mediation in MSEM

Mediation poses problem in a standard multilevel setting, as it's hard to isolate cross-level dynamics ( $2 \rightarrow 1 \rightarrow 1$ ): we can't disentangle within-group and between-group effects (Zhang, Zyphur, & Preacher, 2009).

MSEM allows for tests of quite diverse linkages:  $1 \rightarrow 1 \rightarrow 2$ ,  $2 \rightarrow 1 \rightarrow 2$ , or  $1 \rightarrow 2 \rightarrow 2$  (level-2 constructs can be outcomes in SEM).

# Multilevel Factor Models

# Multiple-group CFA (MG-CFA)

A precursor to multilevel CFA: checking for measurement *invariance* by checking factor loadings across multiple groups, e.g. women and men, East and West Germany.

*Invariance*: individuals from two different populations with the same level on the latent construct have the same scores on the measured indicators.

This property is important if we want to apply the same measurement instrument (political efficacy, political trust, populist attitudes) across contexts.

Without it, cross-context differences might be just due to measurement error.

# Multilevel CFA

Multilevel CFA aims at the same insights as MG-CFA, but has the toolbox to pursue these questions across many more groups:

- ✓ it can test whether the within-group structure of measurement matches the between-group one
- ✓ level-2 covariates can be used to explain variance in level-2 constructs (becoming a full structural regression model)
- ✓ allows for testing of measurement invariance across many groups at the same time

## 2-level CFA: random intercepts (I)

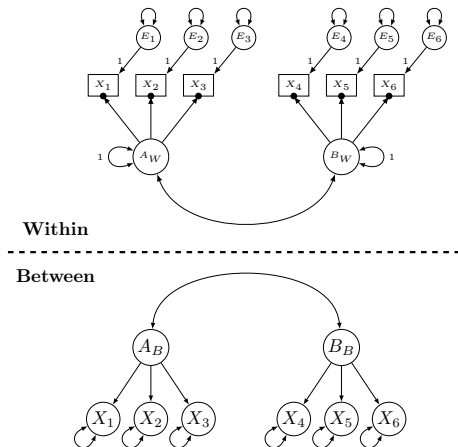
The variance of an observed indicator is split into 2 components, within- and between-, which are additive and orthogonal:

$$Var(Y) = Var(Y)_B + Var(Y)_W \quad (1)$$

The “within” variance is explained by the level-1 latent factors. The “between” part is explained with the level-2 latent factors.

These two sets of specifications together comprise the 2-level measurement structure.

## 2-level CFA: random intercepts (II)



At the “between” part we’re explaining the intercepts from the “within” part using 2 factors.

We have to assume that the  $g_1, g_2, \dots, g_j$  covariance matrices for the groups are identical  $\Rightarrow$  measurement invariance.



## Between-group variance

As an initial stage, just as in a standard MLM, we would check that there is sufficient between-group variance in the indicators.

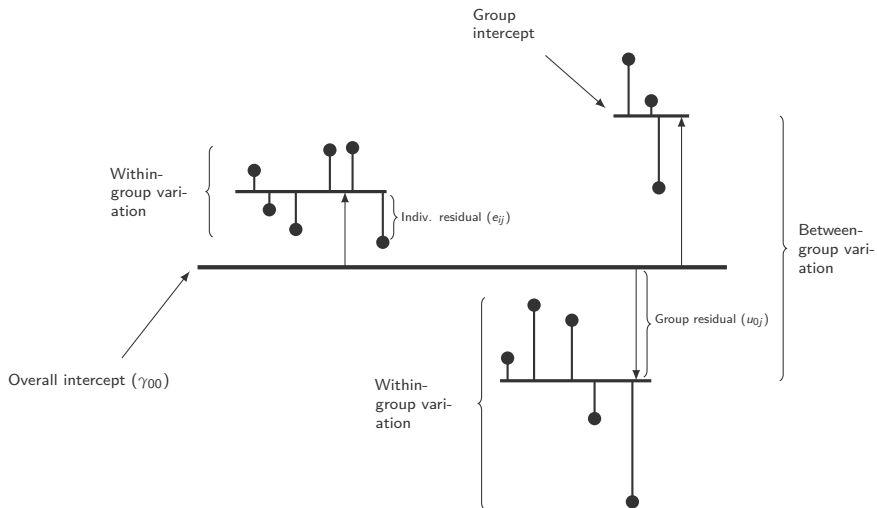
The standard tool for this is the ICC (*intra-class correlation coefficient*):

$$\rho = \frac{\sigma_{\alpha}^2}{\sigma_{\alpha}^2 + \sigma_y^2} \quad (2)$$

where  $\sigma_{total}^2 = \sigma_{\alpha}^2 + \sigma_y^2$

With insufficient variance, there is not much for group-level factors to explain.

# ICC decomposition



Adapted from Merlo et al. (2005).

# Identification

The rule presented in the beginning is that there need to be fewer free parameter than the number of observations:  $\frac{p(p+1)}{2}$ .

These are also the off-diagonal elements in the variance-covariance matrix of observed indicators.

In a multilevel setting, we have 2 such matrices: for the “within” and the “between” components.

Therefore, the maximum number of estimable parameters is  $p(p+1) + k$ , where  $k$  is the number of indicator intercepts.

## 2-level CFA: random loadings

The assumption of measurement invariance is limiting. What if specific groups have a different measurement structure than others, e.g. employees from certain companies have different patterns in their scores?

To check for this, we would allow both intercepts and factor loadings to vary across groups.

The implication is that we now consider the  $g_1, g_2, \dots, g_j$  group covariance matrices to be different.

As before, we can add group-level indicators to explain the cross-group variance in loadings.

## Final point on software

The capabilities of **lavaan** are still limited here, though growing fast.

Currently, **lavaan** can only run multilevel CFA with random intercepts.

Furthermore, only with continuous indicators!

For more advanced models, **Mplus** is, by and large, the most capable software. It also has a **R** package to send data for estimation: **MplusAutomation** (Hallquist & Wiley, 2018).

Thank **you** for the kind attention!

# References I

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