Quiz Problem II

$X_n \sim Exp(\lambda)$

$f(x) = \lambda e^{-\lambda x} \rightarrow L(\lambda) = \prod_n \lambda e^{-\lambda x} = \lambda^n e^{-\lambda \Sigma x}$

$H_0: \lambda = \lambda_0$, $H_A = \lambda \neq \lambda_0$

$$\lambda(\underset{\sim}{x}) = \frac{\max_{\lambda \in \lambda_0} L(\lambda)}{\max_{\lambda \in \lambda} L(\lambda)} = \frac{L(\lambda_0)}{\max_{\lambda \in \lambda} L(\lambda)}$$ ← this is MLE

$\log(L(\lambda)) = n \log - \lambda \Sigma x$

$\frac{d}{d\lambda}(\log(L(\lambda)) = \frac{n}{\lambda} - \Sigma x$

Set to zero $\frac{n}{\lambda} - \Sigma x = \lambda = \frac{1}{\bar{x}}$

$$\lambda(\underset{\sim}{x}) = \frac{L(\lambda_0)}{L(1/\bar{x})} = \frac{\lambda_0^n e^{-\lambda_0 \Sigma x}}{(\frac{1}{\bar{x}})^n e^{-n}} = (\lambda_0 \bar{x})^n e^{n - \lambda_0 \Sigma x}$$

$(\lambda_0 \bar{x})(e^{n - \lambda_0 \Sigma x}) \leq c$ when

We reject if this is the case where c is a chosen constant between 0 and 1