Quiz Problem 6

First prove estimator umbiase

$$\hat{\beta} = \frac{\bar{Y}}{x} \quad \text{then} \quad E[\hat{\beta}] = \frac{1}{x} E[\bar{Y}] = \frac{\mu}{x} = \beta$$

so its umbiased for $T(\beta) = \beta$

Find $I_N(\beta)$

$$F(x) = \frac{1}{6\sqrt{2\pi}} e^{\frac{-1}{26^2}(y - x\beta)^2}$$

log: $\log\left(\frac{1}{6\sqrt{2\pi}}\right) - \frac{1}{26^2}(y^2 - 2yBx + \beta^2 x^2)$

$\partial\beta$: $\frac{yx}{6^2} - \frac{\beta x^2}{6^2}$

$\partial\beta$: $-\frac{x^2}{6^2}$

$$I(\beta) = -E\left[-\frac{x^2}{6^2}\right] = \frac{x^2}{6^2} \rightarrow I_N(\beta) = \frac{Nx^2}{6^2}$$

CRLB : $\frac{\left(\frac{\partial J}{\partial B}\right)^2}{I_N(\beta)} = \frac{(1)^2}{\frac{Nx^2}{6^2}} = \frac{6^2}{Nx^2}$

To prove $\beta = \frac{\bar{Y}}{x}$ is UmVUE . $Var(\beta) = \frac{6^2}{Nx^2}$

$$Var\left(\frac{\bar{Y}}{x}\right) = \frac{1}{x^2} Var(\bar{Y}) = \frac{1}{x^2}\left(\frac{6^2}{\cancel{x}.N}\right)$$

$$= \frac{6^2}{x^2 N}$$

$\leftarrow$ since it is equal to CRLB $\beta = \frac{\bar{Y}}{\bar{x}}$ is UMVUE