Quiz Problem 6

we need to find indicator Functions

$$\delta_c(x) = \frac{-\mu_c^2}{2\sigma^2} + \log \pi_c + \frac{\mu_c}{\sigma^2} x$$ ← From notes

so

$$\delta_{-1} = \frac{-\mu_{-1}^2}{2\sigma^2} + \log \pi_{-1} + \frac{\mu_{-1}}{\sigma^2} x$$

$$\delta_1 = \frac{-\mu_1^2}{2\sigma^2} + \log \pi_1 + \frac{\mu_1}{\sigma^2} x$$

σ is shared

$\pi_1 = \pi_{-1}$ since # in classes is same

First direction: If $\delta_1 > \delta_{-1}$ then $\alpha_0 + \alpha \cdot x > 0$

$$\delta_1 - \delta_{-1} > 0$$

$$\frac{-\mu_1^2}{2\sigma^2} + \log \pi_1 + \frac{\mu_1 x}{\sigma^2} + \frac{\mu_{-1}^2}{2\sigma^2} - \log \pi_{-1} - \frac{\mu_{-1} x}{\sigma^2} > 0$$

$$-\frac{\mu_1^2}{2} + \mu_1 x + \frac{\mu_{-1}^2}{2} - \mu_{-1} x > 0$$

$$-\frac{1}{2}(\mu_1^2 - \mu_{-1}^2) + (\mu_1 - \mu_{-1})x > 0$$

$$\underbrace{-\frac{1}{2}(\mu_1 - \mu_{-1})(\mu_1 + \mu_{-1})}_{\alpha_0} + \underbrace{(\mu_1 - \mu_{-1})}_{\alpha_1} x > 0$$

Since all the algebra we have done is invertible meaning we can work backwards then it is true

$\alpha_0 + \alpha x > 0 \rightarrow \delta_1 > \delta_{-1}$ must be true too

2. We need to First find sums:

- $\bar{x} = \frac{1}{N}\sum x_n = \frac{1}{N}\left(\sum_{n:y_n=1} x_n + \sum_{n:y_n=-1} x_n\right)$

$$= \frac{1}{2}\hat{\mu}_1 + \frac{1}{2}\mu_{-1} = \frac{1}{2}(\mu_1 + \mu_{-1})$$

- $\bar{y} = 0$

- $\sum x_n y_n = \sum_{n:y_n=1} x_n - \sum_{n:y_n=-1} x_n = \frac{N}{2}(\hat{\mu}_1 - \hat{\mu}_{-1})$

So

$$\bar{y} - \hat{\beta}\bar{x} + \hat{\beta}x_n > 0 \quad , \quad \bar{y} = 0$$

$$\cancel{-\hat{\beta}} \quad \hat{\beta} = \frac{\sum_{n=1}^{N} x_n(y_n - \bar{y})}{\sum_{n=1}^{N}(x_n - \bar{x})^2} = \frac{\sum_{n=1}^{N} x_n y_n}{\sum_{n=1}^{N}(x_n - \bar{x})^2}$$

$\cancel{-\hat{\beta}\bar{x}}$

$$-\frac{\sum_{n=1}^{N} x_n y_n}{\sum_{n=1}^{N}(x_n - \bar{x})^2}\bar{x} + \frac{\sum_{n=1}^{N} x_n y_n}{\sum_{n=1}^{N}(x_n - \bar{x})^2} x_n > 0$$

$$-\left(\frac{N}{2}(\hat{\mu}_1 - \hat{\mu}_{-1})\right)\left(\frac{1}{2}\mu_1 + \mu_{-1}\right) + \frac{N}{2}(\hat{\mu}_1 - \hat{\mu}_{-1})x > 0$$

$$\underbrace{-\frac{1}{2}(\mu_1 - \mu_{-1})(\mu_1 + \mu_{-1})}_{\alpha_0} + \underbrace{(\hat{\mu}_1 - \hat{\mu}_{-1})}_{\alpha_1}x > 0$$

We showed previously that if $\alpha_0 + \alpha_1 x > 0$ than the point is classified as 1.
Since Algebraicly invertible then the other direction is true as well.