Quiz Problem 8

$$\Sigma = \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & 1 \\ -1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$\det(\Sigma - \lambda I) = 0$$

$$(\lambda - 6)(\lambda - 4) = 0$$

$$\lambda_1 = 6, \quad \lambda_2 = 4$$

V_1 :

$$\begin{bmatrix} 5-6 & 1 \\ 1 & 5-6 \end{bmatrix} v_1 = 0$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} v_1 = 0$$

$$v = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

V_2 :

$$\begin{bmatrix} 5-4 & 1 \\ 1 & 5-4 \end{bmatrix} V_2 = 0$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} V_2 = 0$$

$$V_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Since $\lambda_1 > \lambda_2$, than V_2 corresponds to second PC.

$$\text{Prog} = X_1 \cdot V_2 = \begin{bmatrix} 0 & 1 \\ -1 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$