Quiz Problem 10

$$\hat{f}(x) = \frac{1}{B}\sum_{b=1}^{B} \hat{f}_b(x)$$

We can think of $\hat{f}(x)$ as the mean of $\hat{f}_b(x)$ so

$$Var(f(x)) = Var\left(\frac{1}{N}\sum_{b=1}^{B} \hat{f}_b(x)\right) = \frac{1}{N^2} Var\left(\sum \hat{f}_b(x)\right)$$

$$= \frac{1}{N^2}\left[\underbrace{\sum_{n} var(\hat{f}_b(x))}_{N\sigma^2} + \underbrace{\sum_{b \neq b_i} cov(f_{b_i}, f_{b_j})}_{N(N-1)\rho\sigma^2}\right]$$

$$= \frac{1}{N^2}\left[N\sigma^2 + N(N-1)\rho\sigma^2\right]$$

$$= \boxed{\frac{\sigma^2}{N} + \frac{(N-1)}{N}\rho\sigma^2}$$

$$\frac{\sigma^2}{N} + \frac{(N-1)}{N}\rho\sigma^2 \geq 0$$

$$1 + (N-1)\rho \geq 0$$

$$\boxed{\rho \geq \frac{-1}{N-1} \qquad \rho \geq \frac{1}{1-N}}$$

~~could never be -1 since N=0 if it did (which makes no sense)~~

the lower bound is $N=2$ so $\rho \geq -1$