

Quiz Problem 12

$$X \sim F_\theta$$

$$f_\theta(x) = \frac{2x}{\theta^2} \quad 0 < x < \theta$$

IF $T = \frac{X}{\theta}$ and $f(t) = 2t$ then

$$F_T(t) = \int_0^t 2t \, dt = t^2 \Big|_0^t = t^2$$

$$g(t) = t^2 = \left(\frac{x}{\theta}\right)^2 \quad \text{if } y = \frac{x^2}{\theta^2}$$

$$g^{-1}(y) = \sqrt{\frac{x^2}{y}} = \frac{x}{\sqrt{y}}$$

Since g is ~~increasing~~ decreasing in θ

$$L = g^{-1}\left(1 - \frac{\alpha}{2}\right) = \frac{x}{\sqrt{1 - \frac{\alpha}{2}}}$$

$$U = g^{-1}\left(\frac{\alpha}{2}\right) = \frac{x}{\sqrt{\frac{\alpha}{2}}}$$

$$\frac{x}{\sqrt{1 - \frac{\alpha}{2}}} \leq \theta \leq \frac{x}{\sqrt{\frac{\alpha}{2}}}$$