Quiz Problem 4

$X_n \sim Exp(\lambda)$

$Var(X_n) = 1/\lambda^2$, $Var(\bar{x}) = \frac{1}{N\lambda^2}$

$E[\bar{x}] = E[x] = 1/\lambda$

CLB: $\sqrt{N}(\bar{x} - 1/\lambda) \xrightarrow{d} N(0, 1/N\lambda^2)$

Delta:

$$\sqrt{N}(g(\bar{x}) - g(1/\lambda)) \xrightarrow{d} N(0, (g'(1/\lambda))^2 (1/N\lambda^2))$$

$g(t) = 1/t$, $g'(t) = -1/t^2$, $g'(1/\lambda) = -\lambda^2$

$g(1/\lambda) = \lambda$

$g(\bar{x}) = 1/\bar{x}$

so

$$\sqrt{N}(1/\bar{x} - \lambda) \xrightarrow{d} N(0, \frac{\lambda^4}{N\lambda^2})$$

$$\sqrt{N}(1/\bar{x} - \lambda) \xrightarrow{d} N(0, \frac{\lambda^2}{N})$$