Quiz Problem 8

Want to show $Y_N \xrightarrow{P} \beta$.

$$P(|Y_N - \beta| < \varepsilon) \xrightarrow{N} 1$$

$$= P(\beta - \varepsilon < Y_N < \beta + \varepsilon) = F_Y(\beta+\varepsilon) - F_Y(\beta-\varepsilon)$$

since $\beta + \varepsilon \geq \beta$ then $F_Y(\beta+\varepsilon) = 1 - \left(\frac{\beta}{\beta+\varepsilon}\right)^{\alpha N}$

since $\beta - \varepsilon \leq \beta$ then $F_Y(\beta-\varepsilon) = 0$

So

$$P(\beta - \varepsilon < Y_n < \beta + \varepsilon) = 1 - \underbrace{\left(\frac{\beta}{\beta+\varepsilon}\right)}_{<1}^{\alpha N}$$

since $\frac{\beta}{\beta+\varepsilon}$ is less than 1 so as N grows $P(|Y_N - \beta| < \varepsilon)$ goes to 1 which implies $Y_n \xrightarrow{P} \beta$.