$$\frac{\begin{array}{c}[A]^x \\ \vdots \\ B\end{array}}{A \to B}\ (\to \mathrm{I}^x) \qquad \frac{A \to B \quad A}{B}\ (\to \mathrm{E})$$

$$\frac{A \quad B}{A\ \&\ B}\ (\&\mathrm{I}) \qquad \frac{A\ \&\ B}{A}\ (\&\mathrm{E}_l) \qquad \frac{A\ \&\ B}{B}\ (\&\mathrm{E}_r)$$

$$\frac{\dfrac{[B\ \&\ A]^z}{A}\ (\&\mathrm{E}_r) \quad \dfrac{[B\ \&\ A]^z}{B}\ (\&\mathrm{E}_l)}{\dfrac{A\ \&\ B}{(B\ \&\ A) \to (A\ \&\ B)}\ (\to \mathrm{I}^z)}\ (\&\mathrm{I})$$

$$\frac{\dfrac{\begin{array}{c}[A]^x \\ \vdots \\ B\end{array}}{A \to B}\ (\to \mathrm{I}^x) \quad \begin{array}{c}\vdots \\ A\end{array}}{B}\ (\to \mathrm{E}) \qquad \Rightarrow \qquad \begin{array}{c}\vdots \\ A \\ \vdots \\ B\end{array}$$

$$\frac{\dfrac{\begin{array}{cc}\vdots & \vdots \\ A & B\end{array}}{A\ \&\ B}\ (\&\mathrm{I})}{A}\ (\&\mathrm{E}_l) \qquad \Rightarrow \qquad \begin{array}{c}\vdots \\ A\end{array}$$

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$$\dfrac{\dfrac{[B \,\&\, A]^z}{A}\;(\&\mathrm{E}_r) \quad \dfrac{[B \,\&\, A]^z}{B}\;(\&\mathrm{E}_l)}{\dfrac{\dfrac{A \,\&\, B}{(B \,\&\, A) \to (A \,\&\, B)}\;(\to \mathrm{I}^z) \quad \dfrac{[B]^y \quad [A]^x}{B \,\&\, A}\;(\&\mathrm{I})}{A \,\&\, B}\;(\to \mathrm{E})}$$

$$\Downarrow$$

$$\dfrac{\dfrac{\dfrac{[B]^y \quad [A]^x}{B \,\&\, A}\;(\&\mathrm{I})}{A}\;(\&\mathrm{E}_r) \quad \dfrac{\dfrac{[B]^y \quad [A]^x}{B \,\&\, A}\;(\&\mathrm{I})}{B}\;(\&\mathrm{E}_l)}{A \,\&\, B}\;(\&\mathrm{I})$$

$$\Downarrow$$

$$\dfrac{[A]^x \quad [B]^y}{A \,\&\, B}\;(\&\mathrm{I})$$