MECE-410: Flight Dynamics **Project #1**
Spring 2014-2015 (2145) **Due:** *Feb 19th, 2015*

Instructions: Use Matlab to solve all problems. Save and program your work in an m-file. Hand-in:

1. a project write-up with clearly defined answers to each problem.
2. a print out copy of your m-file program.
3. a print out copy of clearly labeled Figures and Plots

#1) Define the following matrices in the Matlab workspace:

$$A = \begin{bmatrix} 3 & 7 & -1 & 5 \\ 4 & 3 & 2 & 1 \\ 12 & -3 & -8 & 9 \\ 8 & 6 & 7 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -2 & 8 & 3 \\ 2 & 4 & 1 & 2 \\ 8 & -1 & 2 & 1 \\ 18 & 6 & 2 & -9 \end{bmatrix}$$

a) Find $A \times B$, $A + B$, $A - B$
b) Find the matrix transpose of A
c) Define $C = \left[A \,\vdots\, B \right]$, C should be a 4×8 matrix
d) Find A^{-1}
e) Find the determinant of A
f) Find Eigenvectors and Eigenvalues of A
g) Find the rank of A
h) Find matrix exponential of A
i) Find $\log_{10}\left[C(1,2) \right]$
j) Find X that satisfies the following equation: $AX = B$
k) Set up a nested "for-loop" to redefine negative elements in A. Replace negative elements in A with a zero.

#2) Define the following state-space model as a state-space system in the Matlab workspace:

$$\dot{\mathbf{z}} = \begin{bmatrix} 0 & 1 \\ -6 & -1 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \mathbf{u}$$
$$\mathbf{y} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 0.25 & 0 \\ 0.1 & 2 \end{bmatrix} \mathbf{u}$$

a) Determine the system's transfer function using the "ss2tf" Matlab command. What are the transfer functions? **Hint**: there are four transfer functions!!
- Use [num1,den]=ss2tf(A,B,C,D,1) to obtain the first set due to the first input
- Use [num2,den]=ss2tf(A,B,C,D,2) to obtain the second set due to the second input

b) Find the system's eigenvalues, natural frequencies, and damping ratio
c) Find the poles of the transfer functions. How do they compare to the eigenvalues?

#3) The longitudinal state-space model for an aircraft initial flying straight and level is defined by:

$$\begin{Bmatrix} \dot{V}_t \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{Bmatrix} = \begin{bmatrix} -0.0111 & -0.0788 & -0.0033 & -0.5615 \\ -0.0092 & -0.7531 & 0.9951 & 0 \\ 0.0062 & -1.5765 & -0.7453 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} V_t \\ \alpha \\ q \\ \theta \end{Bmatrix} + \begin{bmatrix} 0.0721 \\ -0.1178 \\ -9.0991 \\ 0 \end{bmatrix} \{\delta e\}$$

$$\begin{Bmatrix} V_t \\ \alpha \\ q \\ \theta \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} V_t \\ \alpha \\ q \\ \theta \end{Bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \{\delta e\}$$

where δe is the elevator deflection. The lateral/directional state-space model is

$$\begin{Bmatrix} \dot{\beta} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{Bmatrix} = \begin{bmatrix} -0.2316 & 0.0633 & -0.9956 & 0.0510 \\ -29.4924 & -3.0169 & 0.0201 & 0 \\ 6.2346 & -0.0274 & -0.4169 & 0 \\ 0 & 1 & 0.0631 & 0 \end{bmatrix} \begin{Bmatrix} \beta \\ p \\ r \\ \phi \end{Bmatrix} + \begin{bmatrix} 0.0052 & 0.0310 \\ -36.4909 & 8.1090 \\ -0.4916 & -2.8274 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \delta a \\ \delta r \end{Bmatrix}$$

$$\begin{Bmatrix} \beta \\ p \\ r \\ \phi \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \beta \\ p \\ r \\ \phi \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \delta a \\ \delta r \end{Bmatrix}$$

where δa is the aileron deflection and δr is the rudder deflection. In your m-file:

a) Define the A,B,C,D matrices for the two state-space models for the workspace.

b) Use the "damp" command to find the eigenvalues, natural frequencies and damping ratios for each both the longitudinal and lateral/directional system models?

c) Using the "lsim" command simulate the response of the longitudinal model for the time interval of $[0,10]$ sec with $\Delta t = 0.01$ sec for (-0.5 deg) step input to the elevator with the step starting at time 1 sec. Plot the True Velocity, V_t vs time; Angle-of-Attack, α vs time; Pitch Rate, q vs time; and Pitch Angle, θ vs time for the step input (assume the initial conditions for each state are zero).

d) Using the "lsim" command simulate the response of the lateral/directional model for the time interval of $[0,10]$ sec with $\Delta t = 0.01$ sec for (-0.5 deg) step input to the aileron with the step starting at time 1 sec. Plot the Sideslip Angle, β vs time; Roll Rate, p vs time; Yaw Rate, r vs time; and Bank Angle, ϕ vs time for the step input (assume the initial conditions for each state are zero).

e) Using the "lsim" command simulate the response of the lateral/directional model for the time interval of $[0,10]$ sec with $\Delta t = 0.01$ sec for (-0.5 deg) step input to the rudder with the step starting at time 1 sec. Plot the Sideslip Angle, β vs time; Roll Rate, p vs time; Yaw Rate, r vs time; and Bank Angle, ϕ vs time for the step input (assume the initial conditions for each state are zero).

f) Repeat c) – e) using Simulink. Overlay your time history plots for each part. Match?