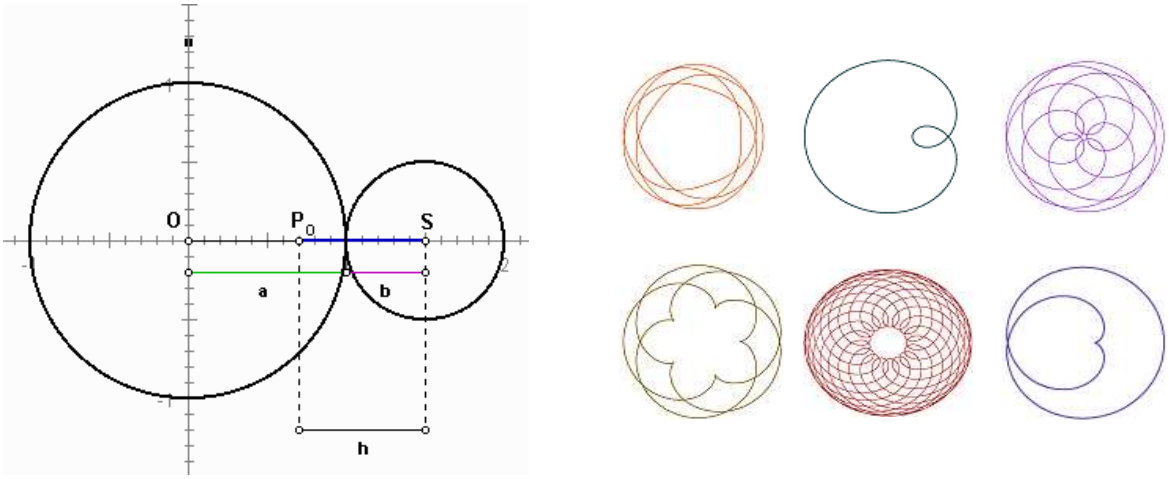
Worked Example: Graphing the Epitrochoid

The epitrochoid is a plane curve specified by the following Cartesian parametric equations:

$$x = m \cdot cos(t) - h \cdot cos(mt / b)$$
$$y = m \cdot sin(t) - h \cdot sin(mt / b)$$
$$where: m \equiv a + b$$

The independent parameter is *t*, and *a*, *b*, and *h* are other geometric input parameters, as derived by Franziska von Herrath at:
http://online.redwoods.cc.ca.us/instruct/darnold/CalcProj/Sp99/Fran/epitrochoid.htm.

Von Herrath explains: "The epitrochoid is a special case of a roulette. The curve is the locus of a point *P* that is rigidly attached to a small circle of radius *b* which rolls without slippage on the outside of a bigger circle with radius *a*. If the ratio of the curves' radii is rational, the curve is periodic. In the case that the ratio is irrational, the epitrochoid will not repeat itself. The ratio of the circles' radii also determines the number of cusps the epitrochoid has. Since the distance from the center of the smaller circle to the point *P* is variable, the epitrochoid is a parent to many other more specialized legendary plane curves like the epicycloid, whose point *P* is located on the circumference of the smaller circle with radius *b*, or, for example, the limacon whose two circles are of equal radius."



On the right in the image above, all six curves are examples of the epitrochoid, generated by the exact same parametric equations with different input parameters. To create the curves, an arbitrary set of values for *t* are generated, and the corresponding values of *x* and *y* are evaluated. For every cycle of *t* through 2π, the epitrichoid makes one cycle around the inner circle—in other words, *t* acts as the

angular location of the center of the outer circle with respect to the x-axis. The curves as shown above are created with an XY Graph in LabVIEW, which is capable of displaying arbitrary sets of paired data points (exactly like a "scatter plot" in Excel). In contrast, Waveform Charts and Waveform Graphs do not display data points that are arbitrarily placed along the horizontal axis, but instead require monotonic, uniformly-spaced progressions of data. The VI shown below emphasizes the features of all three graphical indicator types.

$$x = m \cdot \cos(t) - h \cdot \cos(mt / b)$$
$$y = m \cdot \sin(t) - h \cdot \sin(mt / b)$$
$$where: m \equiv a + b$$