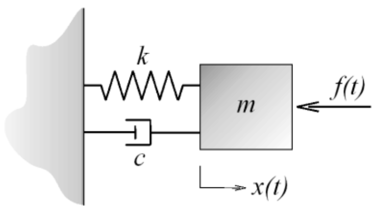
Homework #1: The Driven van der Pol Oscillator
Due: Wednesday, March 13 by 4:00 PM

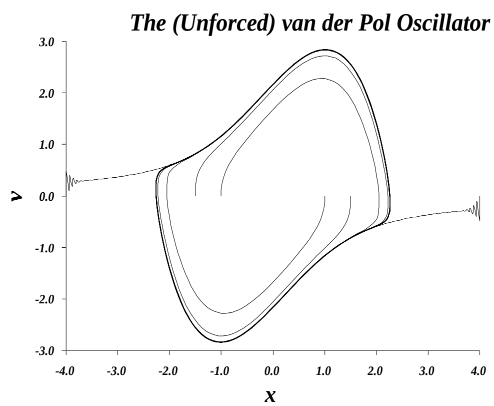
Consider the mechanical spring-mass-dashpot system shown, to which some external forcing function *f(t)* is applied. Suppose the dashpot is nonlinear in behavior, such that the governing ordinary differential equation generally becomes (after substitution of all constants):



$$\ddot{x} + (x^2 - 1)\dot{x} + x = f(t)$$

$$where: \ddot{x} \equiv \frac{d^2x}{dt^2}, \quad \dot{x} \equiv \frac{dx}{dt}$$

This equation is often referred to as the *driven van der Pol oscillator*, after the simple model studied by van der Pol in which no external forcing is present. **Without** forcing [$f(t)=0$], a numerical solution for several different initial conditions yields the "phase plots" of velocity vs. position shown on the right. **With** forcing, the solution is obviously expected to differ, depending upon the degree of forcing. Suppose now that we subject the system to the following forcing function:



$$f(t) = -20e^{-t/5}\sin(5t)$$

Note that we can generally re-write the single 2nd-order equation as two coupled 1st-order equations, where the intermediate variable *v* is actually the velocity of the mass itself:

$$\dot{x} = v$$
$$\dot{v} = -(x^2 - 1)v - x + f(t)$$

Assume that the system starts at $t = 0$ and $x = -3$ **from rest**, and that we are interested in the system's behavior only up to and including $t = 30$.

Using a simple finite difference technique (as in the Projectile Motion example) and 1000 divisions in time, solve the given system of 1st-order differential equations shown above using an <u>**Excel** spreadsheet approach</u>, and using a <u>**Matlab** scripting approach</u>.

Submit one page that explains your respective Excel and Matlab formulations of the solution, and a second **separate** page with the respective solution graphics of position and velocity vs. time on the same set of axes for each solution scheme. Appropriately label all axes and curves on the respective graphs.