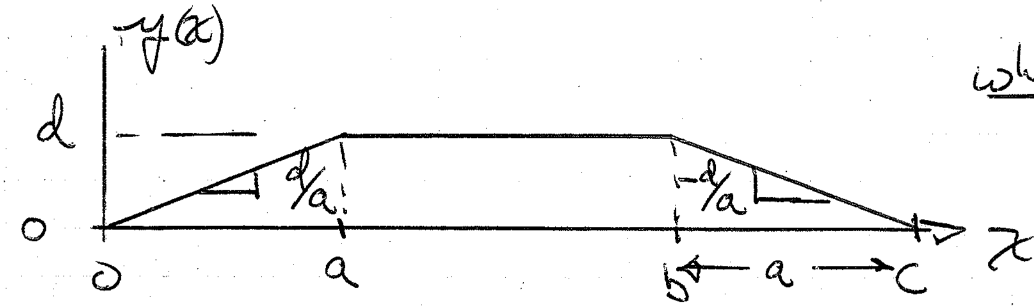
RIT Speed Bump Revisited

Recall we discussed the cross-sectional model

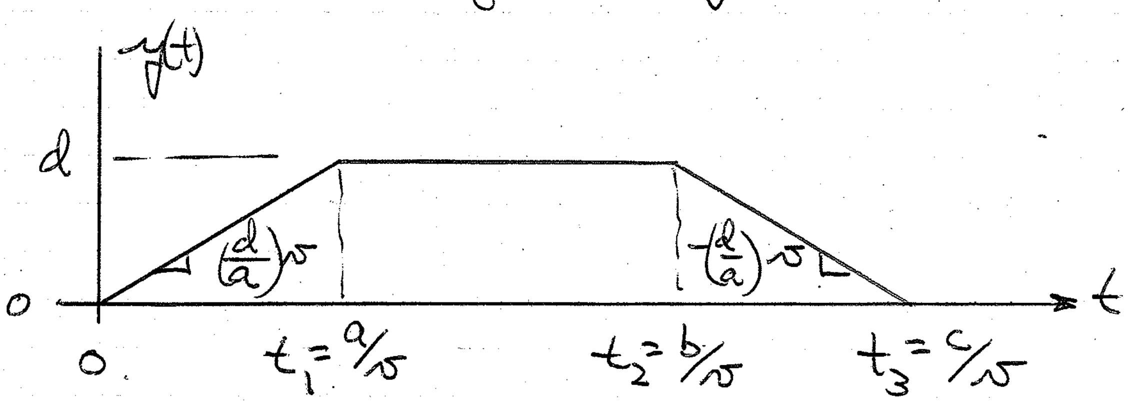


where: $x(t) = v\,t$

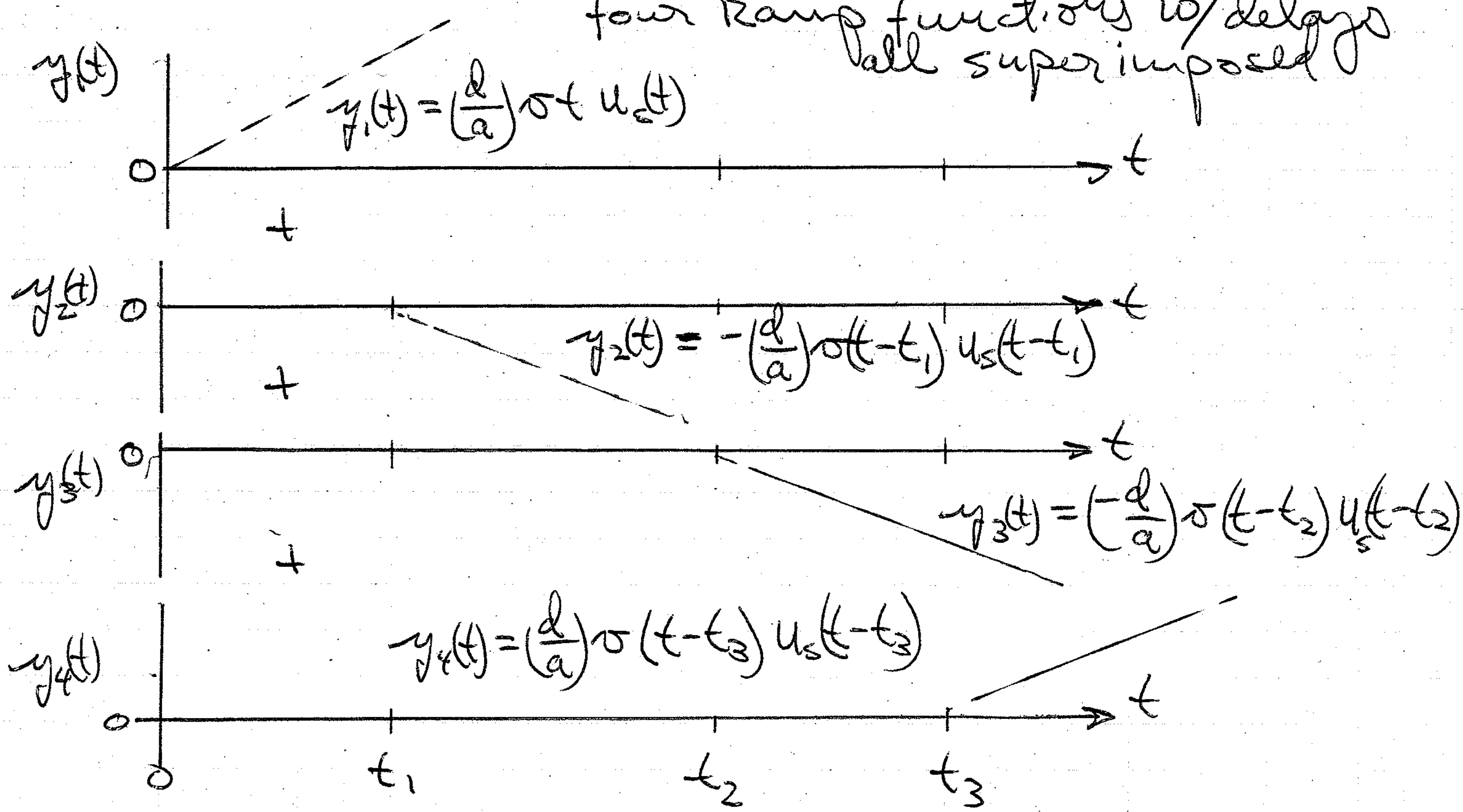
and

v = const approach velocity

mapping into $y(t)$



= four ramp functions w/ delays all superimposed



Hence

$$y(t) = y_1(t) + y_2(t) + y_3(t) + y_4(t)$$

$$= \left(\frac{d}{a}\right) v \left\{ \underbrace{t\,u_s(t)} - \underbrace{(t-t_1)\,u_s(t-t_1)} - \underbrace{(t-t_2)\,u_s(t-t_2)} + \underbrace{(t-t_3)\,u_s(t-t_3)} \right\}$$

$\mathcal{L}$

$$Y(s) = \left(\frac{d}{a}\right) v \left\{ \frac{1}{s^2} - \frac{1}{s^2} e^{-t_1 s} - \frac{1}{s^2} e^{-t_2 s} + \frac{1}{s^2} e^{-t_3 s} \right\}$$

$$Y(s) = \underbrace{\frac{d}{a} v \frac{1}{s^2}}_{\text{"cookie cutter" function}} \left\{ \underbrace{1}_{\text{initial 'cookie'}} - \underbrace{e^{-t_1 s}}_{\text{delayed "cookie"}} - \underbrace{e^{-t_2 s}}_{\text{delayed more "cookie"}} + \underbrace{e^{-t_3 s}}_{\text{most delayed "cookie"}} \right\}$$

(*) So now, any system into which $Y(s)$ is an input, will have 4 similar responses to the "cookie cutter" input and the corresponding replicates (initial, delayed, delayed more, most delayed)

say

$$\xrightarrow{Y(s)} \boxed{G(s)} \longrightarrow W(s)$$

Transformed input — $Y(s)$

system TF — $G(s)$

transformed output — $W(s)$

say $G(s) = \frac{1}{s+2}$ for example purposes

then
$$W(s) = G(s) \cdot Y(s)$$
$$= \frac{1}{(s+2)} \cdot \left(\frac{d}{a}\right) v \frac{1}{s^2} \left\{ 1 - e^{-t_1 s} - e^{-t_2 s} + e^{-t_3 s} \right\}$$
$$= W_1(s) \underbrace{\mp W_2(s) \mp W_3(s) + W_4(s)}_{\text{delayed versions of } W_1(s)}$$

and
$$w(t) = w_1(t) - \overbrace{w_2(t) - w_3(t) + w_4(t)}$$ after inverse Laplace

we only need to take
$$W_1(s) \xrightarrow{\mathcal{L}^{-1}} w_1(t)$$, then $w_2(t) = w_1(t - t_1)$

i.e., where ever t occurs in w_1, replace with $t - t_1$

"cookie cutter" Replicates

similarly $w_3(t) = w_1(t - t_2)$

and $w_4(t) = w_1(t - t_3)$

so the only 'work' involved is finding $w_1(t)$ from $W_1(s)$

so if $\omega_1(s) = \left(\frac{d}{a}\right) v \frac{1}{s^2(s+2)}$

then via PFE $\stackrel{?}{=} \frac{A}{s^2} + \frac{B}{s} + \frac{C}{(s+2)}$

so $\left(\frac{d}{a}\right) v \frac{1}{s^2(s+2)} \stackrel{?}{=} \frac{As + Bs^2}{s^2 \cdot s} + \frac{C}{(s+2)}$

$$= \frac{A(s+2)\cancel{s} + Bs\cancel{s}(s+2) + Cs^2\cancel{s}}{s^2 \cdot \cancel{s}(s+2)}$$

comparing numerators:

$$\therefore 0s^2 + 0s^1 + \left(\frac{d}{a}\right) v s^0 \stackrel{?}{=} (B+C)s^2 + (A+2B)s^1 + 2As^0$$

s^2: $0 = B + C$

s^1: $0 = A + 2B$

s^0: $\left(\frac{d}{a}\right) v = 2A$

$A = \left(\frac{d}{2a}\right) v = \left(\frac{d}{a}\right) v \left(\frac{1}{2}\right)$

$B = -\left(\frac{d}{4a}\right) v = -\left(\frac{d}{a}\right)(v)\frac{1}{4}$

$C = \left(\frac{d}{4a}\right) v = +\left(\frac{d}{a}\right) v \frac{1}{4}$

$$\therefore \omega_1(s) = \left(\frac{d}{a}\right) v \left(\frac{1/2}{s^2} - \frac{1/4}{s} + \frac{1/4}{s+2}\right)$$

$\mathcal{L}^{-1}$

$$\omega_1(t) = \left(\frac{d}{a}\right) v \left(\frac{1}{2}t - \frac{1}{4} + \frac{1}{4}e^{-2t}\right) u_s(t)$$

and the cookie cutter Replicates are:

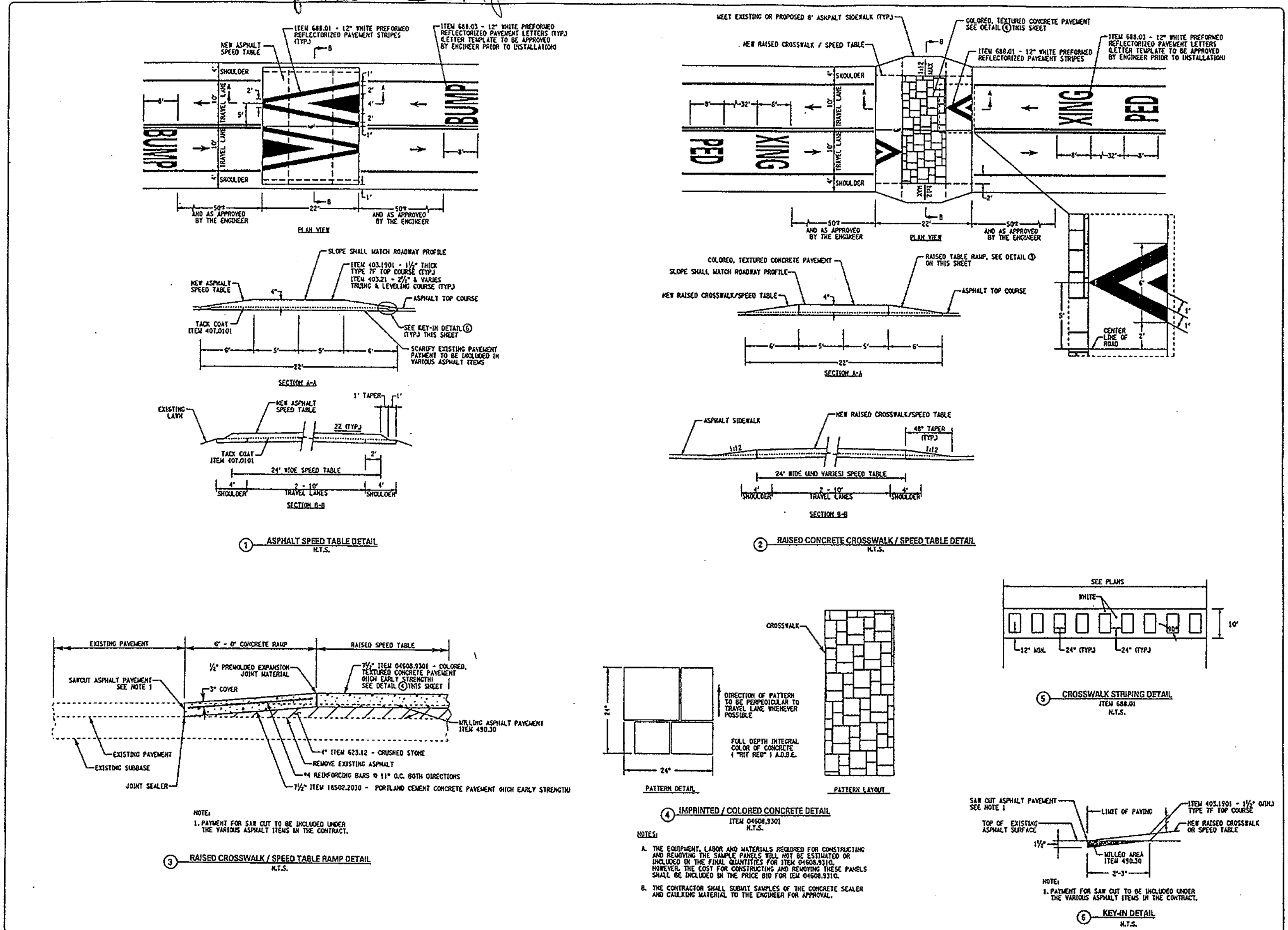
$$\omega_2(t) = \left(\frac{d}{a}\right) v \left(\frac{1}{2}(t-t_1) - \frac{1}{4} + \frac{1}{4}e^{-2(t-t_1)}\right) u_s(t-t_1)$$

$$\omega_3(t) = \left(\frac{d}{a}\right) v \left(\frac{1}{2}(t-t_2) - \frac{1}{4} + \frac{1}{4}e^{-2(t-t_2)}\right) u_s(t-t_2)$$

$$\omega_4(t) = \left(\frac{d}{a}\right) v \left(\frac{1}{2}(t-t_3) - \frac{1}{4} + \frac{1}{4}e^{-2(t-t_3)}\right) u_s(t-t_3)$$

and $\omega(t) = \omega_1(t) - \omega_2(t) - \omega_3(t) + \omega_4(t)$ ANS

The RIT Speed Bump



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Key Plan

Consultant Project Information
Drawn By SH Checked By JB
Proj Mgr WRF Proj # 1725704
CADD File

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AUGUST 2003

Approval Signatures & Dates
Consultant ____ Date ____
Consultant ____ Date ____
Owner ____ Date ____
Client ____ Date ____

Architect/Engineering Stamps

RIT
Rochester Institute of Technology
Design & Construction Svcs
Rochester, NY 14623
RIT Building Name

RIT Bldg No

RIT Project Manager
CHIC BRUNO

Project Area
PEDESTRIAN CROSSING IMPROVEMENTS

Project Name
RIT Proj # - CE 036734

Revisions

No	Date	By	Description

Sheet Title
MISCELLANEOUS DETAILS

Sheet Number
MD-1