Homework #3: A Second-Order System with Harmonic Input
Due: Thursday, March 28 by 4:00 PM

In **System Dynamics**, a linear second-order system is that which satisfies the following generic ODE:

$$\ddot{x} + 2\zeta\omega_n \cdot \dot{x} + \omega_n^2 \cdot x = \omega_n^2 \cdot f(t)\,; \quad x(0) \equiv x_{ic}\ , \quad \dot{x}(0) \equiv \dot{x}_{ic} \tag{1}$$

Where ζ is the *damping coefficient*, ω_n is the *natural frequency*, *f(t)* is an input *forcing function*, dots indicate time derivatives, and a subscript "*ic"* indicates an initial condition. Assuming that $f(t) = A+B\cdot sin(\omega t)$, where A and B are *input amplitudes* (constants), and ω is the *driving frequency,* the following solution results for $t \geq 0$:

$$x(t) = C e^{-\zeta\omega_n t} \sin(\omega_{nd} t + \lambda) + A + \Delta \cdot B \sin(\omega t - \phi) \tag{2}$$

Where the intermediates C, ω_{nd}, λ, Δ, and ϕ are defined via:

$$\Delta \equiv \left[(2\zeta\omega/\omega_n)^2 + (1-(\omega/\omega_n)^2)^2\right]^{-\frac{1}{2}}\,; \quad \omega_{nd} \equiv \omega_n\sqrt{1-\zeta^2} \tag{3}$$

$$\phi \equiv Arc\tan\left[\frac{2\zeta\omega/\omega_n}{1-(\omega/\omega_n)^2}\right] \tag{4}$$

$$\tan(\lambda) = \frac{-\omega_{nd}\,(A + \Delta \cdot B\sin(-\phi))}{-\zeta\omega_n A - \zeta\omega_n \Delta \cdot B\sin(-\phi) - \Delta \cdot B\omega\cos(-\phi)} \tag{5}$$

$$C\sin(\lambda) = -A - \Delta \cdot B\sin(-\phi) \tag{6}$$

You are to compare this exact solution with the numerical results determined from **both** Euler's method **and** a high-order Runge-Kutta method with $\zeta = 0.12$, $\omega_n = 25$ rad/sec, zero initial conditions (system completely at rest), $A = 20$, $B = 3.4$, $\omega = 38$ rad/sec, time step $h = 0.01$ sec, and a final time of 3 sec.

Your submission is to contain **no more than two pages** (no cover page) with your:

a) Euler solution done twice: via Matlab script done in Matlab, and via LabVIEW Mathscript;
b) Runge-Kutta solution done in Matlab using the *ode45* function;
c) The exact solution should be performed in each algorithm as a functional consistency check

Your submission should present comparative graphs of the exact, Euler, and Runge-Kutta solutions done using Matlab plotting:

a) export your LabVIEW generated results to a tab-delimited text file and import into Matlab
b) On your graph indicate the exact solution with a solid curve and no markers, and each numerical solution with respective markers and no lines
c) Properly label axes, title, and legend entries
d) Show appropriate code and block-diagram/front-panel graphics.

Include a brief (2-3 sentence) interpretation: **What do you observe from the solutions generated?**