

**Problem:** The “divide and average” method, an old-time method for approximating the square root of any positive number  $a$ , can be formulated as

$$x = \frac{x + \frac{a}{x}}{2} \quad (2)$$

Write a well-structured function to implement this algorithm based on the algorithm outlined in Figure 3.3.

**Solution:** I created a Matlab function for this problem.

Here is my function call:

**>> Problem\_2(1,10,.01,200)** “where  $x = 1$ ,  $a = 10$ , error percent desired = .01, and max iterations = 200”

Here is my answer:

**$X = 3.162278$  EA =0.005628 Iterations =5.000000**

Here is my code:

```
function Problem_2(x,a,ed,maxit)
iter = 1;
xold = -1;
ea = 100;
while xold ~= x
    xold = x;
    x = (x+a/x)/2;
    if x ~= 0
        ea = abs((x-xold)/x)*100;
        if ea <= ed || iter >= maxit
            fprintf('X = %f EA =%f Iterations =%f \n',x,ea,iter);
            break;
        end
    end
    iter = iter+1;
end
end
```

**Problem:** The Maclaurin series expansion for  $\cos x$  is:

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots \quad (1)$$

Starting with the simplest version,  $\cos x = 1$ , add terms one at a time to estimate  $\cos(\pi/3)$ . After each new term is added, compute the true and approximate percent relative errors. Use your pocket calculator to determine the true value. Add terms until the absolute value of the approximate error estimate falls below an error criterion conforming to two significant figures.

**Solution:** I used matlab coding in a script file to solve this problem.

Here are my answers:

#### **Approximate Error**

100	121.391441302213	9.98563898380532	0.366353197463768	0.00717361458189889
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#### **True relative Error**

1	9.66227112321509	0.359240300036179	0.00708693421748174	8.66865829785013e-05
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#### **F(x) approximation**

1	0.451688644383925	0.501796201500181	0.499964565328913	0.500000433432915
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Here is my code:

```
%% Problem 3(book 4.1)

clear all; close all; clc;
trueval = cos(pi/3);
ed = .01;
ea = 100;
et = 1;
n = 2;
xold = 1;
x = 1;
xa(n/2)=x;
eaa(n/2) = ea;
eta(n/2) = et;
while ea>ed
    x = x+(-1)^(n/2)*((pi/3)^n)/factorial(n);
    xa(n/2+1) = x;
    ea = abs((x-xold)/x)*100;
    eaa(n/2+1) = ea;
    et = abs((x-trueval)/(trueval))*100;
    eta(n/2+1) = et;
    n = n + 2;
    xold = x;
end
n = n/2;
fprintf('x=%u ea=%u et=%u n=%u\n',x, ea, et, n);
```

**Problem:** Use zero through fourth-order Taylor series expansions to predict  $f(2.5)$ . for  $f(x) = \ln(x)$  using a base point at  $x = 1$ . Compute the true percent relative error  $E_t$  for each approximation. Discuss the meaning of the results

**Solution:** I used Matlab script files to write the code and for the Taylor series order I did hand calculations.

Here is the solution I got:

True relative error zero through 4<sup>th</sup> order.

**Inf      161.086048791610      344.344195166441      161.086048791610      490.950712266306**

Estimates

The estimates fluctuate

**0.916290731874155      -1.500000000000000      -0.375000000000000      -1.500000000000000  
-0.234375000000000**

This is caused by the change in sign from one order to the next and the negation of terms when the signs are opposite, but the values are the same.

Here is my code:

**Problem:** The Stefan-Boltzmann law can be employed to estimate the rate of radiation of energy  $H$  from a surface, as in

$$H = Ae\sigma T^4 \quad (1)$$

Where  $H$  is in watts,  $A$  = the surface area ( $\text{m}^2$ ),  $e$  = the emissivity that characterizes the emitting properties of the surface (dimensionless),  $\sigma$  = a universal constant called the Stefan-Boltzmann constant ( $= 5.67 \times 10^{-8} \text{ W m}^{-2}\text{K}^{-4}$ ), and  $T$  = absolute temperature (K). Determine the error of  $H$  for a copper sphere with radius  $0.15 \pm 0.01 \text{ m}$ ,  $e = 0.90 \pm 0.05$ , and  $T = 550 \pm 20$ . Repeat the computation but with  $T = 650 \pm 40$ .

**Solution:** I used Matlab script files to write the code and output the answers.

Here are my answers:

$$H \text{ at } 550^\circ = 1320 \pm 441, \text{ and } H \text{ at } 650^\circ = 2576 \pm 633$$

Here is my code:

```
%% Problem 5(book4.9)
clear all; close all; clc
%e = emissivity, t = absolute temp, sigma = Boltzman constant,
e = 0.90;
eerror = .05;
t = 550;
terror = 20;
r = 0.15;
rerror = .01;
a = 4*pi*r^2;
aerror = abs(8*pi*r)*rerror;
sigma = 5.67e-8;
H = a*e*sigma*t^4;
et = abs(e*sigma*t^4)*aerror + abs(a*sigma*t^4)*eerror +
abs(a*e*sigma^4*t^3)*terror;
t2 = 650;
t2error = 40;
H2 = a*e*sigma*t2^4;
et2 = abs(e*sigma*t2^4)*aerror + abs(a*sigma*t2^4)*eerror +
abs(a*e*sigma^4*t2^3)*t2error;
fprintf('H at 550° = %0.0f+/-%0.0f, and H at 650°=%0.0f+/-
%0.0f\n',H,et,H2,et2);
```

**Problem:** In a fashion similar to that in Fig 3.11, write a short program to determine the smallest number,  $x_{\min}$ , used on the computer you will be employing with this book. Note that your computer will be unable to reliably distinguish between zero and a quantity that is smaller than this number.

**Solution:** I used Matlab for the solution.

The smallest number that the computer I used can interpret is: **9.881313e-324**

Here is my script:

```
%% Problem 1(book 3.4)
% Create a program that determines the smallest number, xmin, that the
computer can distinguish from zero
%Clear all previous programs, commands, and windows
clear all; close all; clc;

xtest = 1;
xmin = 1;

while xtest ~= 0
    xtest = xtest/10;
    if xtest == 0
        else
            xmin = xtest;
        end
    end
end

fprintf('The smallest number is: %e\n',xmin);
```