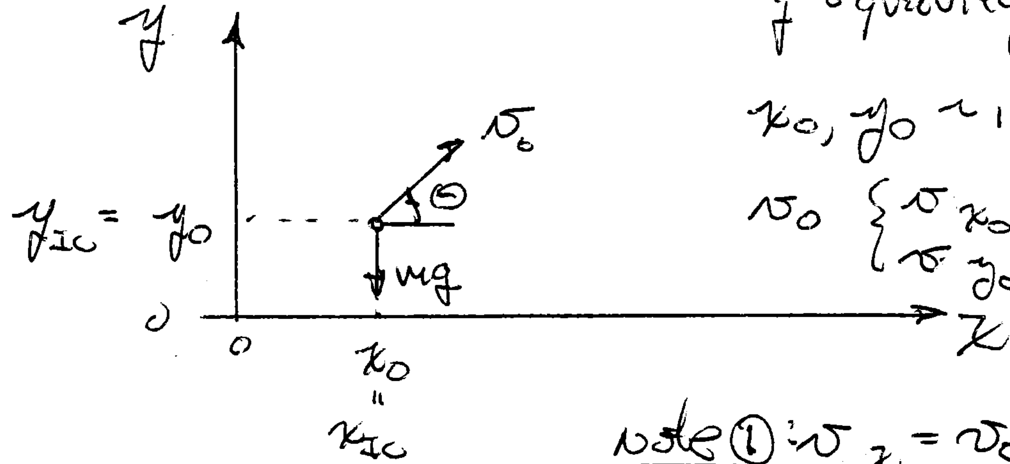
Basic Projectile Motion

$m \sim$ point mass (const)

$g \sim$ gravity (const)



$x_0, y_0 \sim$ initial displacement coords

$v_0 \begin{cases} v_{x_0} \\ v_{y_0} \end{cases}$ initial velocity coords

$\Theta \sim$ launch angle (Rad)

note ①: $v_{x_0} = v_0 \cos\Theta$

$v_{y_0} = v_0 \sin\Theta$

note ②: $\dfrac{v_{0y}}{v_{0x}} = \dfrac{\cancel{v_0} \sin\Theta}{\cancel{v_0} \cos\Theta} = \tan\Theta$

No Drag case: only mg is active force on m.

a) $\xrightarrow{+} \Sigma F_x = m a_x$

$0 = m \dfrac{dv_x}{dt} \implies \therefore \dfrac{dv_x}{dt} = 0 \qquad \therefore v_x = \text{const}$ for all time $= v_{x0}$

$\therefore \boxed{v_x(t) = v_{x0} = \text{const}}$

but $\dfrac{dx}{dt} = v_x = v_{x0}$

$\int dx = \int v_{x0}\, dt$

$x - x_0 = v_{x0} t \implies \therefore \boxed{x(t) = v_{x0} t + x_0}$

b) $+\uparrow \Sigma F_y = m a_y$

$$-mg = m\frac{dv_y}{dt} \Rightarrow \frac{dv_y}{dt} = -g$$

$$\therefore \int dv_y = \int -g\,dt$$

$$v_y - v_{y0} = -gt$$

$$\therefore \boxed{v_y(t) = -gt + v_{y0}}$$

but $\frac{dy}{dt} = v_y(t) = -gt + v_{y0}$

$$\int dy = \int (-gt + v_{y0})\,dt$$

$$y(t) - y_0 = -\frac{1}{2}gt^2 + v_{y0}t$$

$$\therefore \boxed{y(t) = -\frac{1}{2}gt^2 + v_{y0}t + y_0}$$

$\therefore$ No Aerodynamic Drag Summary Eqns:

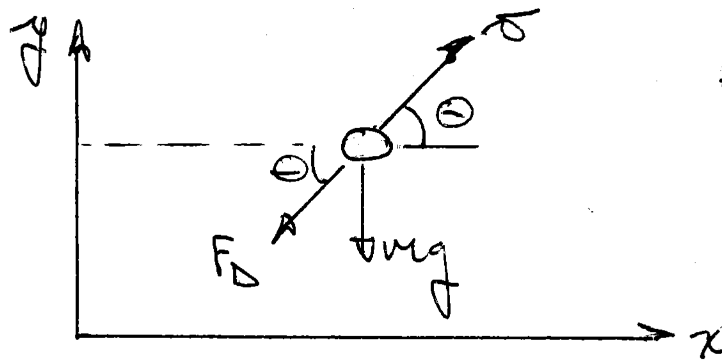
$v_x(t) = v_{x0}$ $\qquad$ $v_y(t) = -gt + v_{y0}$

$x(t) = v_{x0}t + x_0$ $\qquad$ $y(t) = -\frac{1}{2}gt^2 + v_{y0}t + y_0$

$v_{x0} = v_0 \cos\theta$ $\qquad$ $v_{y0} = v_0 \sin\theta$

· where: x_0, y_0 are initial spacial coords
v_0 is initial launch velocity vector mag.
θ is initial launch angle

with Aerodynamic Drag, FBD changes



Here: $F_D = \frac{1}{2}\rho A v^2$

acting opposite v

ρ ~ ~~mass~~ effective density of air

A ~ effective c.s. area perpendicular to vel

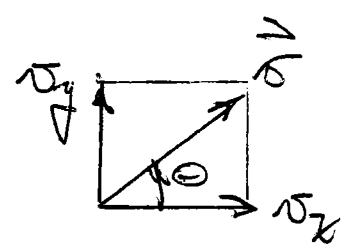
Let: $\frac{1}{2}\rho A \triangleq C$ constant @ given altitude & temperature (STP say)

$\overset{+}{\rightarrow}\sum F_x = m a_x$

$-F_D \cos\Theta = m \frac{d}{dt}(v_x)$

$\hookrightarrow C v^2 = C(v_x^2 + v_y^2)$

rewrite $\therefore \frac{dv_x}{dt} = -\frac{C}{m}\overset{v^2}{\cancel{(v_x^2+v_y^2)}}\cos\Theta$ eqn(1)



$+\uparrow \sum F_y = m a_y$

$-F_D \sin\Theta - mg = m \frac{d}{dt}(v_y)$

$\hookrightarrow C v^2$

$\sin\Theta = v_y/v$

$\cos\Theta = v_x/v$

rewrite $\therefore \frac{dv_y}{dt} = \left(-\frac{C}{m}v^2\right)\sin\Theta - g$ eqn(2) adapt eqn (1) $\Rightarrow$ $\dfrac{dv_x}{dt} = -\dfrac{c}{m} v^{\cancel{2}} \cdot \left(\dfrac{v_x}{\cancel{v}}\right)$

$= -\dfrac{c}{m} v$

$\hookrightarrow \sqrt{v_x^2 + v_y^2}$

adapt eqn (2) $\Rightarrow$ $\dfrac{dv_y}{dt} = -\dfrac{c}{m} v^{\cancel{2}} \left(\dfrac{v_y}{\cancel{v}}\right) - g$

$\therefore$ velocity gradient eqns become

eqn (1a): $\boxed{\dfrac{dv_x}{dt} = -\dfrac{c}{m} \sqrt{v_x^2 + v_y^2} \cdot v_x}$

eqn (2a): $\boxed{\dfrac{dv_y}{dt} = -\dfrac{c}{m} \sqrt{v_x^2 + v_y^2} \cdot v_y - g}$

<u>note</u>: interdependent eqns