Calculus BC - Worksheet on 5.2

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Work the following on **notebook paper**. Do not use your calculator.

1. $\int \frac{1}{2x+5}\,dx$

   Let $u = 2x + 5 \therefore du = 2dx \rightarrow dx = \frac{du}{2}$

   $\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|2x+5| + C$

2. $\int \left(x^3+1\right)^5 x^2\,dx$

   Let $u = x^3 + 1 \therefore du = 3x^2dx \rightarrow dx = \frac{du}{3x^2}$

   $\frac{1}{3}\int u^5\,du = \frac{u^6}{18} + C = \frac{1}{18}(x^3+1)^6 + C$

3. $\int \frac{x}{x^2+4}\,dx$

   Let $u = x^2 + 4 \therefore du = 2xdx \rightarrow dx = \frac{du}{2x}$

   $\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+4| + C$

4. $\int x\sin\left(3x^2\right)dx$

   Let $\theta = 3x^2 \therefore d\theta = 6xdx \rightarrow dx = \frac{d\theta}{6x}$

   $\frac{1}{6}\int \sin\theta\,d\theta = -\frac{1}{6}\cos\theta + C = -\frac{1}{6}\cos\left(3x^2\right) + C$

5. $\int \frac{x^2}{\sqrt{x^3+2}}\,dx$

   Let $u = x^3 + 2 \therefore du = 3x^2dx \rightarrow dx = \frac{du}{3x^2}$

   $\frac{1}{3}\int \frac{1}{\sqrt{u}}\,du = \frac{2}{3}\sqrt{u} + C = \frac{2}{3}\sqrt{x^3+2} + C$

6. $\int \tan^5 x\sec^2 x\,dx$

   Let $u = \tan x \therefore du = \sec^2 xdx \rightarrow dx = \frac{du}{\sec^2 x}$

   $\int u^5\,du = \frac{u^6}{6} + C = \frac{1}{6}\tan^6 x + C$

7. $\int \frac{\cos x}{\sqrt{\sin x}}\,dx$

   Let $u = \sin x \therefore du = \cos xdx \rightarrow dx = \frac{du}{\cos x}$

   $\int \frac{1}{\sqrt{u}}\,du = 2\sqrt{u} + C = 2\sqrt{\sin x} + C$

8. $\int \frac{\sin x}{1+\cos x}\,dx$

   Let $u = 1 + \cos x \therefore du = -\sin x dx \rightarrow dx = \frac{du}{-\sin x}$

   $-\int \frac{1}{u}\,du = -\ln|u| + C = -\ln|1 + \cos x| + C$

9. $\int_0^1 x\left(x^2+1\right)^3\,dx$

   $\int_0^1 x^7 dx + 3\int_0^1 x^5 dx + 3\int_0^1 x^3 dx + \int_0^1 x dx = [\frac{x^8}{8}]_0^1 + 3[\frac{x^6}{6}]_0^1 + 3[\frac{x^4}{4}]_0^1 + [\frac{x^2}{2}]_0^1 = \frac{1}{8} + \frac{1}{2} + \frac{3}{4} + \frac{1}{2} = \frac{15}{8} = 1.875$

10. $\int_0^4 \frac{2x}{\sqrt{x^2+9}}\,dx$

    Let $u = x^2 + 9 \therefore du = 2x dx \rightarrow dx = \frac{du}{2x}$

    $\int_9^{25} \frac{1}{\sqrt{u}}\,du = [2\sqrt{u}]_9^{25} = 4$

11. $\int_0^2 x\sqrt[3]{4+x^2}\,dx$

    Let $u = 4 + x^2 \therefore du = 2x dx \rightarrow dx = \frac{du}{2x}$

    $\frac{1}{2}\int_4^8 \sqrt[3]{u}\,du = [\frac{3}{8}u^{\frac{4}{3}}]_4^8 = \frac{3}{8}(16 - 4\sqrt[3]{4}) = \frac{1}{2}(12 - 3\sqrt[3]{4})$

12. $\int_1^2 \frac{x-2}{x}\,dx$

    $\int_1^2 dx - 2\int_1^2 \frac{1}{x}\,dx = [x - 2\ln|x|]_1^2 = (2 - 2\ln 2) - (1 - 2\ln 1) = 1 - 2\ln 2$

13. $\int_0^{\frac{\pi}{2}} \sin^3 x \cos x\,dx$

    Let $u = \sin x \therefore du = \cos x dx \rightarrow dx = \frac{du}{\cos x}$

    $\int_0^1 u^3\,du = [\frac{u^4}{4}]_0^1 = \frac{1}{4}$

14. $\int_1^{e^3} \frac{\ln x}{x}\,dx$

    $\int_1^{e^3} \frac{\ln x}{x}\,dx = \int_1^{e^3} \frac{1}{x}\ln x\,dx$

    Let $u = \ln x \therefore du = \frac{dx}{x} \rightarrow dx = x du$

    $\int_0^3 u\,du = [\frac{u^2}{2}]_0^3 = \frac{9}{2} = 4.5$

15. $\int_0^2 \frac{x^2-2}{x+1}\,dx$

    Let $u = x + 1 \rightarrow x = u - 1 \therefore du = dx$

    $\int_1^3 \frac{u^2-2u-1}{u}\,du = \int_1^3 u\,du - 2\int_1 3\,du - \int_1^3 \frac{1}{u}\,du = [\frac{u^2}{2} - 2u - \ln|u|]_1^3 = -\ln|3| \approx -1.099$

16. $\int_0^{\frac{\pi}{2}} \cos\left(\frac{2x}{3}\right) dx$

    Let $\theta = \frac{2x}{3} \therefore d\theta = \frac{2dx}{3} \rightarrow dx = \frac{3d\theta}{2}$

    $\frac{3}{2}\int_0^{\frac{\pi}{3}} \cos\theta \, d\theta = \frac{3}{2}[\sin\theta]_0^{\frac{\pi}{3}} = \frac{3\sqrt{3}}{4} \approx 1.299$

17. $\int_2^6 \frac{x-4}{x+1} \, dx$

    Let $u = x + 1 \rightarrow x = u - 1 \therefore du = dx$

    $\int_3^7 \frac{u-5}{u} \, du = \int_3^7 dx - 5\int_3^7 \frac{1}{u} \, du = [x - 5\ln|u|]_3^7 = 4 + 5\ln|3| - 5\ln|7| = 4 - 5\ln\left(\frac{3}{7}\right)$

18. $\int_{e^2}^{e^3} \frac{1}{x\ln x} \, dx$

    Let $u = \ln x \therefore du = \frac{dx}{x} \rightarrow dx = xdu$

    $\int_2^3 \frac{1}{u} \, du = [\ln|u|]_2^3 = \ln 3 - \ln 2 = \ln\frac{3}{2} \approx 0.405$