Calculus BC – Worksheet on Arc Length and Review

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2 March 2022

<u>Relevant Formulas and Notes:</u>

Area Bounded by Two Graphs:

$$A = \int_a^b f(x) - g(x)\, dx$$

$$A = \int_c^d f(y) - g(y)\, dy$$

Arc Length:

$$s = \int_a^b \sqrt{1 + (f\,'\,(x))^2}\, dx$$

$$s = \int_c^d \sqrt{1 + (f\,'\,(y))^2}\, dy$$

Work the following on **notebook paper**.
On problems 1 – 3, find the arc length of the graph of the function over the indicated interval. Do **not** use your calculator on problems 1 - 3.

1. $y = \frac{2}{3}x^{\frac{3}{2}} + 1$, $[0, 1]$

   $\frac{dy}{dx} = \frac{2*3}{3*2}x^{\frac{1}{2}} = \sqrt{x}$

   $s = \int_0^1 \sqrt{1 + (\sqrt{x})^2}\,dx = \int_0^1 \sqrt{1+x}\,dx$

   Let $u = 1 + x \therefore du = dx$

   $s = \int_0^1 \sqrt{1+x}\,dx = \int_1^2 \sqrt{u}\,du = \left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^2 = \frac{2}{3}\left(2\sqrt{2} - 1\sqrt{1}\right) = \frac{4\sqrt{2}-2}{3} \approx \frac{5.656-2}{3} = \frac{3.656}{3} \approx 1.219$

2. $y = \frac{3}{2}x^{\frac{2}{3}}$, $[1, 8]$

   $\frac{dy}{dx} = \frac{3*2}{2*3}x^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{x}}$

3. $y = \ln(\sin x)$, $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

   $\frac{dy}{dx} = \frac{1}{\sin x}\frac{d}{dx}(\sin x) = \frac{\cos x}{\sin x} = \cot x$

   $s = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sqrt{1 + \cot^2 x}\,dx = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \csc x\,dx$

   Let $u = \tan\left(\frac{x}{2}\right)$

   $\csc x = \frac{1}{\sin x} = \frac{\cos^2\left(\frac{x}{2}\right)+\sin^2\left(\frac{x}{2}\right)}{\sin x} = \frac{1}{2\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)} = \frac{1+u^2}{2u}$

   $du = \frac{1+u^2}{2}dx \rightarrow dx = \frac{2}{1+u^2}du$