Calculus BC – Worksheet 1 on Taylor Polynomials

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<u>Relevant Formulas and Notes:</u>

Taylor Series:

$$P_n(x) = \sum_{n=0}^{n} \frac{f^{(n)}(x)(x-c)^n}{n!}$$

(A Maclaurin Series is a Taylor Series wherein $c = 0$.)

Common Maclaurin Series Formats:

$$e^u = \sum_{n=0}^{\infty} \frac{(u)^n}{n!}$$

$$\cos u = \sum_{n=0}^{\infty} \frac{(-1)^n (u)^{2n}}{(2n)!}$$

$$\sin u = \sum_{n=0}^{\infty} \frac{(-1)^n (u)^{2n+1}}{(2n+1)!}$$

$$\frac{1}{1+u} = \sum_{n=0}^{\infty} (-1)^n (u)^n$$

$$\frac{1}{1-u} = \sum_{n=0}^{\infty} (u)^n$$

Work the following on **notebook paper**. Use your calculator only on problem 8(b), 9(b), and 10(a). Show all work.

1. Find a fourth-degree Maclaurin polynomial for $f(x) = e^{3x}$.

   $P_4(x) = \sum_{n=0}^{4} \frac{(3x)^n}{n!} = 1 + \frac{3x}{1!} + \frac{9x^2}{2!} + \frac{27x^3}{3!} + \frac{81x^4}{4!} = 1 + 3x + \frac{9}{2}x^2 + \frac{9}{2}x^3 + \frac{27}{8}x^4$

2. Find a sixth-degree Maclaurin polynomial for $f(x) = \cos x$.

   $P_6(x) = \sum_{n=0}^{6} \frac{(-1)^n (x)^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \frac{x^{12}}{12!} = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320} - \frac{x^{10}}{3628800} + \frac{x^{12}}{479001600}$

3. Find a fifth-degree Maclaurin polynomial for $f(x) = \frac{1}{x+1}$.

   $P_5(x) = \sum_{n=0}^{5} (-1)^n (x)^n = 1 - x + x^2 - x^3 + x^4 - x^5$

4. Find a third-degree polynomial for $f(x) = \sin x$, centered at $x = \frac{\pi}{6}$.

   $P_3(x) = \sum_{n=0}^{3} \frac{f^{(n)}(x)(x-\frac{\pi}{6})^n}{n!} = \frac{\frac{1}{2}(x-\frac{\pi}{6})^0}{0!} + \frac{\frac{\sqrt{3}}{2}(x-\frac{\pi}{6})^1}{1!} - \frac{\frac{1}{2}(x-\frac{\pi}{6})^2}{2!} - \frac{\frac{\sqrt{3}}{2}(x-\frac{\pi}{6})^3}{3!} =$

   $\frac{1}{2} + \frac{\sqrt{3}(x-\frac{\pi}{6})}{2} - \frac{(x-\frac{\pi}{6})^2}{4} - \frac{\sqrt{3}(\frac{\pi}{6})^3}{12}$

5. Find a fifth-degree Taylor polynomial for $f(x) = \frac{1}{1-x}$, centered at $x = 2$.

   $P_5(x) = \sum_{n=0}^{5} \frac{f^{(n)}(x)(x-2)^n}{n!} = \frac{1}{1-2} + \frac{x-2}{1!} + \frac{-2(x-2)^2}{2!} + \frac{6(x-2)^3}{3!} + \frac{-24(x-2)^4}{4!} + \frac{120(x-2)^5}{5!} =$

   $-1 + (x-2) - (x-2)^2 + (x-2)^3 - (x-2)^4 + (x-2)^5$

6. Find a third-degree Taylor polynomial for $f(x) = e^{(x-4)}$, centered at $x = 4$.

   $P_3(x) = \sum_{n=0}^{3} \frac{f^{(n)}(x)(x-2)^n}{n!} = \frac{e^0(x-4)^0}{0!} + \frac{e^0(x-4)}{1!} + \frac{e^0(x-4)^2}{2!} + \frac{e^0(x-4)^3}{3!} = 1 + (x-4) + \frac{1}{2}(x-4)^2 + \frac{1}{6}(x-4)^3$

7. Find a fifth-degree Taylor polynomial for $f(x) = \ln(x-1)$, centered at $x = 2$.

   $P_5(x) = \sum_{n=0}^{5} \frac{f^{(n)}(x)(x-2)^n}{n!} = \ln 1 + \frac{x-2}{1!} - \frac{(x-2)^2}{2!} + \frac{2(x-2)^3}{3!} - \frac{6(x-2)^4}{4!} + \frac{24(x-2)^5}{5!} =$

   $(x-2) - \frac{1}{2}(x-2)^2 + \frac{1}{3}(x-2)^3 - \frac{1}{4}(x-2)^4 + \frac{1}{5}(x-2)^5$