Calculus BC - 8.1 Basic Integration Formulas and Review

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Ex. Evaluate the given integrals.

(a) $\int \left(3x^2 + \frac{5}{x^4} + 2\sin x\right) dx$

$\int \left(3x^2 + \frac{5}{x^4} + 2\sin x\right) dx = 3\int \left(x^2\right) dx + \int \left(\frac{dx}{x^4}\right) + 2\int \sin x\, dx = x^3 - \frac{5}{3x^3} - 2\cos x + C$

(b) $\int_1^2 x^2(x^3+1)^2\, dx$

Let $u = x^3 + 1 \therefore du = 3x^2 dx$

$\int_1^2 x^2(x^3+1)^2\, dx = \frac{1}{3}\int_2^9 u^2\, du = [\frac{1}{9}u^3]_2^9 = \frac{721}{9}$

(c) $\int e^{3x}\, dx$

Let $u = 3x \therefore du = 3dx$

$\int e^{3x}\, dx = \frac{1}{3}\int e^u\, du = \frac{1}{3}e^u + C = \frac{1}{3}e^{3x} + C$

(d) $\int_0^{\frac{\pi}{6}} \sin(3x)\, dx$

Let $\theta = 3x \therefore d\theta = 3dx$

$\int_0^{\frac{\pi}{6}} \sin(3x)\, dx = \frac{1}{3}\int_0^{\frac{\pi}{2}} \sin\theta\, d\theta = \frac{1}{3}[-\cos x]_0^{\frac{\pi}{2}} = \frac{1}{3}$

(e) $\int_0^{\frac{\pi}{20}} \sec^2(5x)\, dx$

Let $\theta = 5x \therefore d\theta = 5dx$

$\int_0^{\frac{\pi}{20}} \sec^2(5x)\, dx = \frac{1}{5}\int_0^{\frac{\pi}{4}} sec^2\theta\, d\theta = \frac{1}{5}[\tan x]_0^{\frac{\pi}{4}} = \frac{1}{5}$

(f) $\int \frac{4}{x^2+9}\, dx$

$\int \frac{4}{x^2+9}\, dx = 4\int \frac{dx}{x^2+9} = \frac{4}{3}\arctan\left(\frac{x}{3}\right) + C$

(g) $\int \frac{4x}{x^2+9}\, dx$

Let $u = x^2 + 9 \therefore du = 2xdx$

$\int \frac{4x}{x^2+9}\, dx = 2\int \frac{du}{u} = 2\ln|u| + C = 2\ln|x^2+9| + C$

(h) $\int \frac{4x^2}{x^2+9}\, dx$

$\int \frac{4x^2}{x^2+9}\, dx = 4\int \frac{x^2}{x^2+9}\, dx = 4\int \frac{(x^2+9)-9}{x^2+9}\, dx = 4\int 1 - \frac{9}{x^2+9}\, dx =$

$4\int dx - 36\int \frac{du}{x^2+9} = 4x - 12\arctan\left(\frac{x}{3}\right) + C$

(i) $\int \frac{dx}{\sqrt{8-2x-x^2}}$

$\int \frac{dx}{\sqrt{8-2x-x^2}} = \int \frac{dx}{\sqrt{9-(x+1)^2}} = \arcsin\left(\frac{x+1}{3}\right) + C$

(j) $\int \frac{dx}{x^2-6x+34}$

$\int \frac{dx}{x^2-6x+34} = \int \frac{dx}{(x-3)^2+25} = \frac{1}{5}\arctan\left(\frac{x-3}{5}\right) + C$

(k) $\int \frac{2x+7}{x^2+4x+13}\,dx$

Let $u = x^2 + 4x + 13 \therefore du = 2x + 4dx$

$\int \frac{2x+7}{x^2+4x+13}\,dx = \int \frac{du}{u} + 3\int \frac{du}{x^2+4x+13} = \int \frac{du}{u} + 3\int \frac{du}{(x+2)^2+9} = \ln|u| + \arctan\left(\frac{x+2}{3}\right) + C =$

$\ln|x^2 + 4x + 13| + \arctan\left(\frac{x+2}{3}\right) + C$