Relevant Formulas and Notes:

8.1: Basic Integration Formulas & Review

$\int_a^b f(x)\,dx = F(b) - F(a)$

$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$

$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$

$\int ax^n\,dx = a\frac{x^{n+1}}{n+1} + C, n \neq 1$

$\int \frac{a}{x+n}\,dx = a\ln|x+n| + C$

$\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + C$

$\int ae^{nx}\,dx = nae^{nx} + C$

$\int a\cos(bx)\,dx = \frac{a}{b}\sin(bx) + C$

$\int a\sin(bx)\,dx = -\frac{a}{b}\cos(bx) + C$

$\int a\tan(bx)\,dx = -\frac{a}{b}\ln|\cos(bx)| + C$

$\int a\sec(bx) = \frac{a}{b}\ln|\sec(bx) + \tan(bx)| + C$

$\int a\sec^2(bx)\,dx = \frac{a}{b}\tan(bx) + C$

$\int a\sec(bx)\tan(bx)\,dx = \frac{a}{b}\sec x + C$

$\int a\csc(bx)\cot(bx)\,dx = -\frac{a}{b}\csc(bx) + C$

$\int a\csc^2(bx)\,dx = -\frac{a}{b}\cot(bx) + C$

$\int \frac{n}{\sqrt{a^2-u^2}}\,dx = n\arcsin\left(\frac{u}{a}\right) + C$

$\int \frac{n}{a^2+u^2}\,dx = \frac{n}{a}\arctan\left(\frac{u}{a}\right) + C$

$\int \frac{n}{u\sqrt{u^2-a^2}}\,dx = \frac{n}{a}\operatorname{arcsec}\left(\frac{u}{a}\right) + C$

8.2: Integration by Parts

$\int u\,dv = uv - \int v\,du$

LIATE for u value:

1. Logarithms
2. Inverse Trig Functions
3. Algebraic Functions
4. Trig Functions
5. Exponentials

8.3: Trigonometric Integrals

$\sin^2\theta + \cos^2\theta = x^2 + y^2 = 1$

$\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \to \tan^2\theta + 1 = \sec^2\theta$

$\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} \to 1 + \cot^2\theta = \csc^2\theta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \to$

$\cos(2\theta) = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

$\frac{1}{2}(\cos(\alpha-\beta) + \cos(\alpha+\beta)) = \cos\alpha\cos\beta$

$\frac{1}{2}(\cos(\alpha-\beta) - \cos(\alpha+\beta)) = \sin\alpha\sin\beta$

$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$, $\cos^2\theta = \frac{1+\cos(2\theta)}{2}$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \to$

$\frac{1}{2}(\sin(\alpha-\beta) + \sin(\alpha+\beta)) = \sin\alpha\cos\beta$

8.4: Trigonometric Substitution

For integrals using $\sqrt{a^2-u^2}$: $\sin\theta = \frac{u}{a}$, $\cos\theta = \frac{\sqrt{a^2-u^2}}{a}$

For integrals using $\sqrt{a^2+u^2}$: $\tan\theta = \frac{u}{a}$, $\sec\theta = \frac{\sqrt{a^2+u^2}}{a}$

For integrals using $\sqrt{u^2-a^2}$: $\sec\theta = \frac{u}{a}$, $\tan\theta = \frac{\sqrt{u^2-a^2}}{a}$

8.5: Integration by Partial Fractions

$\int \frac{u}{(x+a)(x+b)}\,dx = \int \frac{A}{x+a}\,dx + \int \frac{B}{x+b}\,dx$

$\int \frac{u}{(x+a)^2(x+b)}\,dx = \int \frac{A}{x+a}\,dx + \int \frac{B}{(x+a)^2}\,dx + \int \frac{C}{x+b}\,dx$

$\int \frac{u}{(x+a)(x^2+b)}\,dx = \int \frac{A}{x+a}\,dx + \int \frac{Bx+C}{x^2+b}\,dx$

Volume of solid rotated around x-axis: $V = \pi\int_a^b f^2(x)\,dx$

8.8: Improper Integrals

$\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$

$\int_{-\infty}^b f(x)\,dx = \lim_{a\to-\infty}\int_a^b f(x)\,dx$

$\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^c f(x)\,dx + \int_c^\infty f(x)\,dx$

Personal Notes Below: