Calculus BC – Worksheet on 7.1

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Relevant Formulas and Notes:

Area of a Region Between Two Curves:

$$A = \int_a^b \left(f(x) - g(x)\right) dx$$

$$A = \int_c^d \left(f(y) - g(y)\right) dy$$

where a, b, c, and d are the intersections of the functions to one another, $f(x)$ is the upper bound of the graph, $g(x)$ is the lower bound of the graph, $f(y)$ is the rightmost bound, and $g(y)$ is the leftmost bound.

Work the following on **notebook paper**.

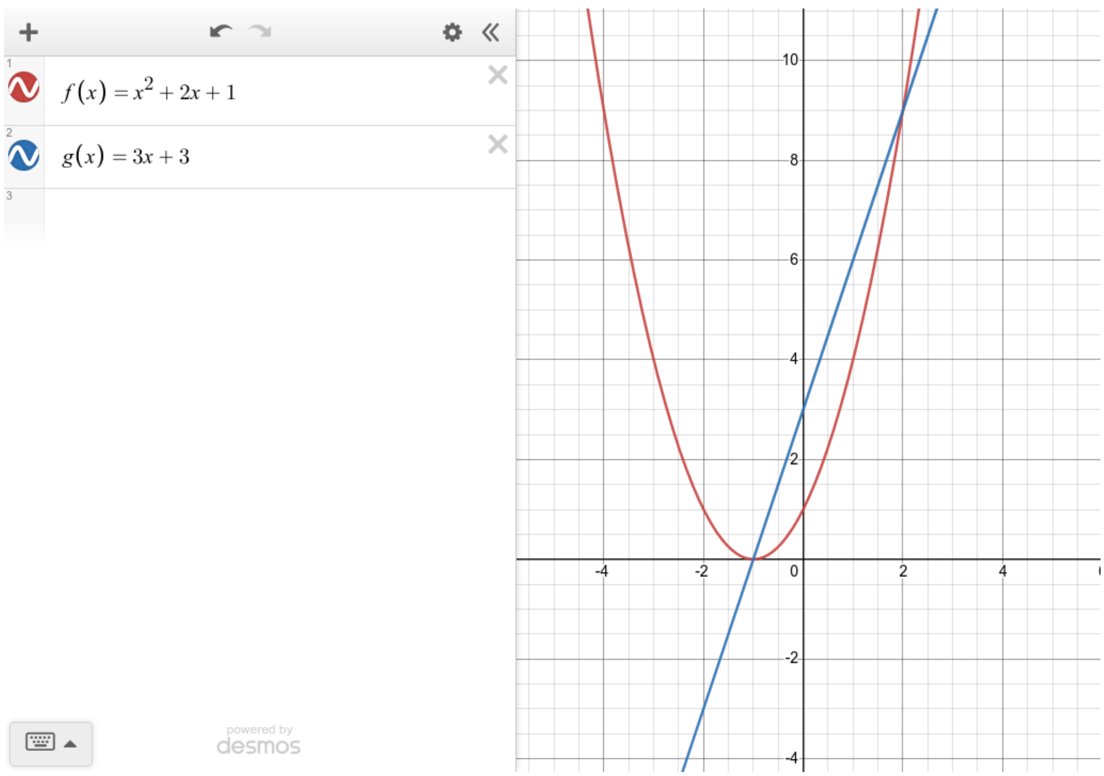
Find the area bounded by the given curves. Draw and label a figure for each problem, and show all work. **Do not use your calculator on problems 1-2.**

1. $f(x) = x^2 + 2x + 1$, $g(x) = 3x + 3$

   $x^2 + 2x + 1 = 3x + 3 \to x^2 - x - 2 = 0 \to x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)} = \frac{1 \pm 3}{2} = -1, 2$

   $\int_{-1}^{2} \left((3x+3) - \left(x^2 + 2x + 1\right)\right) dx = \int_{-1}^{2} \left(-x^2 + x + 2\right) dx = [-\frac{1}{3}x^3]_{-1}^{2} + [\frac{1}{2}x^2]_{-1}^{2} + [2x]_{-1}^{2} =$

   $-3 + \frac{3}{2} + 6 = \frac{12-6+3}{2} = \frac{9}{2}$

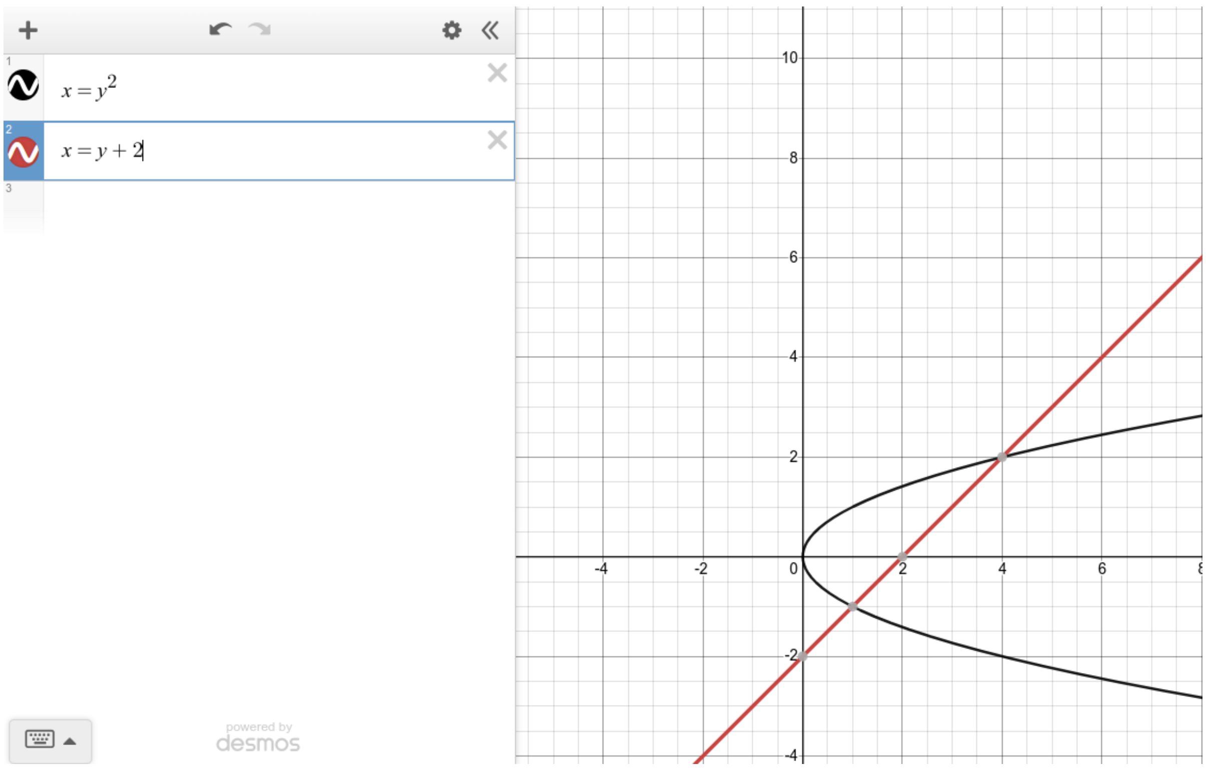


2. $x = y^2$, $x = y + 2$

$y^2 = y + 2 \to y^2 - y - 2 = 0 \to y = \frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-2)}}{2(1)} = \frac{1\pm 3}{2} = -1, 2$

$\int_{-1}^{2} \left((y+2) - (y^2)\right) dy = \int_{-1}^{2} \left(-y^2 + y + 2\right) dy = [-\frac{1}{3}y^3]_{-1}^{2} + [\frac{1}{2}y^2]_{-1}^{2} + [2y]_{-1}^{2} =$

$-3 + \frac{3}{2} + 6 = \frac{12-6+3}{2} = \frac{9}{2}$

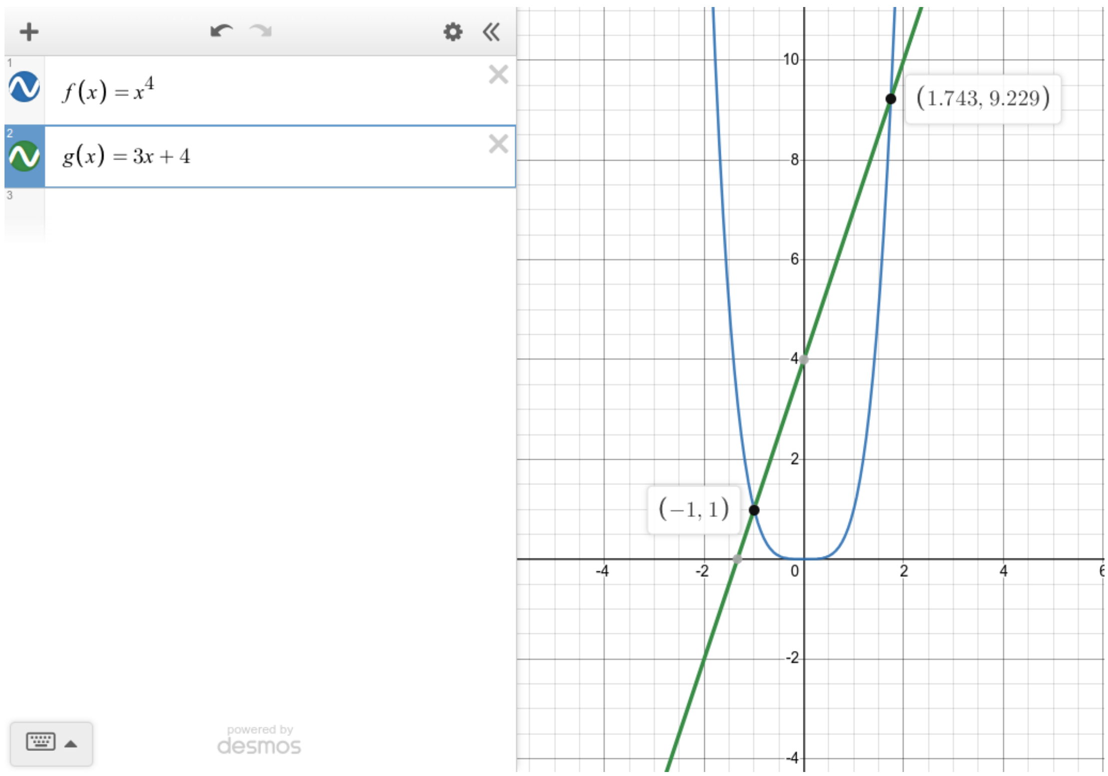


Find the area bounded by the given curves. Draw and label a figure for each problem, set up the integral(s) needed, and then evaluate on your calculator.

3. $f(x) = x^4$, $g(x) = 3x + 4$

$x^4 = 3x + 4 \rightarrow x^4 - 3x - 4 = 0 \rightarrow x = -1, 1.743$

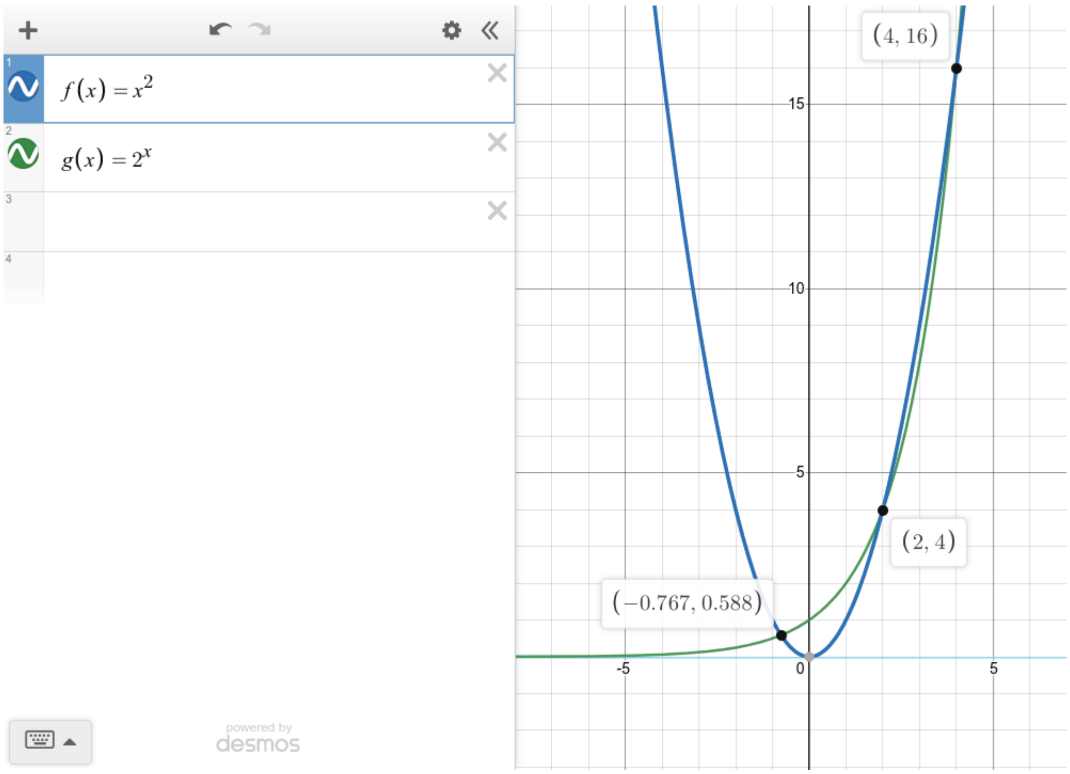
$\int_{-1}^{1.743} \left((3x+4) - \left(x^4\right)\right) dx = [\frac{3}{2}x^2]_{-1}^{1.743} + [4x]_{-1}^{1.743} - [\frac{1}{5}x^5]_{-1}^{1.743} \approx 3.057 + 10.972 - 3.417 = 10.612$



4. $f(x) = x^2$, $g(x) = 2^x$

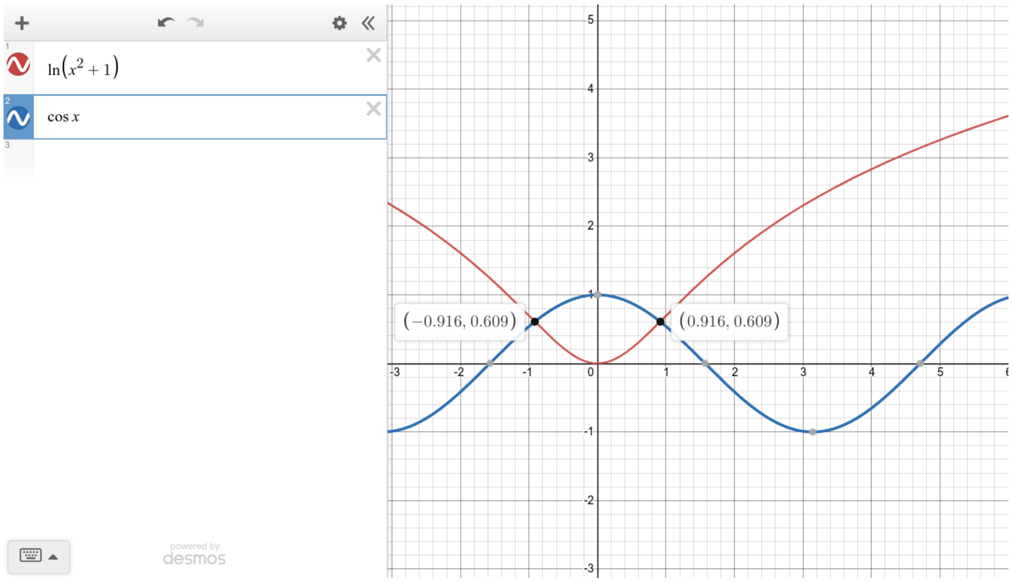
$x^2 = 2^x \rightarrow x = -0.767, 2, 4$

$\int_{-0.767}^{2} \left((2^x) - (x^2)\right) dx + \int_{2}^{4} \left((x^2) - (2^x)\right) dx = [\frac{2^x}{\ln 2}]_{-0.767}^{2} - [\frac{1}{3}x^3]_{-0.767}^{2} + [\frac{1}{3}x^3]_{2}^{4} - [\frac{2^x}{\ln 2}]_{2}^{4} \approx 3.46$



5. $y = \ln\left(x^2+1\right)$, $y = \cos x$
   $\ln\left(x^2+1\right) = \cos x \rightarrow x = \pm 0.916$

   $\int_{-0.916}^{0.916} \left((\cos x) - \left(\ln\left(x^2+1\right)\right)\right) dx \approx 1.168$



6. (No calculator)
   Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \sqrt{x}$ and $y = 6 - x$ as shown in the figure on the right.

   (a) Find the area of R by working in x's.

   $\sqrt{x} = 6 - x \rightarrow x = x^2 - 12x + 36 \rightarrow x^2 - 13x + 36 = 0 \rightarrow x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(1)(36)}}{2(1} = \frac{13 \pm 5}{2} = 4, 9$

   Because $x = 9$ is out of bounds, we will only use the intersection $x = 4$.

   $\int_0^4 \sqrt{x}\, dx + \int_4^6 6 - x\, dx = [\frac{2}{3}x^{\frac{3}{2}}]_0^4 + [6x - \frac{1}{2}x^2]_4^6 = \frac{16}{3} + (36 - 24) - (18 - 8) = \frac{16}{3} + 12 - 10 = \frac{22}{3}$

   (b) Find the area of R by working in y's.

   $y = \sqrt{x} \rightarrow x = y^2$

   $y = 6 - x \rightarrow -x = y - 6 \rightarrow x = 6 - y$

   $y^2 = 6 - y \rightarrow y^2 + y - 6 = 0 \rightarrow y = -3, 2$

   Because $y = -3$ is out of bounds, we will use the lower bound $y = 0$.

   $\int_0^2 \left((6 - y) - \left(y^2\right)\right) dy = [6y - \frac{1}{2}y^2]_0^2 - [\frac{1}{3}y^3]_0^2 = 10 - \frac{8}{3} = \frac{22}{3}$

   (c) When you found your answers to (a) and (b), was it less work to work in terms of x or in terms of y?

   It was less work overall to work in terms of x, since the formulas were given in terms of x. However, it was easier to integrate in terms of y because the solution was in simpler exponents.

7. (Calculator)
   Let R be the region bounded by the graphs of $y = 2^x$, $y = 4^x$, and $y = \frac{1}{x}$, as shown in the figure on the right. Find the area of R.

   $2^x = 4^x \rightarrow x = 0$

   $2^x = \frac{1}{x} \rightarrow x = 0.641$

   $4^x = \frac{1}{x} \rightarrow x = 0.5$

   $\int_0^{0.5} (4^x - 2^x)\, dx + \int_{0.5}^{0.641} \frac{1}{x} - (2^x) = 0.124 + 0.039 = 0.163$