Calculus BC – Worksheet on Alternating Series and Remainders

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Relevant Formulas and Notes:

Work the following on **notebook paper**.

1. Approximate the sum, S, of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ by using its first five terms, and explain why your estimate differs from the actual value by less than .009. Then use your results to find an interval in which S must lie.

   $S = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \approx \sum_{n=0}^{4} \frac{(-1)^n}{n!} = 1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} = \frac{3}{8}$

   $|\text{error}| = |S - S_4| < |a_5|$

   $|\text{error}| < |\frac{(-1)^5}{5!}| = |-\frac{1}{120}| \approx 0.00833 < 0.009$

   $\therefore |\text{error}| < 0.009$ by the Alternatng Series Remainder.

   $S_4 - \frac{1}{120} = 0.366 < S < S_4 + \frac{1}{120} = 0.383 \therefore S$ lies in the interval $(0.366, 0.383)$ or $(\frac{11}{30}, \frac{23}{60})$

2.

3. Approximate the sum of the convegent series $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!}$ so that the error will be less than $\frac{1}{1000}$? How many terms were needed? What are the properties of the terms of this series that guarantee that your approximation is within $\frac{1}{1000}$ of the exact value? Justify your answer.

   $S = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!} \approx \sum_{n=0}^{4} \frac{(-1)^n}{2^n n!} = 1 - \frac{1}{2} + \frac{1}{8} - \frac{1}{48} + \frac{1}{384} \approx \frac{233}{384}$

   $|\text{error}| = |S - S_4| < |\frac{(-1)^5}{32*120}| = |-\frac{1}{3840}| < \frac{1}{1000}$

   $\therefore |\text{error}| < 1000$ by the Alternating Series Remainder.

4.

5. Approximate the sum of the convegent series $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n^3}$ so that the error will be less than $\frac{1}{1200}$? How many terms were needed? What are the properties of the terms of this series that guarantee that your approximation is within $\frac{1}{1200}$ of the exact value? Justify your answer.