Calculus BC - Worksheet on Inverse Trig Functions and Review

Craig Cabrera

26 January 2022

Formulas:

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$$
$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$$

Work the following on **notebook paper**. Evaluate. SHOW ALL WORK.

1. $\int \frac{dx}{\sqrt{25-x^2}}$

   $\int \frac{dx}{\sqrt{25-x^2}} = \arcsin\left(\frac{x}{5}\right) + C$

2. $\int \frac{dx}{x^2-6x+34}$

   $\int \frac{dx}{x^2-6x+34} = \int \frac{dx}{(x-3)^2+25} = \frac{1}{5}\arctan\left(\frac{x-3}{5}\right) + C$

3. $\int \frac{dx}{4+x^2}$

   $\int \frac{dx}{4+x^2} = \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

4. $\int \frac{dx}{\sqrt{8-2x-x^2}}$

   $\int \frac{dx}{\sqrt{8-2x-x^2}} = \int \frac{dx}{\sqrt{9-(x^2+2x+1)}} = \int \frac{dx}{\sqrt{9-(x+1)^2}} = \arcsin\left(\frac{x+1}{3}\right) + C$

5. $\int \frac{dx}{x\sqrt{x^2-9}}$

   $\int \frac{dx}{x\sqrt{x^2-9}} = \frac{1}{3}\operatorname{arcsec}\left(\frac{|x|}{3}\right) + C$

6. $\int \frac{2x+7}{x^2+4x+13}\,dx$

   Let $u = x^2+4x+13 \therefore du = 2x+4dx$

   $\int \frac{du}{u} + 3\int \frac{dx}{x^2+4x+13} = \ln|u| + 3\int \frac{dx}{9+(x+2)^2} = \ln|x^2+4x+13| + \arctan\left(\frac{x+2}{3}\right) + C$

7. $\int \frac{x+3}{\sqrt{16-x^2}}\,dx$

   $\int \frac{x+3}{\sqrt{16-x^2}}\,dx = \int \frac{xdx}{\sqrt{16-x^2}} + 3\int \frac{dx}{\sqrt{16-x^2}}$

   Let $u = 16-x^2 \therefore du = -2xdx$

   $-\frac{1}{2}\int \frac{du}{\sqrt{u}} + 3\int \frac{dx}{\sqrt{16-x^2}} = 3\arcsin\left(\frac{x}{4}\right) - \sqrt{16-x^2} + C$

8. $\int \frac{3-2x}{\sqrt{10x-x^2-9}}$

$3\int \frac{dx}{\sqrt{10x-x^2-9}} - 2\int \frac{xdx}{\sqrt{10x-x^2-9}} = 3\int \frac{dx}{16-(x-5)^2} - 2\int \frac{xdx}{\sqrt{16-(x-5)^2}}$

Let $u = x - 5 \to x = u + 5 \therefore du = dx$

$3\int \frac{dx}{16-(x-5)^2} - 2\int \frac{(u+5)dx}{\sqrt{16-u^2}} = 3\arcsin\left(\frac{x-5}{4}\right) - 2\left(\int \frac{(u)}{\sqrt{16-u^2}}\,du + 5\int \frac{du}{\sqrt{16-u^2}}\right)$

Let $v = 16 - u^2 \therefore dv = -2udu$

$3\arcsin\left(\frac{x-5}{4}\right) - 2\left(-\frac{1}{2}\int \frac{dv}{\sqrt{v}} + 5\int \frac{du}{\sqrt{16-u^2}}\right) = 3\arcsin\left(\frac{x-5}{4}\right) - 2\left(5\arcsin\left(\frac{u}{4}\right) - \sqrt{v}\right) + C =$

$2\sqrt{10x-x^2-9} - 7\arcsin\left(\frac{x-5}{4}\right) + C$