Calculus BC - Worksheet on 8.1

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Work the following on **notebook paper**. **No calculator.** Evaluate the given integrals.

1. $\int \left(9x + \frac{2}{x^3} + 3\sec x \tan x - 7\sec^2 x\right) dx$

   $\int \left(9x + \frac{2}{x^3} + 3\sec x \tan x - 7\sec^2 x\right) dx = 9\int x\, dx + 2\int \frac{dx}{x^3} + 3\int \sec x \tan x\, dx - 7\int \sec^2 x\, dx =$

   $\frac{9}{2}x^2 - \frac{1}{x^2} + 3\sec x - 7\tan x + C$

2. $\int \frac{x-4}{\sqrt{x^2-8x+1}}\, dx$

   Let $u = x^2 - 8x + 1 \therefore du = (2x-8)dx$

   $\int \frac{x-4}{\sqrt{x^2-8x+1}}\, dx = \int \frac{x-4}{\sqrt{u}} \frac{du}{2x-8} = \int \frac{du}{\sqrt{u}} \frac{x-4}{2x-8} = \int \frac{du}{2\sqrt{u}} = \frac{4}{3}u^{\frac{3}{2}} + C = \frac{4}{3}\left(x^2 - 8x + 1\right)^{\frac{3}{2}} + C$

3. $\int x^3 \cos\left(5x^4\right) dx$

   Let $\theta = 5x^4 \therefore du = 20x^3 dx$

   $\int x^3 \cos\left(5x^4\right) dx = \frac{1}{20}\int \cos\theta\, d\theta = \frac{1}{20}\sin\theta + C = \frac{1}{20}\sin\left(5x^4\right) + C$

4. $\int \sin^5(3x)\cos(3x)\, dx$

   Let $u = \sin 3x \therefore du = 3\cos x dx$

   $\int \sin^5(3x)\cos(3x)\, dx = \frac{1}{3}\int u^5\, du = \frac{1}{18}u^6 + C = \frac{1}{18}\sin^6 x + C$

5. $\int_1^2 x\left(x^2+1\right)^3 dx$

   Let $u = x^2 + 1 \therefore du = 2xdx$

   $\int_1^2 x\left(x^2+1\right)^3 dx = \frac{1}{2}\int_2^5 u^3\, du = \frac{1}{8}[u^4]_2^5 = \frac{609}{8} = 76.125$

6. $\int_{\frac{\pi}{12}}^{\frac{\pi}{9}} \sin(3x)\, dx$

   Let $\theta = 3x \therefore d\theta = 3dx$

   $\int_{\frac{\pi}{12}}^{\frac{\pi}{9}} \sin(3x)\, dx = \frac{1}{3}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin\theta\, d\theta = -\frac{1}{3}[\cos\theta]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = -\frac{1}{3}[\frac{1}{2} - \frac{\sqrt{2}}{2}] = \frac{\sqrt{2}-1}{6} \approx 0.069$

7. $\int_e^{e^2} \frac{(\ln x)^4}{x}\, dx$

   Let $u = \ln x \therefore du = \frac{dx}{x}$

   $\int_e^{e^2} \frac{(\ln x)^4}{x}\, dx = \int_1^2 u^4\, du = \frac{1}{5}[u^5]_1^2 = \frac{32-1}{5} = 6.2$

8. $\int_0^3 \frac{x^2-5}{x+2}\,dx$

   Skip; need reference for solving.

9. $\int_{-1}^0 e^x \cos e^x\,dx$

   Let $\theta = e^x \therefore d\theta = e^x dx$

   $\int_{-1}^0 e^x \cos e^x\,dx = \int_{\frac{1}{e}}^1 \cos\theta\,d\theta = [\sin\theta]_{\frac{1}{e}}^1 = \sin\frac{1}{e} - \sin 1 \approx 0.482$

10. $\int x^3 7^{x^4}\,dx$

    Let $u = 7^{x^4} \therefore du = 4\ln 7 * 7^{x^4} * x^3 dx$

    $\int x^3 7^{x^4}\,dx = \frac{1}{4\ln 7}\int\,du = \frac{u}{4\ln 7} + C = \frac{7^{x^4}}{4\ln 7} + C$

11. $\int \frac{6}{\sqrt{10x-x^2}}\,dx$

    $\int \frac{6}{\sqrt{10x-x^2}}\,dx = 6\int \frac{dx}{\sqrt{25-(x-5)^2}} = 6\arcsin\left(\frac{x-5}{5}\right) + C$

12. $\int \frac{1}{x^2-4x+9}\,dx$

    $\int \frac{1}{x^2-4x+9}\,dx = \int \frac{1}{(x-2)^2+5}\,dx = \frac{1}{\sqrt{5}}\arctan\left(\frac{x-2}{\sqrt{5}}\right) + C$

13. $\int \frac{2x+7}{x^2+4x+13}\,dx$

    Let $u = x^2+4x+13 \therefore du = 2x+4dx$

    $\int \frac{2x+7}{x^2+4x+13}\,dx = \int \frac{du}{u} + 3\int \frac{du}{x^2+4x+13} = \int \frac{du}{u} + 3\int \frac{du}{(x+2)^2+9} = \ln|u| + \arctan\left(\frac{x+2}{3}\right) + C =$

    $\ln|x^2+4x+13| + \arctan\left(\frac{x+2}{3}\right) + C$

14. $\int \frac{x+3}{\sqrt{16-x^2}}\,dx$

    $\int \frac{x+3}{\sqrt{16-x^2}}\,dx = \int \frac{x}{\sqrt{16-x^2}}\,dx + 3\int \frac{dx}{\sqrt{16-x^2}}$

    Let $u = 16-x^2 \therefore du = -2xdx$

    $\int \frac{x}{\sqrt{16-x^2}}\,dx + 3\int \frac{dx}{\sqrt{16-x^2}} = 3\int \frac{dx}{\sqrt{16-x^2}} - \int \frac{dx}{2\sqrt{u}} = 3\arcsin\frac{x}{4} - \sqrt{u} + C = 3\arcsin\frac{x}{4} - \sqrt{16-x^2} + C$

Multiple Choice. All work must be shown.

15. Which of the following represents the area of the shaded region in the figure on the page?

(a) $\int_c^d f(y)\,dy$
(b) $\int_a^b d - f(x)\,dx$
(c) $f'(b) - f'(a)$
(d) $(b-a)[f(b) - f(a)]$
(e) $(d-c)[f(b) - f(a)]$

For every infintesimal length in which we calculate within an integral, the area is represented by the length dx, an infinite amount of which fill the interval $[b, a]$, and height of the integrand (in most cases, this would be $f(x)$). Because we are measuring the area from height d to the curve of height $c = y$, we can then define the integrand of the function as $d - c = d - f(x)$. Therefore, the representative formula for the area of the shaded region is $\int_a^b d - f(x)\,dx$.

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16. If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y, $\frac{dy}{dx} =$

(a) $-\frac{x^2+y}{x+2y^2}$
(b) $-\frac{x^2+y}{x+y^2}$
(c) $-\frac{x^2+y}{x+2y}$
(d) $-\frac{x^2+y}{2y^2}$
(e) $-\frac{x^2}{1+2y^2}$

$x^3 + 3xy + 2y^3 = 17$

$3x^2 + 3y + 3x\frac{dy}{dx} + 6y^2\frac{dy}{dx} = 0$

$3x^2 + 3y = -3x\frac{dy}{dx} - 6y^2\frac{dy}{dx}$

$-3x^2 - 3y = 3x\frac{dy}{dx} + 6y^2\frac{dy}{dx}$

$-x^2 - y = \frac{dy}{dx}(x + 2y^2)$

$-\frac{x^2+y}{x+2y^2} = \frac{dy}{dx}$

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17. $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx =$

Let $u = x^3 + 1 \therefore du = 3x^2dx$

$\int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C = 2\sqrt{x^3+1} + C$

18. For what value of x does the function $f(x) = (x-2)(x-3)^2$ have a relative maximum?

To find the maximum of the function f, we must first conduct an Candidates Test using the critical points of f. Let us first solve for the critical points.

$f(x) = (x-2)(x-3)^2 = x^3 - 8x^2 + 21x - 18 = 0$ at $x = 2$ and $x = 3$. $(2 < x < 3)$

$f'(x) = 3x^2 - 16x + 21 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{16 \pm \sqrt{256-4*3*21}}{2*3} = \frac{16 \pm 2}{6} = 3$ and $\frac{14}{3}$

Because $\frac{14}{3}$ is the only value within range for a relative extrema, we can determine that this value is our only extrema. Let us conduct an Intervals Test to determine if this value is a relative maximum.

Intervals	$(2, \frac{14}{3})$	$(\frac{14}{3}, 3)$
$f'(x)$	Positive	Negative
$f(x)$	Increasing	Decreasing

From the results in the Intervals Test, we can conclude that f has a relative maximum at $x = \frac{14}{3}$.