

Social Network Analysis Review

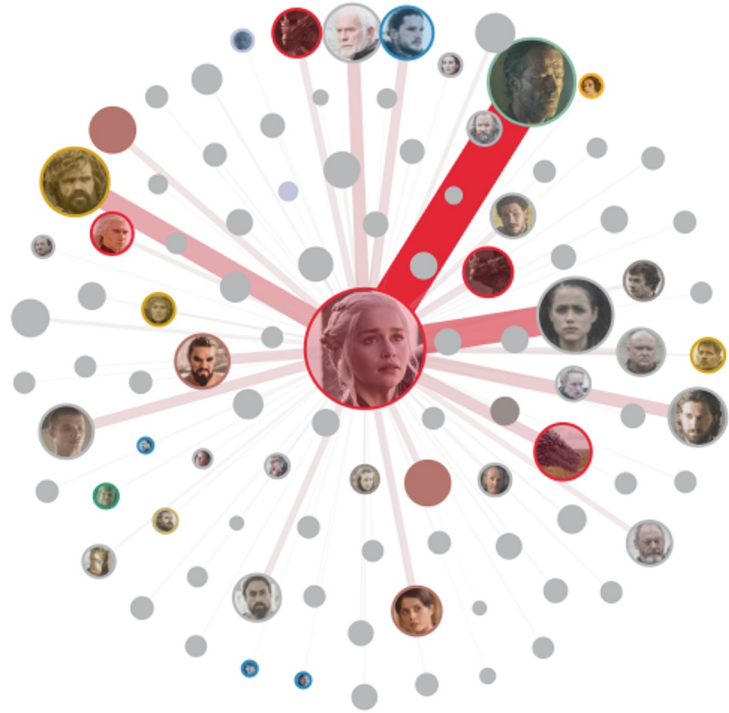
Let's get started from the basics!

- Node/Vertex
- Tie/Edge
 - Tie direction
 - Tie weights
- Network Size

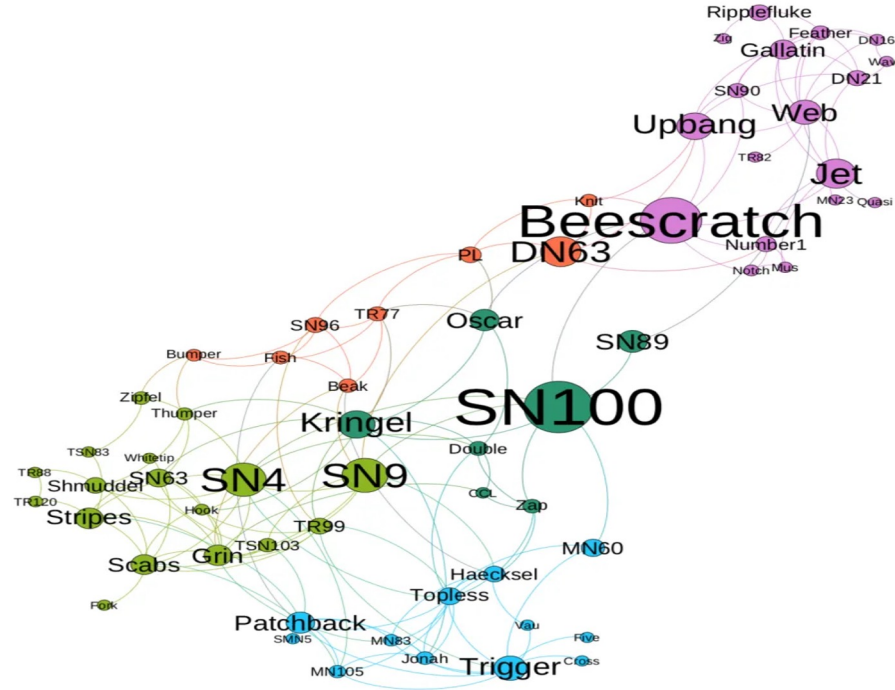


Image source <https://news.northeastern.edu/2019/04/11/the-game-of-thrones-social-network/>

Ego Network

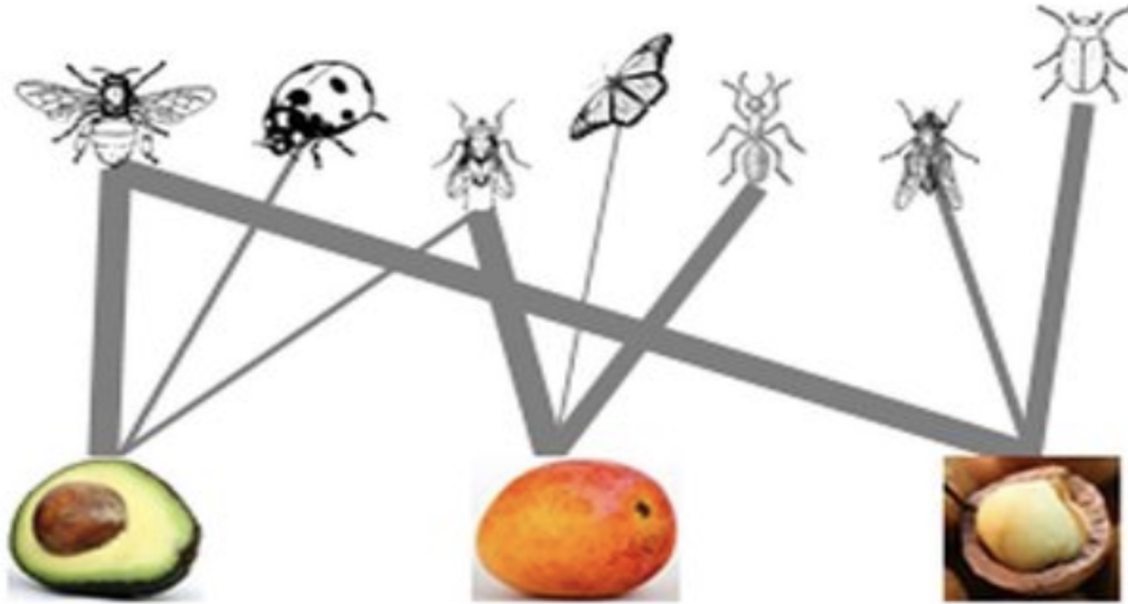


One mode networks



Dolphins swimming network From:
<https://studentwork.prattsi.org/infovis/labs/mapping-the-social-structure-of-dolphins-using-gephi/>

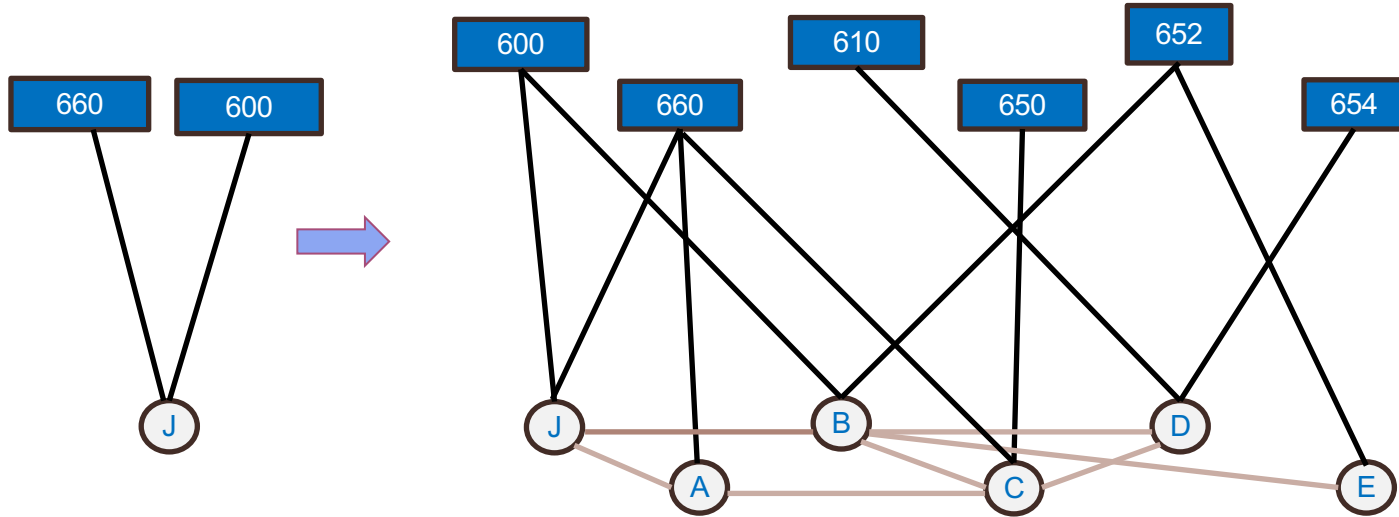
Two mode networks



Pollination network
Willcox et al. 2019

Multilevel networks

- Students and classes



How to represent networks?

- Adjacency matrix
- Edgelist

Adjacency matrices

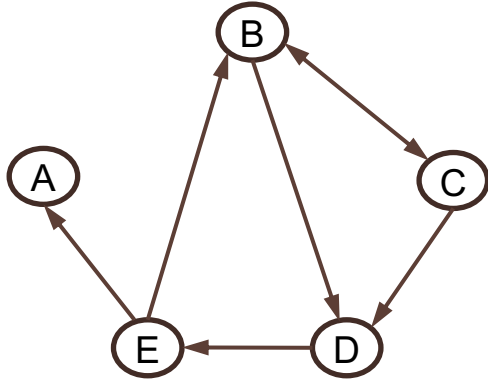
- Represent edges (who is adjacent to whom) as a matrix
 - $A_{ij} = 1$ if node i has an edge to node j ; $A_{ij} = 0$ if node i doesn't have an edge to node j .



- $A_{ii} = 0$ unless the network has self loops.
- $A_{ij} = A_{ji}$ if the network is undirected or i and j share reciprocated edges.



Adjacency matrix example



	A	B	C	D	E
A	0	0	0	0	0
B	0	0	1	1	0
C	0	1	0	1	0
D	0	0	0	0	1
E	1	1	0	0	0

How to calculate density?

- Network Density:
 - The number of observed ties/Maximum number of possible ties

	A	B	C	D	E
A	0	0	0	0	0
B	0	0	1	1	0
C	0	1	0	1	0
D	0	0	0	0	1
E	1	1	0	0	0

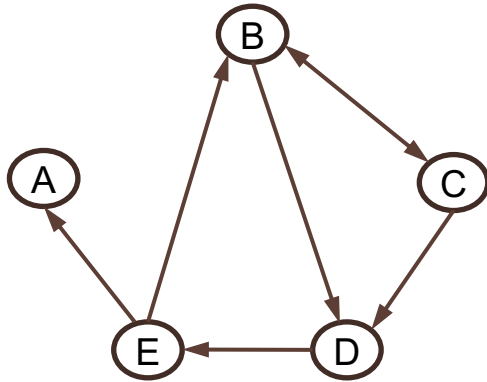
Node degrees

The outdegree of a node i is the number of nodes that are adjacent *from* i

Outdegree $A = \{a_{ij}\} =$

	A	B	C	D	E
A	0	0	0	0	0
B	0	0	1	1	0
C	0	1	0	1	0
D	0	0	0	0	1
E	1	1	0	0	0

$\Rightarrow a_{i+} = \sum_{j=1}^n a_{ij}$



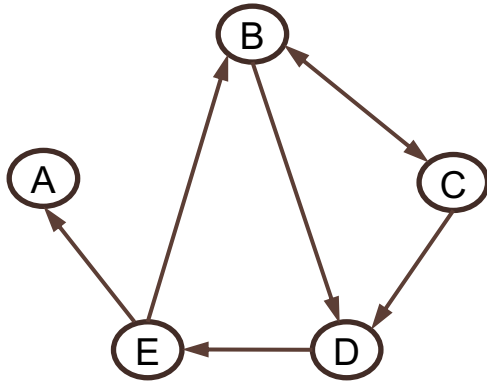
Indegree $A = \{a_{ij}\} =$

	A	B	C	D	E
A	0	0	0	0	0
B	0	0	1	1	0
C	0	1	0	1	0
D	0	0	0	0	1
E	1	1	0	0	0

$a_{+j} = \sum_{i=1}^n a_{ij}$

The indegree of a node i is the number of nodes that are adjacent *to* i

Edge-list example



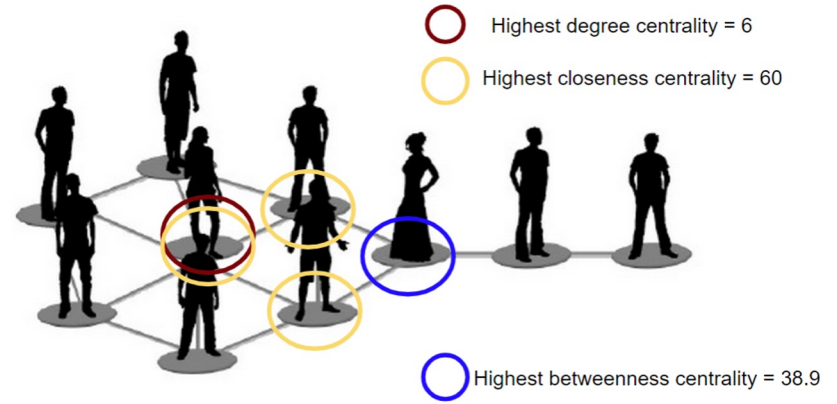
- B, C
- B, D
- C, B
- C, D
- D, E
- E, A
- E, B

Local measures

Centrality

Several types of centrality measurements:

- Degree centrality
- Closeness centrality
- Betweenness centrality
- Eigenvector centrality



Krackhardt kite graph. Image source: Slides by Hagen, 2018

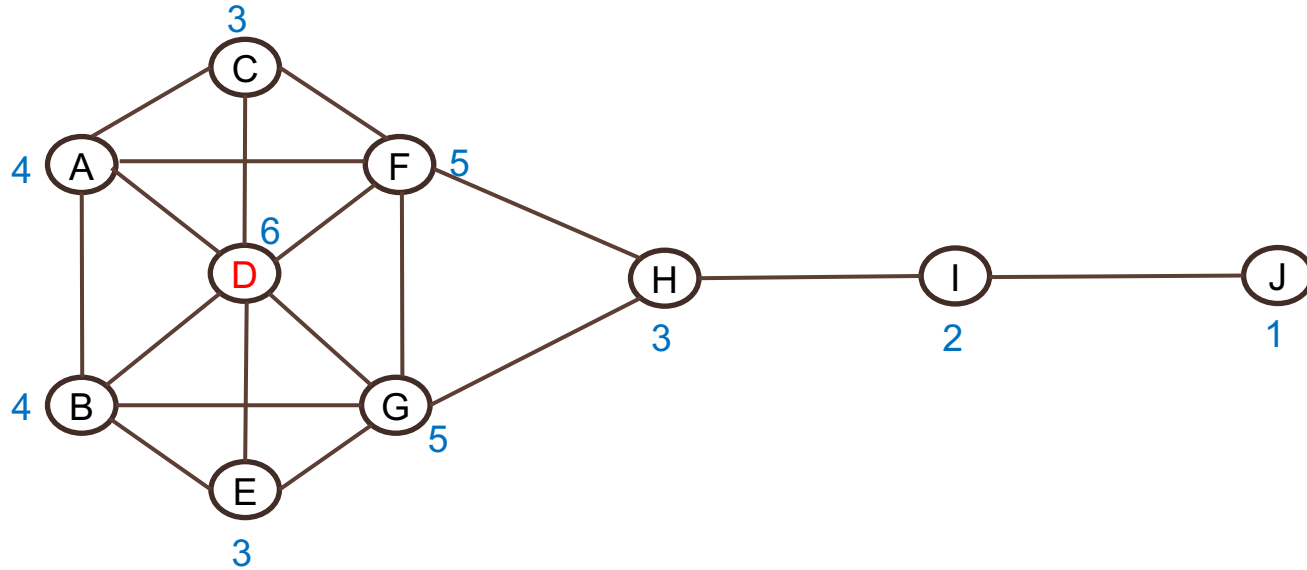
Degree centrality

“Being central means having ties to many others.”

- Degree centrality is the count of all ties an actor has with all other actors:

$$c_i = \sum_{\substack{j=1, \\ j \neq i}}^n x_{ij} \quad \text{or} \quad c_i = x_{i+}$$

Degree centrality



Degree centrality

- Represents an index of *exposure* to what flows through the network;
- It indicates the opportunity to *influence others*
- but also *be influenced* by others.
- *Centrality and...*
 - *Creativity*
 - *Leadership*
 - *Job satisfaction*
 - *Task performance*

Closeness centrality

“Being well-connected means being able to reach all others quickly / be reached by others quickly.”

- E.g., *gossip* – central actors hear any rumour quickly!
- Various definitions(!)
 - E.g., inverse of the sum of social distances to everybody else.
 - (occasionally computed as eigenvector centrality based on distance matrix)
- ***Work here with “sum of distances to all other nodes”*** (which is a negative measure of closeness, actually: “farness”).

Closeness centrality

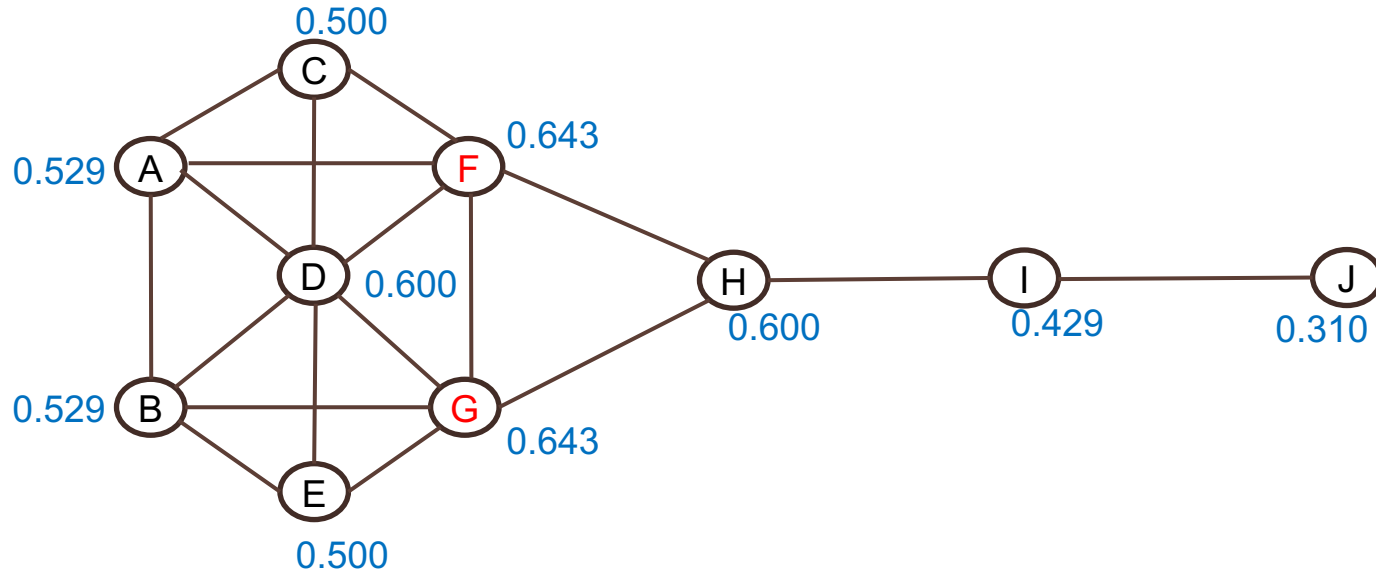
- “Central” nodes in a closeness are important, as they can reach (and be reached by) the whole network more quickly than non-central nodes
- Importance measured by **how close a node is to other nodes**

- Average Distance:
$$D_{avg}(v_i) = \frac{1}{n-1} \sum_{j \neq i}^n g(v_i, v_j)$$

- **Closeness Centrality**

$$C_C(v_i) = \left[\frac{1}{n-1} \sum_{j \neq i}^n g(v_i, v_j) \right]^{-1} = \frac{n-1}{\sum_{j \neq i}^n g(v_i, v_j)}$$

Closeness centrality



Betweenness centrality

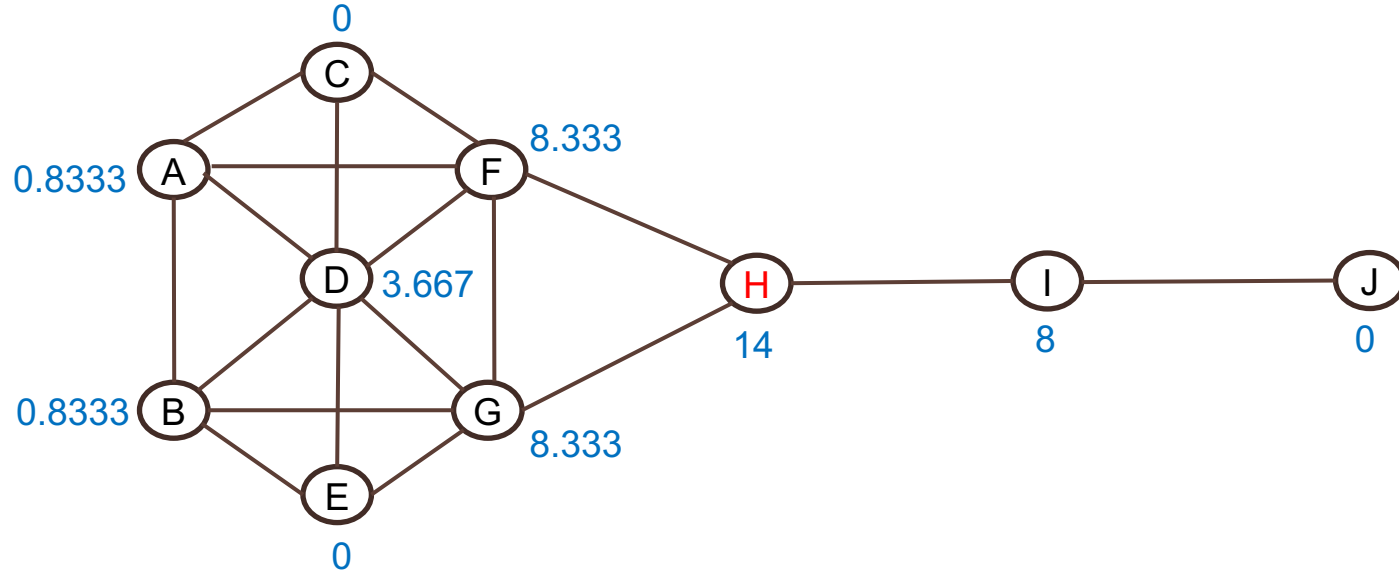
“Being well-connected means being able to hinder (or at least complicate) communication between two others.”

- Sum of fractions of shortest paths between any two nodes that pass through a given node;
- Formula:

$$c_i = \sum_{\substack{j=1, k=1, \\ j \neq i}}^n \sum_{\substack{k=1, \\ k \neq i, j}}^n \frac{g_{jk \sim i}}{\tilde{g}_{jk}}$$

Here $g_{jk \sim i}$ is the number of shortest paths from j to k passing through i , while \tilde{g}_{jk} is the total number of shortest paths from j to k (others might not pass through i).

Betweenness centrality



Betweenness centrality

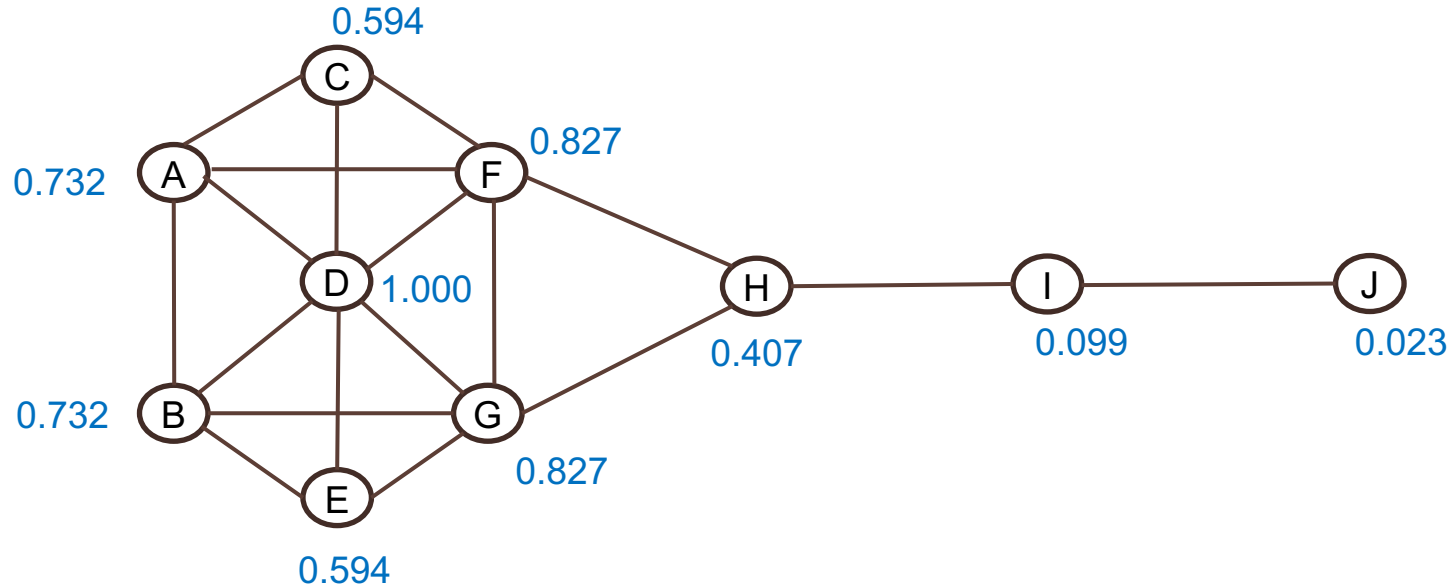
- The betweenness of a node counts the number of shortest paths that go through that node
- **Intuition:** how many pairs of individuals would have to go through you in order to reach one another in the minimum number of hops?
- Global network cohesion and connectivity are dependent on nodes connecting different parts (components) of the network
- Betweenness is typically associated with “influence,” “control” and the possibility of having access to information that is not generally shared

Eigenvector centrality

"Being well-connected means having ties to well-connected others." others will interpret as value.

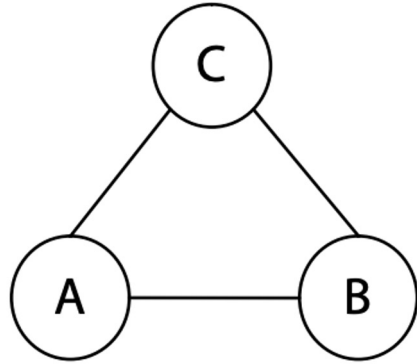
- Self-referencing definition:
 - Your centrality is proportional to the sum of your network neighbours' centrality.
 - Formula:
$$c_i = \lambda \sum_{\substack{j=1, \\ j \neq i}}^n x_{ij} c_j$$
 - Here λ is the proportionality factor.
 - Mathematically, λ is the first eigenvalue of the adjacency matrix (x_{ij}) and the vector of centralities is the corresponding eigenvector (hence the name).

Eigenvector centrality

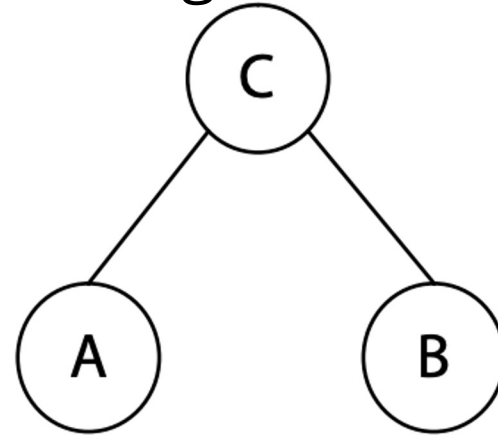


Social Capital and Networks

Closure

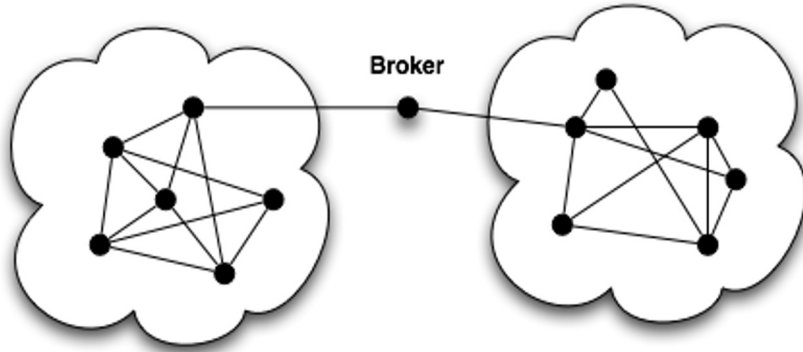


Brokerage



Network brokerage leads to good ideas

Network brokerage offers opportunity to exploit information advantage and access to novel information.



Artists' communication network and novelty

How structural holes can help entrepreneurs?

Animal + Ramen =

Threadless, Design by Vincent Trinidad Art



Weak Ties and Finding a Job (Granovetter, 1973)

A strong tie is someone you know well and a weak tie is someone you know but the interaction between you two is minimum.

No strong tie is a bridge.

Is it still true in the digital context?

Garg & Telang (2018) found weak ties from SNS are not effective leading to substantial job seeking outcomes.

