1. Provide a definition for the following concepts, accompanied by mathematical notation:
   1. Probability
   2. Joint probability
   3. Conditional probability
   4. Marginal probability
   5. The probability triple
2. Coin flipping part A. Suppose you flip a fair coin 100 times. Write down a formula representing the problem, and then use ‘pbinom’ function in R to help answer the following questions:
   1. What is the probability of observing exactly 50 heads?
   2. What is the probability of observing more than 50 heads?
3. Data from the 1981-1983 NFL football seasons suggest that the *point spread* from betting marketsprovides a well-calibrated estimate of the probability that the favored team will win. We can model the distribution of score outcomes (i.e. difference in scores between winning and losing teams) minus the predicted point spread as Normal(0,142). That is, the difference between outcome *y* and predicted point spread *x:*

*y – x* ~ Normal(0,142)

1. What is the probability that a team favored by 3.5 points loses? (HINT: Use ‘pnorm’ function in R)
2. What is the probability that a team favored by 9 points loses?
3. {Insert image of data}: If your goal is to place well-calibrated bets, should you be more confident in your bets on games with small *point spreads* or large *point spreads*?
4. Dice in an urn. Suppose you have two dice in an urn – one of them a D6, the other a D12. Your fun gamer friend randomly draws one of the dice, keeping it carefully concealed from you. Hidden from your view, she then rolls the die and records the number. Define the following components:
   1. What is the sample space?
   2. What are the possible events in this sample space?
   3. Using the rules of probability discussed above, write down a function for assigning probabilities to each of these events (technically called a “*probability measure*”), and compute the probability for each event in b.
   4. How do your answers in b and c change if your friend performs the entire die-sampling process twice (with replacement) and sums the two numbers?
5. Hemophilia problem. Hemophilia is an X-chromosome linked disorder of blood coagulation, that displays recessive inheritance. Since males have only one X-chromosome, if a male inherits the allele, he will display the hemophiliac phenotype, whereas a female with only one copy of the gene will not have hemophilia. We are told that a certain woman has a brother with hemophilia, but her father is not hemophiliac.
   1. Given this information alone, what is the probability the woman carries a copy of the gene for hemophilia?
   2. Suppose you are now told that the woman has two sons, neither of whom suffers from hemophilia. Treat your answer to ‘a’ as a prior probability, and use Bayes Theorem to work out the posterior probability that the woman carries the gene.
   3. Now suppose she has a third son, who also does not have hemophilia. Apply Bayes Theorem to update your answer in ‘b’ and provide a new probability.
6. **Challenge**: While supervising an outdoor recreation summer camp in rural Massachusetts, you come down with a wicked case of flu-like symptoms, high fever, fatigue and delirium. You muster your last energy to bring yourself to your friends who then bring you to hospital. There, the doctors run a test for Lyme Disease and you come back negative. Undeterred the doctors prescribe Doxycycline, and diagnose you with Lyme Disease based on symptom presentation.
   1. Use Bayes Theorem to work out the probability you have Lyme disease. What information do you need to use to construct your prior? How do you condition on the test result? What assumptions are you making?
   2. More generally, is treating you as though you have Lyme disease the right decision? Why or why not?