EEE-6512: Image Processing and Computer Vision

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Lecture #7: Morphological Image Processing
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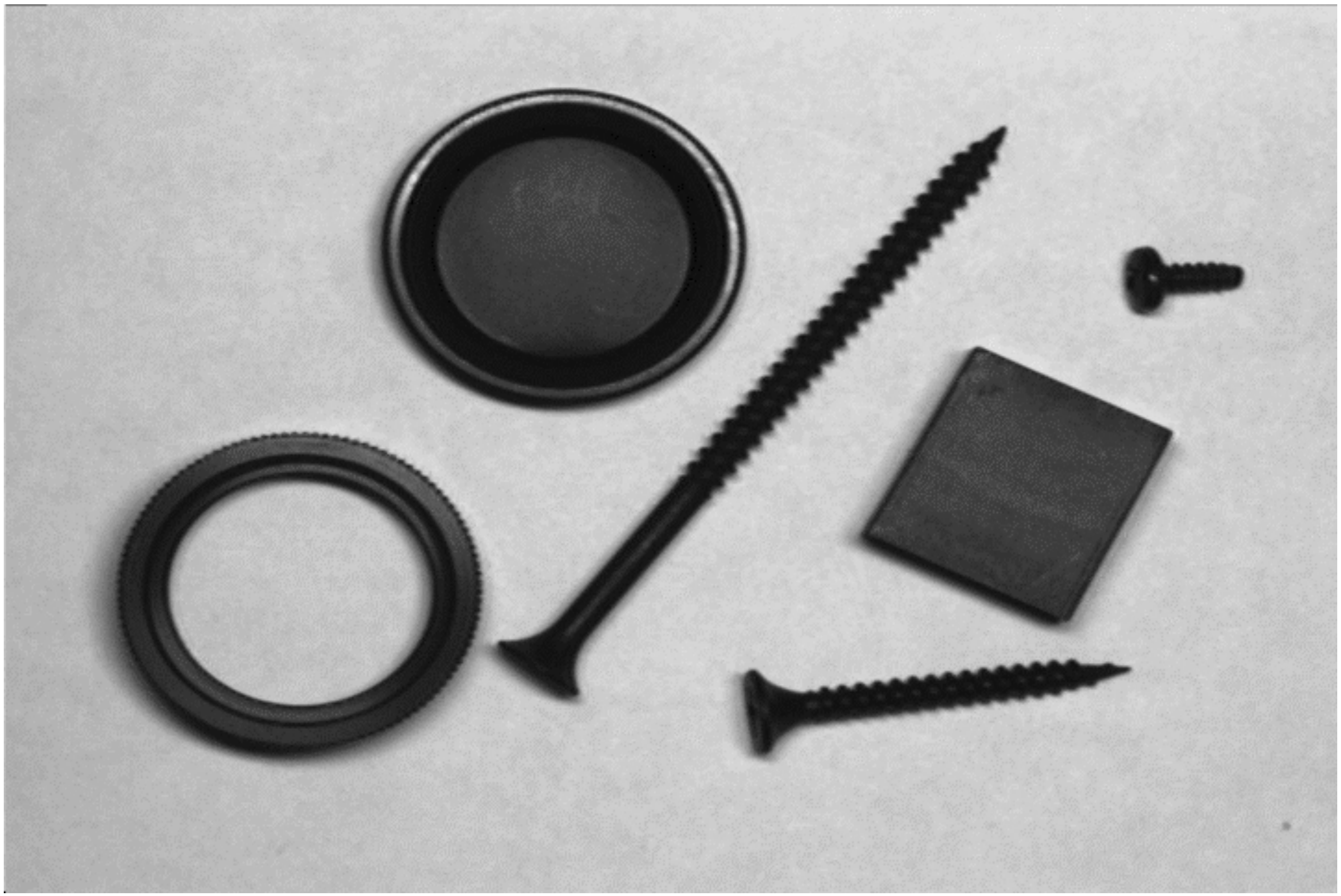
Outline

- Preliminaries
- Erosion and Dilation
- Opening and Closing
- Hit-or-Miss Transformation
- Basic Morphological Algorithms
- Summary of Binary Morphology
- Grayscale Morphology
- Morphology Summary

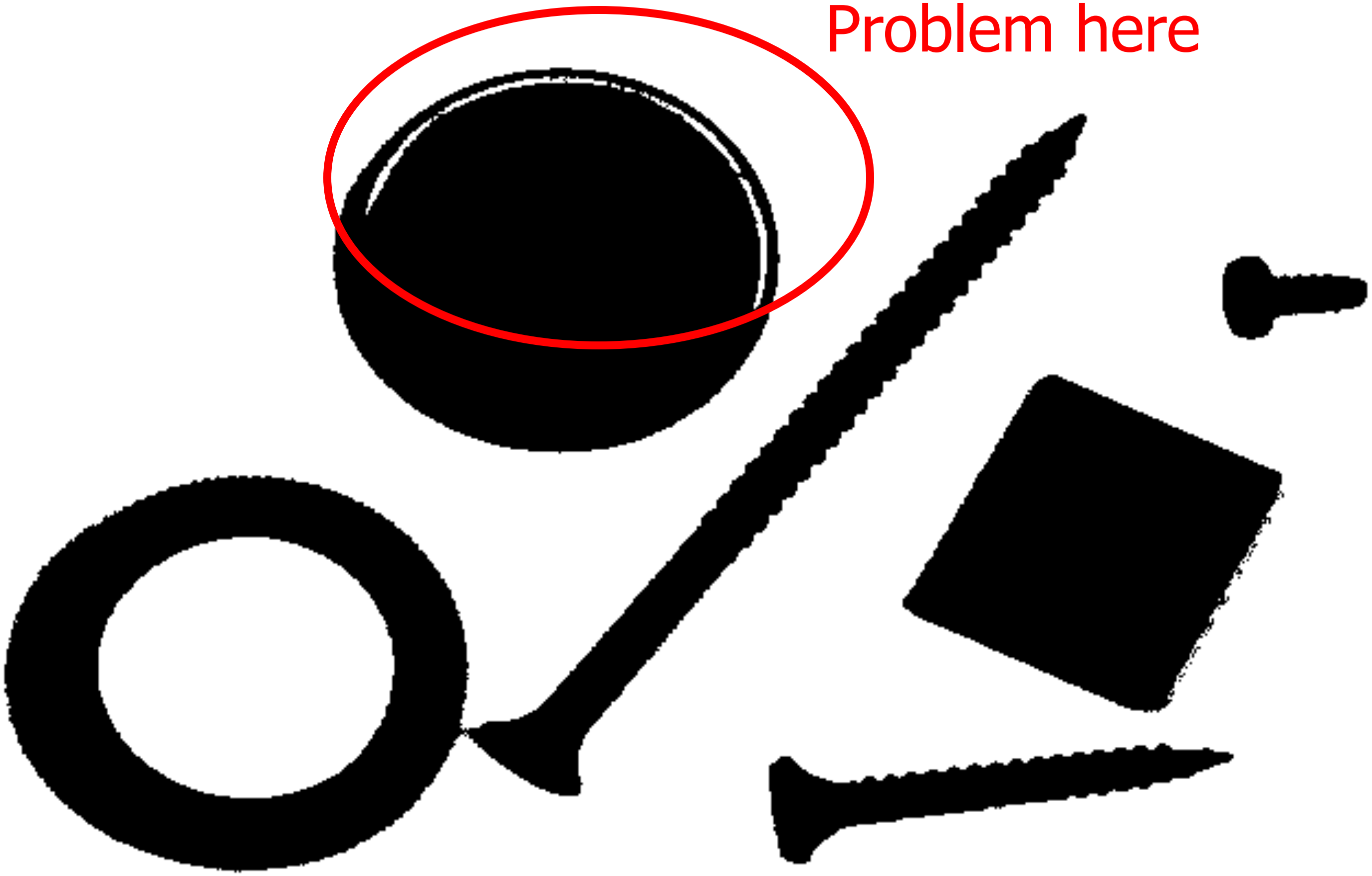
Preliminaries

- **Morphology**: a branch of biology that deals with the form and structure of animals and plants
- Morphological image processing is used to extract image components for representation and description of region shape, such as boundaries, and skeletons.

Example Application



Example (cont.)



How do we fill "missing pixels"?

Preliminaries

- **Sets** in mathematical morphology represent **objects** in an image

- Example
  - Binary image: the elements of a set is the coordinate (x,y) of the pixels, in $\mathbf{Z}^2$
  - Gray-level image: the element of a set is the triple, (x, y, gray-value), in $\mathbf{Z}^3$

Preliminaries

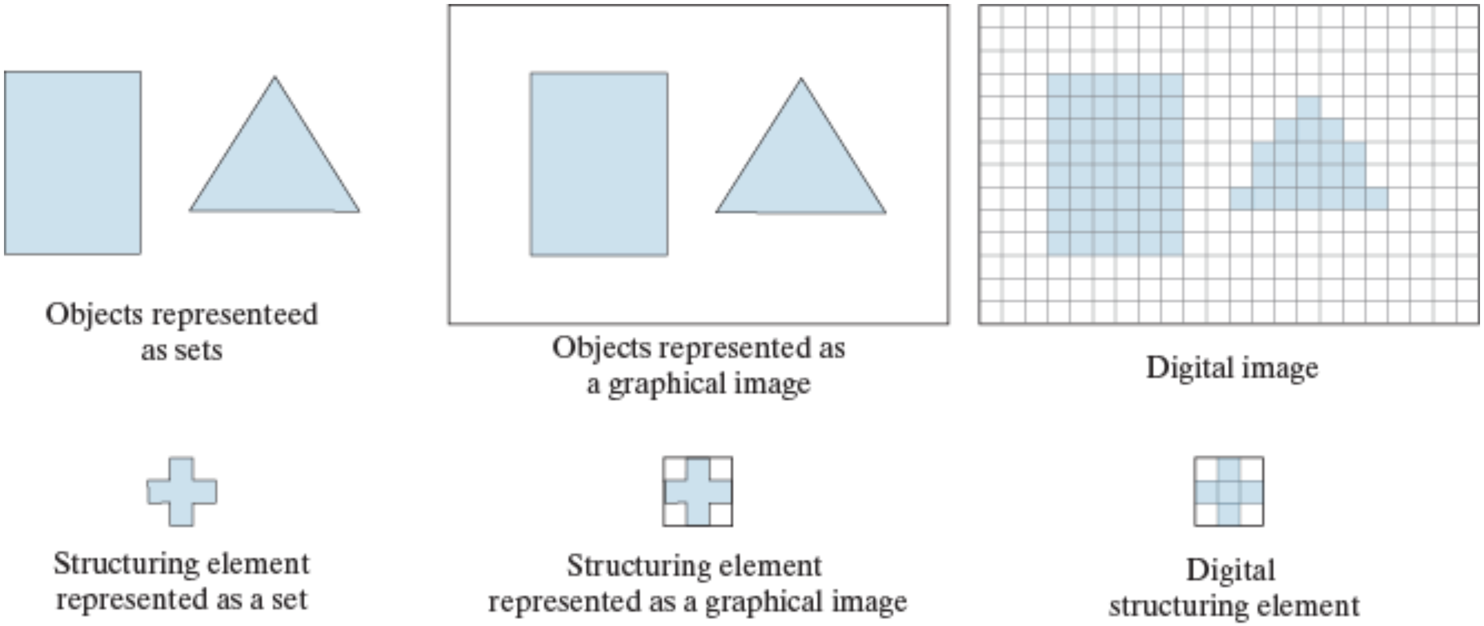


FIGURE 9.1 Top row. *Left:* Objects represented as graphical sets. *Center:* Objects embedded in a background to form a graphical image. *Right:* Object and background are digitized to form a digital image (note the grid). Second row: Example of a structuring element represented as a set, a graphical image, and finally as a digital SE.

Preliminaries – set theory

- A be a set in $\mathbf{Z}^2$.
- a = (a_1, a_2) is an element of A. $a \in A$
- a is not an element of A $a \notin A$
- Null (empty) set: $\varnothing$

Preliminaries – set theory (cont.)

- Explicit expression of a set

$$A = \{a_1, a_2, \ldots, a_n\}$$

$$A = \left\{ element \,\middle|\, \text{condition for set elements} \right\}$$

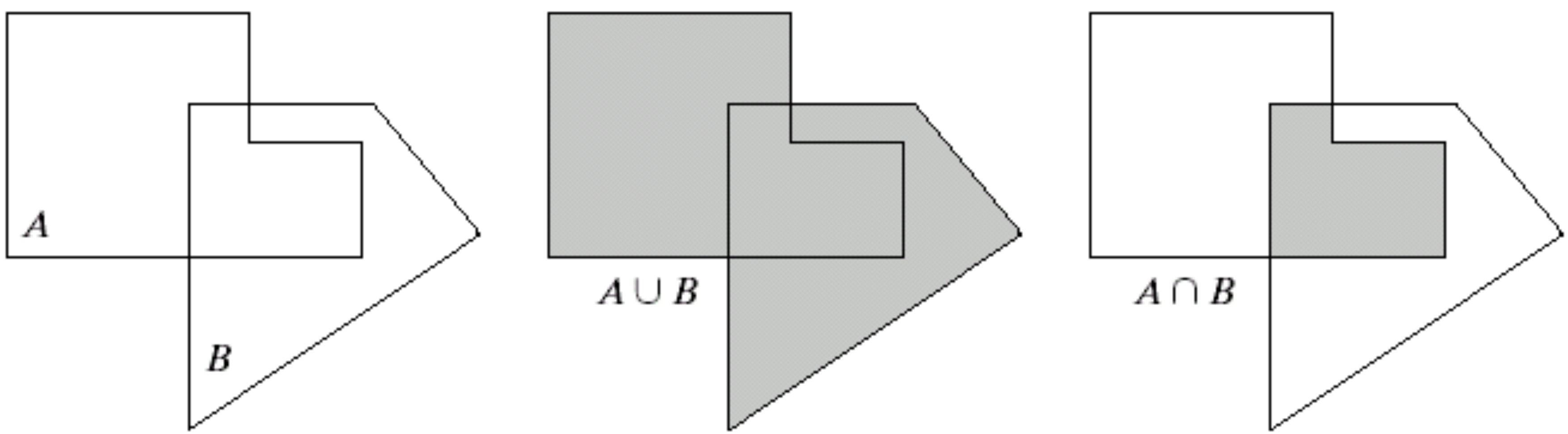
- Example:

$$C = \{w \mid w = -d, \ \text{for } d \in D\}$$

Preliminaries - set operations

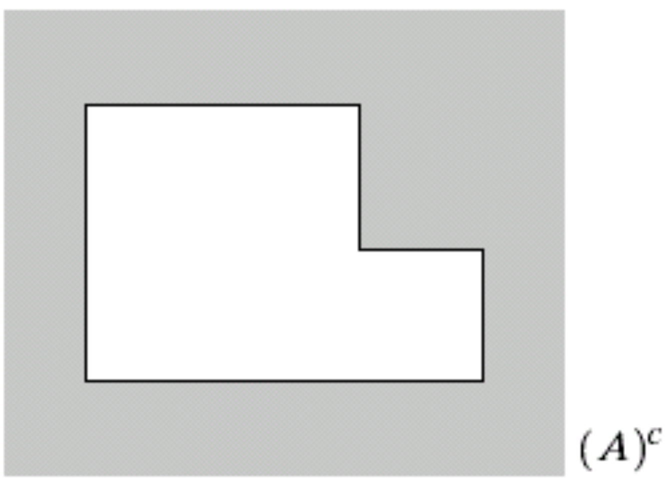
- A is a subset of B: every element of A is an element of another set B $A \subseteq B$
- Union $C = A \cup B$
- Intersection $C = A \cap B$
- Mutually exclusive $A \cap B = \varnothing$

Graphical examples

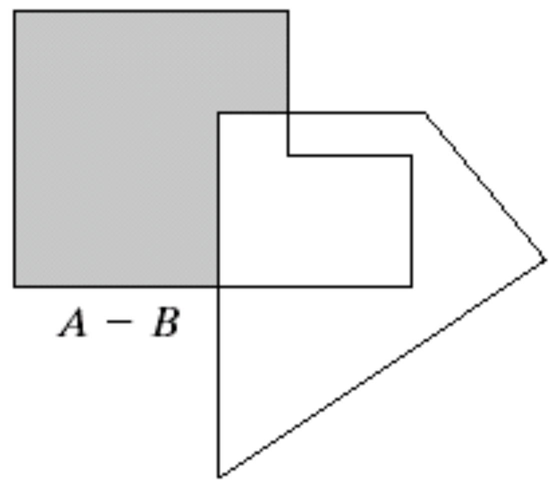


Graphical examples (cont.)

$$A^c = \left\{ w \middle| w \notin A \right\}$$



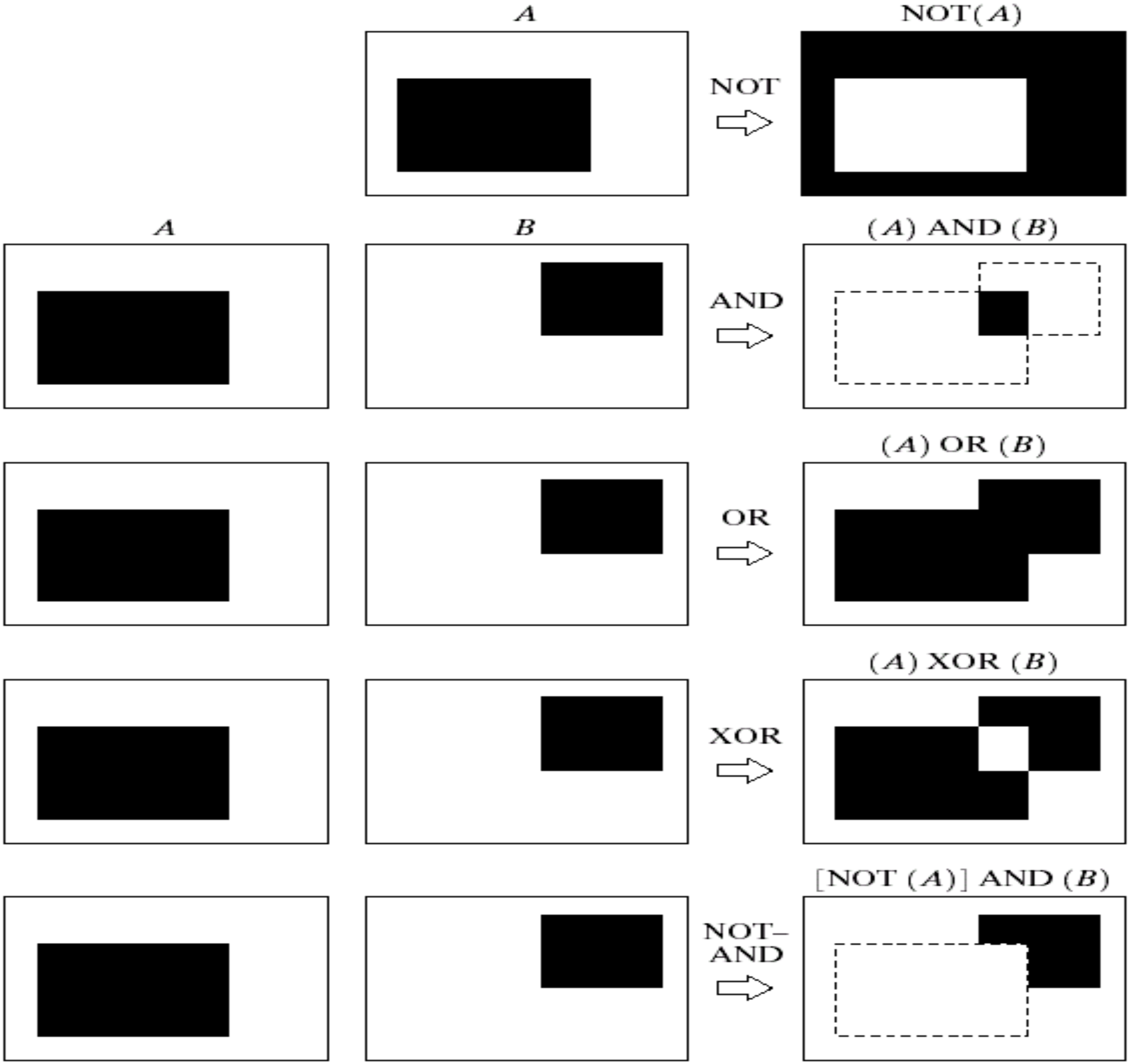
$$A - B = \left\{ w \middle| w \in A, w \notin B \right\}$$



Preliminaries: Logic operations on binary images

- Functionally complete operations
  - AND, OR, NOT

p	q	p **AND** q **(also** $p \cdot q$**)**	p **OR** q **(also** $p + q$**)**	**NOT (**p**) (also** $\bar{p}$**)**
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0



$A \cap B$

$A \cup B$

$B - A$

Preliminaries

- **Reflection**

The reflection of a set B, denoted $\hat{B}$, is defined as

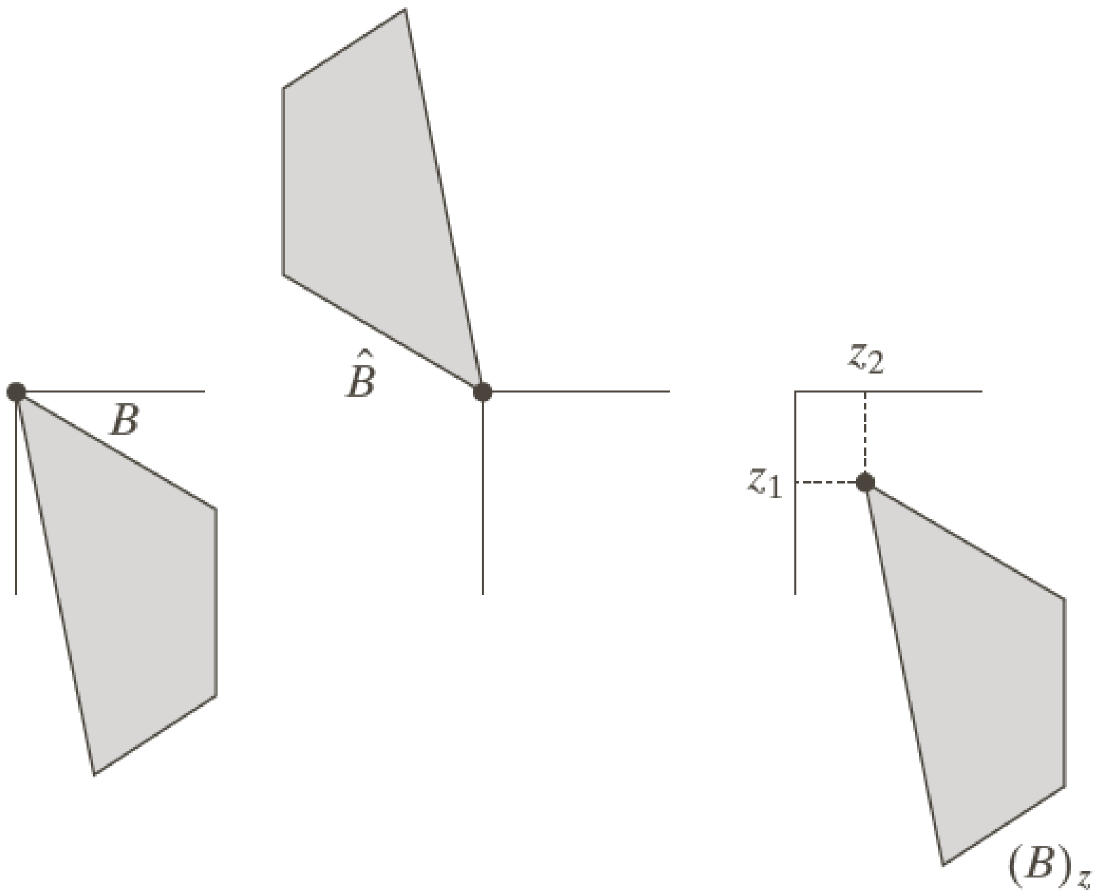
$$\hat{B} = \{w \mid w = -b, \text{for } b \in B\}$$

- **Translation**

The translation of a set B by point $z = (z_1, z_2)$, denoted $(B)_Z$, is defined as

$$(B)_Z = \{c \mid c = b + z, \text{for } b \in B\}$$

Example: Reflection and Translation



a b c

FIGURE 9.1
(a) A set, (b) its reflection, and (c) its translation by z.

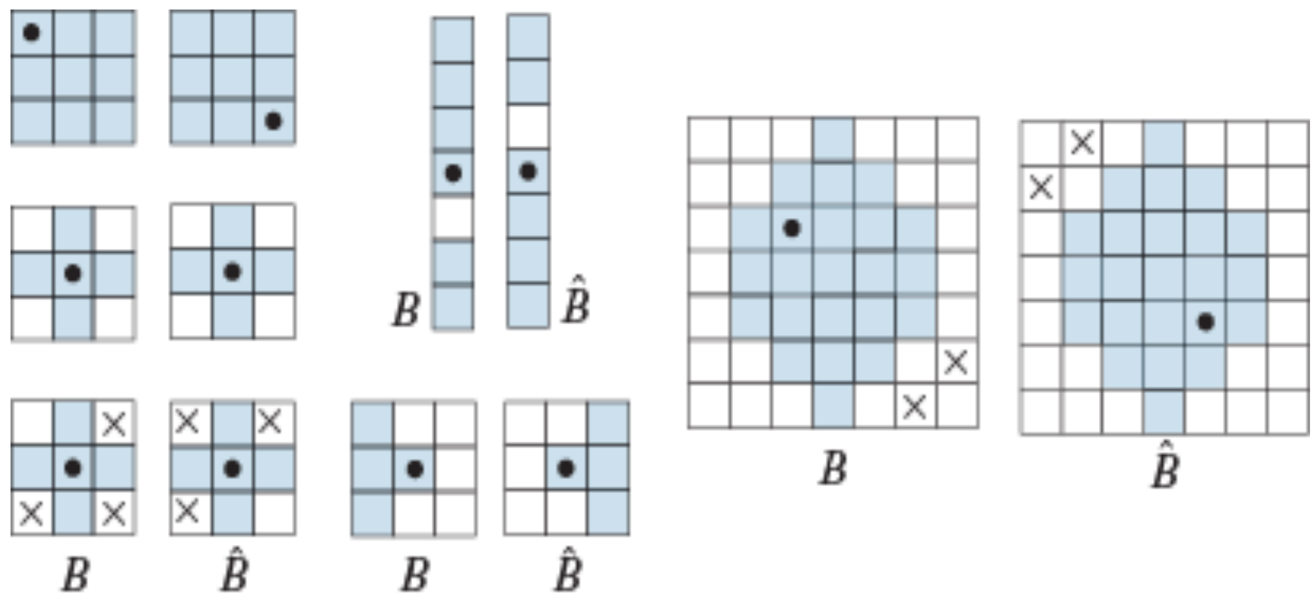
Preliminaries

- **Structure elements (SE)**

Small sets or sub-images used to probe an image under study for properties of interest

Examples: Structuring Elements (1)

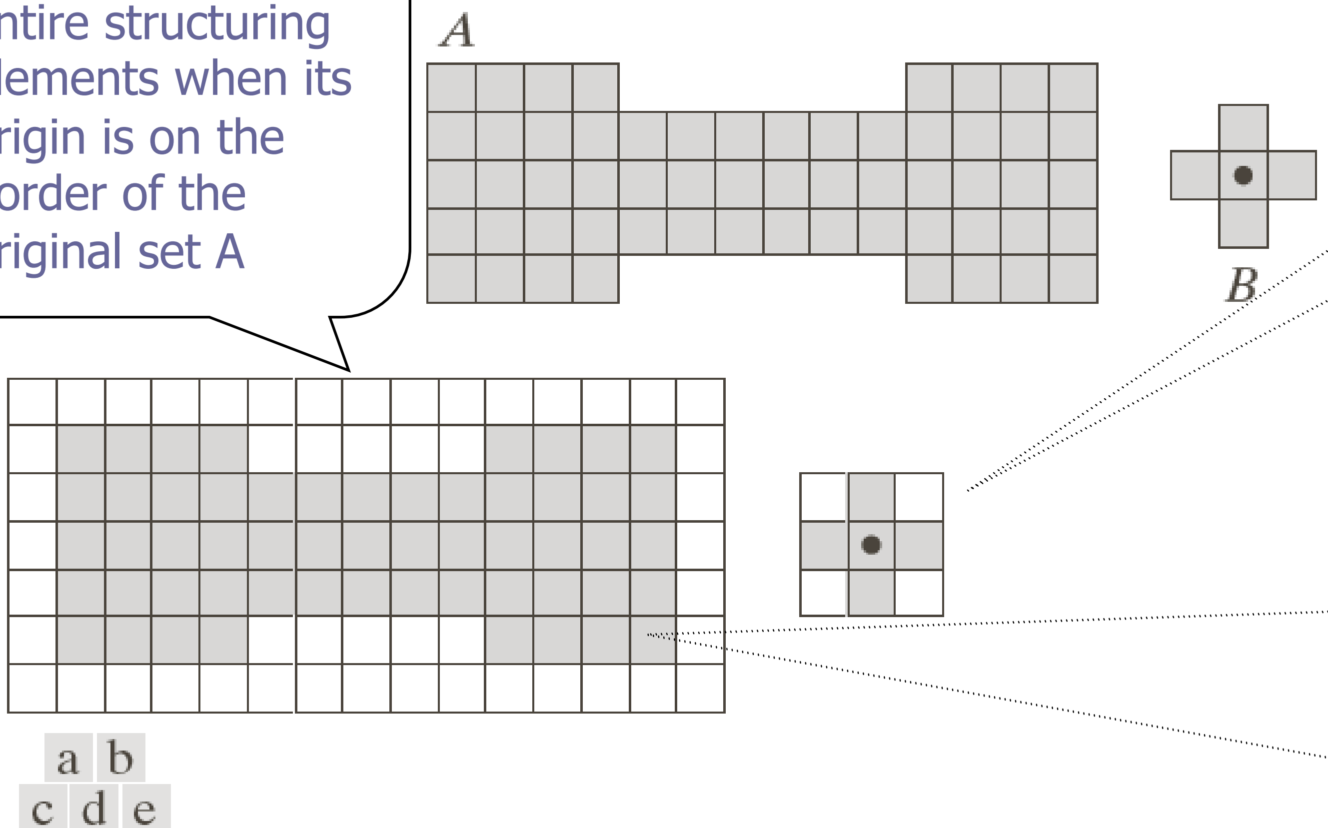
FIGURE 9.2 Structuring elements and their reflections about the origin (the ×'s are don't care elements, and the dots denote the origin). Reflection is rotation by 180° of an SE about its origin.



Examples: Structuring Elements (2)

Accommodate the entire structuring elements when its origin is on the border of the original set A

Origin of B visits every element of A



At each location of the origin of B, if B is completely contained in A, then the location is a member of the new set, otherwise it is not a member of the new set.

FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

Erosion and Dilation

What are they used for?

- Erosion

  Removal of structures of certain shape and size given by structuring element

- Dilation

  Filling of holes of certain shape and size given by the structuring element

Erosion

With A and B as sets in Z^2, the erosion of A by B, denoted $A \ominus B$, defined

$$A \ominus B = \left\{ z \mid (B)_Z \subseteq A \right\}$$

The set of all points z such that B, translated by z, is contained by A.

$$A \ominus B = \left\{ z \mid (B)_Z \cap A^c = \varnothing \right\}$$

Example of Erosion (1)

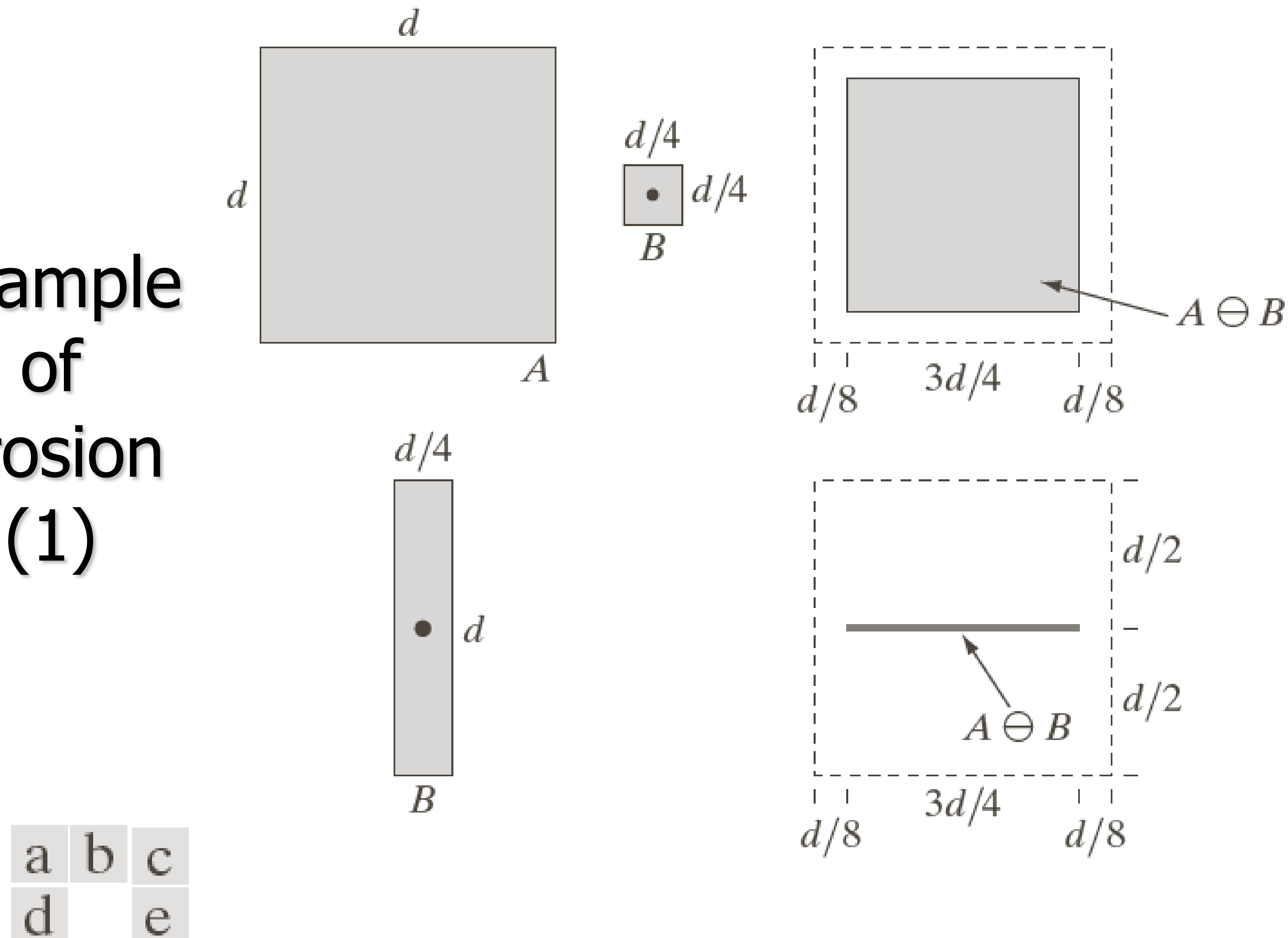
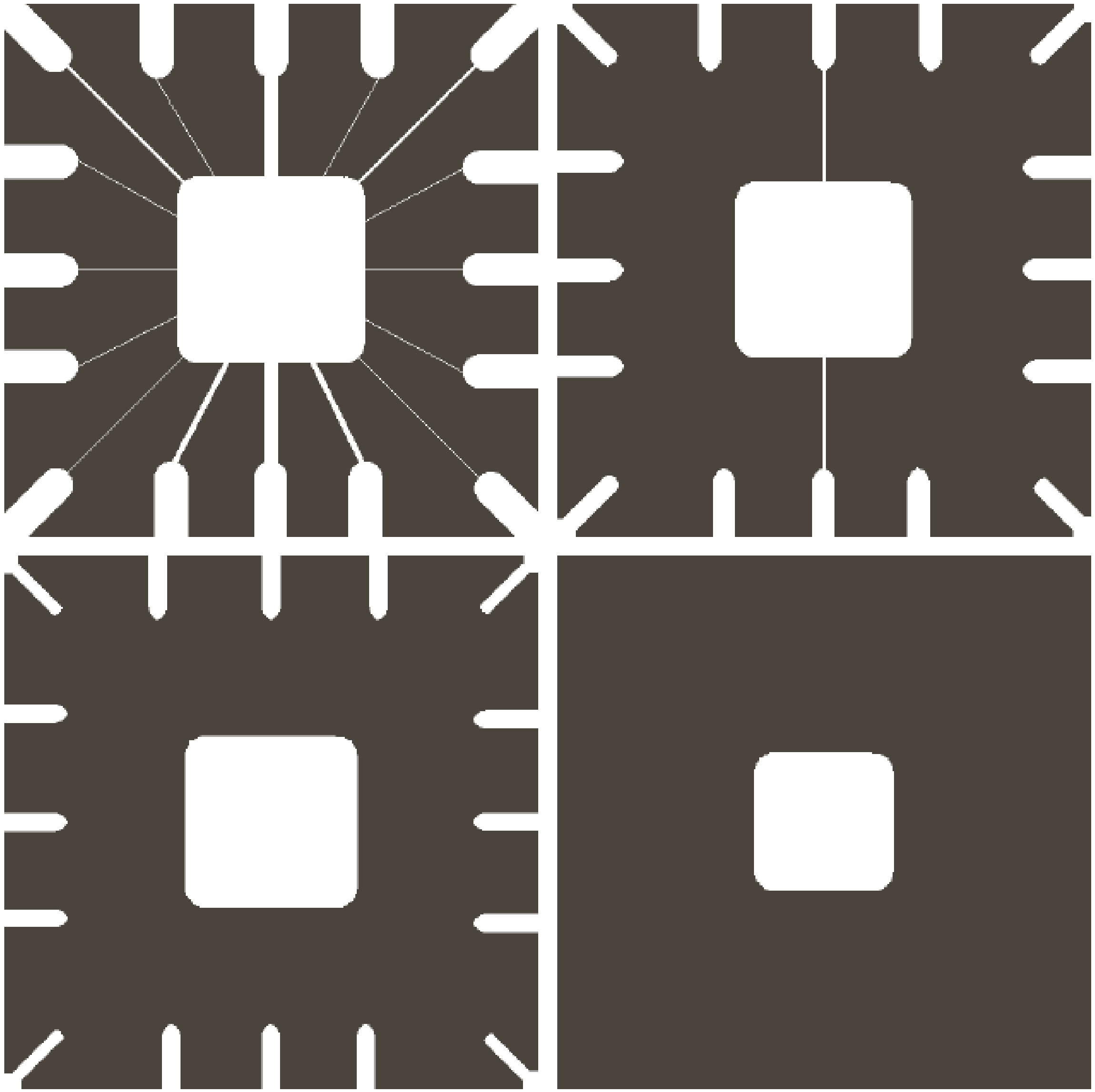


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

Example of Erosion (2)



a b
c d

FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.

Dilation

With A and B as sets in Z^2, the dilation of A by B, denoted $A \oplus B$, is defined as

$$\mathrm{A} \oplus \mathrm{B} = \left\{ z \mid \left(\hat{B}\right)_z \cap A \neq \varnothing \right\}$$

The set of all displacements z, the translated $\hat{B}$ and A overlap by at least one element.

$$A \oplus B = \left\{ z \mid \left[\left(\hat{B}\right)_z \cap A \right] \subseteq A \right\}$$

Examples of Dilation (1)

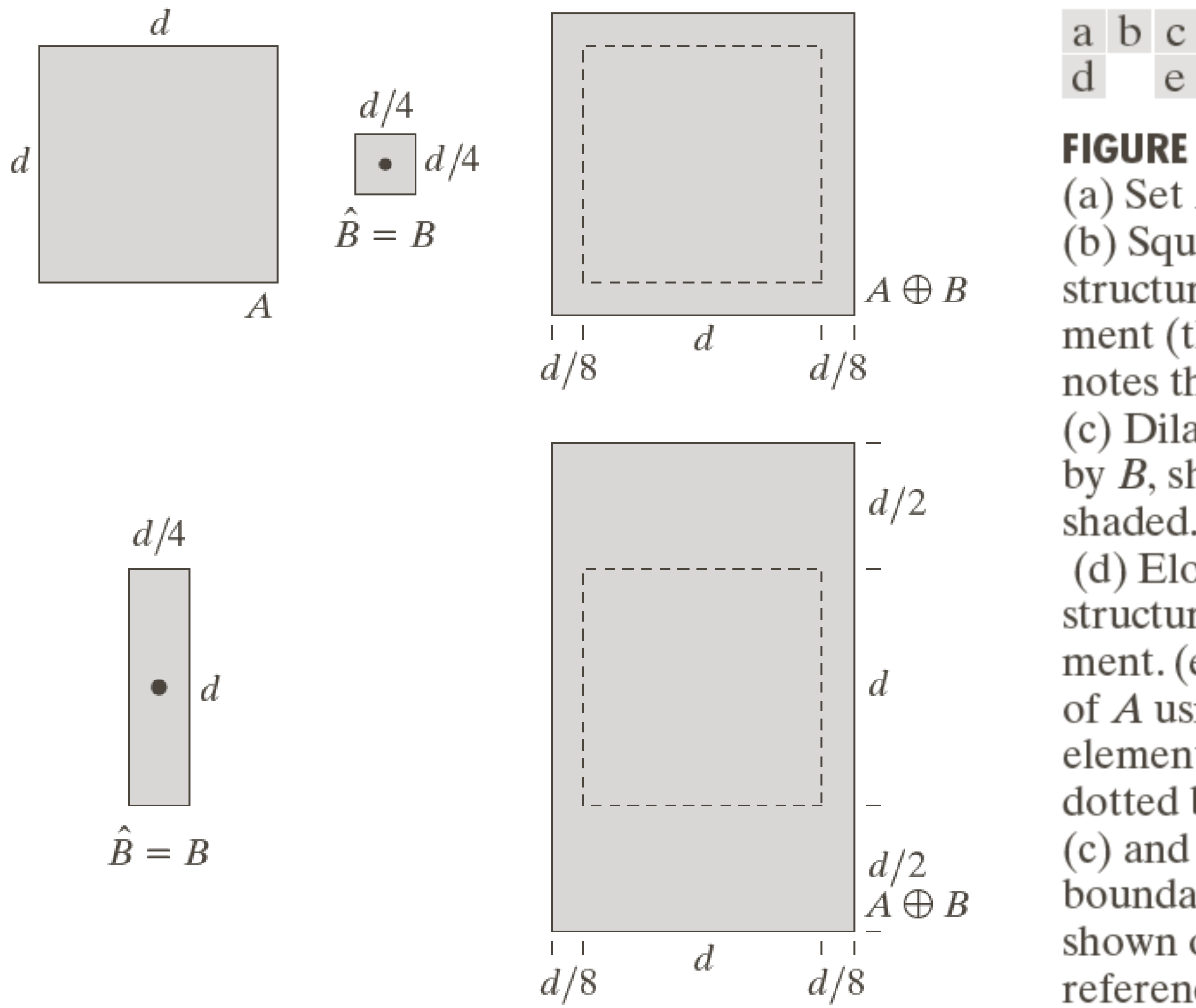
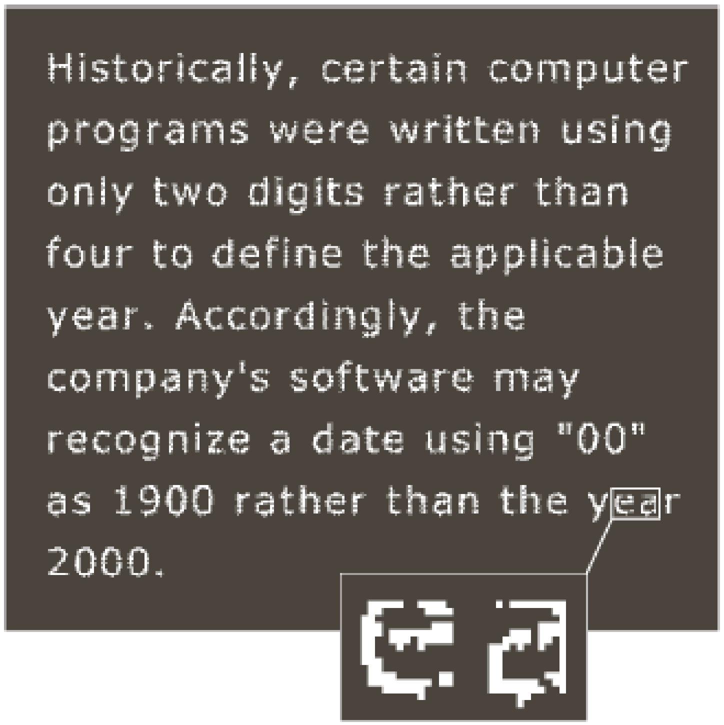
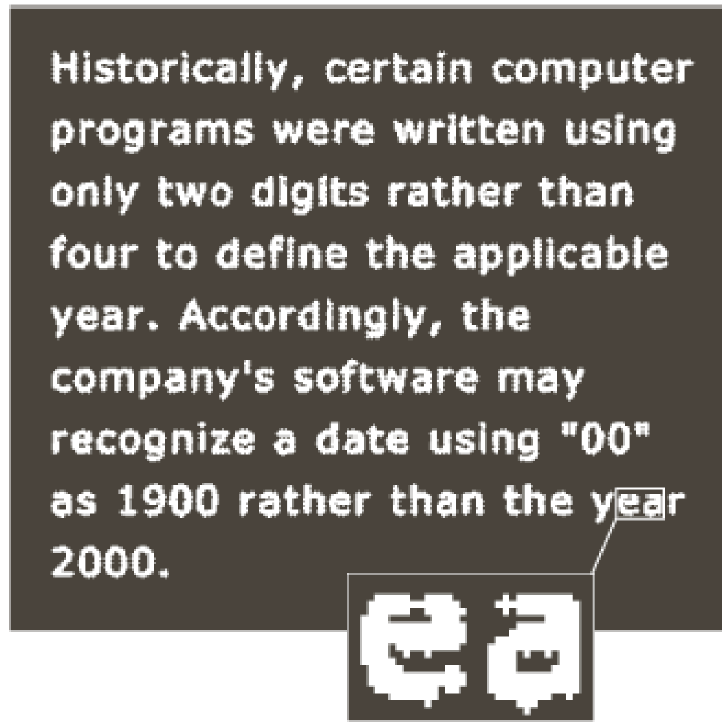


FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

Examples of Dilation (2)





0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Duality

- Erosion and dilation are duals of each other with respect to set complementation and reflection

$$\left(A \ominus B\right)^c = A^c \oplus \hat{B}$$

$$and$$

$$\left(A \oplus B\right)^c = A^c \ominus \hat{B}$$

Opening and Closing

What are they used for?

- Opening

  generally smooths the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions. Key feature is that it eliminates regions smaller than the structuring element.

- Closing

  tends to smooth sections of contours but it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour. Key feature is that fills objects smaller than the structuring element.

Opening and Closing

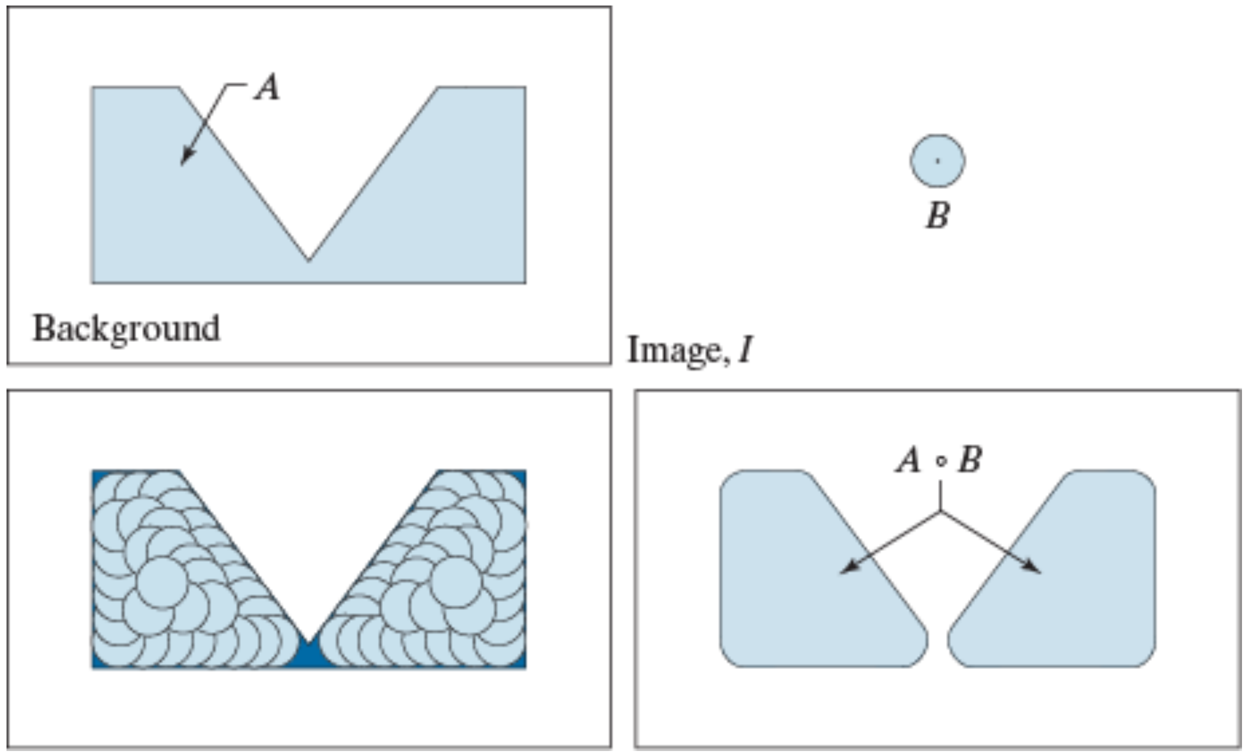
The opening of set A by structuring element B, denoted $A \circ B$, is defined as

$$A \circ B = (A \ominus B) \oplus B$$

The closing of set A by structuring element B, denoted $A \bullet B$, is defined as

$$A \bullet B = (A \oplus B) \ominus B$$

Example: Opening



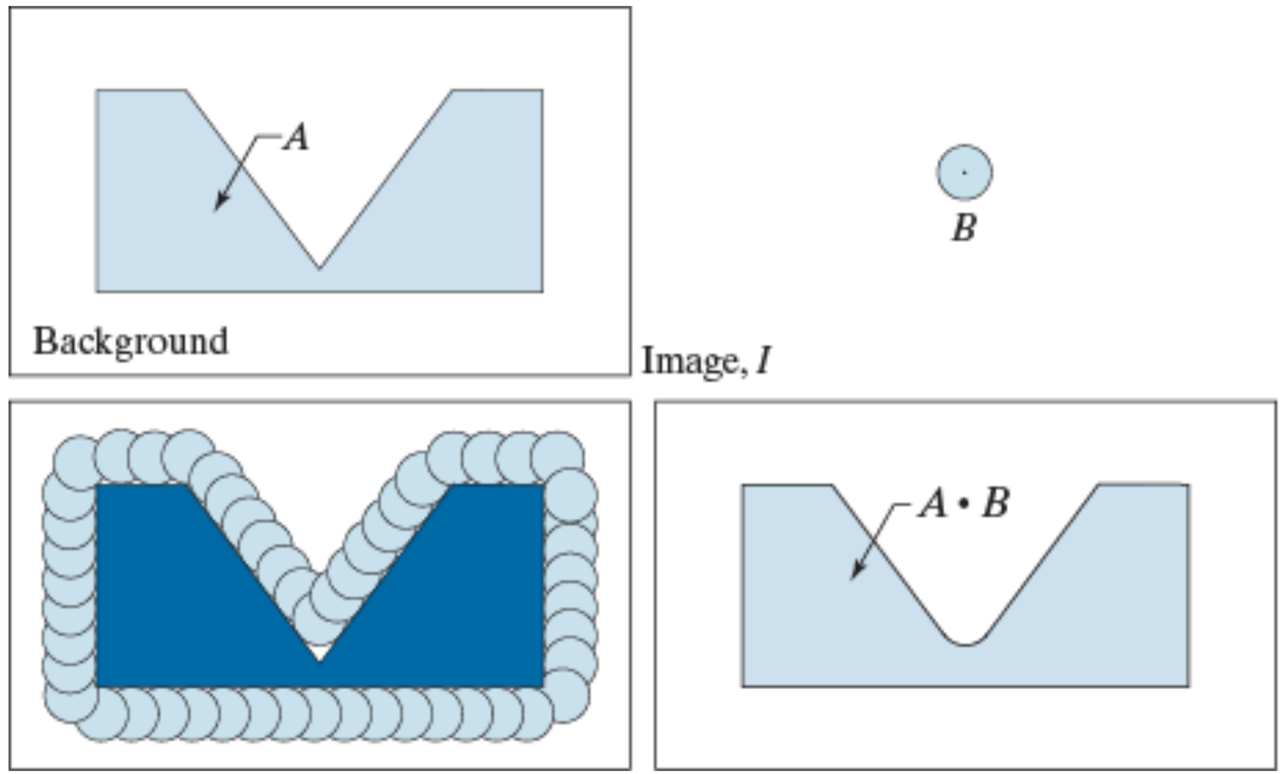
a b
c d

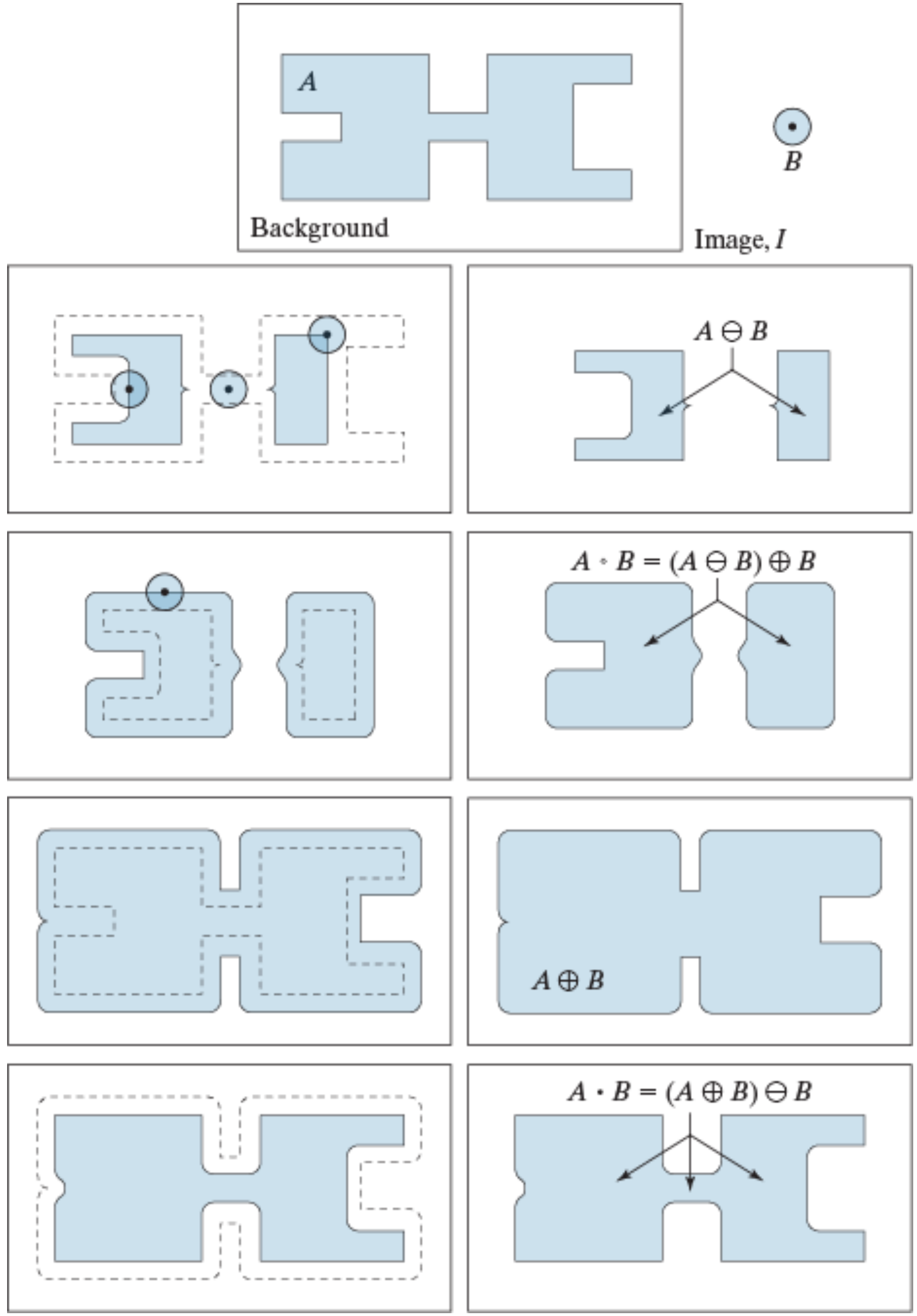
FIGURE 9.8
(a) Image I, composed of set (object) A and background.
(b) Structuring element, B.
(c) Translations of B while being contained in A. (A is shown dark for clarity.)
(d) Opening of A by B.

Example: Closing

a b
c d

FIGURE 9.9
(a) Image I, composed of set (object) A, and background.
(b) Structuring element B.
(c) Translations of B such that B does not overlap any part of A. (A is shown dark for clarity.)
(d) Closing of A by B.





a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. (a) Image *I*, composed of a set (object) *A* and background; a solid, circular structuring element is shown also. (The dot is the origin.) (b) Structuring element in various positions. (c)-(i) The morphological operations used to obtain the opening and closing.

Duality of Opening and Closing

- Opening and closing are duals of each other with respect to set complementation and reflection

$$\left(A \bullet B\right)^c = (A^c \circ \hat{B})$$

$$\left(A \circ B\right)^c = (A^c \bullet \hat{B})$$

The Properties of Opening and Closing

- Properties of Opening

(a) $A \circ B$ is a subset (subimage) of A

(b) if C is a subset of D, then $C \circ B$ is a subset of $D \circ B$

(c) $(A \circ B) \circ B = A \circ B$

- Properties of Closing

(a) A is subset (subimage) of $A \bullet B$

(b) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$

(c) $(A \bullet B) \bullet B = A \bullet B$

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

Hit-or-Miss Transformation

The Hit-or-Miss Transformation

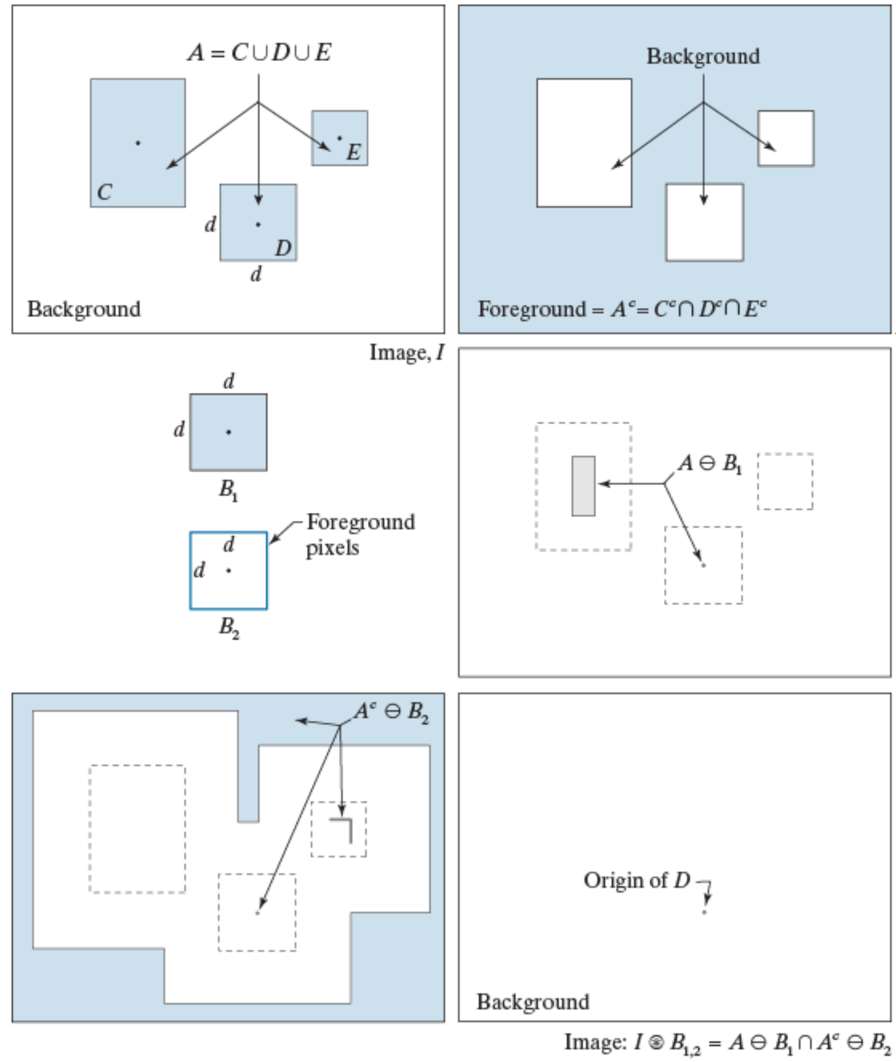


FIGURE 9.12 (a) Image consisting of a foreground (1's) equal to the union, A, of set of objects, and a background of 0's. (b) Image with its foreground defined as A^c. (c) Structuring elements designed to detect object D. (d) Erosion of A by B_1. (e) Erosion of A^c by B_2. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origin of their respective components. Each dot is a single pixel.

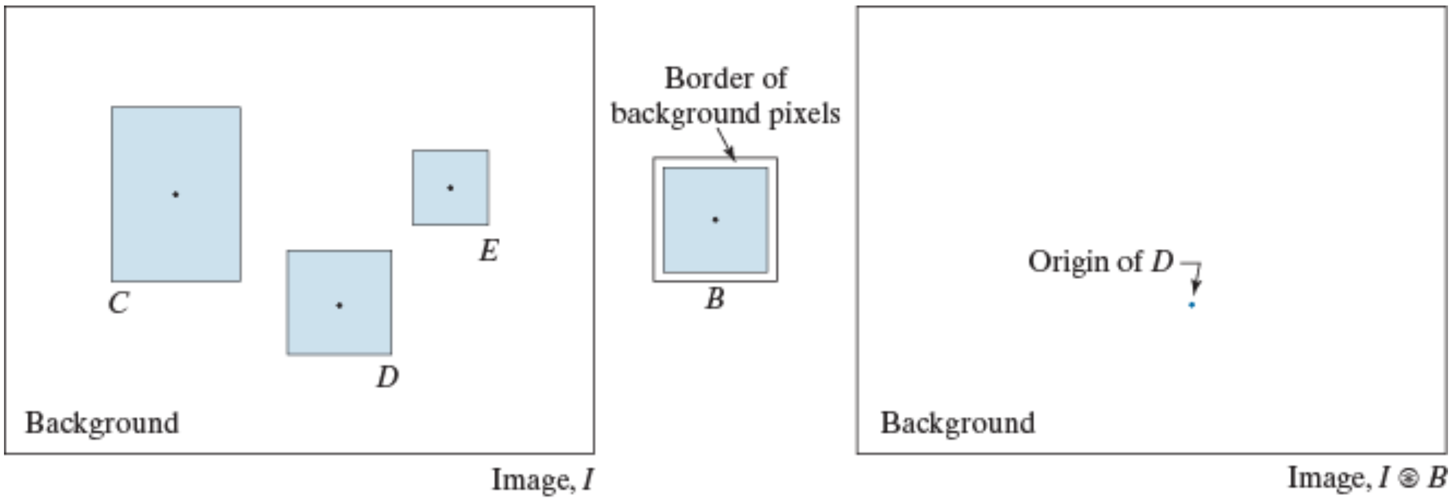
$$B = \left(B_1, B_2\right)$$

B_1 : object

B_2 : background

$$A \circledast B = \left(A \ominus B_1\right) \cap \left(A^c \ominus B_2\right)$$

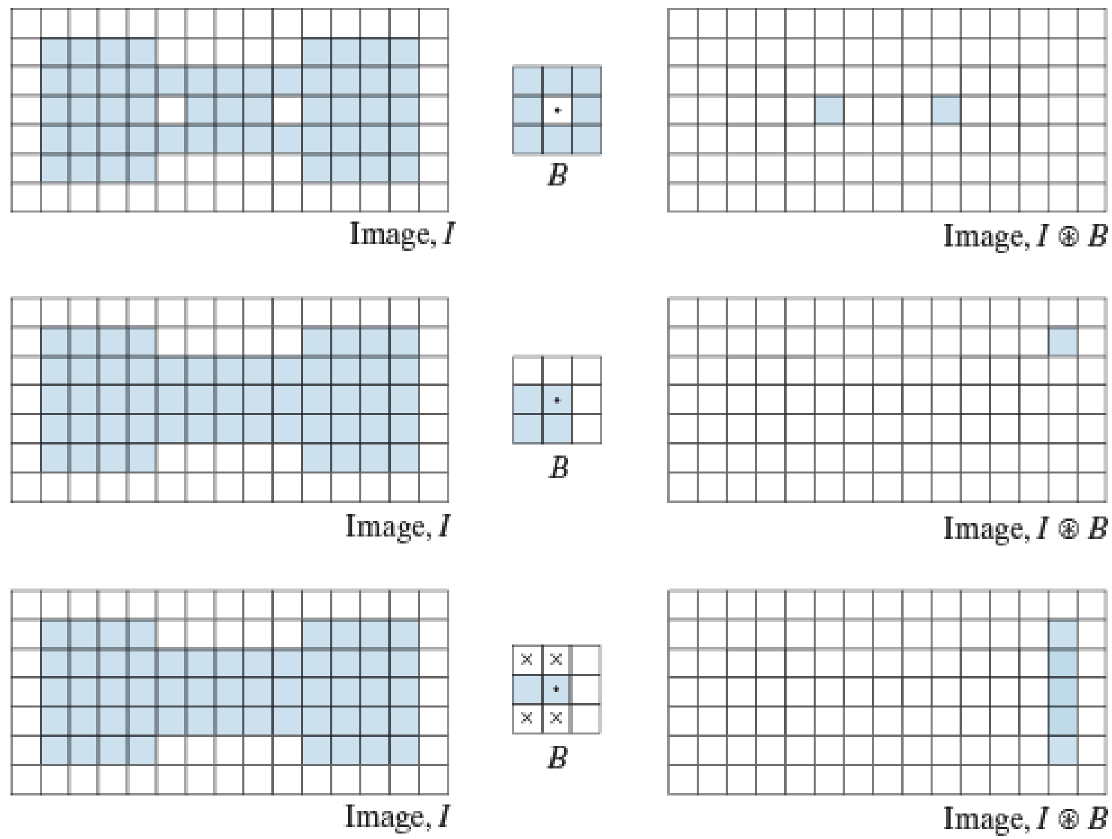
The Hit-or-Miss Transformation



a b c

FIGURE 9.13 Same solution as in Fig. 9.12, but using Eq. (9-17) with a single structuring element.

The Hit-or-Miss Transformation



a b c
d e f
g h i

FIGURE 9.14 Three examples of using a single structuring element and Eq. (9-17) to detect specific features. First row: detection of single-pixel holes. Second row: detection of an upper-right corner. Third row: detection of multiple features.

Basic Morphological Algorithms

Some Basic Morphological Algorithms (1)

- **Boundary Extraction**

  The boundary of a set A, can be obtained by first eroding A by B and then performing the set difference between A and its erosion.

$$\beta(A) = A - \left(A \ominus B \right)$$

Example 1

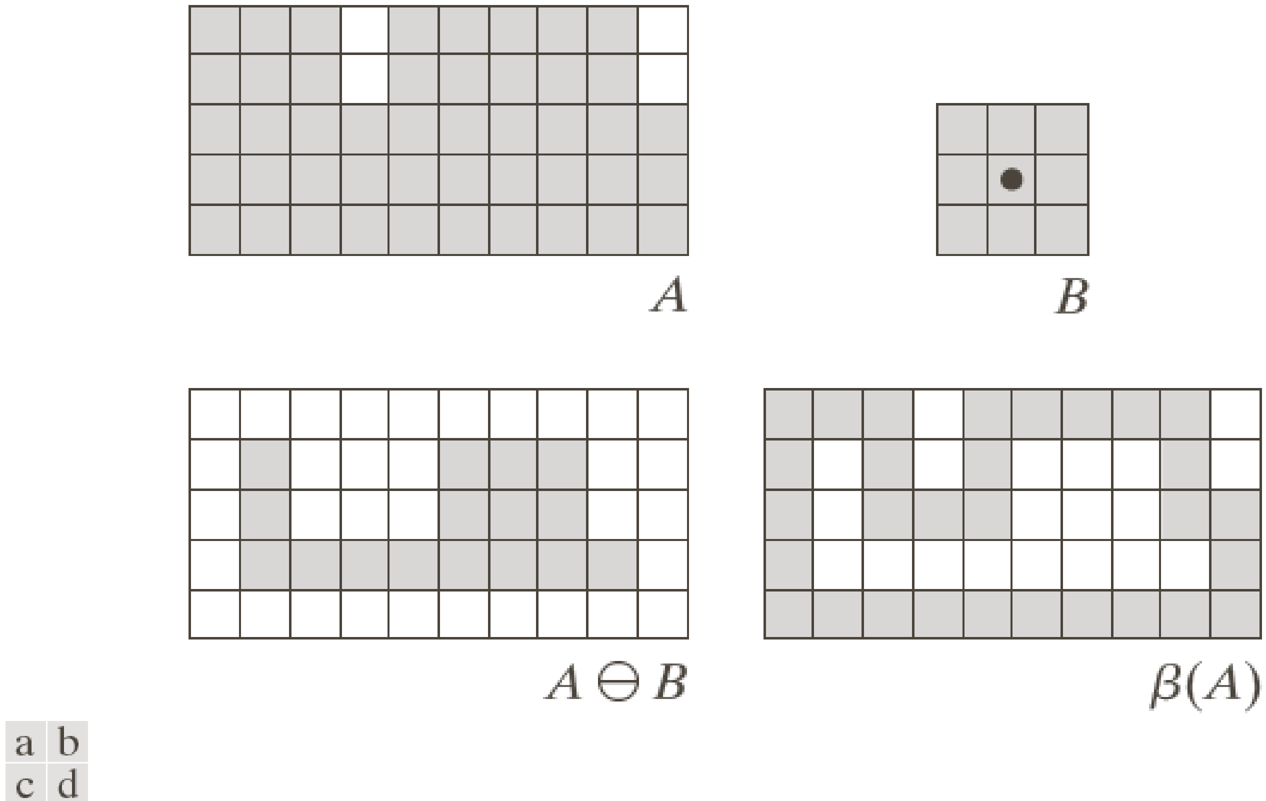


FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

Example 2

FIGURE 9.14 (a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Some Basic Morphological Algorithms (2)

- **Hole Filling**

  A hole may be defined as a background region surrounded by a connected border of foreground pixels.

  Let A denote a set whose elements are 8-connected boundaries, each boundary enclosing a background region (i.e., a hole). Given a point in each hole, the objective is to fill all the holes with 1s.

Some Basic Morphological Algorithms (2)

- **Hole Filling**

  1. Forming an array X_0 of 0s (the same size as the array containing A), except the locations in X_0 corresponding to the given point in each hole, which we set to 1.

  2. $X_k = (X_{k-1} \oplus B) \cap A^c \quad k=1,2,3,\ldots$

  Stop the iteration if $X_k = X_{k-1}$

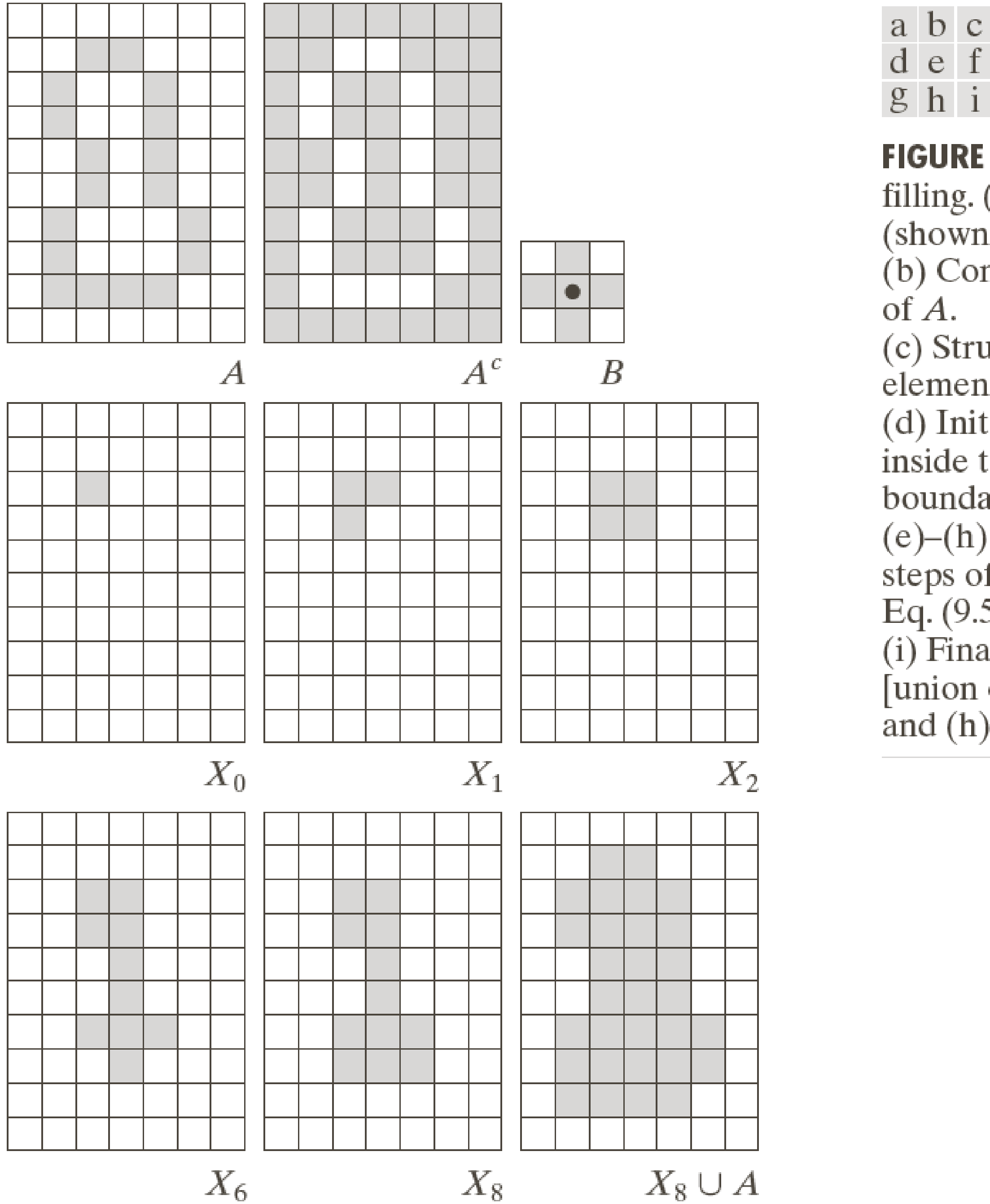
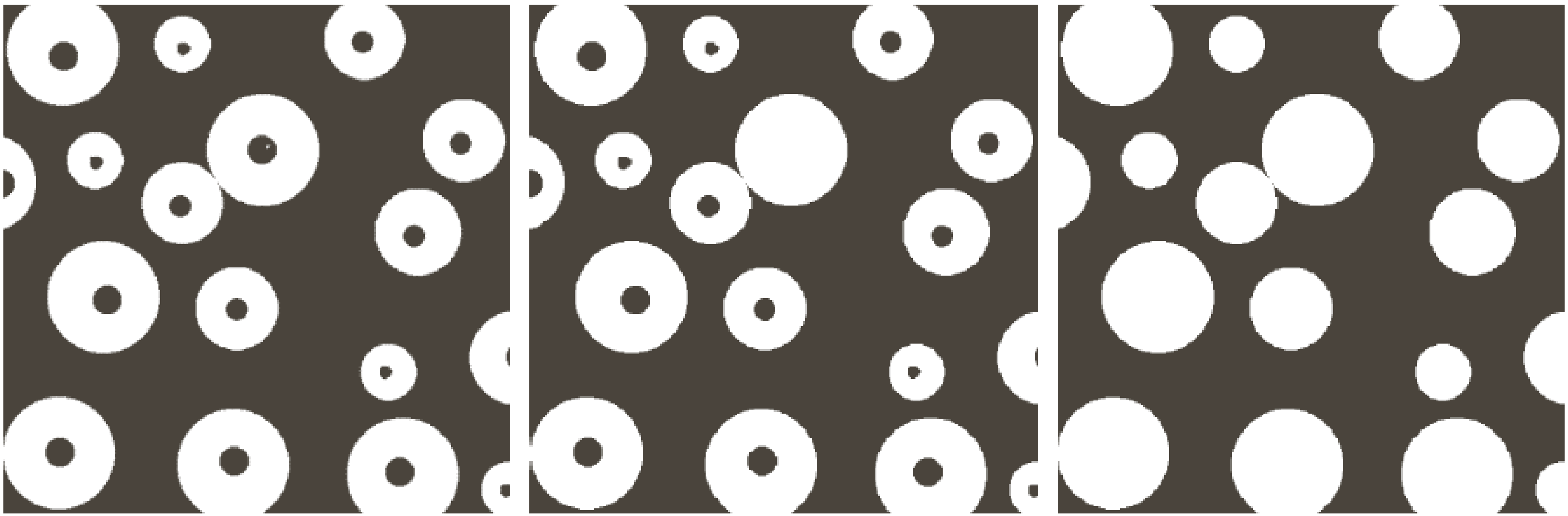


FIGURE 9.15 Hole filling. (a) Set A (shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.

Some Basic Morphological Algorithms (3)

- **Extraction of Connected Components**

  Central to many automated image analysis applications.

  Let A be a set containing one or more connected components, and form an array X_0 (of the same size as the array containing A) whose elements are 0s, except at each location known to correspond to a point in each connected component in A, which is set to 1.

Some Basic Morphological Algorithms (3)

- **Extraction of Connected Components**

  Central to many automated image analysis applications.

$$X_k = (X_{k-1} \oplus B) \cap A$$

$$B : \text{structuring element}$$

$$\text{until } X_k = X_{k\text{-}1}$$

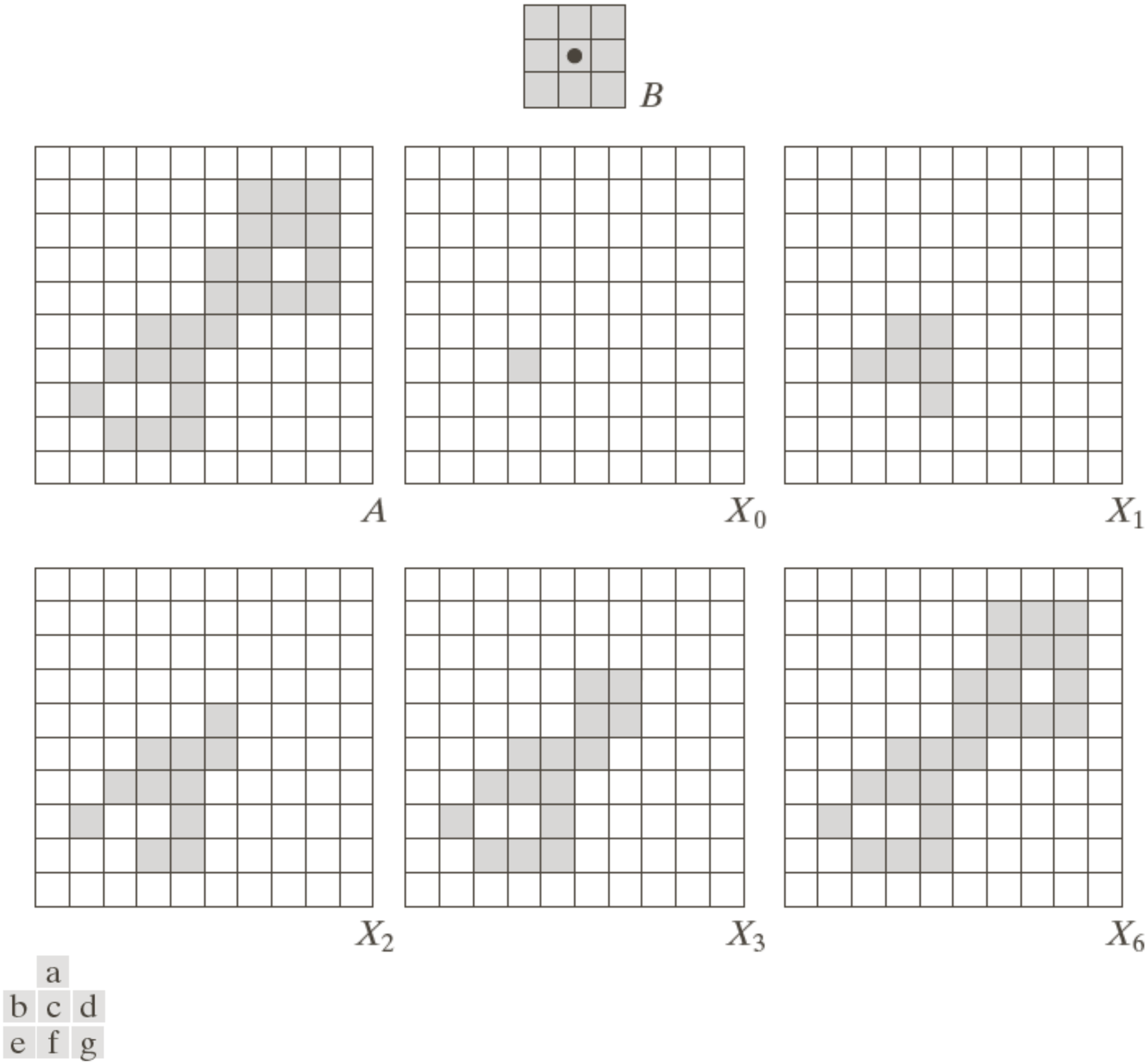
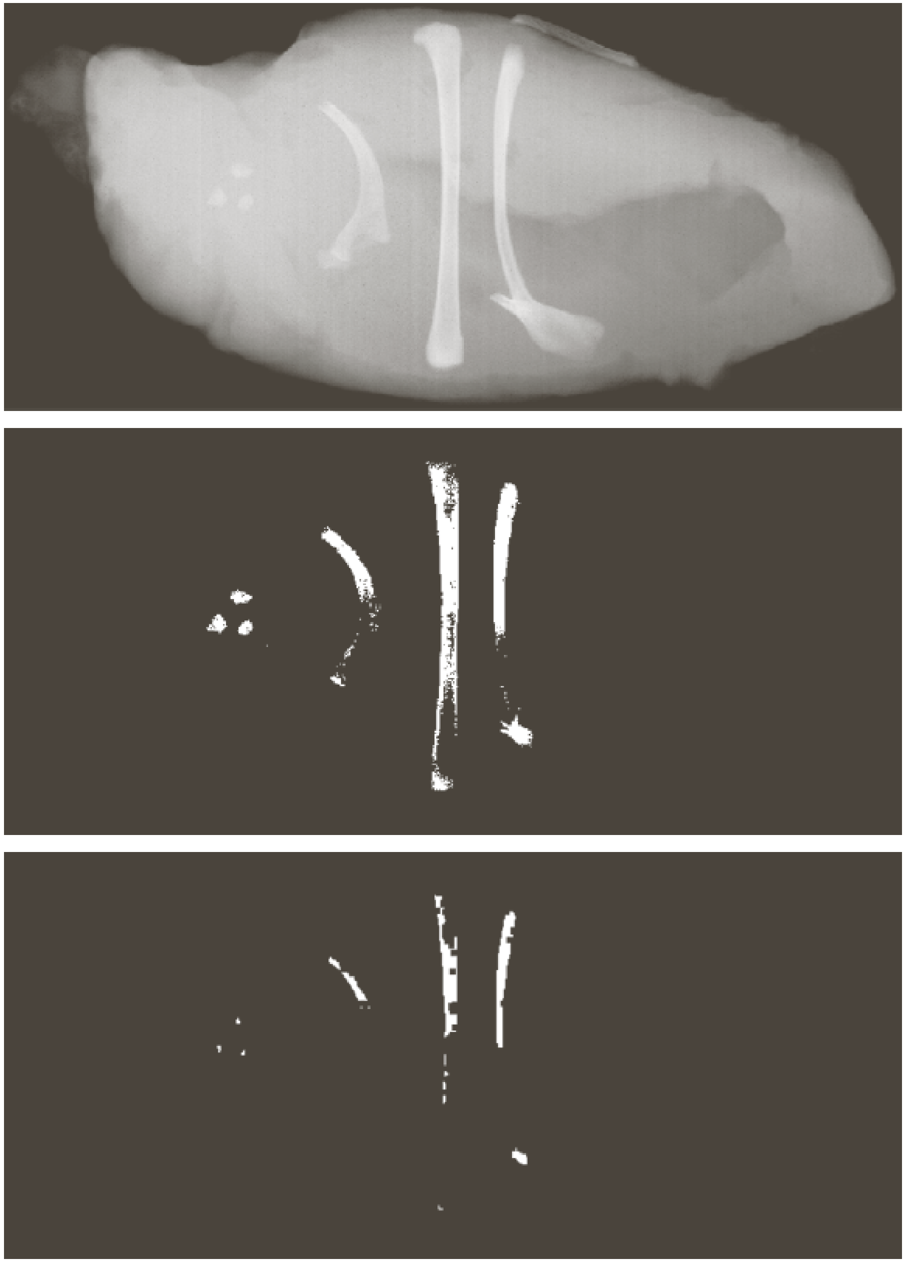


FIGURE 9.17 Extracting connected components. (a) Structuring element. (b) Array containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9.5-3).



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1s. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Some Basic Morphological Algorithms (4)

- **Convex Hull**

  A set A is said to be ***convex*** if the straight line segment joining any two points in A lies entirely within A.

  The ***convex hull*** H or of an arbitrary set S is the smallest convex set containing S.

Some Basic Morphological Algorithms (4)

- **Convex Hull**

  Let B^i, $i = 1, 2, 3, 4$, represent the four structuring elements.

  The procedure consists of implementing the equation:

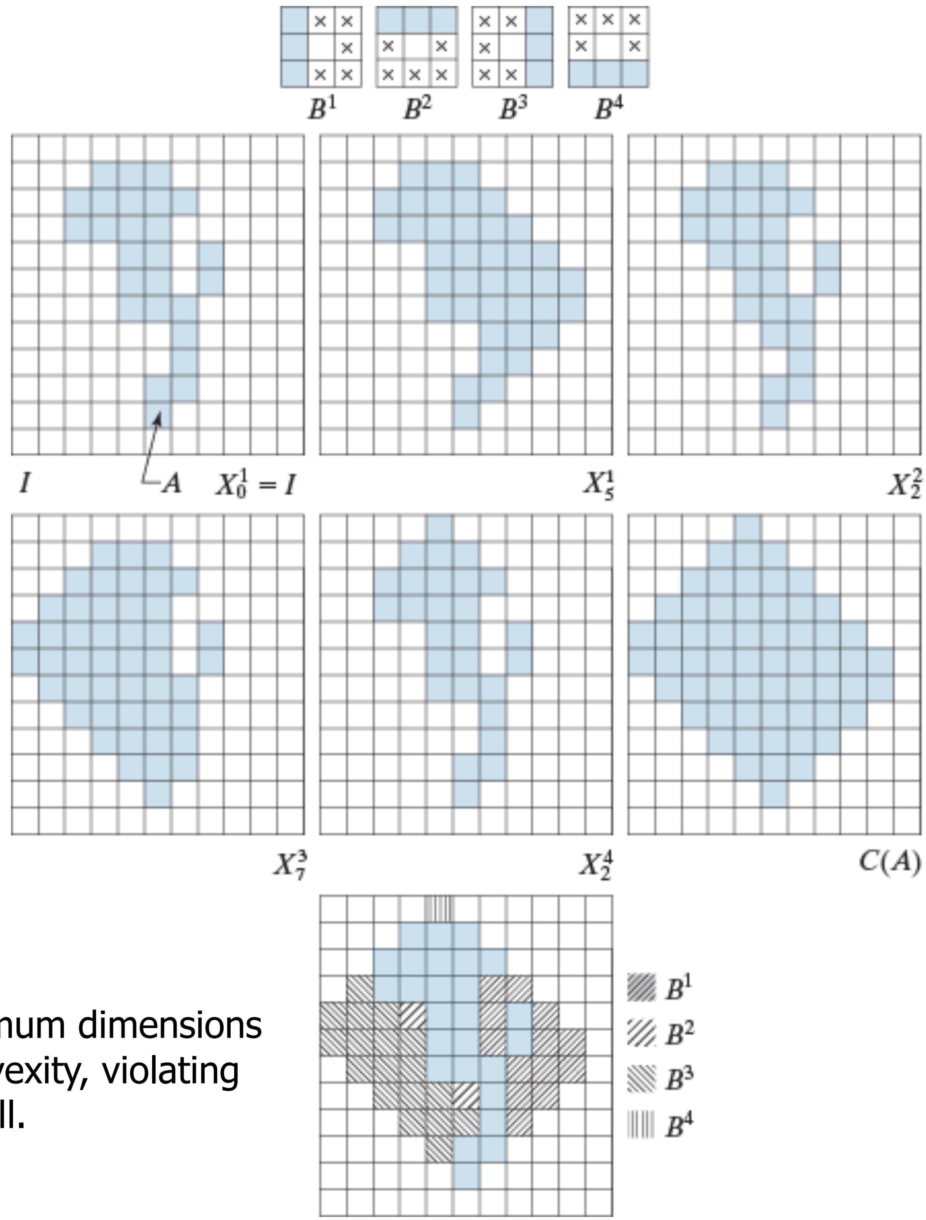
$$X_k^i = (X_{k-1} \circledast B^i) \cup A$$
$$i = 1, 2, 3, 4 \text{ and } k = 1, 2, 3, \ldots$$

  with $X_0^i = A$.

  When the procedure converges, or $X_k^i = X_{k-1}^i$, let $D^i = X_k^i$,

  the convex hull of A is

$$C(A) = \bigcup_{i=1}^{4} D^i$$

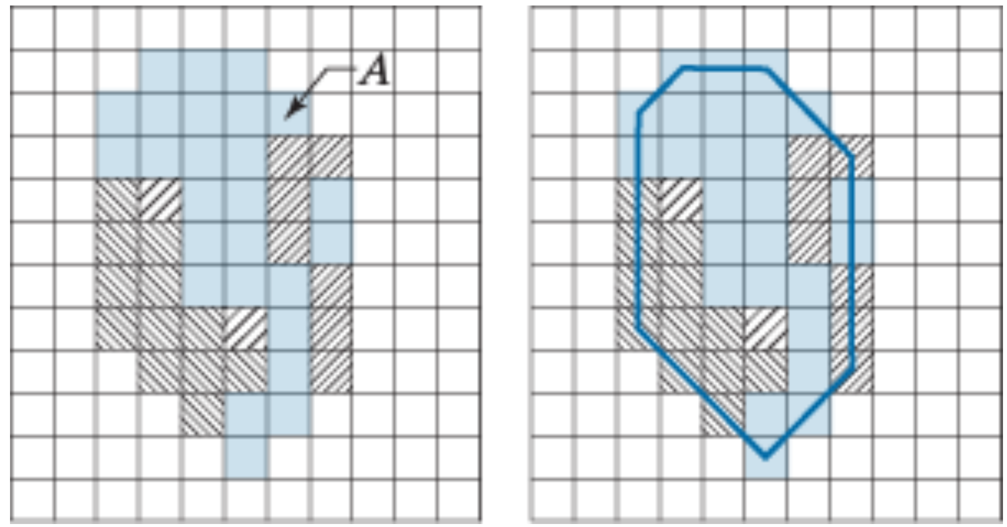


a
b c d
e f g
h

FIGURE 9.21
(a) Structuring elements.
(b) Set A.
(c)–(f) Results of convergence with the structuring elements shown in (a).
(g) Convex hull.
(h) Convex hull showing the contribution of each structuring element.

Shape grows beyond minimum dimensions required to guarantee convexity, violating the definition of convex hull.

Convex Hull



a b

FIGURE 9.22
(a) Result of limiting growth of the convex hull algorithm.
(b) Straight lines connecting the boundary points show that the new set is convex also.

Solution: Place limitation that grow can not be beyond horizontal and vertical dimensions of set A.

Some Basic Morphological Algorithms (5)

- Thinning

  The thinning of a set A by a structuring element B, defined

$$A \otimes B = A - (A \circledast B)$$

$$= A \cap (A \circledast B)^c$$

Some Basic Morphological Algorithms (5)

- A more useful expression for thinning A symmetrically is based on a sequence of structuring elements:

$$\{B\} = \{B^1, B^2, B^3, ..., B^n\}$$

where B^i is a rotated version of B^{i-1}

The thinning of A by a sequence of structuring element $\{B\}$

$$A \otimes \{B\} = ((...((A \otimes B^1) \otimes B^2)...) \otimes B^n)$$

Thinning

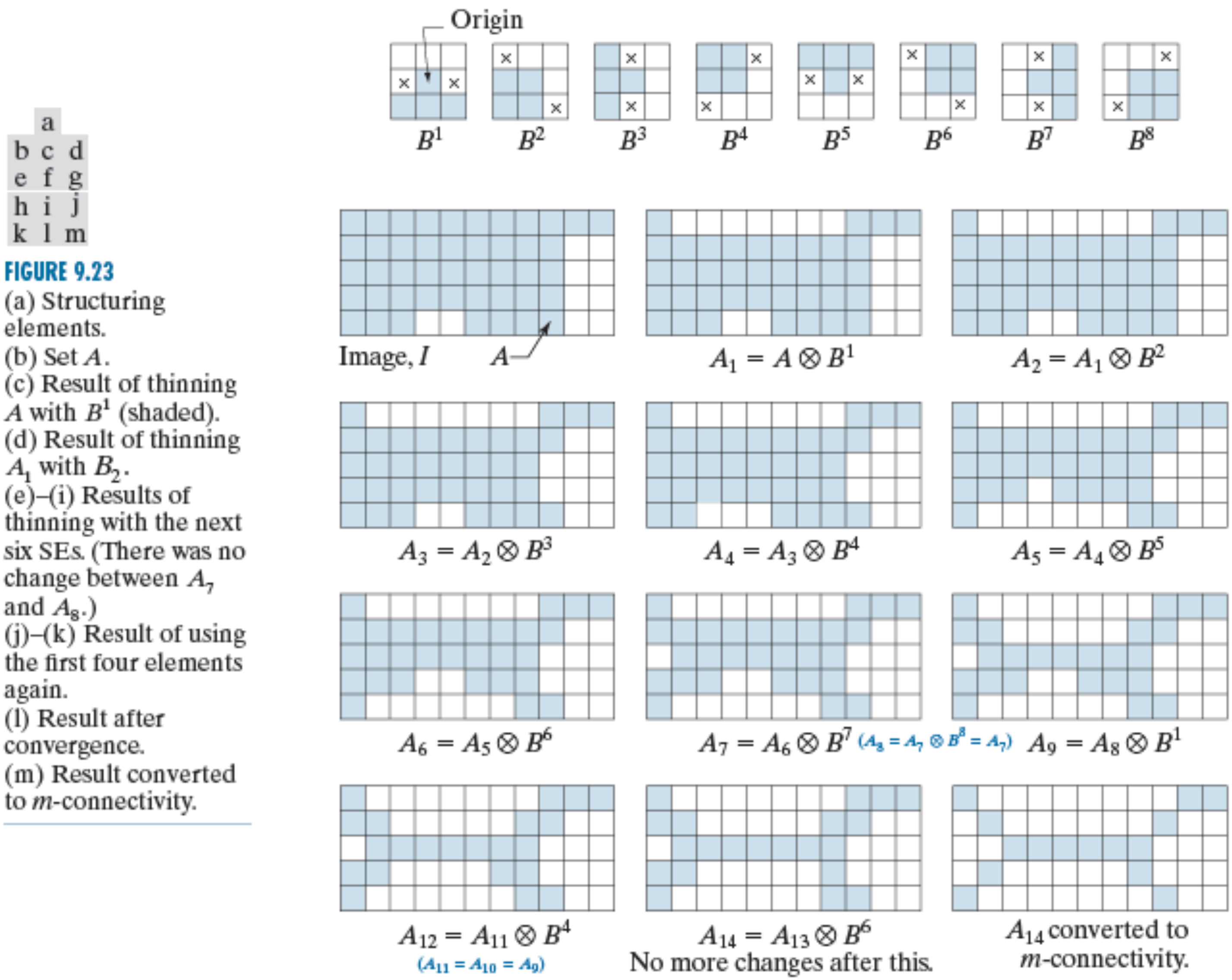


FIGURE 9.23 (a) Structuring elements. (b) Set A. (c) Result of thinning A with B^1 (shaded). (d) Result of thinning A_1 with B_2. (e)–(i) Results of thinning with the next six SEs. (There was no change between A_7 and A_8.) (j)–(k) Result of using the first four elements again. (l) Result after convergence. (m) Result converted to m-connectivity.

Some Basic Morphological Algorithms (6)

- Thickening:

The thickening is defined by the expression

$$A \odot B = A \cup \left(A \circledast B \right)$$

The thickening of A by a sequence of structuring element $\{B\}$

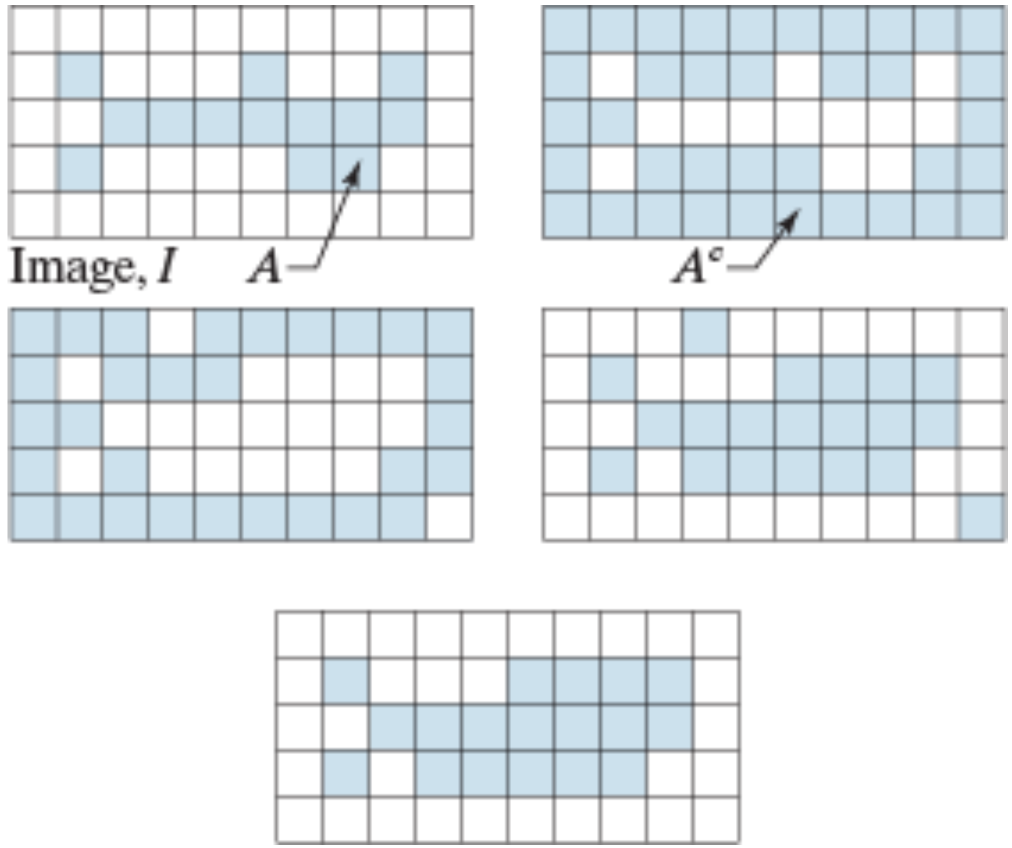
$$A \odot \{B\} = ((...((A \odot B^1) \odot B^2)...) \odot B^n)$$

In practice, the usual procedure is to thin the background of the set and then complement the result.

Some Basic Morphological Algorithms (6)

a b
c d
e

FIGURE 9.24
(a) Set A.
(b) Complement of A.
(c) Result of thinning the complement.
(d) Thickened set obtained by complementing (c).
(e) Final result, with no disconnected points.



Some Basic Morphological Algorithms (7)

- Skeletons

A skeleton, $S(A)$ of a set A has the following properties

a. if z is a point of $S(A)$ and $(D)_z$ is the largest disk centered at z and contained in A, one cannot find a larger disk containing $(D)_z$ and included in A. The disk $(D)_z$ is called a maximum disk.

b. The disk $(D)_z$ touches the boundary of A at two or more different places.

Some Basic Morphological Algorithms (7)

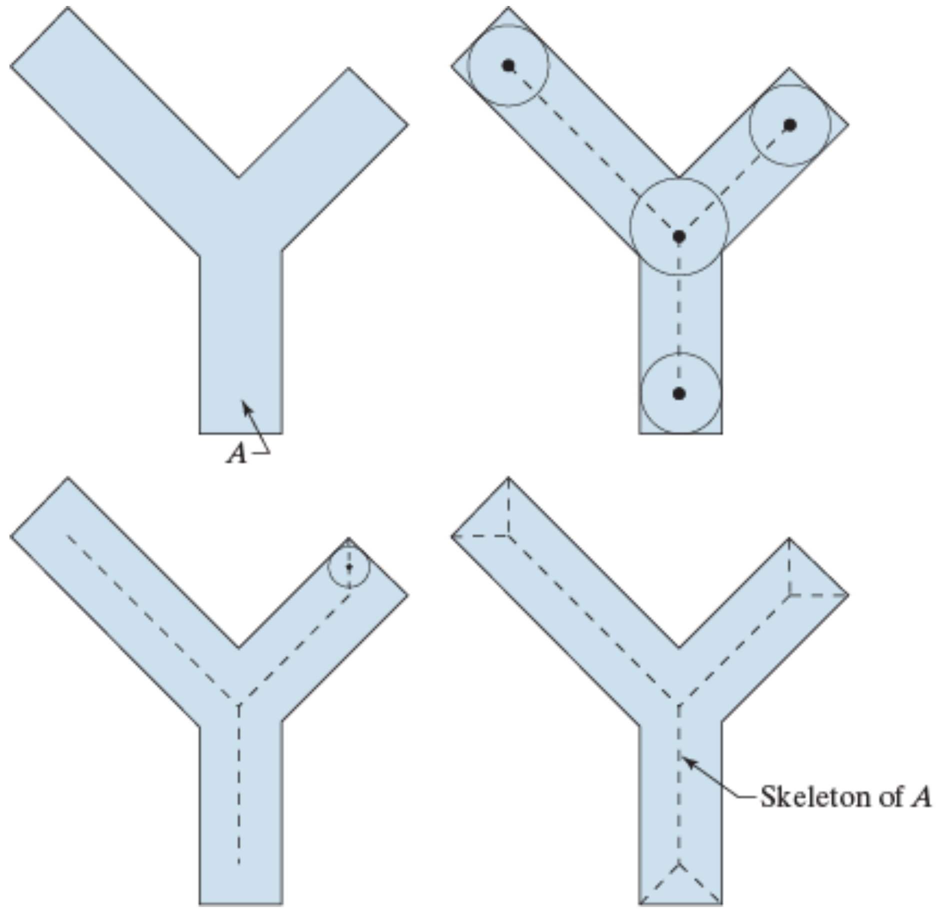


FIGURE 9.25
(a) Set A.
(b) Various positions of maximum disks whose centers partially define the skeleton of A.
(c) Another maximum disk, whose center defines a different segment of the skeleton of A.
(d) Complete skeleton (dashed).

Some Basic Morphological Algorithms (7)

The skeleton of A can be expressed in terms of erosion and openings.

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

with $K = \max\{k \mid A \ominus kB \neq \phi\}$;

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

where B is a structuring element, and

$$(A \ominus kB) = ((..((A \ominus B) \ominus B) \ominus ...) \ominus B)$$

k successive erosions of A.

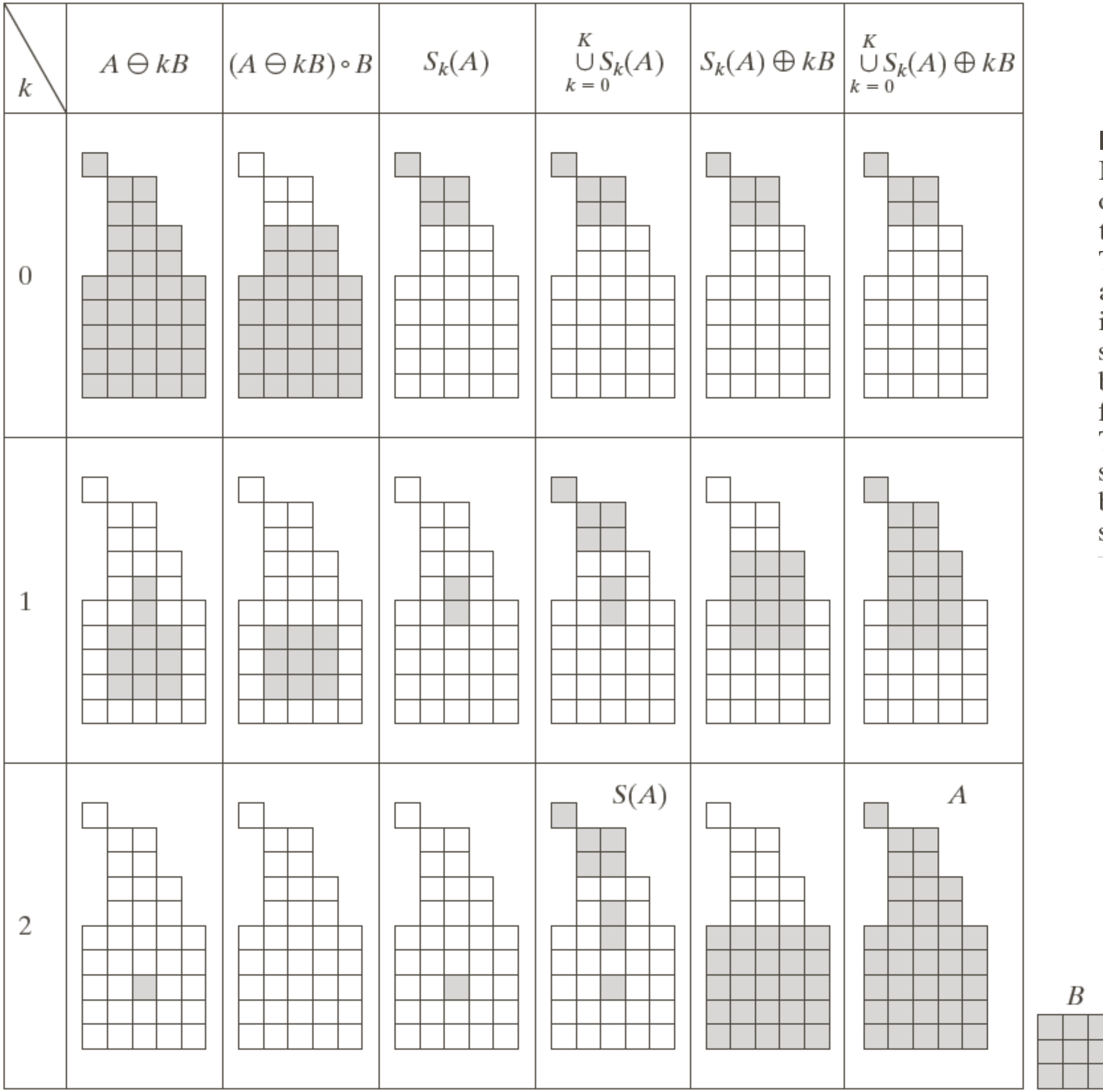


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

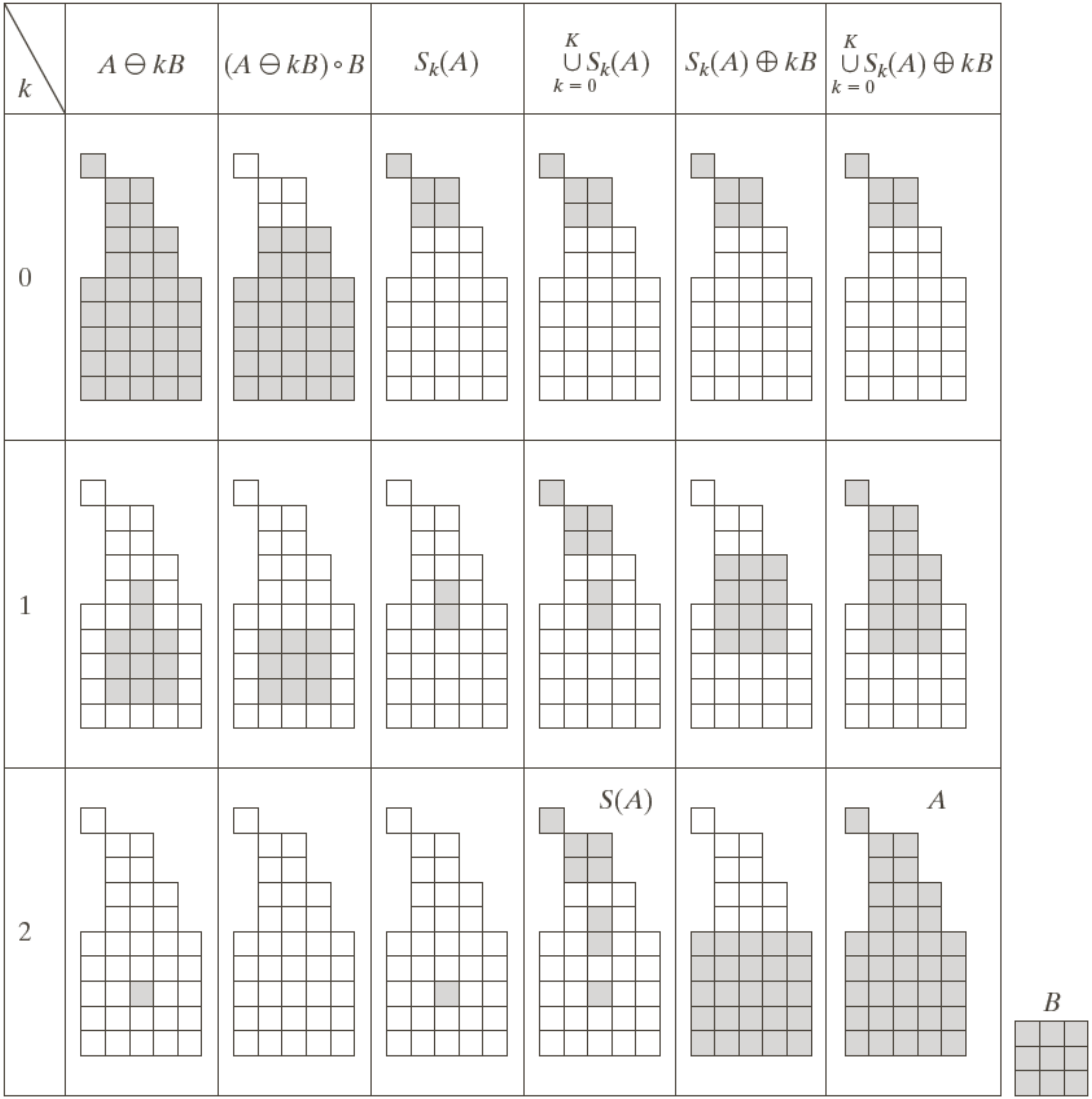


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

Some Basic Morphological Algorithms (7)

A can be reconstructed from the subsets by using

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

where $S_k(A) \oplus kB$ denotes k successive dilations of A.

$$(S_k(A) \oplus kB) = ((\ldots((S_k(A) \oplus B) \oplus B)\ldots \oplus B)$$

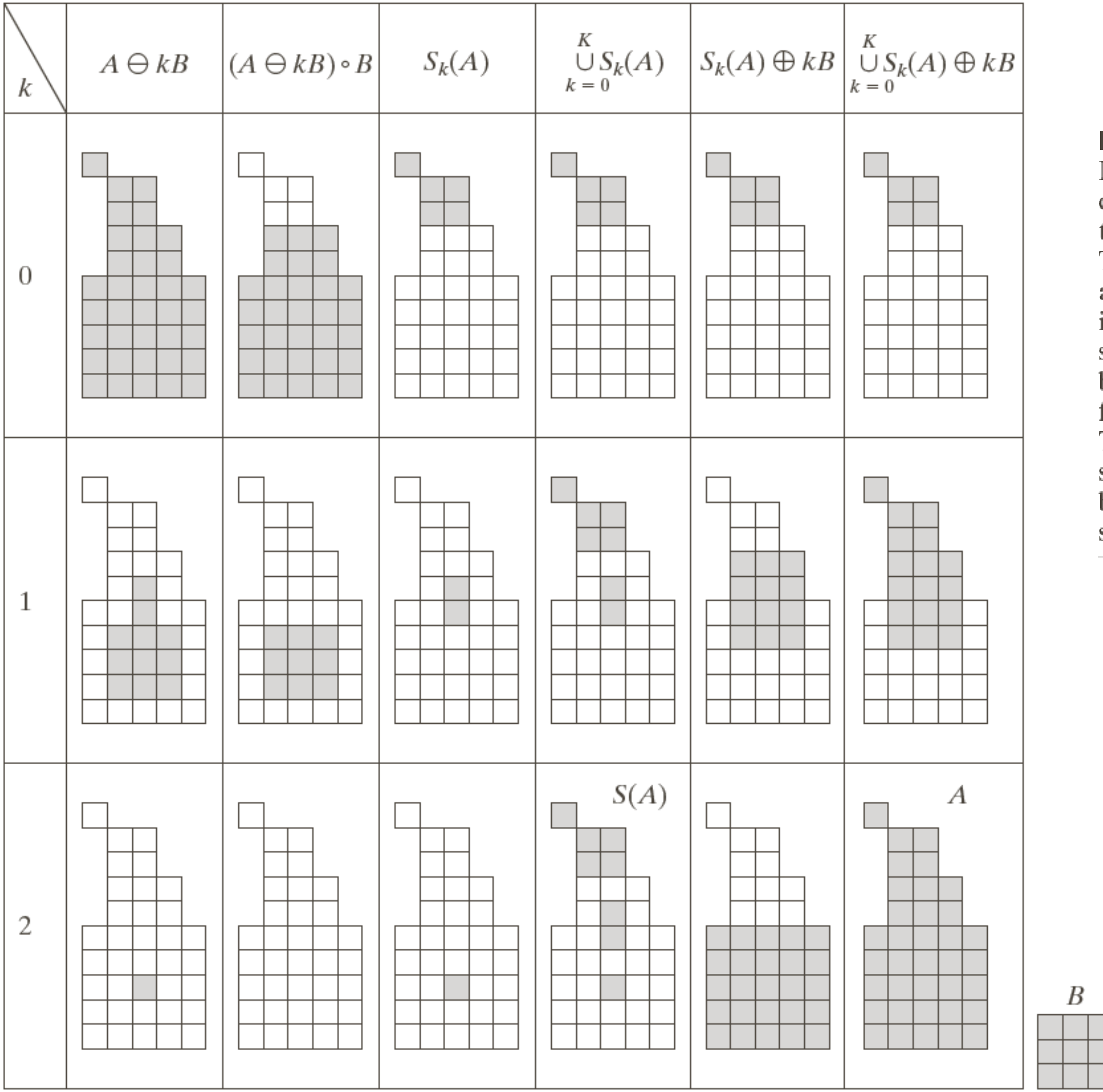


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

Summary of Binary Morphology

Summary (1)

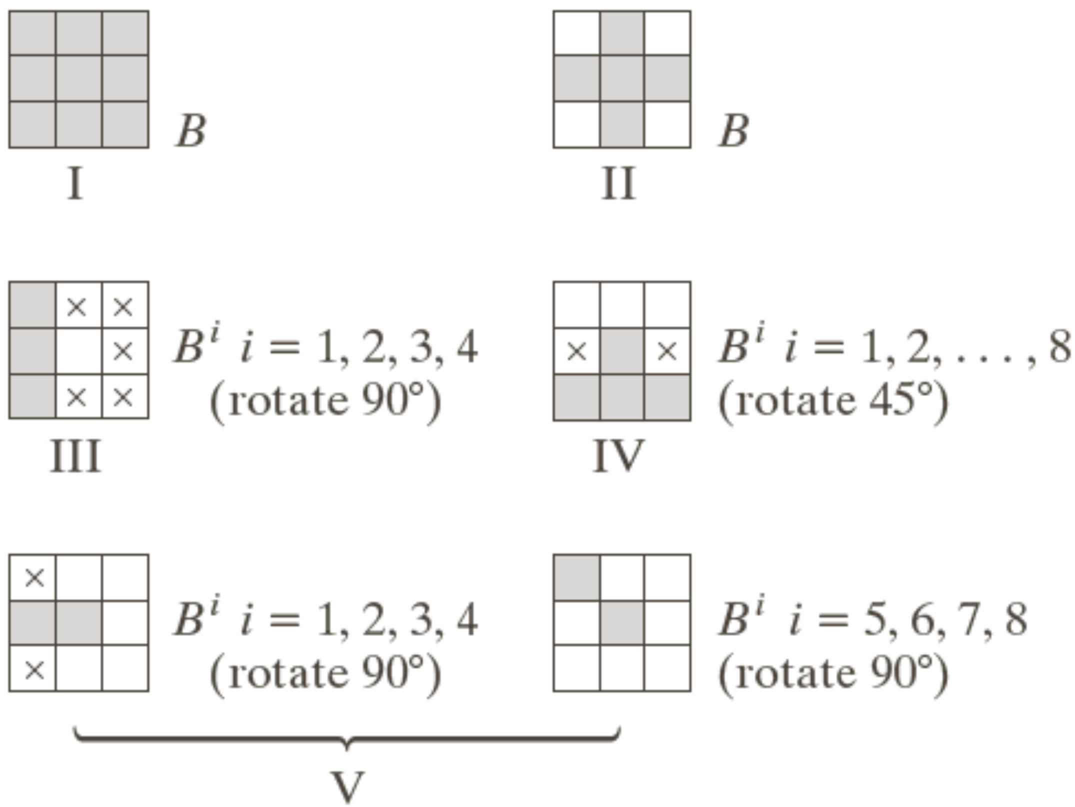


FIGURE 9.33 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

Summary (2)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Translation	$(B)_z = \{w \mid w = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \left\{z \mid (\hat{B}_z) \cap A \neq \varnothing\right\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \left\{z \mid (B)_z \subseteq A\right\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)

(*Continued*)

TABLE 9.1 Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $k = 1, 2, 3, \ldots$	Fills holes in A; X_0 = array of 0s with a 1 in each hole. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $k = 1, 2, 3, \ldots$	Finds connected components in A; X_0 = array of 0s with a 1 in each connected component. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0s and 1s reversed.
Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB)$ $- [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosions of A by B. (I)

(Continued)

TABLE 9.1 *(Continued)*

Grayscale Morphology

Gray-Scale Morphology

$f(x, y)$: gray-scale image

$b(x, y)$: structuring element

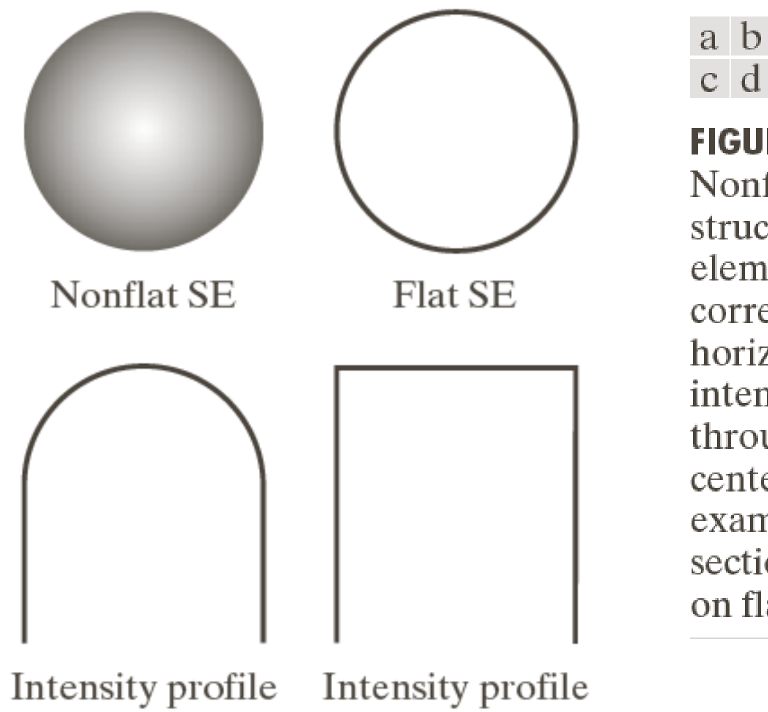
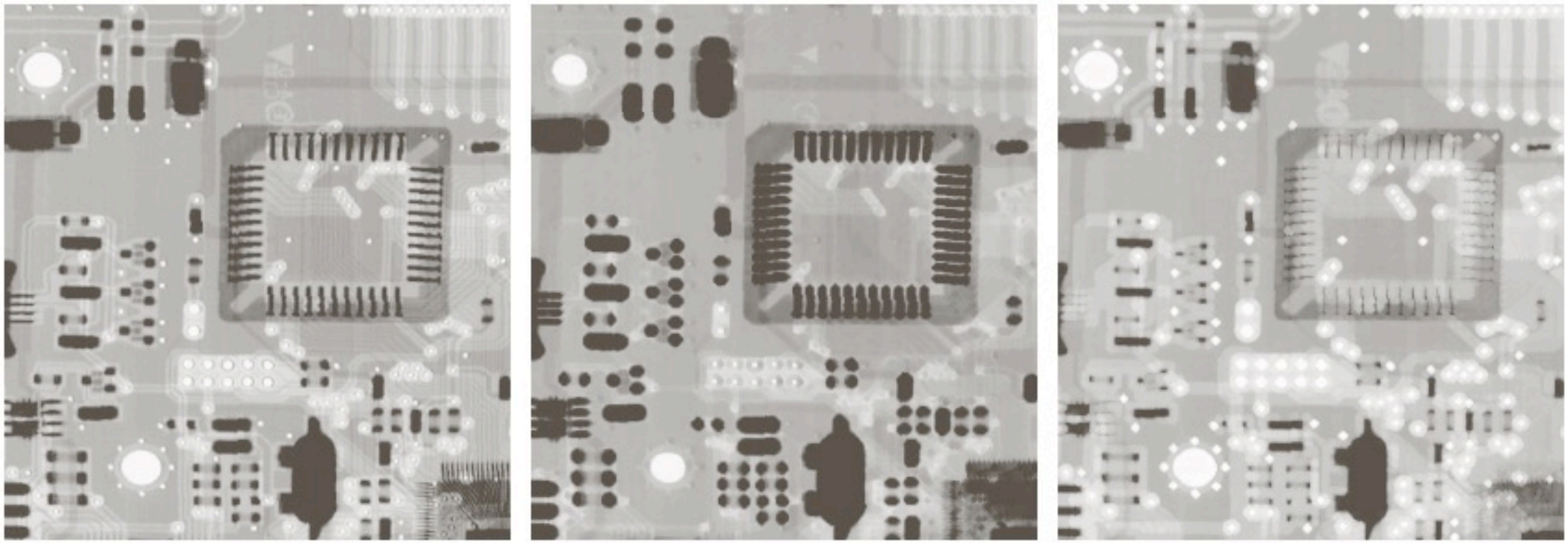


FIGURE 9.34 Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their center. All examples in this section are based on flat SEs.

Gray-Scale Morphology: Erosion and Dilation by Flat Structuring

$$\left[f \ominus b\right](x, y) = \min_{(s,t)\in b}\left\{f(x+s, y+t)\right\}$$

$$\left[f \oplus b\right](x, y) = \max_{(s,t)\in b}\left\{f(x-s, y-t)\right\}$$



a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448 $\times$ 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

Gray-Scale Morphology: Erosion and Dilation by Nonflat Structuring

$$\left[f \ominus b_N \right](x, y) = \min_{(s,t)\in b} \left\{ f(x+s, y+t) - b_N(s,t) \right\}$$

$$\left[f \oplus b_N \right](x, y) = \max_{(s,t)\in b} \left\{ f(x-s, y-t) + b_N(s,t) \right\}$$

Duality: Erosion and Dilation

$$\left[f \ominus b \right]^c (x, y) = \left(f^c \oplus \hat{b} \right)(x, y)$$

where $f^c = -f(x, y)$ and $\hat{b} = b(-x, -y)$

$$\left[f \ominus b \right]^c = \left(f^c \oplus \hat{b} \right)$$

$$\left(f \oplus b \right)^c = (f^c \ominus \hat{b})$$

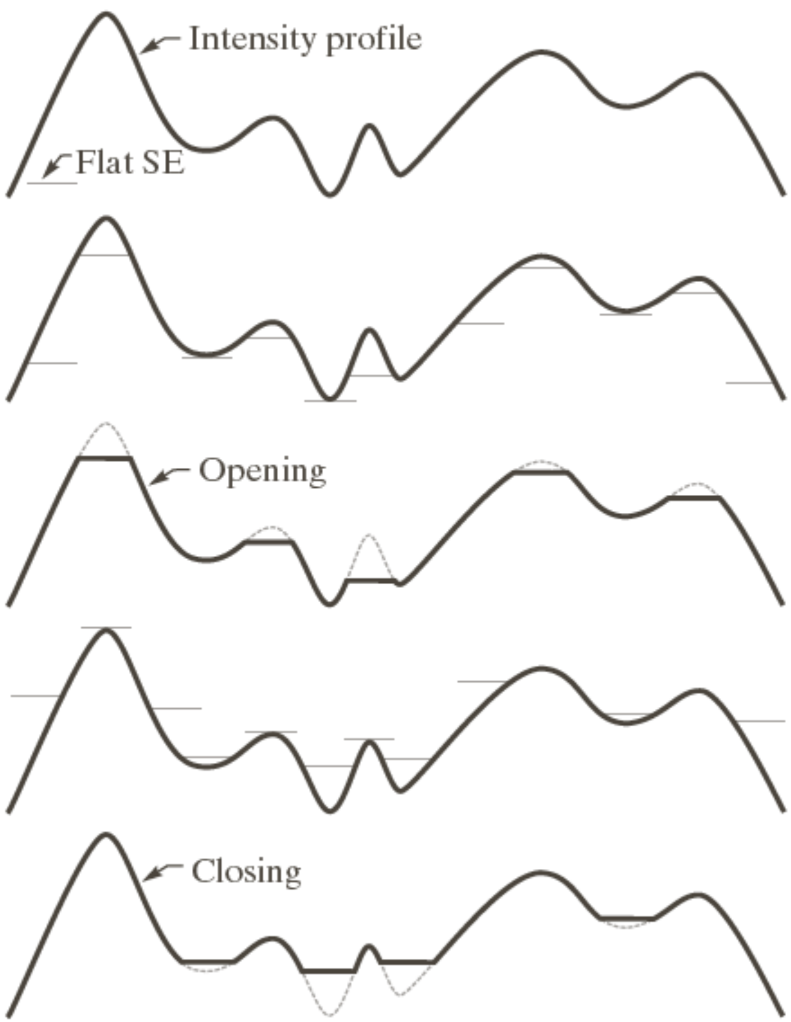
Opening and Closing

$$f \circ b = \left(f \ominus b \right) \oplus b$$

$$f \bullet b = \left(f \oplus b \right) \ominus b$$

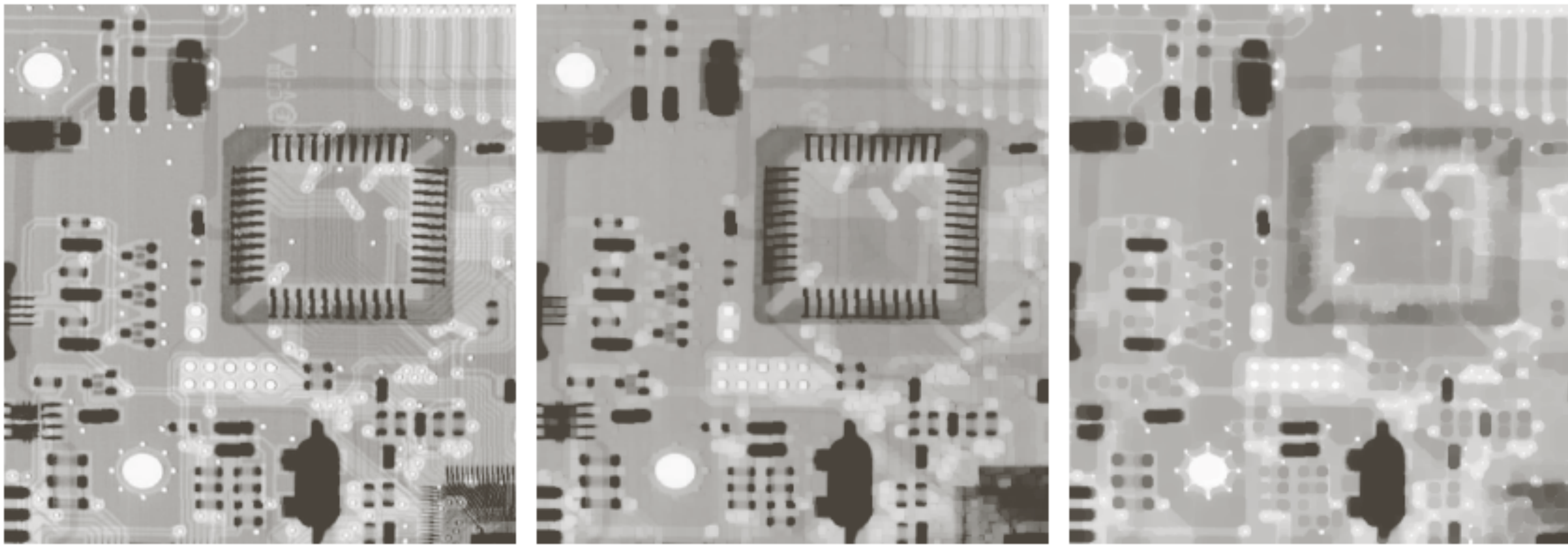
$$\left(f \bullet b \right)^{c} = f^{c} \circ \hat{b} = -f \circ \hat{b}$$

$$\left(f \circ b \right)^{c} = f^{c} \bullet \hat{b} = -f \bullet \hat{b}$$



a
b
c
d
e

FIGURE 9.36 Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal.
(c) Opening.
(d) Flat structuring element pushed down along the top of the signal.
(e) Closing.



a b c

FIGURE 9.37 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.

Morphological Smoothing

- Opening suppresses bright details smaller than the specified SE, and closing suppresses dark details.

- Opening and closing are used often in combination as morphological filters for image smoothing and noise removal.

Morphological Smoothing

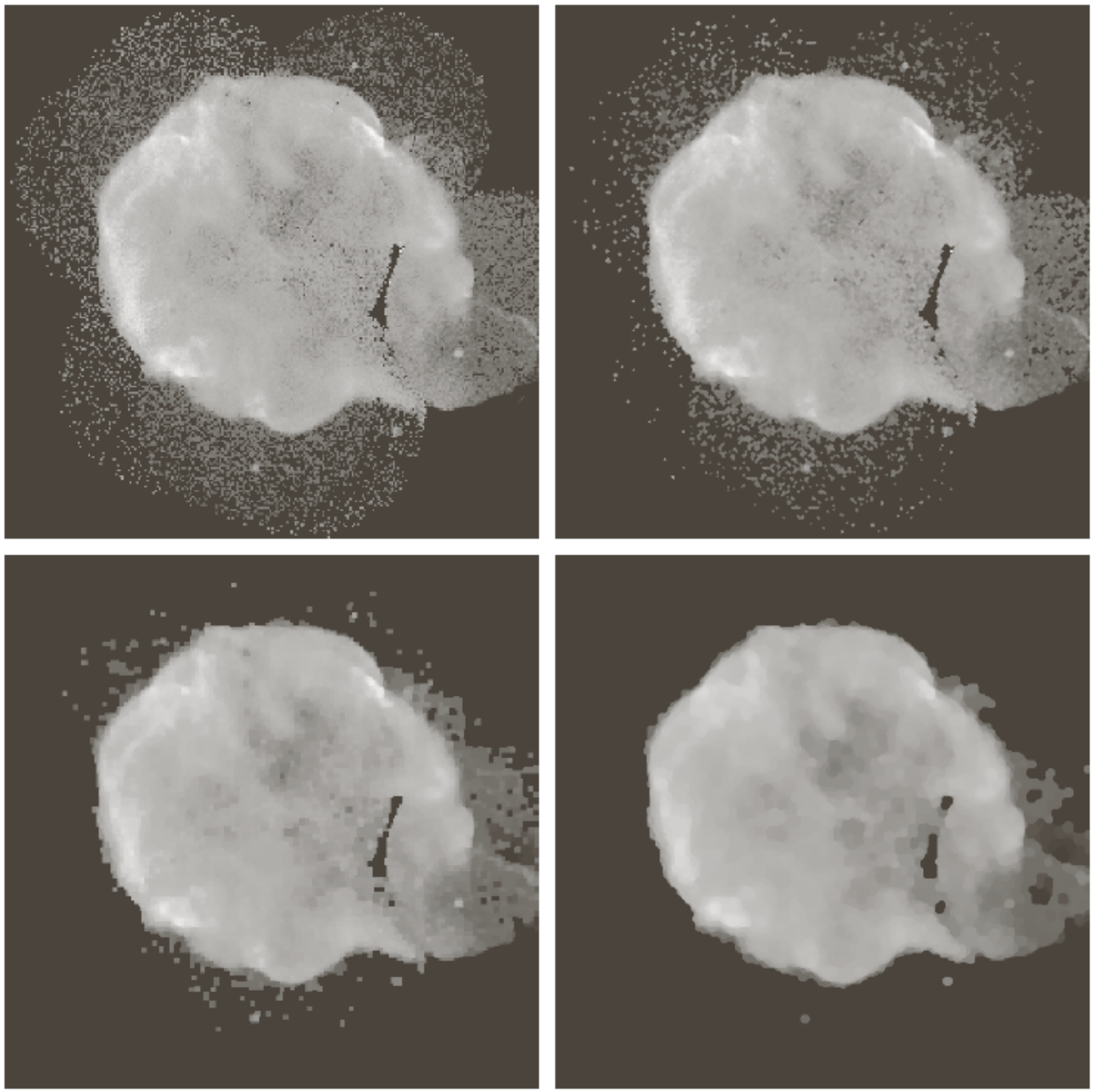


FIGURE 9.38 (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

Morphological Gradient

- Dilation and erosion can be used in combination with image subtraction to obtain the morphological gradient of an image, denoted by g,

$$g = (f \oplus b) - (f \ominus b)$$

- The edges are enhanced and the contribution of the homogeneous areas are suppressed, thus producing a "derivative-like" (gradient) effect.

Morphological Gradient

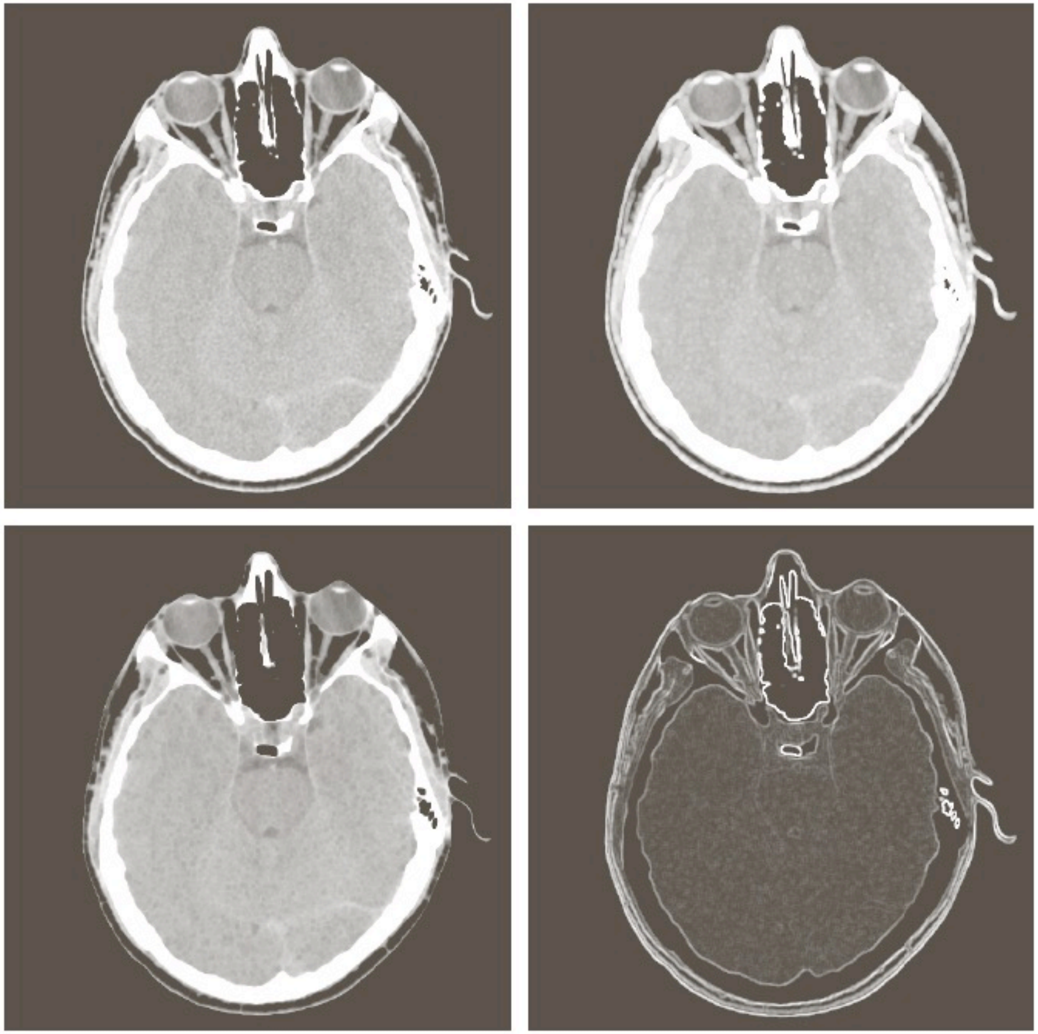


FIGURE 9.39
(a) 512 × 512 image of a head CT scan.
(b) Dilation.
(c) Erosion.
(d) Morphological gradient, computed as the difference between (b) and (c).
(Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

Top-hat and Bottom-hat Transformations

- The top-hat transformation of a grayscale image f is defined as f minus its opening:

$$T_{hat}(f) = f - (f \circ b)$$

- The bottom-hat transformation of a grayscale image f is defined as its closing minus f:

$$B_{hat}(f) = (f \bullet b) - f$$

Top-hat and Bottom-hat Transformations

- One of the principal applications of these transformations is in removing objects from an image by using structuring element in the opening or closing operation
- Top-hat (white top-hat) transform extracts small elements and details from an image. Leaves only details that are smaller than the structuring element. Used to enhance bright objects.
- Bottom-hat transform (black top-hat) is used to do the opposite, i.e. enhance dark objects of interest in a bright background.

Summary

- Morphological image processing is based upon set theory.
- Morphological algorithms are useful for extracting features of interest in a binary image
  - Filtering out large or small regions
  - Finding the boundary of regions
  - Finding connected regions
  - Filling in interior of regions
  - Finding the skeleton of a region
- Morphological algorithms can be extended to grayscale images.

Questions?

Next Time: Chapter 10 Image Segmentation I, Edge Detection, and Region Detection

Slide Credits

Images taken from Digital Image Processing by Gonzalez and Woods Text.

Material taken from Jen-Chang Liu lecture slides