EEE-6512: Image Processing and Computer Vision

September 11, 2018

Lecture #4: Intensity Transformations and Spatial Filtering

Damon L. Woodard, Ph.D.
Dept. of Electrical and Computer Engineering
dwoodard@ece.ufl.edu

Outline

- **Section 3.4:** Fundamentals of Spatial Filtering
- **Section 3.5:** Smoothing (Lowpass) Spatial Filters
- **Section 3.6:** Sharpening (Highpass) Spatial Filters

Fundamentals of Spatial Filtering

Fundamentals of Spatial Filtering

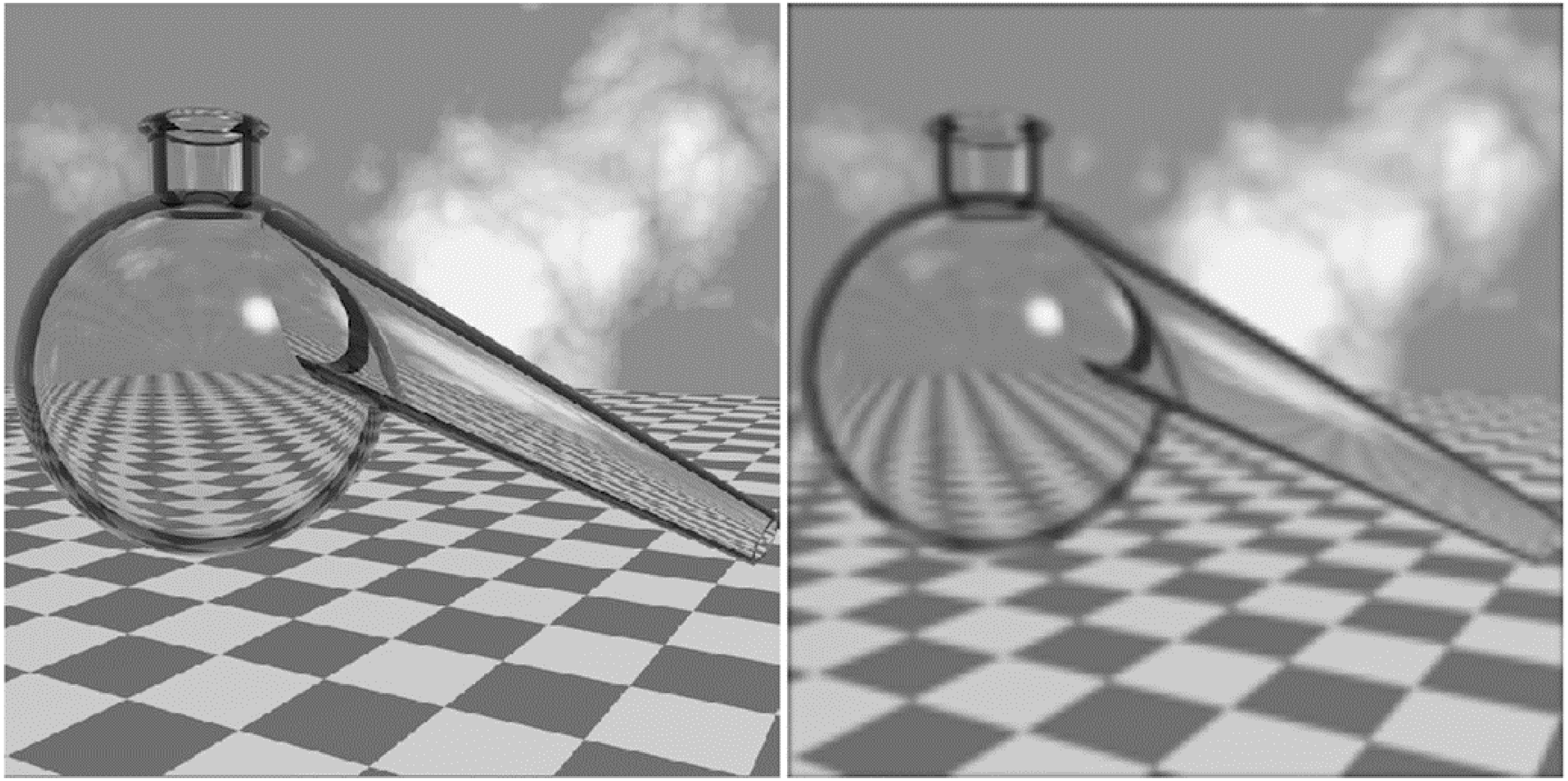
- The name *filter* is borrowed from frequency domain processing where “filtering’ refers to passing, modifying, or rejecting specified frequency components of an image.

- For example, a filter that passes low frequencies of an image is a *low pass filter*. The net effect of a low pass filter is to blur the image.

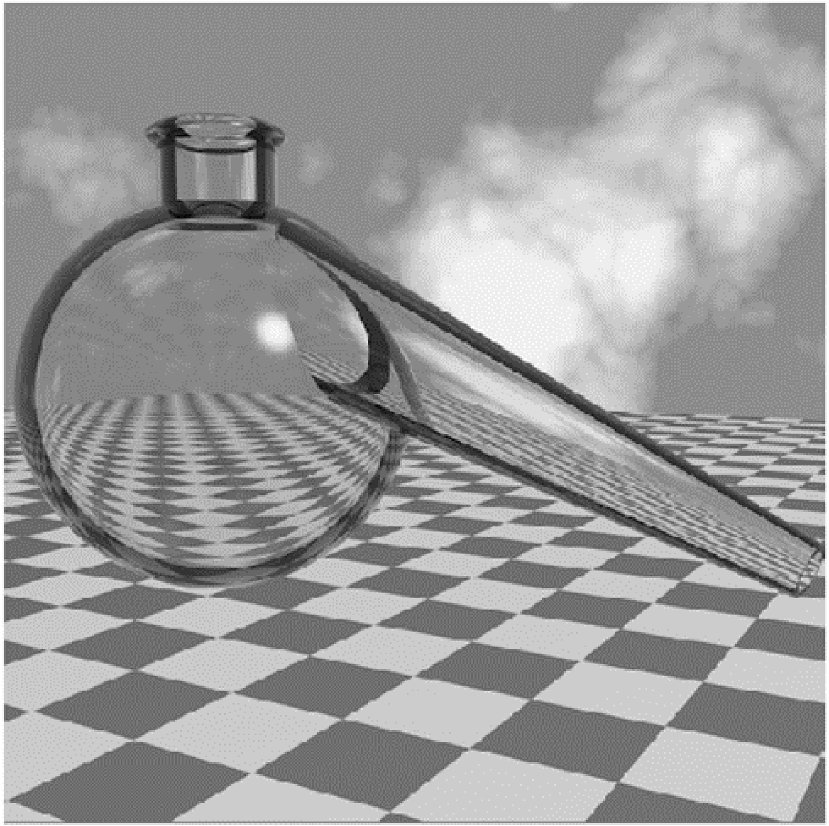
- This can be accomplished similarly by smoothing directly on the image itself using spatial filters.

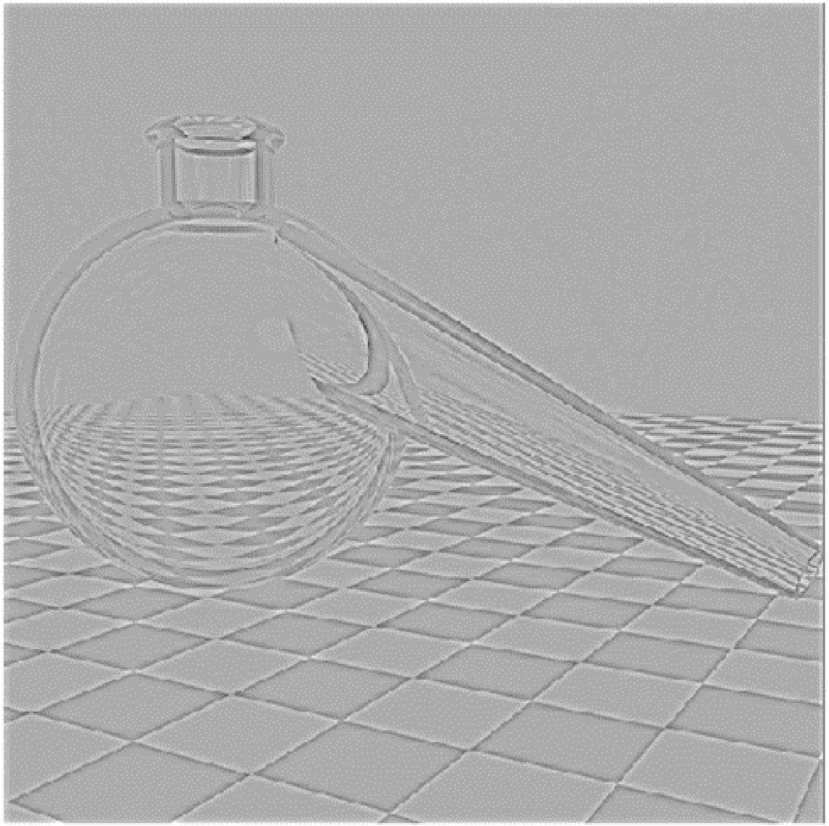
- Spatial filters modify an image by replacing the value of each pixel by a function of the values of the pixel and its neighbors.

Fundamentals of Spatial Filtering
Low-Pass Filtered Image



Fundamentals of Spatial Filtering
High-Pass Filtered Image





Fundamentals of Spatial Filtering

- If the operation performed on the image pixels is linear, then the filter is called a linear spatial filter.

- Otherwise the filter is a nonlinear spatial filter.

- Linear spatial filter performs a sum-of-products operation between image *f* and filter kernel *w*.

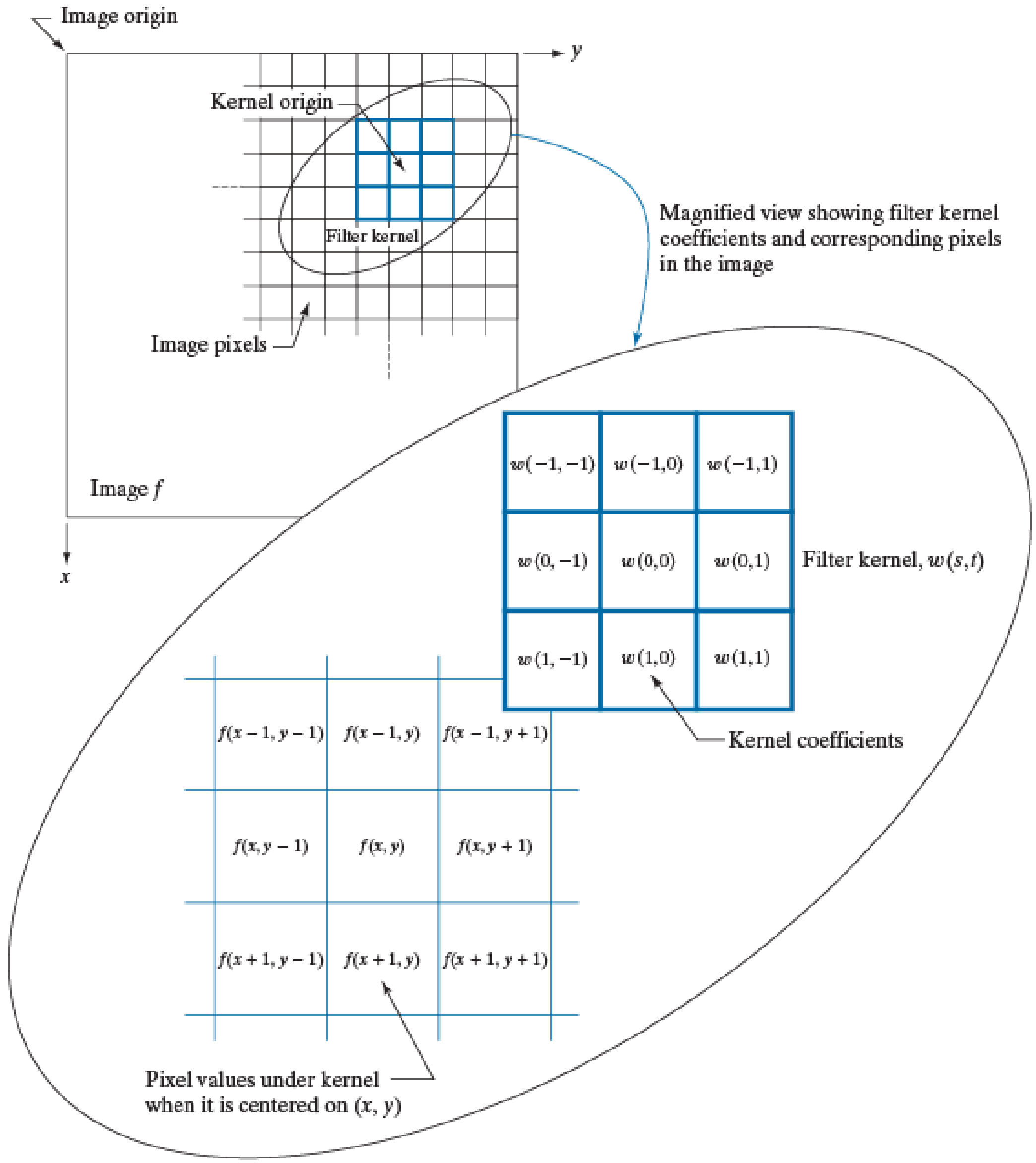
- In general, linear spatial filtering of an image of size *M* x *N* with a kernel of size *m* x *n* is given by the expression:

$$g(x,y) = \sum_{y=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

where *x* and *y* are varied so that the center of the kernel visits each pixel in *f* once.

Fundamentals of Spatial Filtering

FIGURE 3.34 The mechanics of linear spatial filtering using a 3 × 3 kernel. The pixels are shown as squares to simplify the graphics. Note that the origin of the image is at the top left, but the origin of the kernel is at its center. Placing the origin at the center of spatially symmetric kernels simplifies writing expressions for linear filtering.



Fundamentals of Spatial Filtering

- Two different spatial filtering operations: **Spatial Correlation** and **Spatial Convolution**

- **Spatial Correlation** consists of moving the center of a kernel over an image, and computing the sum of products at each location. (Previous slide)

- **Spatial Convolution** is the same except that the correlation kernel is flipped 180 degrees.

Fundamentals of Spatial Filtering

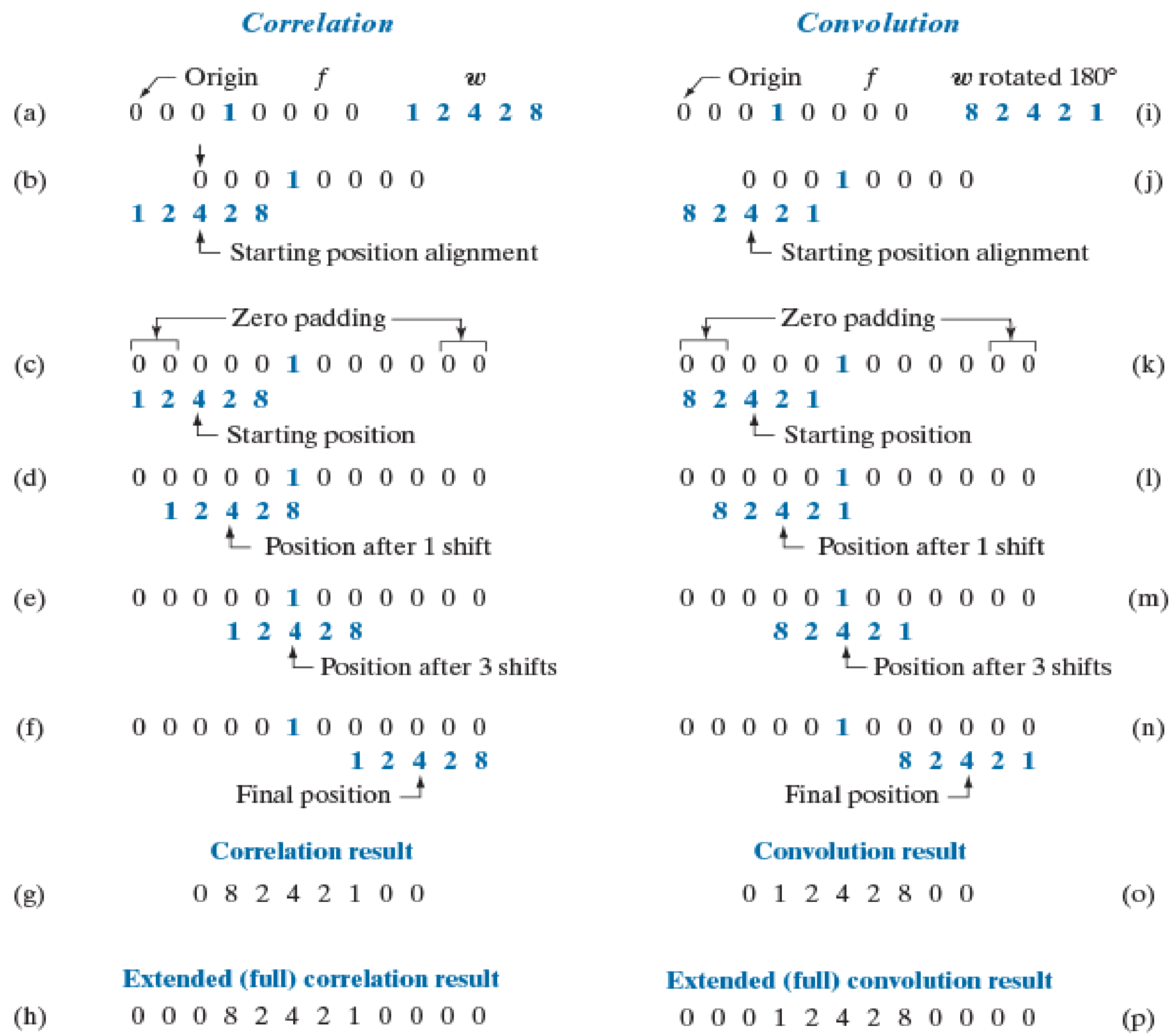
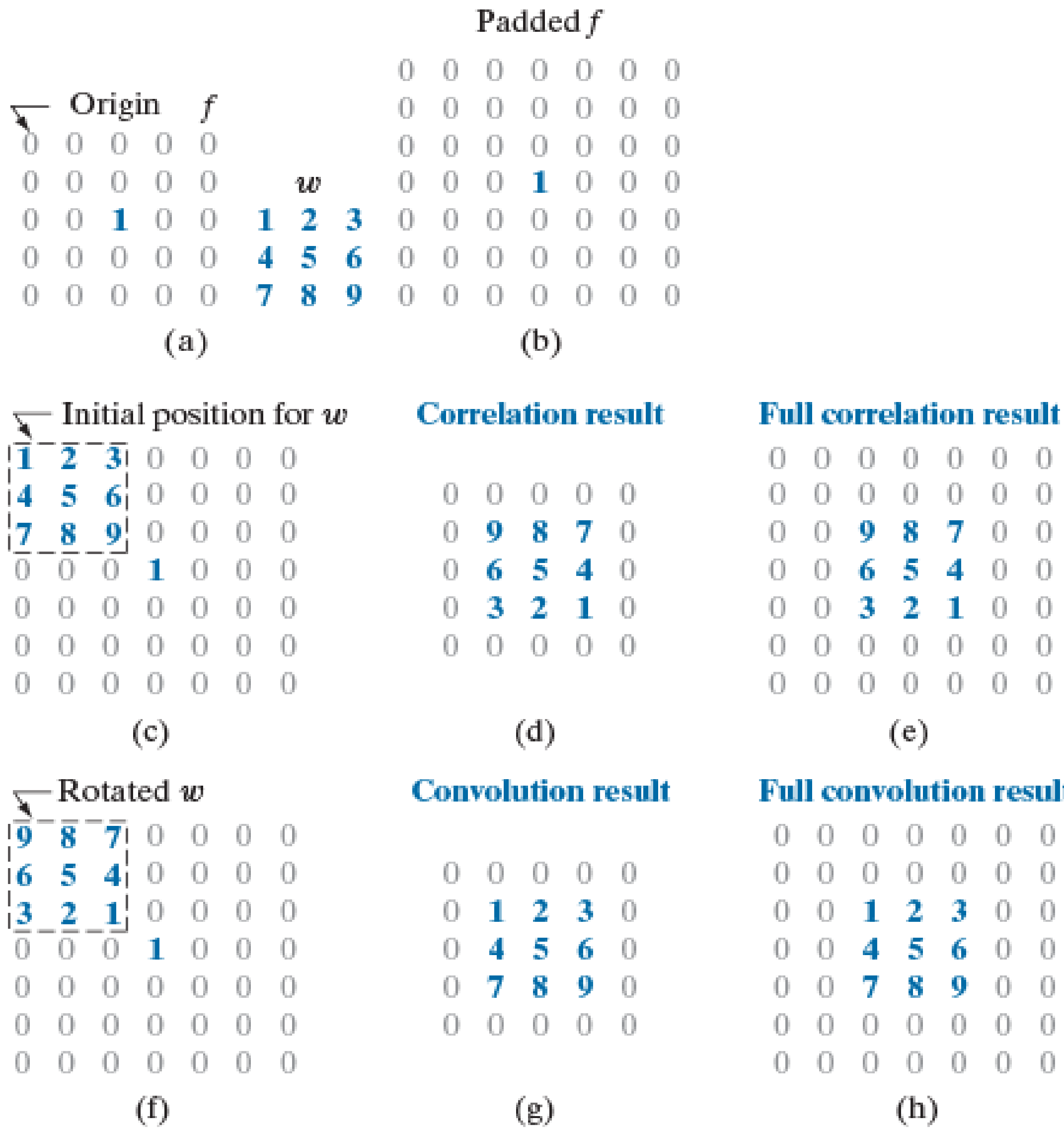


FIGURE 3.35 Illustration of 1-D correlation and convolution of a kernel, w, with a function f consisting of a discrete unit impulse. Note that correlation and convolution are functions of the variable x, which acts to *displace* one function with respect to the other. For the extended correlation and convolution results, the starting configuration places the right-most element of the kernel to be coincident with the origin of f. Additional padding must be used.

Fundamentals of Spatial Filtering

FIGURE 3.36 Correlation (middle row) and convolution (last row) of a 2-D kernel with an image consisting of a discrete unit impulse. The 0's are shown in gray to simplify visual analysis. Note that correlation and convolution are functions of x and y. As these variable change, they *displace* one function with respect to the other. See the discussion of Eqs. (3-45) and (3-46) regarding full correlation and convolution.



Fundamentals of Spatial Filtering

- Typical terms used in the literature are convolutional filter, convolutional mask, or convolutional kernel do not specifically denote that convolution rather than correlation is being performed.

- In the text, when using the term linear spatial filtering, it means convolving a kernel with an image.

TABLE 3.5 Some fundamental properties of convolution and correlation. A dash means that the property does not hold.

Property	Convolution	Correlation
Commutative	$f \star g = g \star f$	—
Associative	$f \star (g \star h) = (f \star g) \star h$	—
Distributive	$f \star (g + h) = (f \star g) + (f \star h)$	$f \,☆\, (g + h) = (f \,☆\, g) + (f \,☆\, h)$

Fundamentals of Spatial Filtering

- A 2-D kernel that can be written as a convolution of two 1-D kernels is called separable.

- The advantage of using a separable kernel is that for an image of size M x N and a kernel size of m x n, requires $MNmn$ multiplications and additions where as a separable version would require $MN(m + n)$ multiplications and additions.

- How do we know the kernel is separable?

Smoothing (Lowpass) Spatial Filtering

Smoothing (Lowpass) Spatial Filtering

- Smoothing spatial filters are used to reduce sharp transitions in intensity.

- Random noise typically consists of sharp transitions in intensity.

- Smoothing is typically used for noise reduction or to reduce irrelevant detail in an image.

Smoothing (Lowpass) Spatial Filtering

a b

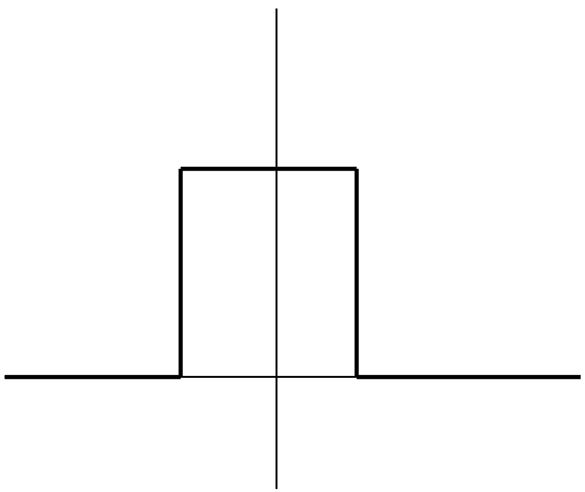
FIGURE 3.37 Examples of smoothing kernels: (a) is a *box* kernel; (b) is a *Gaussian* kernel.

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

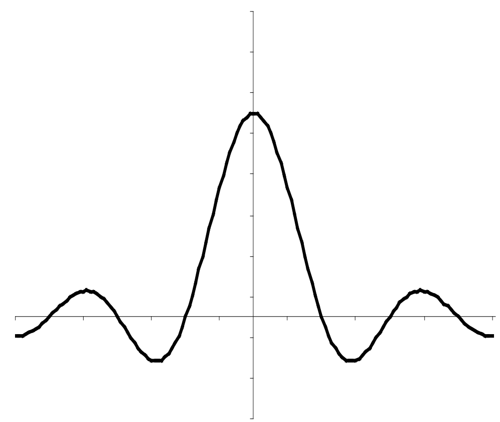
$$\frac{1}{4.8976} \times \begin{array}{|c|c|c|} \hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline \end{array}$$

Smoothing (Lowpass) Spatial Filtering

- Box (mean) filters smooth by averaging neighbors
- In frequency domain, keeps low frequencies and attenuates (reduces) high frequencies, so clearly a low-pass filter



Spatial: Box

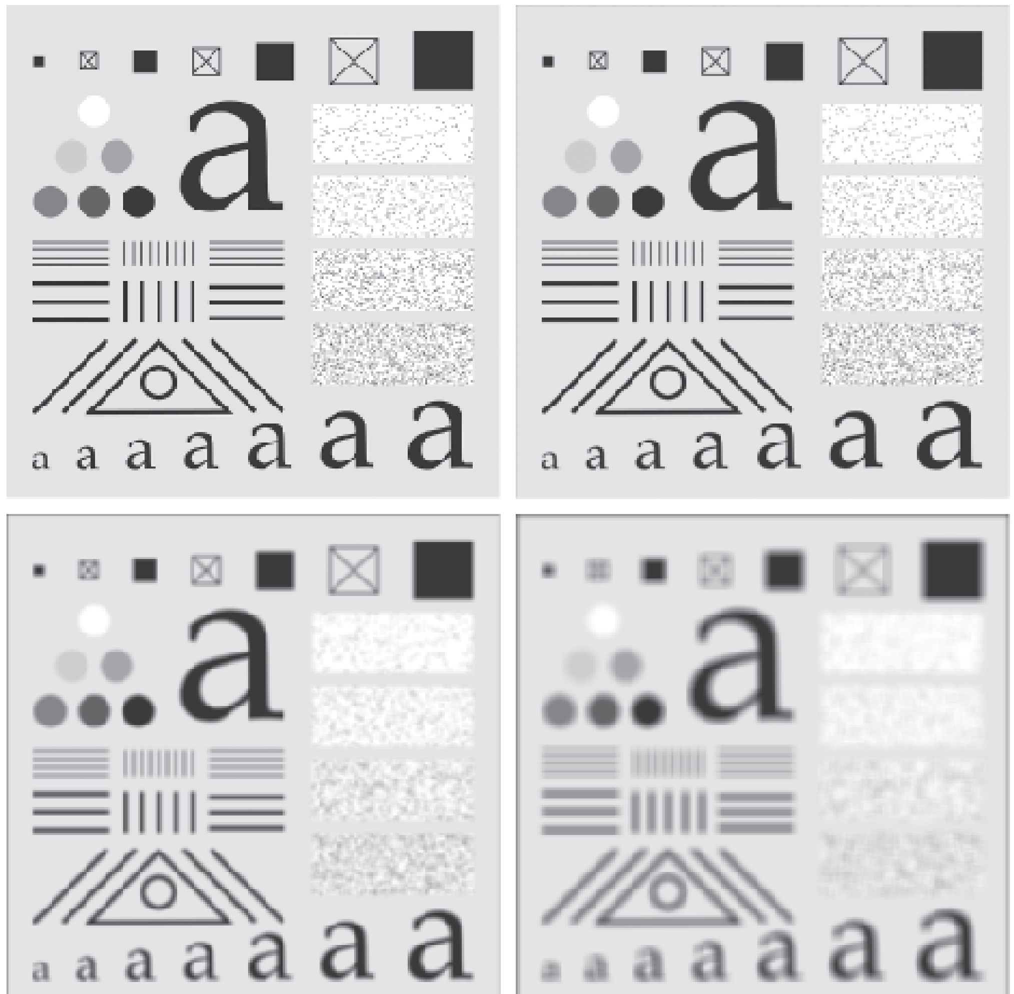


Frequency: sinc

Smoothing (Lowpass) Spatial Filtering

a b
c d

FIGURE 3.39
(a) Test pattern of size 1024×1024 pixels.
(b)-(d) Results of lowpass filtering with box kernels of sizes 3×3, 11×11, and 21×21, respectively.



Smoothing (Lowpass) Spatial Filtering

Box (Mean) Filter

- **Advantages**
  - Usable for quick experimentation
  - Yield smoothing results that are visibly acceptable
  - They are useful to reduce the effects of smoothing on edges*
- **Disadvantages**
  - They are poor approximations to the blurring characteristics of lens.
  - Favor blurring along perpendicular directions. The directionality of box filters often produce undesirable results.

Smoothing (Lowpass) Spatial Filtering

- Gaussian filter attenuates high frequencies even further
- Rotational symmetric in 2D so fewer artifacts (Isotropic), response of filter is independent of orientation.
- Only isotropic kernel that is separable
- Nothing to be gained by using Gaussian kernel which is larger than $\lceil 6\sigma \rceil \times \lceil 6\sigma \rceil$.
- Gaussian kernels have to be larger than box filters to achieve the same amount of blurring.

Smoothing (Lowpass) Spatial Filtering

Padding

- **Zero padding:** Boundary values extending using zero values.
- **Mirror padding:** Boundary values are extended using mirror reflected values of the image.

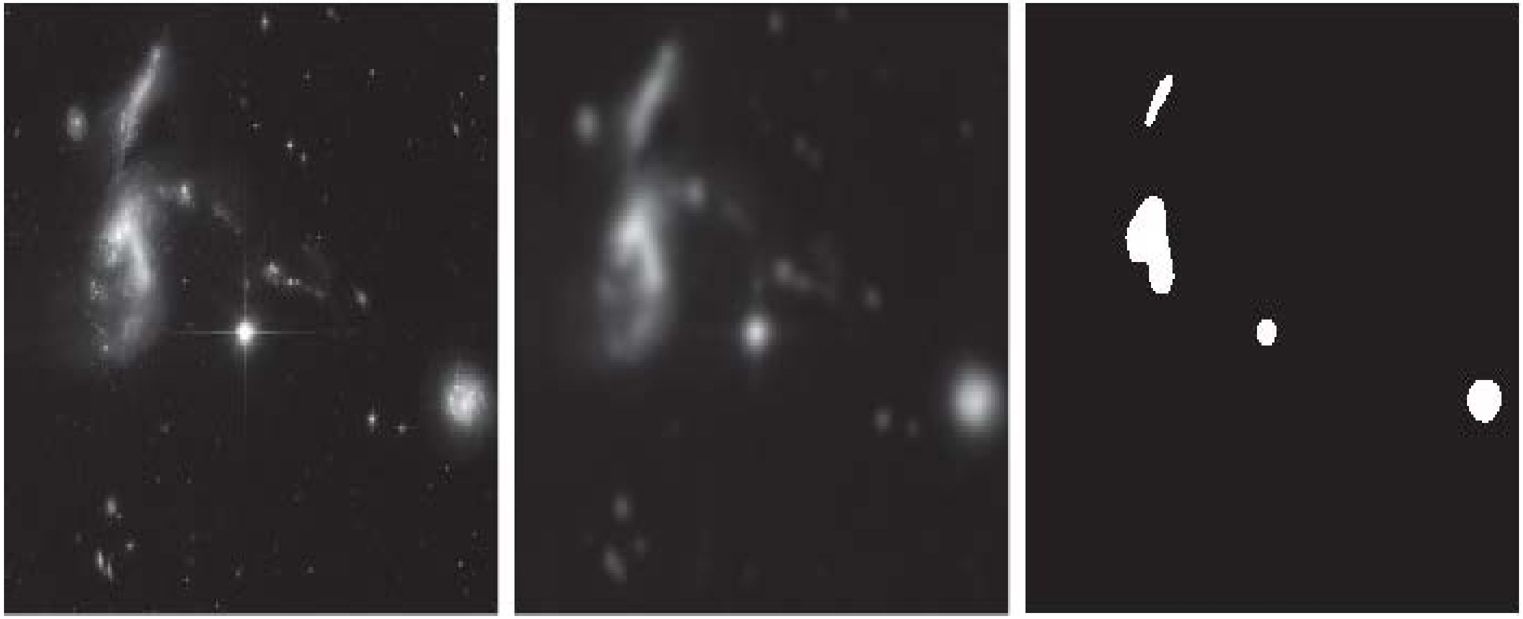
- **Replicate padding:** Boundary values are extended using the nearest image border value.

Smoothing (Lowpass) Spatial Filtering

a b c

FIGURE 3.45 Result of filtering the test pattern in Fig. 3.42(a) using (a) zero padding, (b) mirror padding, and (c) replicate padding. A Gaussian kernel of size 187×187, with $K = 1$ and $\sigma = 31$ was used in all three cases.

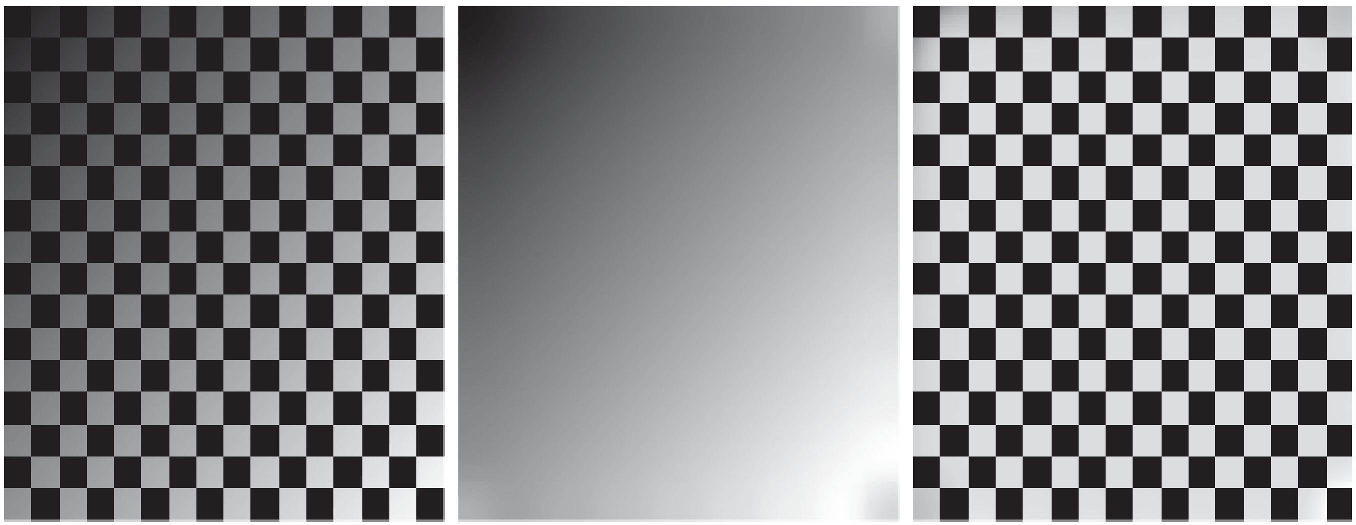
Smoothing (Lowpass) Spatial Filtering: Applications



a b c

FIGURE 3.47 (a) A 2566 × 2758 Hubble Telescope image of the *Hickson Compact Group*. (b) Result of lowpass filtering with a Gaussian kernel. (c) Result of thresholding the filtered image (intensities were scaled to the range [0, 1]). The Hickson Compact Group contains dwarf galaxies that have come together, setting off thousands of new star clusters. (Original image courtesy of NASA.)

Smoothing (Lowpass) Spatial Filtering: Applications



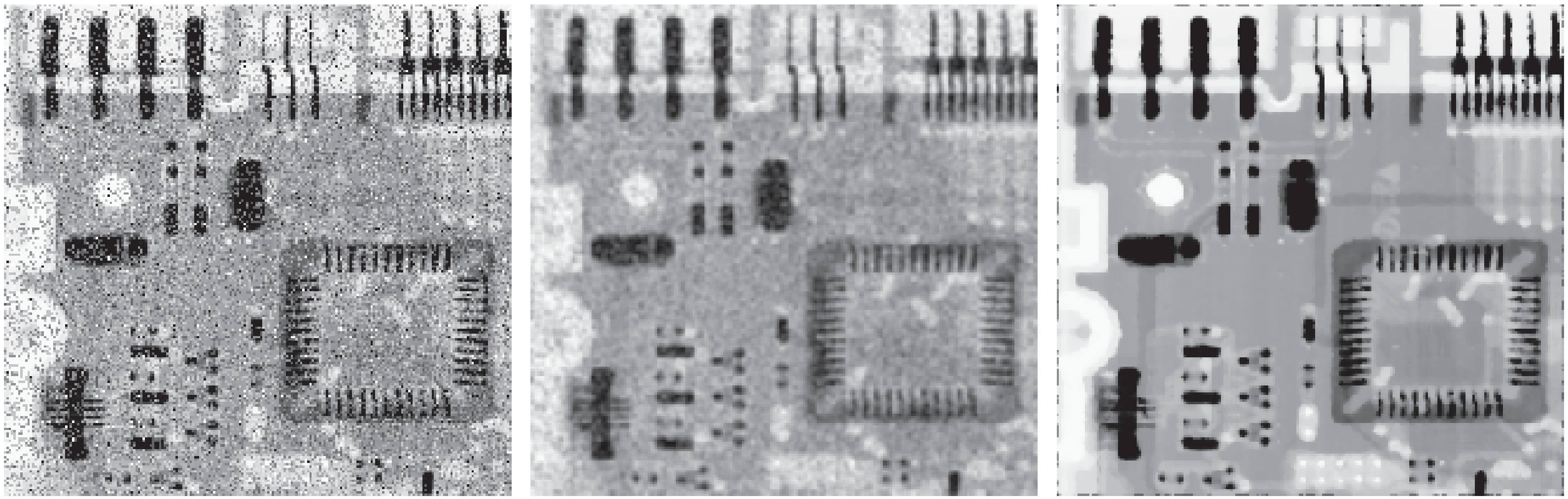
a b c

FIGURE 3.48 (a) Image shaded by a shading pattern oriented in the −45° direction. (b) Estimate of the shading patterns obtained using lowpass filtering. (c) Result of dividing (a) by (b). (See Section 9.8 for a morphological approach to shading correction).

Order-Statistic (Nonlinear Filters)

- Not considered convolutions. Why?
- Examples: Median, Max, and Min filters.
- Median filters are useful in applications requiring the removal of impulse noise and the preservation of edges.

Order-Statistic (Nonlinear Filters)



a b c

FIGURE 3.49 (a) X-ray image of a circuit board, corrupted by salt-and-pepper noise. (b) Noise reduction using a 19×19 Gaussian lowpass filter kernel with $\sigma = 3$. (c) Noise reduction using a 7×7 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Smoothing Spatial Filters Summary

Purpose:

Blur or noise reduction

- Lowpass/Smoothing spatial filtering
  - Sum of the mask coefficients is 1
  - Visual effect: reduced noise but blurred edge as well

- Smoothing linear filters
  - Averaging filter
  - Weighted average (e.g., Gaussian)

- Smoothing nonlinear filters

  Order statistics filters (e.g., median filter)

Sharpening (Highpass) Spatial Filters

Sharpening Spatial Filters

- Sharpening filters are based on computing spatial derivatives of an image.

- The first-order derivative of a one-dimensional function *f*(*x*) is

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

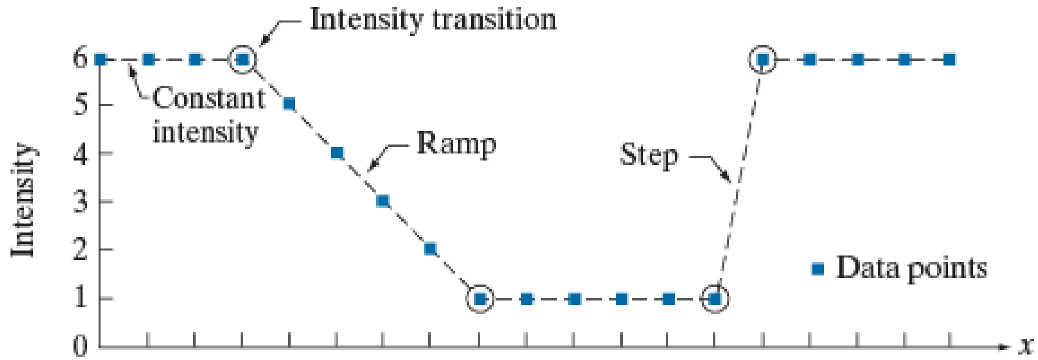
- The second-order derivative of a one-dimensional function *f*(*x*) is

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

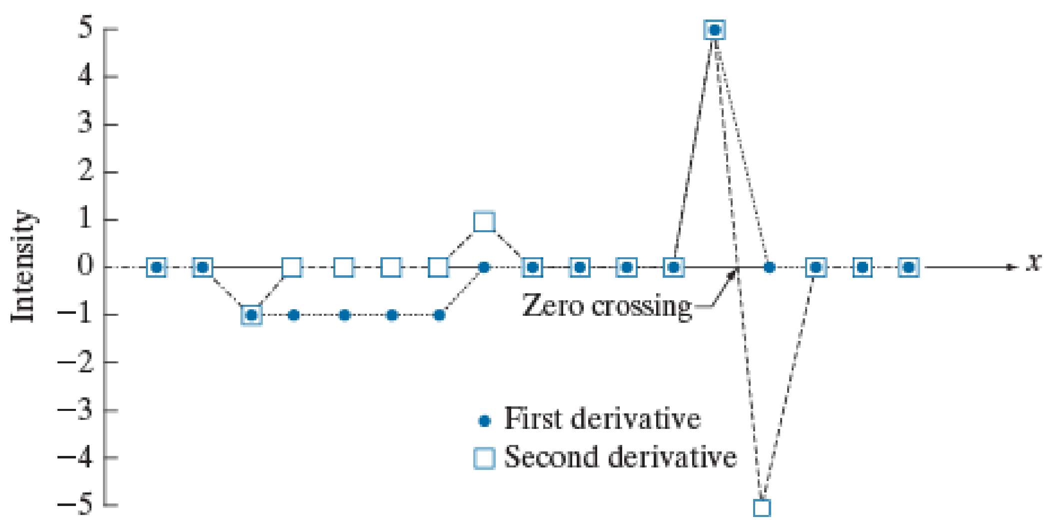
Spatial Filters: First Derivatives

- Must be zero in areas of constant intensity
- Must be nonzero at the onset of an intensity step or ramp
- Must be nonzero along intensity ramps

Sharpening (Highpass) Spatial Filters



Values of scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6
1st derivative		0	0	−1	−1	−1	−1	−1	0	0	0	0	0	5	0	0	0	0	
2nd derivative		0	0	−1	0	0	0	0	1	0	0	0	0	5	−5	0	0	0	



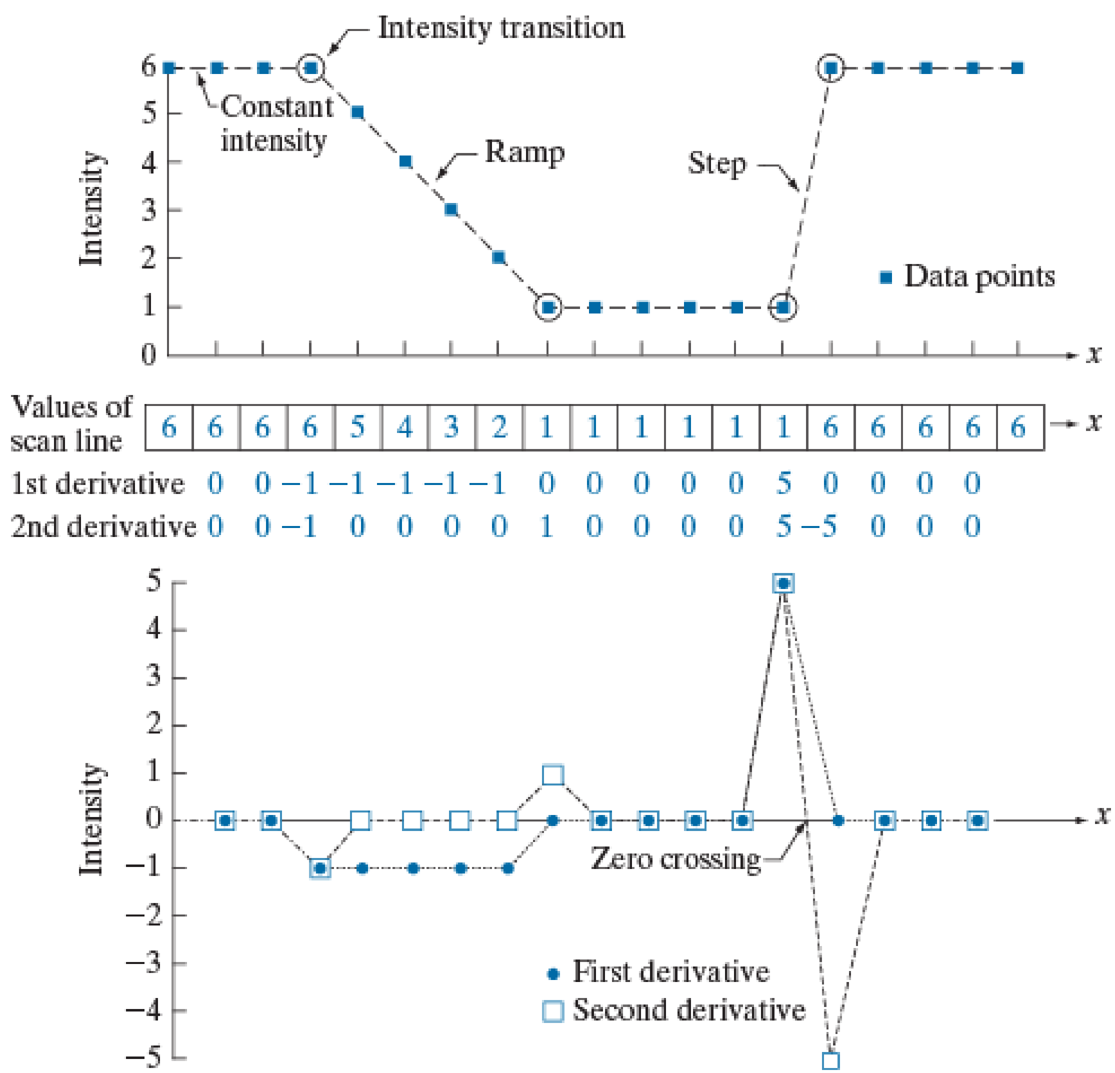
a
b
c

FIGURE 3.50 (a) A section of a horizontal scan line from an image, showing ramp and step edges, as well as constant segments. (b) Values of the scan line and its derivatives. (c) Plot of the derivatives, showing a zero crossing. In (a) and (c) points were joined by dashed lines as a visual aid.

Spatial Filters: Second Derivatives

- Must be zero in areas of constant intensity
- Must be nonzero at the onset and end of an intensity step or ramp
- Must be zero along intensity ramps

Sharpening (Highpass) Spatial Filters



a
b
c

FIGURE 3.50
(a) A section of a horizontal scan line from an image, showing ramp and step edges, as well as constant segments.
(b) Values of the scan line and its derivatives.
(c) Plot of the derivatives, showing a zero crossing. In (a) and (c) points were joined by dashed lines as a visual aid.

First vs. Second Derivatives

- Edges detected by first derivatives are thick along ramps
- Second derivatives detect single pixel wide double edges separated by zeros.
- Therefore it can be concluded that the second derivative would enhance fine detail much better than the first derivative.
- Also, second derivatives require fewer operations to implement than the first derivative.

Sharpening (Highpass) Spatial Filters: Laplacian

Development of the Laplacian method for image sharpening

- The two dimensional Laplacian operator for continuous functions:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- The Laplacian is a linear operator.

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\Rightarrow \nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x)$$

Sharpening (Highpass) Spatial Filters: Laplacian

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

a b c d

FIGURE 3.51 (a) Laplacian kernel used to implement Eq. (3-62). (b) Kernel used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other Laplacian kernels.

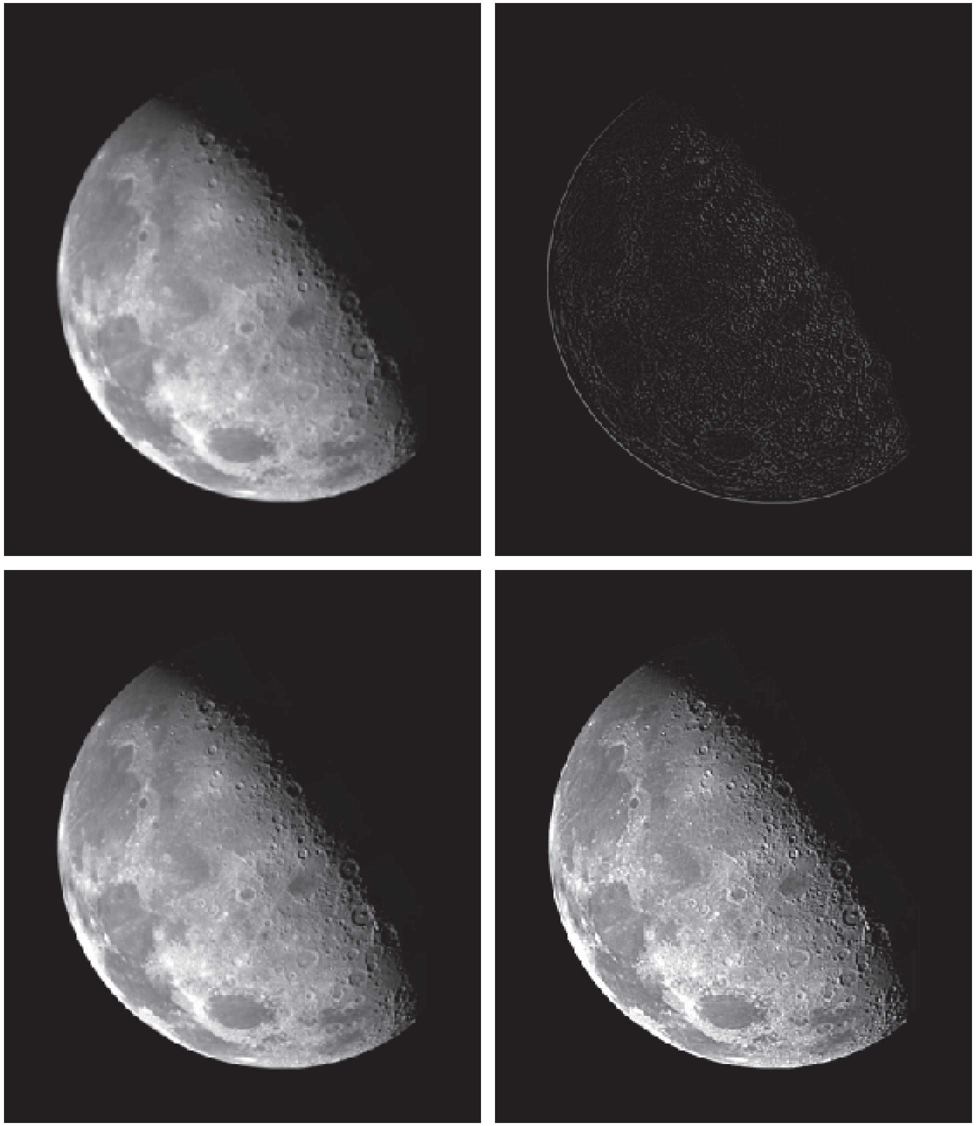
Sharpening (Highpass) Spatial Filters: Laplacian

Because the Laplacian is a derivative operator, it highlights sharp intensity transitions in an image and de-emphasizes regions of slowly varying intensities.

To sharpen an image, the Laplacian of the image is subtracted from the original image.

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f & \text{if the center coefficient of the Laplacian mask is negative.} \\ f(x,y) + \nabla^2 f & \text{if the center coefficient of the Laplacian mask is positive.} \end{cases}$$

Sharpening (Highpass) Spatial Filters: Laplacian

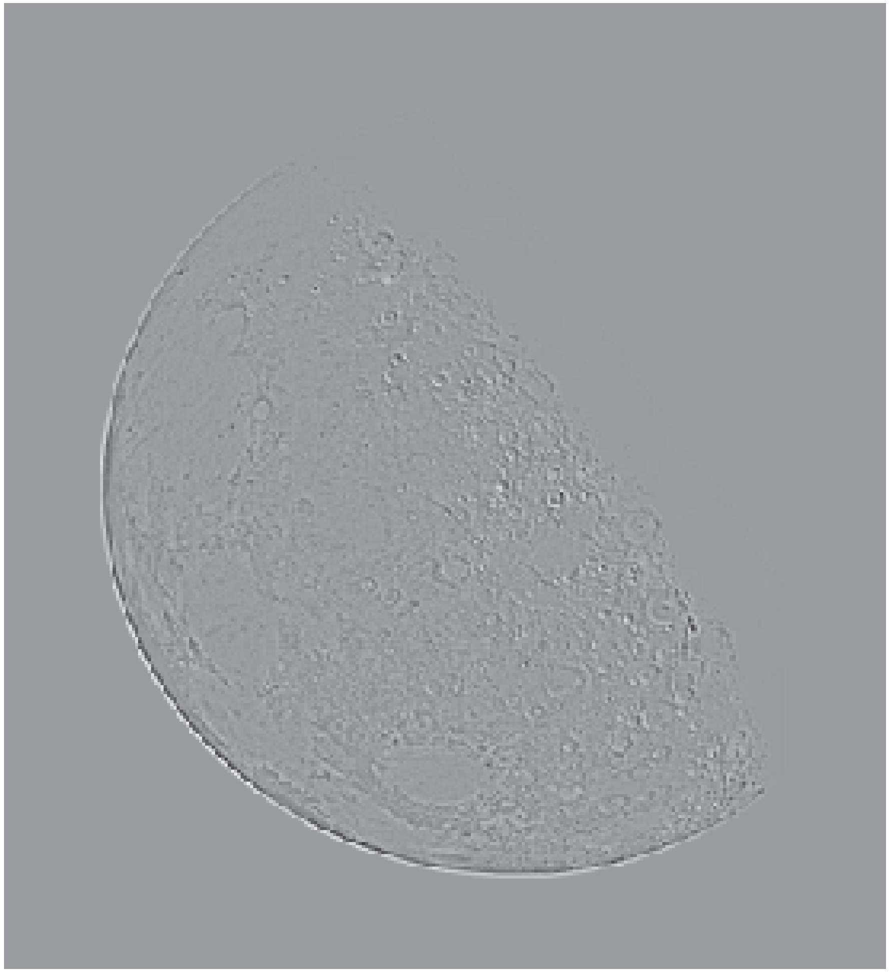


a b
c d

FIGURE 3.52
(a) Blurred image of the North Pole of the moon.
(b) Laplacian image obtained using the kernel in Fig. 3.51(a).
(c) Image sharpened using Eq. (3-63) with $c = -1$.
(d) Image sharpened using the same procedure, but with the kernel in Fig. 3.51(b).
(Original image courtesy of NASA.)

Sharpening (Highpass) Spatial Filters: Laplacian

FIGURE 3.53 The Laplacian image from Fig. 3.52(b), scaled to the full [0, 255] range of intensity values. Black pixels correspond to the most negative value in the unscaled Laplacian image, grays are intermediate values, and white pixels corresponds to the highest positive value.



Sharpening (Highpass) Spatial Filters: Unsharp Masking / Highboost

- Blur the original image
- Subtract the blurred image from the original (the resulting difference is called the mask) (Unsharp)

$$g_{mask}(x,y) = f_{input}(x,y) - f_{blurred}(x,y)$$

- Add the mask to the original

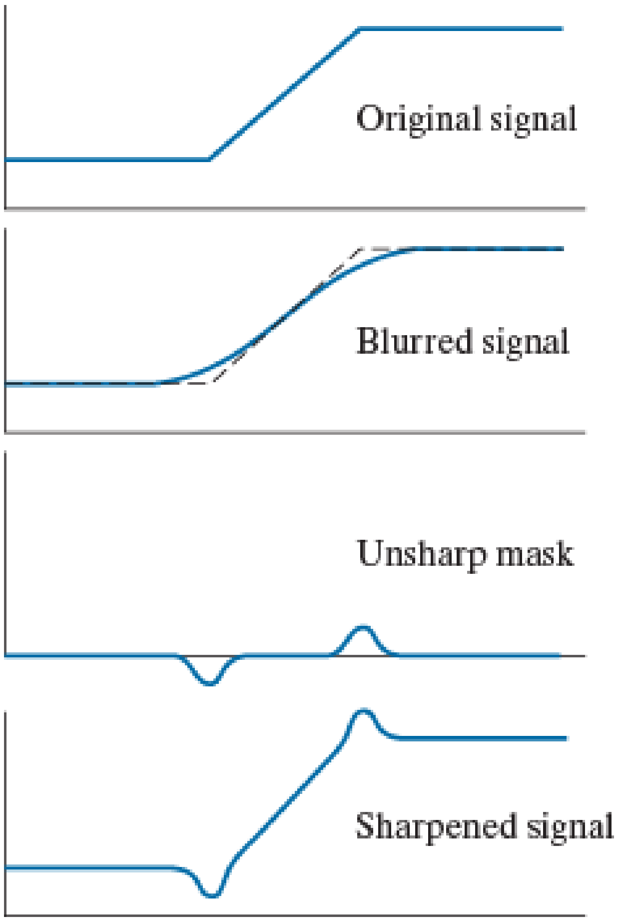
$$g(x,y) = f(x,y) + kg_{mask}(x,y)$$

- When k > 1, the process is referred to as highboost filtering

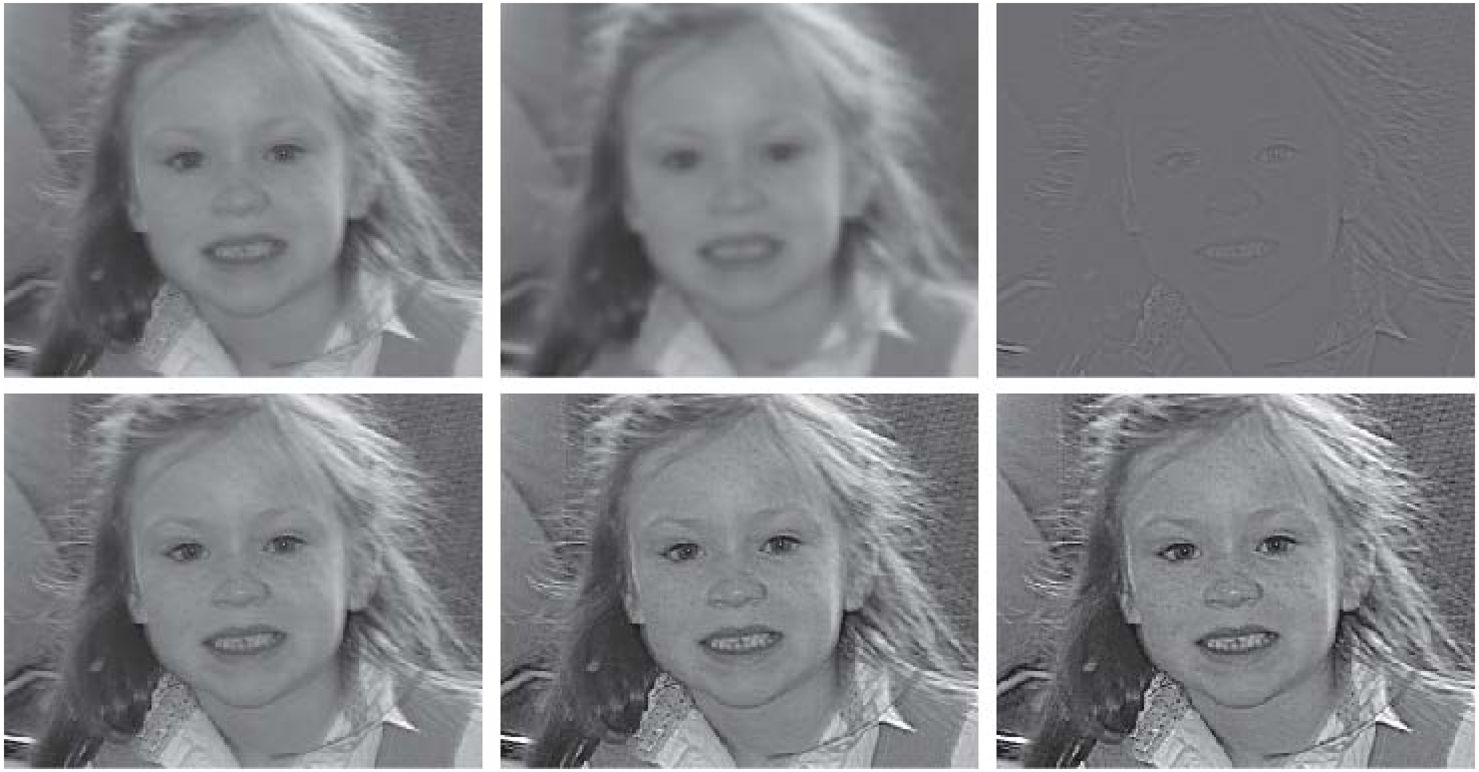
Sharpening (Highpass) Spatial Filters: Unsharp Masking / Highboost

a
b
c
d

FIGURE 3.54 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).



Sharpening (Highpass) Spatial Filters: Unsharp Masking / Highboost



a	b	c
d	e	f

FIGURE 3.55 (a) Unretouched "soft-tone" digital image of size 469×600 pixels. (b) Image blurred using a 31×31 Gaussian lowpass filter with $\sigma = 5$. (c) Mask. (d) Result of unsharp masking using Eq. (3-65) with $k = 1$. (e) and (f) Results of highboost filtering with $k = 2$ and $k = 3$, respectively.

First-Order Derivatives for Sharpening

Development of the Gradient method

- The gradient of function *f* at coordinates (*x*,*y*) is defined as the two-dimensional column vector:

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

- The magnitude of this vector is given by

$$\nabla f = \mathrm{mag}(\nabla \mathbf{f}) = \left[G_x^2 + G_y^2\right]^{\frac{1}{2}} = \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{\frac{1}{2}}$$

$$\nabla f \approx \left|G_x\right| + \left|G_y\right|$$

First-Order Derivatives for Sharpening

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

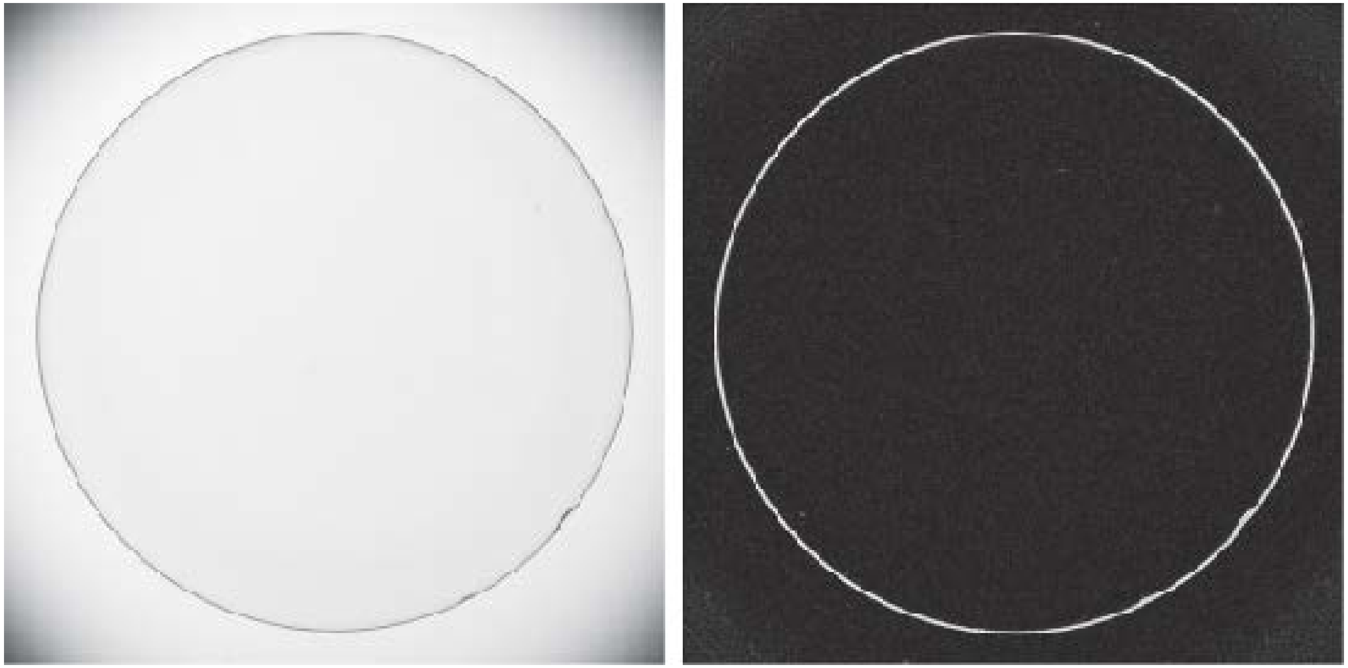
a
b c
d e

FIGURE 3.56
(a) A 3×3 region of an image, where the zs are intensity values. (b)–(c) Roberts cross-gradient operators. (d)–(e) Sobel operators. All the kernel coefficients sum to zero, as expected of a derivative operator.

First-Order Derivatives for Edge Enhancement

a b

FIGURE 3.57
(a) Image of a contact lens (note defects on the boundary at 4 and 5 o'clock).
(b) Sobel gradient.
(Original image courtesy of Perceptics Corporation.)



Sharpening Spatial Filters Summary

Purpose

Highlight fine detail or enhance detail blurred

- Highpass/Sharpening spatial filter
  - Sum of the mask coefficients is 0
  - Visual effect: enhanced edges on a dark background

- High-boost filtering and unsharp masking

- Derivative filters
  - 1st
  - 2nd

Questions?