EEE-6512: Image Processing and Computer Vision

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Lecture #5: Frequency Domain Filtering

Damon L. Woodard, Ph.D.
Dept. of Electrical and Computer Engineering
dwoodard@ece.ufl.edu

Why Frequency Domain Filtering?

- Filtering is used primarily for two applications: restoration and enhancement.
- **Restoration**: the goal is to remove the effects of noise that has degraded the image quality from its original condition.
- **Enhancement:** involves accentuating or sharpening features to make the image more useful, going beyond simply a pure, noise-free image.

Background

- Any function that **periodically** repeats itself can be expressed as the **sum** of sines and/or cosines of different frequencies, each multiplied by a different coefficient (**Fourier series**).
- Even functions that are **not periodic** (but whose area under the curve is finite) can be expressed as the *integral* of sines and/or cosines multiplied by a weighting function (**Fourier transform**).

Background

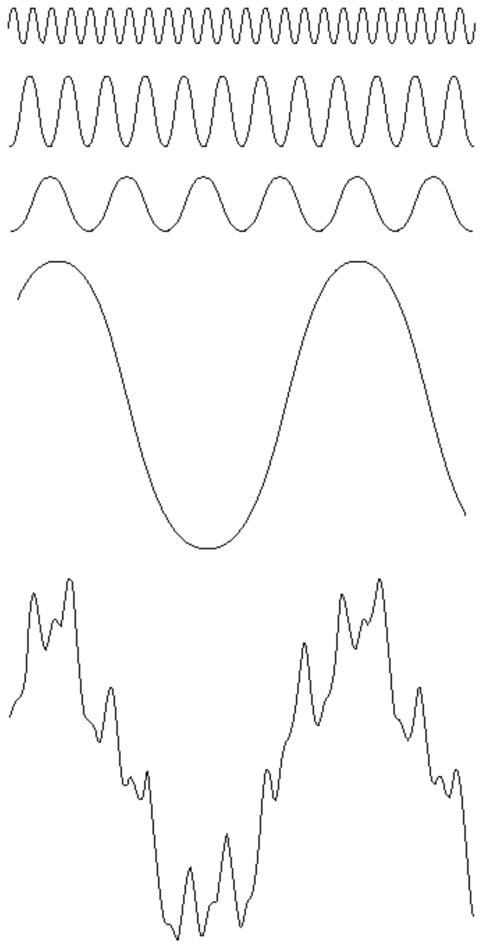


FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

- The **frequency domain** refers to the plane of the two dimensional discrete Fourier transform of an image.
- The purpose of the Fourier transform is to represent a signal as a linear combination of sinusoidal signals of various frequencies.

Introduction to the Fourier Transform and the Frequency Domain

- The **one-dimensional** Fourier transform and its inverse
  - Fourier transform (**continuous case**)

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx \quad \text{where } j = \sqrt{-1}$$

  - Inverse Fourier transform:

$$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$$

- The **two-dimensional** Fourier transform and its inverse
  - Fourier transform (**continuous case**)

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dxdy$$

  - Inverse Fourier transform:

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v) e^{j2\pi(ux+vy)} dudv$$

Introduction to the Fourier Transform and the Frequency Domain

- The **one-dimensional** Fourier transform and its inverse
  - Fourier transform (**discrete case**) DTC

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M} \quad \text{for } u = 0,1,2,\ldots,M-1$$

  - Inverse Fourier transform:

$$f(x) = \sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M} \quad \text{for } x = 0,1,2,\ldots,M-1$$

Introduction to the Fourier Transform and the Frequency Domain

- Since $e^{j\theta} = \cos\theta + j\sin\theta$ and the fact $\cos(-\theta) = \cos\theta$

  then discrete Fourier transform can be redefined

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)[\cos 2\pi ux/M - j\sin 2\pi ux/M]$$

$$\text{for } u = 0,1,2,\ldots,M-1$$

  – **Frequency (time) domain**: the domain (values of u) over which the values of $F(u)$ range; because u determines the frequency of the components of the transform.
  – **Frequency (time) component**: each of the M terms of $F(u)$.

Introduction to the Fourier Transform and the Frequency Domain

- $F(u)$ can be expressed in polar coordinates:

$$F(u) = |F(u)| e^{j\phi(u)}$$

$$\text{where } |F(u)| = \left[R^2(u) + I^2(u)\right]^{\frac{1}{2}} \quad \text{(magnitude or spectrum)}$$

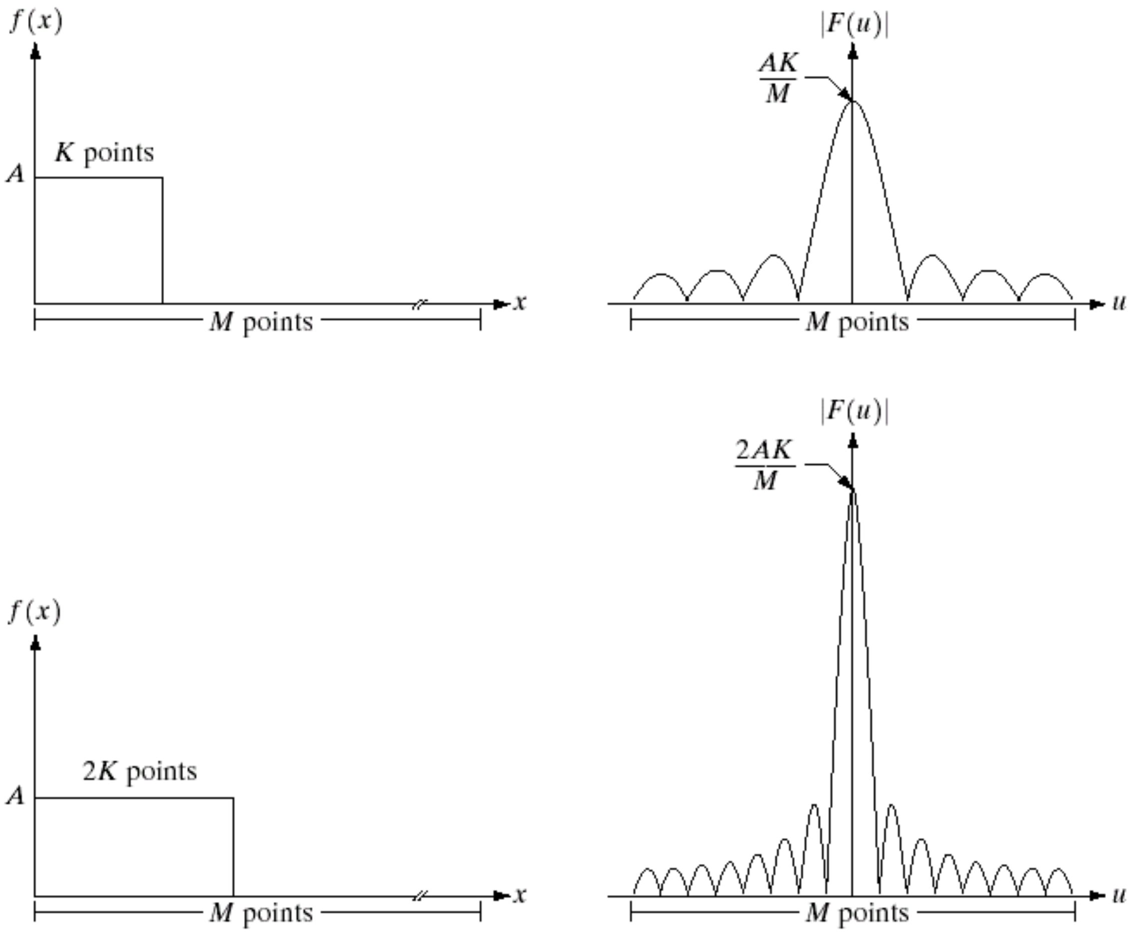
$$\phi(u) = \tan^{-1}\left[\frac{I(u)}{R(u)}\right] \quad \text{(phase angle or phase spectrum)}$$

  - $R(u)$: the real part of $F(u)$
  - $I(u)$: the imaginary part of $F(u)$

- Power spectrum:

$$P(u) = |F(u)|^2 = R^2(u) + I^2(u)$$

The One-Dimensional Fourier Transform Example



a	b
c	d

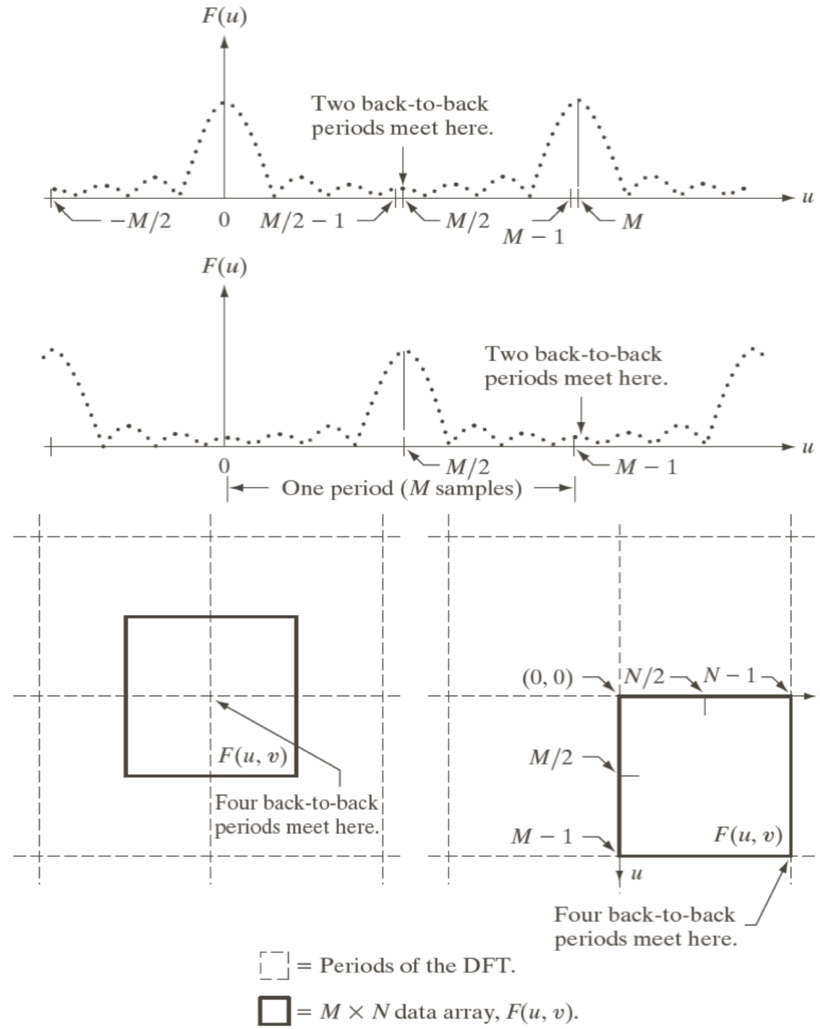
FIGURE 4.2 (a) A discrete function of M points, and (b) its Fourier spectrum. (c) A discrete function with twice the number of nonzero points, and (d) its Fourier spectrum.

Periodicity of the DFT

- The range of frequencies of the signal is between

[$-M/2$, $M/2$].

- The DFT covers two back-to-back half periods of the signal as it covers [0, $M-1$].



Periodicity of the DFT (cont…)

- For display and computation purposes it is convenient to shift the DFT and have a complete period in [*0, M-1*].

- From DFT properties:

$$f[n]e^{j2\pi(N_0 n/M)} \Leftrightarrow F(k-N_0)$$

Letting $N_0=M/2$: $f[n](-1)^n \Leftrightarrow F(k-M/2)$

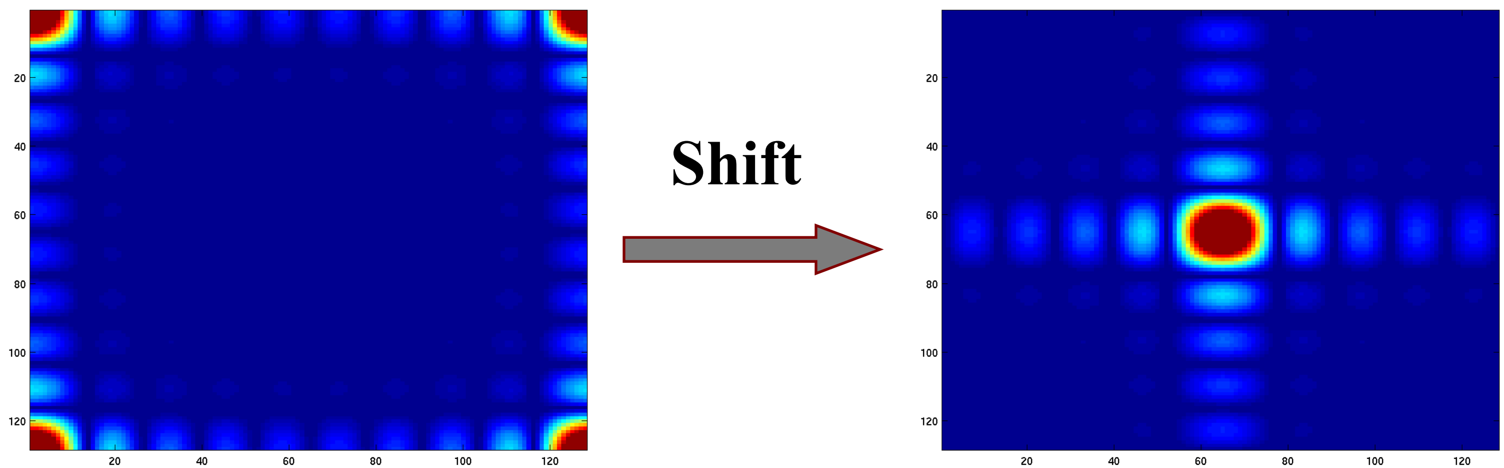
And F(0) is now located at *M/2*.

Periodicity of the DFT (cont...)

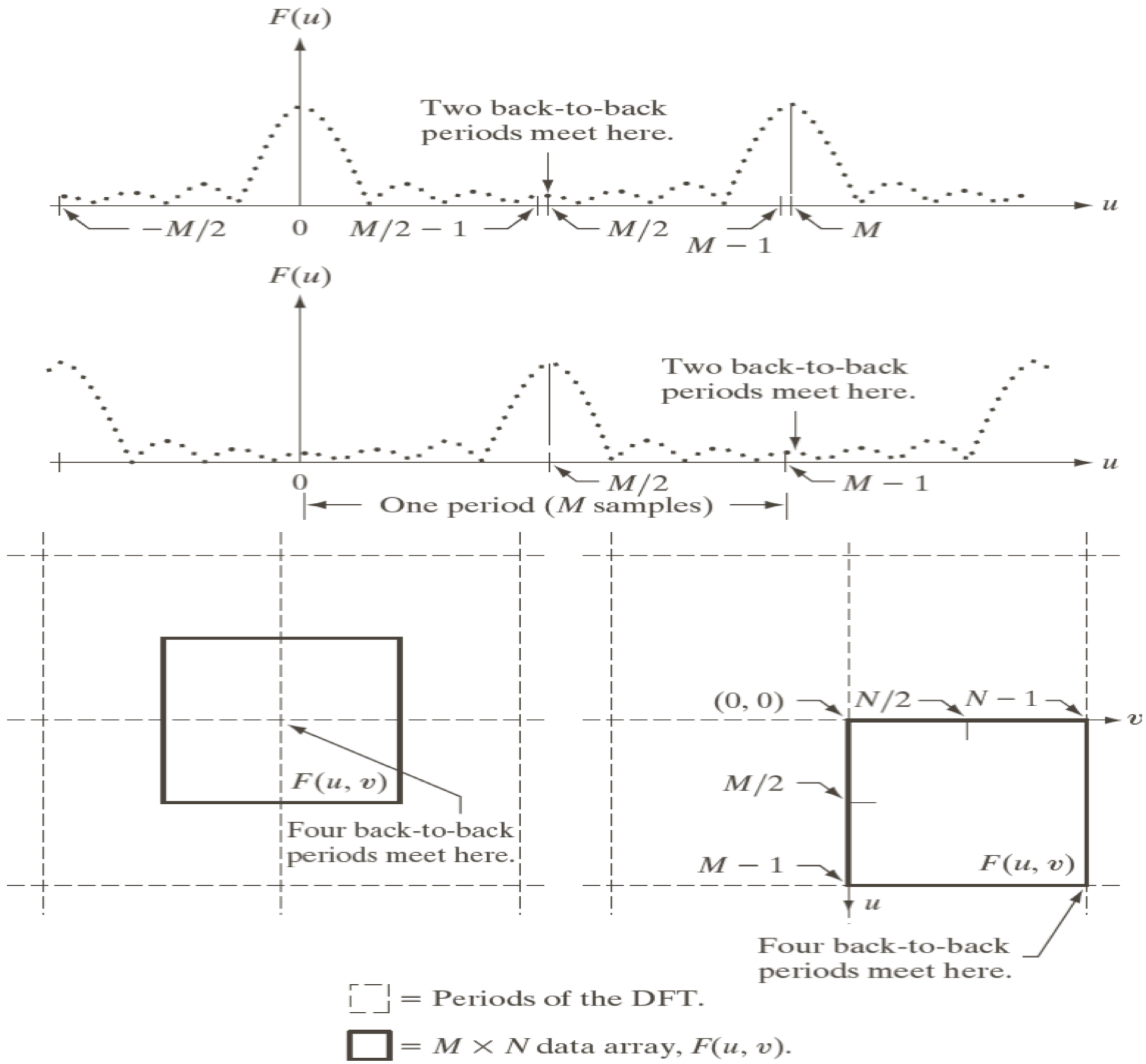
- In two dimensions:

$$f[m,n](-1)^{m+n} \Leftrightarrow F(k-M/2, l-N/2)$$

and F(0,0) is now located at (*M/2, N/2*).

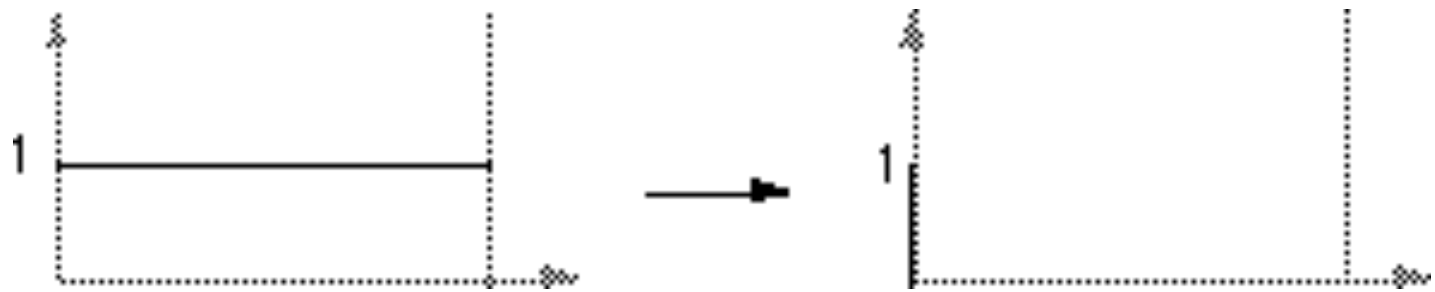


Periodicity of the DFT (cont...)

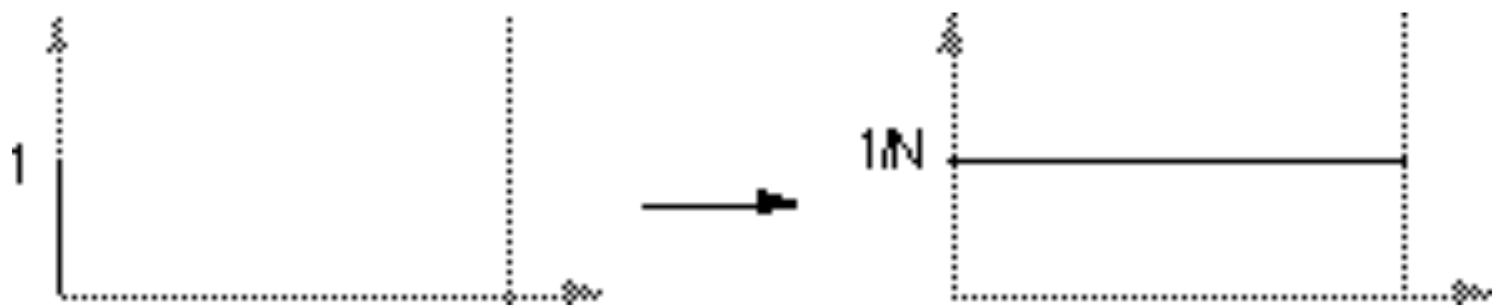


The One-Dimensional Fourier Transform Examples

- The transform of a constant function is a DC value only.

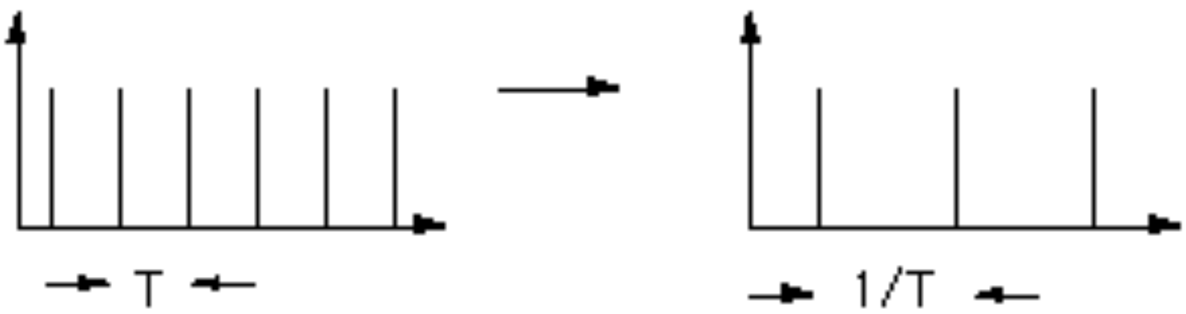


- The transform of a delta function is a constant.



The One-Dimensional Fourier Transform Examples

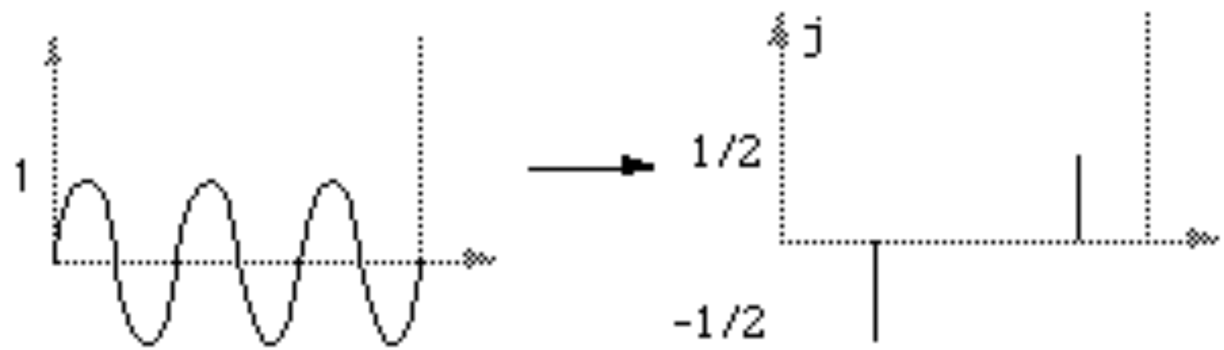
- The transform of an infinite train of delta functions spaced by T is an infinite train of delta functions spaced by 1/T.



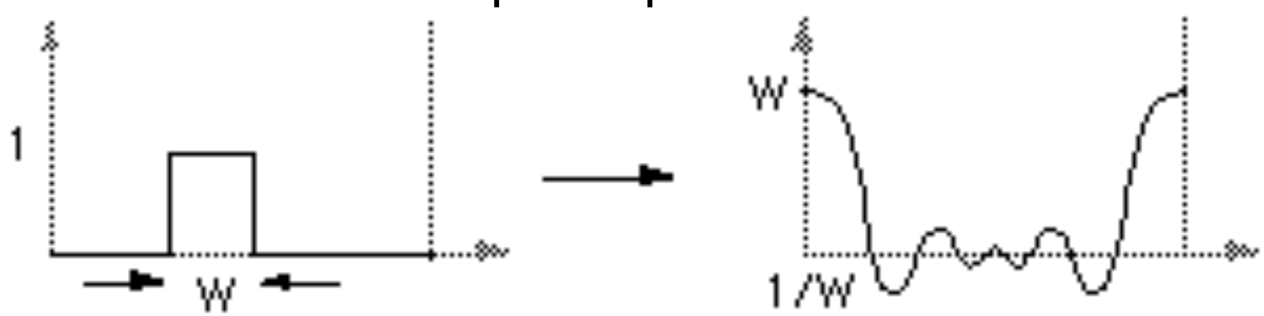
- The transform of a cosine function is a positive delta at the appropriate positive and negative frequency.

The One-Dimensional Fourier Transform Examples

- The transform of a sin function is a negative complex delta function at the appropriate positive frequency and a negative complex delta at the appropriate negative frequency.



- The transform of a square pulse is a sinc function.



Introduction to the Fourier Transform and the Frequency Domain

- The two-dimensional Fourier transform and its inverse
  - Fourier transform (discrete case) DTC

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

$$\text{for } u = 0,1,2,\ldots,M-1,\ v = 0,1,2,\ldots,N-1$$

  - Inverse Fourier transform:

$$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(ux/M+vy/N)}$$

$$\text{for } x = 0,1,2,\ldots,M-1,\ y = 0,1,2,\ldots,N-1$$

    - *u*, *v* : the transform or frequency variables
    - *x*, *y* : the spatial or image variables

Introduction to the Fourier Transform and the Frequency Domain

- Some properties of Fourier transform:

$$\Im\left[f(x,y)(-1)^{x+y}\right] = F\left(u-\frac{M}{2}, v-\frac{N}{2}\right) \quad \text{(shift)}$$

$$F(0,0) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y) \qquad \text{(average)}$$

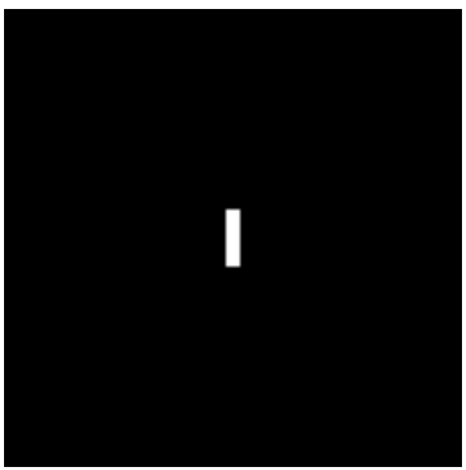
$$F(u,v) = F^*(-u,-v) \qquad \text{(conujgate symmetric)}$$

$$|F(u,v)| = |F(-u,-v)| \qquad \text{(symmetric)}$$

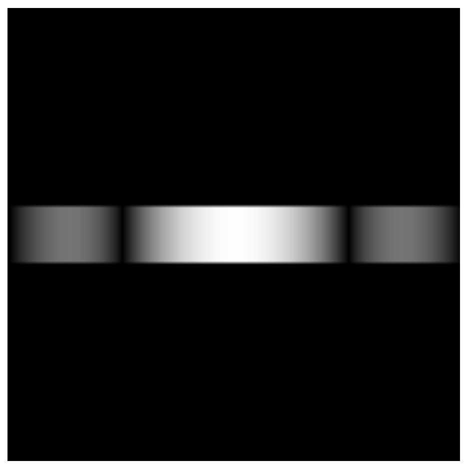
The Two-Dimensional DFT and Its Inverse

The 2D DFT *F(u,v)* can be obtained by:

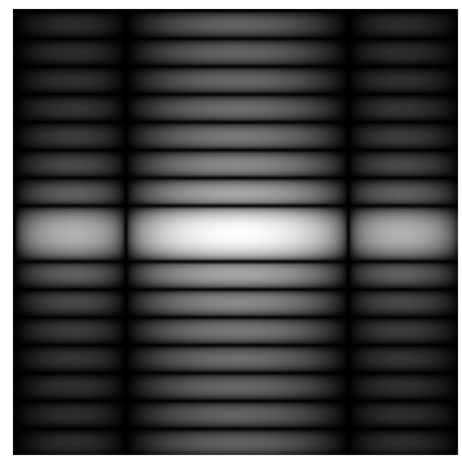
- Taking the 1D DFT of every row of image *f(x,y)*, *F(u,y)*
- Taking the 1D DFT of every column of F(u,y)



(a)f(x,y)



(b)F(u,y)



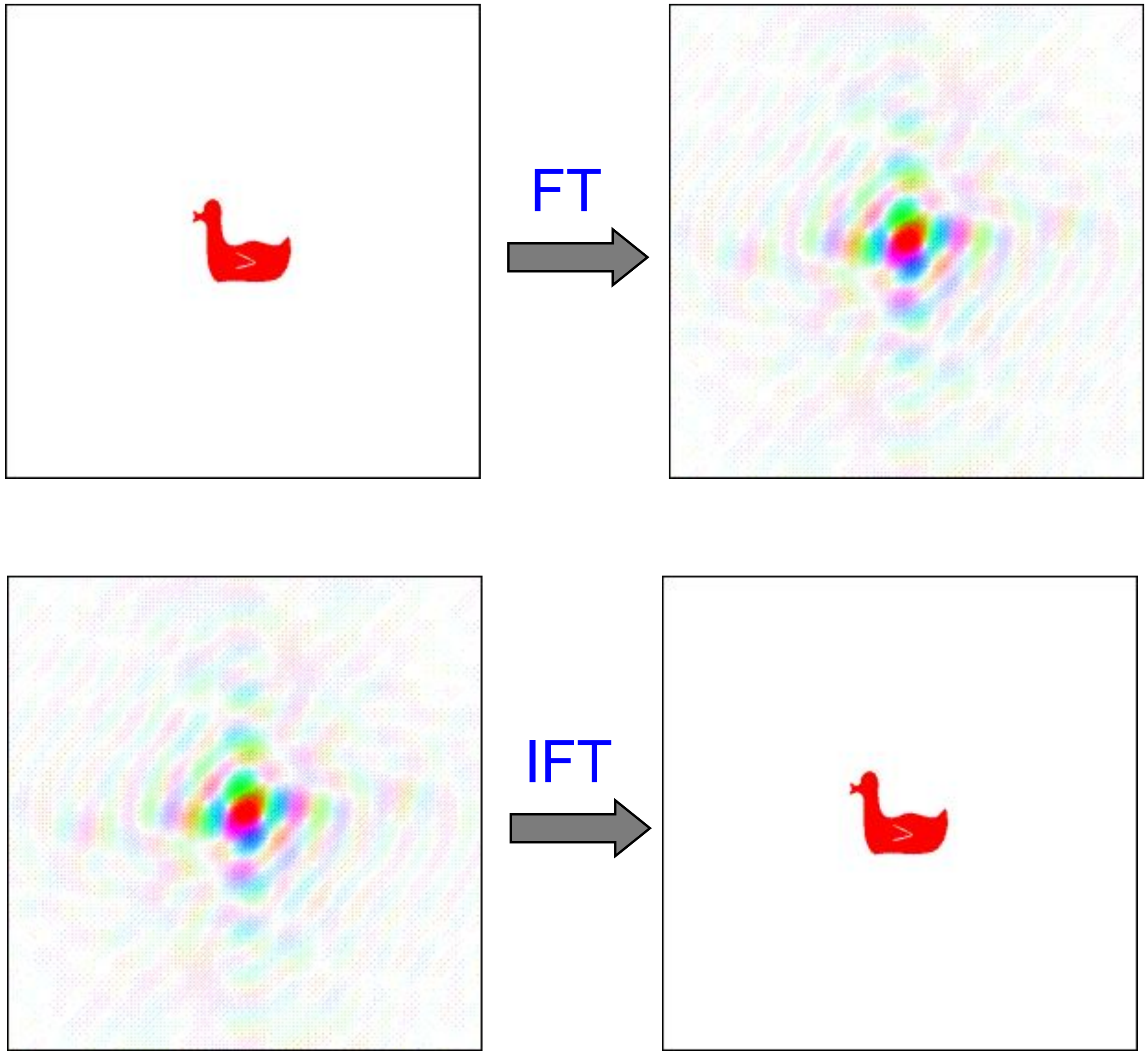
(c)F(u,v)

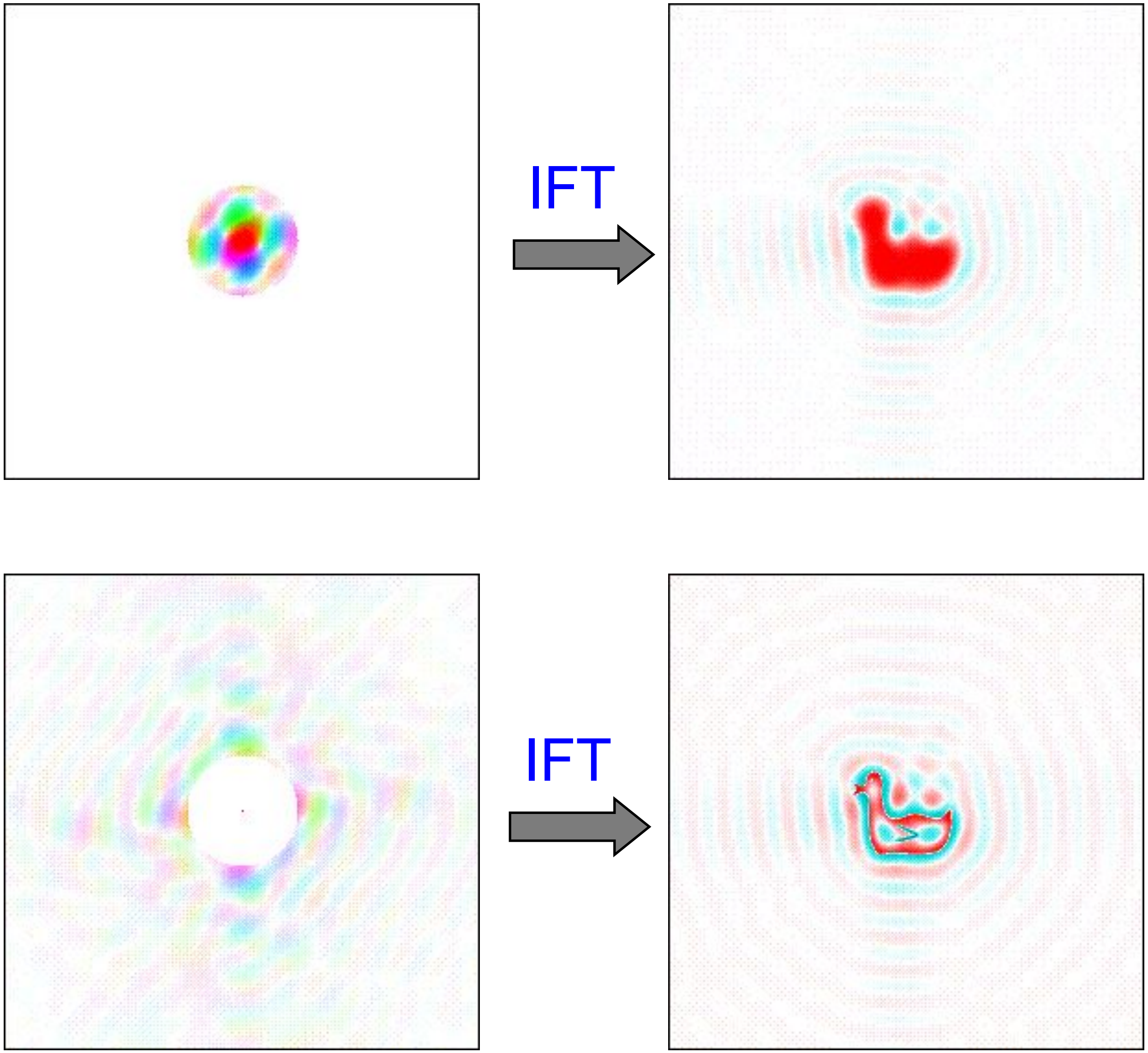
Interpreting the Fourier Spectrum

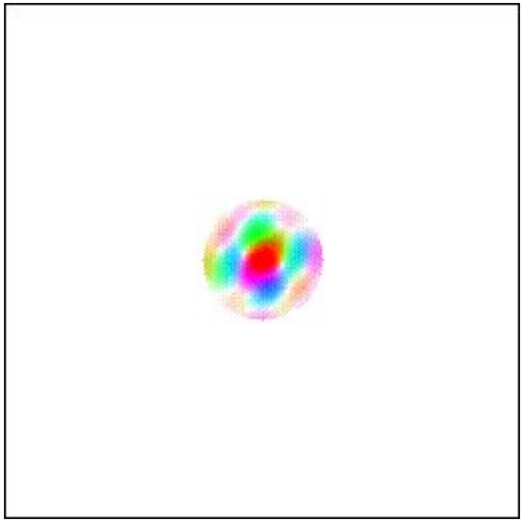
Interpreting the Fourier Spectrum

The Fourier spectrum (i.e., the magnitude of its Fourier transform) of an image shows important characteristics of the image.

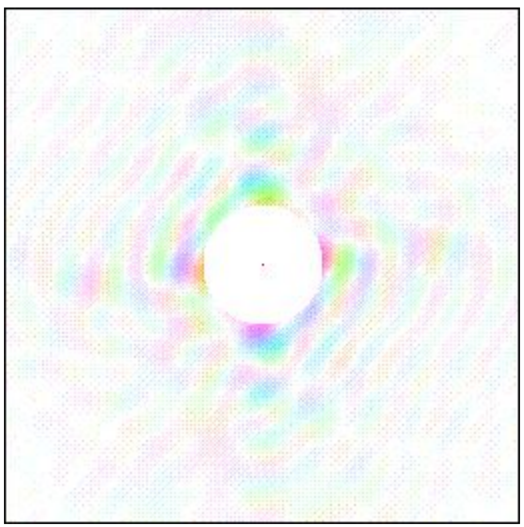
- Periodic structures in the image show up has peaks in the spectrum.
- Energy at high spatial frequencies correspond to small image detail, sharp edges, and noise.
- A sharp edge will result in high energy perpendicular to the edge.


FT
IFT


IFT
IFT



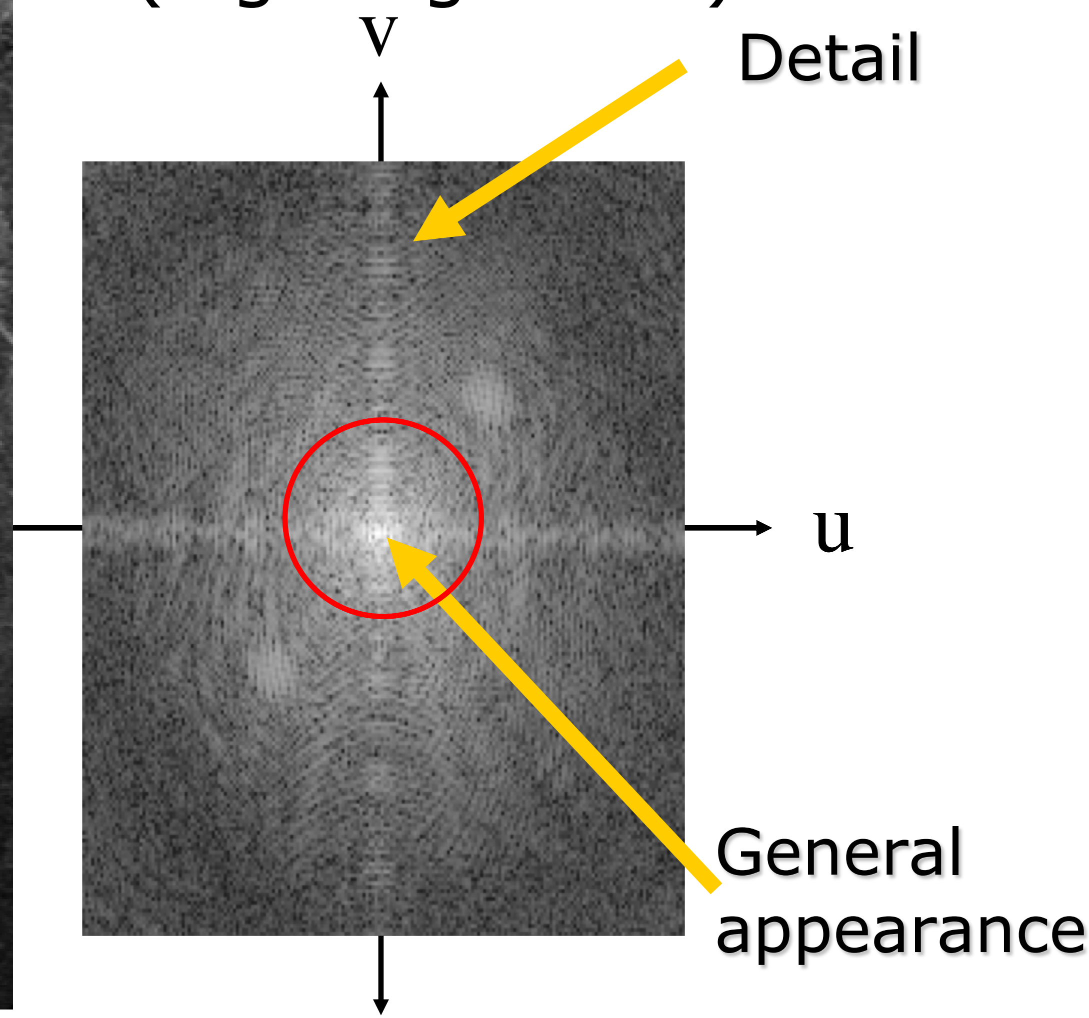
The central part of FT, i.e. the low frequency components are responsible for the general gray-level appearance of an image.

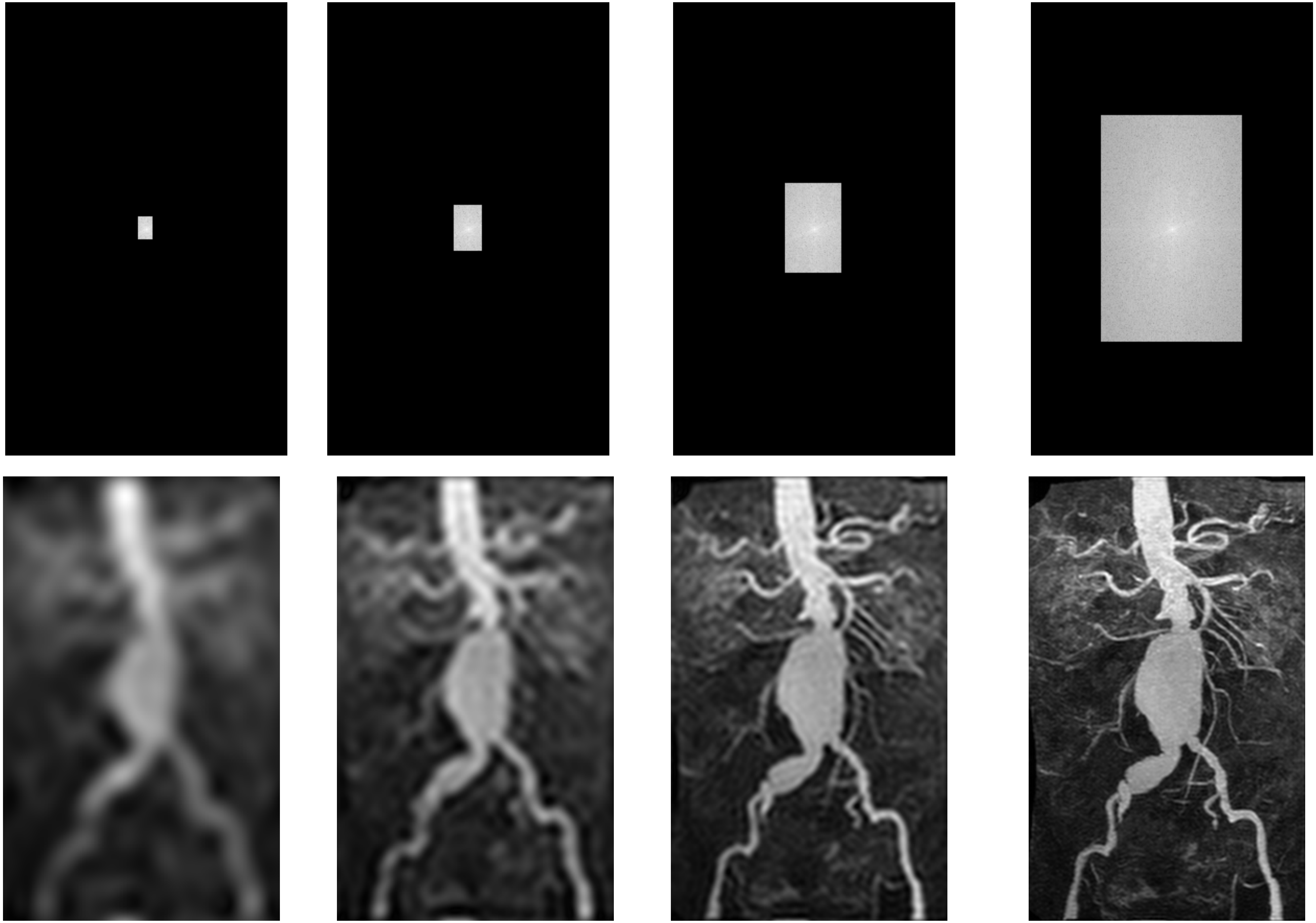


The high frequency components of FT are responsible for the detail information of an image.

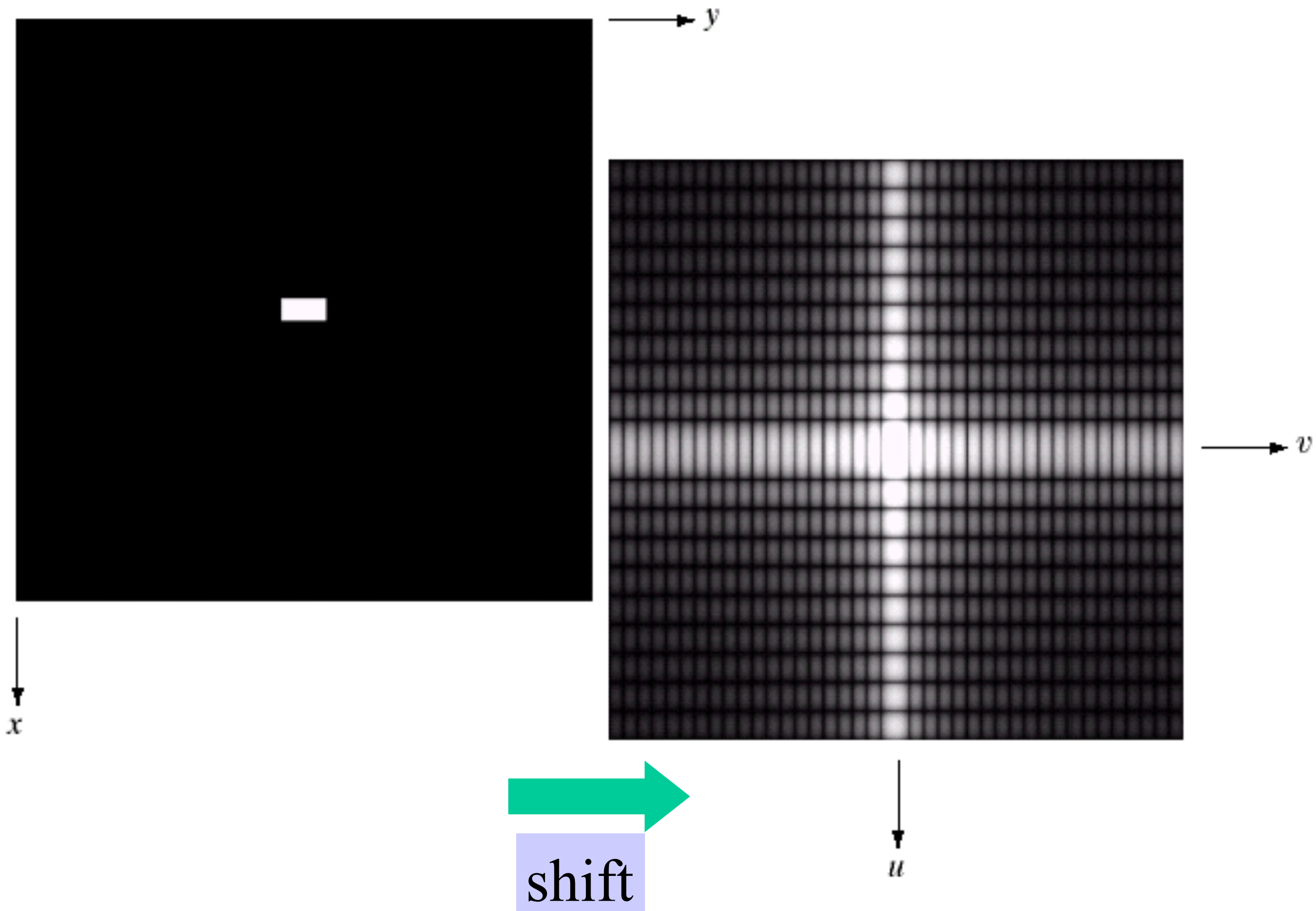
Image

Frequency Domain (log magnitude)





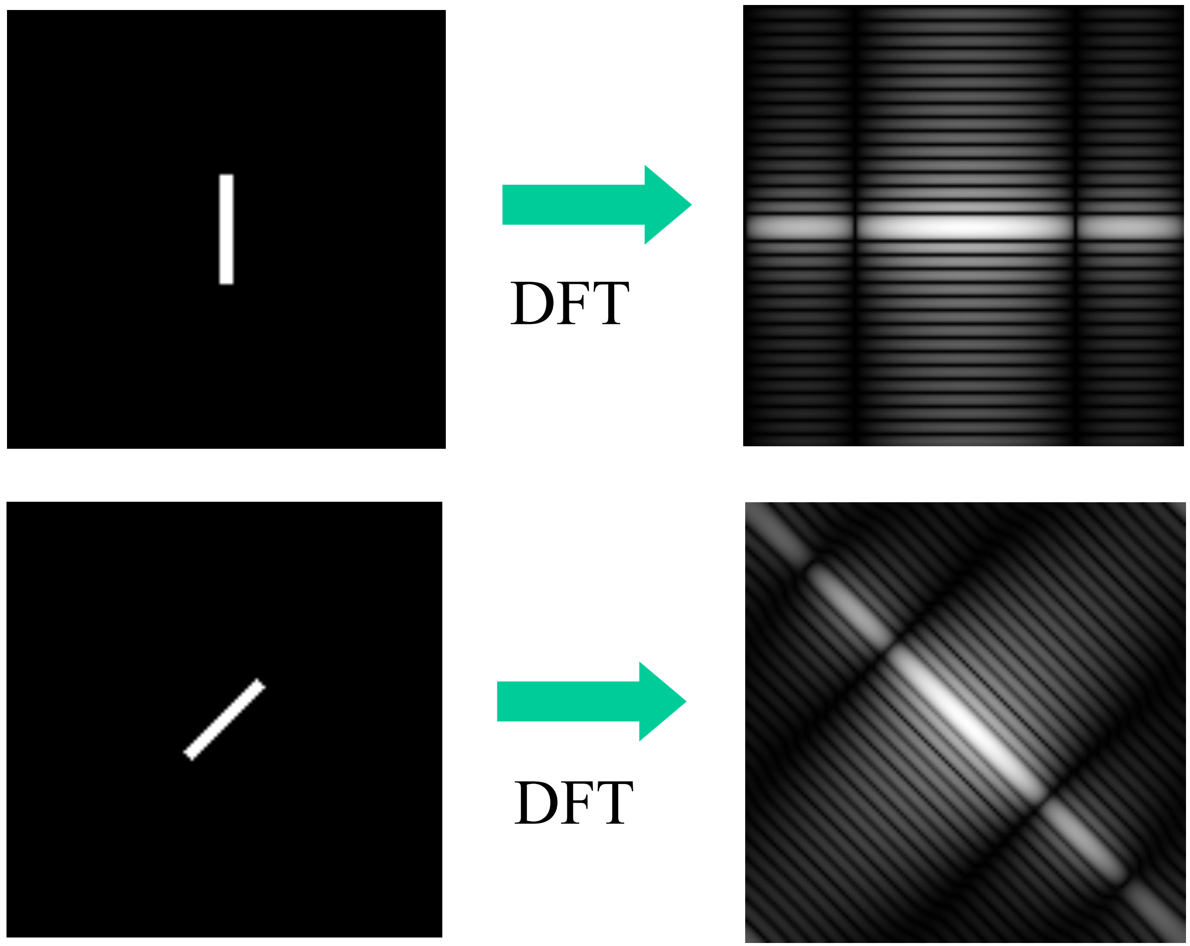
The Two-Dimensional DFT and Its Inverse



a b

FIGURE 4.3
(a) Image of a 20×40 white rectangle on a black background of size 512×512 pixels.
(b) Centered Fourier spectrum shown after application of the log transformation given in Eq. (3.2-2). Compare with Fig. 4.2.

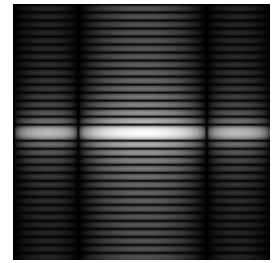
The Property of Two-Dimensional DFT Rotation



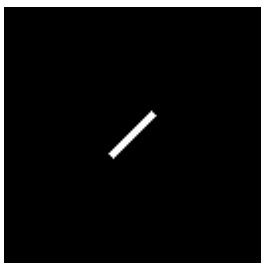
The Property of Two-Dimensional DFT Linear Combination

A

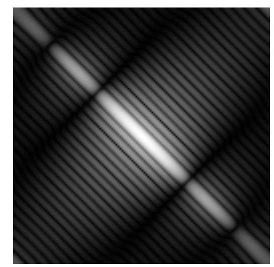
DFT



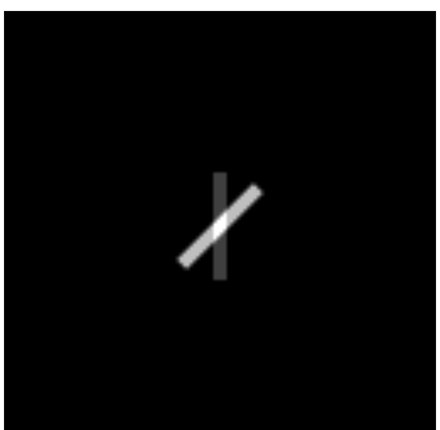
B



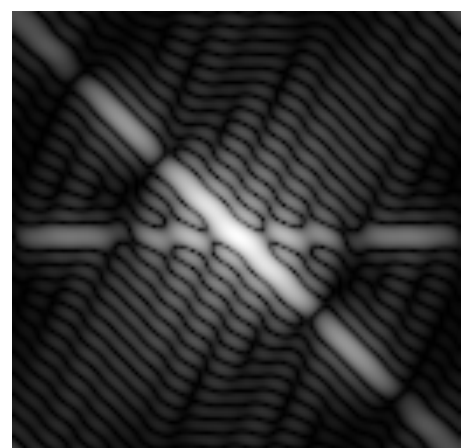
DFT



0.25 * A
+ 0.75 * B

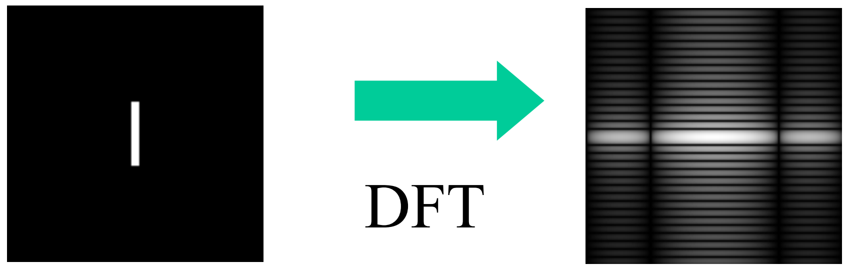


DFT

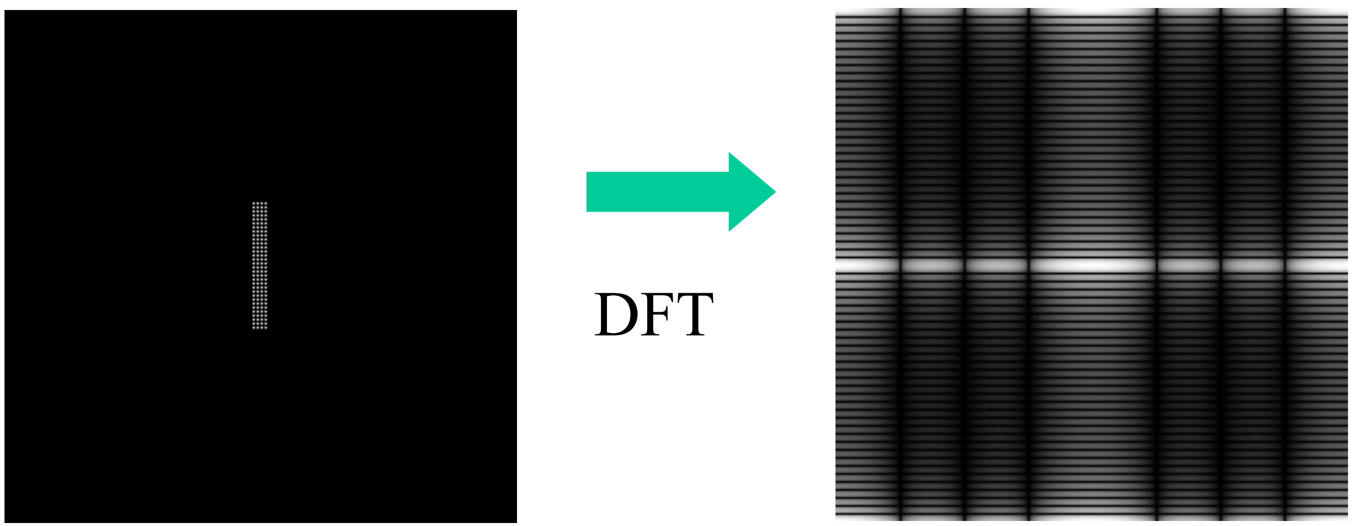


The Property of Two-Dimensional DFT Expansion

A



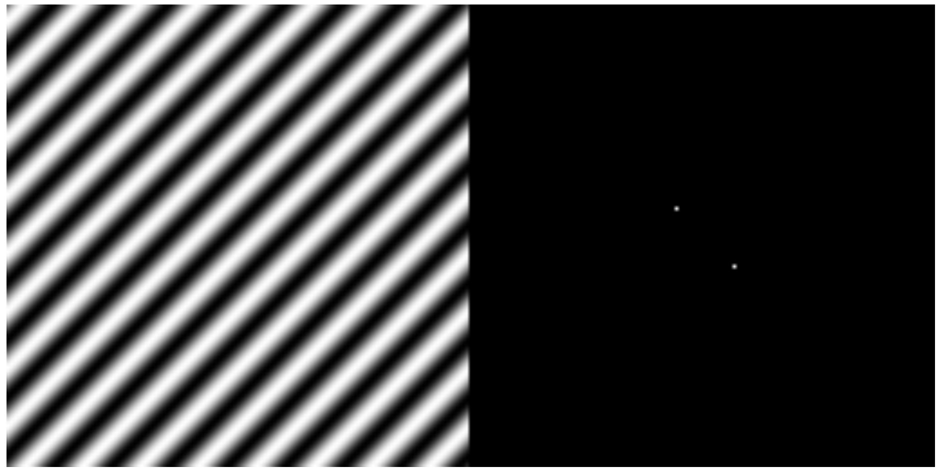
B



Expanding the original image by a factor of n (n=2), filling the empty new values with zeros, results in the same DFT.

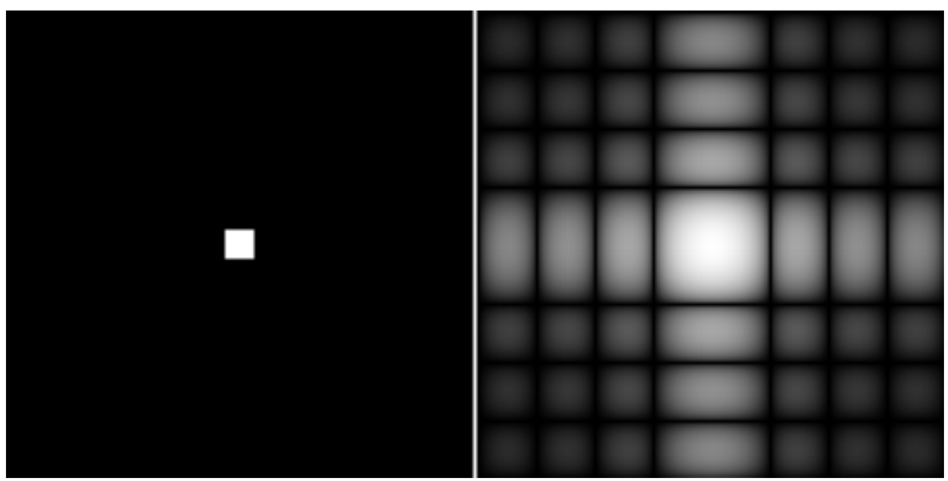
Two-Dimensional DFT with Different Functions

Sine wave



Its DFT

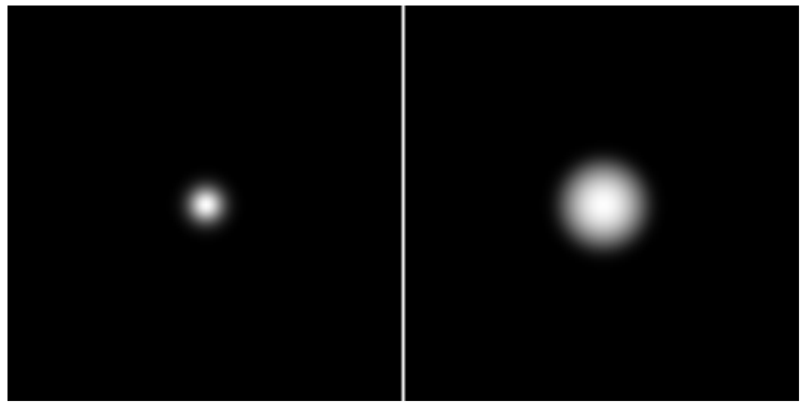
Rectangle



Its DFT

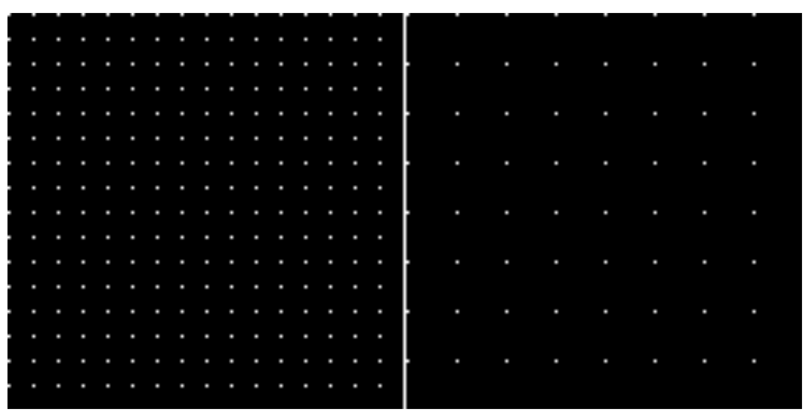
Two-Dimensional DFT with Different Functions

2D Gaussian function



Its DFT

Impulses



Its DFT

Filtering in the Frequency Domain

Advantages of Filtering in the Frequency Domain

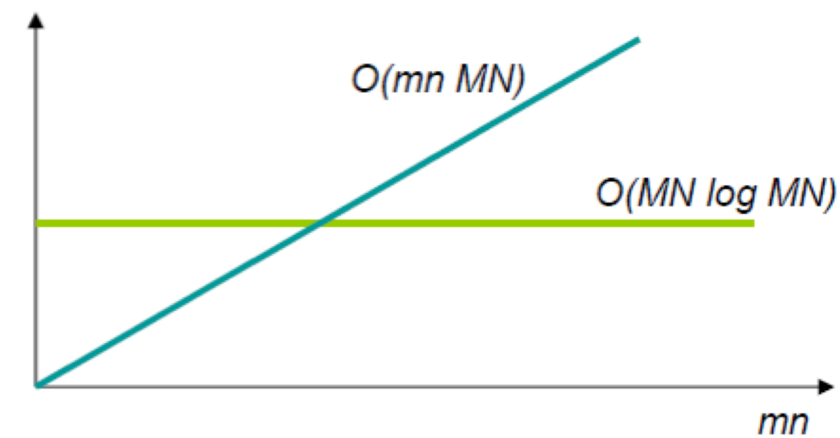
- Cost (number of operations) of the computation of the Fast Fourier Transform is $O(MN \log MN)$ where MN = number of pixels in image.

- The total cost of filtering in the frequency domain is dominated by FFT.

- Compare this to convolution in the spatial domain –it is $O((mn)(MN))$

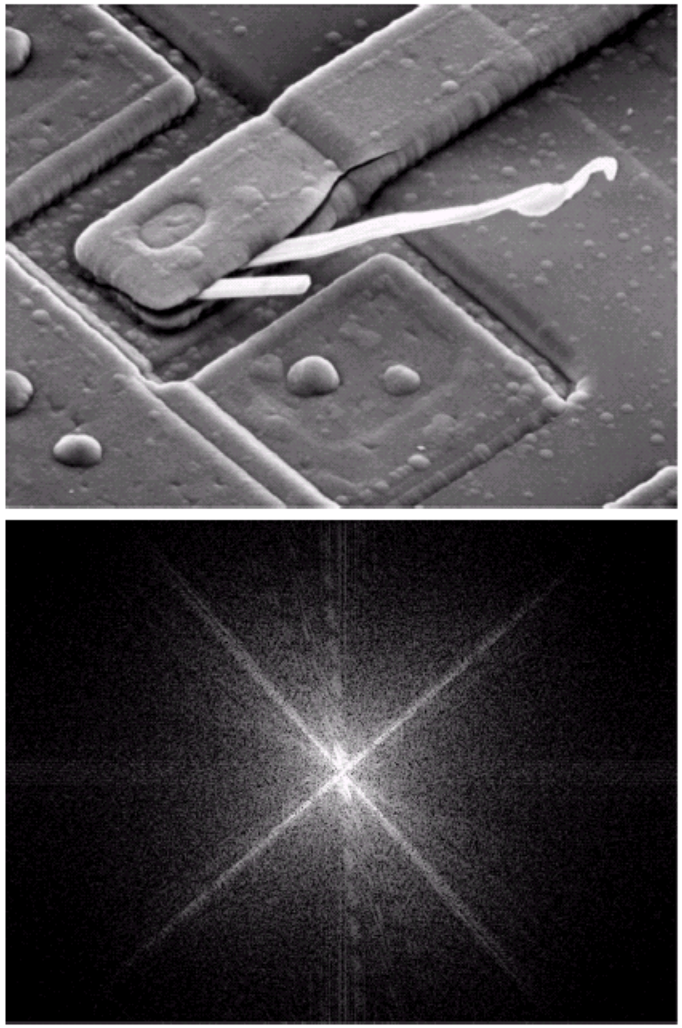
Advantages of Filtering in the Frequency Domain

Plot of cost vs mn, with image size MN fixed



Convolution in the frequency domain faster for large kernels (when mn gets much larger than log(MN))

Filtering in the Frequency Domain



a
b

FIGURE 4.4
(a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

Basics of Filtering in the Frequency Domain

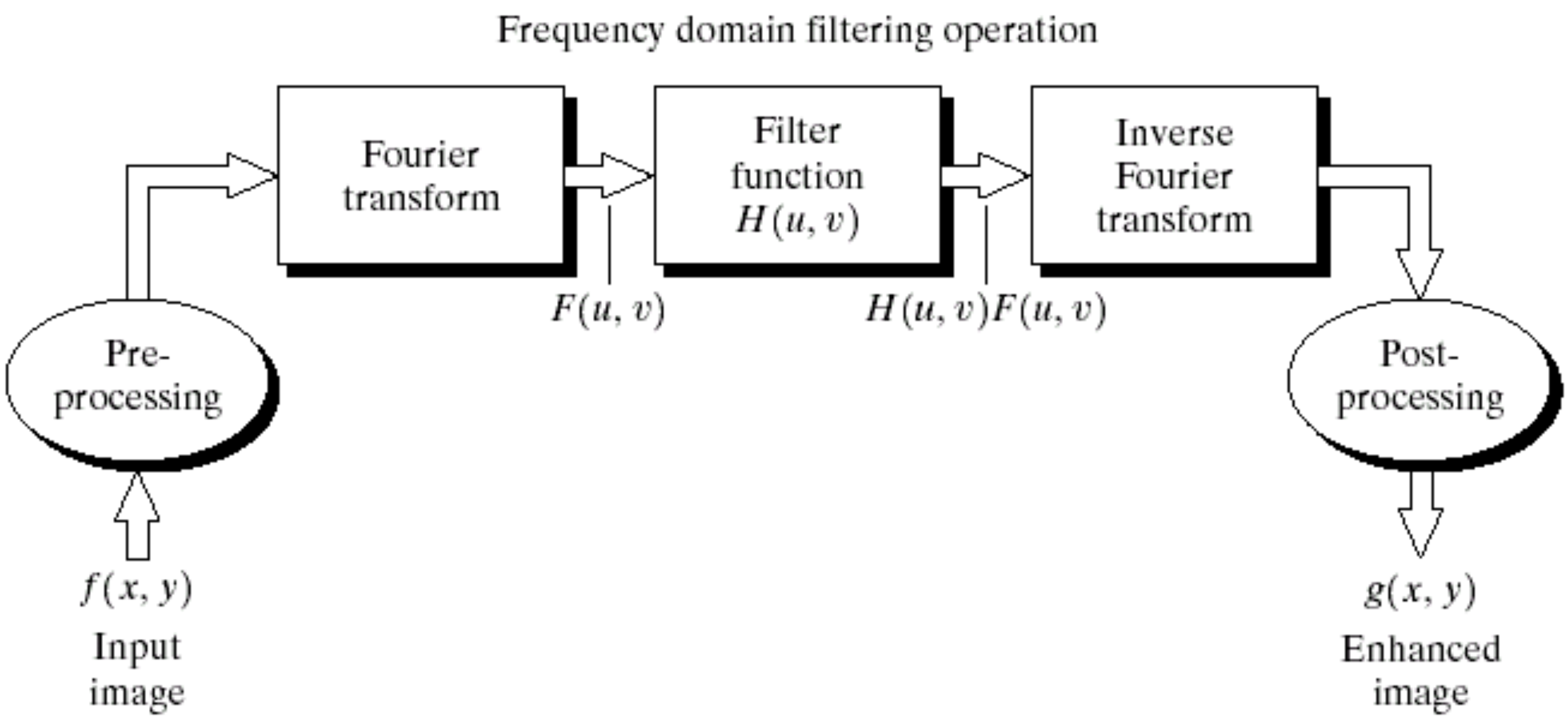


FIGURE 4.5 Basic steps for filtering in the frequency domain.

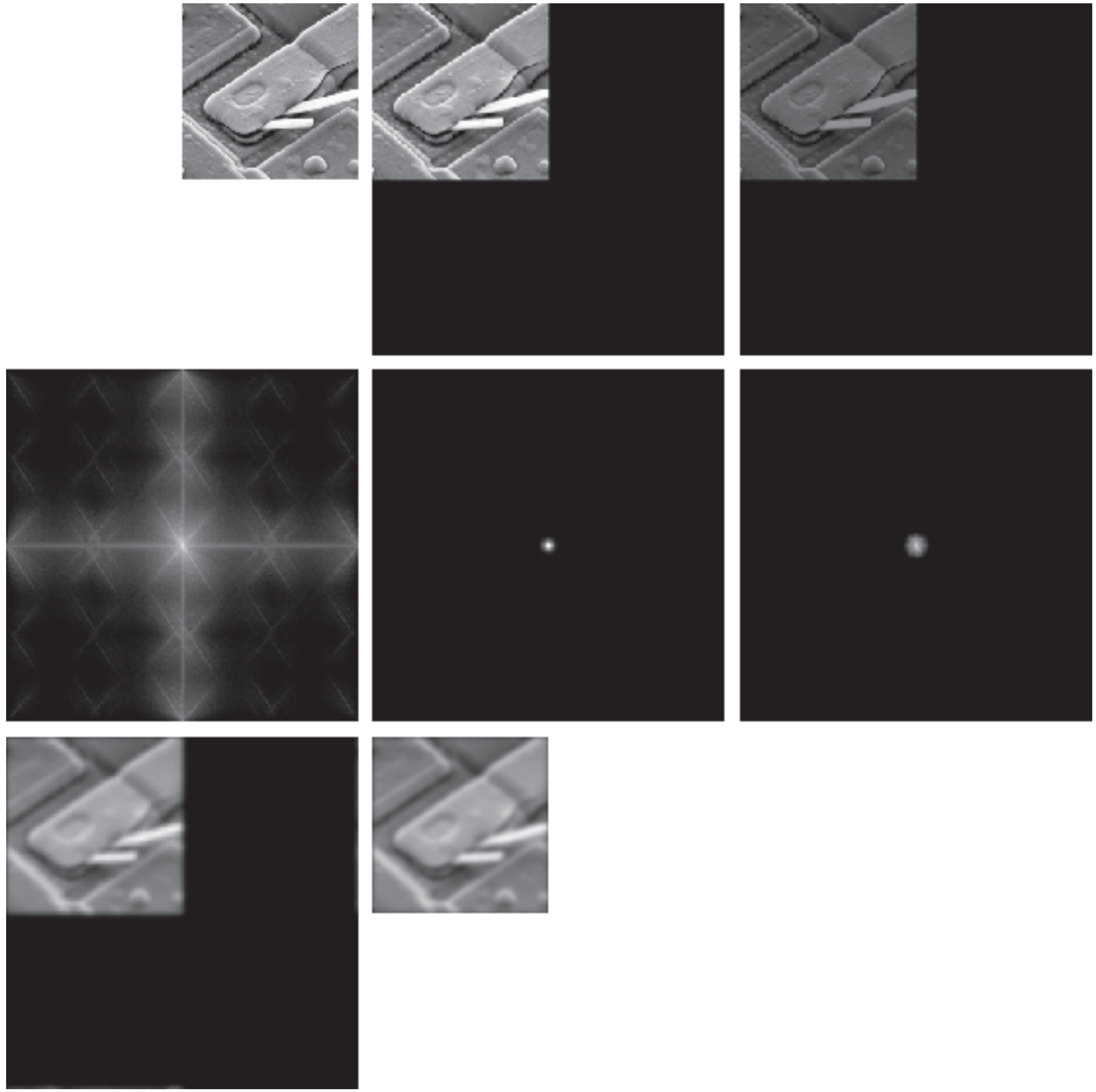
Frequency domain filtering steps

1. Given an input image $f(x,y)$ of size $M \times N$, obtain the padding size of $P = 2M$ and $Q = 2N$.
2. Form a padded image $f_p(x,y)$ of size $P \times Q$
3. Multiply $f_p(x,y)$ by $(-1)^{x+y}$ to center the Fourier transform on the $P \times Q$ frequency rectangle
4. Compute the DFT, of the image from Step 3.
5. Construct a real, symmetric filter transfer function, $H(u,v)$, of size $P \times Q$ with center at $(P/2, Q/2)$.
6. Form the product $G(u,v) = H(u,v)F(u,v)$ using element-wise multiplication
7. Obtain the filtered image (of size P x Q) by computing the IDFT of $G(u,v)$:

$$g_p(x,y) = (real\ [F^{-1}\{G(u,v)\}])(-1)^{x+y}$$

8. Obtain the final filtered result, $g(x,y)$ of the same size input image by extracting the $M \times N$ region from the top left quadrant of $g_p(x,y)$.

Frequency domain filtering steps



a b c
d e f
g h

FIGURE 4.35
(a) An $M \times N$ image, f.
(b) Padded image, f_p of size $P \times Q$.
(c) Result of multiplying f_p by $(-1)^{x+y}$.
(d) Spectrum of F. (e) Centered Gaussian lowpass filter transfer function, H, of size $P \times Q$.
(f) Spectrum of the product HF.
(g) Image g_p, the real part of the IDFT of HF, multiplied by $(-1)^{x+y}$.
(h) Final result, g, obtained by extracting the first M rows and N columns of g_p.

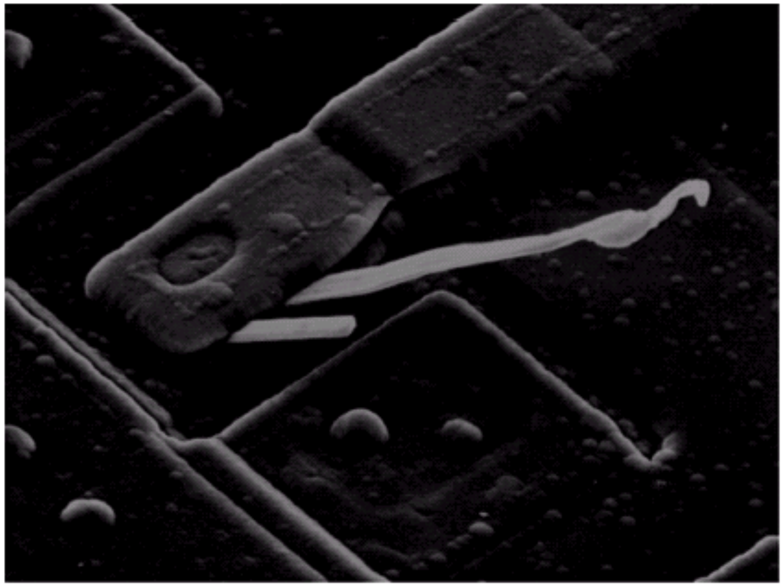
Some Basic Filters and Their Functions

- Multiply all values of $F(u,v)$ by the filter function (notch filter):

$$H(u,v) = \begin{cases} 0 & \text{if } (u,v) = (M/2, N/2) \\ 1 & \text{otherwise.} \end{cases}$$

  – All this filter would do is set $F(0,0)$ to zero (force the average value of an image to zero) and leave all frequency components of the Fourier transform untouched.

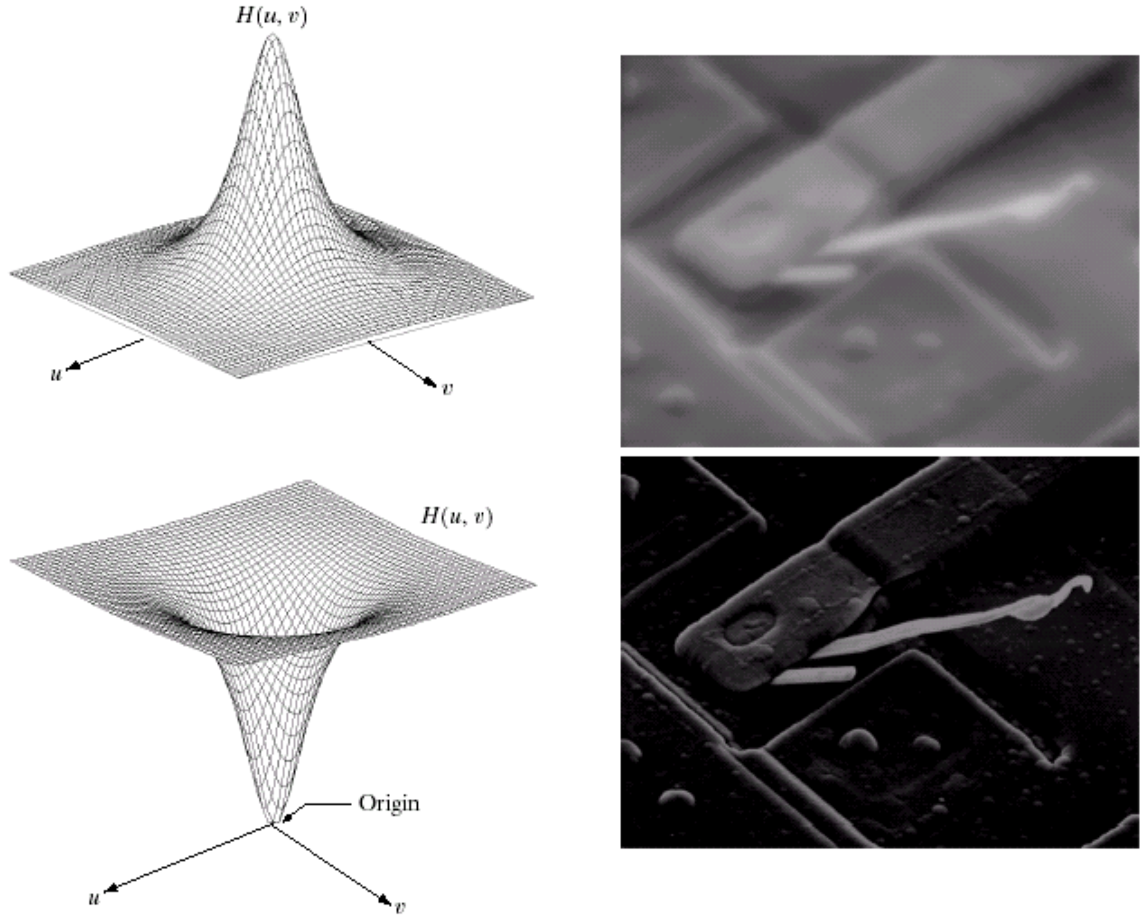
FIGURE 4.6 Result of filtering the image in Fig. 4.4(a) with a notch filter that set to 0 the $F(0,0)$ term in the Fourier transform.



Some Basic Filters and Their Functions

Lowpass filter

Highpass filter

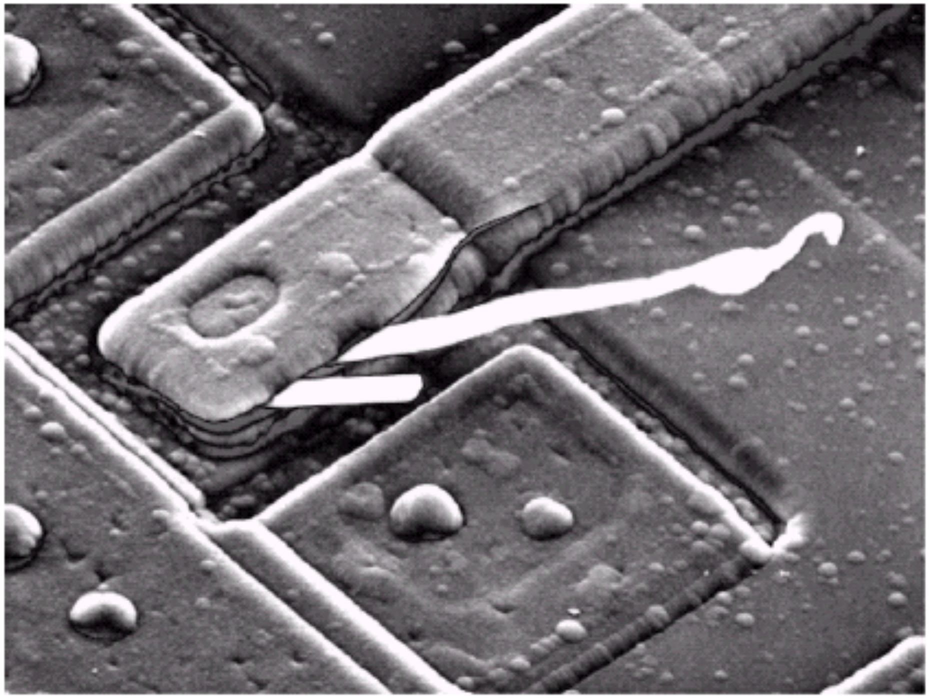


a b
c d

FIGURE 4.7 (a) A two-dimensional lowpass filter function. (b) Result of lowpass filtering the image in Fig. 4.4(a). (c) A two-dimensional highpass filter function. (d) Result of highpass filtering the image in Fig. 4.4(a).

Some Basic Filters and Their Functions

FIGURE 4.8 Result of highpass filtering the image in Fig. 4.4(a) with the filter in Fig. 4.7(c), modified by adding a constant of one-half the filter height to the filter function. Compare with Fig. 4.4(a).



Correspondence between Filtering in the Spatial and Frequency Domain

- Convolution theorem:
  - The discrete convolution of two functions $f(x,y)$ and $h(x,y)$ of size MxN is defined as

$$f(x,y)*h(x,y)=\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}f(m,n)h(x-m,y-n)$$

  - Let $F(u,v)$ and $H(u,v)$ denote the Fourier transforms of $f(x,y)$ and $h(x,y)$, then

$$f(x,y)*h(x,y)\Leftrightarrow F(u,v)H(u,v)$$ Eq. (4.2-31)

$$f(x,y)h(x,y)\Leftrightarrow F(u,v)*H(u,v)$$ Eq. (4.2-32)

Correspondence between Filtering in the Spatial and Frequency Domain

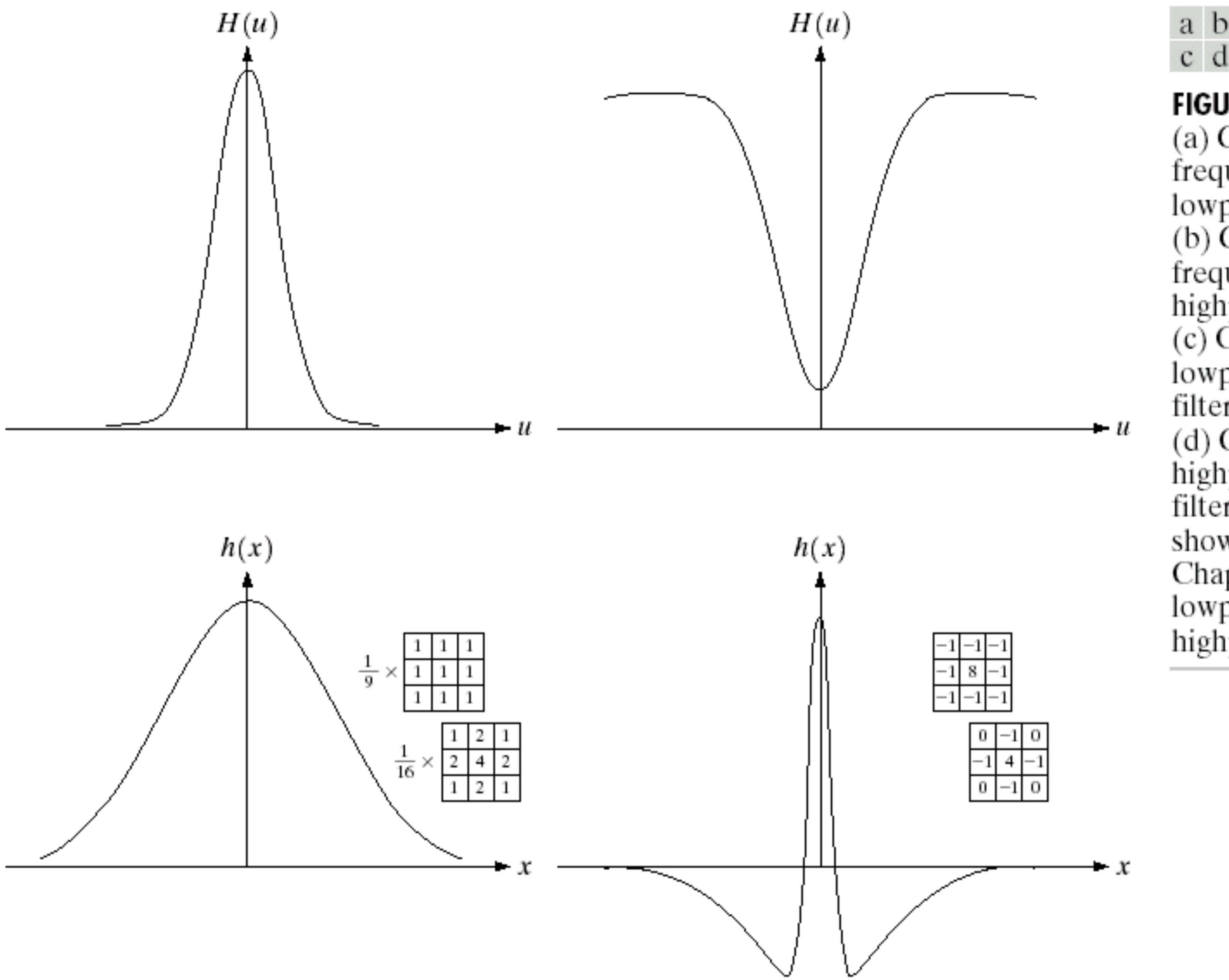


FIGURE 4.9 (a) Gaussian frequency domain lowpass filter. (b) Gaussian frequency domain highpass filter. (c) Corresponding lowpass spatial filter. (d) Corresponding highpass spatial filter. The masks shown are used in Chapter 3 for lowpass and highpass filtering.

Smoothing Frequency-Domain Filters

- The basic model for filtering in the frequency domain

$$G(u,v) = H(u,v)F(u,v)$$

  where $F(u,v)$: the Fourier transform of the image to be smoothed

  $H(u,v)$: a filter transfer function

- Smoothing is fundamentally a lowpass operation in the frequency domain.
- There are several standard forms of lowpass filters (LPF).
  - Ideal lowpass filter
  - Butterworth lowpass filter
  - Gaussian lowpass filter

Frequency Domain Low Pass Filters

Ideal Lowpass Filters (ILPFs)

- The simplest lowpass filter is a filter that "cuts off" all high-frequency components of the Fourier transform that are at a distance greater than a specified distance D_0 from the origin of the transform.
- The transfer function of an ideal lowpass filter

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

where $D(u,v)$: the distance from point (u,v) to the center of the frequency rectangle

$$D(u,v) = \left[(u - M/2)^2 + (v - N/2)^2\right]^{\frac{1}{2}}$$

Ideal Lowpass Filters (ILPFs)

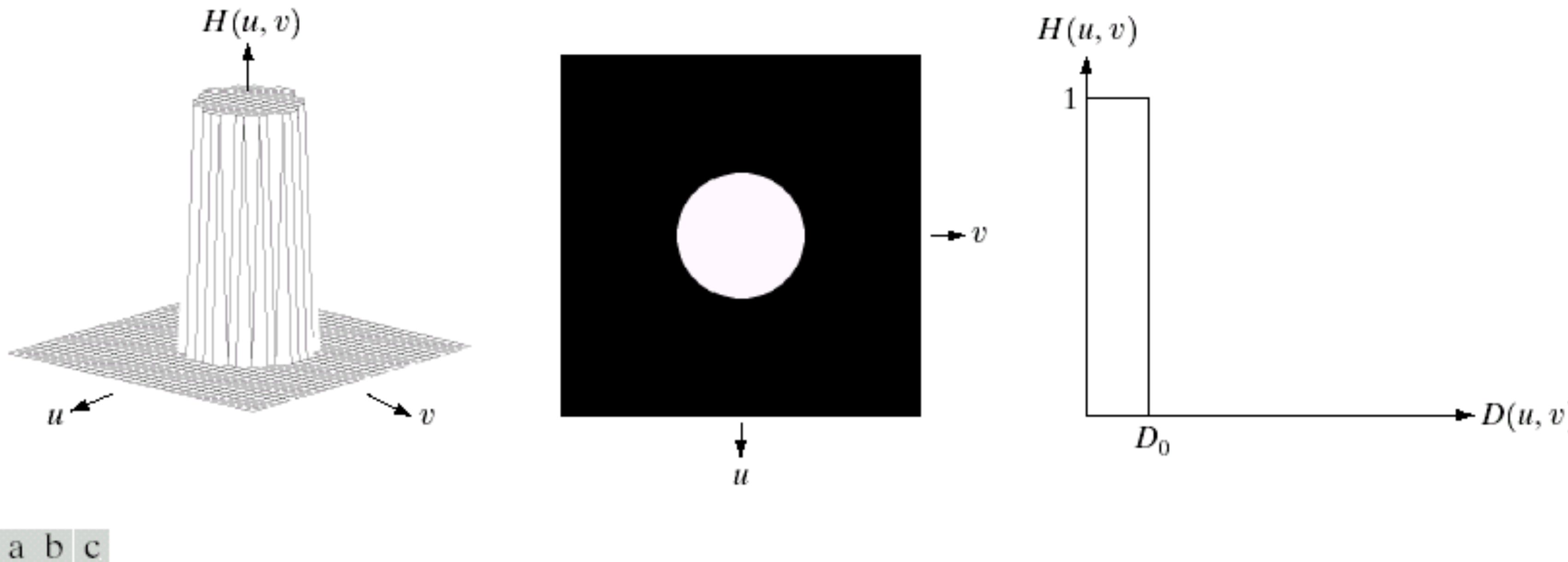
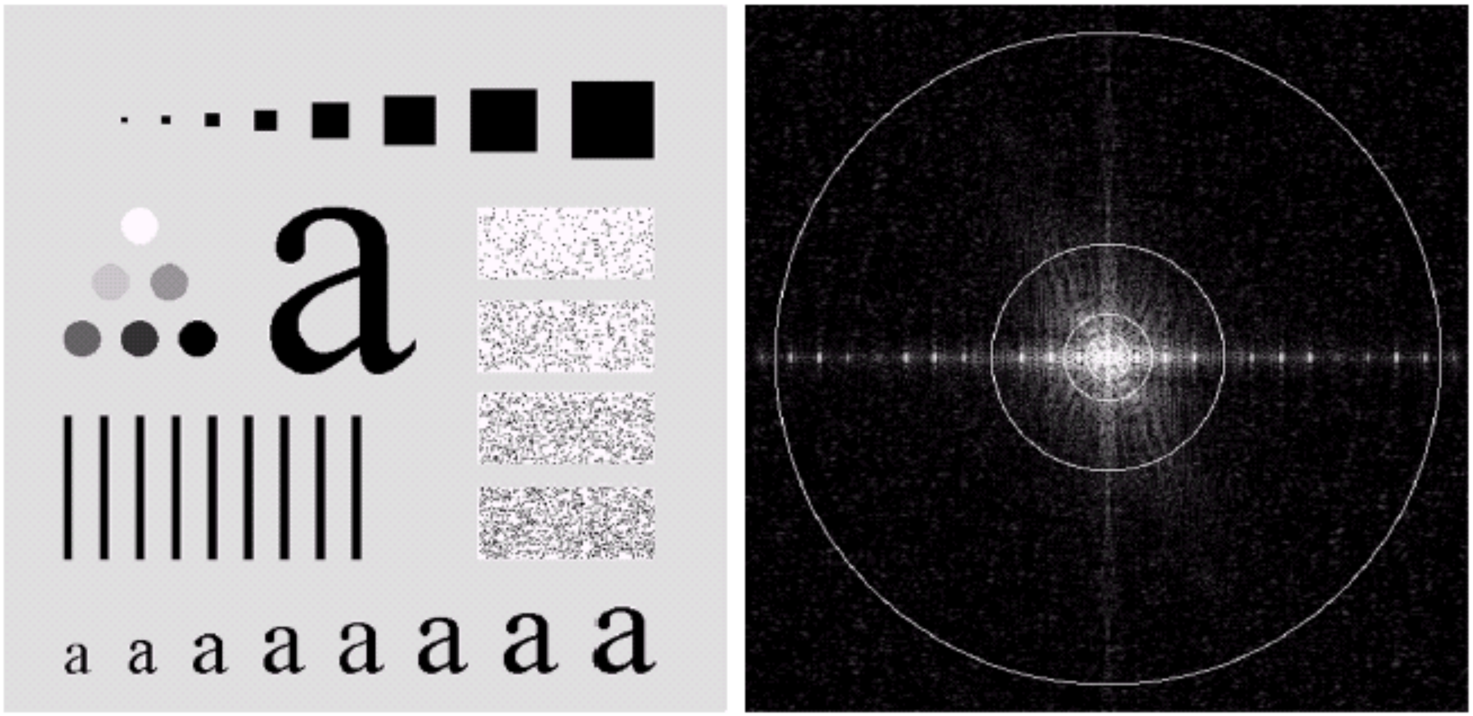


FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

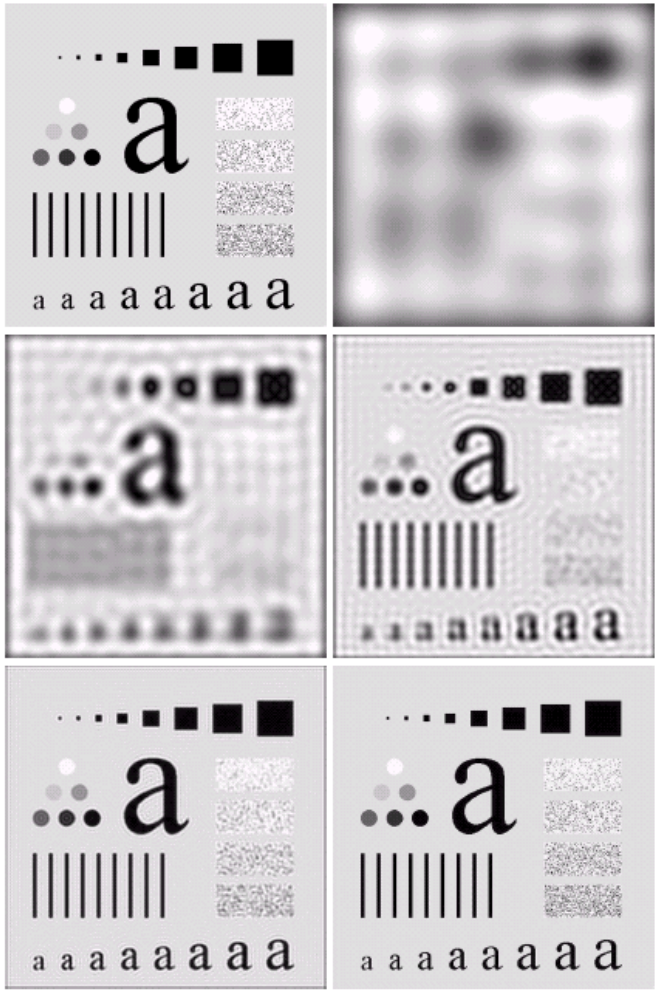
Ideal Lowpass Filters (ILPFs)



a b

FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

Ideal Lowpass Filters

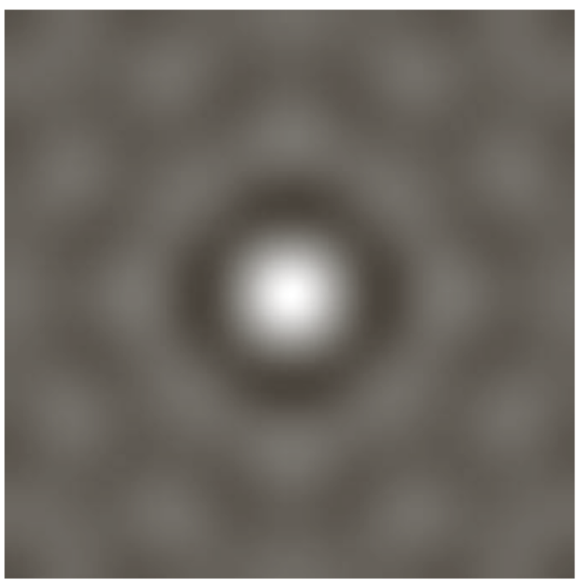


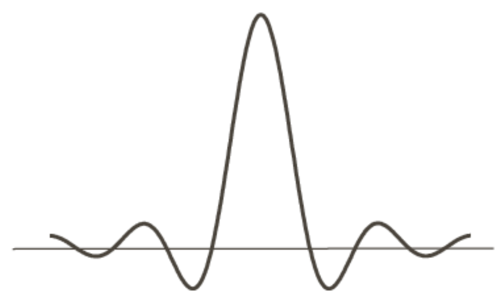
a	b
c	d
e	f

FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

Ideal Lowpass Filters

- ILPF in the spatial domain is a sinc function that has to be truncated and produces ringing effects.
- The main lobe is responsible for blurring and the side lobes are responsible for ringing.





Butterworth Lowpass Filters (BLPFs) With order *n*

$$H(u,v) = \frac{1}{1 + \left[D(u,v) / D_0 \right]^{2n}}$$

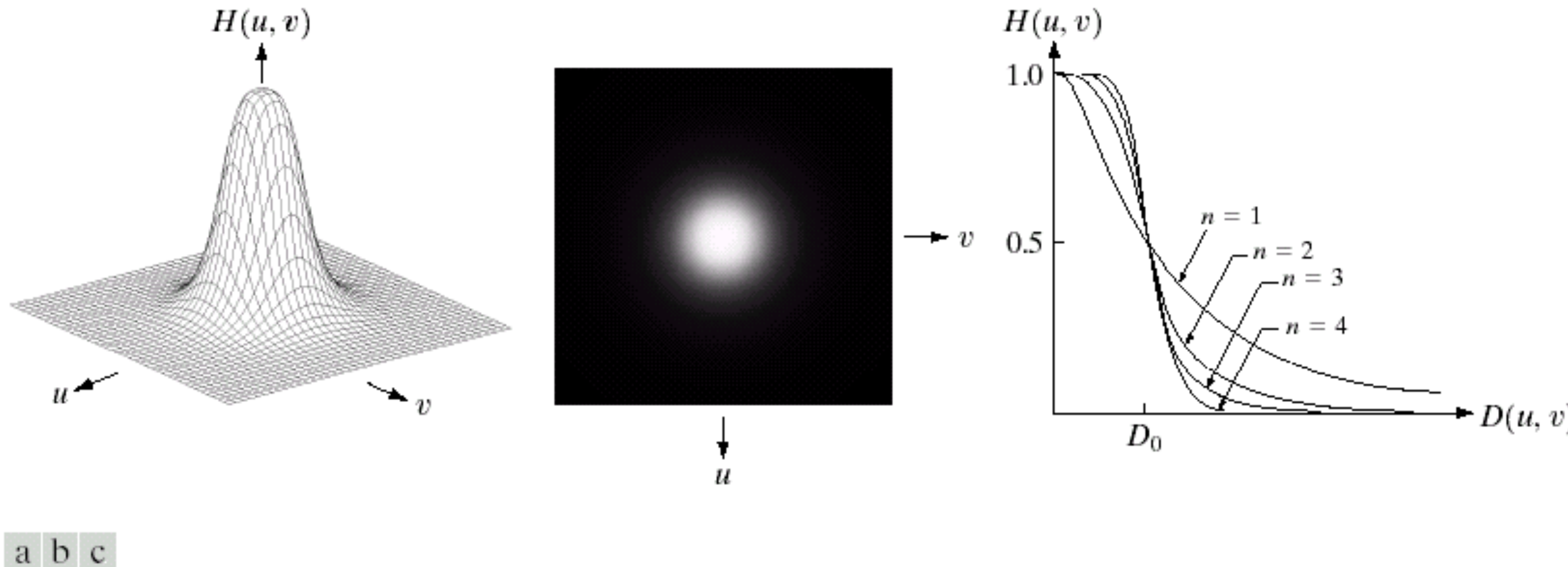
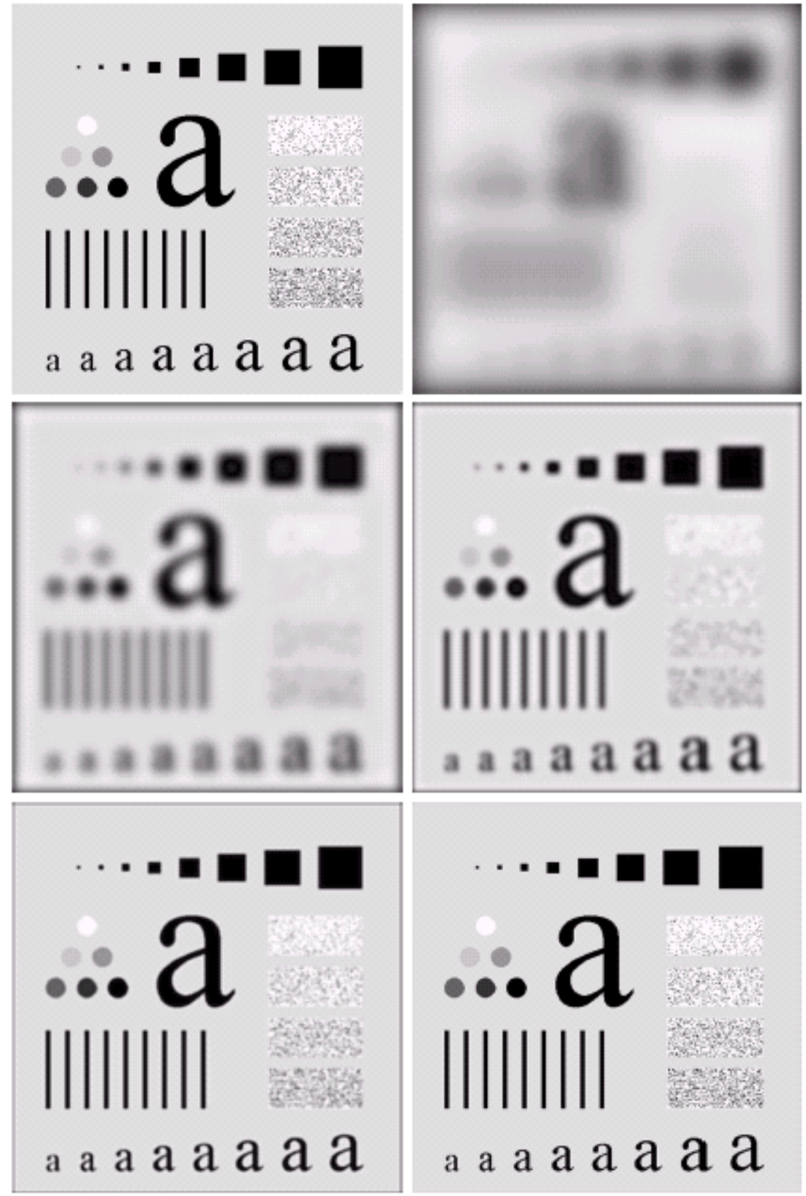


FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

Butterworth Lowpass Filters (BLPFs)

n=2

D_0=5,15,30,80,and 230



a b
c d
e f

FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

Butterworth Lowpass Filters (BLPFs)Spatial Representation

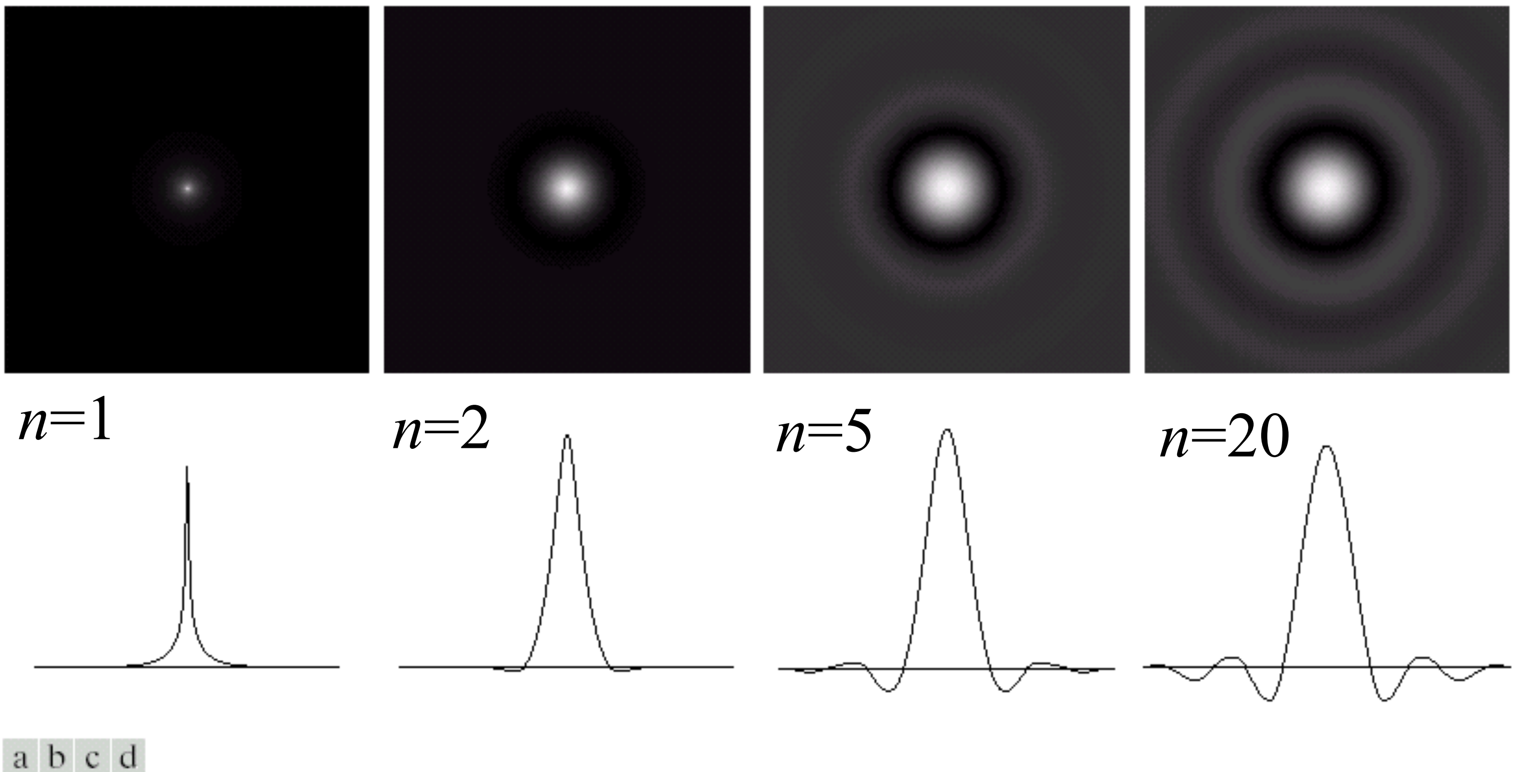
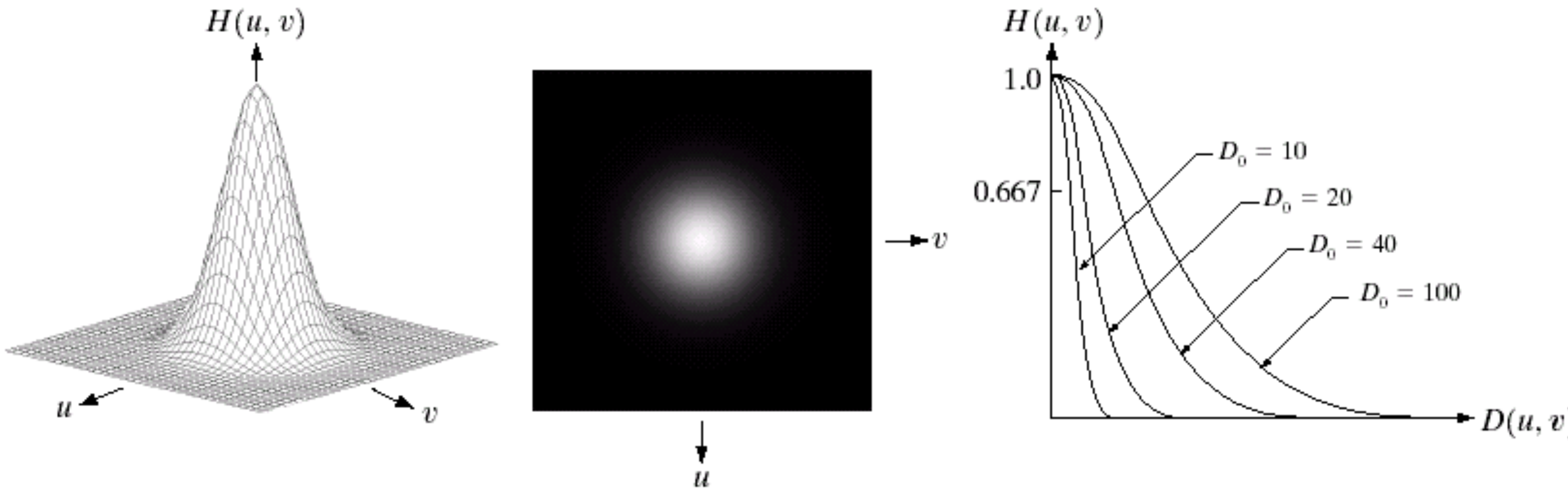


FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

Gaussian Lowpass Filters (FLPFs)

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$



a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

Gaussian Lowpass Filters (FLPFs)

D_0=5,15,30,80,and 230

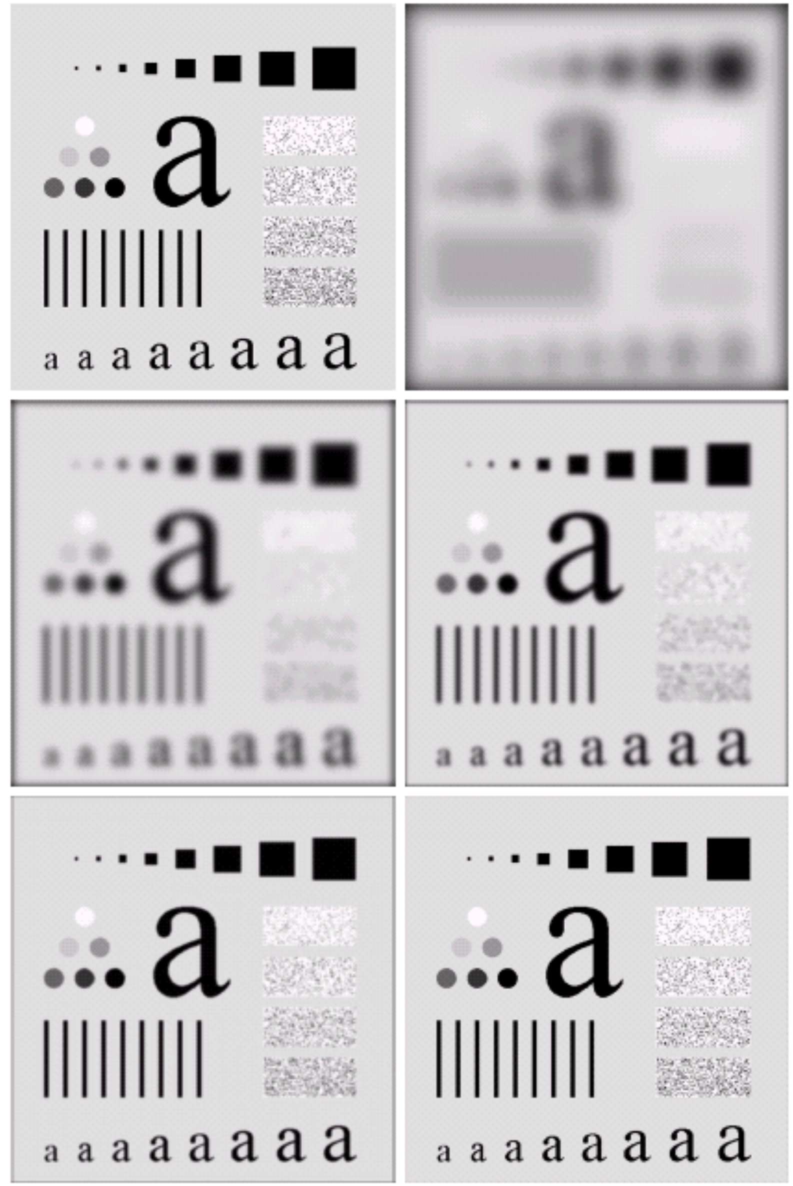


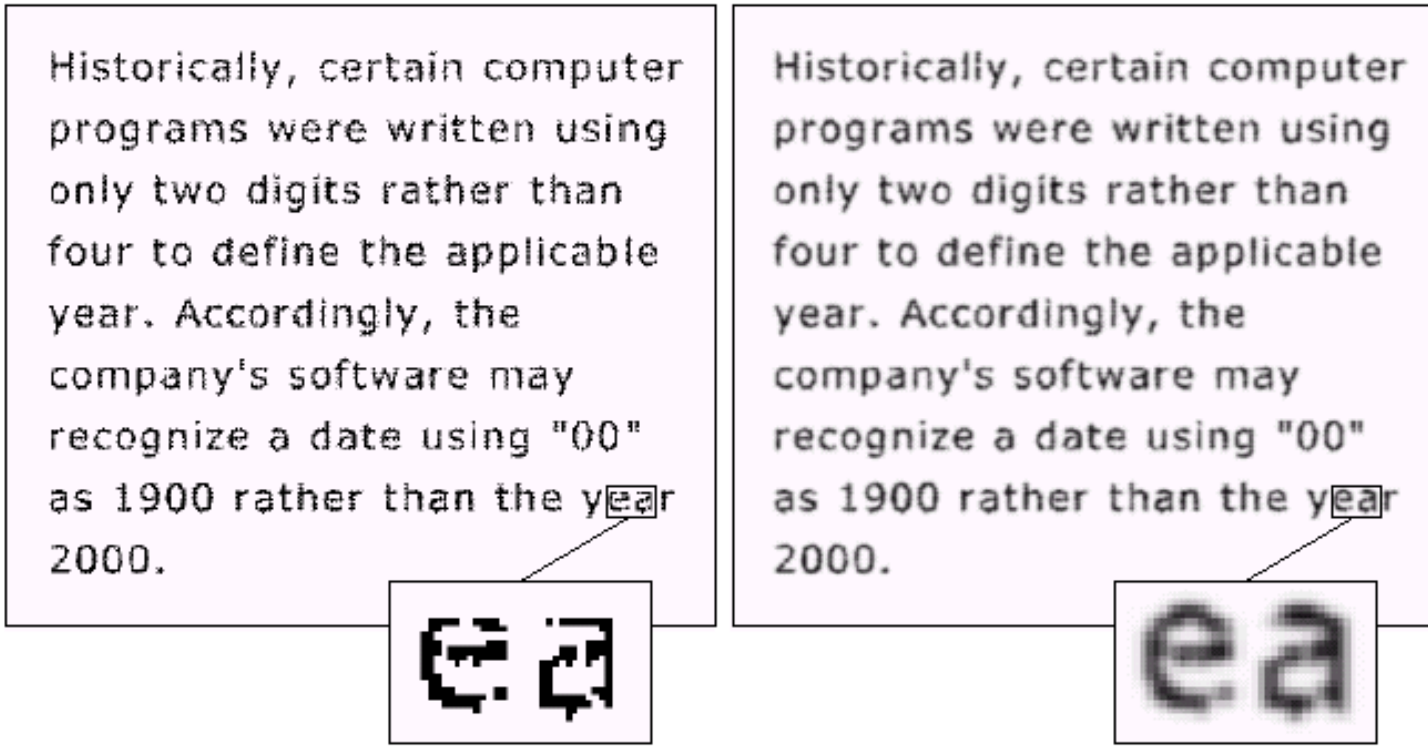
FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

a	b
c	d
e	f

Additional Examples of Lowpass Filtering

a b

FIGURE 4.19
(a) Sample text of poor resolution (note broken characters in magnified view).
(b) Result of filtering with a GLPF (broken character segments were joined).



Additional Examples of Lowpass Filtering

a b c

FIGURE 4.20 (a) Original image (1028 × 732 pixels). (b) Result of filtering with a GLPF with $D_0 = 100$. (c) Result of filtering with a GLPF with $D_0 = 80$. Note reduction in skin fine lines in the magnified sections of (b) and (c).

High Pass Frequency Domain Filters

Sharpening Frequency Domain Filter

$$H_{hp}(u,v) = H_{lp}(u,v)$$

Ideal highpass filter

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

Butterworth highpass filter

$$H(u,v) = \frac{1}{1 + \left[D_0 / D(u,v)\right]^{2n}}$$

Gaussian highpass filter

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

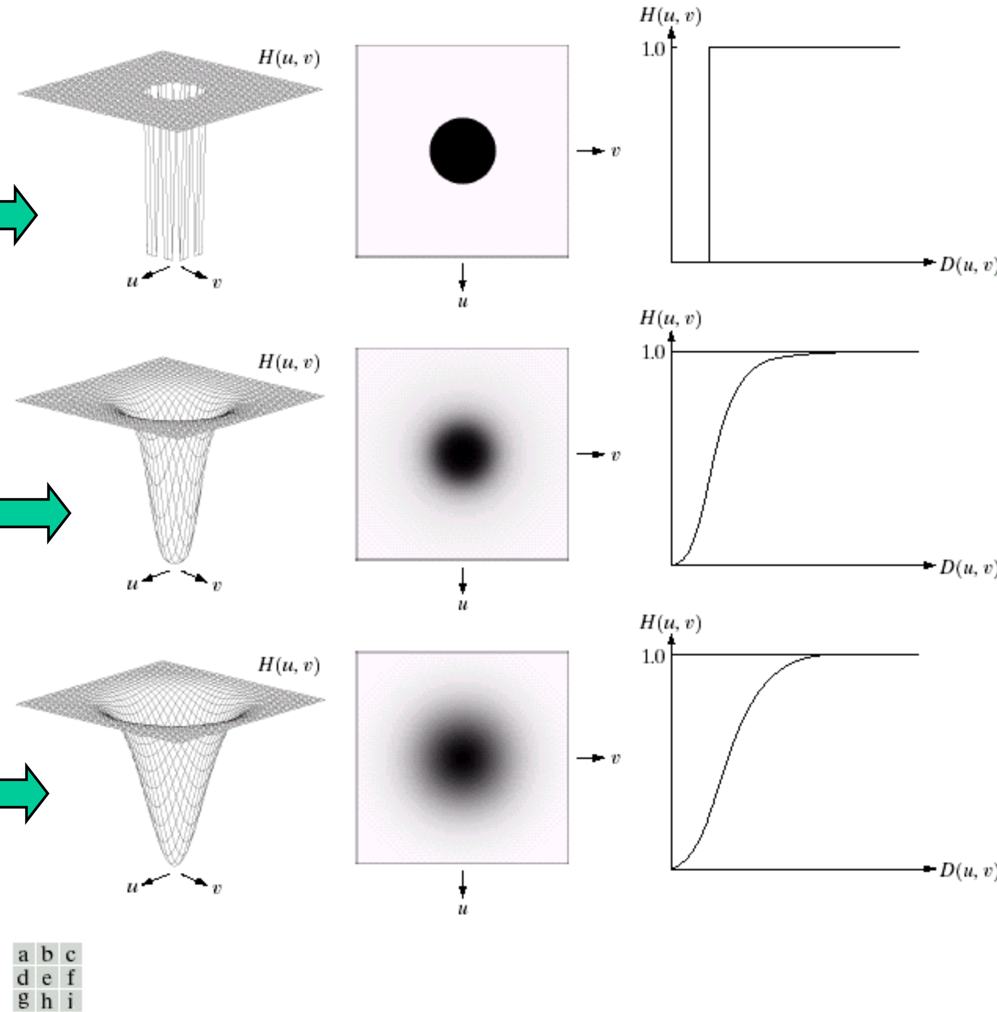


FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

Highpass Filters Spatial Representation

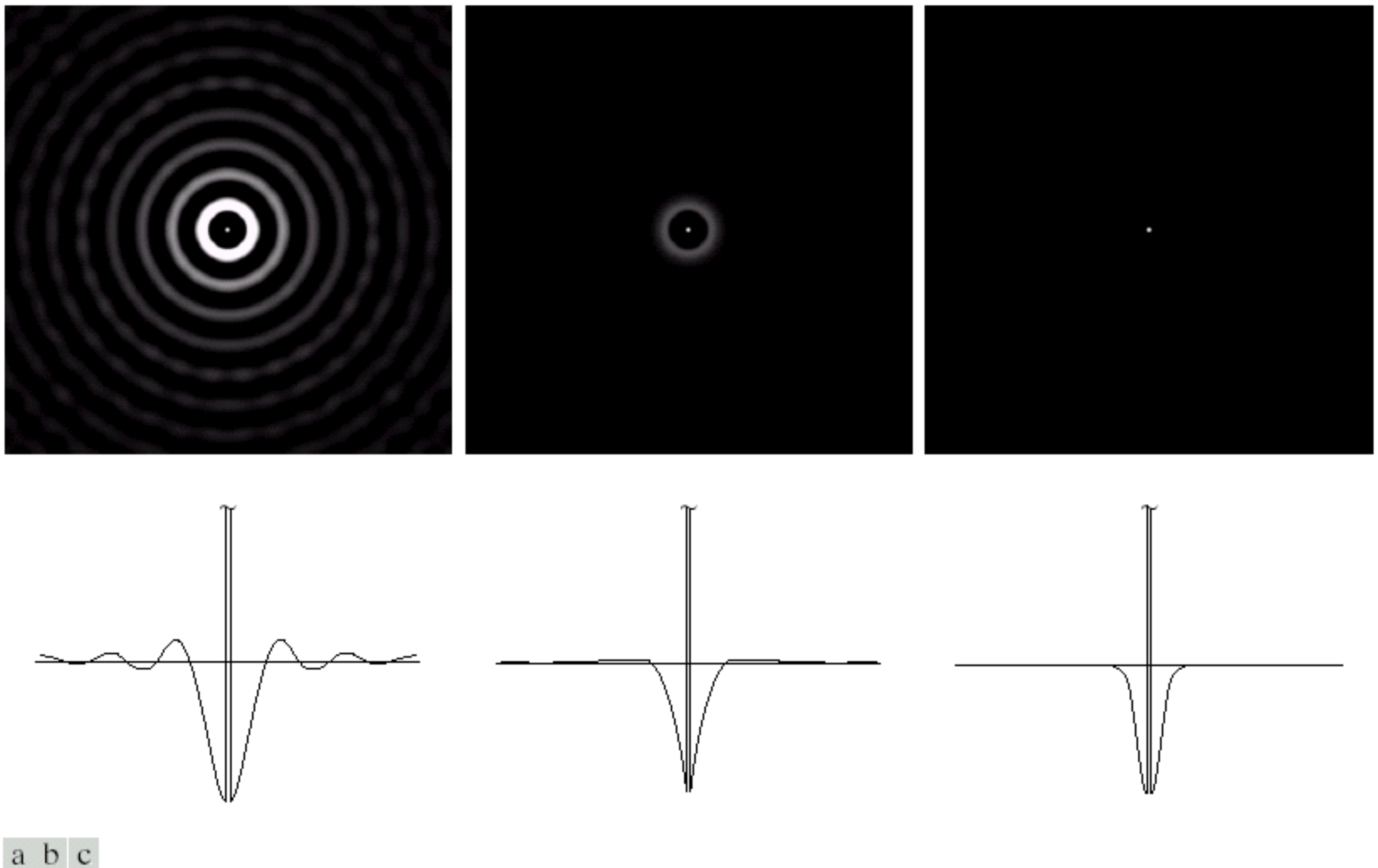
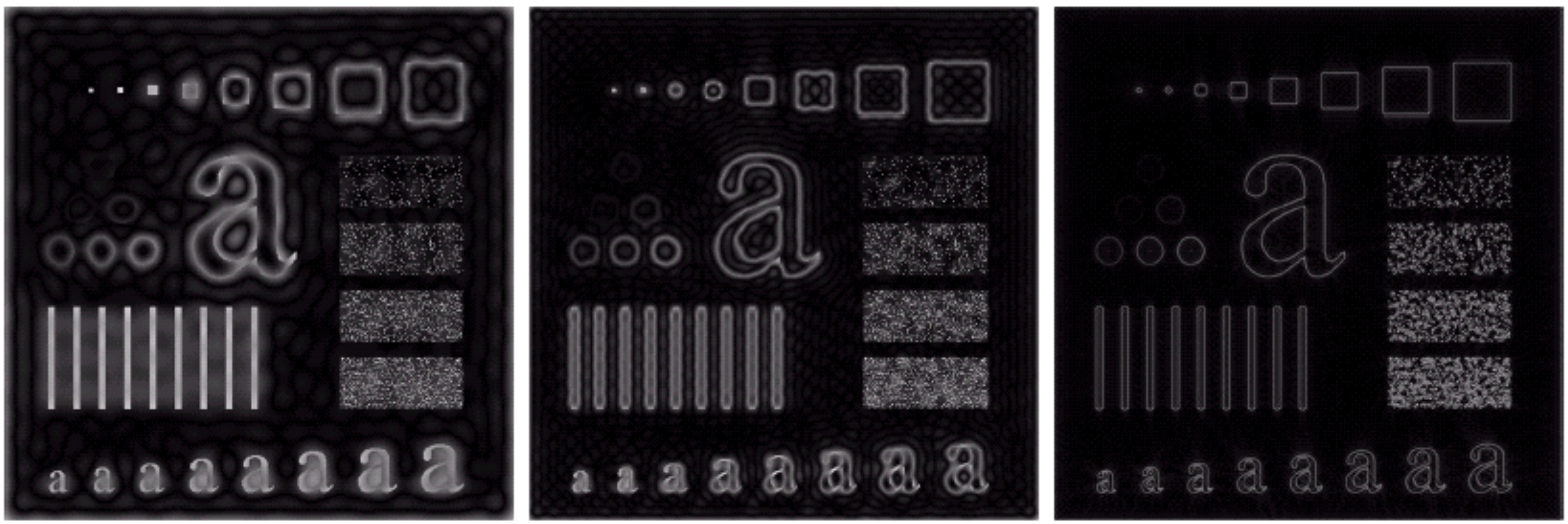


FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

Ideal Highpass Filters

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

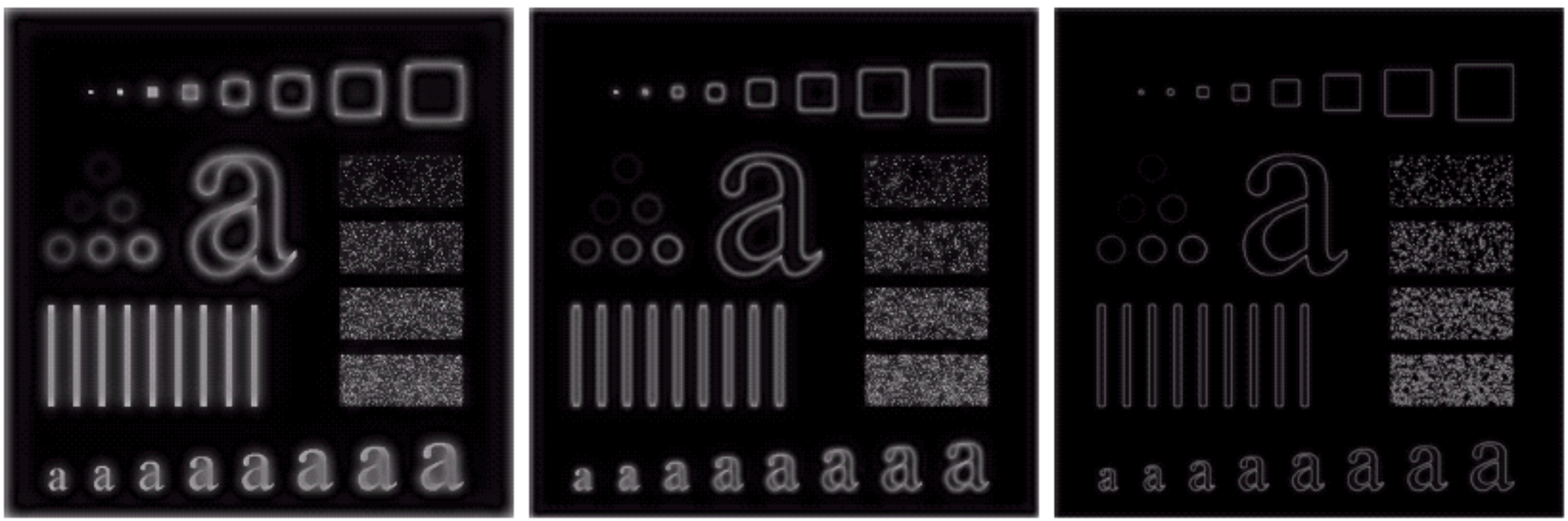


a b c

FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0 = 15$, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).

Butterworth Highpass Filters

$$H(u,v) = \frac{1}{1 + \left[D_0 / D(u,v)\right]^{2n}}$$



a b c

FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

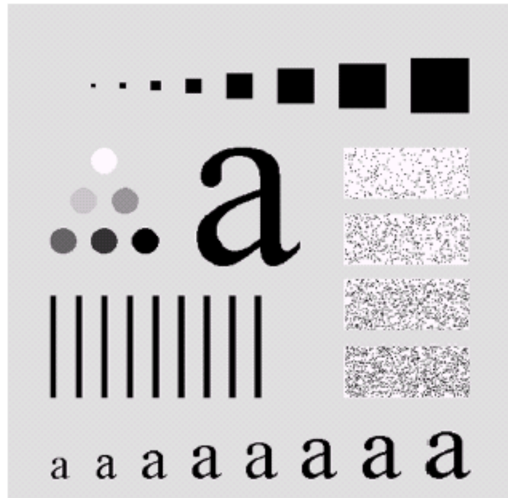
Gaussian Highpass Filters

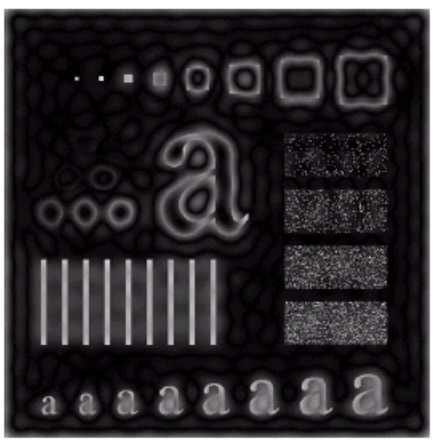
$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

a b c

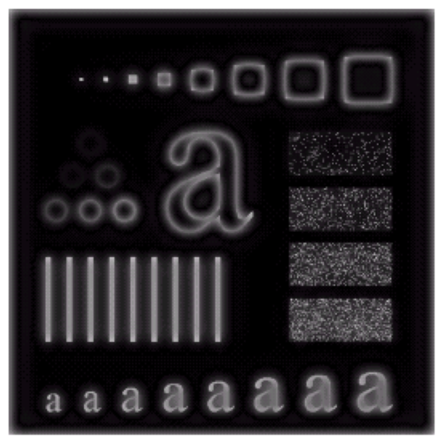
FIGURE 4.26 Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.

Highpass Filter Comparison

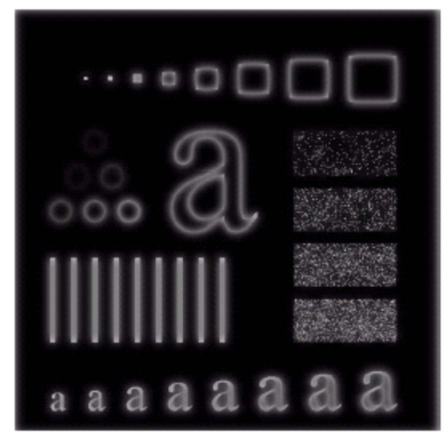




IHPF D_0 = 15



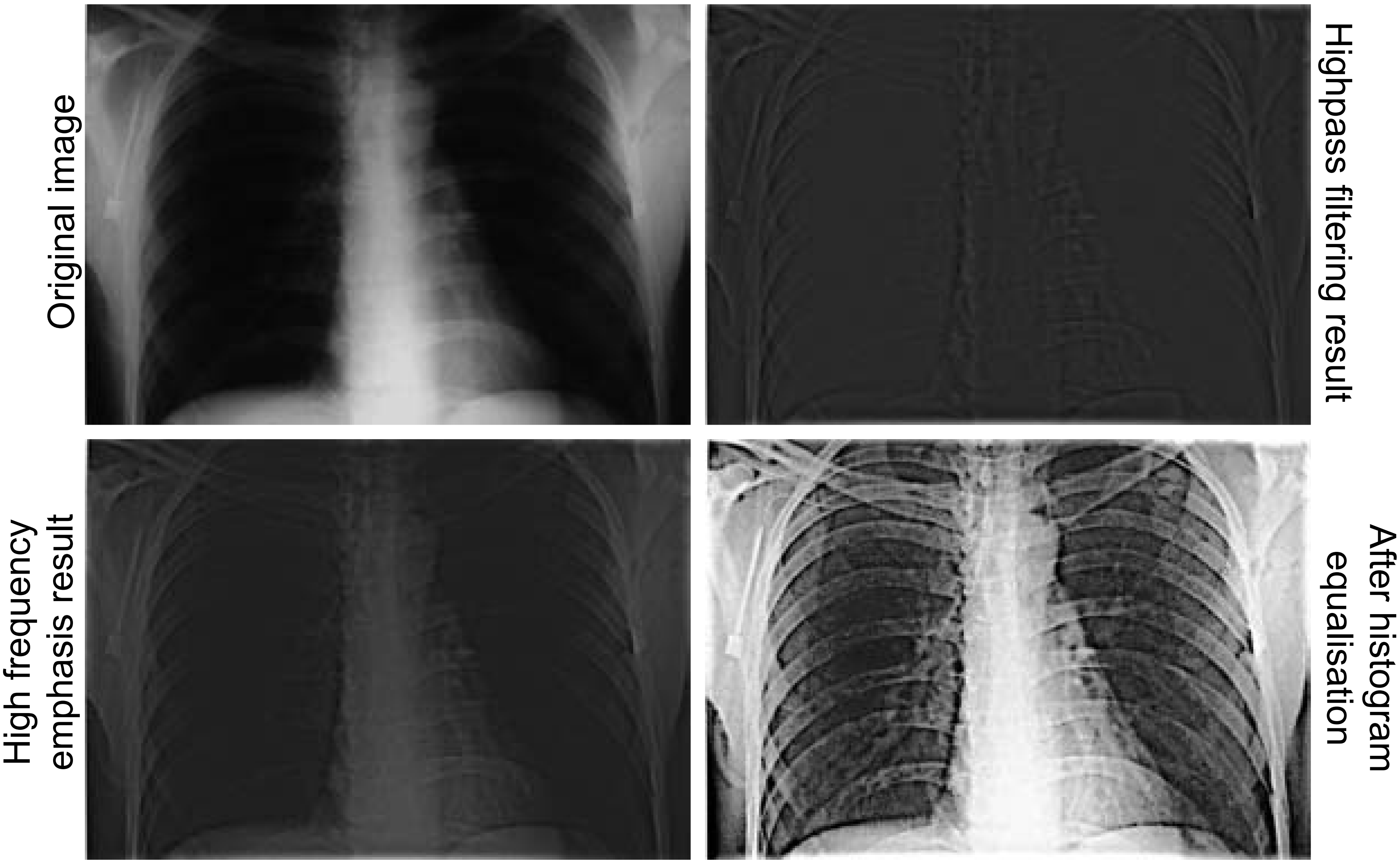
BHPF n=2, D_0 =15



Gaussian HPF

n=2, D_0 =15

Highpass Filtering Example



Laplacian in Frequency Domain

$$\Im\left[\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}\right] = -(u^2 + v^2)F(u,v)$$

$$H_1(u,v) = -(u^2 + v^2)$$

Frequency domain

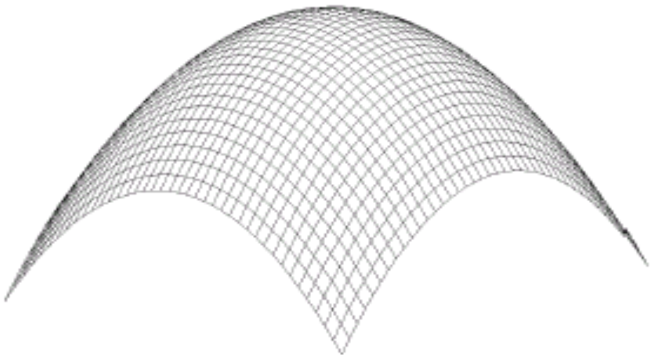
$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

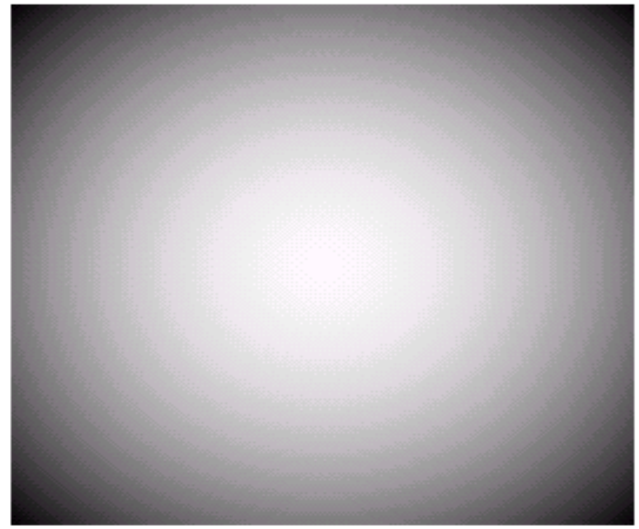
Spatial domain

Laplacian operator

Laplacian In The Frequency Domain

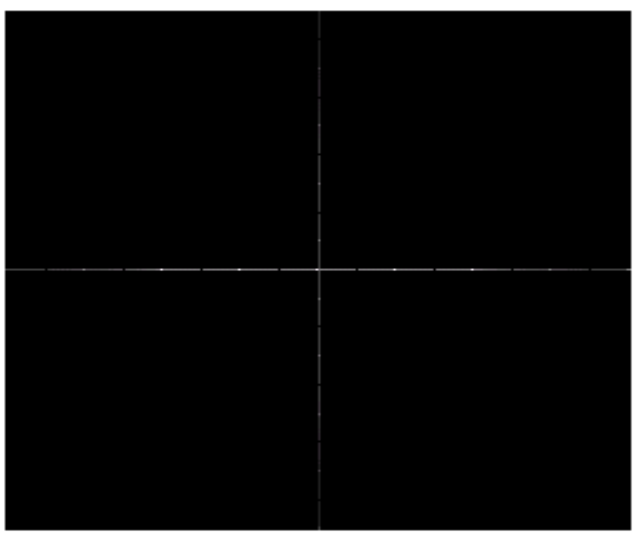
Laplacian in the frequency domain

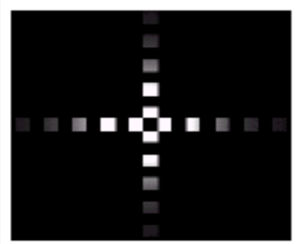




2-D image of Laplacian in the frequency domain

Inverse DFT of Laplacian in the frequency domain





0	1	0
1	−4	1
0	1	0

Zoomed section of the image on the left compared to spatial filter

Subtract Laplacian from the Original Image to Enhance It

enhanced image **Original image** **Laplacian output**

Spatial domain

$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$

Frequency domain

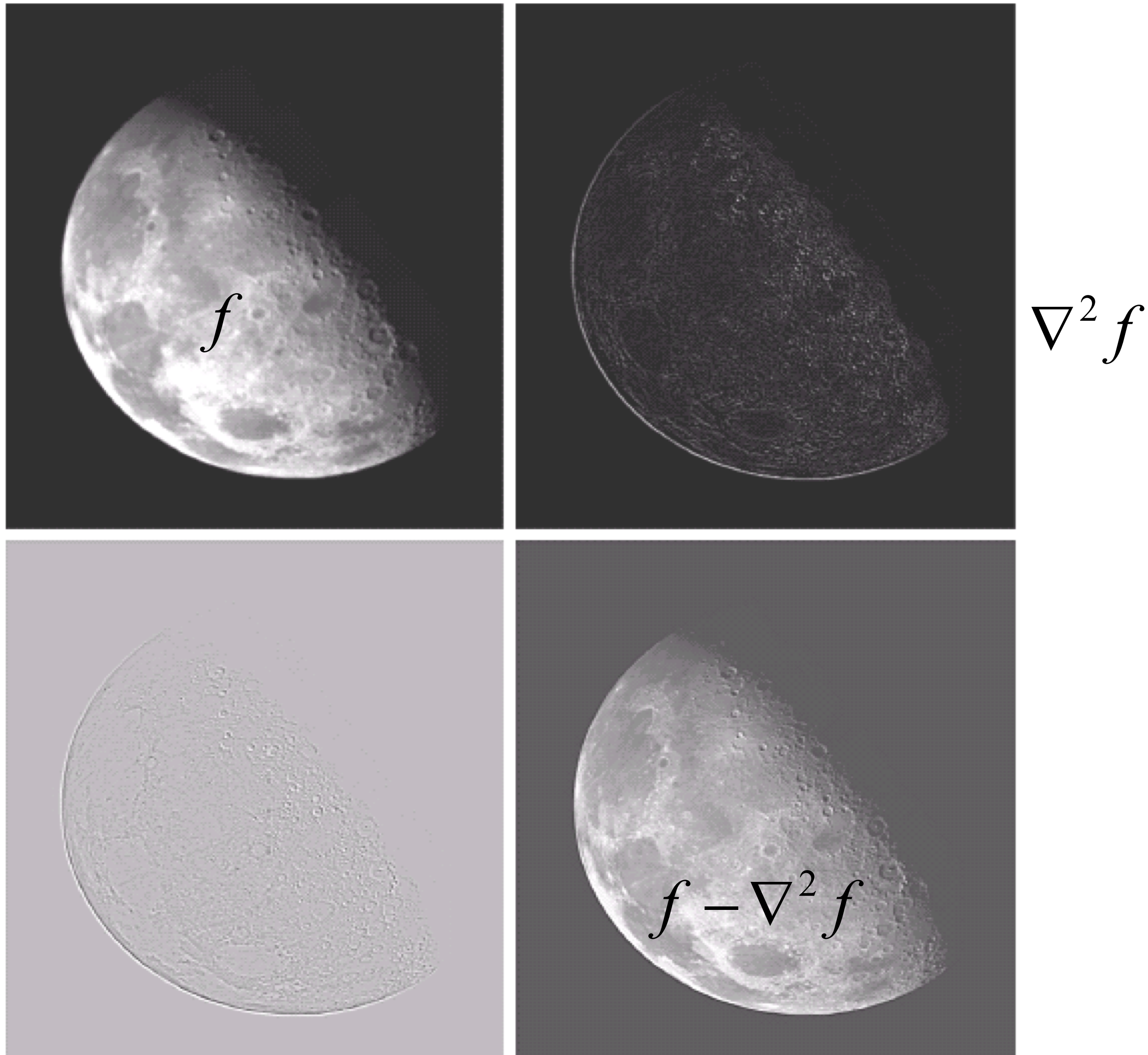
$$G(u,v) = F(u,v) + (u^2 + v^2)F(u,v)$$

new operator $$H_2(u,v) = 1 + (u^2 + v^2) = 1 - \underbrace{H_1(u,v)}_{\text{Laplacian}}$$

Frequency Domain Laplacian Example

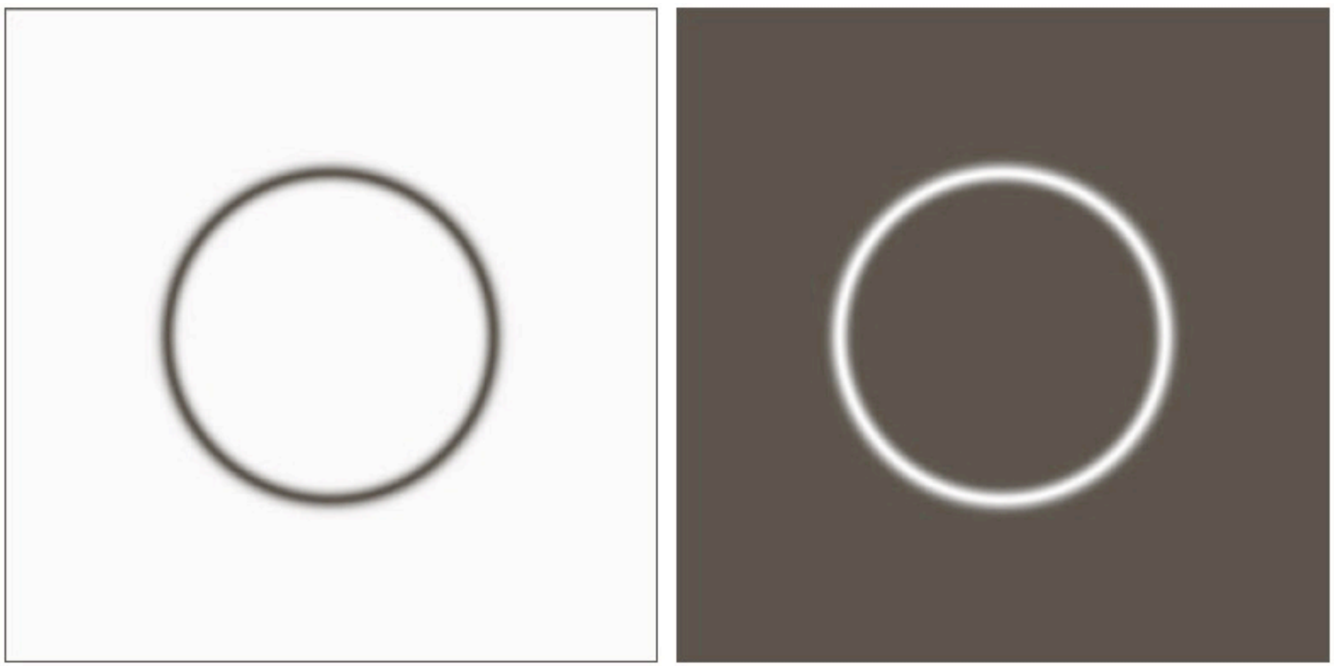
a b
c d

FIGURE 4.28
(a) Image of the North Pole of the moon.
(b) Laplacian filtered image.
(c) Laplacian image scaled.
(d) Image enhanced by using Eq. (4.4-12).
(Original image courtesy of NASA.)



Band Pass and Band Reject Filters

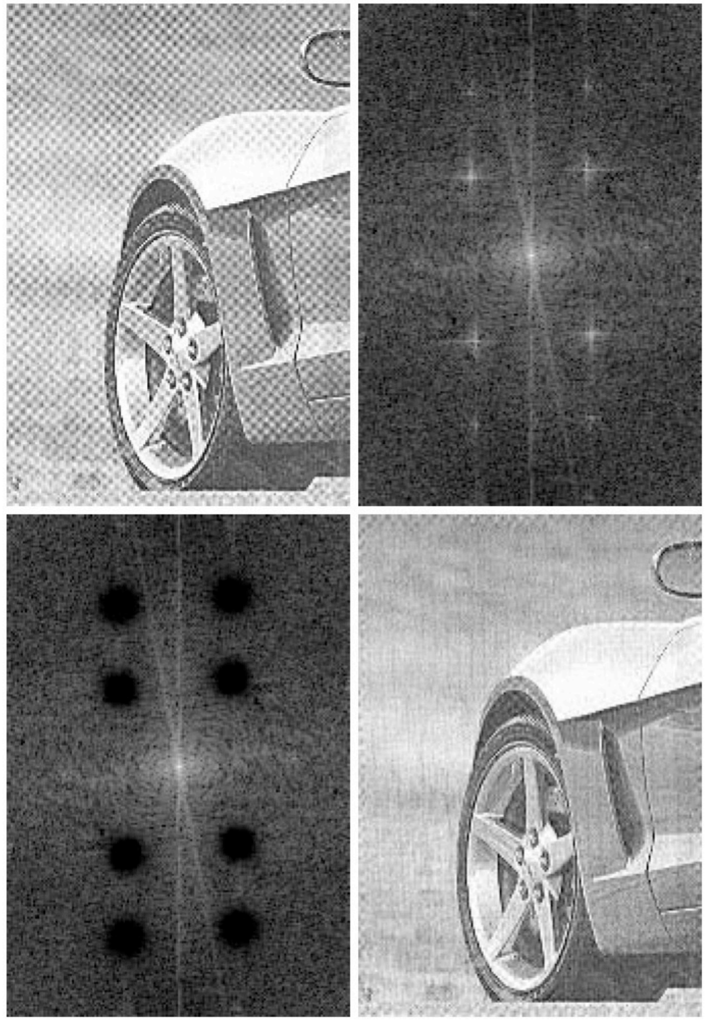
Band pass and Band Reject Filters



a b

FIGURE 4.63 (a) Bandreject Gaussian filter. (b) Corresponding bandpass filter. The thin black border in (a) was added for clarity; it is not part of the data.

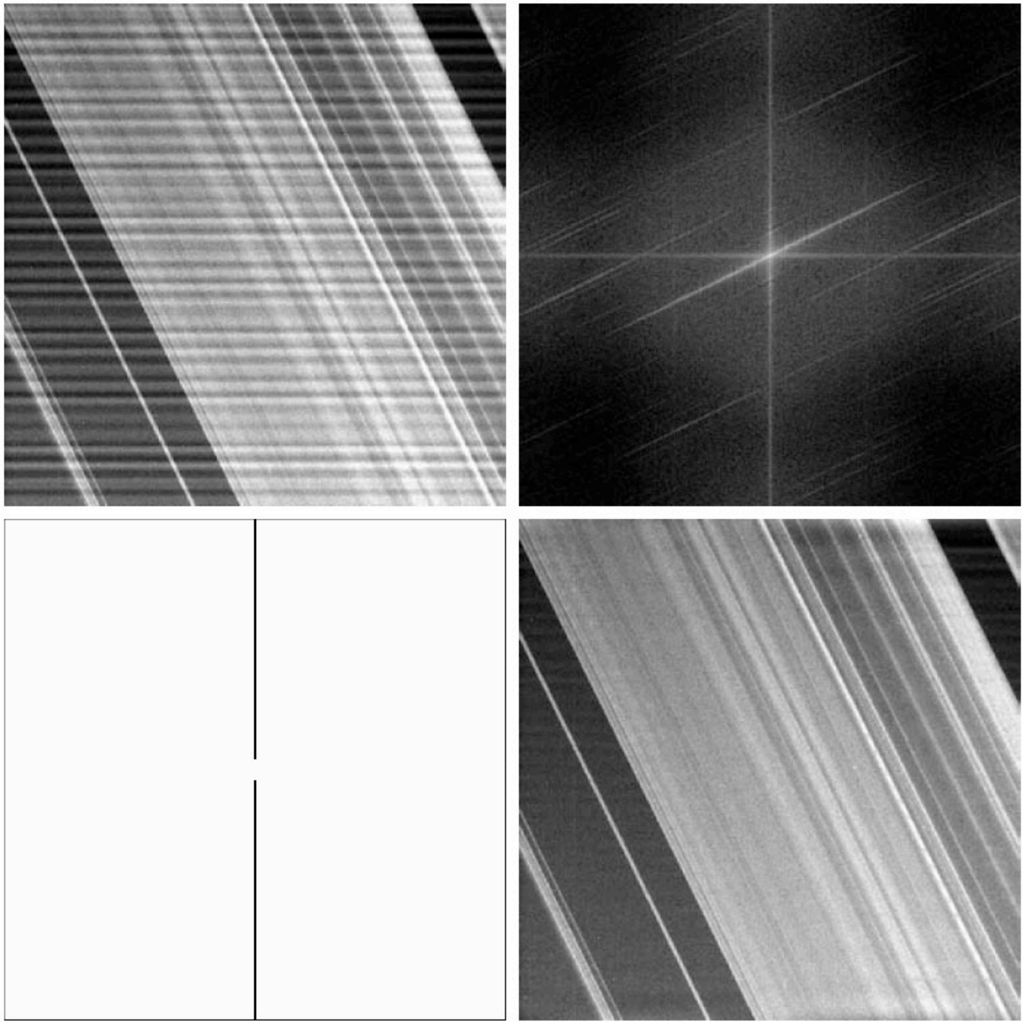
Band-Pass Filters (cont...)



a b
c d

FIGURE 4.64 (a) Sampled newspaper image showing a moiré pattern. (b) Spectrum. (c) Butterworth notch reject filter multiplied by the Fourier transform. (d) Filtered image.

Band-Pass Filters (cont...)



a b
c d

FIGURE 4.65 (a) 674 × 674 image of the Saturn rings showing nearly periodic interference. (b) Spectrum: The bursts of energy in the vertical axis near the origin correspond to the interference pattern. (c) A vertical notch reject filter. (d) Result of filtering. The thin black border in (c) was added for clarity; it is not part of the data. (Original image courtesy of Dr. Robert A. West, NASA/JPL.)

Summary

- Similar filtering can be done in the spatial and frequency domains.
- Spatial domain filtering is typically easier to understand.
- When using large images, filtering in the frequency domain is much faster.

Questions?

Credits

- Some of the slides were adapted from
- Material in Digital Image Processing by Gonzales and Woods
- Slides from Christopher Nikou, University of Ioannina
- Slides from William Hoffman, Colorado School of Mines
- Slides from Xiaolin Wu, McMaster University