EEE-6512: Image Processing and Computer Vision

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Lecture #10: Features/Feature Extraction I

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Outline

- Topics so Far (Image Processing)
- What are doing next? (Computer Vision)
- Introduction
- Global Shape Features

Course Topics So Far (Image Processing)

Focused on low level features

- Digital Image Fundamental (Chapter 2)
- Spatial Filtering (Chapter 3)
- Frequency Domain Filtering (Chapter 4)
- Color Image Processing (Chapter 7)
- Morphological Image Processing (Chapter 9)
- Image Segmentation/Edge Detection (Chapter 10)

Input and Output are Images

What are we doing next? (Computer Vision)

Focus on semantics or geometric descriptions

- Feature Extraction
- Model Fitting
- Classification

Input is an image and output is object (faces, people, chairs, etc.)

Introduction

- The goal in image analysis is to extract useful information for solving application-based problems.

- The first step to this is to reduce the amount of image data using methods that we have discussed before:
  - Image segmentation
  - Filtering in frequency domain

Introduction

- The next step would be to extract features that are useful in solving computer imaging problems.

- What features to be extracted are application dependent.

- After the features have been extracted, then analysis can be done.

Introduction

The goal is to generate features that exhibit high information packing properties:

- Extract the information from the raw data that is most relevant for discrimination between classes.
- Extract features with low within-class variability and high between class variability.
- Discard redundant information

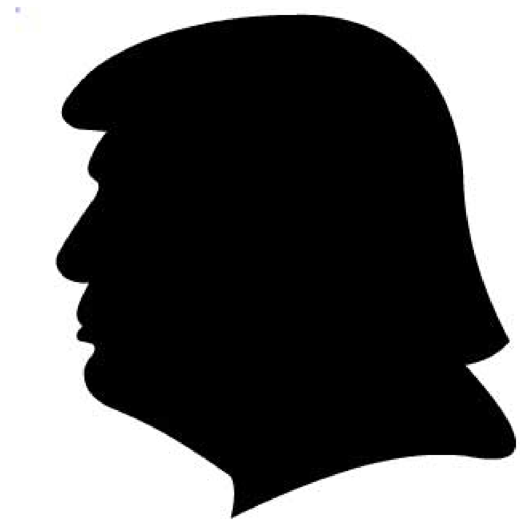
Shape Features

Shape Features

- Depend on a silhouette (outline) of an image
  - Shape is a very powerful feature
  - Objects may be recognized from their shape.

- All that is needed is a binary image

Shape Features



Shape Features

- 2D shape analysis useful in machine vision applications
- Medical image analysis
- Aerial image analysis
- Object detection

Binary Object Features

- In order to extract object features, we need an image that has undergone image segmentation and any necessary morphological filtering.

- This will provide us with a clearly defined object which can be labeled and processed independently.

Binary Object Features

- After all the binary objects in the image are labeled, we can treat each object as a binary image.
  - The labeled object has a value of ‘1’ and everything else is ‘0’.
- The labeling process goes as follows:
  - Define the desired connectivity.
  - Scan the image and label connected objects with the same symbol.

Binary Object Features

- After we have labeled the objects, we have an image filled with object numbers.
- This image is used to extract the features of interest.
- Among the global binary object features include area, center of area, axis of least second moment, perimeter, Euler number, projections, thinness ratio, aspect ratio, and moments.

Global Shape Features

Global Shape Features– Area

- The area of the *i*th object is defined as follows:

$$A_i = \sum_{r=0}^{height-1} \sum_{c=0}^{width-1} I_i(r,c)$$

- The area A_i is measured in pixels and indicates the relative size of the object.

Global Shape Features– Center of Area

- The center of area is defined as follows:

$$\bar{r}_i = \frac{1}{A_i} \sum_{r=0}^{height-1} \sum_{c=0}^{width-1} r I_i(r,c)$$

$$\bar{c}_i = \frac{1}{A_i} \sum_{r=0}^{height-1} \sum_{c=0}^{width-1} c I_i(r,c)$$

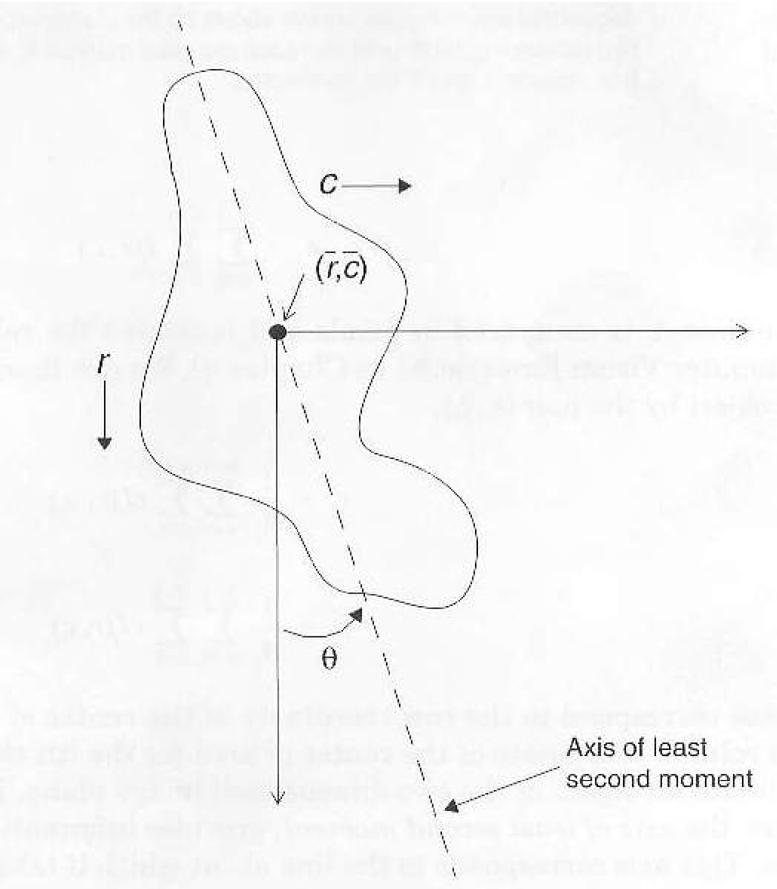
- These correspond to the row and column coordinate of the center of the *i*th object.

Global Shape Features– Axis of Least Second Moment

- The Axis of Least Second Moment is expressed as θ - the angle of the axis relatives to the vertical axis.

$$\theta_i = \frac{1}{2}\tan^{-1}\left(\frac{2\sum_{r=0}^{height-1}\sum_{c=0}^{width-1}(r-\bar{r})(c-\bar{c})I_i(r,c)}{\sum_{r=0}^{height-1}\sum_{c=0}^{width-1}(r-\bar{r})^2 I_i(r,c) - \sum_{r=0}^{height-1}\sum_{c=0}^{width-1}(c-\bar{c})^2 I_i(r,c)} \right)$$

Global Shape Features– Axis of Least Second Moment



- This assumes that the origin is as the center of area.
- This feature provides information about the object's orientation.
- This axis corresponds to the line about which it takes the least amount of energy to spin an object.

Global Shape Features- Perimeter

- The perimeter is defined as the total pixels that constitutes the edge of the object.
- Perimeter can help us to locate the object in space and provide information about the shape of the object.
- Perimeters can be found by counting the number of ‘1’ pixels that have ‘0’ pixels as neighbors.

Global Shape Features - Perimeter

- Perimeter can also be found by applying an edge detector to the object, followed by counting the '1' pixels.

- The two methods above only give an estimate of the actual perimeter.

- An improved estimate can be found by multiplying the results from either of the two methods by $\pi/4$.

Global Shape Features– Thinness Ratio

- The thinness ratio, *T*, can be calculated from perimeter and area.

- The equation for thinness ratio is defined as follows:

$$T_i = 4\pi\left(\frac{A_i}{P_i^{\,2}}\right)$$

Global Shape Features – Thinness Ratio

The thinness ratio is used as a measure of roundness.

- It has a maximum value of 1, which corresponds to a circle.
- As the object becomes thinner and thinner, the perimeter becomes larger relative to the area and the ratio decreases.

Global Shape Features – Irregularity Ratio

- The inverse of thinness ration is called compactness or irregularity ratio, $1/T$.
- This metric is used to determine the regularity of an object:

  Regular objects have less vertices (branches) and hence, less perimeter compare to irregular object of the same area.

Global Shape Features – Aspect Ratio

- The aspect ratio (also called elongation or eccentricity) is defined by the ratio of the bounding box of an object.

- This can be found by scanning the image and finding the minimum and maximum values on the row and column where the object lies.

Global Shape Features – Aspect Ratio

- The equation for aspect ratio is as follows:

$$\frac{c_{\max} - c_{\min} + 1}{r_{\max} - r_{\min} + 1}$$

- It reveals how the object spread in both vertical and horizontal direction.
- High aspect ratio indicates the object spread more towards horizontal direction.

Global Shape Features – Euler Number

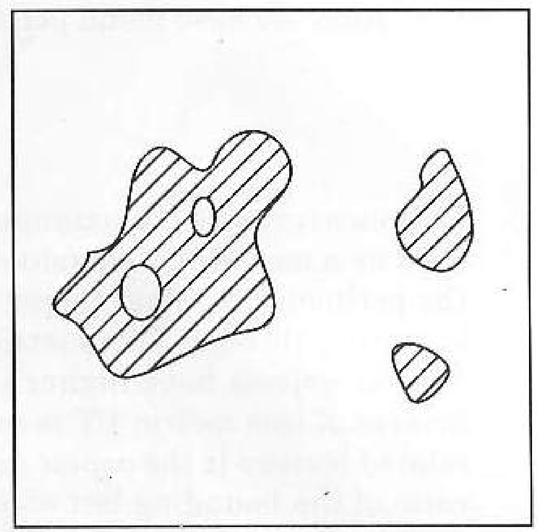
- Euler number is defined as the difference between the number of objects and the number of holes.
  - Euler number = num of object – number of holes

- In the case of a single object, the Euler number indicates how many closed curves (holes) the object contains.

Global Shape Features – Euler Number

- Euler number can be used in tasks such as optical character recognition (OCR).

a. This image has eight objects and one hole, so its Euler number is 8 – 1 = 7. The letter *V* has Euler number of 1, $i = 2$, $s = 1$, $o = 0$, and $n = 1$.



b. This image has three objects and two holes, so the Euler number is 3 – 2 = 1.

Global Shape Features – Euler Number

- Euler number can also be found using the number of convexities and concavities.
  - Euler number = number of convexities – number of concavities
- This can be found by scanning the image for the following patterns:

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Convexities

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

Concavities

Global Shape Features – Projection

- The projection of a binary object, may provide useful information related to object's shape.
- It can be found by summing all the pixels along the rows or columns.
  - Summing the rows give horizontal projection.
  - Summing the columns give the vertical projection.

Global Shape Features – Projection

- We can defined the horizontal projection $h_i(r)$ and vertical projection $v_i(c)$ as:

$$h_i(r) = \sum_{c=0}^{width-1} I_i(r,c)$$

$$v_i(c) = \sum_{r=0}^{height-1} I_i(r,c)$$

- An example of projections is shown in the next slide:

Global Shape Features – Projection

$r \downarrow$ \ $c \rightarrow$									$h(r)$
0	0	0	0	1	0	0	0	0	1
0	0	0	0	1	1	0	0	0	2
0	0	1	1	1	1	1	0	0	5
0	1	1	1	1	1	1	0	0	6
0	0	0	0	0	1	1	0	0	2
0	0	0	0	0	0	0	0	0	0
$v(c) \longrightarrow$ 0	1	2	2	4	4	3	0	0	

To find the projections, we sum the number of 1s in the rows and columns.

Global Shape Feature - Moments

Moments are statistical measures of data.

- They come in integer orders.
- Order 0 is just the number of points in the data.
- Order 1 is the sum and is used to find the average.
- Order 2 is related to the variance, and order 3 to the skew of the data.
- Higher orders can also be used, but don’t have simple meanings.

Global Shape Feature - Moments

- Let r be a random variable, and $g(r_i)$ be normalized (as the probability of value r_i occurring), then the moments are

$$\mu_n(r) = \sum_{k=0}^{K-1} (r_i - m)^n g(r_i)$$

$$\text{where } m = \sum_{i=0}^{K-1} r_i g(r_i)$$

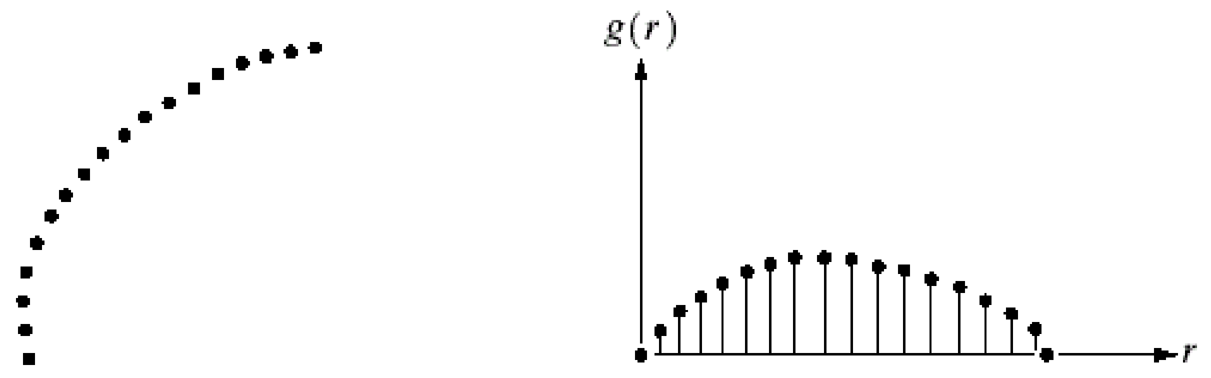


FIGURE 11.15
(a) Boundary segment.
(b) Representation as a 1-D function.

Global Shape Feature - Moments

- For a 2-D continuous function *f*(*x*,*y*), the moment of order (*p*+*q*) is defined as

$$m_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q f(x,y)\,dxdy \quad \text{for } p,q = 1,2,3,\ldots$$

- The central moments are defined as

$$\mu_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\bar{x})^p (y-\bar{y})^q f(x,y)\,dxdy$$

$$\text{where } \bar{x} = \frac{m_{10}}{m_{00}} \text{ and } \bar{y} = \frac{m_{01}}{m_{00}}$$

Global Shape Feature - Moments

- If $f(x,y)$ is a digital image, then

$$\mu_{pq} = \sum_x \sum_y (x-\bar{x})^p (y-\bar{y})^q f(x,y)$$

- The central moments of order up to 3 are

$$\mu_{00} = \sum_x \sum_y (x-\bar{x})^0 (y-\bar{y})^0 f(x,y) = \sum_x \sum_y f(x,y) = m_{00}$$

$$\mu_{10} = \sum_x \sum_y (x-\bar{x})^1 (y-\bar{y})^0 f(x,y) = m_{10} - \frac{m_{10}}{m_{00}}(m_{00}) = 0$$

$$\mu_{01} = \sum_x \sum_y (x-\bar{x})^0 (y-\bar{y})^1 f(x,y) = m_{01} - \frac{m_{01}}{m_{00}}(m_{00}) = 0$$

$$\mu_{11} = \sum_x \sum_y (x-\bar{x})^1 (y-\bar{y})^1 f(x,y) = m_{11} - \frac{m_{10}m_{01}}{m_{00}}$$

$$= m_{11} - \bar{x}m_{01} = m_{11} - \bar{y}m_{10}$$

Global Shape Feature - Moments

- The central moments of order up to 3 are

$$\mu_{20} = \sum_x \sum_y (x-\bar{x})^2 (y-\bar{y})^0 f(x,y) = m_{20} - \bar{x} m_{10}$$

$$\mu_{02} = \sum_x \sum_y (x-\bar{x})^0 (y-\bar{y})^2 f(x,y) = m_{02} - \bar{y} m_{01}$$

$$\mu_{21} = \sum_x \sum_y (x-\bar{x})^2 (y-\bar{y})^1 f(x,y) = m_{21} - 2\bar{x} m_{11} - \bar{y} m_{20} + 2\bar{x} m_{01}$$

$$\mu_{12} = \sum_x \sum_y (x-\bar{x})^1 (y-\bar{y})^2 f(x,y) = m_{12} - 2\bar{y} m_{11} - \bar{x} m_{02} + 2\bar{y} m_{10}$$

$$\mu_{30} = \sum_x \sum_y (x-\bar{x})^3 (y-\bar{y})^0 f(x,y) = m_{30} - 3\bar{x} m_{20} + 2\bar{x}^2 m_{10}$$

$$\mu_{03} = \sum_x \sum_y (x-\bar{x})^0 (y-\bar{y})^3 f(x,y) = m_{03} - 3\bar{y} m_{02} + 2\bar{y}^2 m_{01}$$

Global Shape Feature - Moments

- The normalized central moments are defined as

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}}$$

$$\text{where } \gamma = \frac{p+q}{2} + 1 \quad \text{for} \quad p+q = 2,3,....$$

Global Shape Feature - Moments

- Seven invariant moments can be derived from the second and third moments:

$$\phi_1 = \eta_{20} + \eta_{02}$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$

$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$

$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$

$$\phi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})\left[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2\right] + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})\left[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right]$$

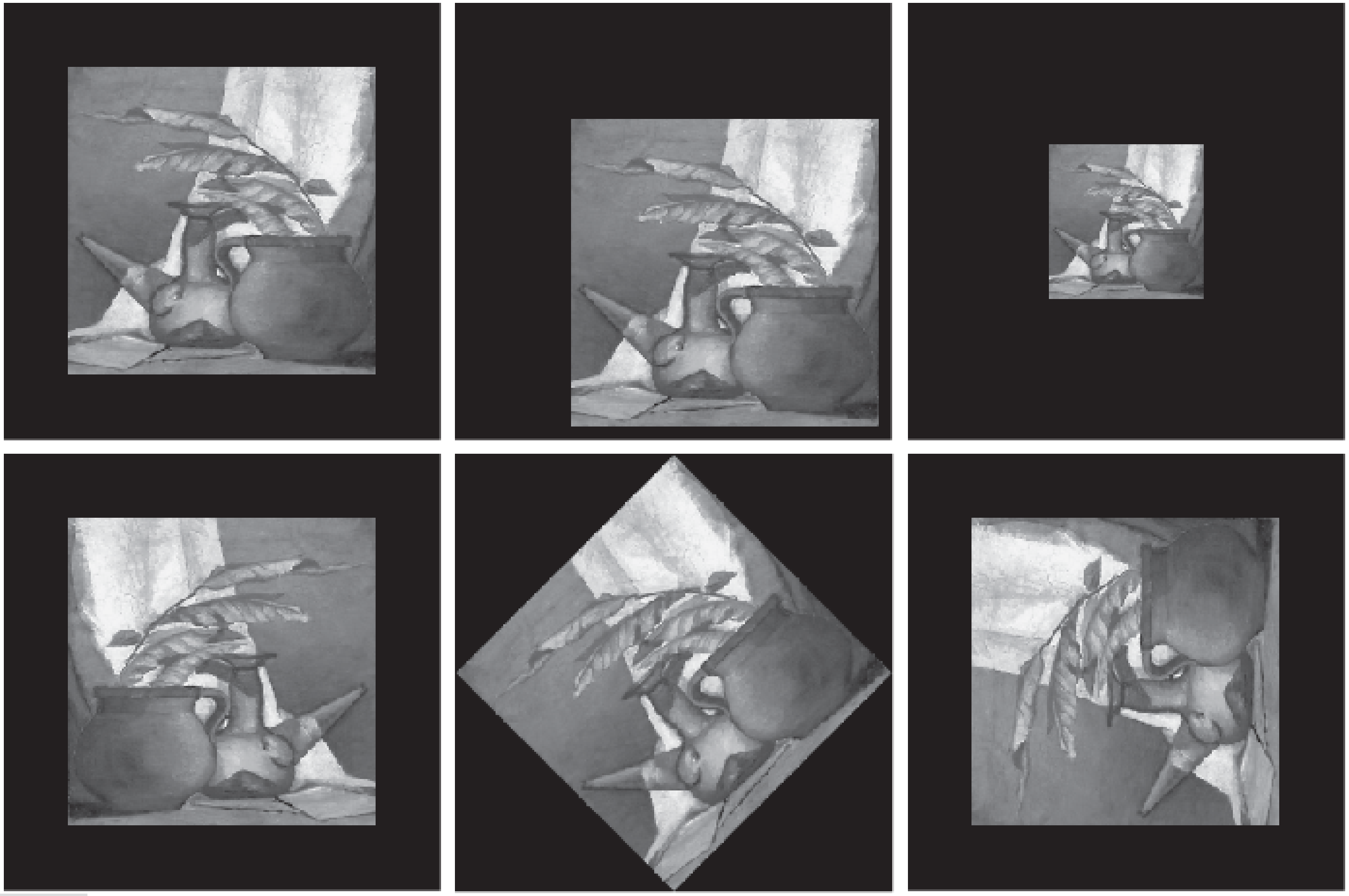
Global Shape Feature - Moments

- Seven invariant moments can be derived from the second and third moments:

- $$\phi_6 = (\eta_{20} - \eta_{02})\left[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03})$$
  $$\phi_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})\left[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2\right] + (3\eta_{12} - \eta_{30})(\eta_{21} + \eta_{03})\left[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right]$$

- This set of moments is invariant to translation, rotation, and scale change.

Global Shape Feature - Moments



a b c
d e f

FIGURE 12.37 (a) Original image. (b)–(f) Images translated, scaled by one-half, mirrored, rotated by 45°, and rotated by 90°, respectively.

Global Shape Feature - Moments

TABLE 12.5
Moment invariants for the images in Fig. 12.37.

Moment Invariant	Original Image	Translated	Half Size	Mirrored	Rotated 45°	Rotated 90°
ϕ_1	2.8662	2.8662	2.8664	2.8662	2.8661	2.8662
ϕ_2	7.1265	7.1265	7.1257	7.1265	7.1266	7.1265
ϕ_3	10.4109	10.4109	10.4047	10.4109	10.4115	10.4109
ϕ_4	10.3742	10.3742	10.3719	10.3742	10.3742	10.3742
ϕ_5	21.3674	21.3674	21.3924	21.3674	21.3663	21.3674
ϕ_6	13.9417	13.9417	13.9383	13.9417	13.9417	13.9417
ϕ_7	−20.7809	−20.7809	−20.7724	20.7809	−20.7813	−20.7809

Boundary Feature Representation

Boundary Feature Representation

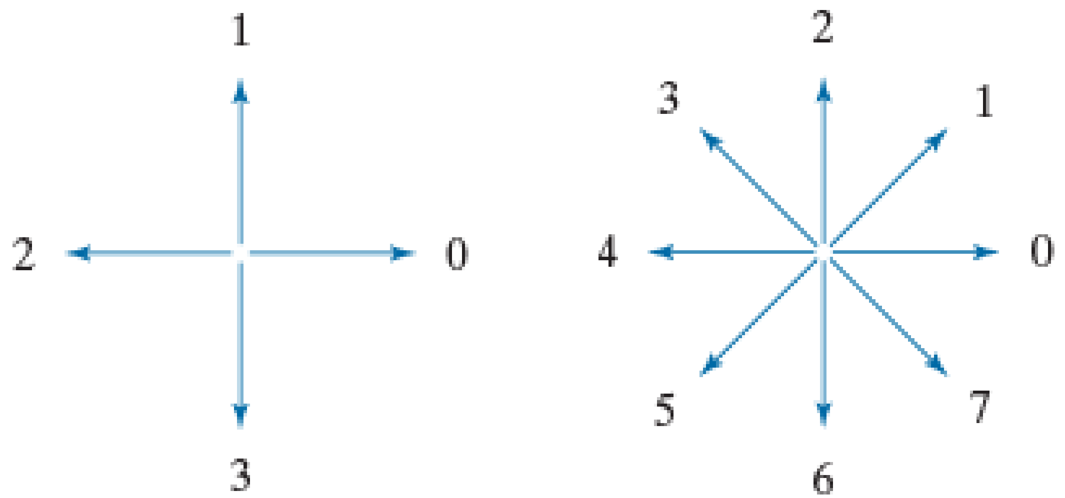
There are several simple geometric measures that can be useful for describing a boundary.

- The length of a boundary: the number of pixels along a boundary gives a rough approximation of its length.

- Curvature: the rate of change of slope
  - To measure a curvature accurately at a point in a digital boundary is difficult
  - The difference between the slops of adjacent boundary segments is used as a descriptor of curvature at the point of intersection of segments

Boundary Feature Representation: Chain Codes

- Image regions (including segments) can be represented by either the border or the pixels of the region. These can be viewed as external or internal characteristics, respectively.
- Chain codes: represent a boundary of a connected region.



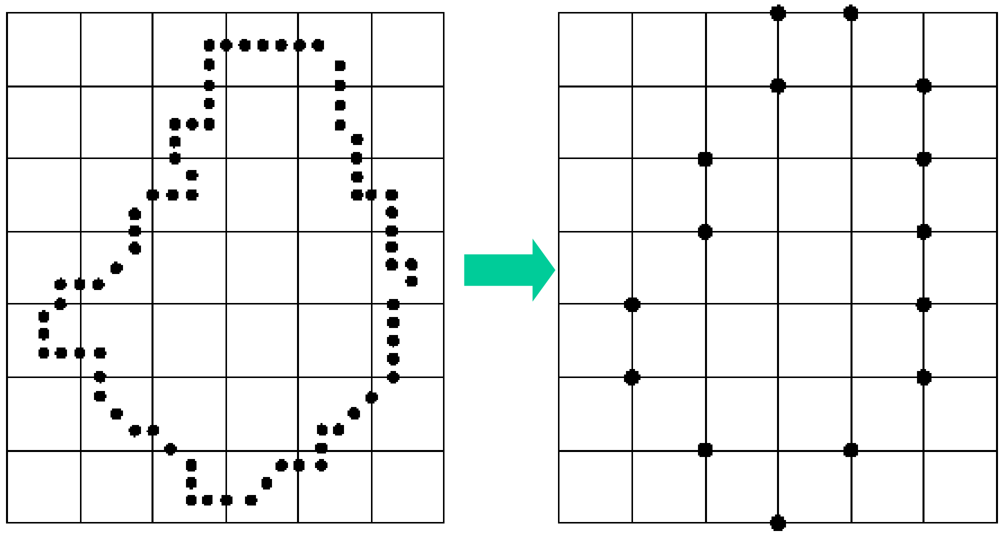
a b

FIGURE 12.3 Direction numbers for (a) 4-directional chain code, and (b) 8-directional chain code.

Boundary Feature Representation: Chain Codes

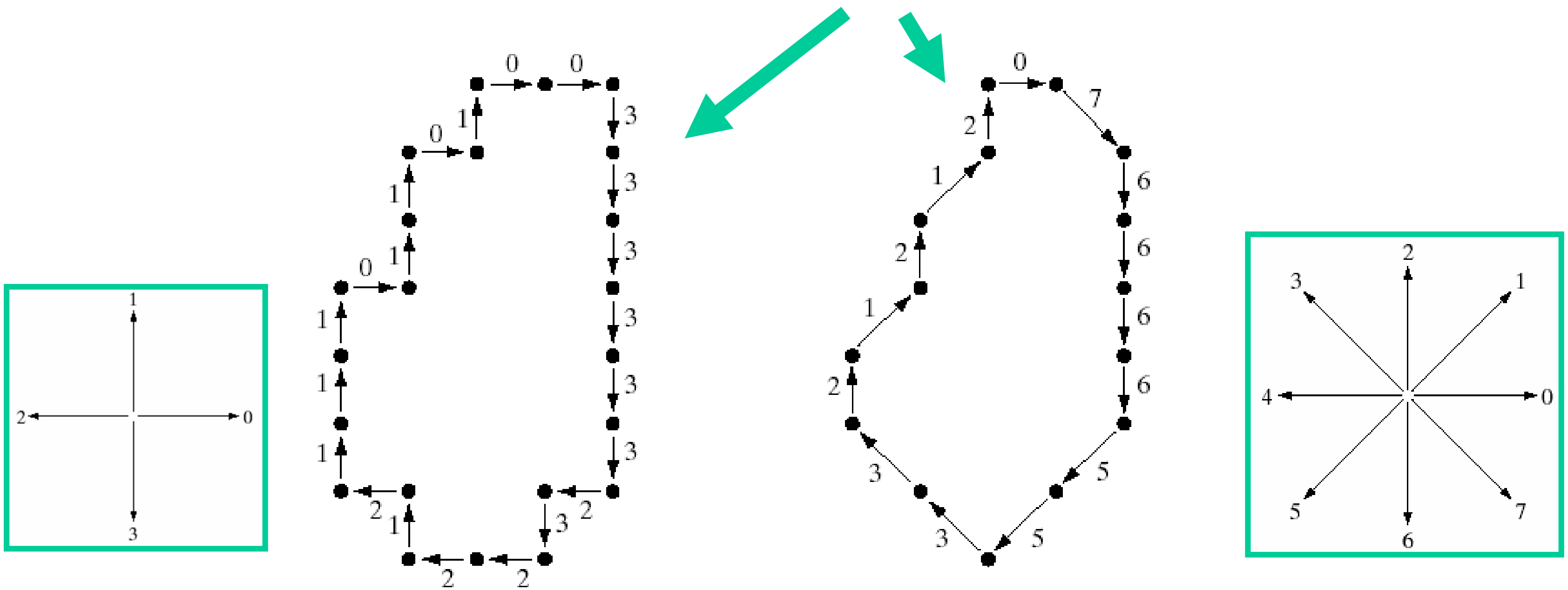
- Problems
  - Chain codes can be long
  - Small disturbances in the boundary can cause large changes in code
- Solution
  - Resample on large grid spacing

Boundary Feature Representation: Chain Code



a b
c d

FIGURE 11.2
(a) Digital boundary with resampling grid superimposed.
(b) Result of resampling.
(c) 4-directional chain code.
(d) 8-directional chain code.



Boundary Feature Representation: Chain Code

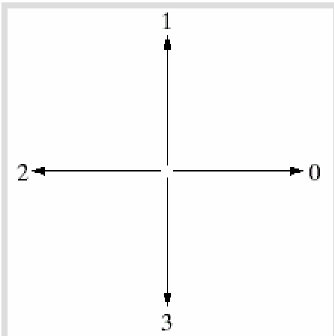
- Chain codes can be based on either 4-connectedness or 8-connectedness.

- The first difference of the chain code:
  - This difference is obtained by counting the number of direction changes (in a counterclockwise direction)
  - For example, the first difference of the 4-direction chain code 10103322 is 3133030.

- Assuming the first difference code represent a closed path, rotation normalization can be achieved by circularly shifting the number of the code so that the list of numbers forms the smallest possible integer.

- Size normalization can be achieved by adjusting the size of the resampling grid.

Boundary Feature Representation



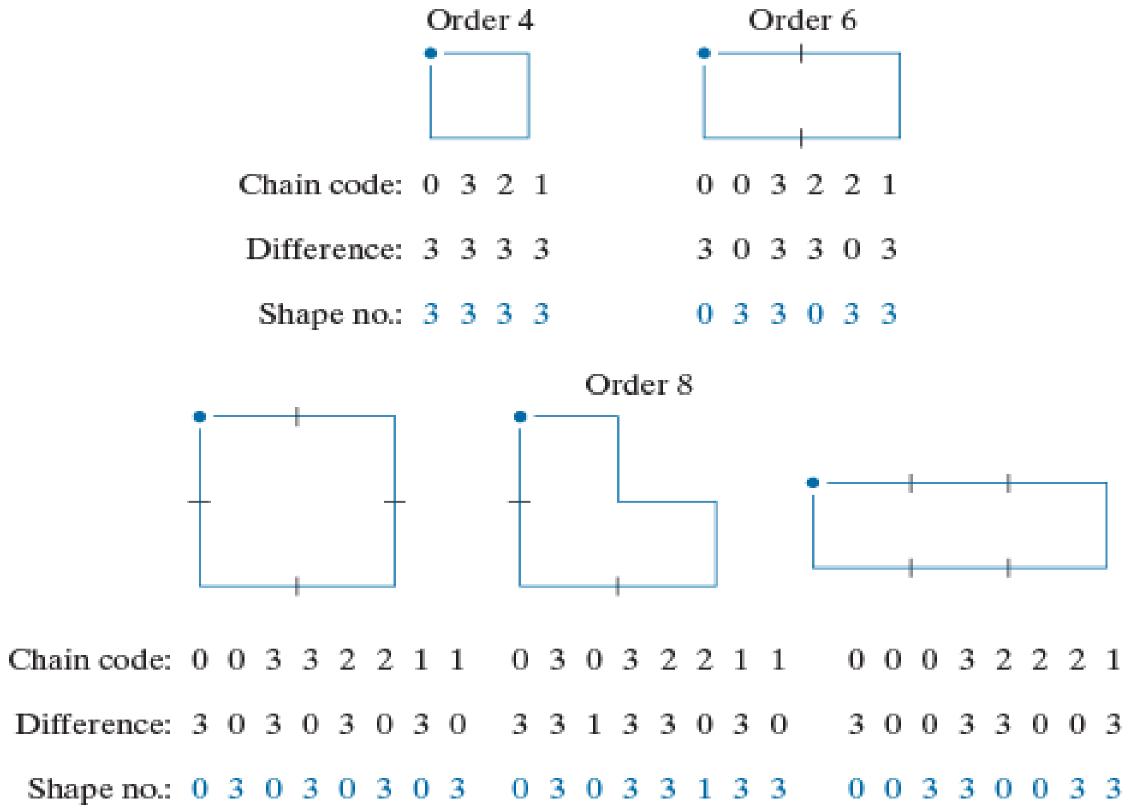


FIGURE 12.16 All shapes of order 4, 6, and 8. The directions are from Fig. 12.3(a), and the dot indicates the starting point.

- Given a chain-coded boundary, its shape number is that particular cyclic permutation of the first difference which is lexicographically smallest among all the cyclic permutations
- The order *n* of a shape number is defined as the number of digits in its representation.

Boundary Feature Representation

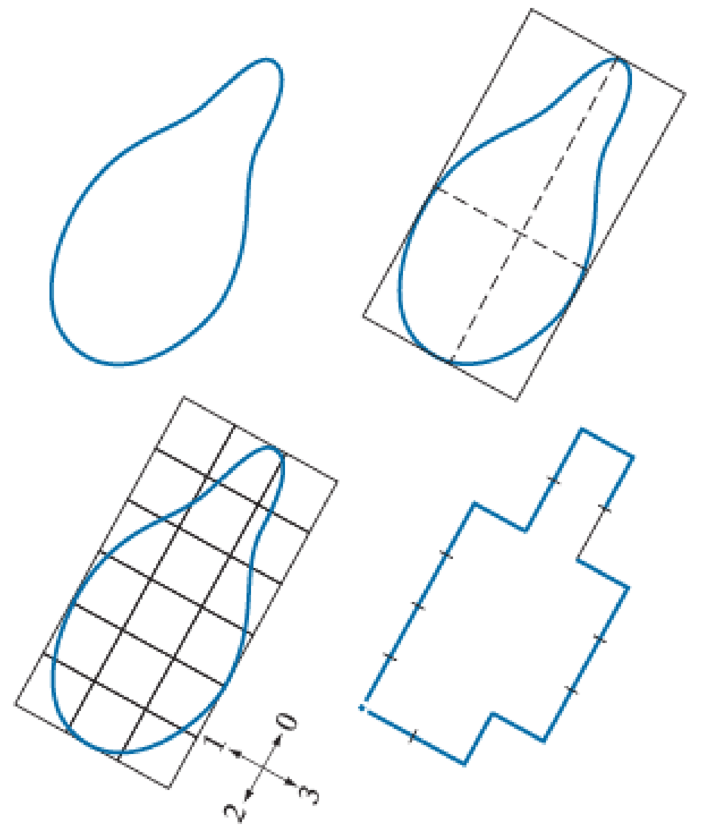


FIGURE 12.17 Steps in the generation of a shape number.

Chain code: 0 0 0 0 3 0 0 3 2 2 3 2 2 2 1 2 1 1

Difference: 3 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0

Shape no.: 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0 3

Boundary Feature Representation: Fourier Descriptor

- This is a way of using the Fourier transform to analyze the shape of a boundary.
  - The *x*-*y* coordinates of the boundary are treated as the real and imaginary parts of a complex number.
  - Then the list of coordinates is Fourier transformed using the DFT (chapter 4).
  - The Fourier coefficients are called the Fourier descriptors.
  - The basic shape of the region is determined by the first several coefficients, which represent lower frequencies.
  - Higher frequency terms provide information on the fine detail of the boundary.

Boundary Feature Representation: Fourier Descriptor

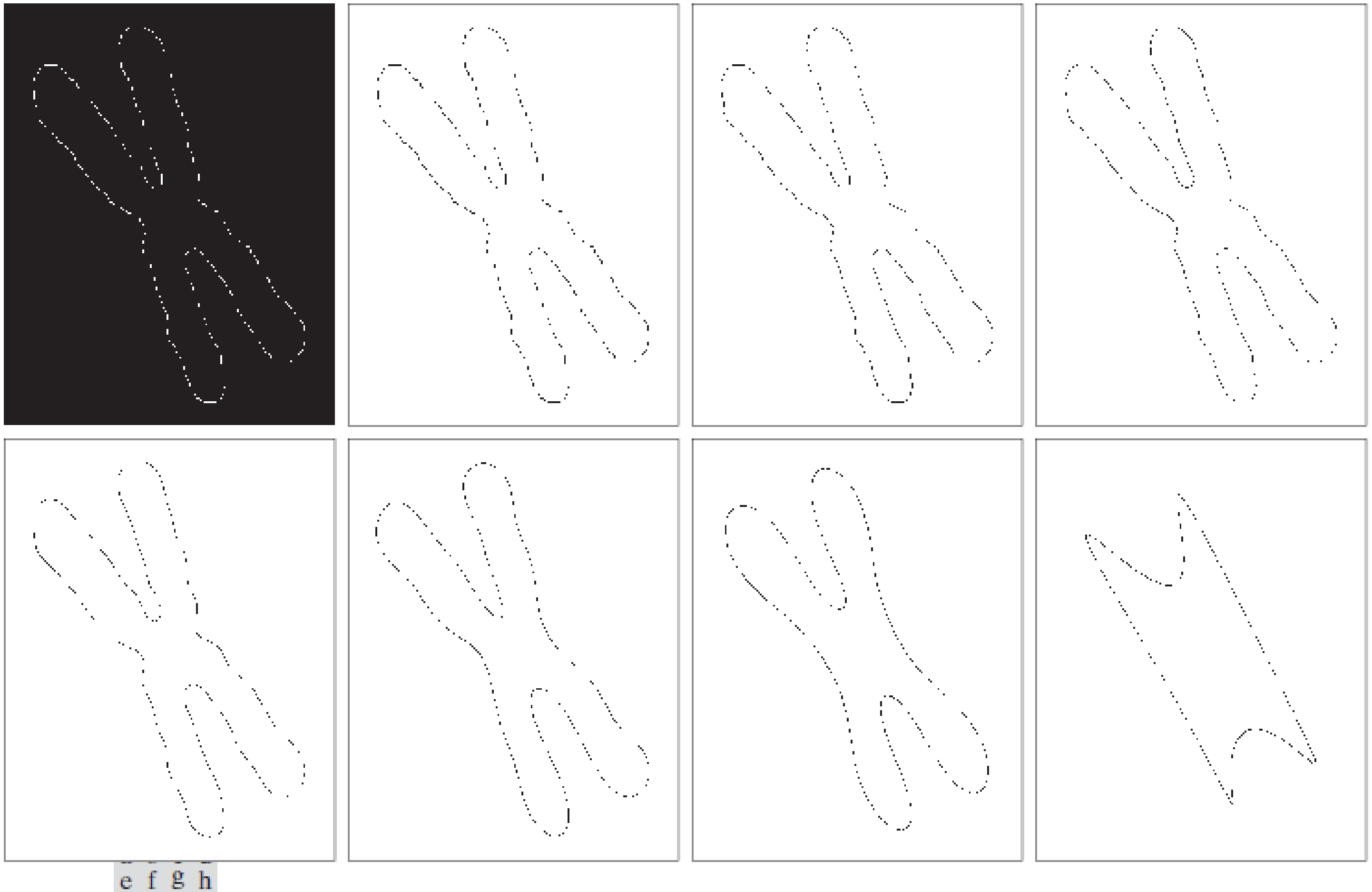


FIGURE 12.19 (a) Boundary of a human chromosome (2868 points). (b)–(h) Boundaries reconstructed using 1434, 286, 144, 72, 36, 18, and 8 Fourier descriptors, respectively. These numbers are approximately 50%, 10%, 5%, 2.5%, 1.25%, 0.63%, and 0.28% of 2868, respectively. Images (b)–(h) are shown as negatives to make the boundaries easier to see.

Summary

- Shape is a powerful cue for object recognition with many applications

- Shape features allow significant data reduction while retaining information relevant to object shape.

Questions?

Next Time: Interest Point Features (Corners), SIFT Features

Slide Credits

Images taken from Digital Image Processing by Gonzalez and Woods Text.

Material taken from Jen-Chang Liu lecture slides