EXPLOITING MEASUREMENT SUBSPACES FOR WIDEBAND ELECTROMAGNETIC INDUCTION PROCESSING

PRESENTER: CHARLES ETHAN HAYES

EVENT: SPIE DEFENSE + SECURITY 2018

CO-AUTHORS: WAYMOND R. SCOTT, JR.
JAMES MCCLELLAN

CREATING THE NEXT

Georgia Institute of Technology

WIDEBAND ELECTROMAGNETIC INDUCTION SENSORS

Processing Wideband EMI data in order to

- Detect buried ferrous targets
- Classify underground targets by their EMI response
- Locate the targets of interest

FREQUENCY-DOMAIN WEMI MEASUREMENTS

- The WEMI system captures measurements

$$\hat{M}(\omega_i, p_j)$$

ω are the measurement frequencies

p is the position of the sensor

- The measurements are grouped into a matrix

$$\hat{\mathbf{M}} = [\hat{M}(\omega_i, p_j)]_{i,j}$$

- A real valued matrix is formed

$$\mathbf{M} = \begin{bmatrix} \Re(\hat{\mathbf{M}}) \\ \Im(\hat{\mathbf{M}}) \end{bmatrix}$$

WEMI MEASUREMENTS MODEL

- Measurement model

$$\mathbf{M} = \mathbf{S} + \mathbf{G} + \mathbf{R} + \mathcal{E}$$

$\mathbf{S}$ is the signal from the desired target

$\mathbf{G}$ is an interferer from the soil response.

$\mathbf{R}$ is an interferer from the sensor self-response.

$\mathcal{E}$ is an additive Gaussian noise term

ISOLATING THE SELF RESPONSE

- The self response model

$$\mathbf{R}(\boldsymbol{\omega}, \boldsymbol{p}) = \boldsymbol{r}(\boldsymbol{\omega})\mathbf{1}^T$$

$\boldsymbol{r}(\boldsymbol{\omega})$ is the unknown self response at each measurement frequency

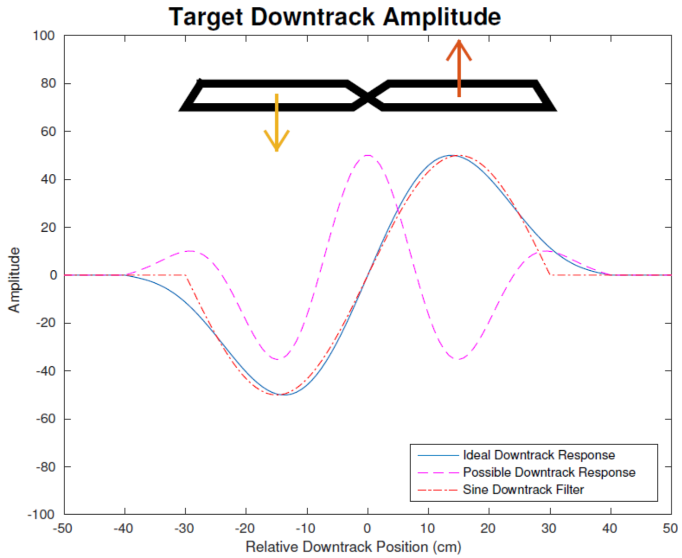
ISOLATING THE SELF RESPONSE

- The self response model

$$\mathbf{R}(\boldsymbol{\omega}, \boldsymbol{p}) = \boldsymbol{r}(\boldsymbol{\omega})\mathbf{1}^T$$

$\boldsymbol{r}(\boldsymbol{\omega})$ is the unknown self response at each measurement frequency

- The sine filter models the target response for a quadrupole receiver

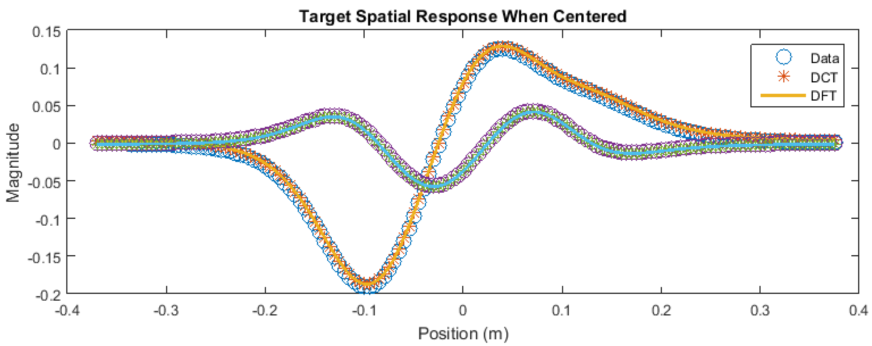


ISOLATING THE SELF RESPONSE (CONTINUED)

- The Discrete Fourier Transform (DFT) and Discrete Cosine Transform (DCT) can isolate specific spatial wavelengths

ISOLATING THE SELF RESPONSE (CONTINUED)

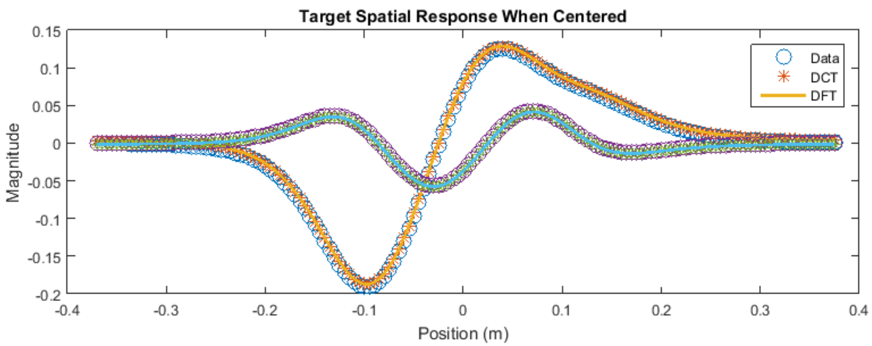
- The Discrete Fourier Transform (DFT) and Discrete Cosine Transform (DCT) can isolate specific spatial wavelengths

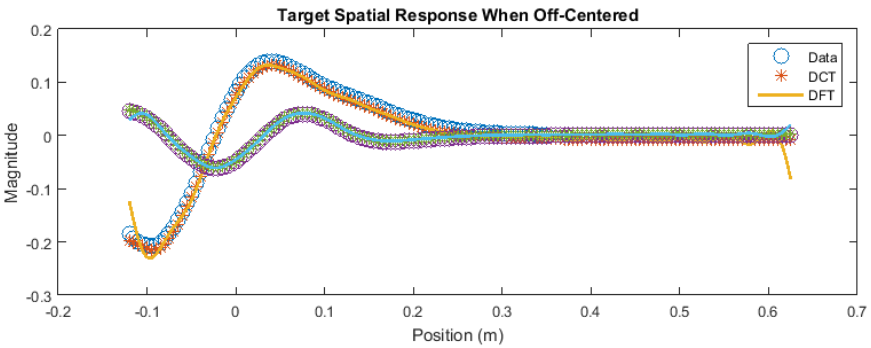


Modeling a simulated target with wavelengths >5cm and no DC

ISOLATING THE SELF RESPONSE (CONTINUED)

- The Discrete Fourier Transform (DFT) and Discrete Cosine Transform (DCT) can isolate specific spatial wavelengths

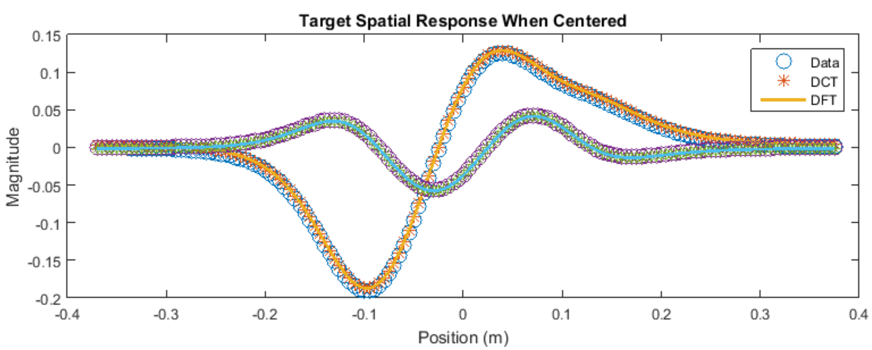


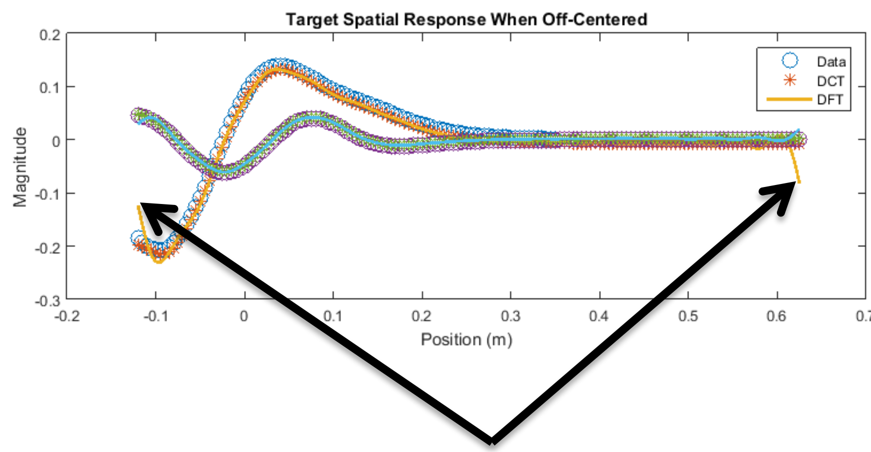


Modeling an off-centered target

ISOLATING THE SELF RESPONSE (CONTINUED)

- The Discrete Fourier Transform (DFT) and Discrete Cosine Transform (DCT) can isolate specific spatial wavelengths

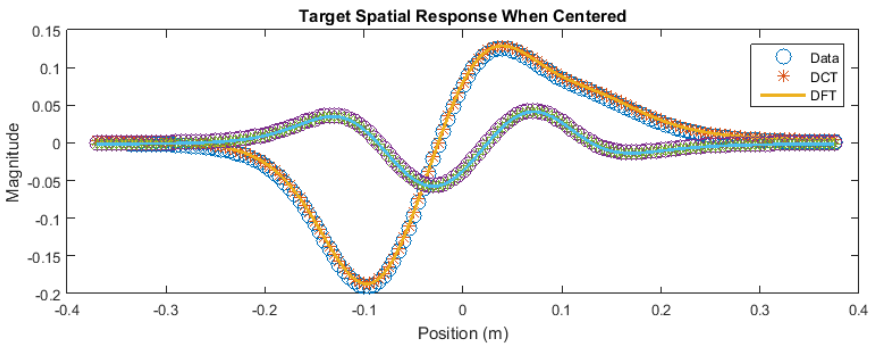


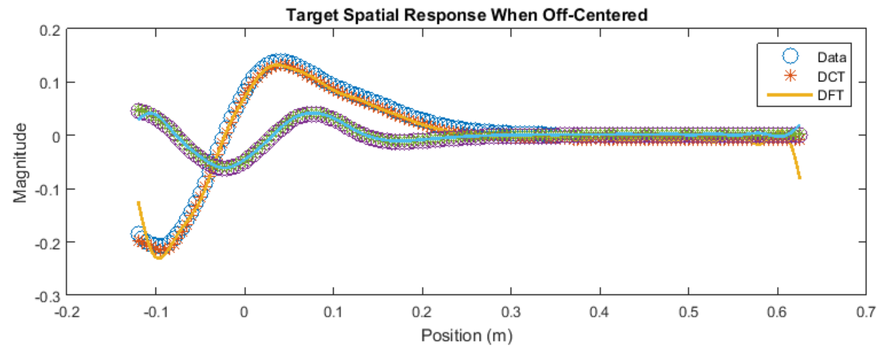


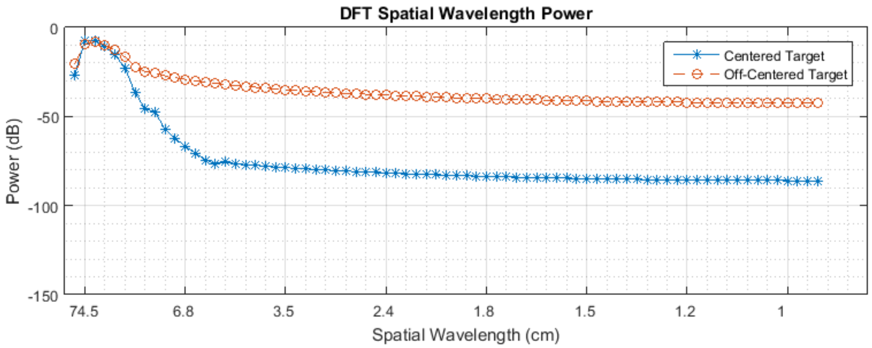
Notice the edge artifact from the DFT

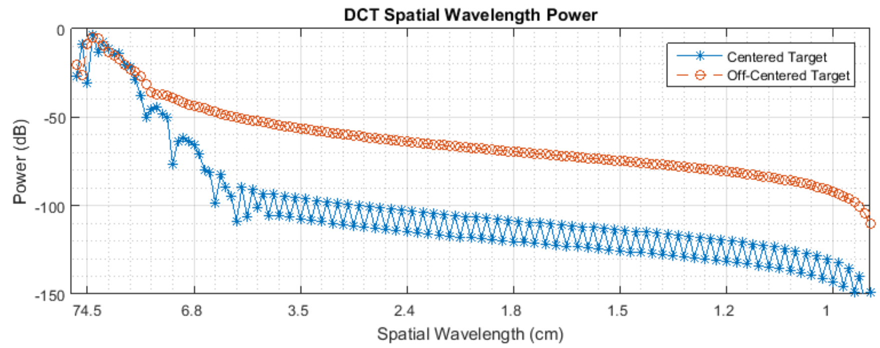
ISOLATING THE SELF RESPONSE (CONTINUED)

- The Discrete Fourier Transform (DFT) and Discrete Cosine Transform (DCT) can isolate specific spatial wavelengths



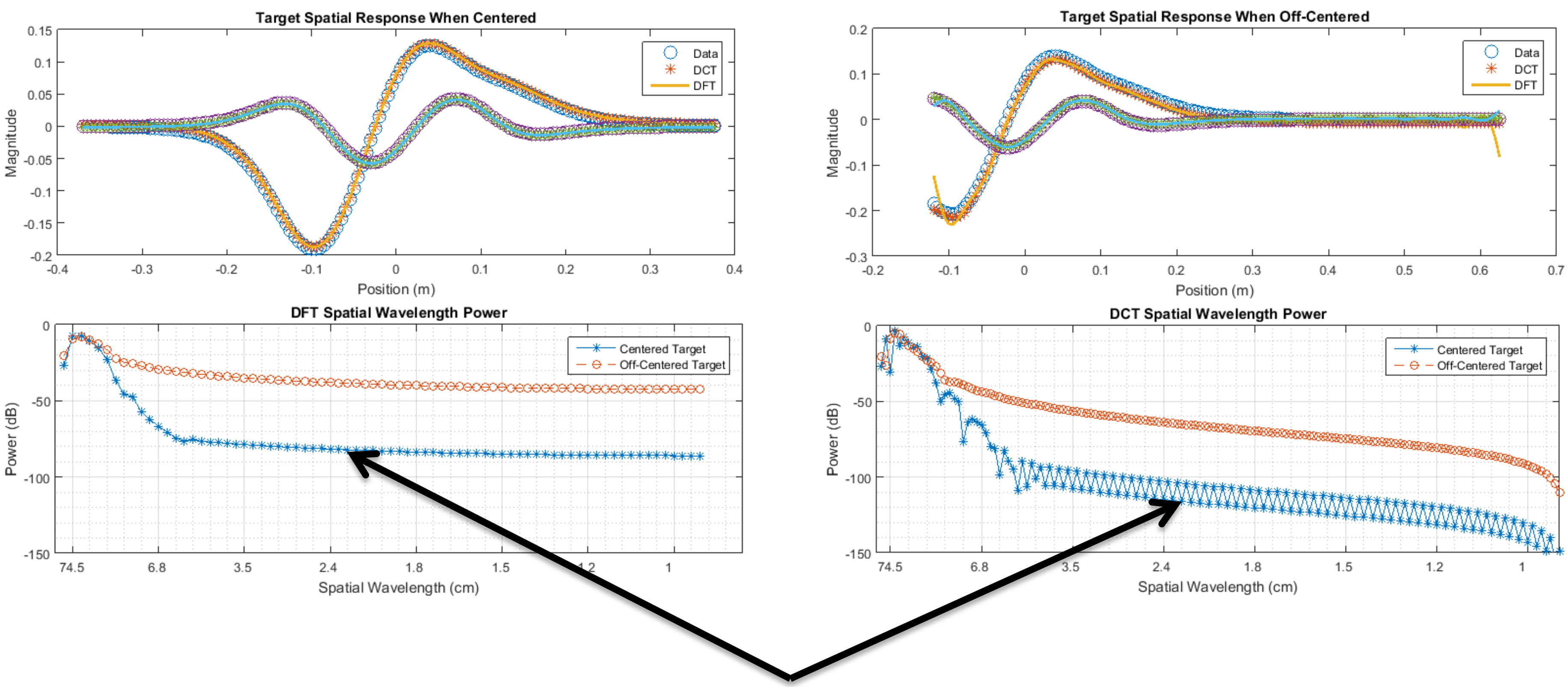






ISOLATING THE SELF RESPONSE (CONTINUED)

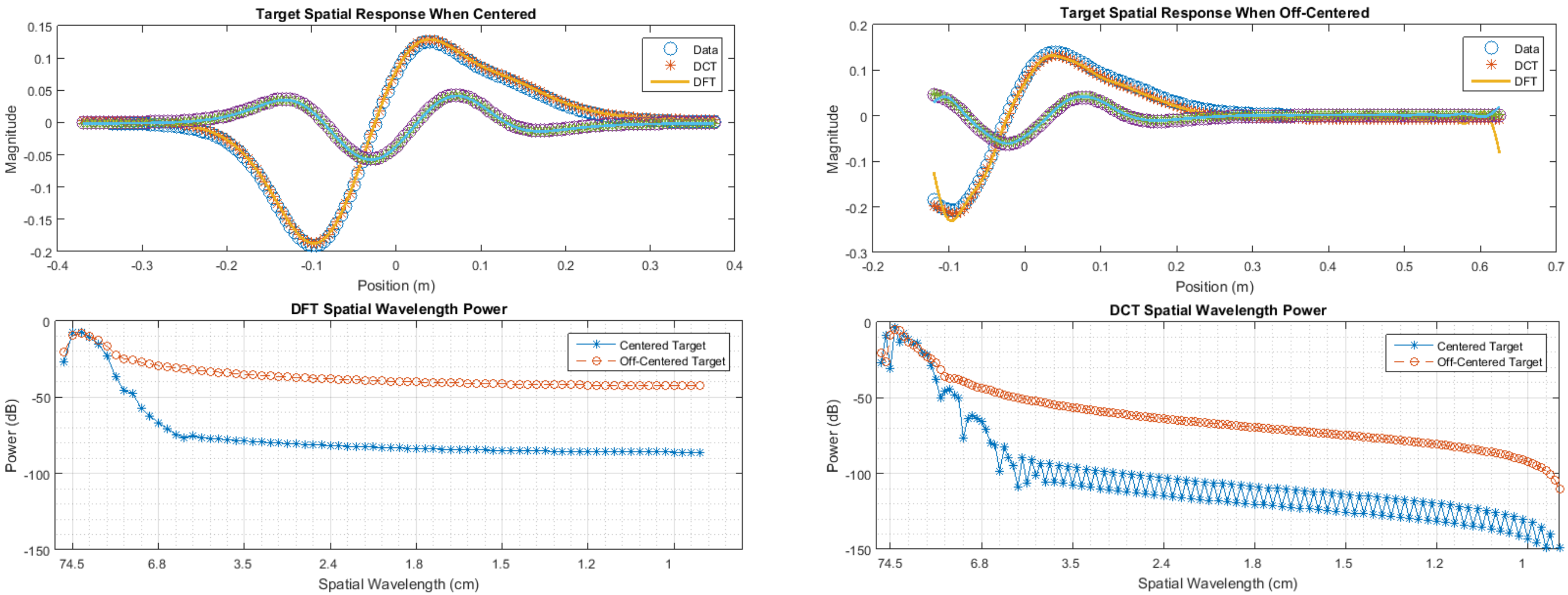
- The Discrete Fourier Transform (DFT) and Discrete Cosine Transform (DCT) can isolate specific spatial wavelengths



The DCT provides better isolation

ISOLATING THE SELF RESPONSE (CONTINUED)

- The Discrete Fourier Transform (DFT) and Discrete Cosine Transform (DCT) can isolate specific spatial wavelengths



The DCT is able to better isolate the target's spatial wavelength and is more robust when the target is on the edge of the data matrix

ISOLATING THE SOIL RESPONSE

- The soil response model

$$\tilde{g}(\omega, p) = \begin{bmatrix} 1 & \log(\frac{\omega}{\omega_c}) + \frac{j\pi}{2} \end{bmatrix} \begin{bmatrix} \xi_1(p) \\ \xi_2(p) \end{bmatrix}$$

$$\xi_1(p) \sim \mathcal{N}(g_1, \sigma_g^2) \qquad \xi_2(p) \sim \mathcal{N}(g_2, \sigma_g^2)$$

$$\boldsymbol{\Psi}(\boldsymbol{\omega}) = \begin{bmatrix} 1 & \log(\frac{\boldsymbol{\omega}}{\omega_c}) + \frac{j\pi}{2} \end{bmatrix}$$

ISOLATING THE SOIL RESPONSE

- The soil response model

$$\tilde{g}(\omega, p) = \begin{bmatrix} 1 & \log(\frac{\omega}{\omega_c}) + \frac{j\pi}{2} \end{bmatrix} \begin{bmatrix} \xi_1(p) \\ \xi_2(p) \end{bmatrix}$$

$$\xi_1(p) \sim \mathcal{N}(g_1, \sigma_g^2) \qquad \xi_2(p) \sim \mathcal{N}(g_2, \sigma_g^2)$$

$$\boldsymbol{\Psi}(\boldsymbol{\omega}) = \begin{bmatrix} 1 & \log(\frac{\boldsymbol{\omega}}{\omega_c}) + \frac{j\pi}{2} \end{bmatrix}$$

- The target response can also be modeled in the measurement frequency domain using the Discrete Spectrum of Relaxation Frequencies (DSRF)

$$\tilde{a}(\omega, \zeta) = \frac{j\omega/\zeta}{1+j\omega/\zeta}$$

ISOLATING THE SOIL RESPONSE (CONTINUED)

- The target's DSRF response is isolated from the soil response

$$\mathbf{A} = \mathbf{P}_{\mathbf{GG}}\mathbf{A} + \mathbf{P}_{\mathbf{GG}}^{\perp}\mathbf{A} \qquad \mathbf{P}_{\mathbf{GG}} = \boldsymbol{\Psi}(\boldsymbol{\Psi}^T\boldsymbol{\Psi})^{-1}\boldsymbol{\Psi}^T$$

ISOLATING THE SOIL RESPONSE (CONTINUED)

- The target's DSRF response is isolated from the soil response

$$\mathbf{A} = \mathbf{P}_{\mathbf{GG}}\mathbf{A} + \mathbf{P}_{\mathbf{GG}}^{\perp}\mathbf{A} \qquad \mathbf{P}_{\mathbf{GG}} = \boldsymbol{\Psi}(\boldsymbol{\Psi}^T\boldsymbol{\Psi})^{-1}\boldsymbol{\Psi}^T$$

DSRF with soil **DSRF without soil**

ISOLATING THE SOIL RESPONSE (CONTINUED)

- The target's DSRF response is isolated from the soil response

$$\mathbf{A} = \mathbf{P_{GG}}\mathbf{A} + \mathbf{P}^{\perp}_{\mathbf{GG}}\mathbf{A} \qquad \mathbf{P_{GG}} = \mathbf{\Psi}(\mathbf{\Psi}^T\mathbf{\Psi})^{-1}\mathbf{\Psi}^T$$

- These parts are sectioned into three components

$$\mathbf{A} = \begin{bmatrix}\mathbf{U}^{\mathbf{A}}_{\bar{\mathbf{G}}\mathbf{S}} & \mathbf{U}^{\mathbf{A}}_{\bar{\mathbf{G}}\mathcal{E}} & \mathbf{U}^{\mathbf{A}}_{\mathbf{G}}\end{bmatrix} \begin{bmatrix}\mathbf{\Sigma}^{\mathbf{A}}_{\bar{\mathbf{G}}\mathbf{S}} & 0 & 0 \\ 0 & \mathbf{\Sigma}^{\mathbf{A}}_{\bar{\mathbf{G}}\mathcal{E}} & 0 \\ 0 & 0 & \mathbf{\Sigma}^{\mathbf{A}}_{\mathbf{G}}\end{bmatrix} \begin{bmatrix}\mathbf{V}^{\mathbf{A}}_{\bar{\mathbf{G}}\mathbf{S}}{}^T \\ \mathbf{V}^{\mathbf{A}}_{\bar{\mathbf{G}}\mathcal{E}}{}^T \\ \mathbf{V}^{\mathbf{A}}_{\mathbf{G}}{}^T\end{bmatrix}$$

ISOLATING THE SOIL RESPONSE (CONTINUED)

- The target's DSRF response is isolated from the soil response

$$\mathbf{A} = \mathbf{P}_{\mathbf{GG}}\mathbf{A} + \mathbf{P}^{\perp}_{\mathbf{GG}}\mathbf{A} \qquad \mathbf{P}_{\mathbf{GG}} = \mathbf{\Psi}(\mathbf{\Psi}^T\mathbf{\Psi})^{-1}\mathbf{\Psi}^T$$

- These parts are sectioned into three components

$$\mathbf{A} = \begin{bmatrix} \mathbf{U}^{\mathbf{A}}_{\bar{\mathbf{G}}\mathbf{S}} & \mathbf{U}^{\mathbf{A}}_{\bar{\mathbf{G}}\mathcal{E}} & \mathbf{U}^{\mathbf{A}}_{\mathbf{G}} \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma}^{\mathbf{A}}_{\bar{\mathbf{G}}\mathbf{S}} & 0 & 0 \\ 0 & \mathbf{\Sigma}^{\mathbf{A}}_{\bar{\mathbf{G}}\mathcal{E}} & 0 \\ 0 & 0 & \mathbf{\Sigma}^{\mathbf{A}}_{\mathbf{G}} \end{bmatrix} \begin{bmatrix} {\mathbf{V}^{\mathbf{A}}_{\bar{\mathbf{G}}\mathbf{S}}}^T \\ {\mathbf{V}^{\mathbf{A}}_{\bar{\mathbf{G}}\mathcal{E}}}^T \\ {\mathbf{V}^{\mathbf{A}}_{\mathbf{G}}}^T \end{bmatrix}$$

The DSRF without soil is separated by how strong the target is expected to be

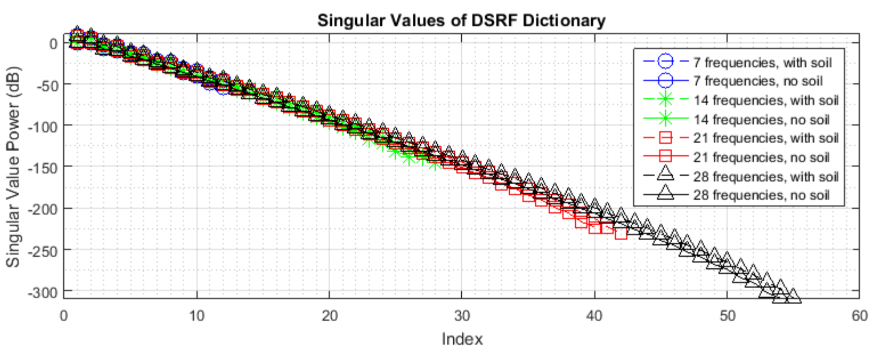
ISOLATING THE SOIL RESPONSE (CONTINUED)

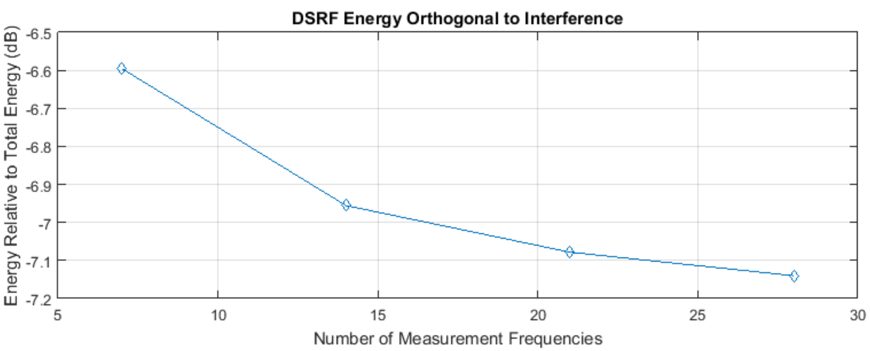
- The target's DSRF response is isolated from the soil response

$$\mathbf{A} = \mathbf{P}_{\mathbf{GG}}\mathbf{A} + \mathbf{P}_{\mathbf{GG}}^{\perp}\mathbf{A} \qquad \mathbf{P}_{\mathbf{GG}} = \boldsymbol{\Psi}(\boldsymbol{\Psi}^T\boldsymbol{\Psi})^{-1}\boldsymbol{\Psi}^T$$

- These parts are sectioned into three components

$$\mathbf{A} = \begin{bmatrix}\mathbf{U}_{\bar{\mathbf{G}}\mathbf{S}}^{\mathbf{A}} & \mathbf{U}_{\bar{\mathbf{G}}\mathcal{E}}^{\mathbf{A}} & \mathbf{U}_{\mathbf{G}}^{\mathbf{A}}\end{bmatrix}\begin{bmatrix}\boldsymbol{\Sigma}_{\bar{\mathbf{G}}\mathbf{S}}^{\mathbf{A}} & 0 & 0 \\ 0 & \boldsymbol{\Sigma}_{\bar{\mathbf{G}}\mathcal{E}}^{\mathbf{A}} & 0 \\ 0 & 0 & \boldsymbol{\Sigma}_{\mathbf{G}}^{\mathbf{A}}\end{bmatrix}\begin{bmatrix}{\mathbf{V}_{\bar{\mathbf{G}}\mathbf{S}}^{\mathbf{A}}}^T \\ {\mathbf{V}_{\bar{\mathbf{G}}\mathcal{E}}^{\mathbf{A}}}^T \\ {\mathbf{V}_{\mathbf{G}}^{\mathbf{A}}}^T\end{bmatrix}$$





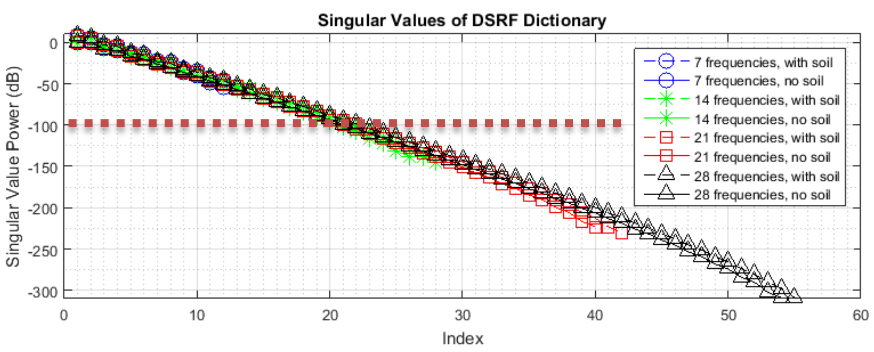
ISOLATING THE SOIL RESPONSE (CONTINUED)

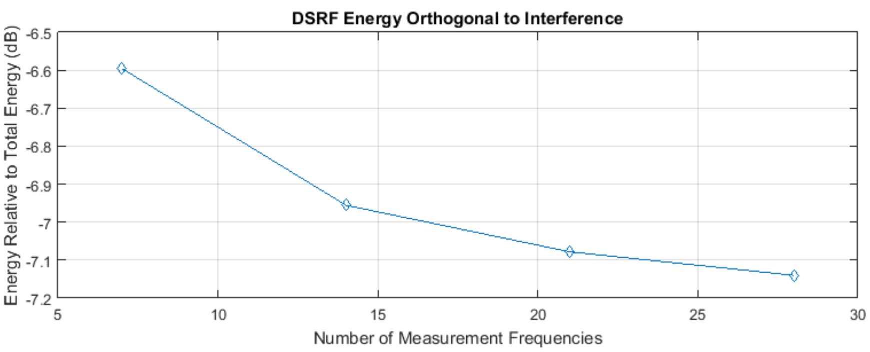
- The target's DSRF response is isolated from the soil response

$$\mathbf{A} = \mathbf{P}_{\mathbf{GG}}\mathbf{A} + \mathbf{P}_{\mathbf{GG}}^{\perp}\mathbf{A} \qquad \mathbf{P}_{\mathbf{GG}} = \mathbf{\Psi}(\mathbf{\Psi}^T\mathbf{\Psi})^{-1}\mathbf{\Psi}^T$$

- These parts are sectioned into three components

$$\mathbf{A} = \begin{bmatrix} \mathbf{U}_{\bar{\mathbf{G}}\mathbf{S}}^{\mathbf{A}} & \mathbf{U}_{\bar{\mathbf{G}}\mathcal{E}}^{\mathbf{A}} & \mathbf{U}_{\mathbf{G}}^{\mathbf{A}} \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma}_{\bar{\mathbf{G}}\mathbf{S}}^{\mathbf{A}} & 0 & 0 \\ 0 & \mathbf{\Sigma}_{\bar{\mathbf{G}}\mathcal{E}}^{\mathbf{A}} & 0 \\ 0 & 0 & \mathbf{\Sigma}_{\mathbf{G}}^{\mathbf{A}} \end{bmatrix} \begin{bmatrix} {\mathbf{V}_{\bar{\mathbf{G}}\mathbf{S}}^{\mathbf{A}}}^T \\ {\mathbf{V}_{\bar{\mathbf{G}}\mathcal{E}}^{\mathbf{A}}}^T \\ {\mathbf{V}_{\mathbf{G}}^{\mathbf{A}}}^T \end{bmatrix}$$





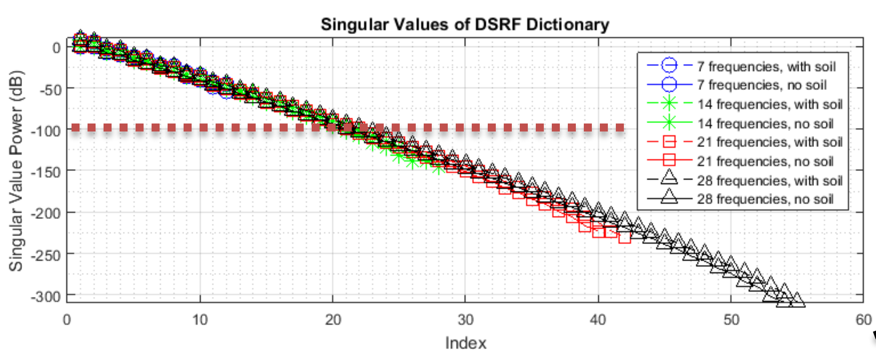
ISOLATING THE SOIL RESPONSE (CONTINUED)

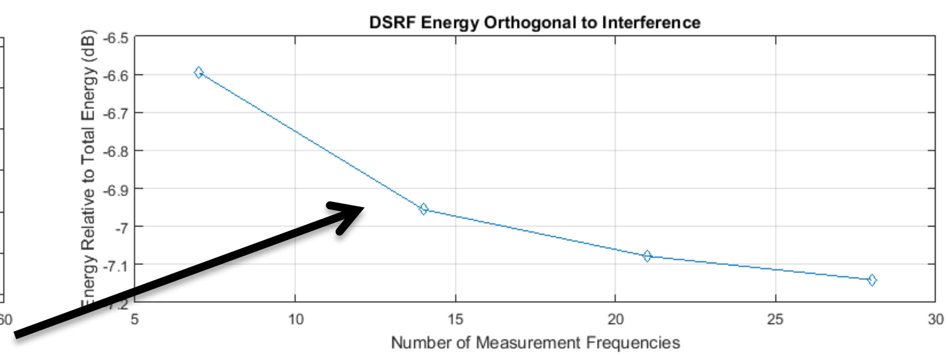
- The target's DSRF response is isolated from the soil response

$$\mathbf{A} = \mathbf{P}_{\mathbf{GG}}\mathbf{A} + \mathbf{P}_{\mathbf{GG}}^{\perp}\mathbf{A} \qquad\qquad \mathbf{P}_{\mathbf{GG}} = \boldsymbol{\Psi}(\boldsymbol{\Psi}^T\boldsymbol{\Psi})^{-1}\boldsymbol{\Psi}^T$$

- These parts are sectioned into three components

$$\mathbf{A} = \begin{bmatrix}\mathbf{U}_{\bar{\mathbf{G}}\mathbf{S}}^{\mathbf{A}} & \mathbf{U}_{\bar{\mathbf{G}}\mathcal{E}}^{\mathbf{A}} & \mathbf{U}_{\mathbf{G}}^{\mathbf{A}}\end{bmatrix} \begin{bmatrix}\boldsymbol{\Sigma}_{\bar{\mathbf{G}}\mathbf{S}}^{\mathbf{A}} & 0 & 0 \\ 0 & \boldsymbol{\Sigma}_{\bar{\mathbf{G}}\mathcal{E}}^{\mathbf{A}} & 0 \\ 0 & 0 & \boldsymbol{\Sigma}_{\mathbf{G}}^{\mathbf{A}}\end{bmatrix} \begin{bmatrix}{\mathbf{V}_{\bar{\mathbf{G}}\mathbf{S}}^{\mathbf{A}}}^T \\ {\mathbf{V}_{\bar{\mathbf{G}}\mathcal{E}}^{\mathbf{A}}}^T \\ {\mathbf{V}_{\mathbf{G}}^{\mathbf{A}}}^T\end{bmatrix}$$





We lose roughly 6-7 dB of the target energy when isolating it from the soil

APPLYING THE PROJECTION MATRICES

- Projection matrix operators are defined

$$\mathbf{P}_{\mathbf{G}} = \begin{bmatrix} \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}} \\ \mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}} \\ \mathbf{P}_{\mathbf{G}\mathbf{G}} \end{bmatrix} = \begin{bmatrix} \mathbf{U}^{\mathbf{A}}_{\bar{\mathbf{G}}\mathbf{S}} & \mathbf{U}^{\mathbf{A}}_{\bar{\mathbf{G}}\mathcal{E}} & \mathbf{U}^{\mathbf{A}}_{\mathbf{G}} \end{bmatrix}^T$$

$$\mathbf{P}_{\mathbf{R}} = \begin{bmatrix} \mathbf{P}_{\mathbf{R}\mathbf{R}} \\ \mathbf{P}_{\bar{\mathbf{R}}\mathbf{S}} \\ \mathbf{P}_{\bar{\mathbf{R}}\mathcal{E}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mathcal{D}}_L \\ \boldsymbol{\mathcal{D}}_M \\ \boldsymbol{\mathcal{D}}_S \end{bmatrix} = \boldsymbol{\mathcal{D}}$$

APPLYING THE PROJECTION MATRICES

- Projection matrix operators are defined

$$\mathbf{P_G} = \begin{bmatrix} \mathbf{P_{\bar{G}S}} \\ \mathbf{P_{\bar{G}\mathcal{E}}} \\ \mathbf{P_{GG}} \end{bmatrix} = \begin{bmatrix} \mathbf{U^A_{\bar{G}S}} & \mathbf{U^A_{\bar{G}\mathcal{E}}} & \mathbf{U^A_{G}} \end{bmatrix}^T$$

$$\mathbf{P_R} = \begin{bmatrix} \mathbf{P_{RR}} \\ \mathbf{P_{\bar{R}S}} \\ \mathbf{P_{\bar{R}\mathcal{E}}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mathcal{D}}_L \\ \boldsymbol{\mathcal{D}}_M \\ \boldsymbol{\mathcal{D}}_S \end{bmatrix} = \boldsymbol{\mathcal{D}}$$

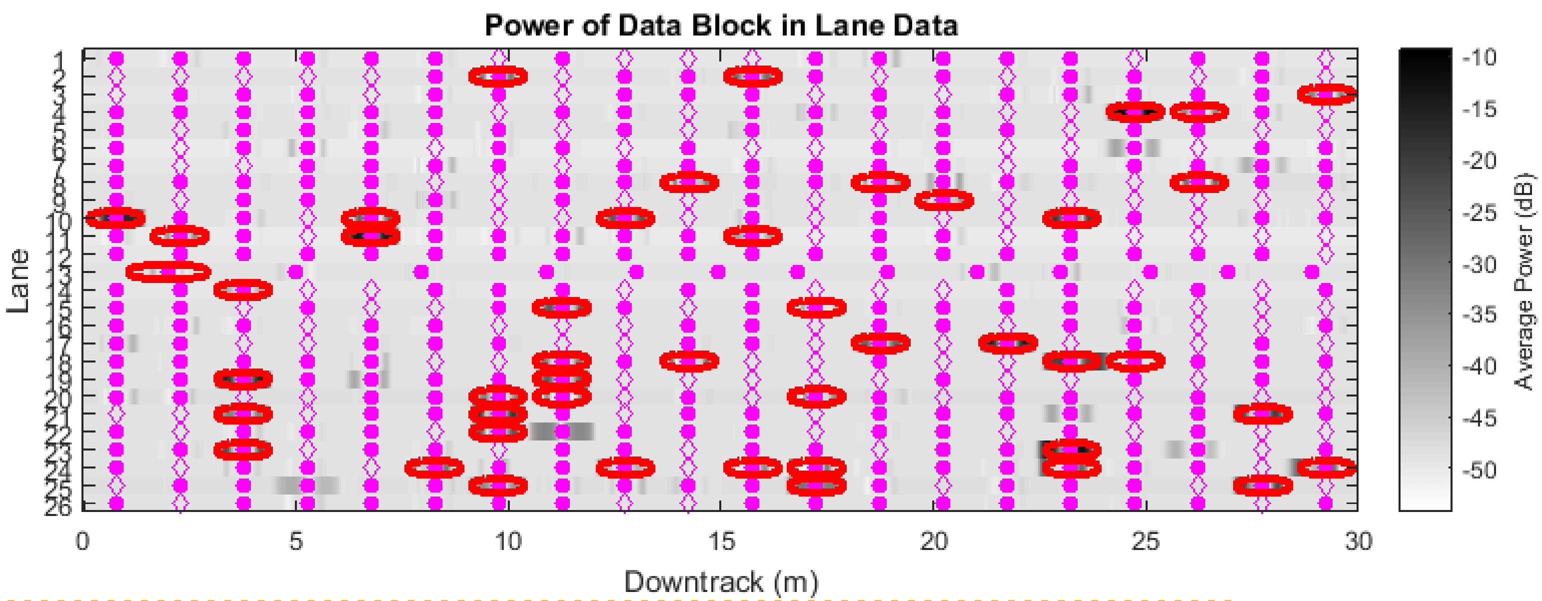
- Projection matrix operations can be used to create nine data blocks

$$\mathbf{P_G M P_R^T} =$$

$\mathrm{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathrm{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathrm{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

INVESTIGATING FIRST DATA BLOCK

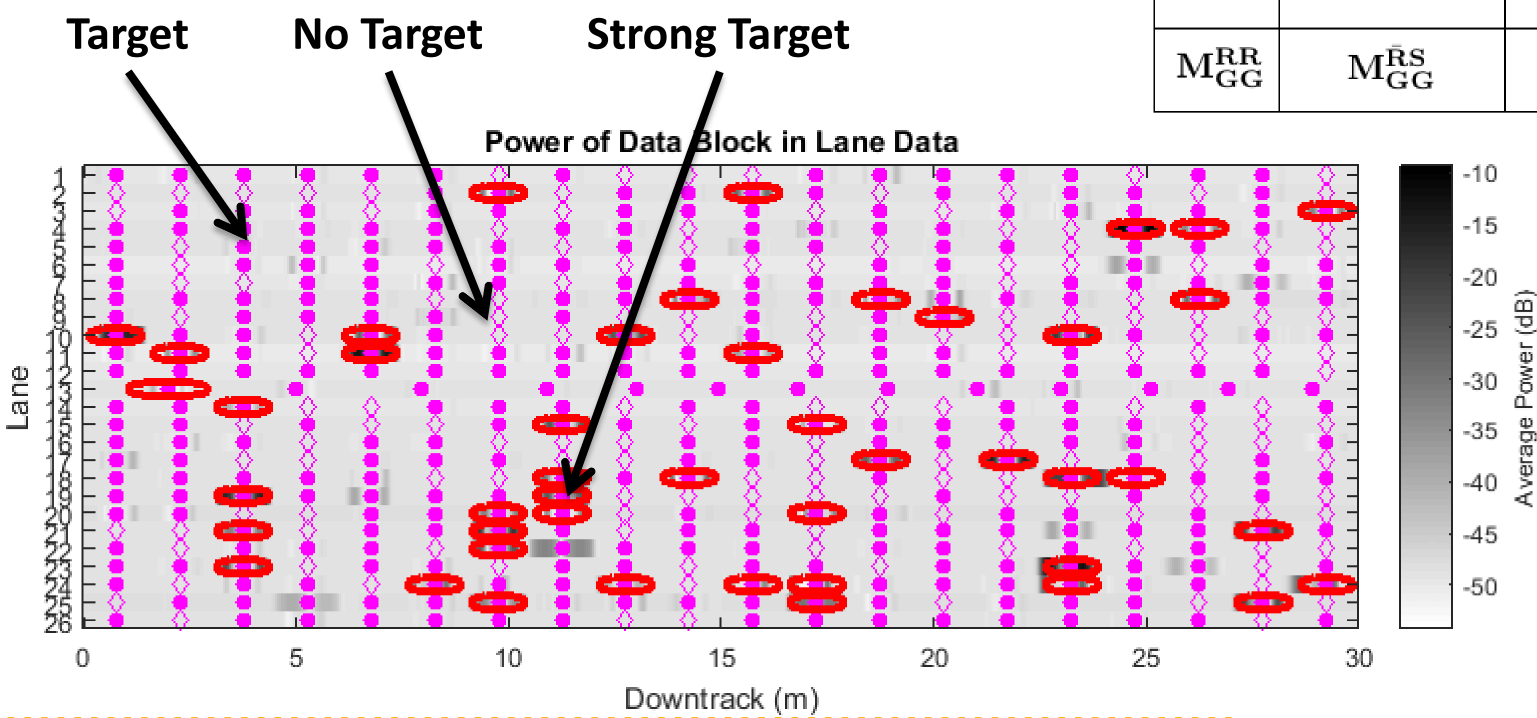
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}$



CREATING THE NEXT

INVESTIGATING FIRST DATA BLOCK

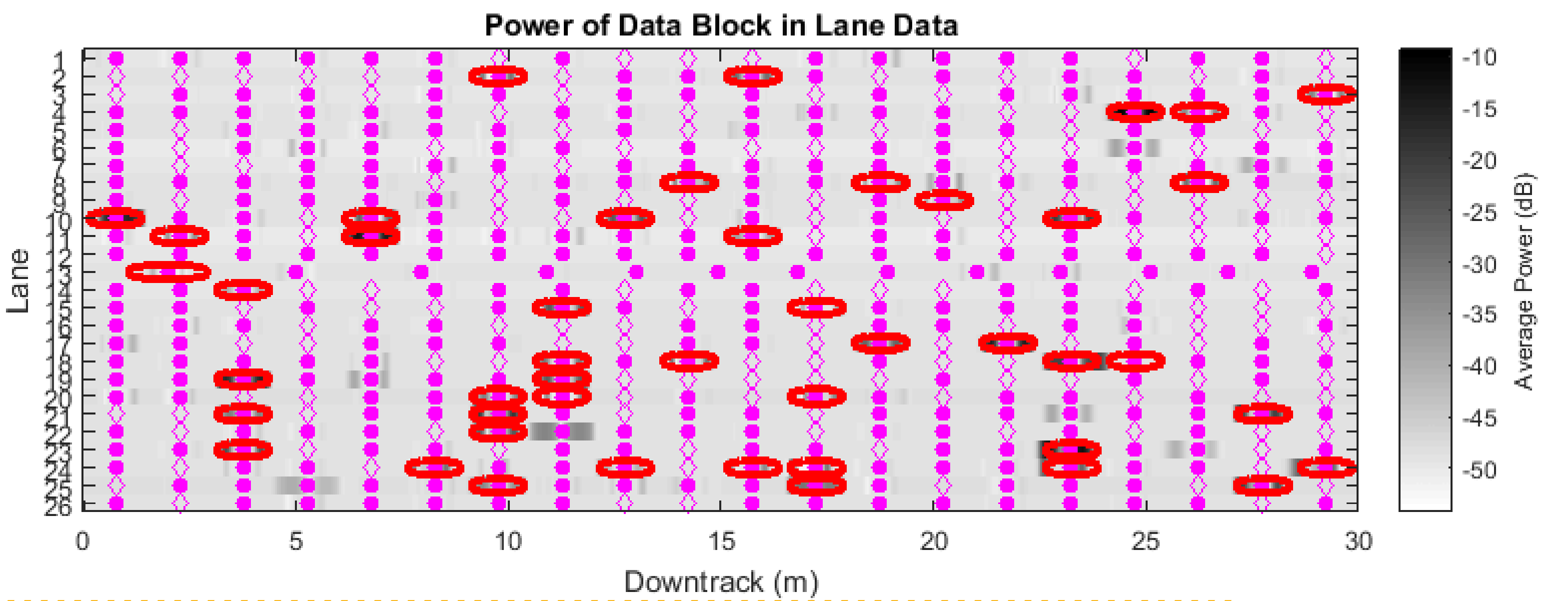
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}$



INVESTIGATING FIRST DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{M}\mathbf{P}^{T}_{\mathbf{RR}} = \cancelto{\epsilon}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{S}\mathbf{P}^{T}_{\mathbf{RR}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{G}\mathbf{P}^{T}_{\mathbf{RR}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{R}\mathbf{P}^{T}_{\mathbf{RR}} + \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathcal{E}\mathbf{P}^{T}_{\mathbf{RR}}$$

$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathbf{R}}_{\bar{\mathbf{G}}\mathbf{S}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{R}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{R}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

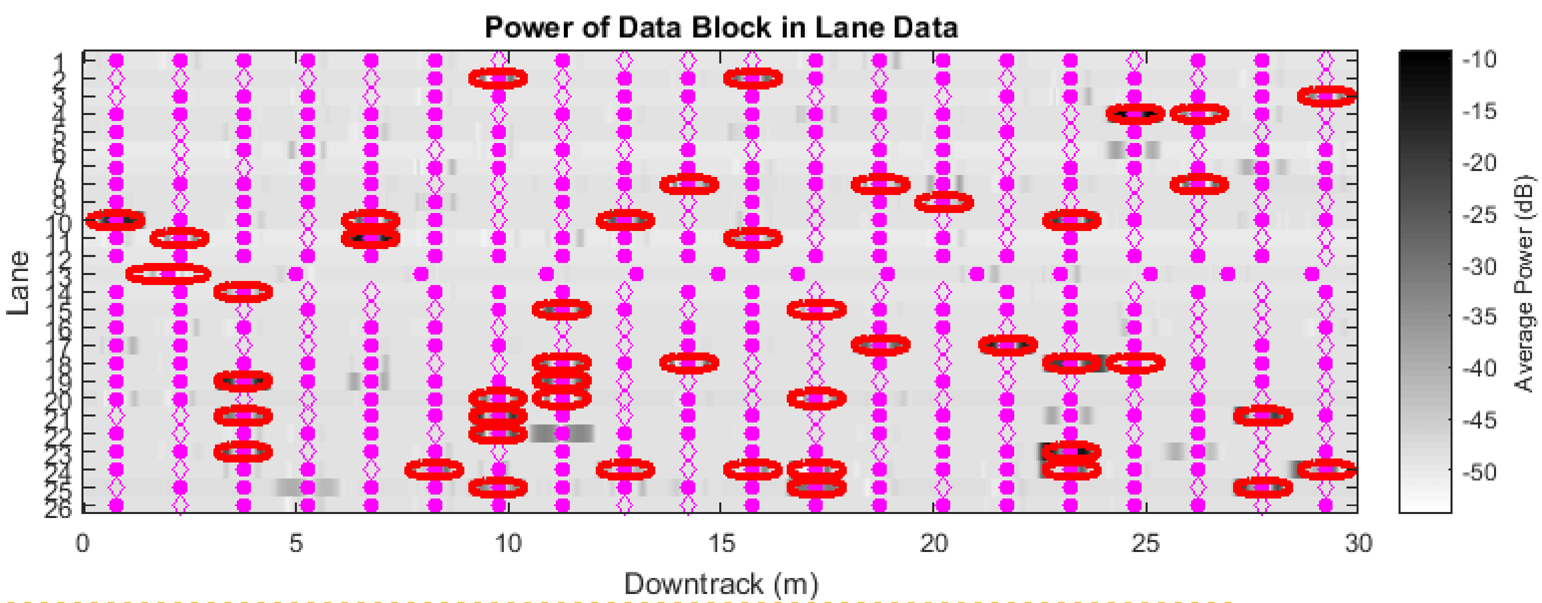


INVESTIGATING FIRST DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{M}\mathbf{P}^{T}_{\mathbf{RR}} = \cancelto{\epsilon}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{S}\mathbf{P}^{T}_{\mathbf{RR}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{G}\mathbf{P}^{T}_{\mathbf{RR}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{R}\mathbf{P}^{T}_{\mathbf{RR}} + \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathcal{E}\mathbf{P}^{T}_{\mathbf{RR}}$$

- This data block is dominated by the **self response**

$\boxed{\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

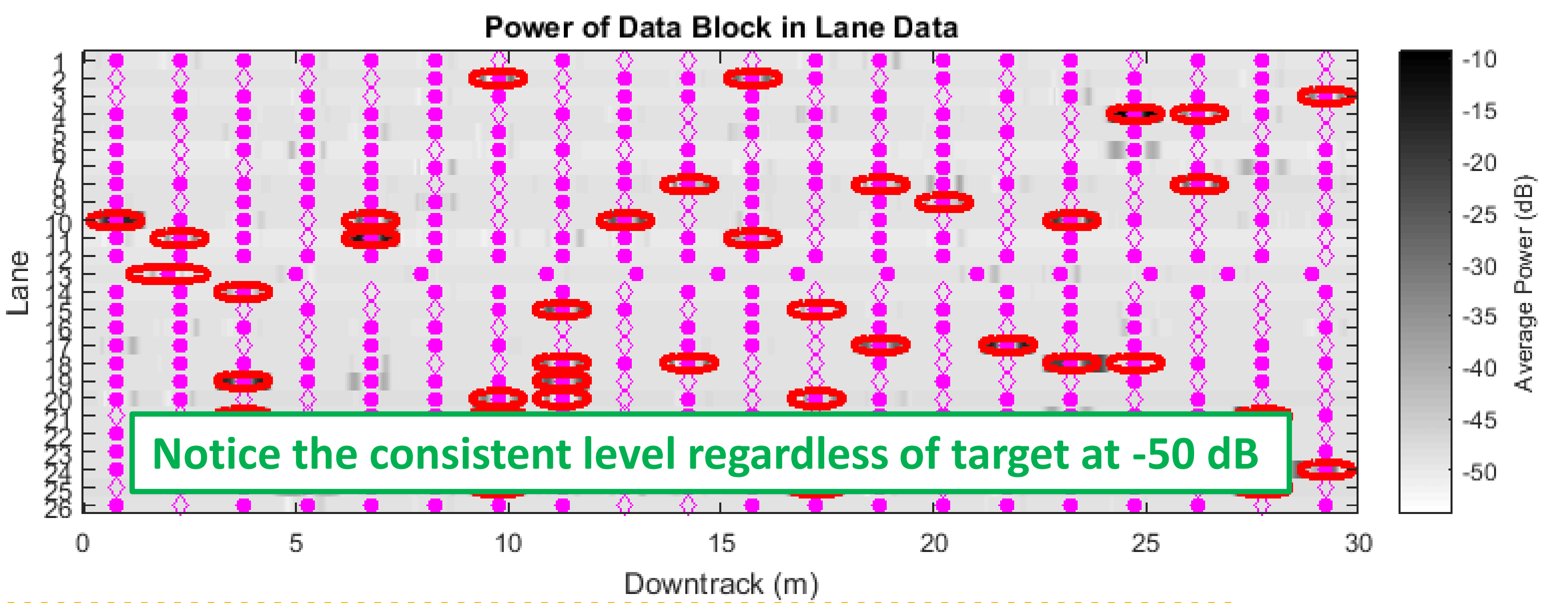


INVESTIGATING FIRST DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{M}\mathbf{P}^T_{\mathbf{RR}} = \cancelto{\epsilon}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{S}\mathbf{P}^T_{\mathbf{RR}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{G}\mathbf{P}^T_{\mathbf{RR}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{R}\mathbf{P}^T_{\mathbf{RR}} + \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathcal{E}\mathbf{P}^T_{\mathbf{RR}}$$

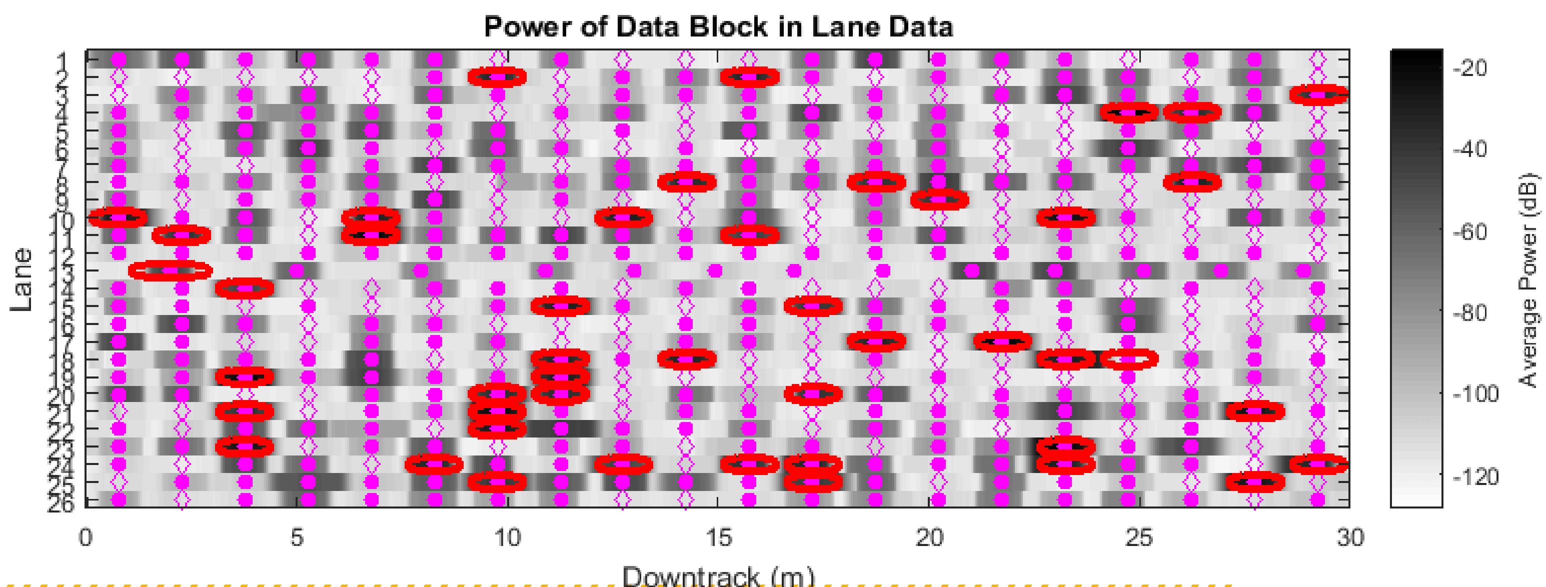
- This data block is dominated by the **self response**
- It is ideal for estimating the quality of the WEMI system

$\boxed{\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$



INVESTIGATING SECOND DATA BLOCK

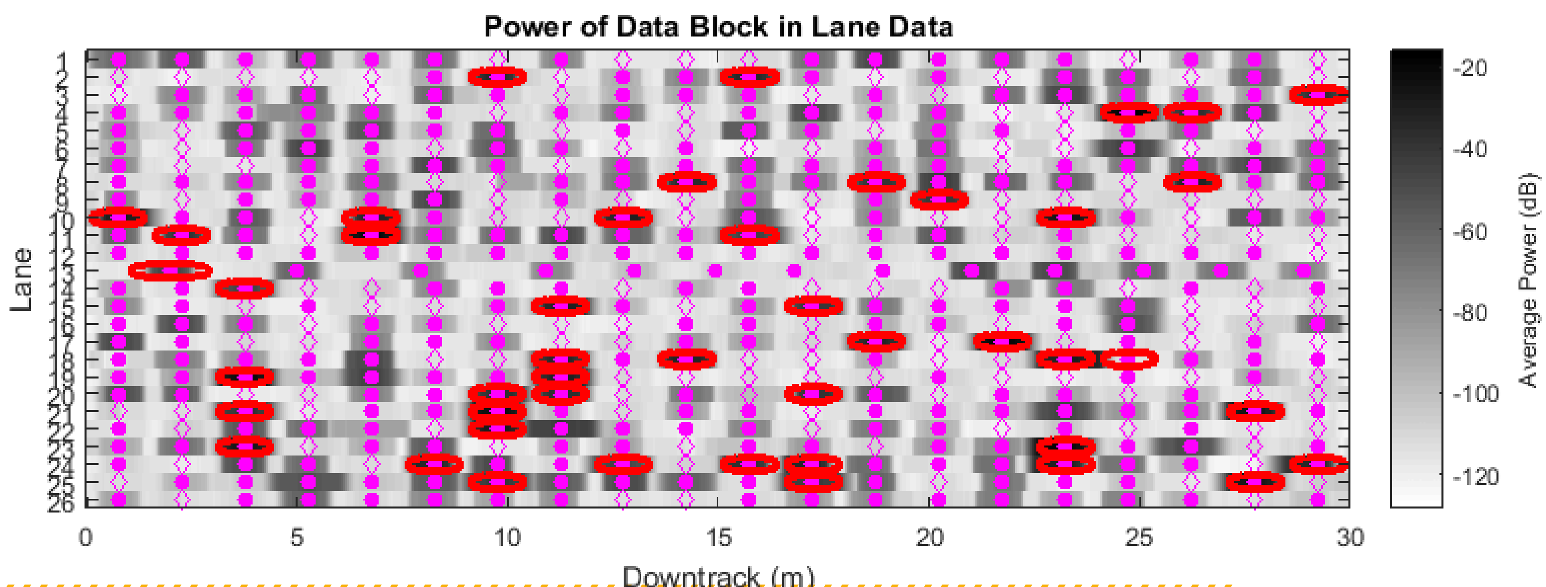
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}$



INVESTIGATING SECOND DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{M}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} = \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{S}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{G}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{R}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathcal{E}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}$$

$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\boxed{\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathrm{S}}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}$

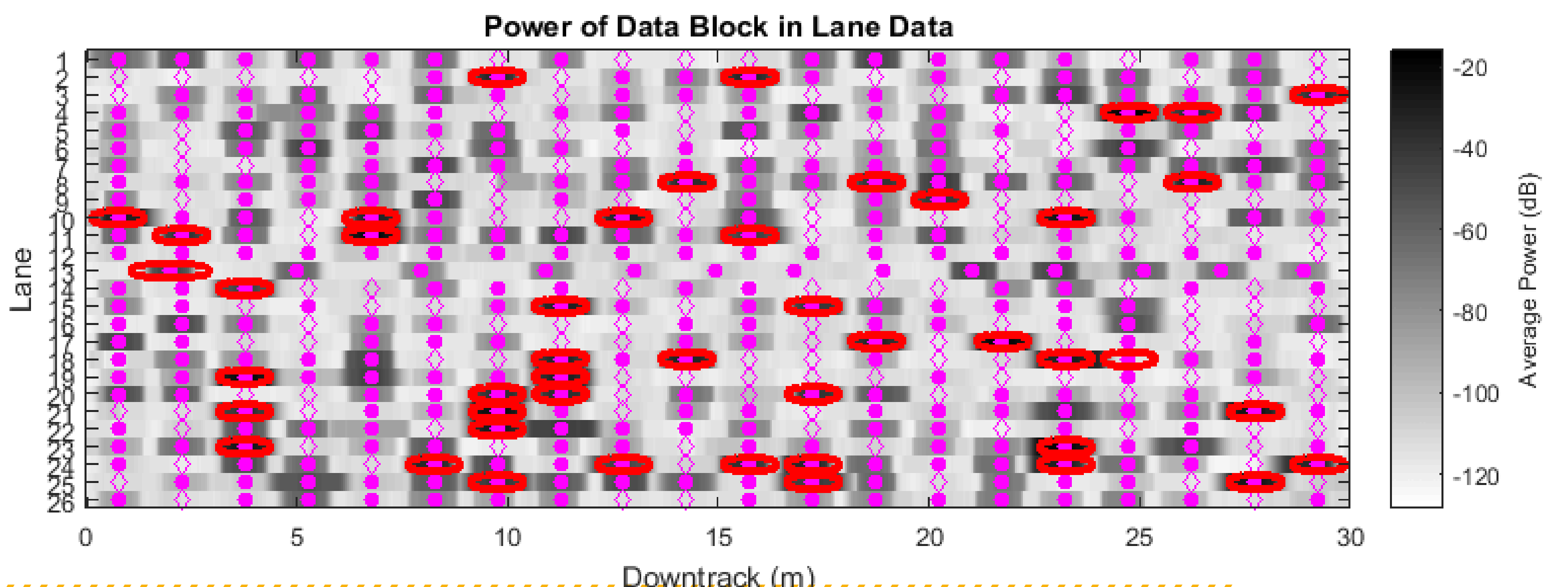


INVESTIGATING SECOND DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{M}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} = \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{S}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{G}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{R}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathcal{E}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}$$

- This data block has a **strong target** response and **no interference**

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$



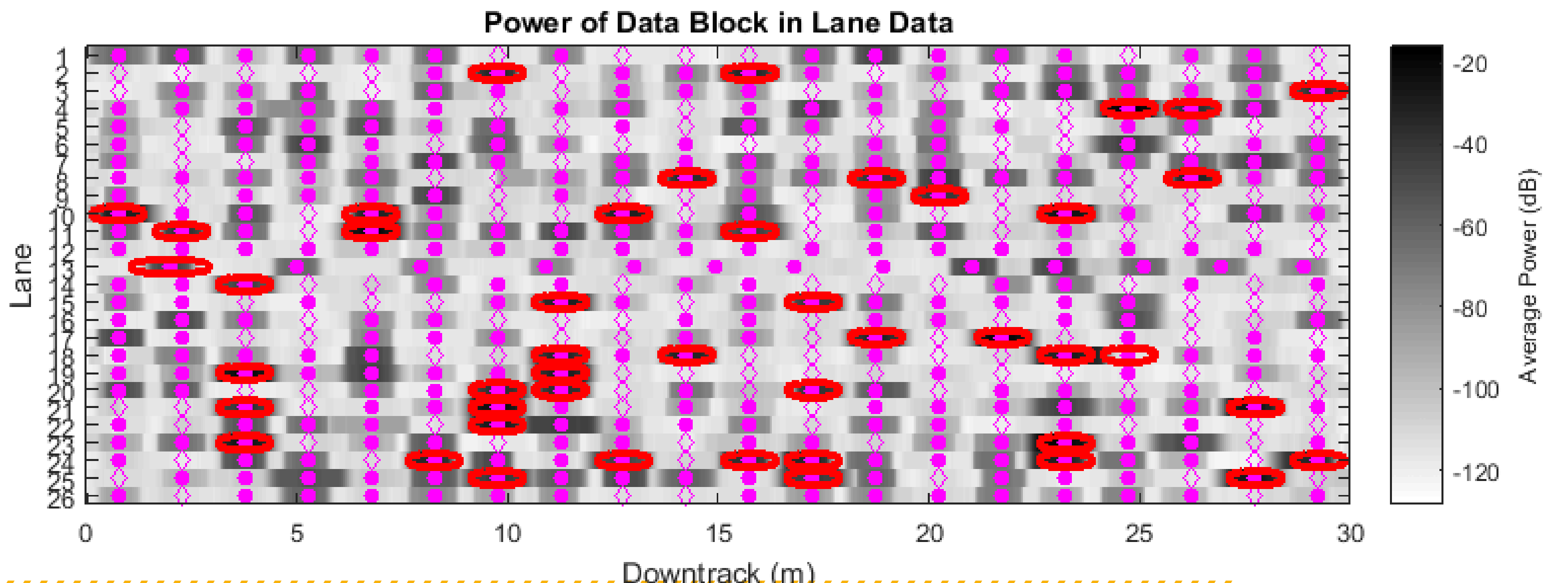
INVESTIGATING SECOND DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{M}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} = \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{S}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{G}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{R}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathcal{E}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}$$

- This data block has a **strong target** response and **no interference**
- This data is ideal for target detection, classification, and localization

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

$$E\left[\frac{\|\mathbf{S}_\mathbf{S}\|_F^2}{\|\mathcal{E}_\mathbf{S}\|_F^2}\right] = \delta_g \frac{2MN}{OP} E\left[\frac{\|\mathbf{S}\|_F^2}{\|\mathcal{E}\|_F^2}\right] \approx \frac{12}{4} E\left[\frac{\|\mathbf{S}\|_F^2}{\|\mathcal{E}\|_F^2}\right]$$



INVESTIGATING SECOND DATA BLOCK

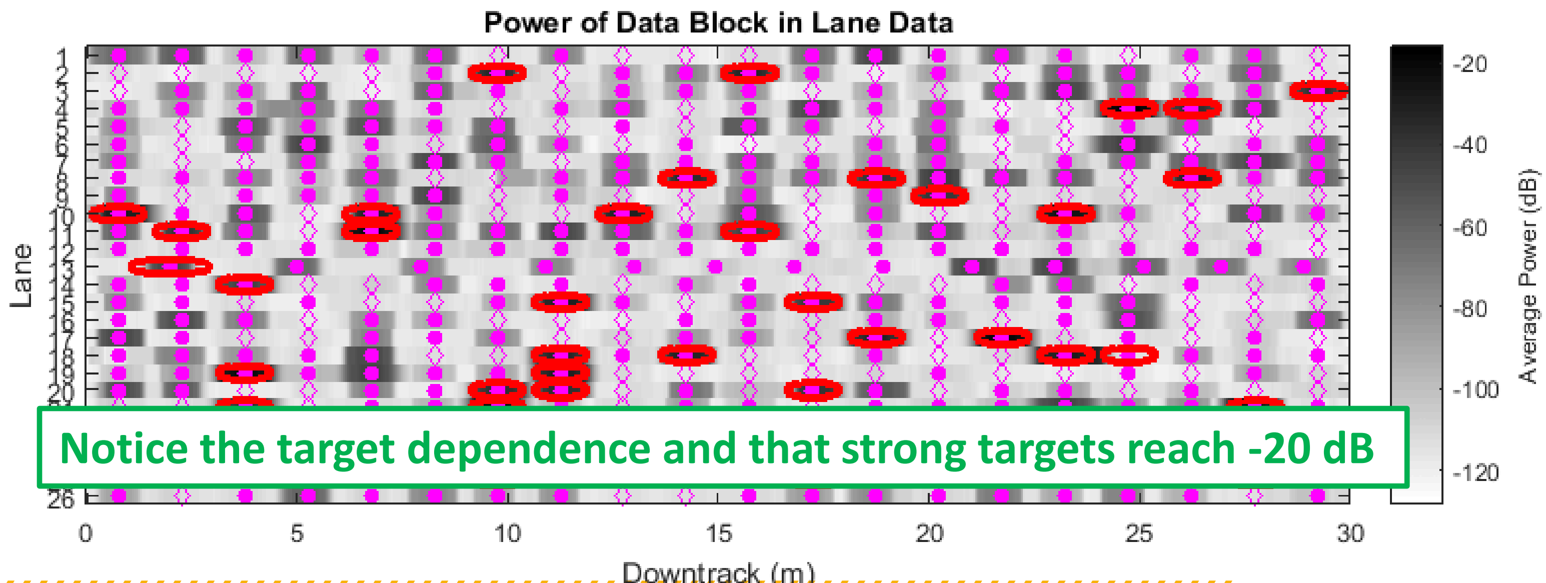
$$\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{M}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} = \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{S}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{G}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{R}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathcal{E}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}$$

- This data block has a **strong target** response and **no interference**
- This data is ideal for target detection, classification, and localization

~9dB SNR gain

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

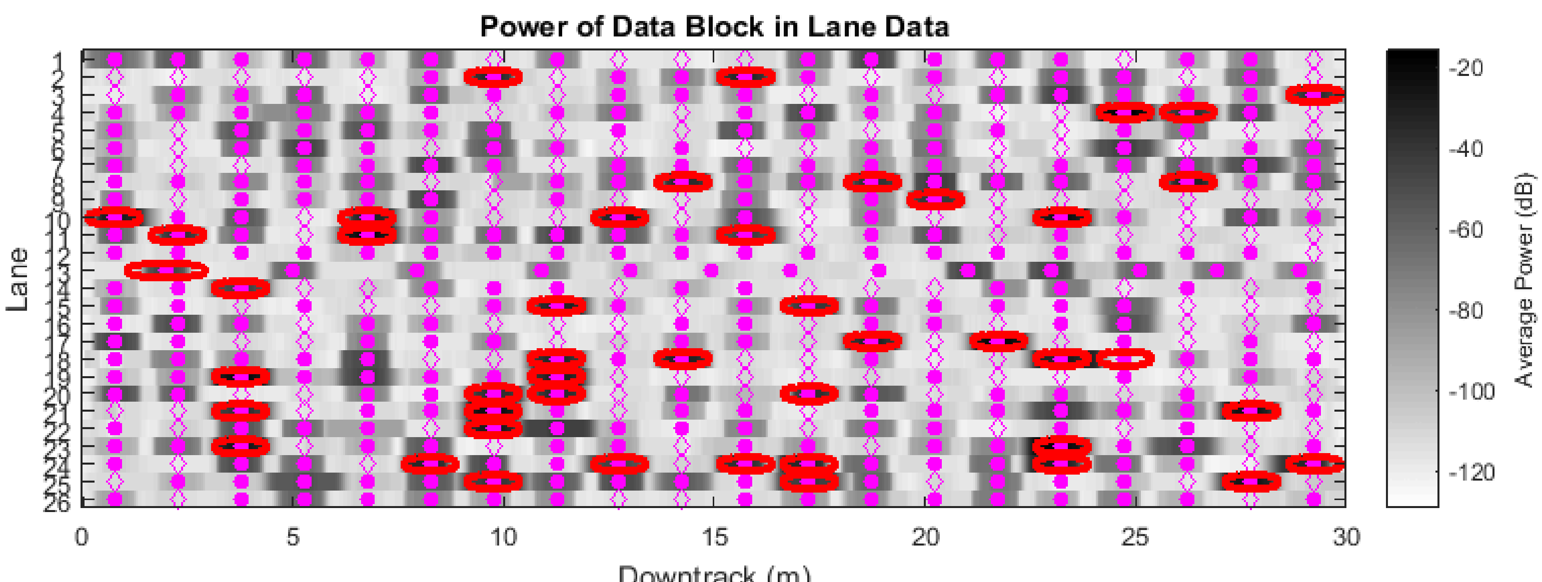
$$E\left[\frac{\|\mathbf{S}_{\mathbf{S}}\|_F^2}{\|\mathcal{E}_{\mathbf{S}}\|_F^2}\right] = \delta_g \frac{2MN}{OP} E\left[\frac{\|\mathbf{S}\|_F^2}{\|\mathcal{E}\|_F^2}\right] \approx \frac{12}{4} E\left[\frac{\|\mathbf{S}\|_F^2}{\|\mathcal{E}\|_F^2}\right]$$



Notice the target dependence and that strong targets reach -20 dB

EXPLOITING LOW-RANK TARGET IN SECOND DATA BLOCK

$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}$

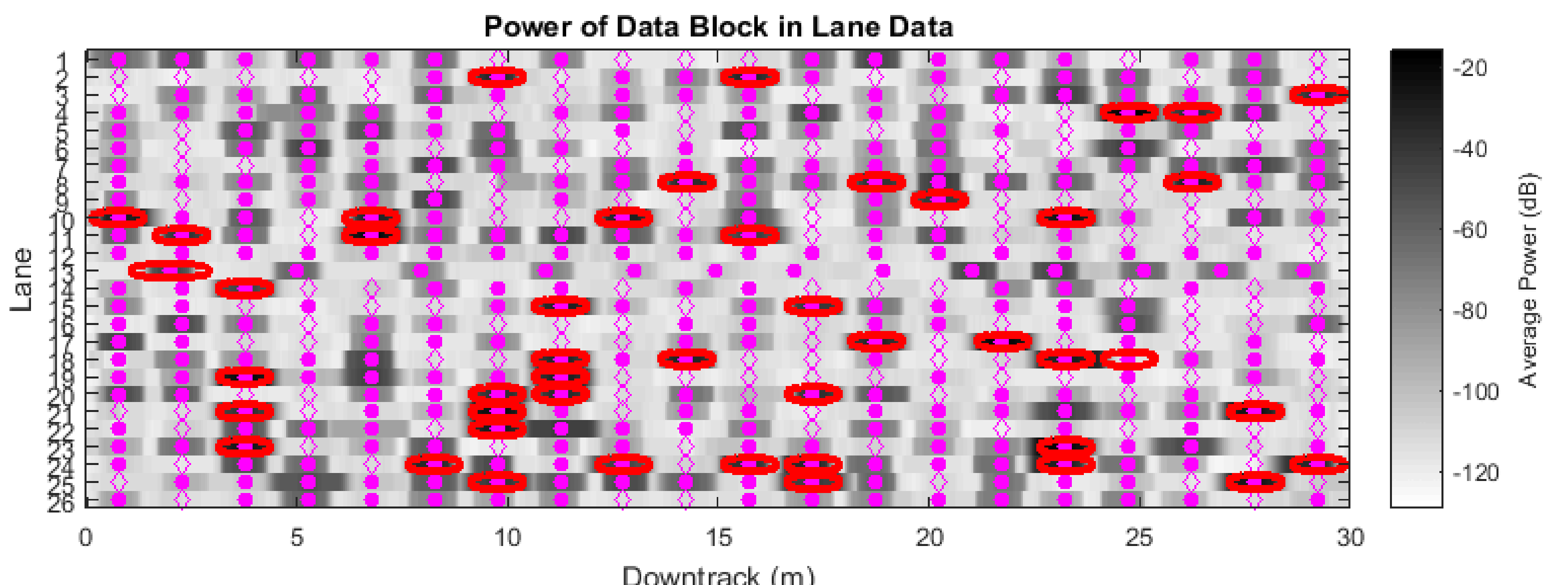


EXPLOITING LOW-RANK TARGET IN SECOND DATA BLOCK

$$\mathbf{M_S} = \begin{bmatrix} \mathbf{U}_{\mathbf{S}}^{\mathbf{M_S}} & \mathbf{U}_{\mathcal{E}}^{\mathbf{M_S}} \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma}_{\mathbf{S}}^{\mathbf{M_S}} & 0 \\ 0 & \mathbf{\Sigma}_{\mathcal{E}}^{\mathbf{M_S}} \end{bmatrix} \begin{bmatrix} {\mathbf{V}_{\mathbf{S}}^{\mathbf{M_S}}}^T \\ {\mathbf{V}_{\mathcal{E}}^{\mathbf{M_S}}}^T \end{bmatrix}$$

$\mathrm{M}_{\bar{\mathrm{G}}\mathrm{S}}^{\mathrm{RR}}$	$\mathrm{M}_{\bar{\mathrm{G}}\mathrm{S}}^{\bar{\mathrm{R}}\mathrm{S}}$	$\mathrm{M}_{\bar{\mathrm{G}}\mathrm{S}}^{\bar{\mathrm{R}}\mathcal{E}}$
$\mathrm{M}_{\bar{\mathrm{G}}\mathcal{E}}^{\mathrm{RR}}$	$\mathrm{M}_{\bar{\mathrm{G}}\mathcal{E}}^{\bar{\mathrm{R}}\mathrm{S}}$	$\mathrm{M}_{\bar{\mathrm{G}}\mathcal{E}}^{\bar{\mathrm{R}}\mathcal{E}}$
$\mathrm{M}_{\mathrm{GG}}^{\mathrm{RR}}$	$\mathrm{M}_{\mathrm{GG}}^{\bar{\mathrm{R}}\mathrm{S}}$	$\mathrm{M}_{\mathrm{GG}}^{\bar{\mathrm{R}}\mathcal{E}}$

- Because the target response is known to be low-rank, the SVD of the data block can be used to further isolate the target.



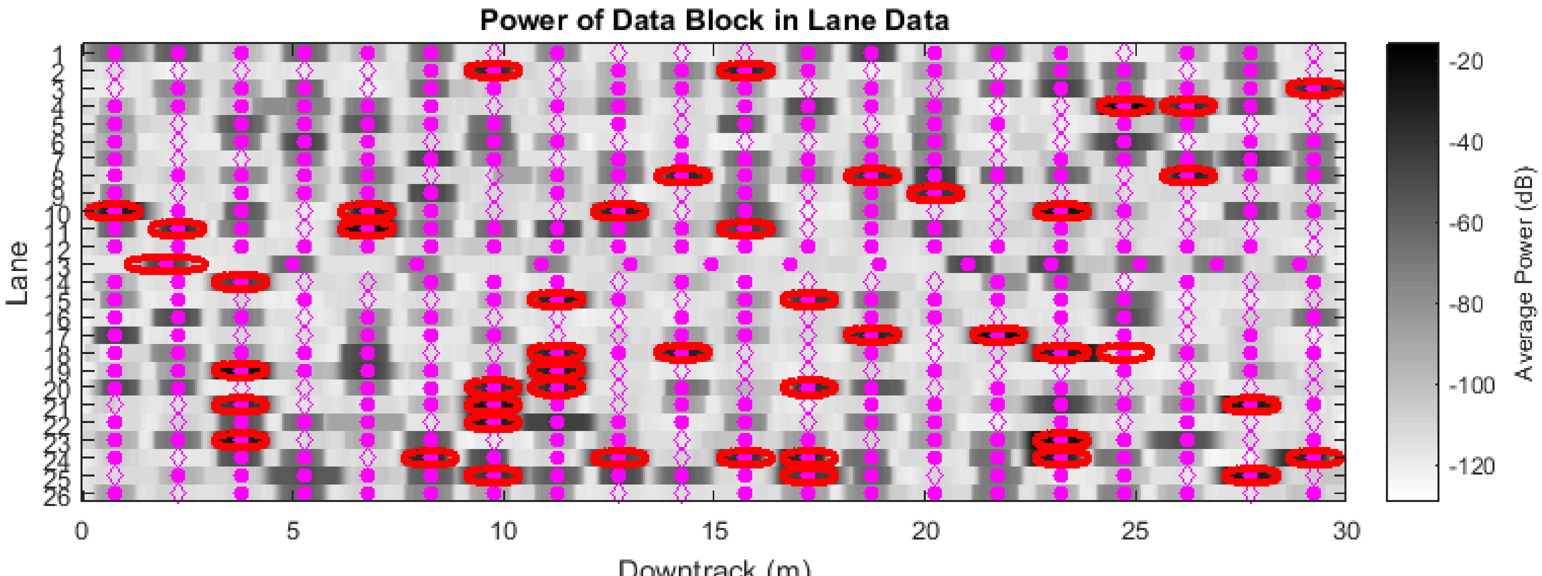
EXPLOITING LOW-RANK TARGET IN SECOND DATA BLOCK

$$\mathbf{M_S} = \begin{bmatrix} \mathbf{U}_{\mathbf{S}}^{\mathbf{M_S}} & \mathbf{U}_{\mathcal{E}}^{\mathbf{M_S}} \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma}_{\mathbf{S}}^{\mathbf{M_S}} & 0 \\ 0 & \mathbf{\Sigma}_{\mathcal{E}}^{\mathbf{M_S}} \end{bmatrix} \begin{bmatrix} {\mathbf{V}_{\mathbf{S}}^{\mathbf{M_S}}}^T \\ {\mathbf{V}_{\mathcal{E}}^{\mathbf{M_S}}}^T \end{bmatrix}$$

$\mathbf{M}_{\bar{\mathbf{G}}\mathbf{S}}^{\mathbf{RR}}$	$\mathbf{M}_{\bar{\mathbf{G}}\mathbf{S}}^{\bar{\mathbf{R}}\mathbf{S}}$	$\mathbf{M}_{\bar{\mathbf{G}}\mathbf{S}}^{\bar{\mathbf{R}}\mathcal{E}}$
$\mathbf{M}_{\bar{\mathbf{G}}\mathcal{E}}^{\mathbf{RR}}$	$\mathbf{M}_{\bar{\mathbf{G}}\mathcal{E}}^{\bar{\mathbf{R}}\mathbf{S}}$	$\mathbf{M}_{\bar{\mathbf{G}}\mathcal{E}}^{\bar{\mathbf{R}}\mathcal{E}}$
$\mathbf{M}_{\mathbf{GG}}^{\mathbf{RR}}$	$\mathbf{M}_{\mathbf{GG}}^{\bar{\mathbf{R}}\mathbf{S}}$	$\mathbf{M}_{\mathbf{GG}}^{\bar{\mathbf{R}}\mathcal{E}}$

- Because the target response is known to be low-rank, the SVD of the data block can be used to further isolate the target.

$$\mathrm{SNR}\left(\mathbf{U}_{\mathbf{S}}^{\mathbf{M_S}}\mathbf{\Sigma}_{\mathbf{S}}^{\mathbf{M_S}}{\mathbf{V}_{\mathbf{S}}^{\mathbf{M_S}}}^T\right) = \delta_g \frac{2MN}{3\cdot\max\{O,P\}} E\left[\frac{\|\mathbf{S}\|_F^2}{\|\mathcal{E}\|_F^2}\right] \approx \frac{84.5}{4} E\left[\frac{\|\mathbf{S}\|_F^2}{\|\mathcal{E}\|_F^2}\right]$$



EXPLOITING LOW-RANK TARGET IN SECOND DATA BLOCK

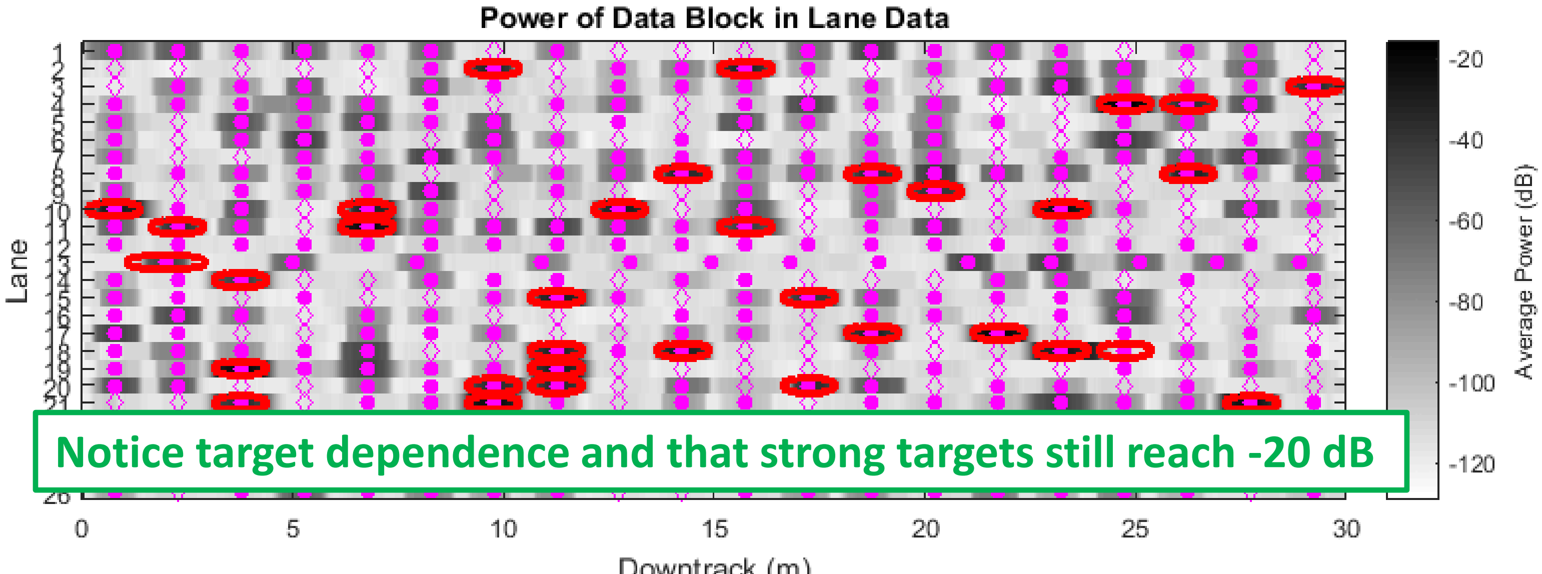
$$\mathbf{M_S} = \left[\mathbf{U}_{\mathbf{S}}^{\mathbf{M_S}} \quad \mathbf{U}_{\mathcal{E}}^{\mathbf{M_S}}\right] \begin{bmatrix} \boldsymbol{\Sigma}_{\mathbf{S}}^{\mathbf{M_S}} & 0 \\ 0 & \boldsymbol{\Sigma}_{\mathcal{E}}^{\mathbf{M_S}} \end{bmatrix} \begin{bmatrix} {\mathbf{V}_{\mathbf{S}}^{\mathbf{M_S}}}^T \\ {\mathbf{V}_{\mathcal{E}}^{\mathbf{M_S}}}^T \end{bmatrix}$$

$\mathbf{M}_{\bar{G}S}^{RR}$	$\mathbf{M}_{\bar{G}S}^{\bar{R}S}$	$\mathbf{M}_{\bar{G}S}^{\bar{R}\mathcal{E}}$
$\mathbf{M}_{\bar{G}\mathcal{E}}^{RR}$	$\mathbf{M}_{\bar{G}\mathcal{E}}^{\bar{R}S}$	$\mathbf{M}_{\bar{G}\mathcal{E}}^{\bar{R}\mathcal{E}}$
$\mathbf{M}_{GG}^{RR}$	$\mathbf{M}_{GG}^{\bar{R}S}$	$\mathbf{M}_{GG}^{\bar{R}\mathcal{E}}$

- Because the target response is known to be low-rank, the SVD of the data block can be used to further isolate the target.

~26dB SNR gain

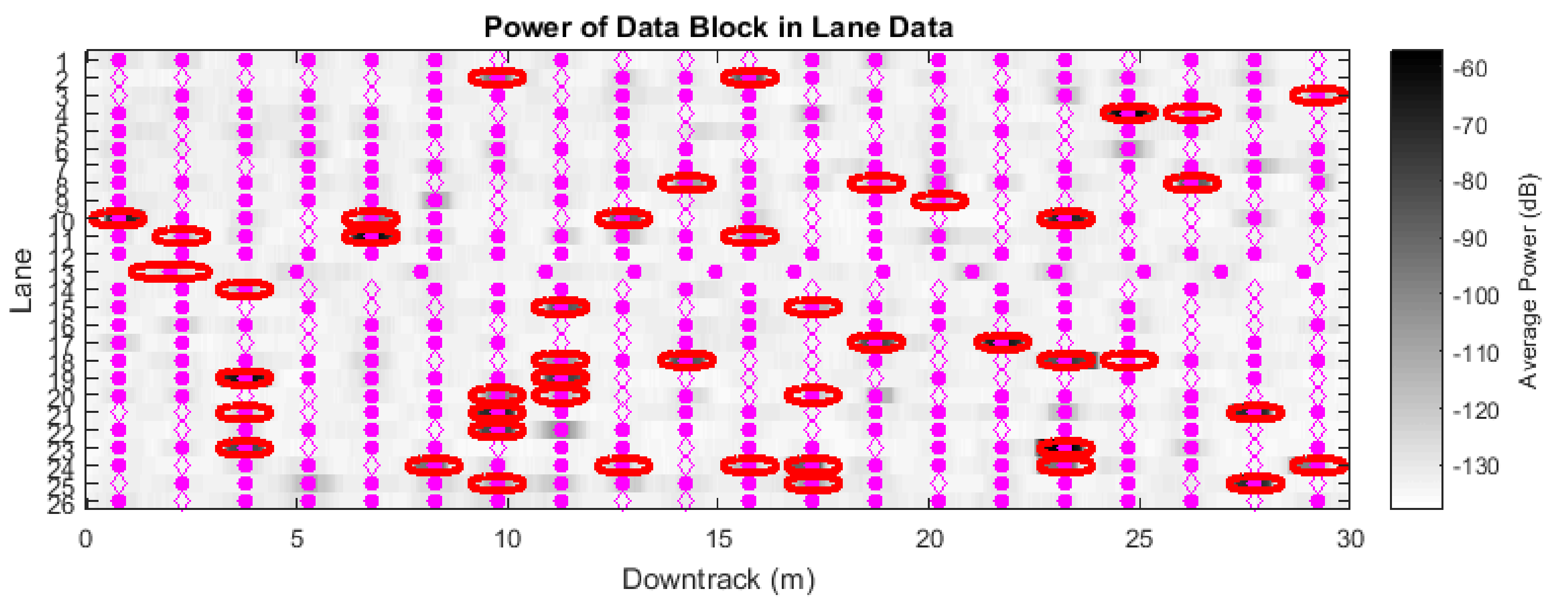
$$\mathrm{SNR}\left(\mathbf{U}_{\mathbf{S}}^{\mathbf{M_S}} \boldsymbol{\Sigma}_{\mathbf{S}}^{\mathbf{M_S}} {\mathbf{V}_{\mathbf{S}}^{\mathbf{M_S}}}^T\right) = \delta_g \frac{2MN}{3 \cdot \max\{O,P\}} E\left[\frac{\|\mathbf{S}\|_F^2}{\|\mathcal{E}\|_F^2}\right] \approx \frac{84.5}{4} E\left[\frac{\|\mathbf{S}\|_F^2}{\|\mathcal{E}\|_F^2}\right]$$



Notice target dependence and that strong targets still reach -20 dB

EXPLOITING NOISE IN SECOND DATA BLOCK

$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}$

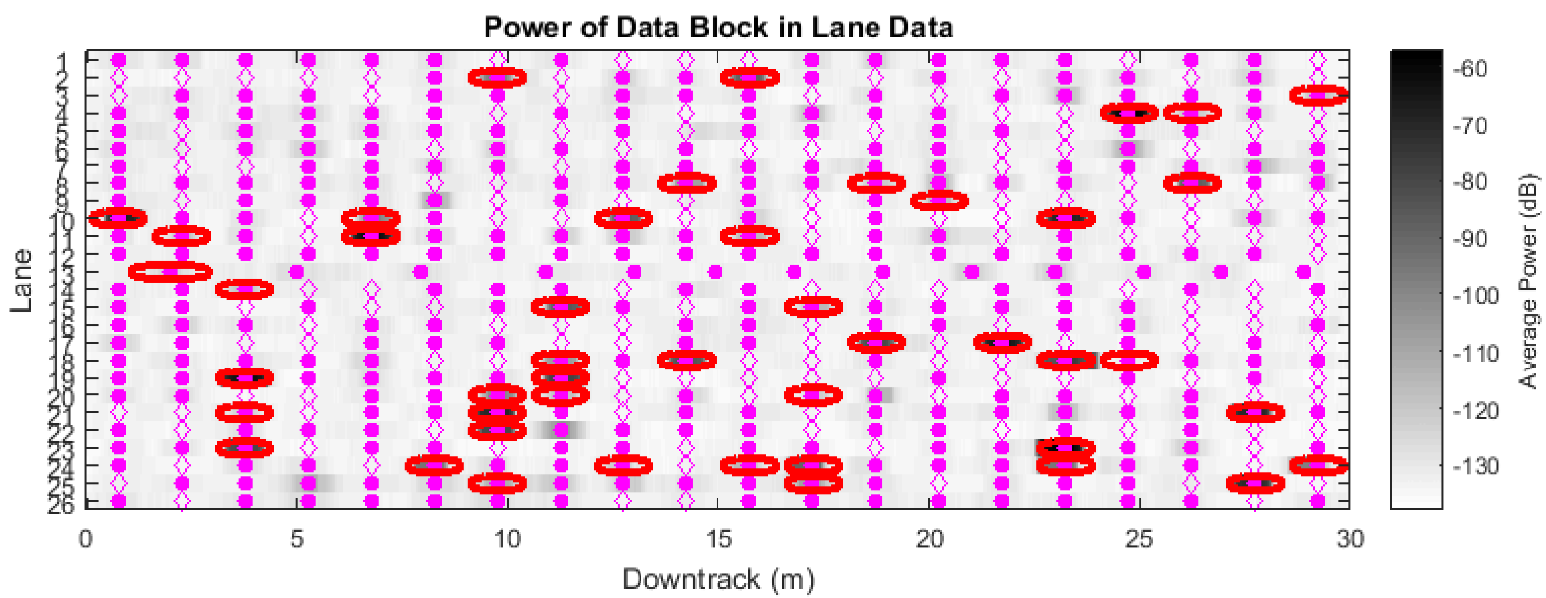


EXPLOITING NOISE IN SECOND DATA BLOCK

$$\mathbf{M_S} = \begin{bmatrix} \mathbf{U_S^{M_S}} & \mathbf{U_{\mathcal{E}}^{M_S}} \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma_S^{M_S}} & 0 \\ 0 & \mathbf{\Sigma_{\mathcal{E}}^{M_S}} \end{bmatrix} \begin{bmatrix} \mathbf{V_S^{M_S}}^T \\ \mathbf{V_{\mathcal{E}}^{M_S}}^T \end{bmatrix}$$

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

- The remaining singular values can be used to estimate the noise power

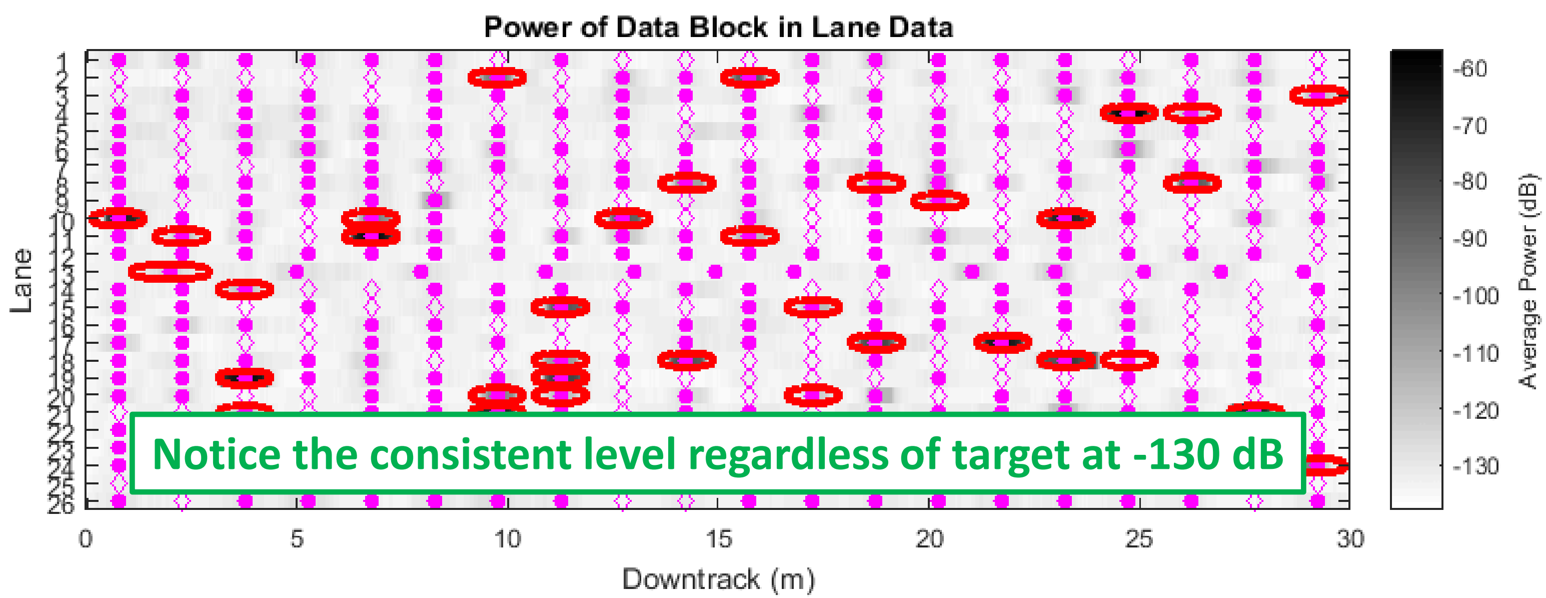


EXPLOITING NOISE IN SECOND DATA BLOCK

$$\mathbf{M_S} = \begin{bmatrix} \mathbf{U_S^{M_S}} & \mathbf{U_{\mathcal{E}}^{M_S}} \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma_S^{M_S}} & 0 \\ 0 & \mathbf{\Sigma_{\mathcal{E}}^{M_S}} \end{bmatrix} \begin{bmatrix} \mathbf{V_S^{M_S}}^T \\ \mathbf{V_{\mathcal{E}}^{M_S}}^T \end{bmatrix}$$

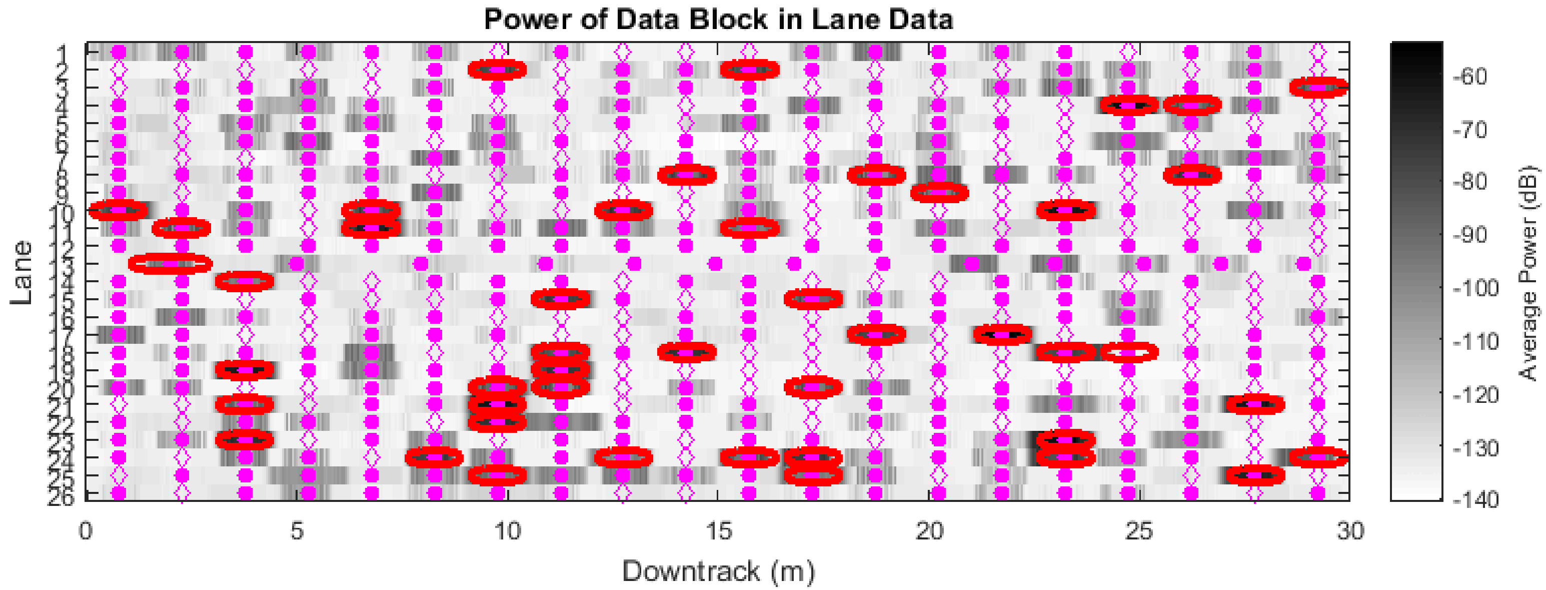
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	[illegible]	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}$

- The remaining singular values can be used to estimate the noise power



INVESTIGATING THIRD DATA BLOCK

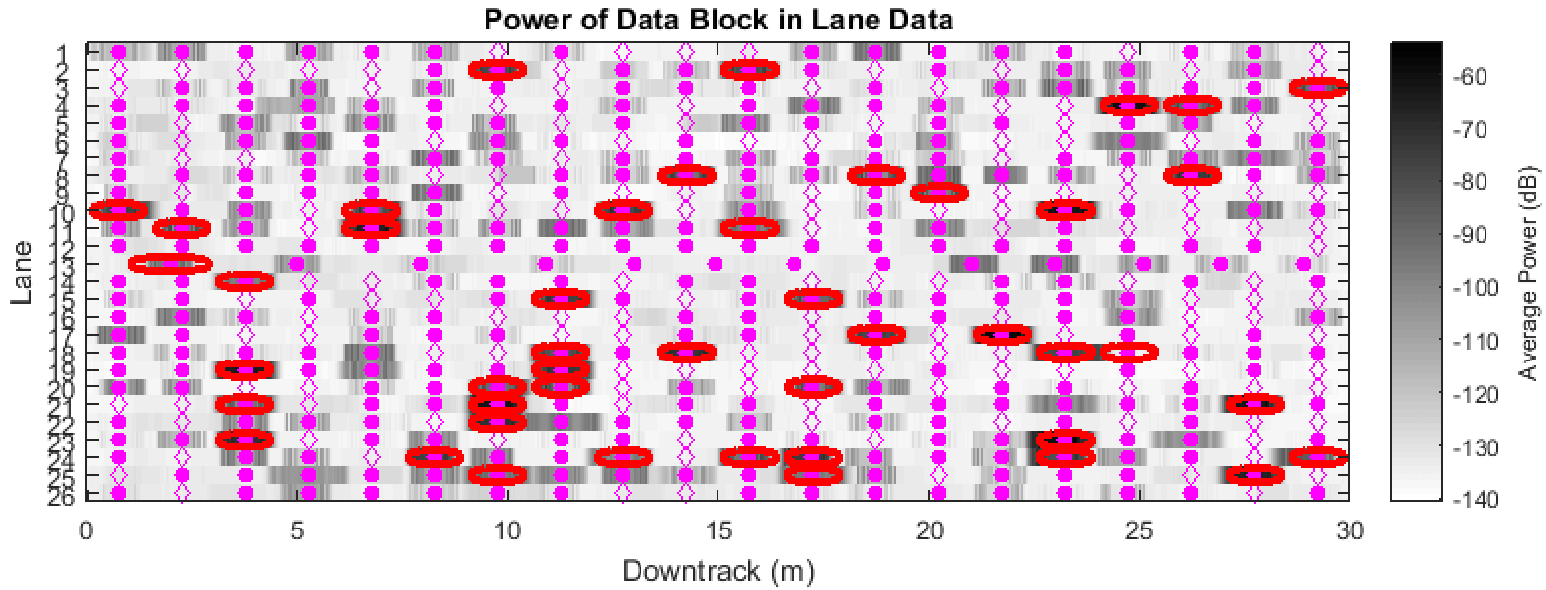
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}$



INVESTIGATING THIRD DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{M}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}} = \cancelto{\epsilon}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{S}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{G}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{R}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathcal{E}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}$$

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

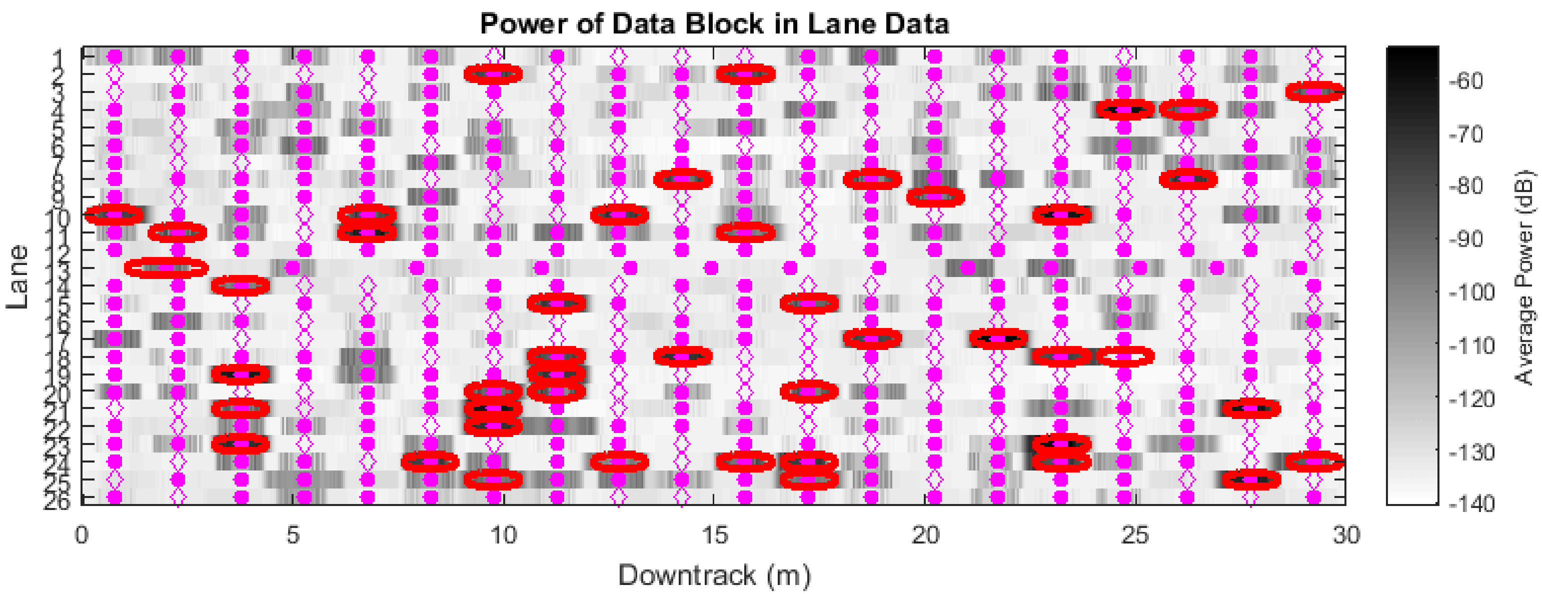


INVESTIGATING THIRD DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{M}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathcal{E}} = \cancelto{\epsilon}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{S}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathcal{E}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{G}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathcal{E}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{R}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathcal{E}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathcal{E}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathcal{E}}$$

- This data block can be used to estimate the **noise** power

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

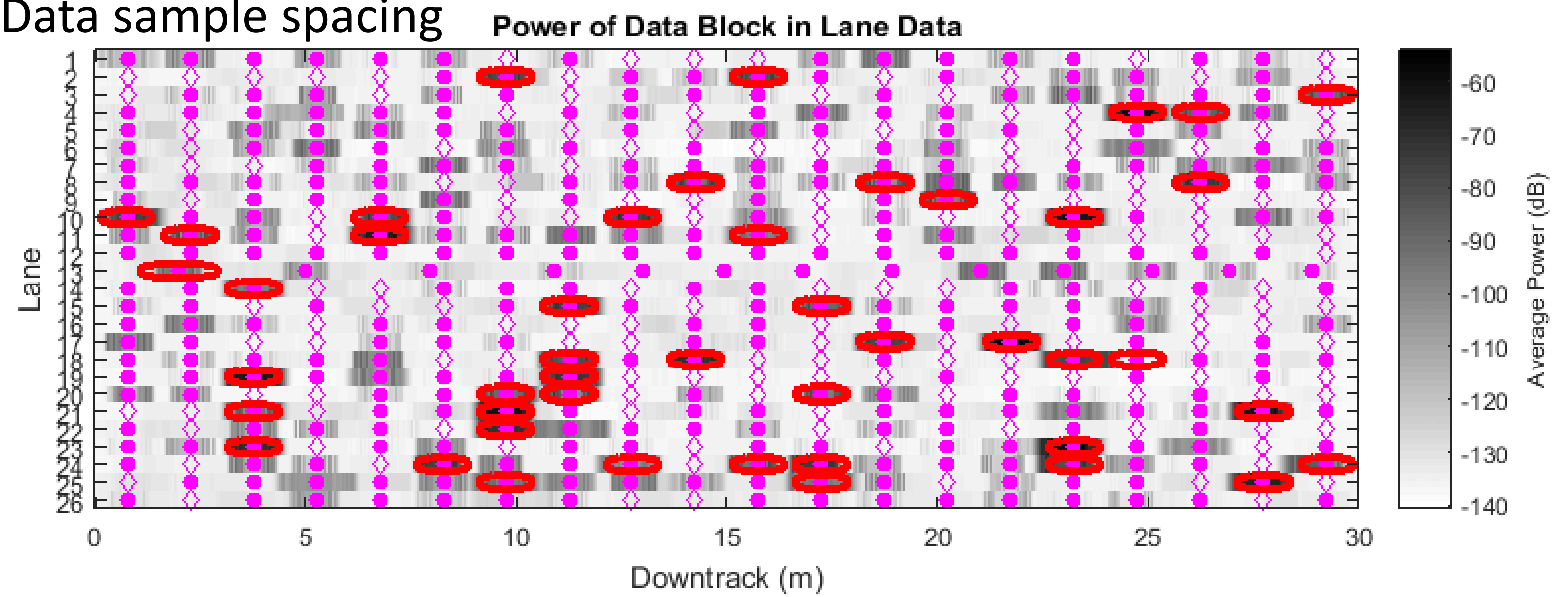


INVESTIGATING THIRD DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{M}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}} = \cancelto{\epsilon}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{S}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{G}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{R}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathcal{E}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}$$

- This data block can be used to estimate the **noise** power
- Stronger targets are expected to bleed into this data block due to:
  - Spatial projection operator (edge artifact)
  - Motion artifacts
  - Data sample spacing

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

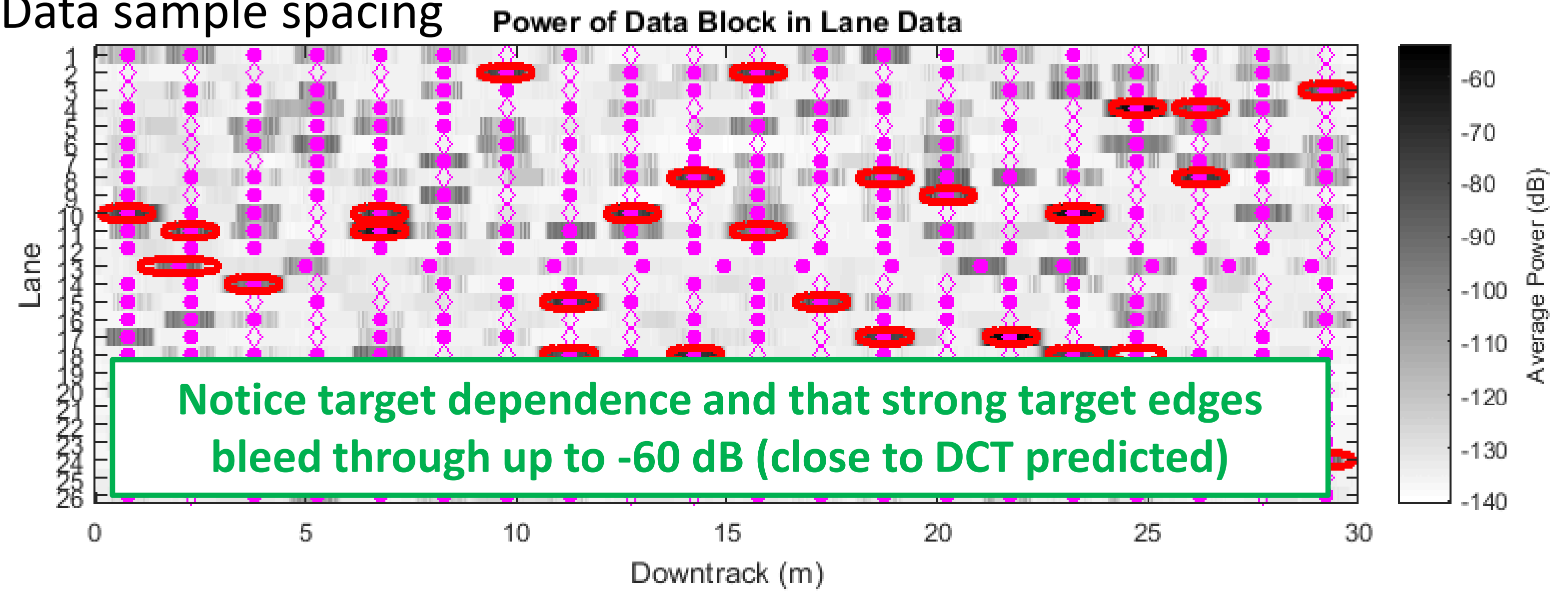


INVESTIGATING THIRD DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{M}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}} = \cancelto{\epsilon}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{S}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{G}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathbf{R}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathbf{S}}\mathcal{E}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}$$

- This data block can be used to estimate the **noise** power
- Stronger targets are expected to bleed into this data block due to:
  - Spatial projection operator (edge artifact)
  - Motion artifacts
  - Data sample spacing

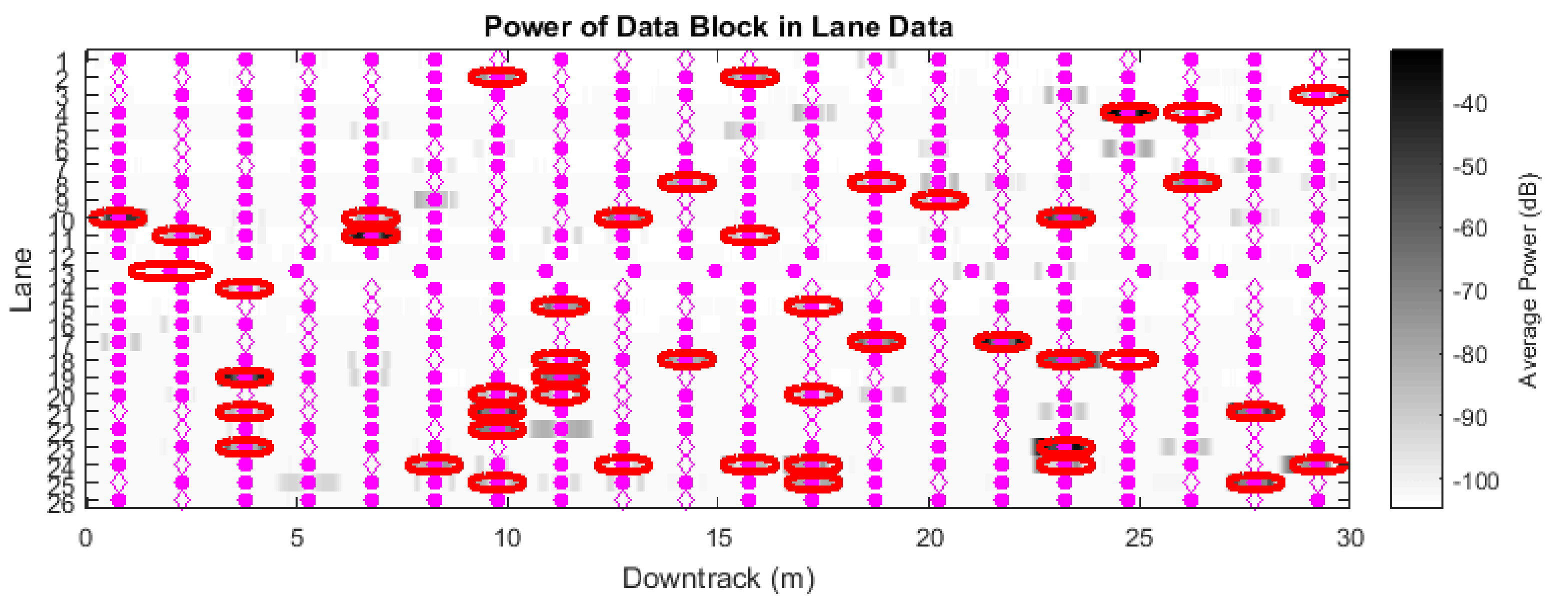
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$



Notice target dependence and that strong target edges bleed through up to -60 dB (close to DCT predicted)

INVESTIGATING FOURTH DATA BLOCK

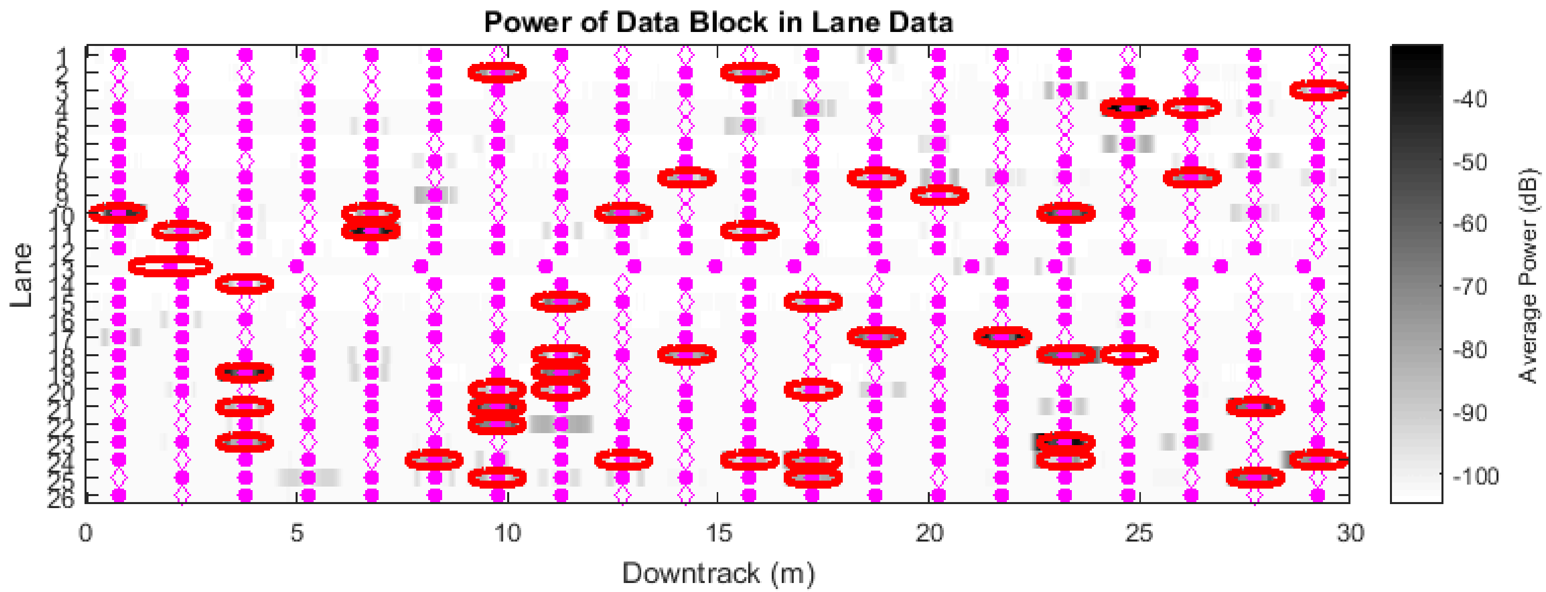
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}$



INVESTIGATING FOURTH DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{M}\mathbf{P}^T_{\mathbf{RR}} = \cancelto{\epsilon\lambda_{\mathbf{G}\mathcal{E}}}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{S}\mathbf{P}^T_{\mathbf{RR}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{G}\mathbf{P}^T_{\mathbf{RR}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{R}\mathbf{P}^T_{\mathbf{RR}} + \mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathcal{E}\mathbf{P}^T_{\mathbf{RR}}$$

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\boxed{\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

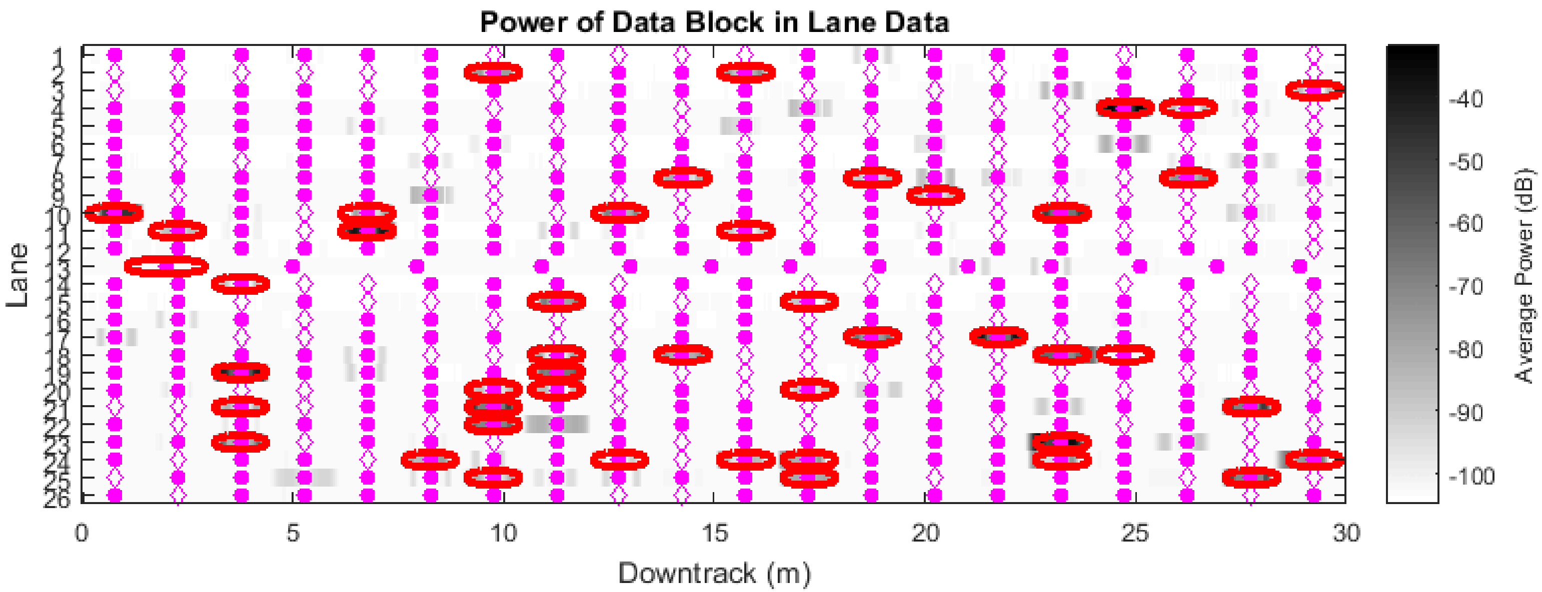


INVESTIGATING FOURTH DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{M}\mathbf{P}_{\mathbf{RR}}^T = \cancelto{\epsilon\lambda_{\mathbf{G}\mathcal{E}}}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{S}\mathbf{P}_{\mathbf{RR}}^T} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{G}\mathbf{P}_{\mathbf{RR}}^T} + \mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{R}\mathbf{P}_{\mathbf{RR}}^T + \mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathcal{E}\mathbf{P}_{\mathbf{RR}}^T$$

- In theory this is an ideal location for estimating the **self** response

$\mathrm{M}_{\bar{\mathrm{G}}\mathrm{S}}^{\mathrm{RR}}$	$\mathrm{M}_{\bar{\mathrm{G}}\mathrm{S}}^{\bar{\mathrm{R}}\mathrm{S}}$	$\mathrm{M}_{\bar{\mathrm{G}}\mathrm{S}}^{\bar{\mathrm{R}}\mathcal{E}}$
$\boxed{\mathrm{M}_{\bar{\mathrm{G}}\mathcal{E}}^{\mathrm{RR}}}$	$\mathrm{M}_{\bar{\mathrm{G}}\mathcal{E}}^{\bar{\mathrm{R}}\mathrm{S}}$	$\mathrm{M}_{\bar{\mathrm{G}}\mathcal{E}}^{\bar{\mathrm{R}}\mathcal{E}}$
$\mathrm{M}_{\mathrm{GG}}^{\mathrm{RR}}$	$\mathrm{M}_{\mathrm{GG}}^{\bar{\mathrm{R}}\mathrm{S}}$	$\mathrm{M}_{\mathrm{GG}}^{\bar{\mathrm{R}}\mathcal{E}}$

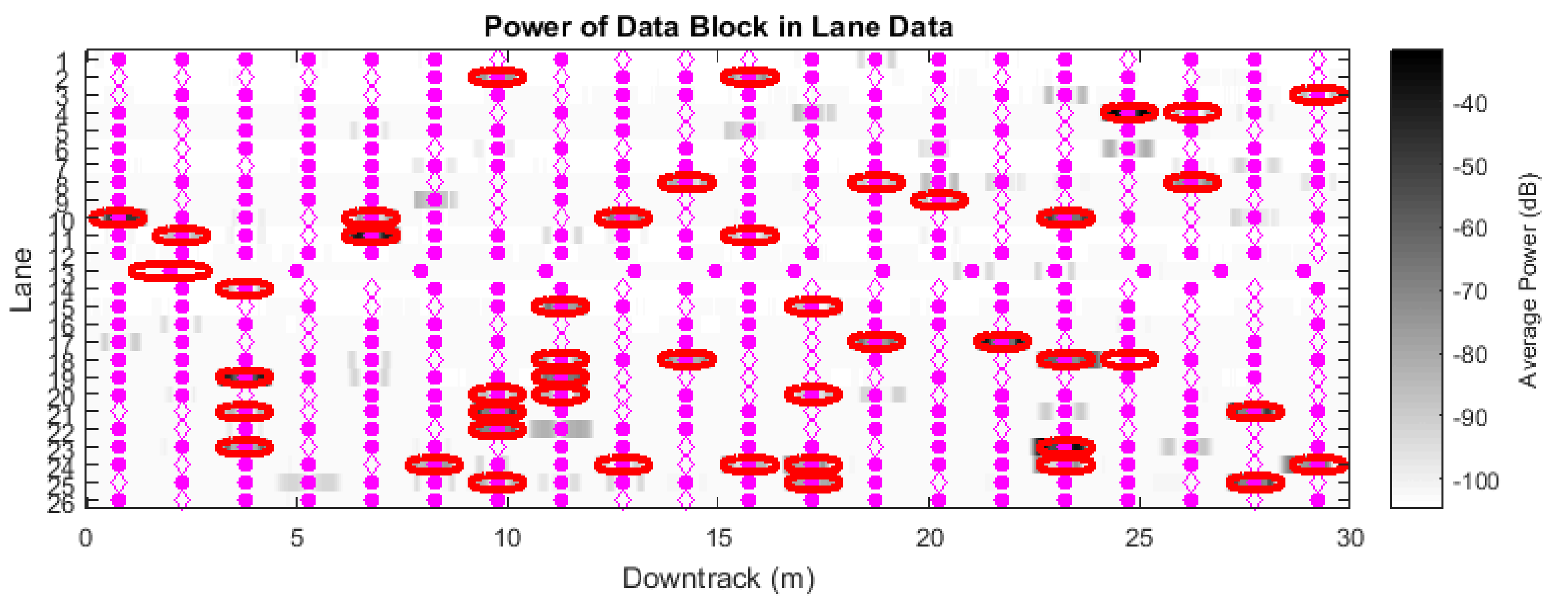


INVESTIGATING FOURTH DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{M}\mathbf{P}_{\mathbf{RR}}^{T} = \cancelto{\epsilon\lambda_{\mathbf{G}\mathcal{E}}}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{S}\mathbf{P}_{\mathbf{RR}}^{T}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{G}\mathbf{P}_{\mathbf{RR}}^{T}} + \mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{R}\mathbf{P}_{\mathbf{RR}}^{T} + \mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathcal{E}\mathbf{P}_{\mathbf{RR}}^{T}$$

- In theory this is an ideal location for estimating the **self** response
- Because the self response can be modeled as the systems EMI response, it is diminished due to the frequency projection
- This serves as a measure of the system calibration

$\mathrm{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\boxed{\mathrm{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathrm{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

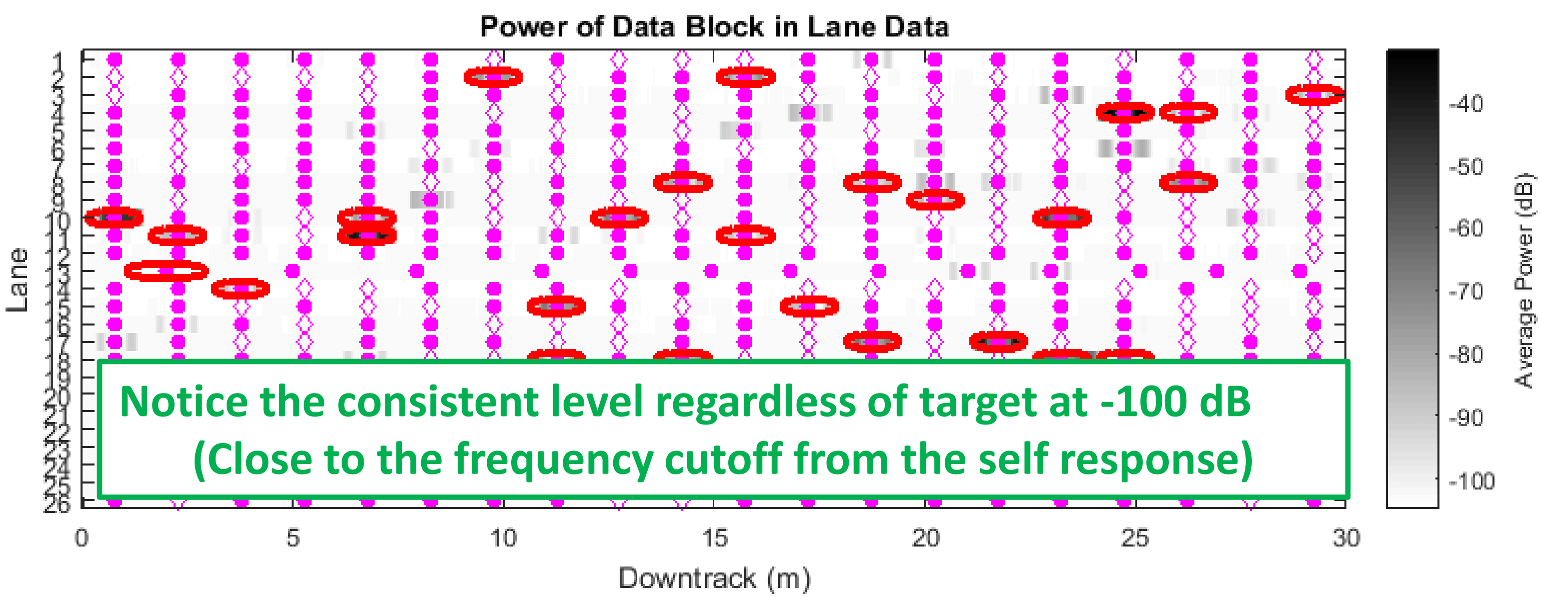


INVESTIGATING FOURTH DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{M}\mathbf{P}^T_{\mathbf{RR}} = \cancelto{\epsilon\lambda_{\mathbf{G}\mathcal{E}}}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{S}\mathbf{P}^T_{\mathbf{RR}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{G}\mathbf{P}^T_{\mathbf{RR}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{R}\mathbf{P}^T_{\mathbf{RR}} + \mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathcal{E}\mathbf{P}^T_{\mathbf{RR}}$$

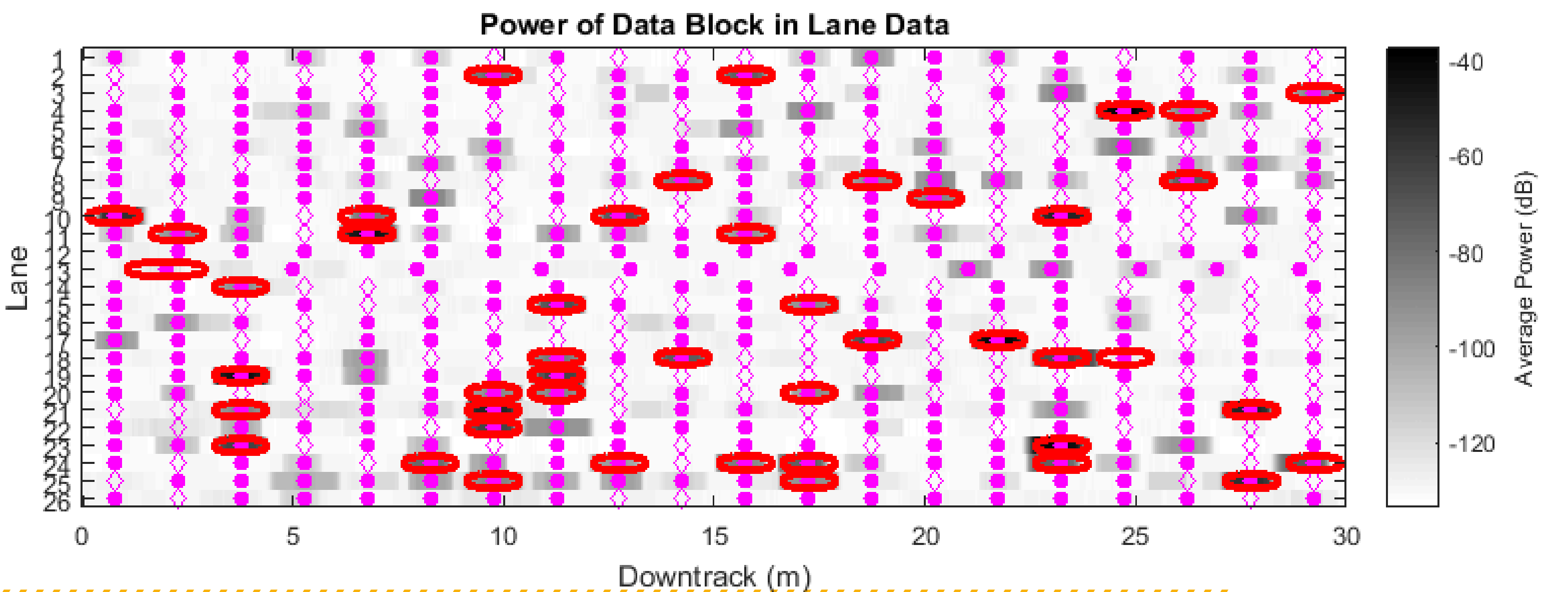
- In theory this is an ideal location for estimating the **self** response
- Because the self response can be modeled as the systems EMI response, it is diminished due to the frequency projection
- This serves as a measure of the system calibration

$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\boxed{\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}$



INVESTIGATING FIFTH DATA BLOCK

$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}$

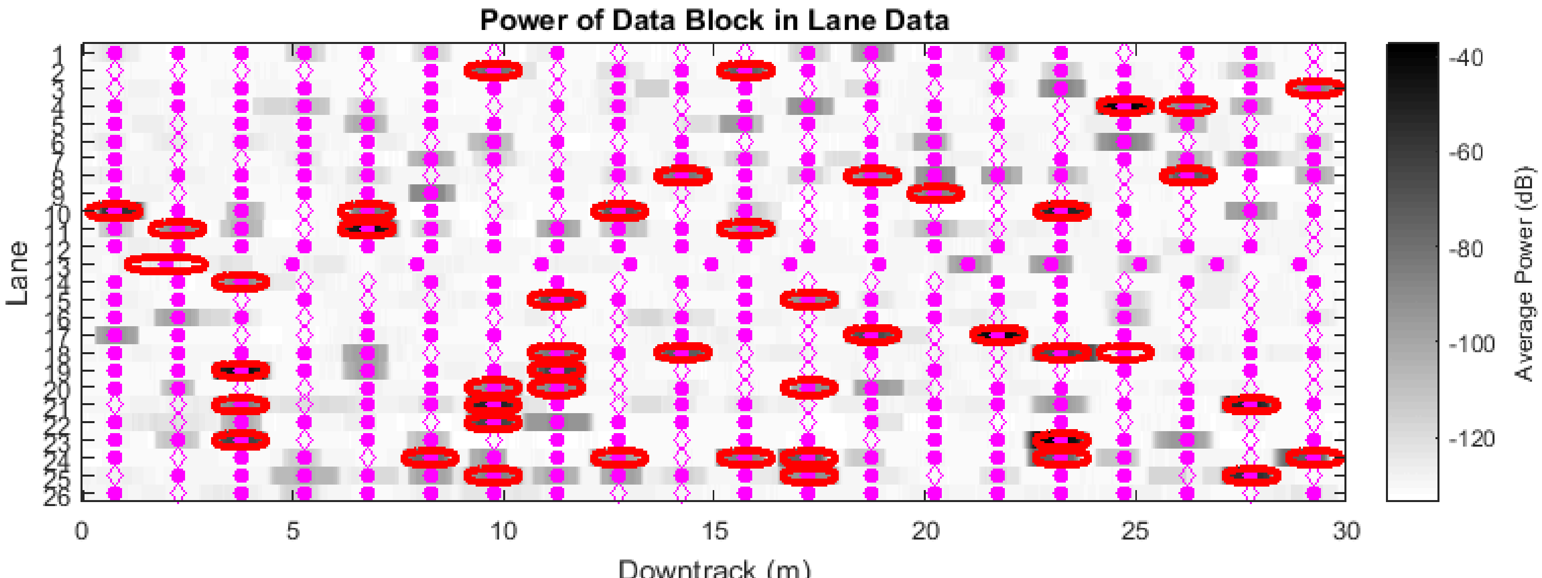


CREATING THE NEXT

INVESTIGATING FIFTH DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{M}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} = \cancelto{\delta_g}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{S}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{G}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{R}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathcal{E}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}$$

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

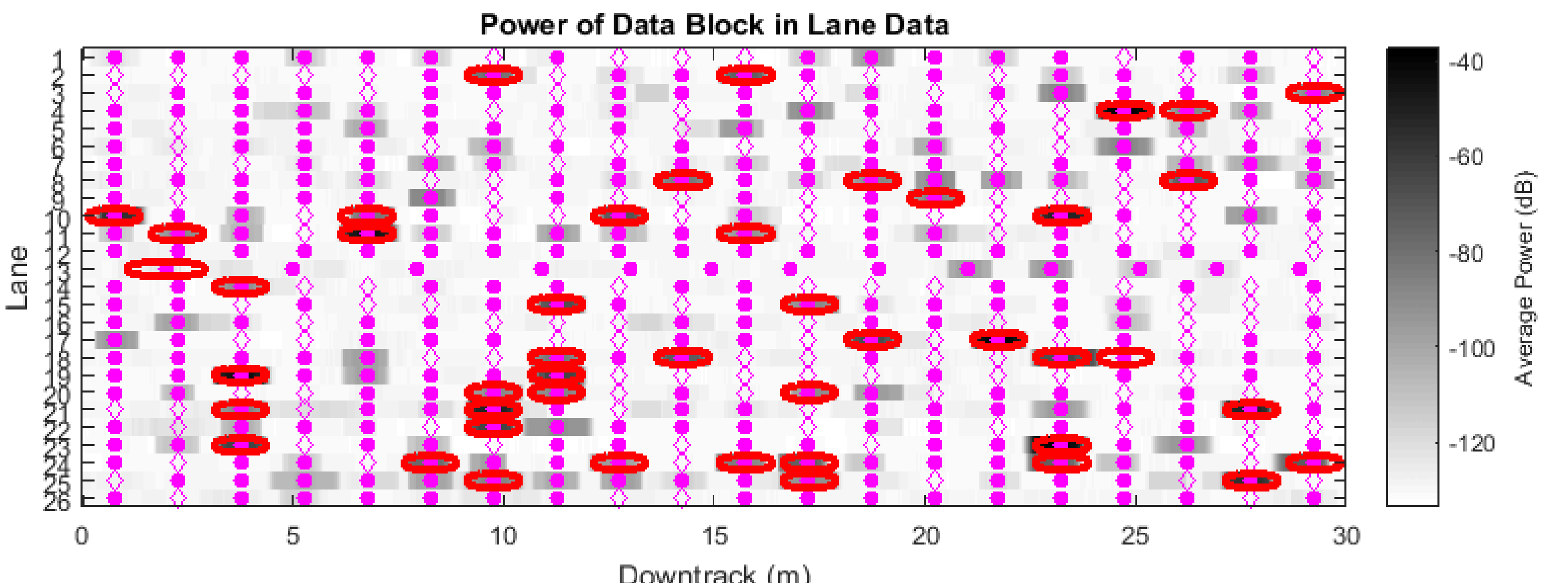


INVESTIGATING FIFTH DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{M}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} = \cancelto{\delta_g}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{S}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{G}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{R}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathcal{E}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}$$

- This data block can be used to estimate the **noise** power

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

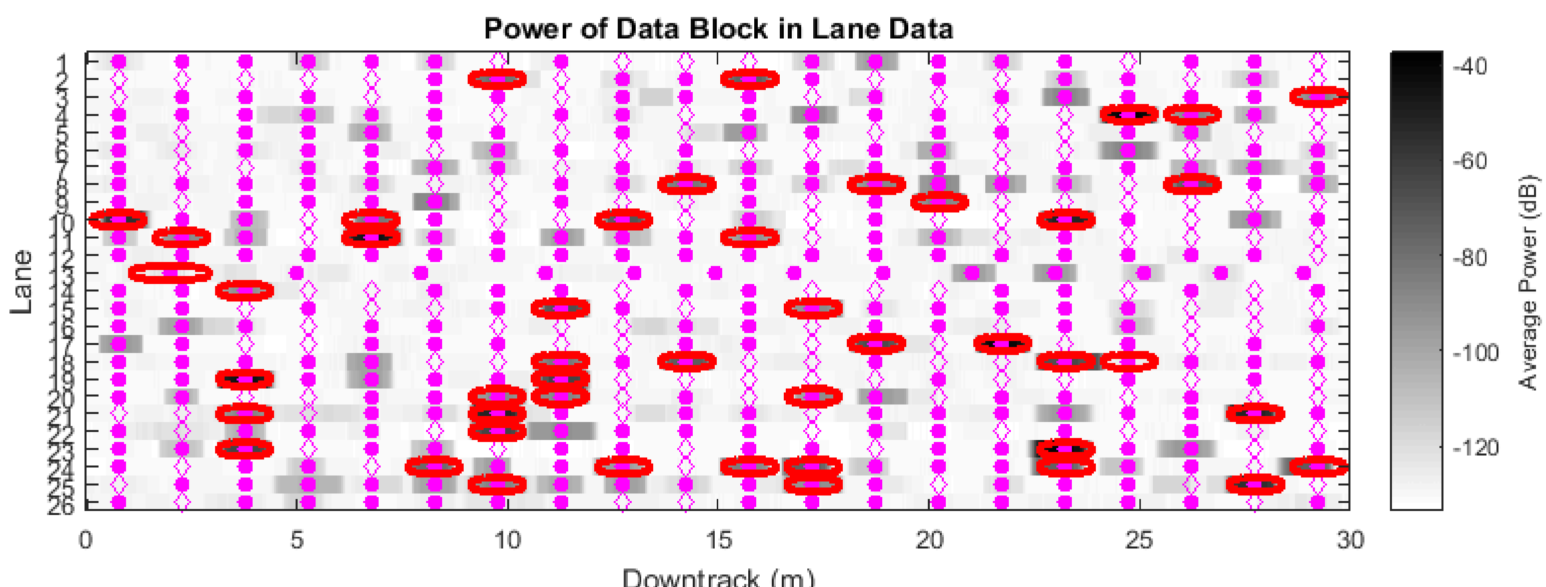


INVESTIGATING FIFTH DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{M}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} = \cancelto{\delta_g}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{S}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{G}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{R}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathcal{E}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}$$

- This data block can be used to estimate the **noise** power
- Stronger targets are expected to bleed into this data block due to:
    - frequency projection operator cutoff
    - Calibration artifacts

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

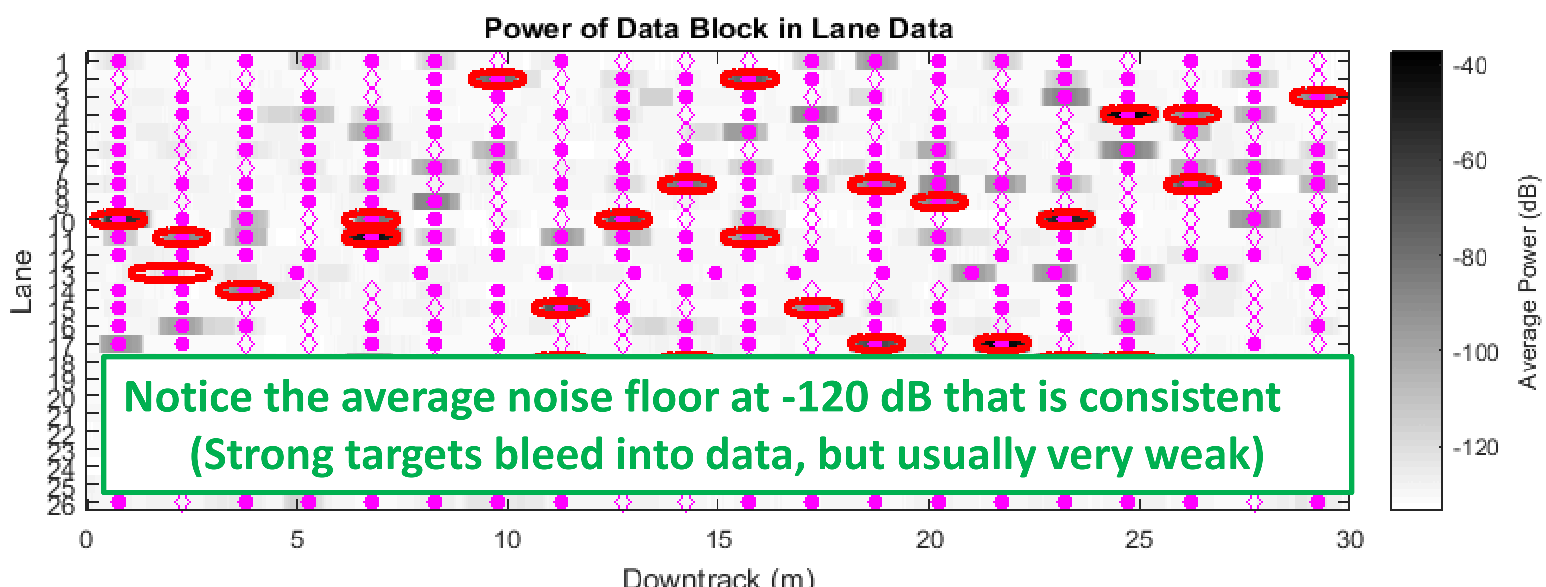


INVESTIGATING FIFTH DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{M}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} = \cancelto{\delta_g}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{S}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{G}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{R}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathcal{E}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}$$

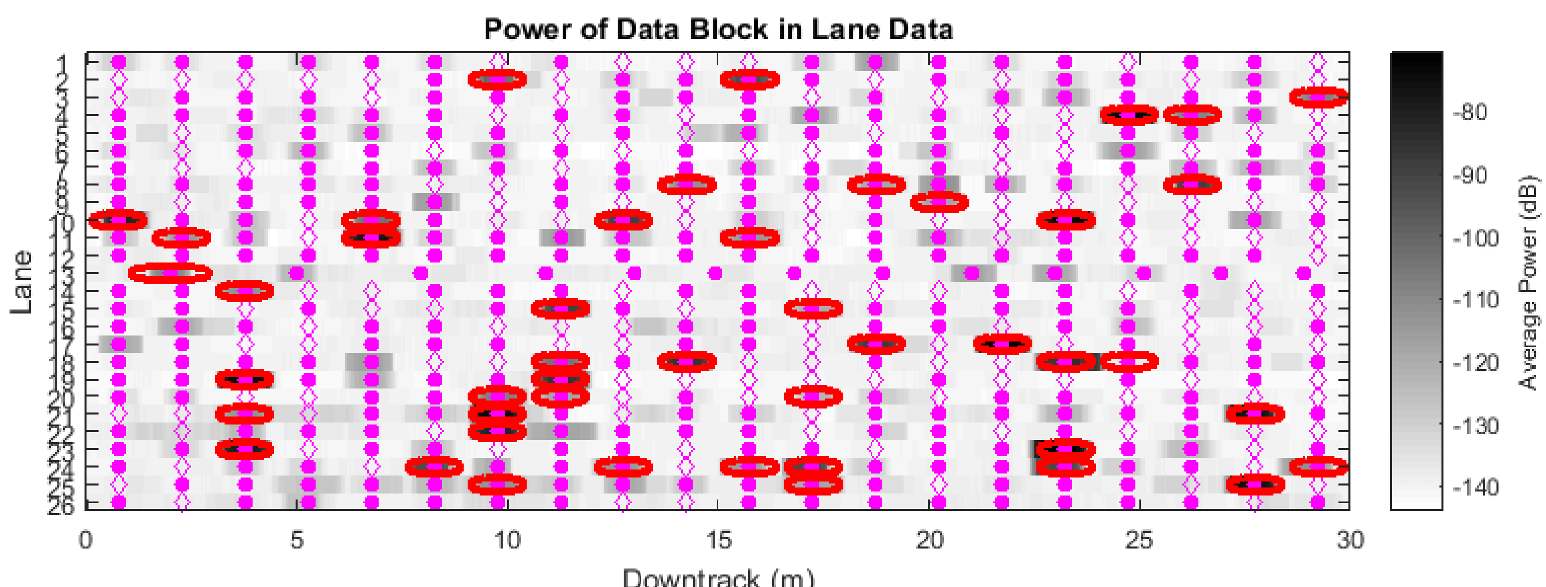
- This data block can be used to estimate the **noise** power
- Stronger targets are expected to bleed into this data block due to:
  - frequency projection operator cutoff
  - Calibration artifacts

$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{R}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{R}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{R}}_{\bar{\mathbf{G}}\mathbf{G}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{G}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{G}}$



INVESTIGATING SIXTH DATA BLOCK

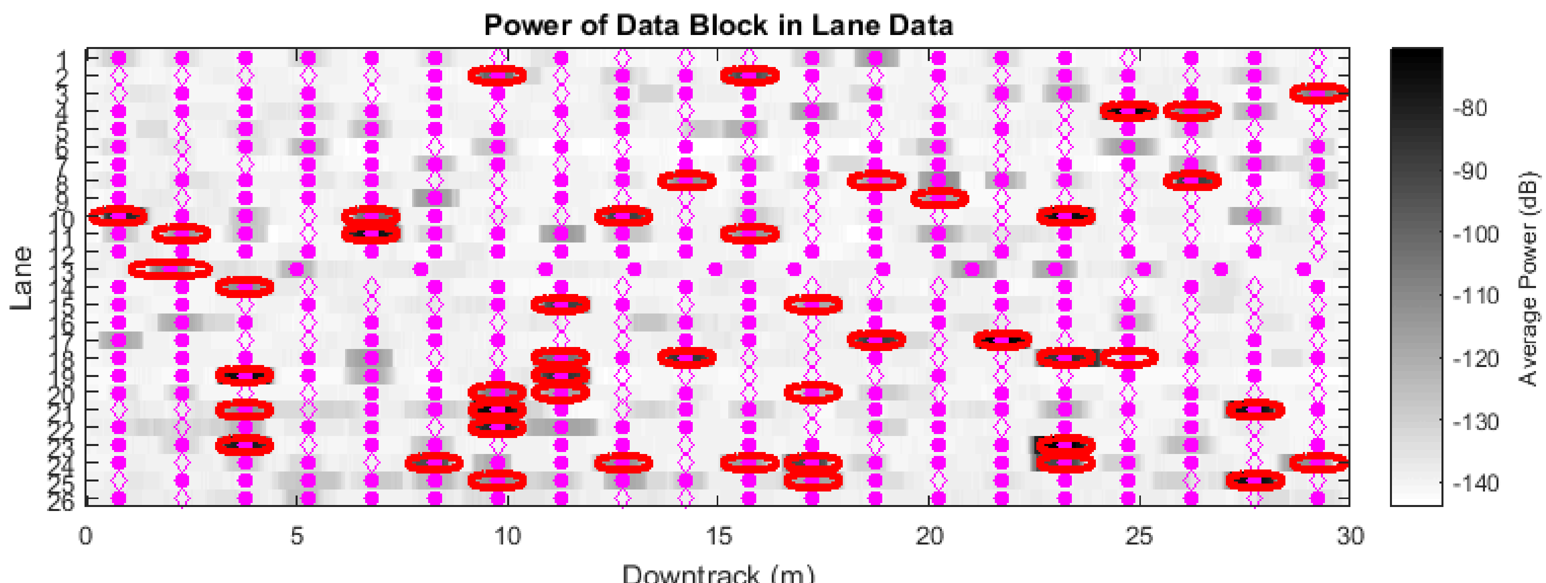
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}$



INVESTIGATING SIXTH DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{M}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}} = \cancelto{\epsilon\delta_g}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{S}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{G}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{R}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathcal{E}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}$$

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

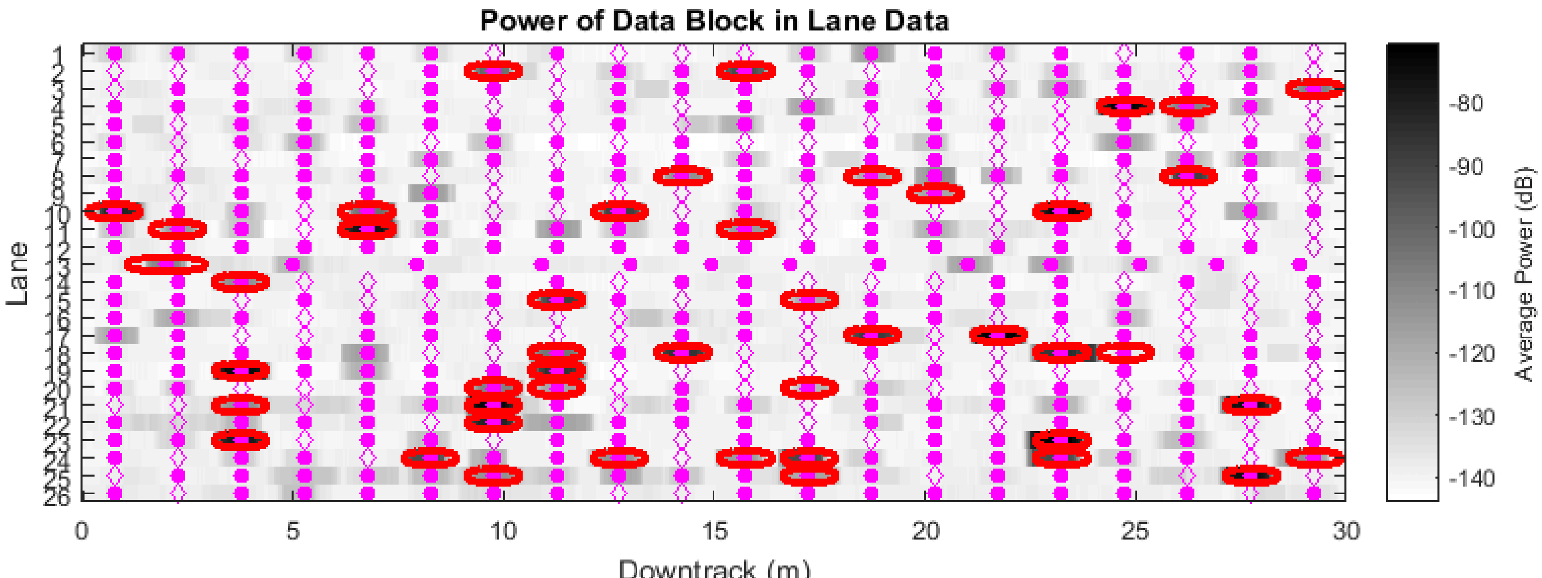


INVESTIGATING SIXTH DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{M}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}} = \cancelto{\epsilon\delta_g}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{S}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{G}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{R}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathcal{E}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}$$

- This is the ideal location to estimate the **noise** power

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

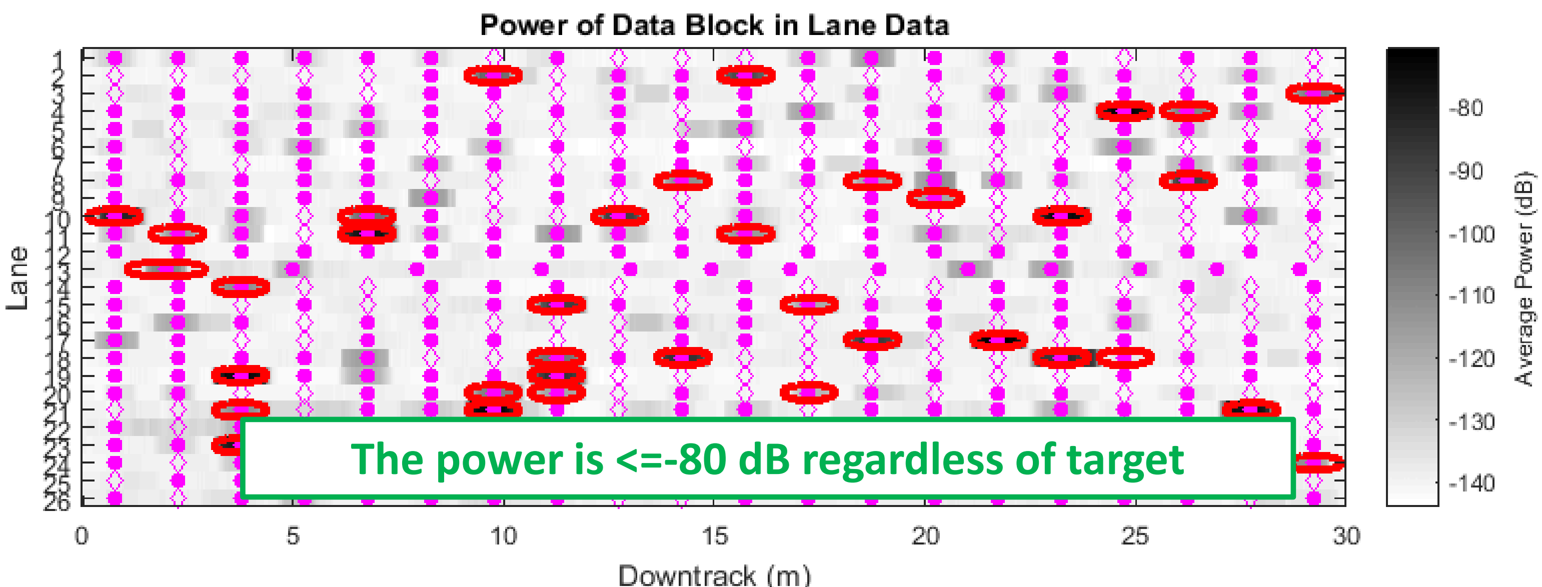


INVESTIGATING SIXTH DATA BLOCK

$$\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{M}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}} = \cancelto{\epsilon\delta_g}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{S}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{G}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \cancelto{0}{\mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathbf{R}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}} + \mathbf{P}_{\bar{\mathbf{G}}\mathcal{E}}\mathcal{E}\mathbf{P}^T_{\bar{\mathbf{R}}\mathcal{E}}$$

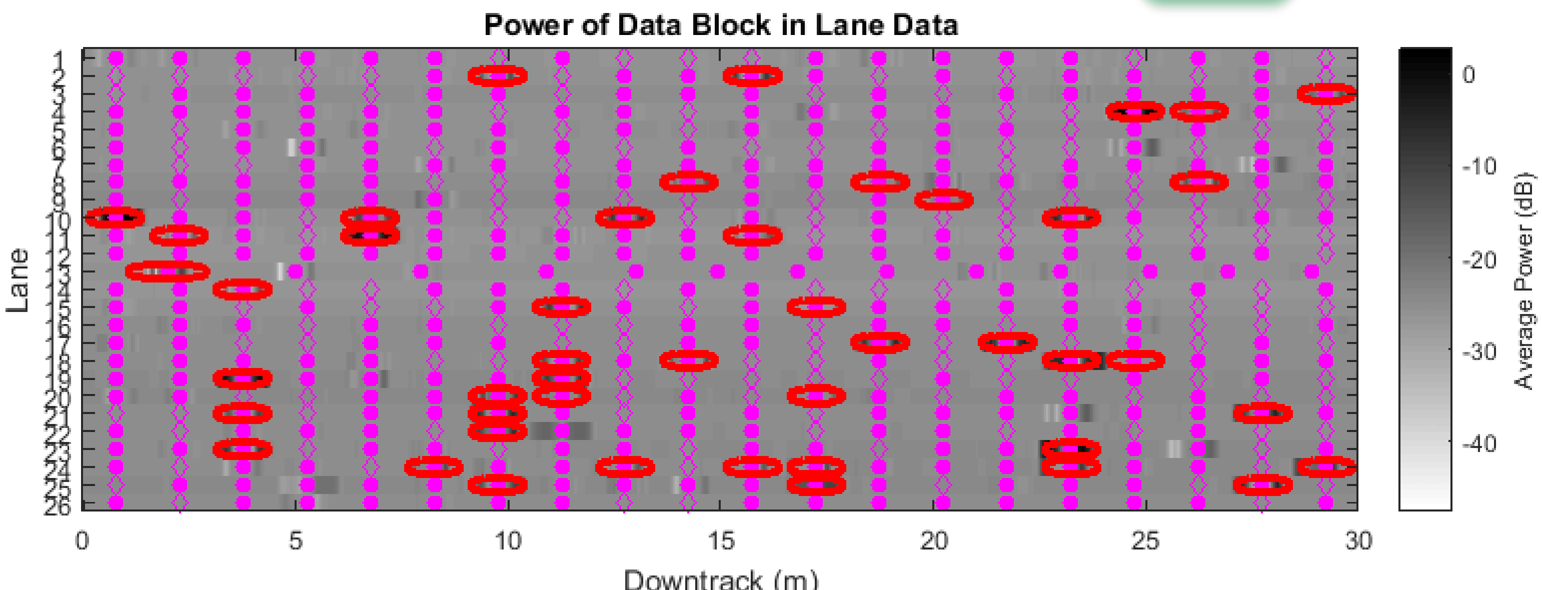
- This is the ideal location to estimate the **noise** power

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$



INVESTIGATING SEVENTH DATA BLOCK

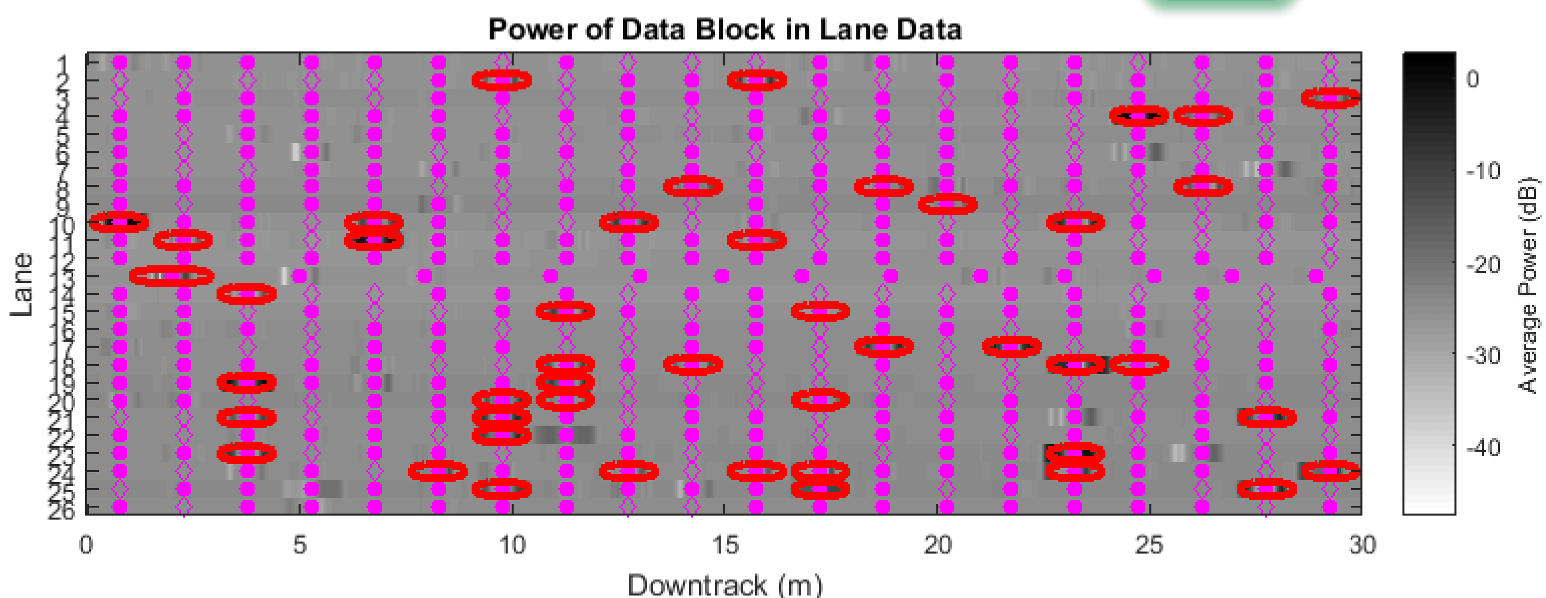
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}$



INVESTIGATING SEVENTH DATA BLOCK

$$\mathbf{P_{GG}MP_{RR}^{\mathit{T}}} = \cancelto{\epsilon}{\mathbf{P_{GG}SP_{RR}^{\mathit{T}}}} + \mathbf{P_{GG}GP_{RR}^{\mathit{T}}} + \mathbf{P_{GG}RP_{RR}^{\mathit{T}}} + \mathbf{P_{GG}}\mathcal{E}\mathbf{P_{RR}^{\mathit{T}}}$$

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\boxed{\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

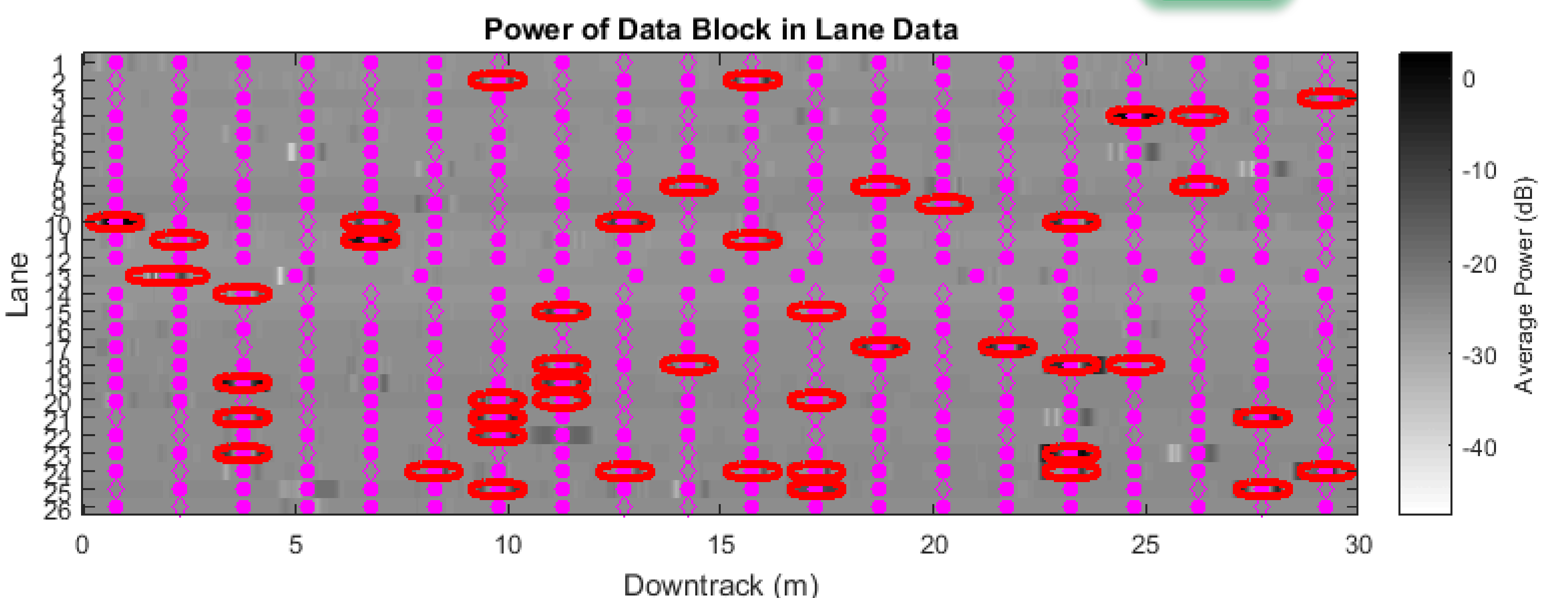


INVESTIGATING SEVENTH DATA BLOCK

$$\mathbf{P_{GG}MP_{RR}^{\mathit{T}}} = \cancelto{\epsilon}{\mathbf{P_{GG}SP_{RR}^{\mathit{T}}}} + \mathbf{P_{GG}GP_{RR}^{\mathit{T}}} + \mathbf{P_{GG}RP_{RR}^{\mathit{T}}} + \mathbf{P_{GG}}\mathcal{E}\mathbf{P_{RR}^{\mathit{T}}}$$

- This data block is dominated by the **interference**
  - The soil means are contained here
  - The self response is largely contained here

$\mathrm{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathrm{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\boxed{\mathrm{M}^{\mathbf{RR}}_{\mathbf{GG}}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathrm{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

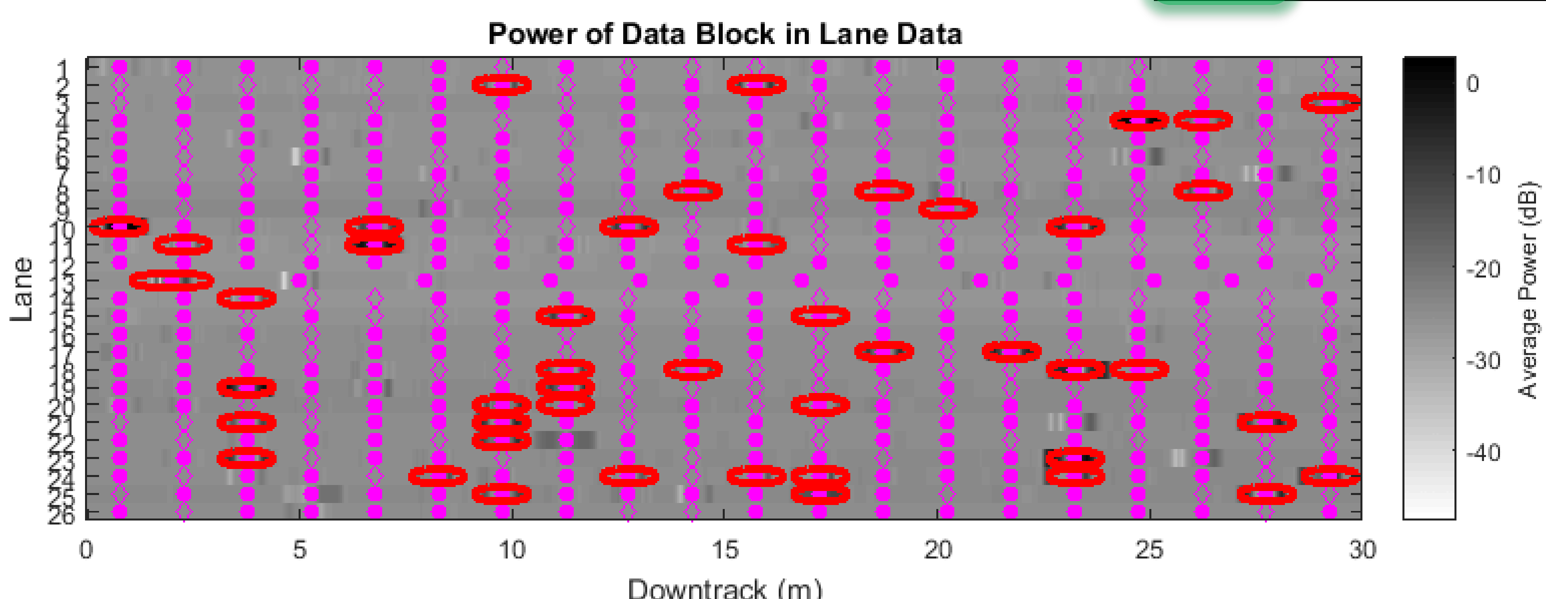


INVESTIGATING SEVENTH DATA BLOCK

$$\mathbf{P_{GG}MP_{RR}^{\mathit{T}}} = \cancelto{\epsilon}{\mathbf{P_{GG}SP_{RR}^{\mathit{T}}}} + \mathbf{P_{GG}GP_{RR}^{\mathit{T}}} + \mathbf{P_{GG}RP_{RR}^{\mathit{T}}} + \mathbf{P_{GG}}\boldsymbol{\mathcal{E}}\mathbf{P_{RR}^{\mathit{T}}}$$

- This data block is dominated by the **interference**
  - The soil means are contained here
  - The self response is largely contained here
- This data block contains the largest amount of measured energy

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

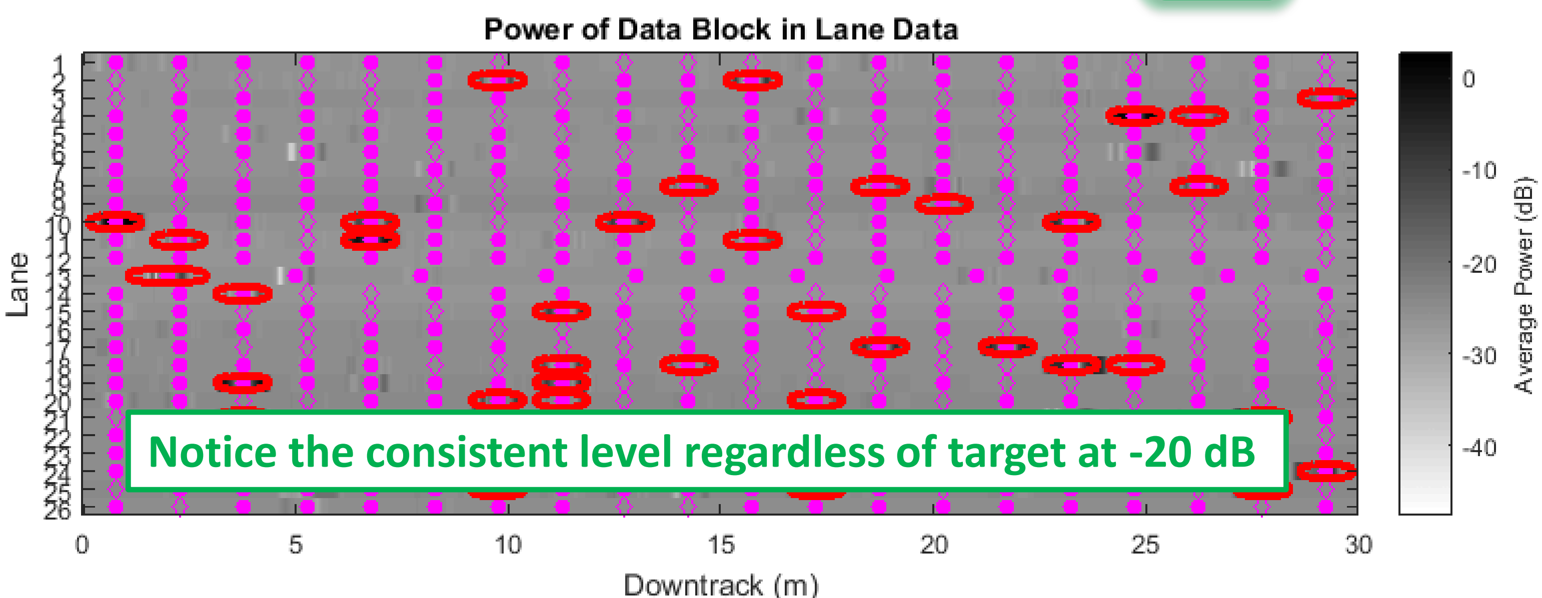


INVESTIGATING SEVENTH DATA BLOCK

$$\mathbf{P_{GG}MP_{RR}^{T}} = \cancelto{\epsilon}{\mathbf{P_{GG}SP_{RR}^{T}}} + \mathbf{P_{GG}GP_{RR}^{T}} + \mathbf{P_{GG}RP_{RR}^{T}} + \mathbf{P_{GG}}\mathcal{E}\mathbf{P_{RR}^{T}}$$

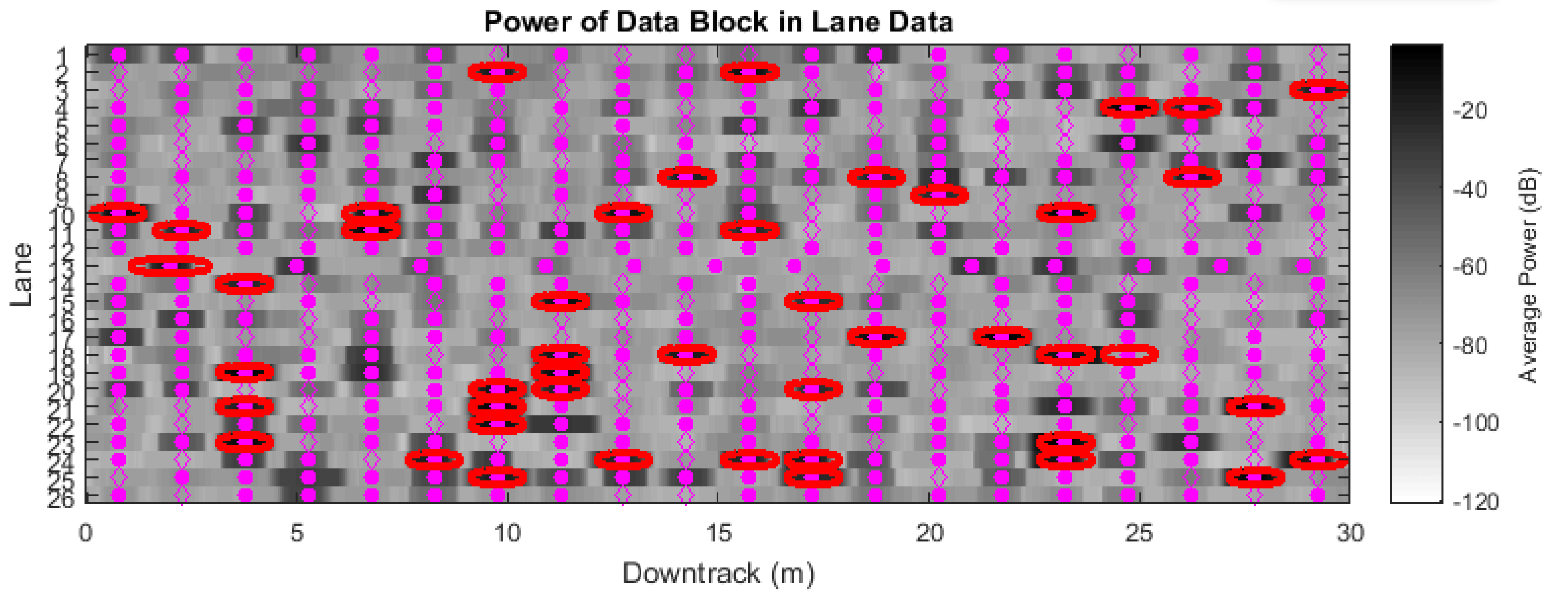
- This data block is dominated by the **interference**
  - The soil means are contained here
  - The self response is largely contained here
- This data block contains the largest amount of measured energy

$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\boxed{\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}$



INVESTIGATING EIGHTH DATA BLOCK

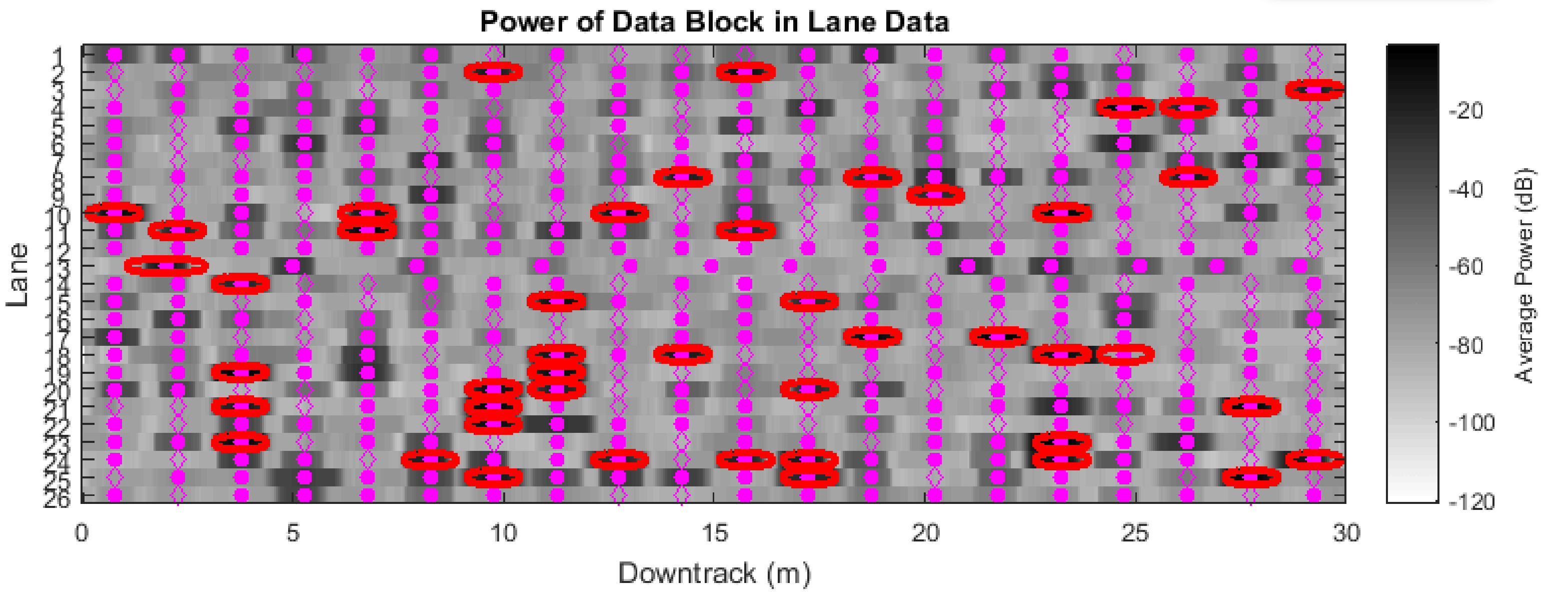
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\boxed{\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}$



INVESTIGATING EIGHTH DATA BLOCK

$$\mathbf{P}_{\mathbf{GG}}\mathbf{M}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathbf{S}} = \mathbf{P}_{\mathbf{GG}}\mathbf{S}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathbf{S}} + \mathbf{P}_{\mathbf{GG}}\mathbf{G}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathbf{S}} + \cancelto{0}{\mathbf{P}_{\mathbf{GG}}\mathbf{R}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathbf{S}}} + \mathbf{P}_{\mathbf{GG}}\mathcal{E}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathbf{S}}$$

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

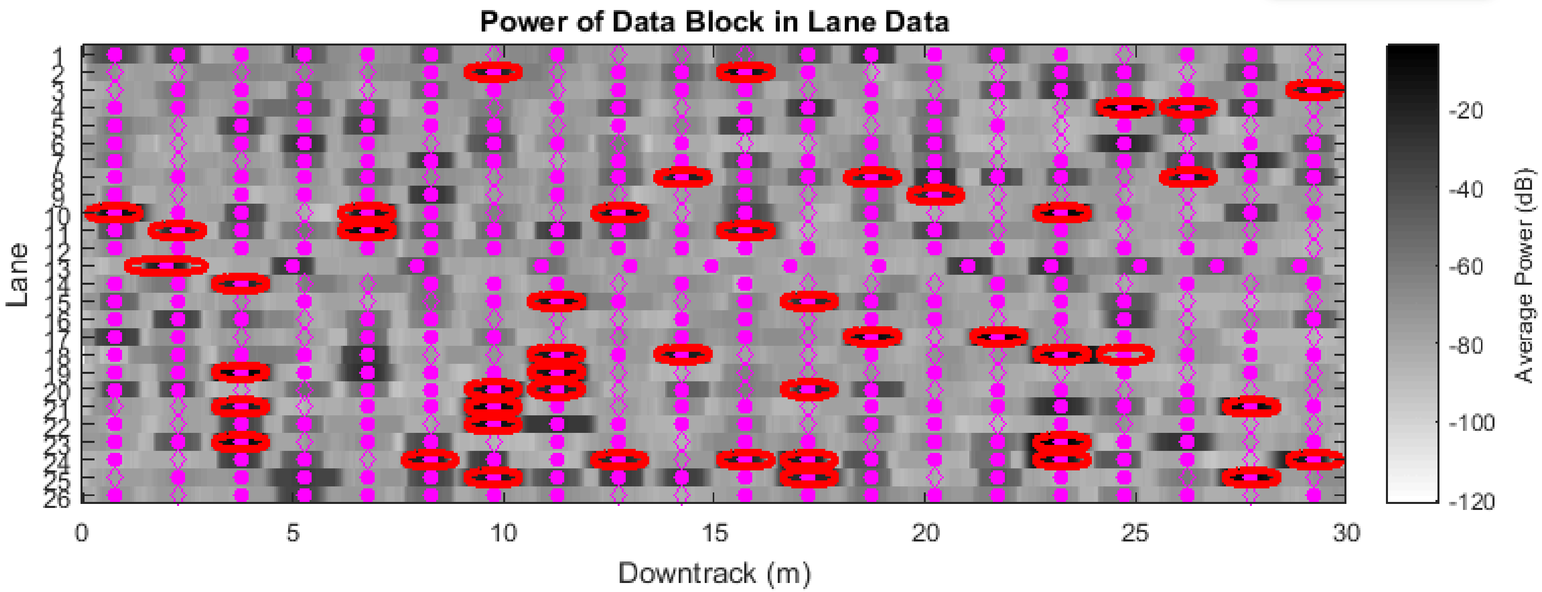


INVESTIGATING EIGHTH DATA BLOCK

$$\mathbf{P}_{\mathbf{GG}}\mathbf{M}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} = \mathbf{P}_{\mathbf{GG}}\mathbf{S}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} + \mathbf{P}_{\mathbf{GG}}\mathbf{G}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} + \cancelto{0}{\mathbf{P}_{\mathbf{GG}}\mathbf{R}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \mathbf{P}_{\mathbf{GG}}\mathcal{E}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}$$

- This data block contains a significant amount of **target** response
- This data is corrupted by the **soil** response

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

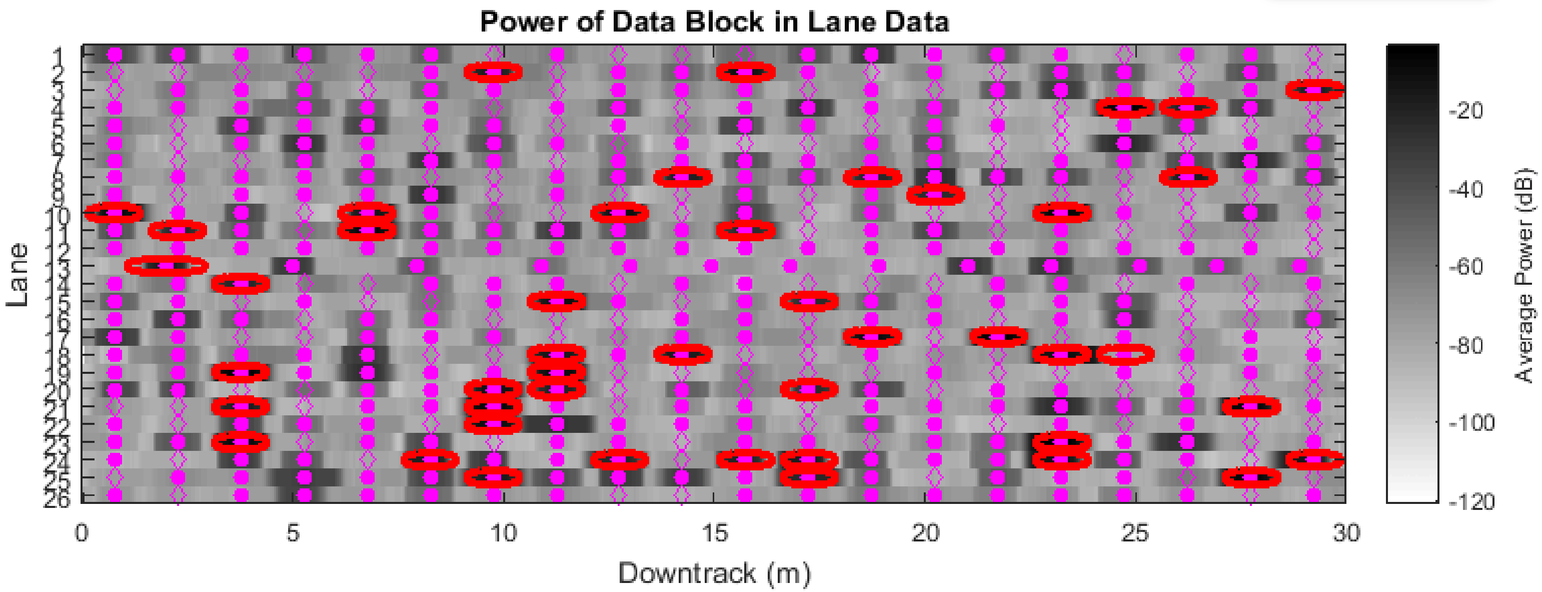


INVESTIGATING EIGHTH DATA BLOCK

$$\mathbf{P}_{\mathbf{GG}}\mathbf{M}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} = \mathbf{P}_{\mathbf{GG}}\mathbf{S}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} + \mathbf{P}_{\mathbf{GG}}\mathbf{G}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}} + \cancelto{0}{\mathbf{P}_{\mathbf{GG}}\mathbf{R}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}} + \mathbf{P}_{\mathbf{GG}}\mathcal{E}\mathbf{P}^T_{\bar{\mathbf{R}}\mathbf{S}}$$

- This data block contains a significant amount of **target** response
- This data is corrupted by the **soil** response
- This data block also has the potential to be used to detect voids

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

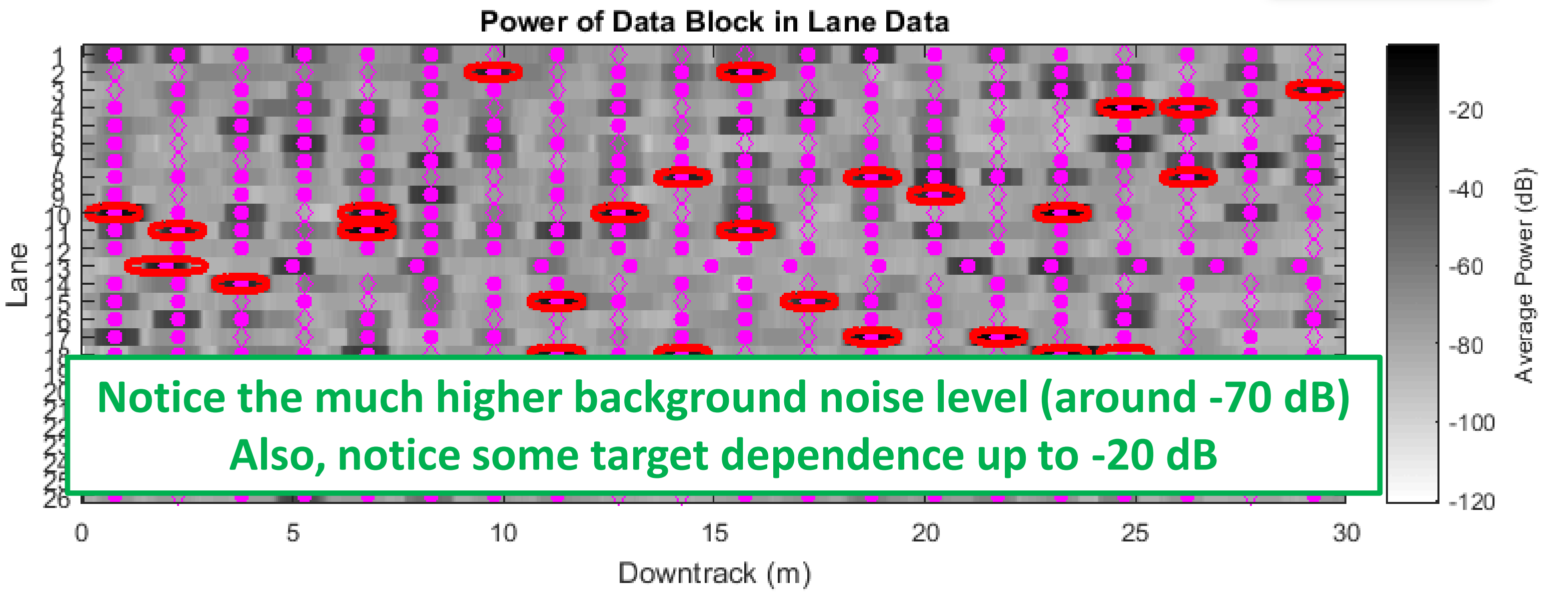


INVESTIGATING EIGHTH DATA BLOCK

$$\mathbf{P}_{\mathbf{GG}}\mathbf{M}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathbf{S}} = \mathbf{P}_{\mathbf{GG}}\mathbf{S}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathbf{S}} + \mathbf{P}_{\mathbf{GG}}\mathbf{G}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathbf{S}} + \cancelto{0}{\mathbf{P}_{\mathbf{GG}}\mathbf{R}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathbf{S}}} + \mathbf{P}_{\mathbf{GG}}\mathcal{E}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathbf{S}}$$

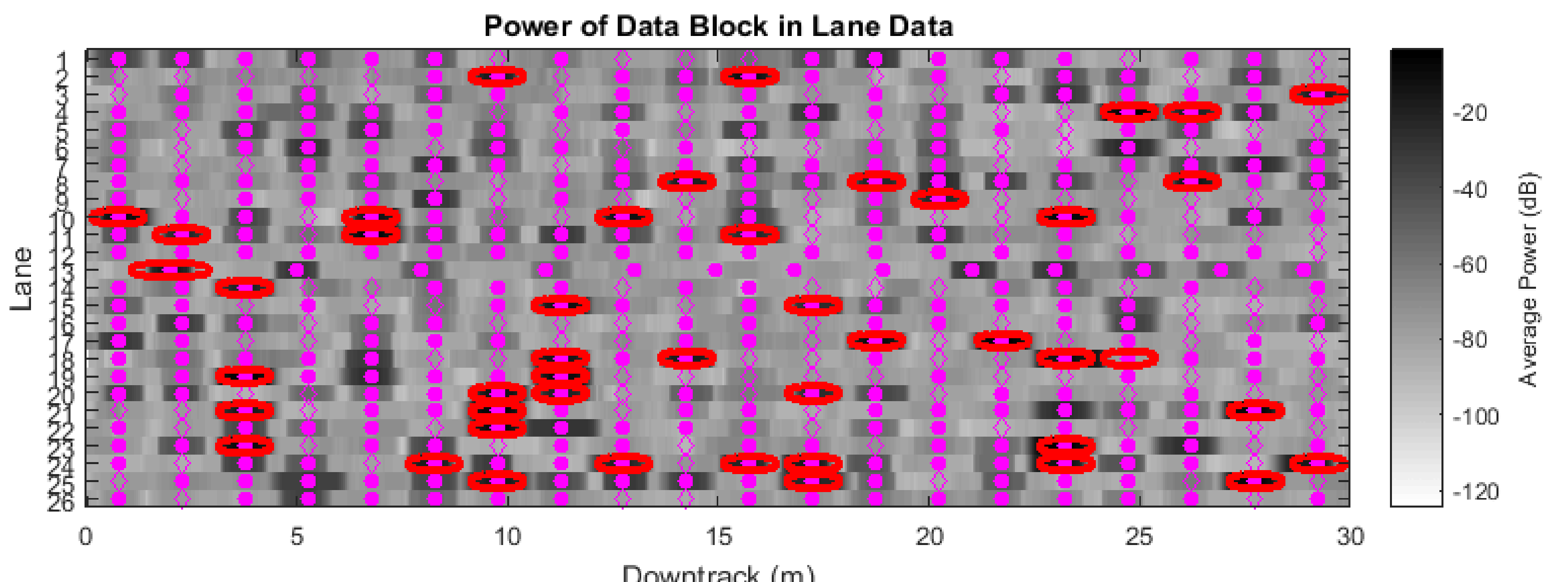
- This data block contains a significant amount of **target** response
- This data is corrupted by the **soil** response
- This data block also has the potential to be used to detect voids

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$



EXPLOITING TARGET IN EIGHTH DATA BLOCK

$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\mathrm{M}^{\mathrm{RS}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}$



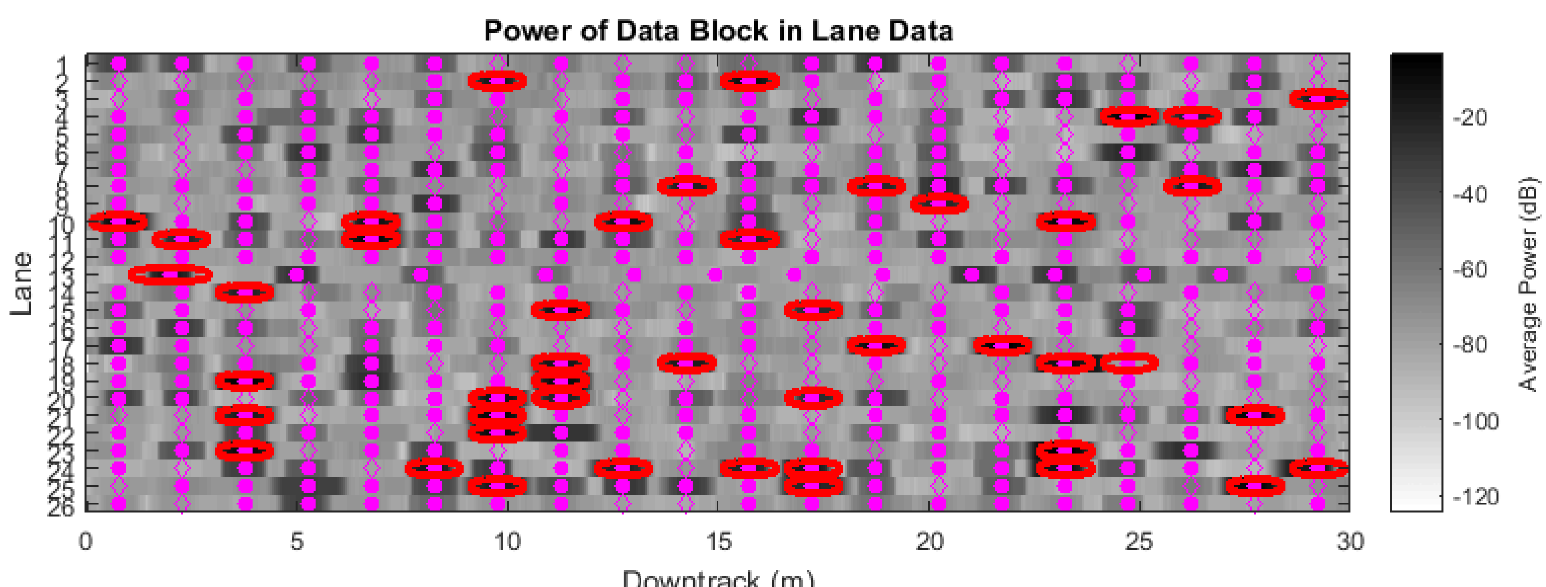
EXPLOITING TARGET IN EIGHTH DATA BLOCK

$$\mathbf{M}_{\mathbf{GG}}^{\bar{\mathbf{R}}\mathbf{S}} = \mathbf{M}_{\mathbf{GG}}^{\bar{\mathbf{R}}\mathbf{S}}\mathbf{P}_{\mathbf{S}}^{T} + \mathbf{M}_{\mathbf{GG}}^{\bar{\mathbf{R}}\mathbf{S}}\mathbf{P}_{\mathbf{S}}^{\perp\,T}$$

$$\mathbf{P}_{\mathbf{S}} = \mathbf{V}_{\mathbf{S}}^{\mathrm{M_S}}(\mathbf{V}_{\mathbf{S}}^{\mathrm{M_S}\,T}\mathbf{V}_{\mathbf{S}}^{\mathrm{M_S}})^{-1}\mathbf{V}_{\mathbf{S}}^{\mathrm{M_S}\,T}$$

- The spatial response of the target found from the SVD of the data can be used to adaptively isolate the target in the this data block

$\mathbf{M}_{\bar{\mathbf{G}}\mathbf{S}}^{\mathbf{RR}}$	$\mathbf{M}_{\bar{\mathbf{G}}\mathbf{S}}^{\bar{\mathbf{R}}\mathbf{S}}$	$\mathbf{M}_{\bar{\mathbf{G}}\mathbf{S}}^{\bar{\mathbf{R}}\mathcal{E}}$
$\mathbf{M}_{\bar{\mathbf{G}}\mathcal{E}}^{\mathbf{RR}}$	$\mathbf{M}_{\bar{\mathbf{G}}\mathcal{E}}^{\bar{\mathbf{R}}\mathbf{S}}$	$\mathbf{M}_{\bar{\mathbf{G}}\mathcal{E}}^{\bar{\mathbf{R}}\mathcal{E}}$
$\mathbf{M}_{\mathbf{GG}}^{\mathbf{RR}}$	$\mathbf{M}_{\mathbf{GG}}^{\bar{\mathbf{R}}\mathbf{S}}$	$\mathbf{M}_{\mathbf{GG}}^{\bar{\mathbf{R}}\mathcal{E}}$



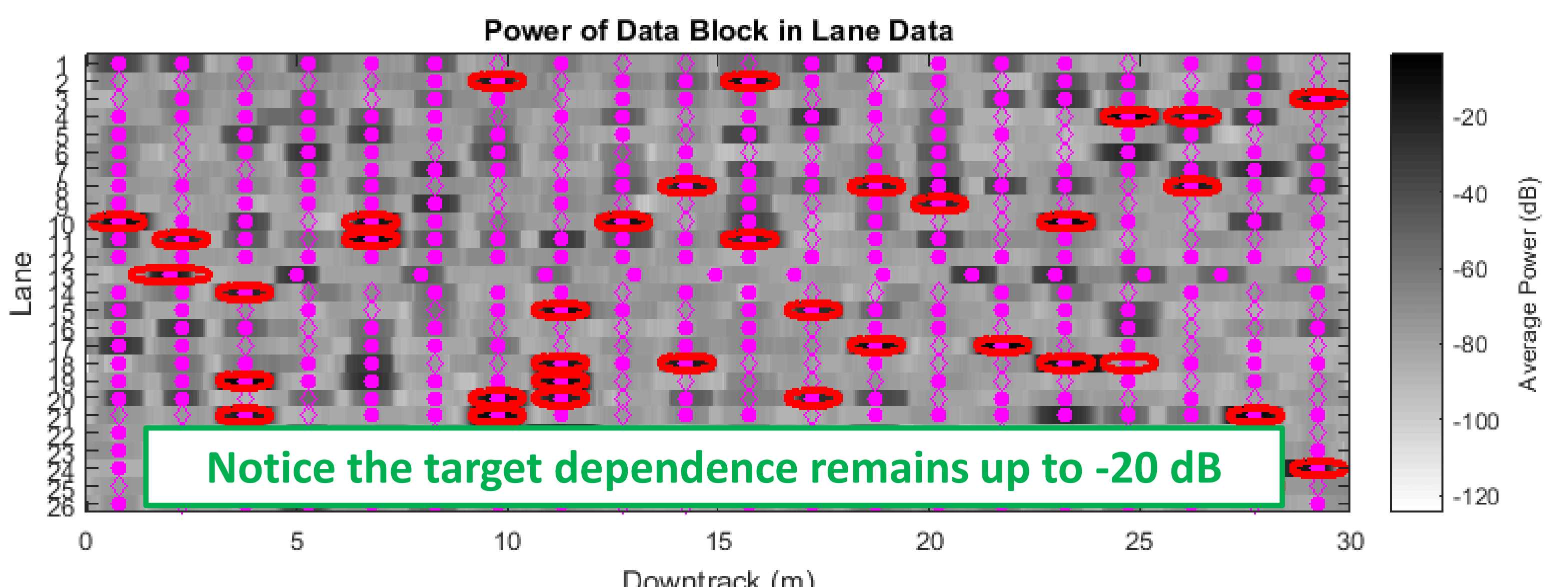
EXPLOITING TARGET IN EIGHTH DATA BLOCK

$$\mathbf{M}_{\mathbf{GG}}^{\mathbf{\bar{R}S}} = \mathbf{M}_{\mathbf{GG}}^{\mathbf{\bar{R}S}}\mathbf{P}_{\mathbf{S}}^{T} + \mathbf{M}_{\mathbf{GG}}^{\mathbf{\bar{R}S}}\mathbf{P}_{\mathbf{S}}^{\perp\,T}$$

$$\mathbf{P}_{\mathbf{S}} = \mathbf{V}_{\mathbf{S}}^{\mathbf{M_S}}(\mathbf{V}_{\mathbf{S}}^{\mathbf{M_S}\,T}\mathbf{V}_{\mathbf{S}}^{\mathbf{M_S}})^{-1}\mathbf{V}_{\mathbf{S}}^{\mathbf{M_S}\,T}$$

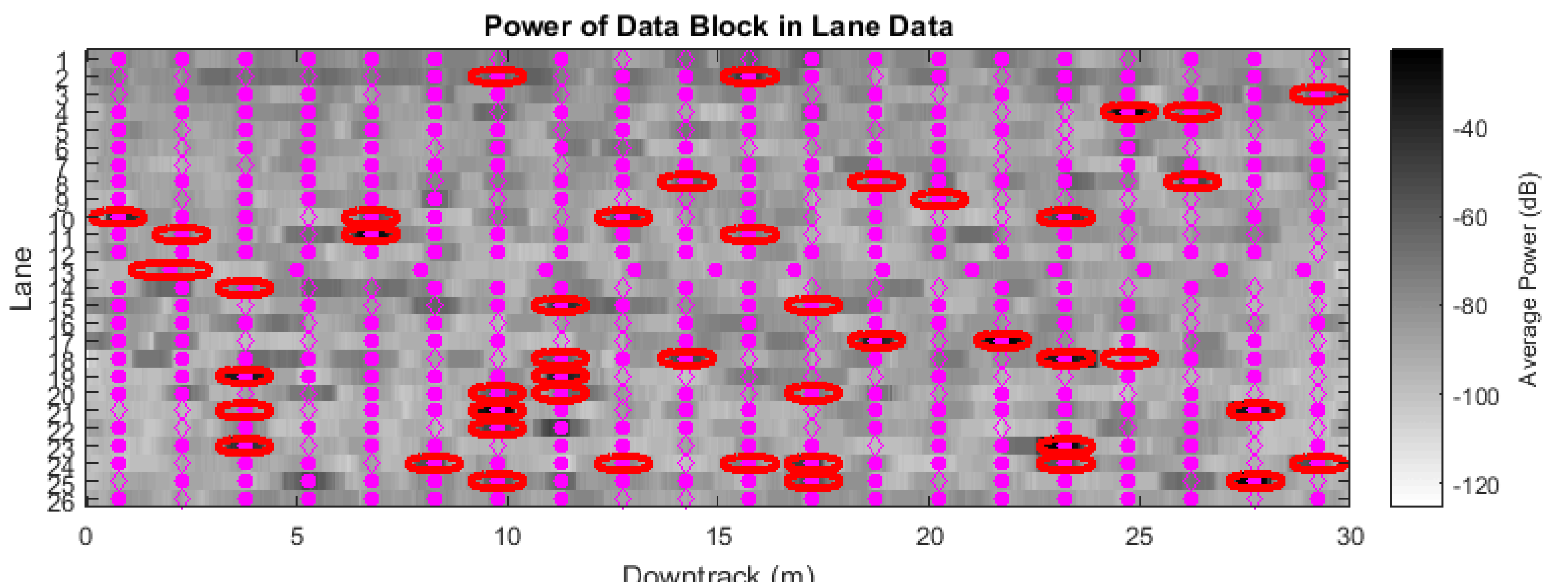
- The spatial response of the target found from the SVD of the data can be used to adaptively isolate the target in the this data block

$\mathrm{M}_{\mathrm{\bar{G}S}}^{\mathrm{RR}}$	$\mathrm{M}_{\mathrm{\bar{G}S}}^{\mathrm{\bar{R}S}}$	$\mathrm{M}_{\mathrm{\bar{G}S}}^{\mathrm{\bar{R}\mathcal{E}}}$
$\mathrm{M}_{\mathrm{\bar{G}\mathcal{E}}}^{\mathrm{RR}}$	$\mathrm{M}_{\mathrm{\bar{G}\mathcal{E}}}^{\mathrm{\bar{R}S}}$	$\mathrm{M}_{\mathrm{\bar{G}\mathcal{E}}}^{\mathrm{\bar{R}\mathcal{E}}}$
$\mathrm{M}_{\mathrm{GG}}^{\mathrm{RR}}$	$\mathrm{M}_{\mathrm{GG}}^{\mathrm{RS}}$	$\mathrm{M}_{\mathrm{GG}}^{\mathrm{\bar{R}\mathcal{E}}}$



EXPLOITING TARGET IN EIGHTH DATA BLOCK

$M^{RR}_{\bar{G}S}$	$M^{\bar{R}S}_{\bar{G}S}$	$M^{\bar{R}\mathcal{E}}_{\bar{G}S}$
$M^{RR}_{\bar{G}\mathcal{E}}$	$M^{\bar{R}S}_{\bar{G}\mathcal{E}}$	$M^{\bar{R}\mathcal{E}}_{\bar{G}\mathcal{E}}$
M^{RR}_{GG}	[illegible]	$M^{\bar{R}\mathcal{E}}_{GG}$



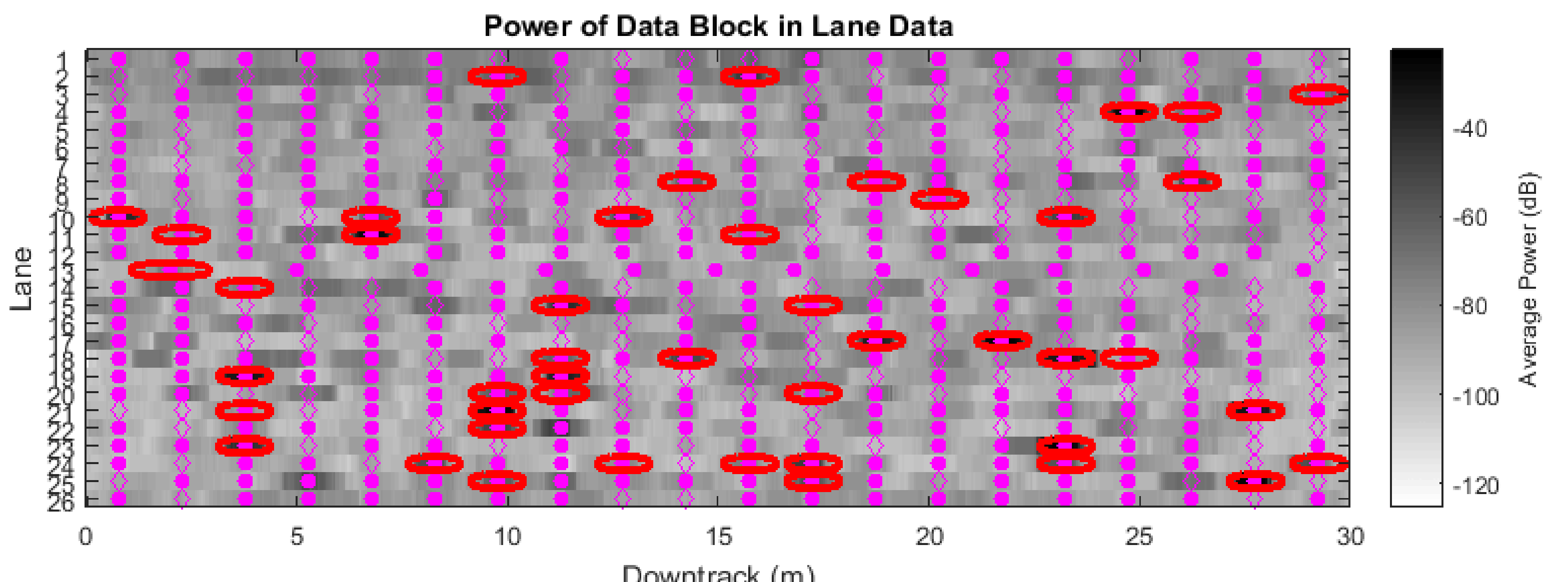
EXPLOITING TARGET IN EIGHTH DATA BLOCK

$$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}} = \mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}} \mathbf{P}^{T}_{\mathbf{S}} + \mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}} \mathbf{P}^{\perp\,T}_{\mathbf{S}}$$

$$\mathbf{P}_{\mathbf{S}} = \mathbf{V}^{\mathbf{M}_{\mathbf{S}}}_{\mathbf{S}} (\mathbf{V}^{\mathbf{M}_{\mathbf{S}}\,T}_{\mathbf{S}} \mathbf{V}^{\mathbf{M}_{\mathbf{S}}}_{\mathbf{S}})^{-1} \mathbf{V}^{\mathbf{M}_{\mathbf{S}}\,T}_{\mathbf{S}}$$

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	[illegible]	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}$

- The remaining portion of this data block can be used to estimate the noise + soil power



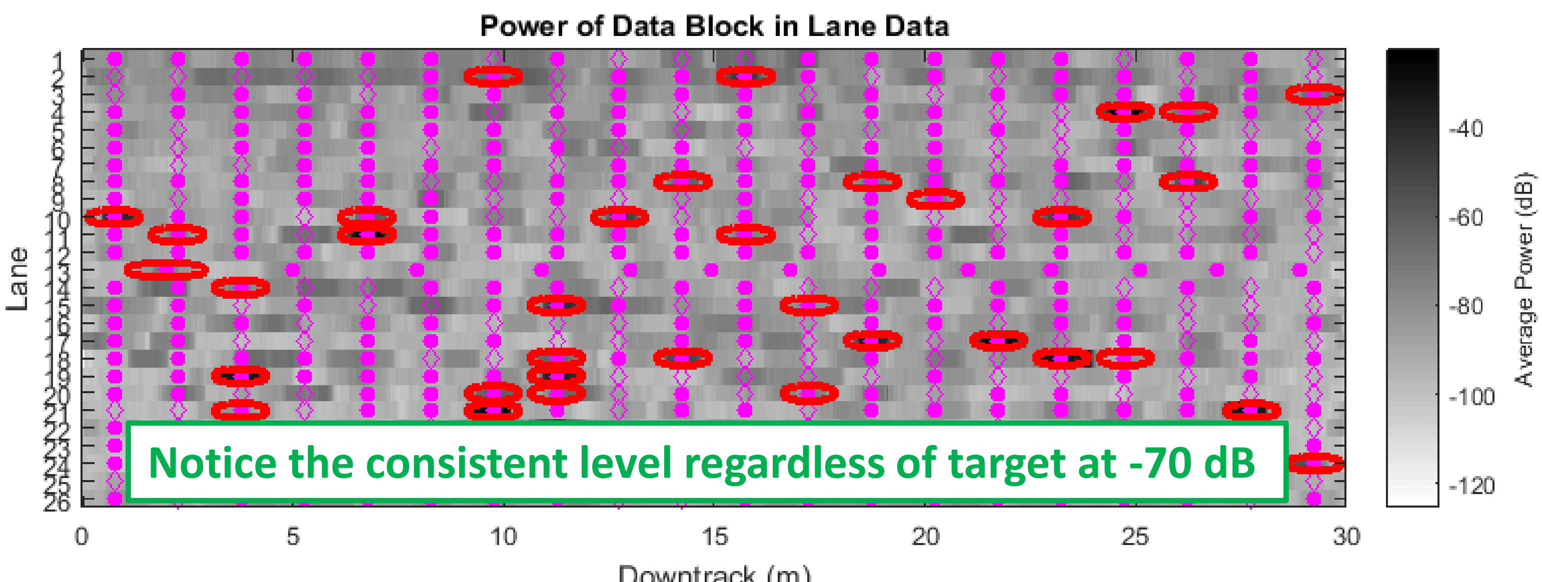
EXPLOITING TARGET IN EIGHTH DATA BLOCK

$$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{G}\mathbf{G}} = \mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{G}\mathbf{G}} \mathbf{P}^{T}_{\mathbf{S}} + \mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{G}\mathbf{G}} \mathbf{P}^{\perp\, T}_{\mathbf{S}}$$

$$\mathbf{P}_{\mathbf{S}} = \mathbf{V}^{\mathbf{M}_\mathbf{S}}_{\mathbf{S}} \left(\mathbf{V}^{\mathbf{M}_\mathbf{S}\, T}_{\mathbf{S}} \mathbf{V}^{\mathbf{M}_\mathbf{S}}_{\mathbf{S}}\right)^{-1} \mathbf{V}^{\mathbf{M}_\mathbf{S}\, T}_{\mathbf{S}}$$

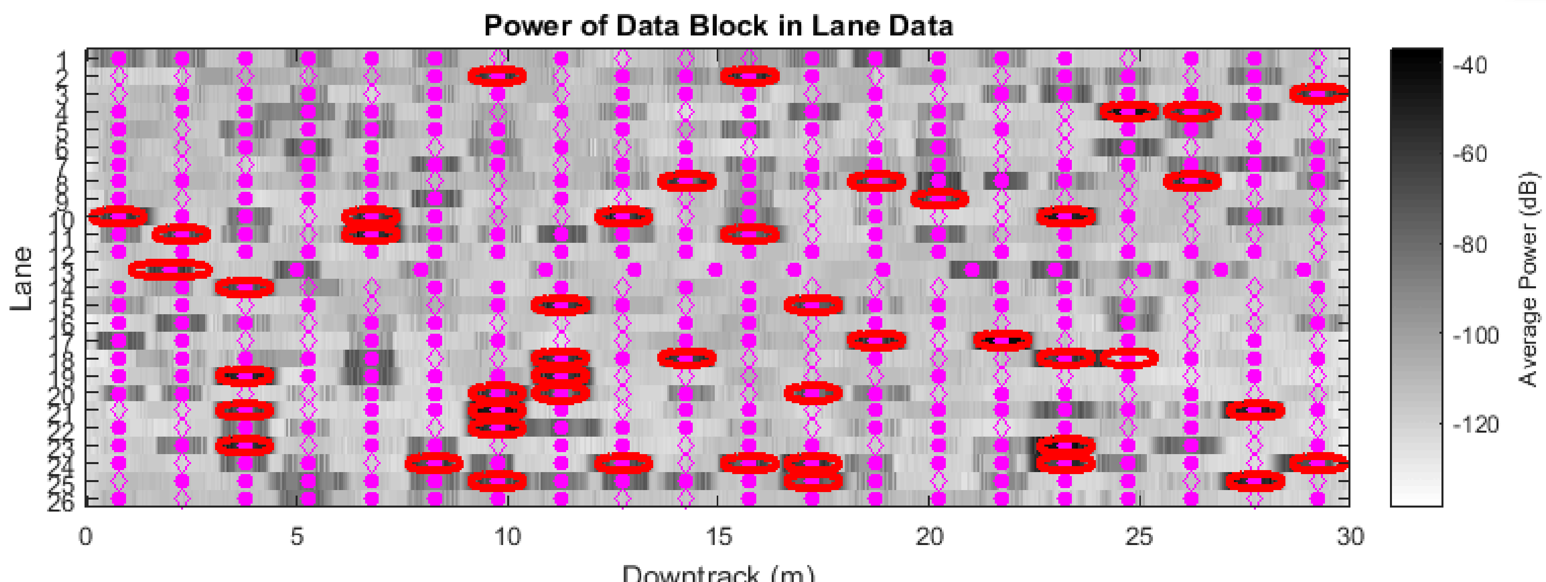
- The remaining portion of this data block can be used to estimate the noise + soil power

$\mathbf{M}^{\mathbf{R}\mathbf{R}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{R}\mathbf{R}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{R}\mathbf{R}}_{\mathbf{G}\mathbf{G}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{G}\mathbf{G}}$



INVESTIGATING NINTH DATA BLOCK

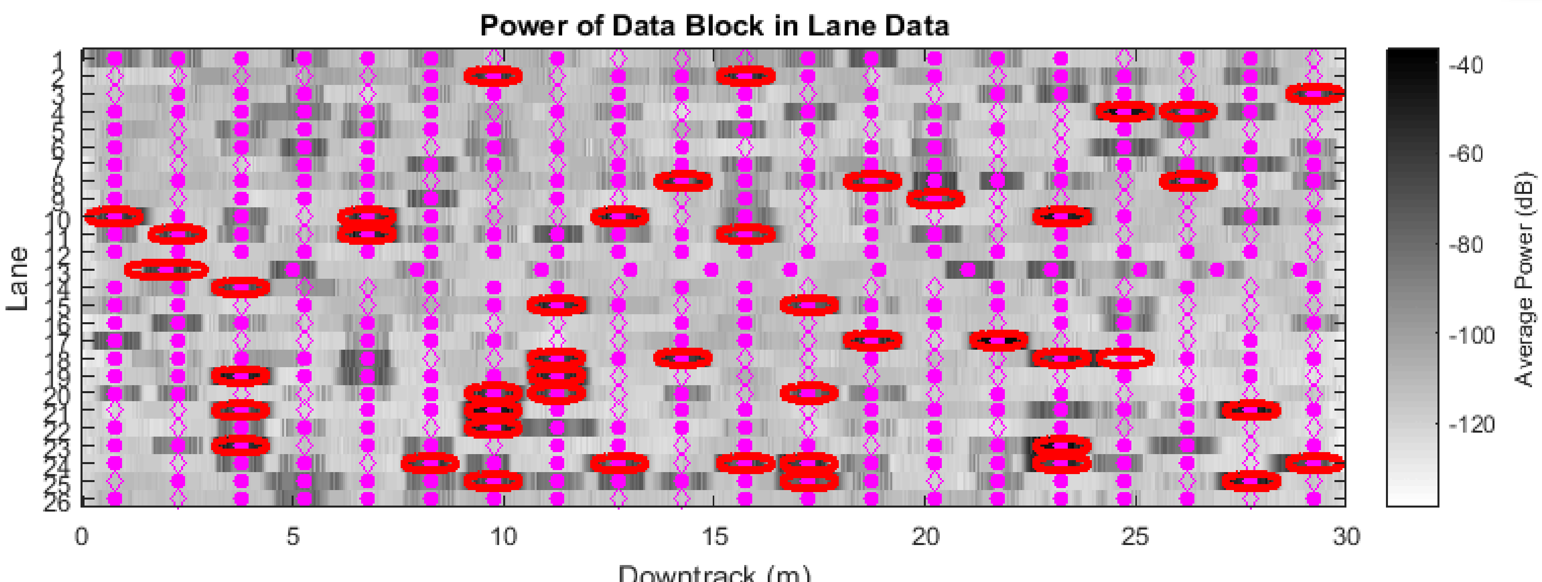
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}$	$\boxed{\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}}$



INVESTIGATING NINTH DATA BLOCK

$$\mathbf{P_{GG}MP^{T}_{\bar{R}\mathcal{E}}} = \cancelto{\epsilon}{\mathbf{P_{GG}SP^{T}_{\bar{R}\mathcal{E}}}} + \mathbf{P_{GG}GP^{T}_{\bar{R}\mathcal{E}}} + \cancelto{0}{\mathbf{P_{GG}RP^{T}_{\bar{R}\mathcal{E}}}} + \mathbf{P_{GG}\mathcal{E}P^{T}_{\bar{R}\mathcal{E}}}$$

$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathrm{S}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathrm{S}}$
$\mathrm{M}^{\mathrm{RR}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\bar{\mathrm{G}}\mathcal{E}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\bar{\mathrm{G}}\mathcal{E}}$
$\mathrm{M}^{\mathrm{RR}}_{\mathrm{GG}}$	$\mathrm{M}^{\bar{\mathrm{R}}\mathrm{S}}_{\mathrm{GG}}$	$\boxed{\mathrm{M}^{\bar{\mathrm{R}}\mathcal{E}}_{\mathrm{GG}}}$

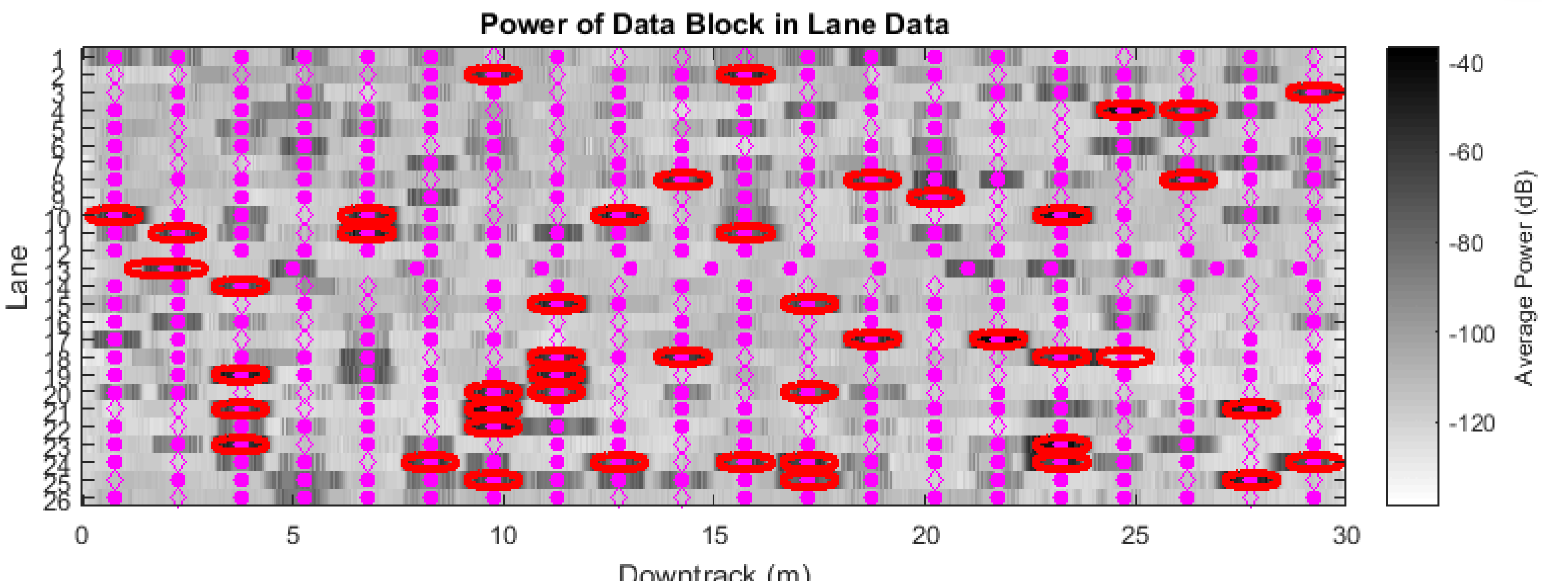


INVESTIGATING NINTH DATA BLOCK

$$\mathbf{P_{GG}MP^{T}_{\bar{R}\mathcal{E}}} = \cancelto{\epsilon}{\mathbf{P_{GG}SP^{T}_{\bar{R}\mathcal{E}}}} + \mathbf{P_{GG}GP^{T}_{\bar{R}\mathcal{E}}} + \cancelto{0}{\mathbf{P_{GG}RP^{T}_{\bar{R}\mathcal{E}}}} + \mathbf{P_{GG}\mathcal{E}P^{T}_{\bar{R}\mathcal{E}}}$$

- This data block can be used to estimate the **noise + soil** power

$\mathbf{M^{RR}_{\bar{G}S}}$	$\mathbf{M^{\bar{R}S}_{\bar{G}S}}$	$\mathbf{M^{\bar{R}\mathcal{E}}_{\bar{G}S}}$
$\mathbf{M^{RR}_{\bar{G}\mathcal{E}}}$	$\mathbf{M^{\bar{R}S}_{\bar{G}\mathcal{E}}}$	$\mathbf{M^{\bar{R}\mathcal{E}}_{\bar{G}\mathcal{E}}}$
$\mathbf{M^{RR}_{GG}}$	$\mathbf{M^{\bar{R}S}_{GG}}$	$\boxed{\mathbf{M^{\bar{R}\mathcal{E}}_{GG}}}$

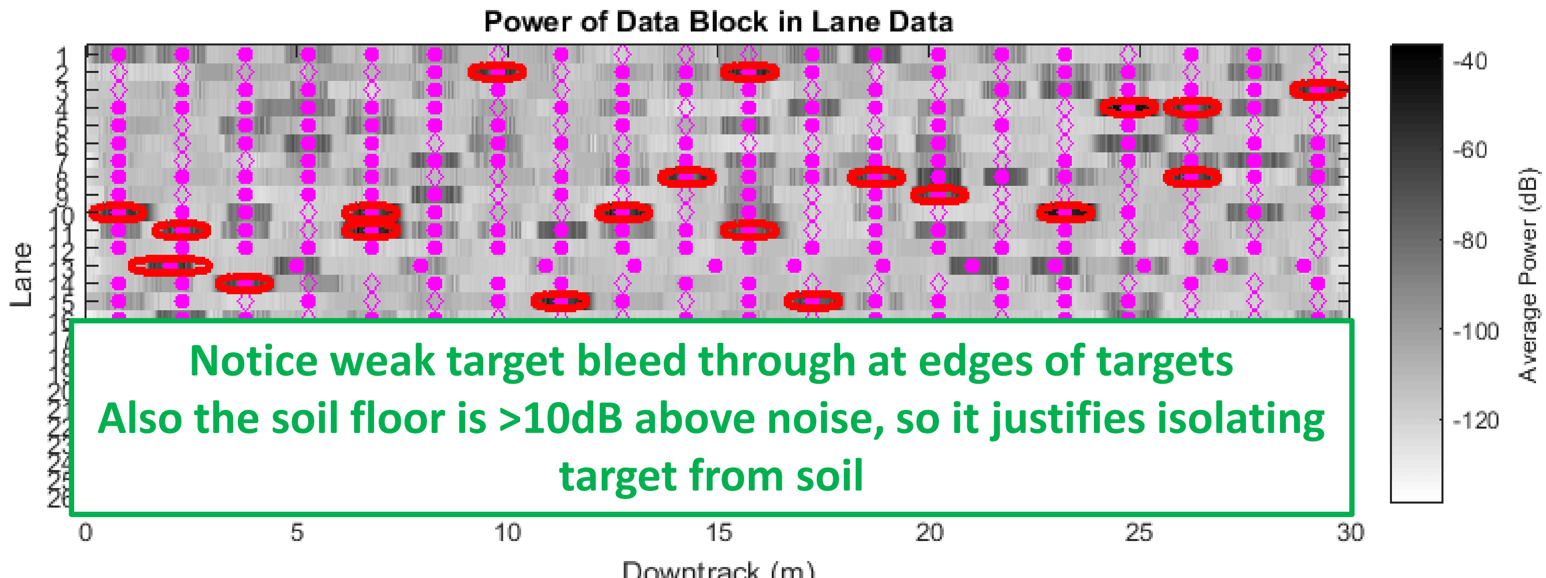


INVESTIGATING NINTH DATA BLOCK

$$\mathbf{P}_{\mathbf{GG}}\mathbf{M}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathcal{E}} = \cancelto{\epsilon}{\mathbf{P}_{\mathbf{GG}}\mathbf{S}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathcal{E}}} + \mathbf{P}_{\mathbf{GG}}\mathbf{G}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathcal{E}} + \cancelto{0}{\mathbf{P}_{\mathbf{GG}}\mathbf{R}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathcal{E}}} + \mathbf{P}_{\mathbf{GG}}\mathcal{E}\mathbf{P}^{T}_{\bar{\mathbf{R}}\mathcal{E}}$$

- This data block can be used to estimate the **noise + soil** power

$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathbf{S}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathbf{S}}$
$\mathbf{M}^{\mathbf{RR}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\bar{\mathbf{G}}\mathcal{E}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\bar{\mathbf{G}}\mathcal{E}}$
$\mathbf{M}^{\mathbf{RR}}_{\mathbf{GG}}$	$\mathbf{M}^{\bar{\mathbf{R}}\mathbf{S}}_{\mathbf{GG}}$	$\boxed{\mathbf{M}^{\bar{\mathbf{R}}\mathcal{E}}_{\mathbf{GG}}}$



Notice weak target bleed through at edges of targets
Also the soil floor is >10dB above noise, so it justifies isolating target from soil

QUESTIONS?

This work is supported in part by the U.S. Army REDCOM CERDEC Night Vision and Electronic Sensors Directorate, Science and Technology Division, Countermine Branch and in part by the U. S. Army Research Office under Contract Number W911NF-11-1-0153