Computer Science
Department

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Geometric Active Contours

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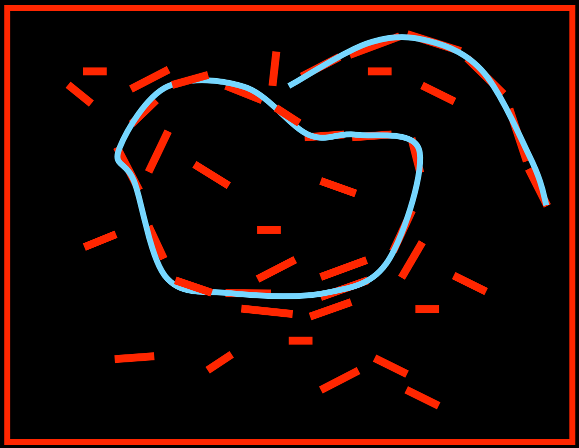
Geometric Image Processing Lab

Edge Detection

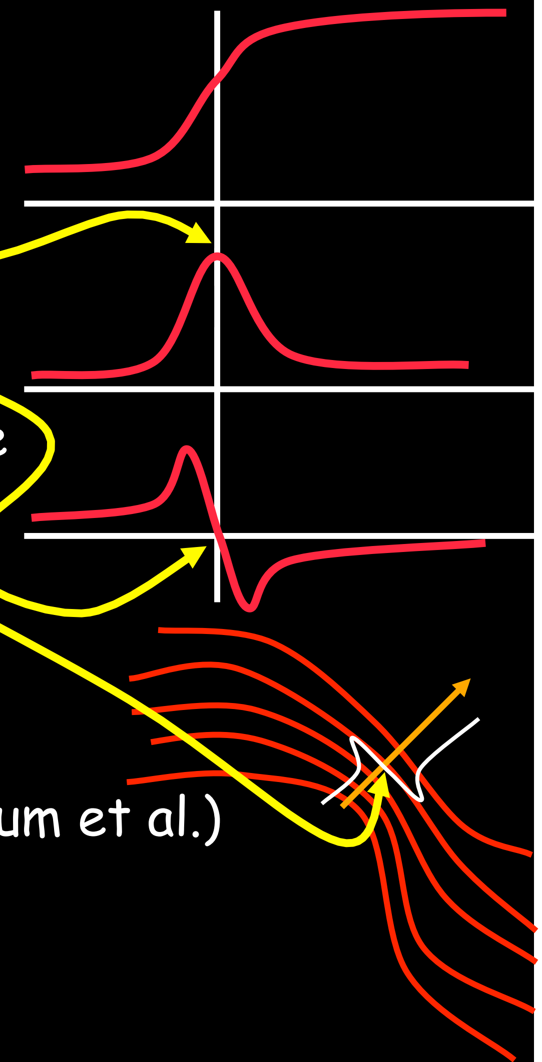
- Edge Detection:
  - The process of labeling the locations in the image where the gray level's "rate of change" is high.
    - OUTPUT: "edgels" locations, direction, strength



- Edge Integration:
  - The process of combining "local" and perhaps sparse and non-contiguous "edgel"-data into meaningful, long edge curves (or closed contours) for segmentation
    - OUTPUT: edges/curves consistent with the local data

The Classics

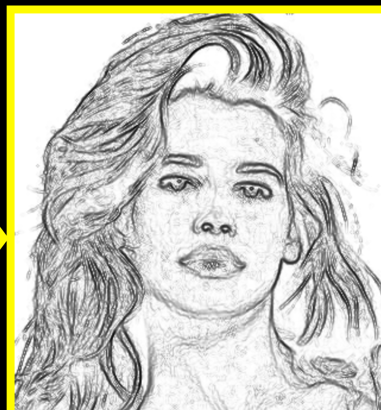
- Edge detection:
  - Sobel, Prewitt, Other gradient estimators
  - Marr Hildreth
    zero crossings of
  - Haralick/Canny/Deriche et al.
    "optimal" directional local max of derivative
- Edge Integration:
  - tensor voting (Rom, Medioni, Williams, …)
  - dynamic programming (Shashua & Ullman)
  - generalized "grouping" processes (Lindenbaum et al.)



The "New-Wave"

- Snakes
- Geodesic Active Contours
- Model Driven Edge Detection

Image



Edge Indicator Function

$$\frac{1}{1+|\nabla(G_\sigma * I)|^2}$$

"nice" curves that optimize a functional of g( ), i.e.

nice: "regularized", smooth, fit some prior information

Edge Curves

Geodesic Active Contours

- Snakes Terzopoulos-Witkin-Kass 88
  - Linear functional ⇨ efficient implementation
  - non-geometric ⇨ depends on parameterization
- Open geometric scaling invariant, Fua-Leclerc 90
- Non-variational geometric flow
  Caselles et al. 93, Malladi et al. 93
  - Geometric, yet does not minimize any functional
- Geodesic active contours Caselles-Kimmel-Sapiro 95
  - derived from geometric functional
  - non-linear ⇨ inefficient implementations:
    - Explicit Euler schemes limit numerical step for stability
- Level set method Ohta-Jansow-Karasaki 82, Osher-Sethian 88
  - automatically handles contour topology
- Fast geodesic active contours Goldenberg-Kimmel-Rivlin-Rudzsky 99
  - no limitation on the time step
  - efficient computations in a narrow band

Laplacian Active Contours

- Closed contours on vector fields
  - Non-variational models Xu-Prince 98, Paragios et al. 01
  - A variational model Vasilevskiy-Siddiqi 01
- Laplacian active contours open/closed/robust

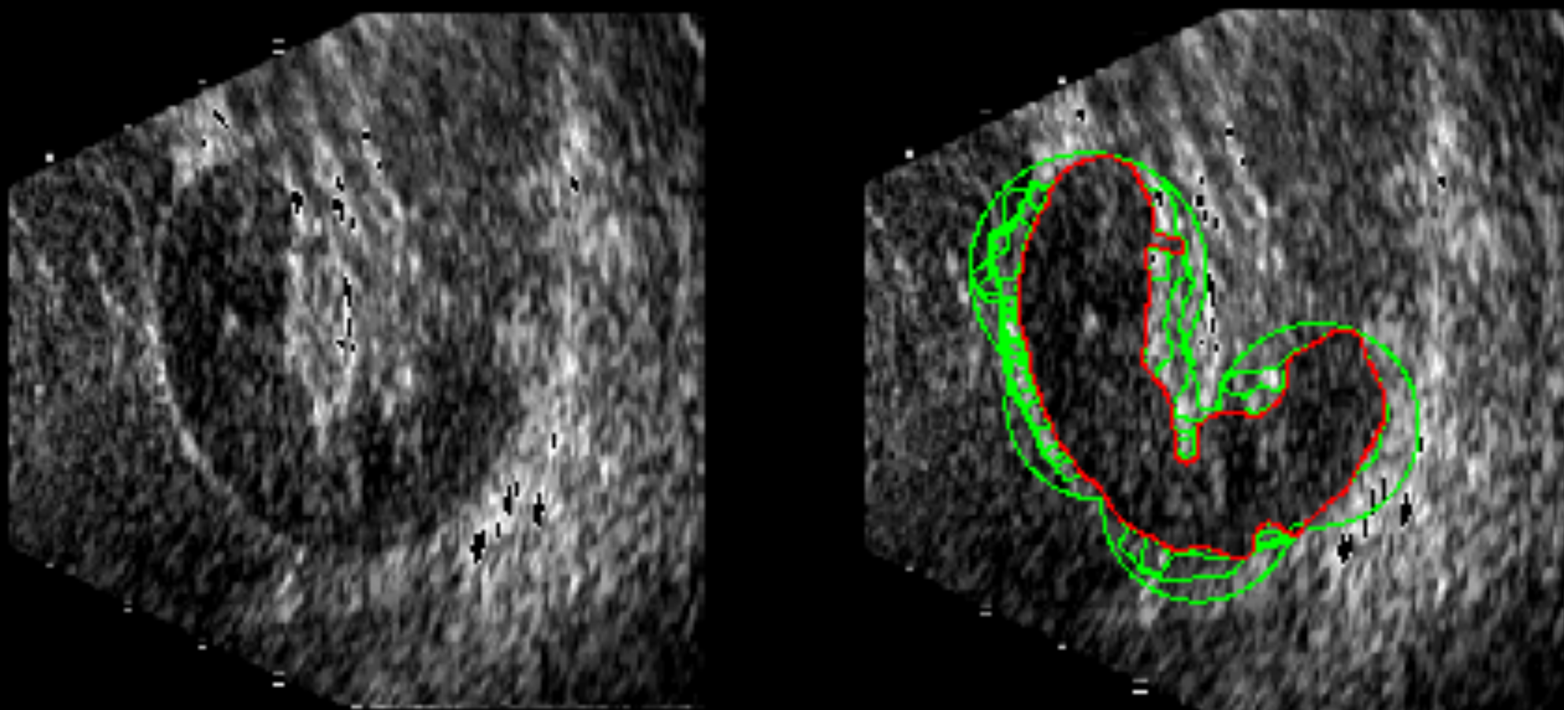
  Kimmel-Bruckstein 01

Most recent:
variational measures
for good old operators
Kimmel-Bruckstein 03

Segmentation

Segmentation

- Ultrasound images

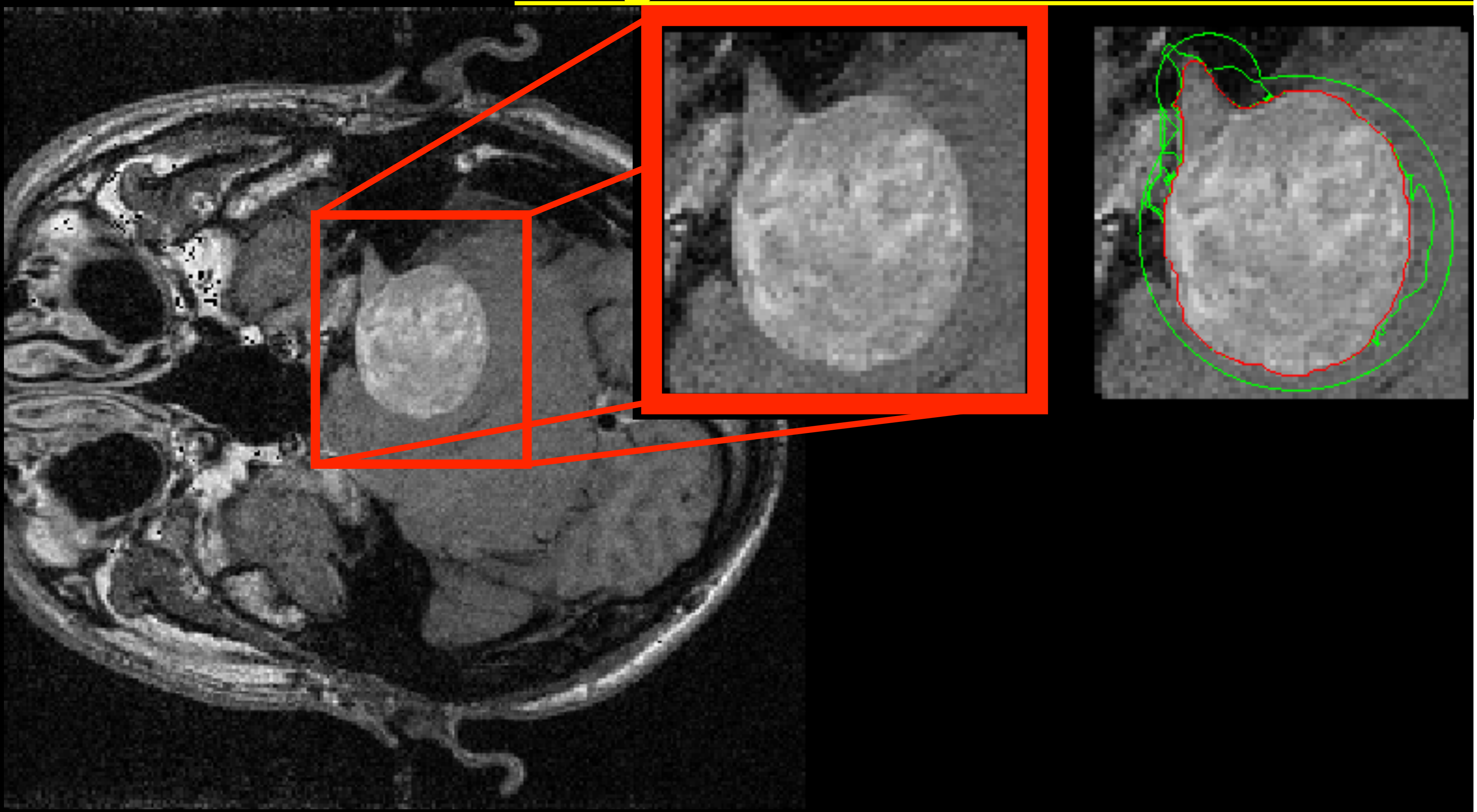


Caselles,Kimmel, Sapiro ICCV'95

Pintos

Woodland Encounter Bev Doolittle 1985

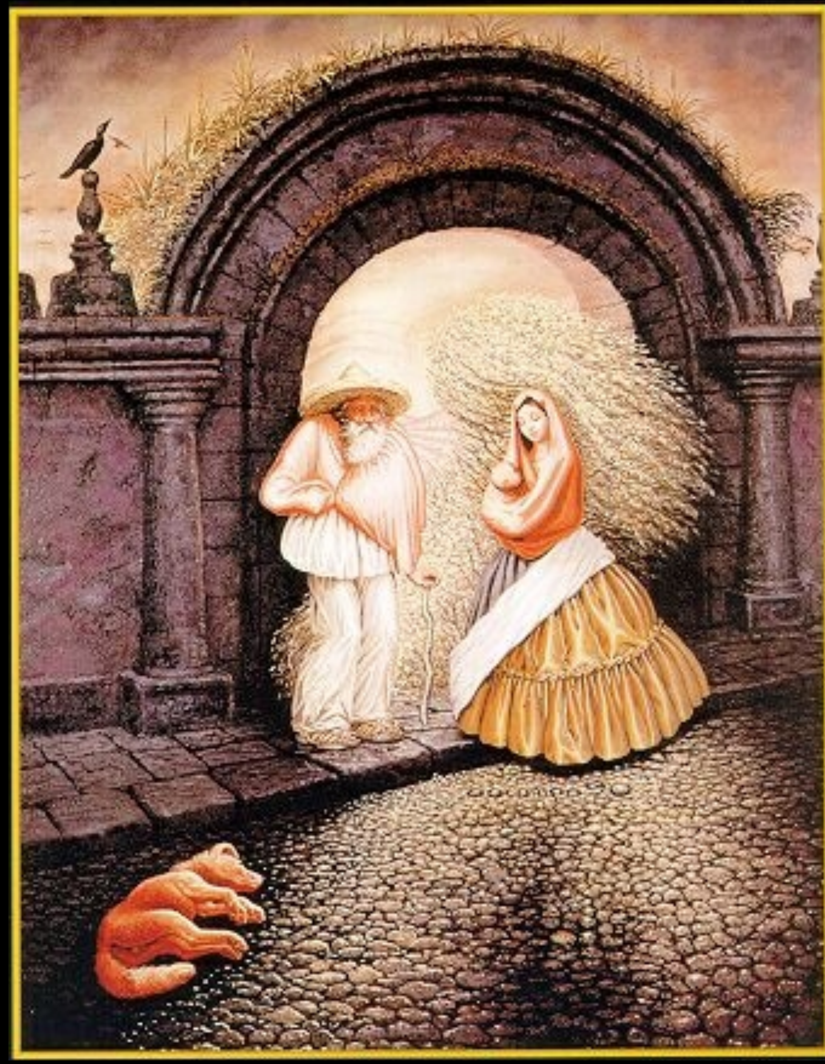
Segmentation

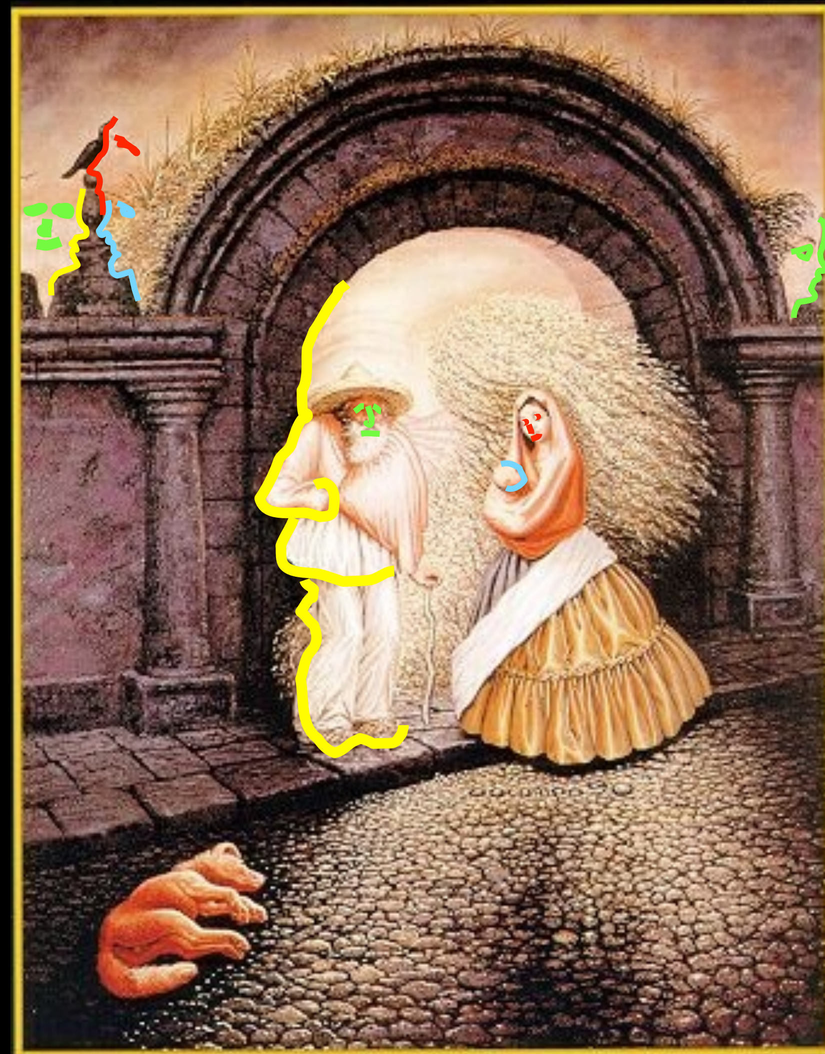


Caselles,Kimmel, Sapiro ICCV'95

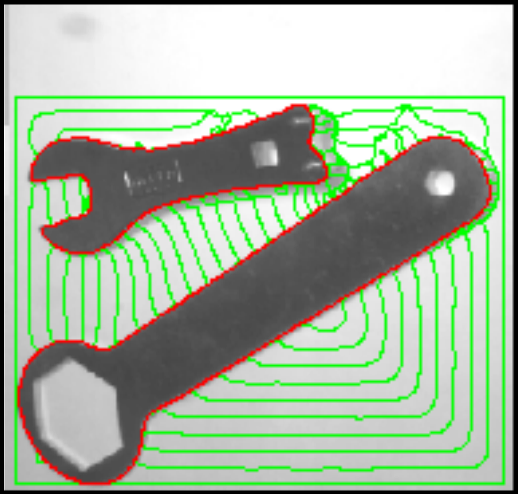
Prior knowledge…

Prior knowledge...





Segmentation

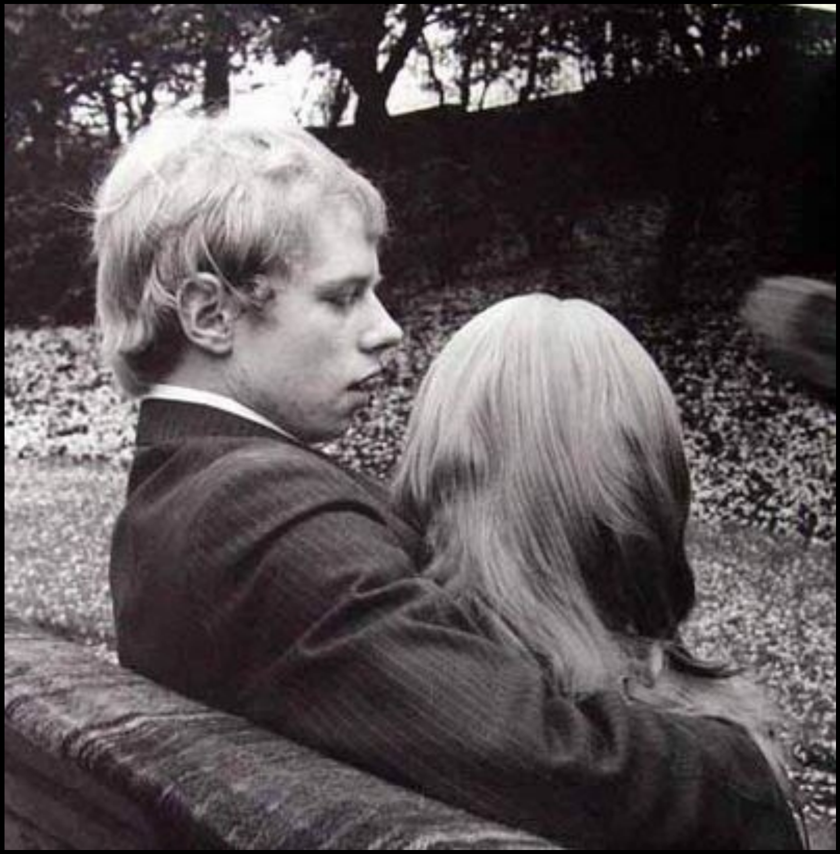


Caselles,Kimmel, Sapiro ICCV'95

Segmentation

- With a good prior who needs the data...

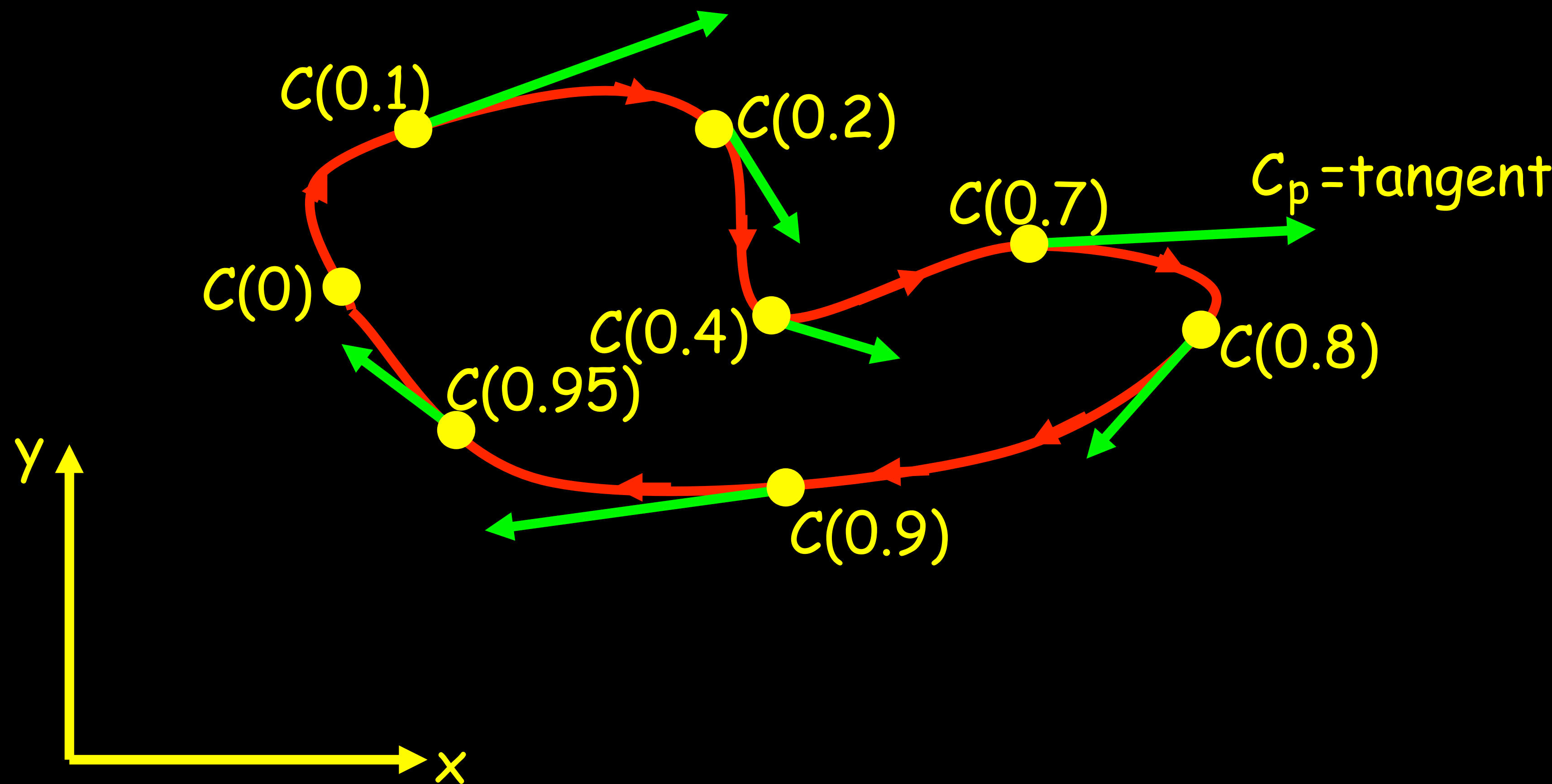
Wrong Prior???



Wrong Prior???

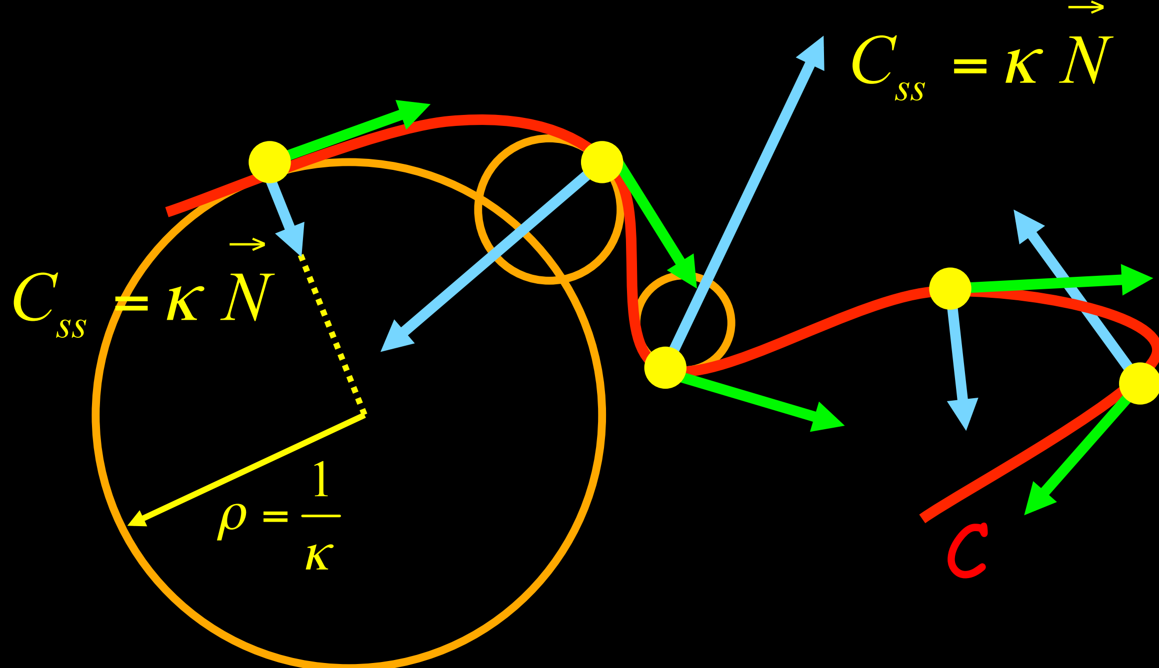
Curves in the Plane

- $C(p)=\{x(p),y(p)\}, \quad p\in[0,1]$



Arc-length and Curvature

$$s(p) = \int_0^p |C_p|\,dp \Longrightarrow |C_s| = 1, \quad \left(C_s = \frac{C_p}{|C_p|} \right)$$



Calculus of Variations

Find C for which $E = \int_0^1 L(C, C_p)\,dp$ is an extremum

$$E(\varepsilon) = \int L(x + \varepsilon\eta, y, x_p + \varepsilon\eta_p, y_p)\,dp$$

$$(\delta E)_x = \left(\frac{\partial E}{\partial \varepsilon}\right)_{\varepsilon=0} = \int \left(L_x - \frac{d}{dp} L_{x_p} \right) \eta\,dp$$

Euler-Lagrange:
$$\left\{ \begin{aligned} L_x - \frac{d}{dp} L_{x_p} &= 0 \\ L_y - \frac{d}{dp} L_{y_p} &= 0 \end{aligned} \right\}$$

Calculus of Variations

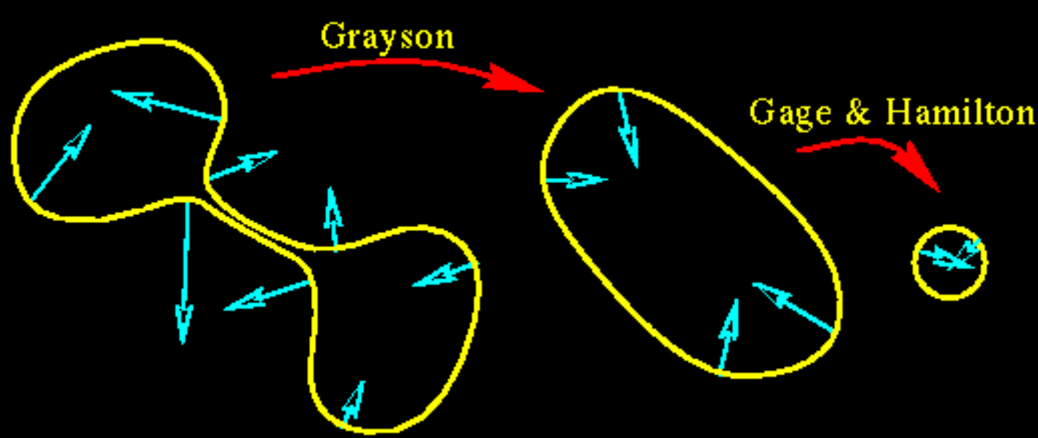
$$E = \int_0^1 |C_p|\, dp$$

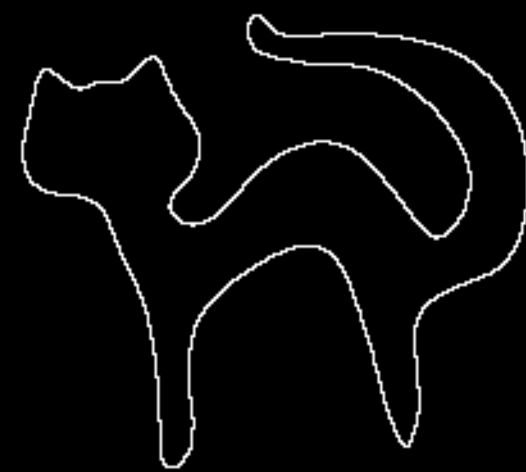
Important Example

⇒ Euler-Lagrange: $\kappa \overrightarrow{N} |C_p| = 0$, setting $ds = |C_p|\, dp$

⇒

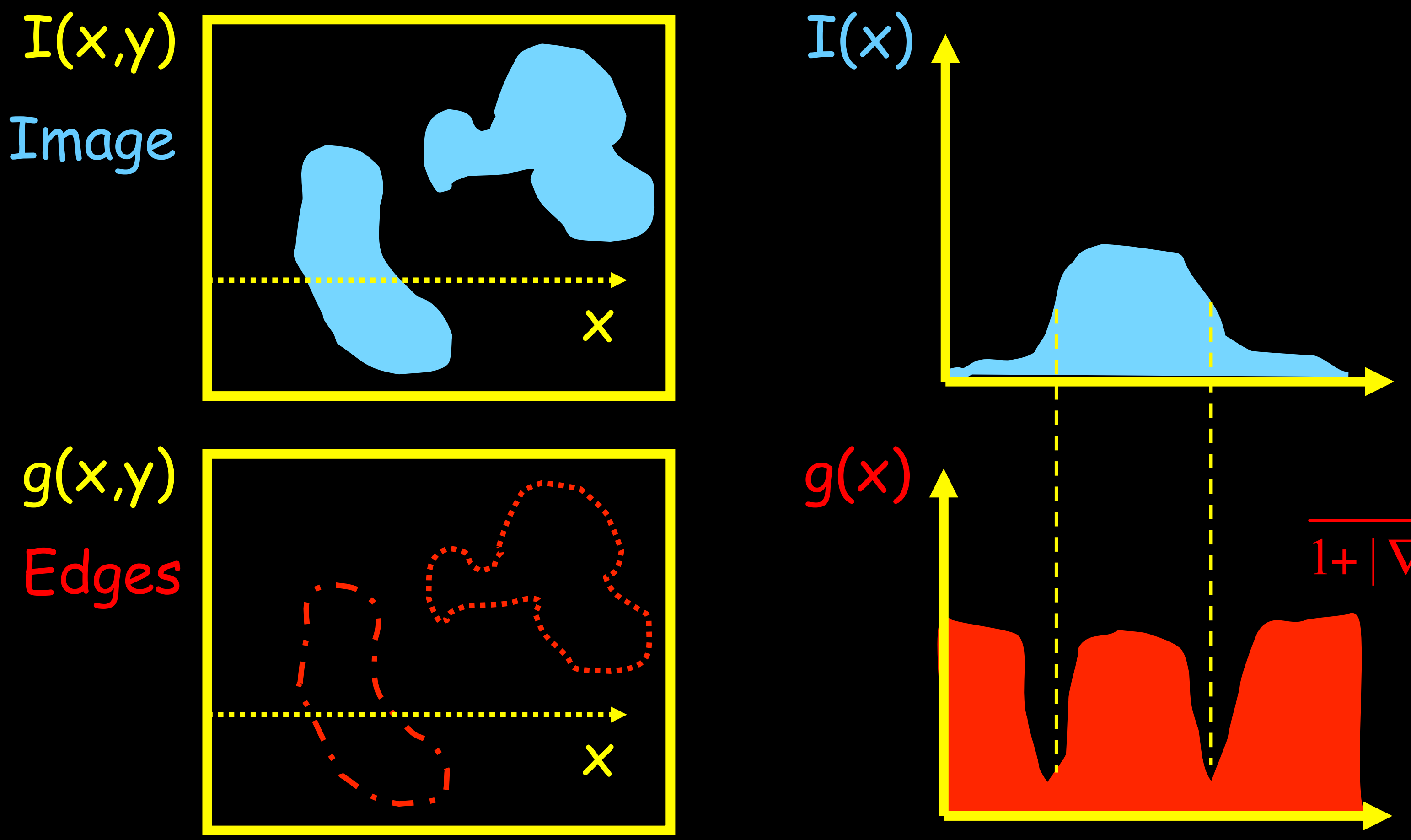
⇒ Curvature flow





$$C_t = C_{ss} \qquad (C_{ss} = \kappa \vec{N})$$

Potential Functions (g)

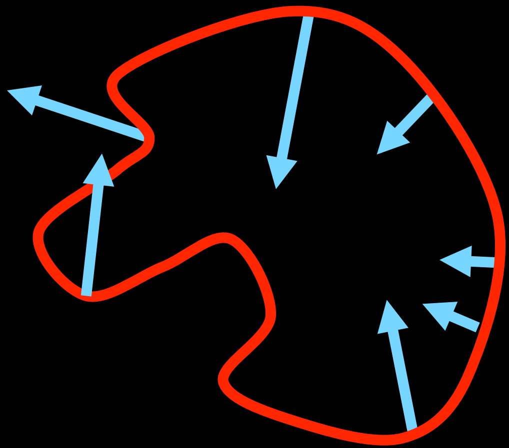


$$\frac{1}{1+|\nabla(G_\sigma * I)|^2}$$

Snakes & Geodesic Active Contours

- **Snake** model
  Terzopoulos-Witkin-Kass 88

- Euler Lagrange as a gradient descent



- **Geodesic active contour** model
  Caselles-Kimmel-Sapiro 95

- Euler Lagrange gradient descent

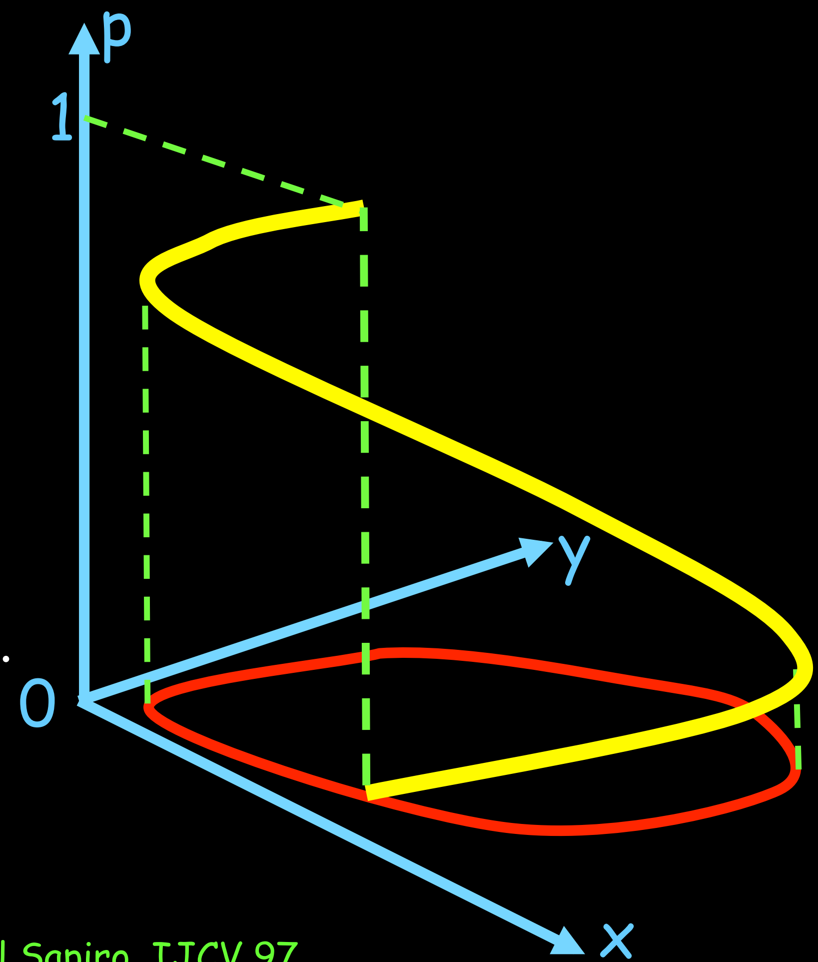
Maupertuis Principle of Least Action

$$\arg\min_{C} \int_0^1 (g(C(p)) + |C_p|^2)\, dp =$$

$$\arg\min_{C} \int_0^L \sqrt{g(C(s)) + E_0}\, ds$$

Snake = Geodesic active contour

up to some E_0, i.e $\tilde{g} = \sqrt{g + E_0}$



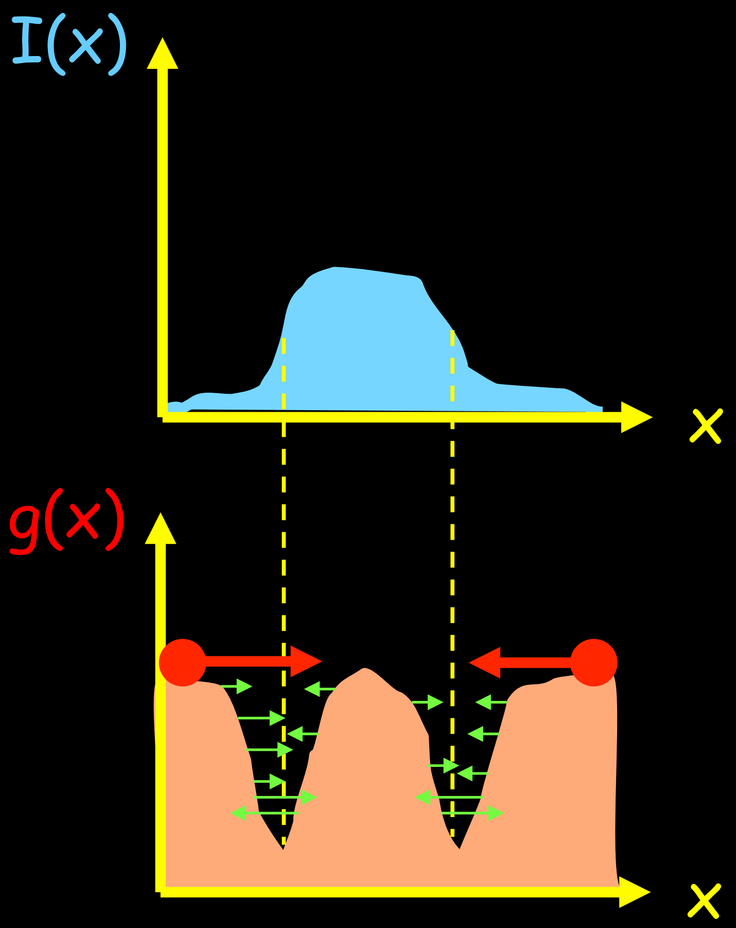
$\Rightarrow$ Snakes depend on parameterization.

$\Rightarrow$ Different initial parameterizations yield solutions for <u>different</u> geometric functionals

Caselles Kimmel Sapiro, IJCV 97

Geodesic Active Contours in 1D

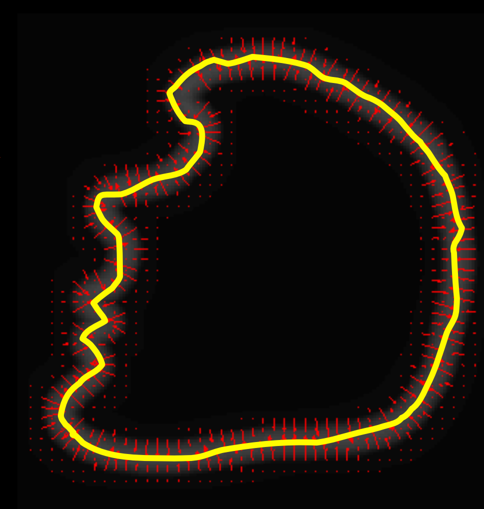
Geodesic active contours are reparameterization invariant



Geodesic Active Contours in 2D

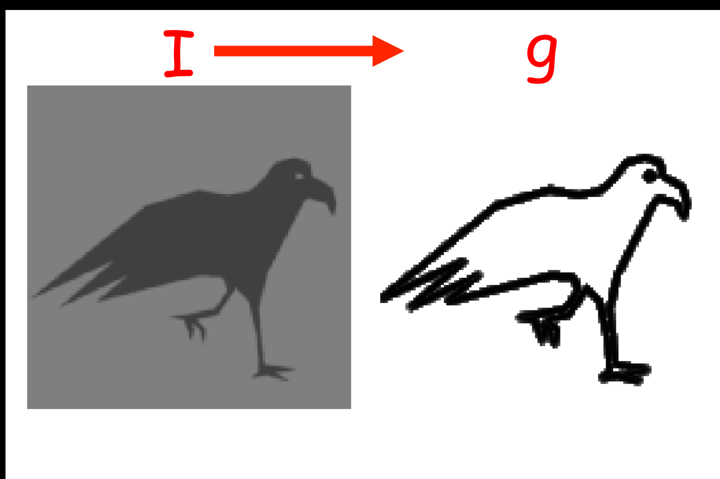
$G_s * I$

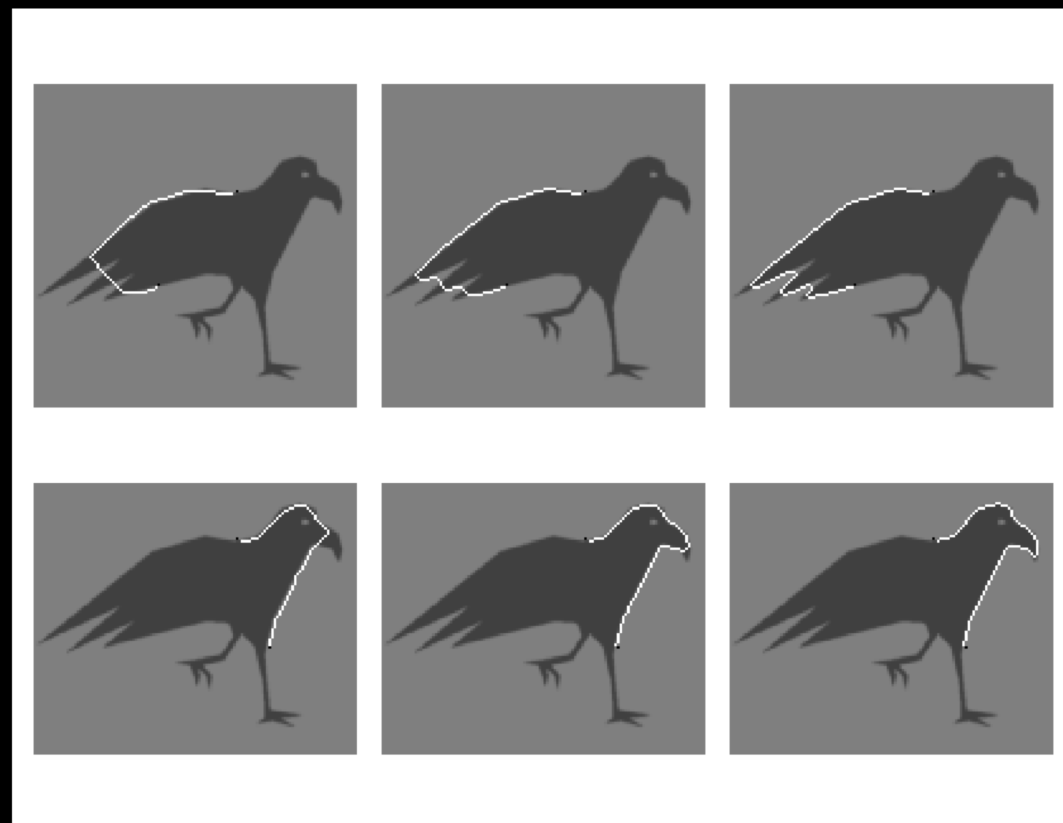
$$g(x) = \frac{1}{1 + |\nabla(G_\sigma * I)|^2}$$



Controlling -max

Smoothness



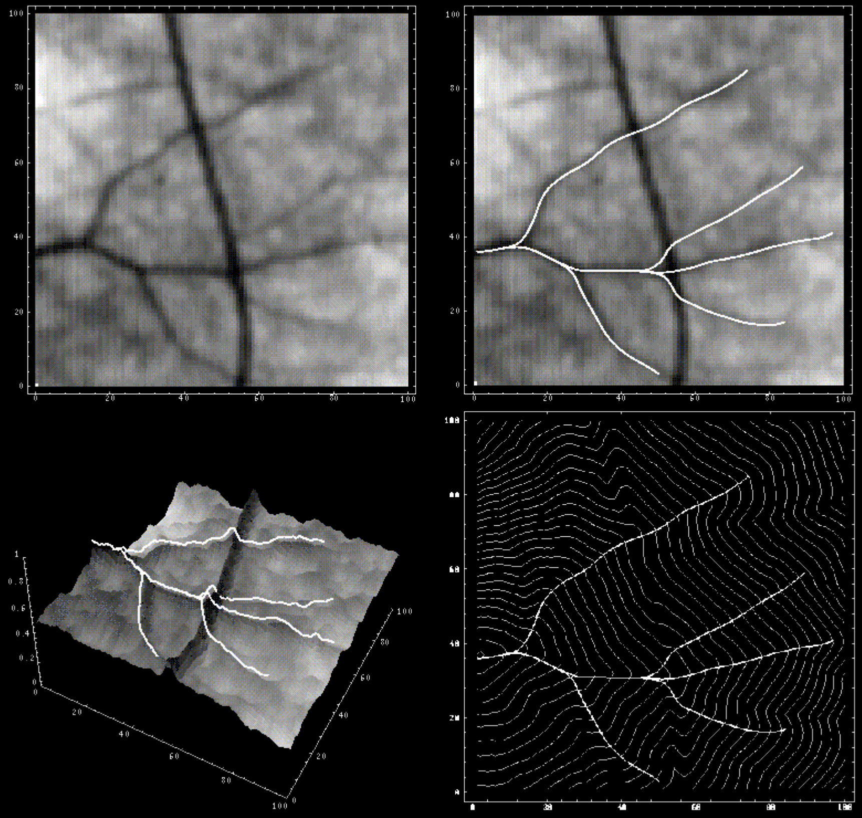


Cohen Kimmel, IJCV 97

Fermat's Principle

In an isotropic medium, the paths taken by light rays are extremal geodesics w.r.t.

i.e.,



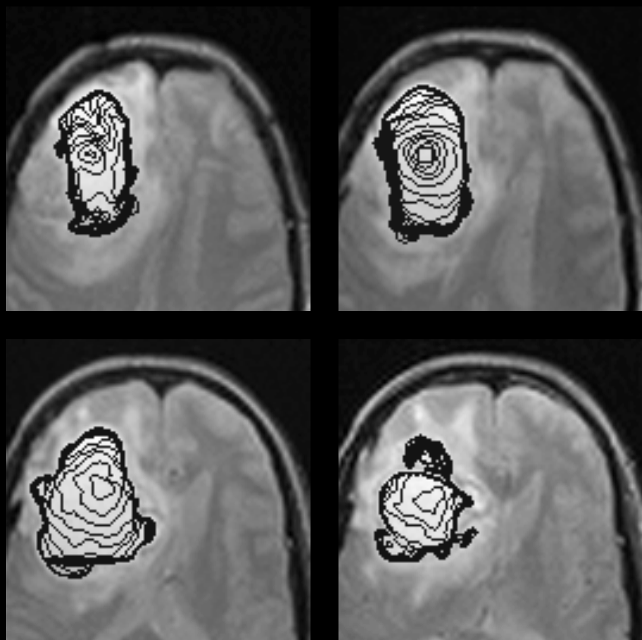
Cohen Kimmel, IJCV 97

Experiments - Color Segmentation

Goldenberg, Kimmel, Rivlin, Rudzsky,

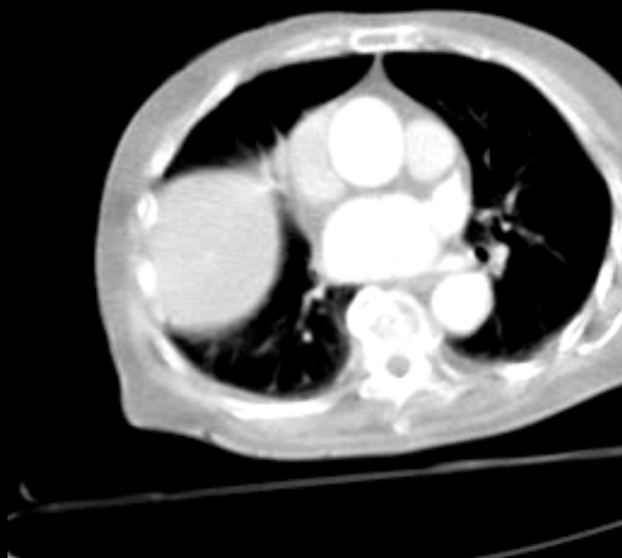
IEEE T-IP 2001

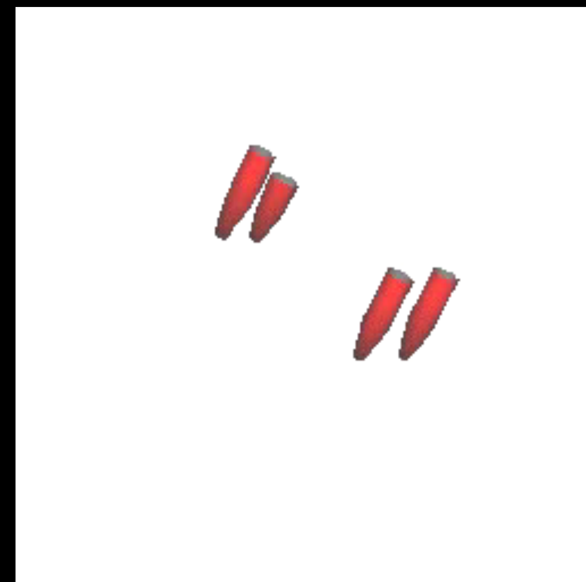
Tumor in 3D MRI



Caselles,Kimmel, Sapiro, Sbert, IEEE T-PAMI 97

Segmentation in 4D





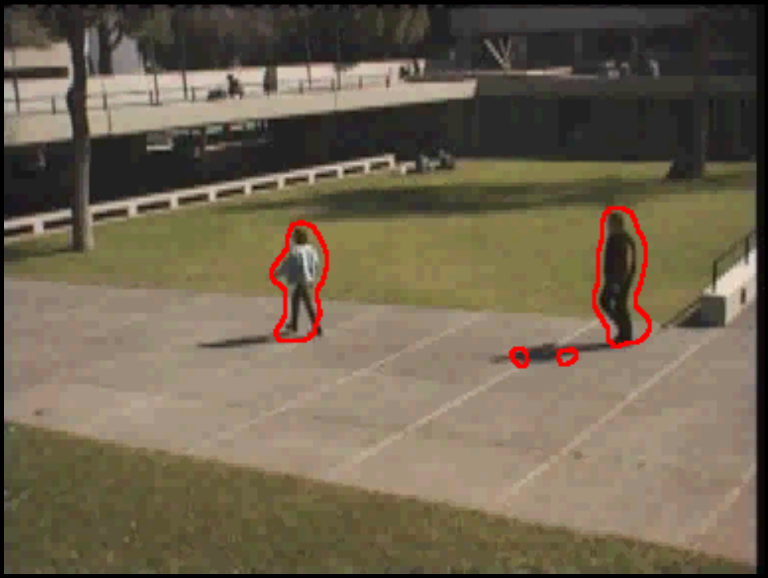
Malladi, Kimmel, Adalsteinsson,
Caselles, Sapiro, Sethian
SIAM Biomedical workshop 96

Tracking in Color Movies

Goldenberg, Kimmel, Rivlin, Rudzsky,

IEEE T-IP 2001

Tracking in Color Movies



Goldenberg, Kimmel, Rivlin, Rudzsky,
IEEE T-IP 2001