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Experiments in Predictive Sensor Fusion

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ABSTRACT

Data fusion is a process of combining evidence from different information sources in order to make a better judgment. However, multiple sources can provide complementary information that can be used to increase the performance in detection and recognition. There are many frameworks within which to combine these pieces into a more meaningful answer. However, new information added might be redundant or even conflicting with the existing information. These questions arise: Can we predict the value added by fusing their outputs together, if we know the general characteristics of a set of sensors? Can we specify the needed characteristics of a new sensor/algorithm to add to an existing suite to gain a desired improvement performance? The characteristic of a new sensor can be in any forms, e.g., the ratio of a target's signal to the clutter's signal (similar in meaning to the signal-to-noise ratio), the position resolution, etc. In this paper, we consider these questions in the context of fuzzy set theory and in particular, a soft decision level fusion scheme we developed for land mine detection scenarios. Here, we primarily consider the ratio of a target's signal to the clutter's signal (called the *d-metric*). We develop a tool to estimate a final *d-metric* when the information from several sensors is fused through the linguistic Choquet fuzzy integral. We utilize this tool in the examination of the performance of *d-metrics* in a simulation environment. The approach is demonstrated for data obtained from an Advanced Technology Demonstration in vehicle-based mine detection.

Keywords: Predictive sensor fusion, Land mine detection, *d-metric*, Fuzzy set theory, Choquet fuzzy integral

1. INTRODUCTION

Sensor fusion is a promising step in increasing the performance in recognition problems. However, a new sensor might produce evidence that is redundant or even conflicting with the existing sensors or it might collect evidence that is truly complementary to the rest and hence, will enhance the combined result. How can we tell if a new information source will provide a benefit or not? A related question is given a particular sensor suite, what characteristics are needed for a new sensor to provide a measurable increase in performance? The bottom line: Can we produce a predictive theory of sensor fusion?

We have explored the use of fuzzy set to build algorithms to create confidence values for a single sensor in several related papers [8-13, 24]. We built algorithms to fuse confidence values or feature values from several algorithms to improve recognition rate as well. But, sometimes the confidence values themselves are uncertain due to several factors, and can only be represented by fuzzy numbers. Nelson et al. [24] and Gader et al. [13] started to use fuzzy logic rule-based systems to deal with this problem. In [4] we described the linguistic Choquet fuzzy integral (LCFI) decision-level fusion of both position and confidence values from 3 detection algorithms operating on 16 passes of the 10 mine lanes at the Aberdeen Proving Ground. An analysis of Receiver Operating Characteristic (ROC) curves showed that the LCFI fusion could produce 100% detection with a false alarm rate lower than all of the individual detectors, and that of 2 numeric fusion approaches.

While a complete theory of sensor fusion remains an open question, a question of predicting a value of a new sensor added arises. In [18], we developed fuzzy set-based tools to study the potential of sensor fusion in the landmine detection situation through position resolution. But, what if we want to predict sensor suite capability through the relationship between the probability of detection and the probability of false alarm? We choose a *d-metric*, i.e., an average ratio of target's signal to the clutter's signal, to represent that relationship. In this paper, we demonstrate a fuzzy set-based system to study the behavior of a final *d-metric* when the information from several sensors is fused. In particular, we look at a quantitative analysis of sensor-based system fusion through an estimated *d-metric*. We model the probability of detection as a fuzzy number and adapt the linguistic version of the Choquet fuzzy integral [14] to estimate the probabilities of detection and false alarm. Then we compute a final *d-metric* in a simulation environment, but with global summary data obtained from an Advanced Technology Demonstration [25] in vehicle-based mine detection. We study the results of a new sensor added by performing this approach for several trials using theoretical knowledge of the distribution of the existing sensor responses. The characteristics of a new sensor are also varied so that the effects can be examined.

2. BACKGROUND

In order to understand how the prediction system works, we need to define the *d-metric*. We also need to discuss the linguistic version of Choquet fuzzy integral which is the basis of our system.

2.1. *d-metric*

In this paper, an analysis of the sensor fusion is done through the *d-metric*. This *d-metric* was defined in [25] as an average ratio of a target's signal to the clutter's signal, and can be derived from the probability of detection (P_d) and probability of false alarm (P_{fa}). Assuming P_d and P_{fa} are known, the *d-metric* is:

$$d = \sqrt{2}\left(\operatorname{erfinv}\left(1 - 2P_{fa}\right) - \operatorname{erfinv}\left(1 - 2P_d\right)\right) \tag{1}$$

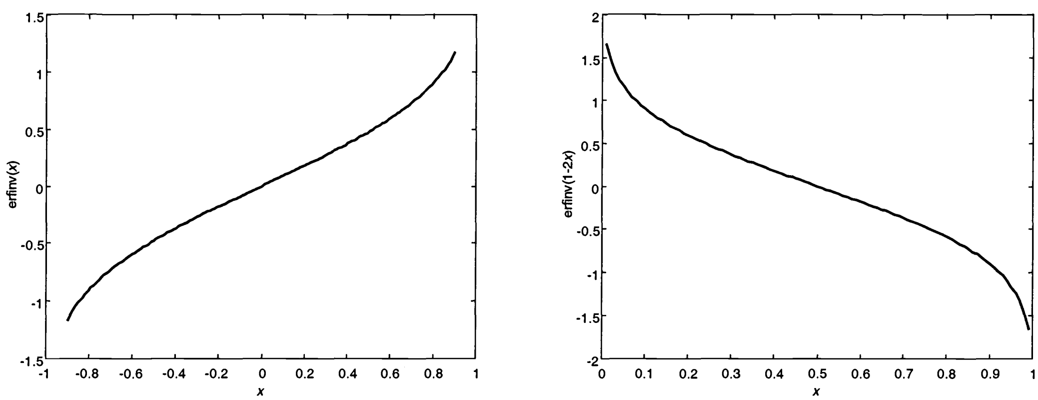
where erfinv(x) is the inverse error function of x and $-1<x<1$. Alternately, if P_d and the *d-metric* value are known, P_{fa} is given by:

$$P_{fa} = \frac{1 - \operatorname{erf}\left(\dfrac{d}{\sqrt{2}} + \operatorname{erfinv}\left(1 - 2P_d\right)\right)}{2}, \tag{2}$$

where the erf(x) is the error function of x,

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt\,. \tag{3}$$

Figure 1(a) shows the inverse error function of x where x = [-0.9, 0.9] and figure 1(b) shows the inverse error function of 1-2x for x = [0, 1].



(a) (b)

Figure 1. (a) Inverse error function of x, (b) Inverse error function of 1-2x

Now, we provide an intuitive understanding of equation (1). Suppose P_d is much greater than P_{fa} then erfinv($1-2P_d$) is much lower than erfinv($1-2P_{fa}$), and hence d will be large. On the other hand, if the values of P_d and P_{fa} are close, then the difference between erfinv($1-2P_{fa}$) and erfinv($1-2P_d$) is small, and so *d-metric* value will be small. Thus, we can view *d-metric* as a separation measure between P_d and P_{fa}. The higher the *d-metric* is, the better the performance of the system.

2.2. Linguistic Choquet Fuzzy Integral (LCFI)

Similar to our fusion of numeric confidence values with the fuzzy integral [27], we can combine fuzzy numbers using a linguistic Choquet fuzzy integral (LCFI). This LCFI has been introduced by Grabisch et al. [14]. The LCFI is based on the concepts of fuzzy algebra. Before we discuss LCFI, we first provide the background theory of fuzzy algebra. Then we review the regular Choquet fuzzy integral.

A fuzzy number $\boldsymbol{A}$ is a normal convex fuzzy set defined on the real line, R [19]. The support of $\boldsymbol{A}$ is bounded in R. Suppose that $\boldsymbol{A}$ and $\boldsymbol{B}$ are two fuzzy numbers. Let the symbol ⊗ denote any of the algebraic operations +, –, * or /. According to the extension principle [28], the algebraic operation maps fuzzy sets in R × R to R producing another fuzzy number $\boldsymbol{Z}$. The result membership function is given by:

$$\mu_Z(y) = \sup_{x_1 \otimes x_2 = y} \min(\mu_A(x_1), \mu_B(x_2)) \tag{4}$$

where x_1 and x_2 satisfy the mapping constraint. If we discretize the continuous support into finite number of points and approximate the fuzzy set on the discrete domain, the resulting fuzzy set is often irregular and inaccurate compared with the exact result. Therefore, the operation is performed using interval arithmetic[5, 6, 22, 23] and the decomposition theorem[19]. Klir and Yuan [19] proved that for a fuzzy set ($\boldsymbol{A} \otimes \boldsymbol{B}$), its α-cut ($[\boldsymbol{A}\otimes\boldsymbol{B}]_\alpha$) equals $[\boldsymbol{A}]_\alpha \otimes [\boldsymbol{B}]_\alpha$ for all $\alpha \in (0, 1]$. Then, by the decomposition theorem,

$$\begin{aligned} \boldsymbol{A} \otimes \boldsymbol{B} &= \bigcup_{\alpha \in [0,1]} [\boldsymbol{A} \otimes \boldsymbol{B}]_\alpha \\ &= \bigcup_{\alpha \in [0,1]} [\boldsymbol{A}]_\alpha \otimes [\boldsymbol{B}]_\alpha . \end{aligned} \tag{5}$$

Since, for any of these operations, $[\boldsymbol{A}\otimes\boldsymbol{B}]_\alpha$ is a closed interval for each $\alpha \in (0, 1]$ and $\boldsymbol{A}$, $\boldsymbol{B}$ are fuzzy numbers, $\boldsymbol{A} \otimes \boldsymbol{B}$ is also fuzzy number.

Let $\mathbf{X} = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$ be a finite set of information sources and let h: $\mathbf{X} \rightarrow [0, 1]$ be a confidence function, i.e., $h(\mathbf{x}_i)$ is the confidence number provided by the feature $\mathbf{x}_i$ that an input sample is from a particular class. The Choquet fuzzy integral of h with respect to a fuzzy measure g (a non-decreasing set function) is

$$\int_c h \circ g = \sum_{i=1}^{n} g(\mathbf{X}_i)[h(\mathbf{x}_i) - h(\mathbf{x}_{i+1})] \text{ or } \int_c h \circ g = \sum_{i=1}^{n} h(\mathbf{x}_i)[g(\mathbf{X}_i) - g(\mathbf{X}_{i-1})] \tag{6}$$

where $h(\mathbf{x}_{n+1}) = 0$, $g(\mathbf{X}_0) = 0$, $h(\mathbf{x}_1) \geq h(\mathbf{x}_2) \geq \ldots \geq h(\mathbf{x}_n)$ and $\mathbf{X}_i = \{\mathbf{x}_1, \ldots, \mathbf{x}_i\}$.

The extended version of fuzzy integral assumes that $h(\mathbf{x}_i)$ is not a real value in [0, 1] but is instead a fuzzy number whose support is bounded. The measure is still defined in the standard sense. We use α-cuts to compute the fuzzy integral in this case. Apostolatos [1] showed that if a function φ is continuous and non-decreasing then when defined on intervals $\mathbf{x} \equiv [\underline{x}, \overline{x}] := \{\tilde{x} \in \mathrm{R} | \underline{x} \leq \tilde{x} \leq \overline{x}\}$, it is equal to function on the lower bound and upper bound of the intervals, i.e., $\varphi(\mathbf{x}) = [\varphi(\underline{x}), \varphi(\overline{x})]$. Grabisch et al. [14] introduced the extended version of the Choquet fuzzy integral as follows:

Let $\boldsymbol{h}$: $\mathbf{X} \rightarrow \Im([0, 1])$ where $\Im([0, 1])$ is the fuzzy power set of [0, 1]. Suppose $\boldsymbol{h}_1, \ldots, \boldsymbol{h}_n$ are confidence fuzzy numbers with respect to $\mathbf{x}_1, \ldots, \mathbf{x}_n$ respectively and let $[\boldsymbol{h}_i]_\alpha = \left[\underline{[\boldsymbol{h}_i]_\alpha}, \overline{[\boldsymbol{h}_i]_\alpha}\right]$ for $1 \leq i \leq n$ and $0 \leq \alpha \leq 1$. The LCFI with respect to g is defined as

$$\left[\int_c \boldsymbol{h} \circ g\right]_\alpha = \left[\int_c \underline{[\boldsymbol{h}]_\alpha} \circ g, \int_c \overline{[\boldsymbol{h}]_\alpha} \circ g\right] \quad \text{for } 0 \leq \alpha \leq 1 \tag{7}$$

In our experiments, we use λ-fuzzy measures [26] which are completely determined by the measures of the singleton subsets (called the fuzzy densities).

3. PREDICTION SYSTEM

Now, we are ready to build a system that can estimate the *d-metric* value of a sensor fusion system. This estimate is calculated by assuming that any fusion algorithm will try to increase P_d while lowering P_{fa} as much as possible. Therefore, we try to first estimate P_d and P_{fa} from fusion system and then compute the *d-metric* value. This estimation system is shown in figure 2. It consists of four parts. First, we compute P_{fa} of each sensor from a given *d-metric* and P_d. Then we fuzzify each P_d, and model each P_{fa} as a singleton fuzzy number. Next, we estimate the probability of detection fuzzy number and the probability of false alarm singleton fuzzy number from the fusion using the LCFI. Finally, we compute the *d-metric* value.

Now, the variables that we have to consider are the fuzzy densities for the estimation of the probability of detection and probability of false alarm. Before we determine the fuzzy densities, we must know that the uncertainty in the probability of detection is determined by assuming that the distribution of the probability of detection is a binomial distribution [25]. A 90-

percent confidence interval (CI_{90}) [15] is computed to determine this uncertainty based on the total number of samples and the number of detections.

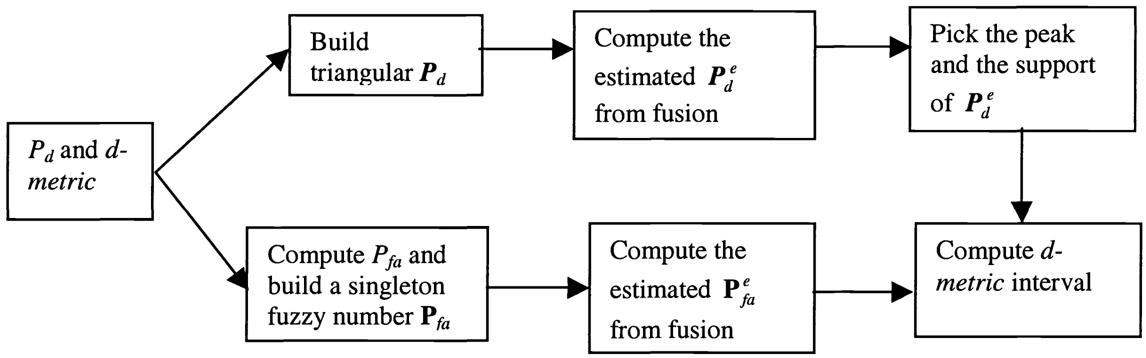


Figure 2. *d-metric* estimation system.

A CI_{90} of an unknown parameter provides limits that one can claim, with a 90-percent confidence, contain the true value of that parameter. We fuzzify the probability of detection of each sensor j as a triangular fuzzy number ($\boldsymbol{P}_{dj}$), shown in figure 3, with the core equals [P_{dj}, P_{dj}]. The lower bound of the support equals that of the CI_{90} and the upper bound of the support equals the upper limit of the CI_{90}.

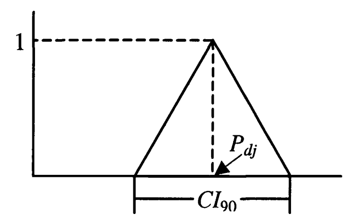


Figure 3. Fuzzified probability of detection for a particular sensor j

In order to compute fuzzy densities in the estimation of probability of detection, we want two properties to hold:

1. The more uncertain a fuzzy number is, the less important it is, and
2. The further away the probability of detection is from the highest value, the less important it is.

For the probability of detection for each sensor j ($\boldsymbol{P}_{dj}$), we compute the fuzzy densities as a product of two real numbers i.e., $g_{pd}^j = g_u^j \times g_d^j$. Since the CI_{90} is used to fuzzify the probability of detection and $\boldsymbol{P}_{dj}$ is more uncertain if the interval is wider, the uncertainty in $\boldsymbol{P}_{dj}$ is modeled by:

$$g_u^j = 1 - \left|\sup\left(\boldsymbol{P}_{dj}\right)\right|. \tag{8}$$

We will now discuss the second property. There are n input fuzzy numbers $\{\boldsymbol{A}_1, \boldsymbol{A}_2, \cdots, \boldsymbol{A}_n\}$ (one for each sensor or algorithm). The highest probability of detection is a singleton fuzzy number **1** whose support is [1, 1]. The distance between a fuzzy number $\boldsymbol{A}_j$ and **1** is defined with respect to α-cut level sets. Let us assume that we have nested α-cut level sets $\left\{[\boldsymbol{A}_j]_{\alpha_1} \subseteq [\boldsymbol{A}_j]_{\alpha_2} \subseteq \cdots \subseteq [\boldsymbol{A}_j]_{\alpha_p}\right\}$. The distance between level set $[\boldsymbol{A}_j]_{\alpha_i}$ and $[\mathbf{1}]_{\alpha_i}$ is [23]:

$$d_{\boldsymbol{A}_j}(\alpha_i) = \max\left(\left|\underline{[\boldsymbol{A}_j]_{\alpha_i}} - 1\right|, \left|\overline{[\boldsymbol{A}_j]_{\alpha_i}} - 1\right|\right) \tag{9}$$

Given a fuzzy set $\boldsymbol{A}_j$, Krishnapuram et al. [20] define a generalized expected value of a property $P(\boldsymbol{A}_j)$ from its level sets by

$$\overline{P}(\boldsymbol{A}_j) = \sum_{i=1}^{p} m\left([\boldsymbol{A}_j]_{\alpha_i}\right) P\left([\boldsymbol{A}_j]_{\alpha_i}\right) = \sum_{i=1}^{p} (\alpha_i - \alpha_{i+1}) P\left([\boldsymbol{A}_j]_{\alpha_i}\right) \tag{10}$$

where $\alpha_1 = 1$ and $\alpha_{p+1} = 0$. The basic probability assignment $m\left([A_j]_{\alpha_i}\right)$ associated with $[A_j]_{\alpha_i}$ can be translated to be the probability mass associated with the statement "$[A_j]_{\alpha_i}$ is the true representation of A_j [7]." In our situation, $P\left([A_j]_{\alpha_i}\right)$ is inversely proportion to the distance between level sets $[A_j]_{\alpha_i}$ and $[\mathbf{1}]_{\alpha_i}$, i.e.,

$$P\left([A_j]_{\alpha_i}\right) = 1 - \frac{d_{A_j}(\alpha_i)}{\sum_k d_{A_k}(\alpha_i)}. \tag{11}$$

The fuzzy densities of the j[th] fuzzy number with respect to $\mathbf{x}_j$ relating to distance from the highest probability of detection are then:

$$g_d^j = \sum_{i=1}^{p} (\alpha_i - \alpha_{i+1}) P\left([A_j]_{\alpha_i}\right). \tag{12}$$

Thus, the fuzzy densities for each sensor *j,* in the sense of the probability of detection, are computed as

$$g_{pd}^j = g_u^j \times g_d^j. \tag{13}$$

These fuzzy densities fulfill both properties for computing the estimated probability of detection ($\boldsymbol{P}_d^e$).

Now, for the estimation of the probability of false alarm, we first model each probability of false alarm *j* as a singleton fuzzy number $\mathbf{P}_{faj}$ because we do not have sensor's accuracy information and we cannot use 90-percent confidence interval. Also, the error in the probability of false alarm is considered small according to [25]. In this estimation, the property needed for computing the fuzzy densities is only "the closer $\mathbf{P}_{faj}$ is to $\mathbf{0}$, the more important it is". For the probability of false alarm of each sensor *j* ($\mathbf{P}_{faj}$), fuzzy densities that satisfy this property are:

$$g_{pf}^j = 1 - \frac{[\mathbf{P}_{faj}]_{\alpha=1}}{\sum_j [\mathbf{P}_{faj}]_{\alpha=1}}. \tag{14}$$

These densities fulfill the property for computing the estimated probability of false alarm ($\mathbf{P}_{fa}^e$).

We then use $\boldsymbol{P}_d^e$ and $\mathbf{P}_{fa}^e$ to compute *d-metric* value. We pick [lower bound of the support ($\underline{P_d^e}$), midpoint of the core (P_d^e), upper bound of the support ($\overline{P_d^e}$)], called Pd-interval, to represent $\boldsymbol{P}_d^e$. Since computing the LCFI on the probability of false alarm singleton fuzzy numbers produces a singleton fuzzy number, we select the midpoint of the core to represent $\mathbf{P}_{fa}^e$, called Pf-final. We then compute the estimated *d-metric* interval [lower bound of *d-metric* ($\underline{d^e}$), middle value of *d-metric* (d^e), upper bound of *d-metric* ($\overline{d^e}$)], from Pd-interval and Pf-final, i.e., $\underline{d^e}$ is computed from $\underline{P_d^e}$ and Pf-final, d^e is calculated from P_d^e and Pf-final, and $\overline{d^e}$ is from $\overline{P_d^e}$ and Pf-final.

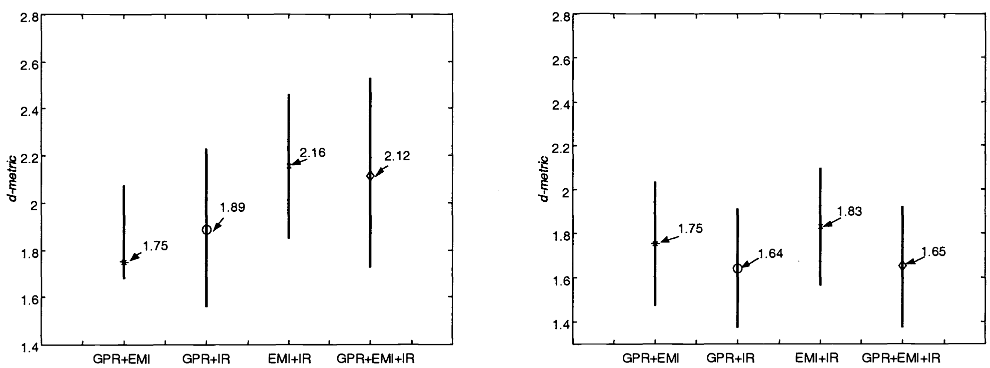
The summary of the *d*-metric estimation system is as follows:

Compute P_{fa} for each sensor from given P_d and *d-metric*
Fuzzify P_d using CI_{90} and model P_{fa} as a singleton fuzzy number
For each sensor
    Compute fuzzy densities for $\boldsymbol{P}_d$ using equations (8), (12), and (13)
    Compute fuzzy densities for $\mathbf{P}_{fa}$ using equation (14)
End For
Compute $\boldsymbol{P}_d^e$ using the LCFI
Compute $\mathbf{P}_{fa}^e$ using the LCFI
Compute *d-metric* interval

4. EXPERIMENTS

We first report on an example of *d-metric* prediction capability on data from 1 contractor on different sensors. We then study the prediction results on the addition of a new sensor to a given suite by performing this prediction for several trials using the knowledge of the probability of detection and *d-metric* of the existing sensor responses. The characteristics of a new sensor are varied so that the effects can be examined. All the data is extracted from summary results reported in [25] on the probability of detection and the *d-metric*.

The following is an example for predicting the *d-metric* interval for a "known" sensor. We use the data from [25] which analyzed performance of a single contractor using ground penetrating radar (GPR), electromagnetic inductance (EMI) sensor, and an infrared (IR) sensor. We predict *d-metric* intervals for the fusion of GPR and EMI, GPR and IR, EMI and IR, and GPR, EMI and IR. For each combination, we generate 2000 trials according to the given binomial distribution. We collect Pd-interval and Pf-final, and then we compute their means. From these means, the *d-metric* interval is computed. Figure 4 shows the *d-metric* intervals from the real fusion data (i.e., the contractor's fusion) and the prediction using linguistic fusion. The lower end and the upper end of the bar represent $\underline{d^e}$ and $\overline{d^e}$, and values in the figure are d^e.



(a) (b)

Figure 4. (a) The real *d-metric* interval from [25] report, (b) the predicted *d-metric* from our prediction system

Although we do not estimate the exact *d-metric* value from our fusion system, our prediction system foretells that adding the third sensor into the fusion does not increase the *d-metric* value. The result from [25] in this case also gives the same information. The fusion of EMI and IR produces the highest middle value of *d-metric* interval in both real report and prediction. Of course, the methods of fusion are different for figure 4(a) and figure 4(b). Hence, the fact that they produce the same trend gives us reason to pursue the simulation trials.

The following is a study where a hypothetical new sensor is added to existing sensors. The probability of detection, total number of mines, and *d-metric* of both GPR and EMI detector systems for two contractors (labeled sensor 1 and sensor 2, here) during the Advanced Technology Demonstrations at the Aberdeen are obtained from Rotondo's report [25]. For the GPR, we choose the on-road information, while for EMI, we picked that of the off-road information. These parameters can be found in each of the tables that follow.

We discuss 4 simulation experiments with these two sensors and an added hypothetical sensor that are typical of the many we ran. Table 1 shows the setup for the first simulation experiment. We set *d-metric* value of the hypothetical sensor 3 to 1.0 and vary its probability of detection (x) from 0.1 to 0.98. For each sensor, we generate 2000 trials according to the given binomial distributions. We collect the Pd-interval and Pf-final, and then compute their means. From these means, the *d-metric* interval is calculated. The plot of this experiment is shown in figure 5.

Table 1. GPR sensor system *d-metric* estimation for the first simulation

sensor	P_d(Total mines = 132)	*d-metric*
1	0.871	2.06
2	0.697	1.41
3	x	1.0

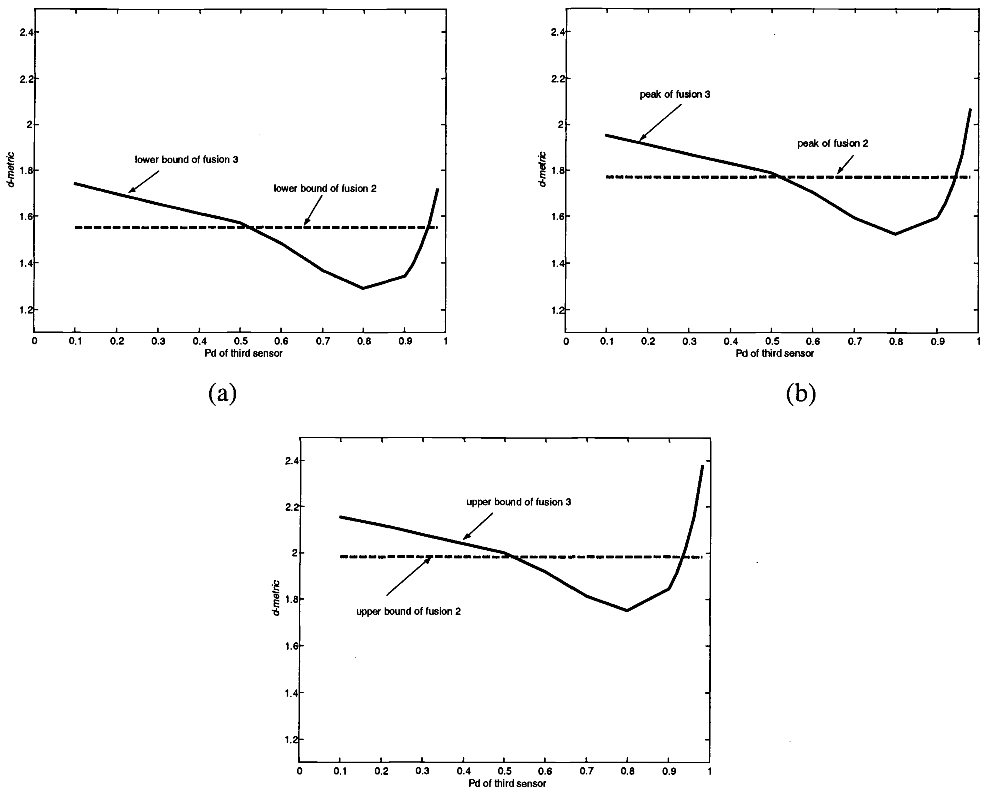


Figure 5. The results of fusion of 2 and 3 GPR sensor systems when the probability of detection of the third is varied (a) the lower bound of *d-metric* interval, (b) the middle value of *d-metric* interval, (c) the upper bound of *d-metric* interval

Like the simulation in the position error prediction, the values for the 2-sensor fusion are represented as horizontal lines representing the means of the 2000 trials (these means are independent of the existence of the third sensor). From figure 5, we see that the values in the estimated *d-metric* interval are decreasing until the probability of detection of the third sensor is around 0.8. It increases when the probability of detection of the third sensor is almost equal to or is greater than the maximum probability of detection of the other two sensors. This may seem unreasonable, but when we consider the estimated Pd-interval and Pf-final, we find that the Pd-interval is almost the same until the probability of detection of the third sensor is more than that of the others. This is caused by the fact that our estimation system gives low priority to the sensor that has low probability of detection, and hence it does not affect the estimated Pd-interval very much. However, when the probability of

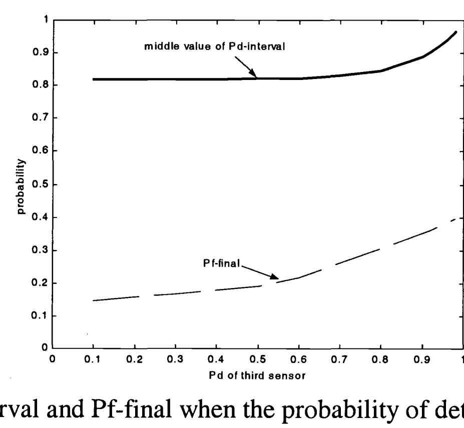


Figure 6. The middle value of Pd-interval and Pf-final when the probability of detection of the third sensor is varied for the first simulation

detection of the third sensor is low, its probability of false alarm is also low, and hence, our system gives high importance to this sensor when computing the Pf-final. The Pf-final is increasing all the time. Thus, the values in *d-metric* interval are decreasing until the values in Pd-interval increase. Figure 6 shows the middle value in Pd-interval and Pf-final for this experiment. From figure 6, it is clear that the middle value of Pd-interval is increasing when the probability of detection of the third sensor is above 0.8 but Pf-final is continually increasing. This explains why the values of *d-metric* interval decrease when the probability of detection of the third sensor is below 0.8.

Next, we set the probability of detection of the hypothetical sensor 3 to 0.7 and vary its *d-metric* value from 0.5 to 2.8. Table 2 shows the simulation input parameters, while the plots of this experiment are shown in figure 7. We see that the values of the *d-metric* interval are increasing with the increasing value of the *d-metric* of the third sensor. If we want a three-sensor suite producing a *d-metric* interval of [1.6, 1.84, 2.01] (with these two existing sensors), we need to find a third sensor that has the probability of detection around 0.7 and a *d-metric* value around 1.6. If the third sensor has a *d-metric* value = 2.2, from figure 7, we predict that the result *d-metric* interval will be [1.71, 1.93, 2.16].

Table 2. GPR sensor system *d-metric* estimation for the second simulation

Sensor	P_d(Total mines = 132)	*d-metric*
1	0.871	2.06
2	0.697	1.41
3	0.7	x

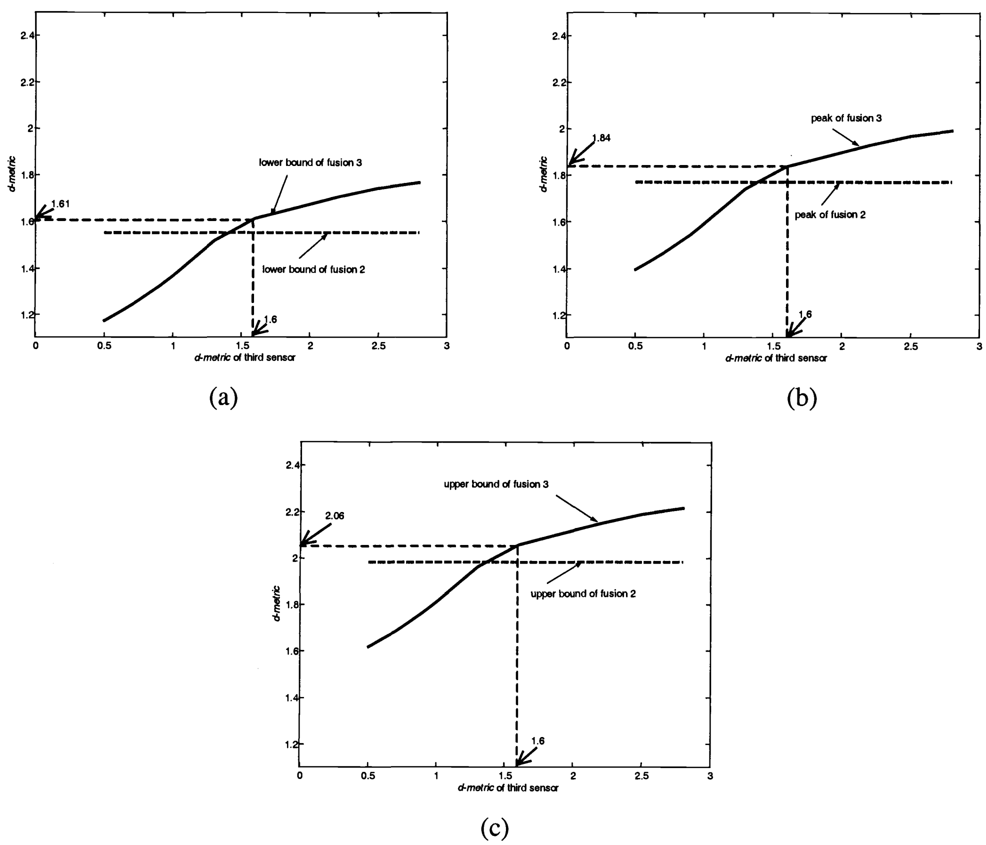
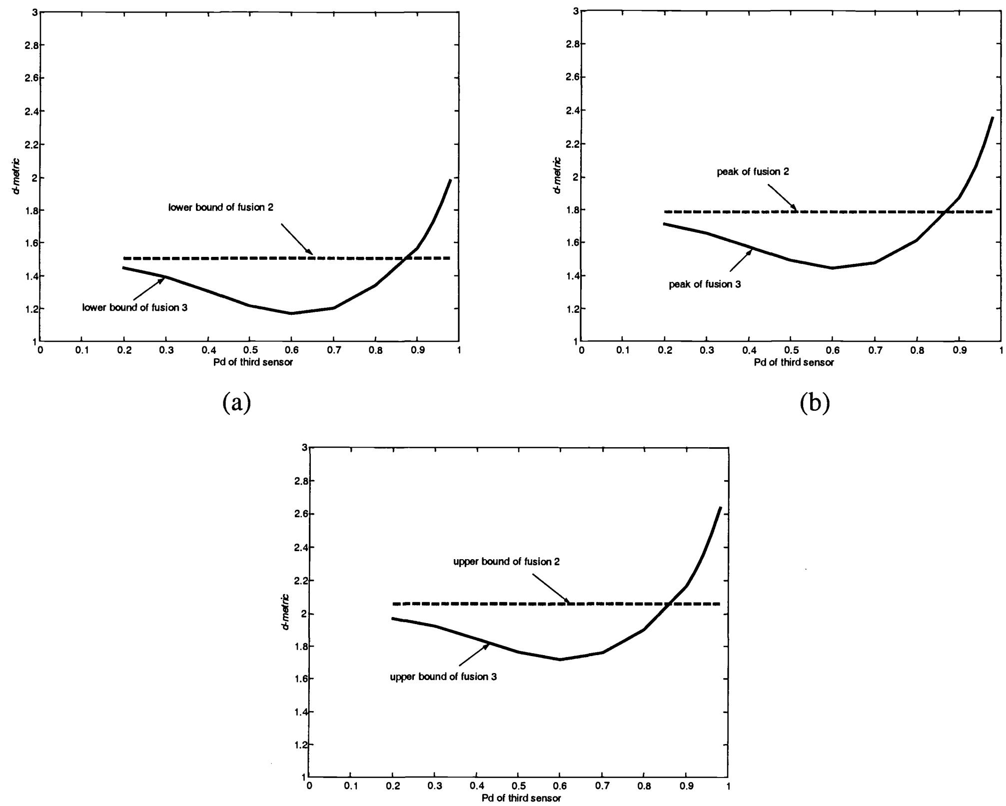


Figure 7. The results of fusion of 2 and 3 GPR sensor systems when the *d-metric* value of the third is varied (a) the lower bound of *d-metric* interval, (b) the middle value of *d-metric* interval, (c) the upper bound of *d-metric* interval

In a similar manner, we simulate the incorporation of an additional sensor to two EMI sensors. Like the GPR trials, we vary the probability of detection of sensor 3 from 0.2 to 0.98 and set the *d-metric* value to 1.0. Table 3 shows the inputs and figure 8 depicts the simulation results.

Table 3. EMI sensor system *d-metric* estimation for the third simulation

Sensor	P_d(Total mines = 68)	*d-metric*
1	0.647	1.76
2	0.588	1.80
3	x	1.0



(a) (b)

(c)

Figure 8. The results of fusion of 2 and 3 EMI sensor systems when the probability of detection of the third is varied (a) the lower bound of *d-metric* interval, (b) the middle value of *d-metric* interval, (c) the upper bound of *d-metric* interval

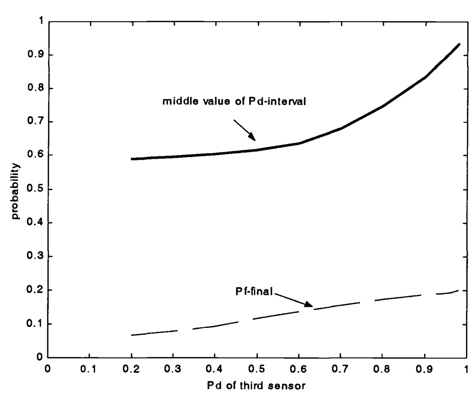


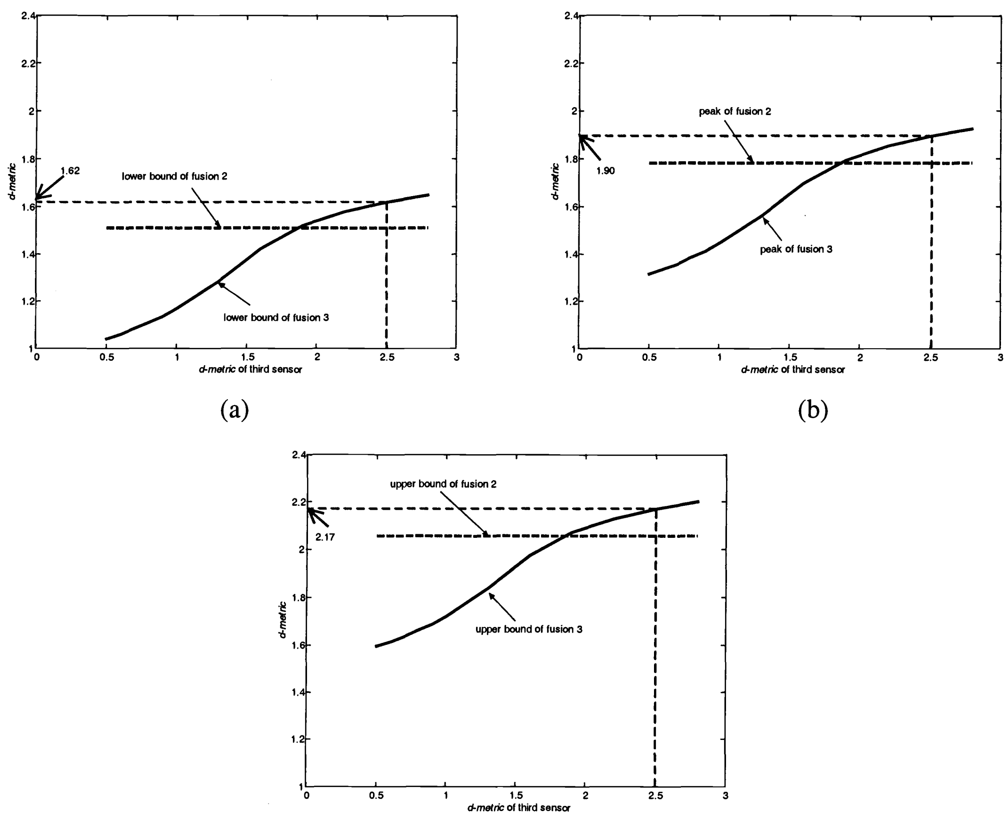
Figure 9. Pd-interval and Pf-final when the probability of detection of the third sensor is varied for the third simulation

Again, the results show that the values of *d-metric* interval increase when the probability of detection of the third sensor is almost equal to or is greater than that of the other two sensors for the same reason as mentioned earlier. To confirm that the Pd-interval is almost the same up to one point but the Pf-final is increasing all the time, we plot the middle values of Pd-interval and Pf-final in figure 9. From this figure, we see that the middle value of Pd-interval increases rapidly when the probability of detection of the third sensor is above 0.6. This confirms the same phenomenon seen in the GPR system simulation. Here, Pf-final increases slowly when the probability of detection of the third sensor is above 0.7,due to the fact that, when the probability of the third sensor is more that 0.7, its probability of false alarm is more than 0.32. However, the probability of false alarms of sensor 1 and 2 are around 0.08 and 0.06, respectively. The prediction system gives low significance to the third sensor when it estimates Pf-final.

Finally, we set the probability of detection of sensor 3 to 0.6 and vary its *d-metric* value from 0.5 to 2.8. Table 4 depicts the input parameters, whereas figure 10 shows the simulation results. Again, the resulting values of *d-metric* interval are increasing when the *d-metric* value of the third sensor is increasing. If we want a three-sensor suite producing a *d-metric* interval of [1.62, 1.90, 2.17] (with these two existing sensors), a third sensor must have a *d-metric* value around 2.5 with the probability of detection = 0.6.

Table 4. EMI Sensor System *d-metric* estimation for the fourth simulation

sensor	P_d(Total mines = 68)	*d-metric*
1	0.647	1.76
2	0.588	1.80
3	0.6	x



(c)

Figure 10. The results of fusion of 2 and 3 EMI sensor systems when the *d-metric* value of the third is varied (a) the lower bound of *d-metric* interval, (b) the middle value of *d-metric* interval, (c) the upper bound of *d-metric* interval.

5. CONCLUSIONS

A critical component in the success of landmine detection programs is a fusion of sensor information. However, there are many forms of combination paradigms around. In this paper, we considered an approach to predictive sensor fusion based on the *d-metric*, i.e., a ratio of a target's signal to the clutter's signal. We described a system to estimate the *d-metric* by first estimating the probability of detection with fuzzy number and the probability of false alarm as a singleton fuzzy number, and then utilizing a linguistic extension of the Choquet fuzzy integral. Initial results with new linguistic tools [2-4, 17, 18] are encouraging, giving us reason to postulate this paradigm to study predictive sensor fusion. With summary probability of detection and *d-metric* information from the ATD tests, we first estimated the *d-metric* on known systems and then studied the effect of adding an additional sensor to the mix.

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