# CS 4124
# Solutions to Homework Assignment 1
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**[25] 1. Textbook Problem 5 in B.1 on Page 520.**

**Prove Lemma A.4: For all alphabets $\Sigma$ and all languages $L \subseteq \Sigma^\star$, the equivalence relation $\equiv_L$ is right-invariant.**

**The lemma and the definition of the equivalence relation are on Page 505.**

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*Proof.* Let $\Sigma$ be an alphabet and $L \subseteq \Sigma^*$ and $\equiv_L$ be an equivalence relation as defined by (A.3). Let $x, y, z, w \in \Sigma^*$ and $x \equiv_L y$ then we have $xz, yz \in L$ as $zw \in \Sigma^*$ we have $xzw, yzw \in L$ which implies $xz \equiv_L yz$. □

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**[25] 2. Textbook Problem 3(c) in B.2 on Page 522.**

**Design an FA $M$, with alphabet $\Sigma = \{0, 1\}$, that recognizes (c) the set of all strings that contain the string $011$, in that order, but not necessarily consecutively.**

**Be certain you understand the set (language) of $M$ before you start designing. Constructing some examples of strings both in $L(M)$ and not in $L(M)$ can be helpful.**

**You may use an algebraic specification or a transition diagram (labeled directed graph) specification to present your design for $M$. Be certain to explain why your design works.**

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**[25] 3. Consider the OA $M_4$ in Figure 3.4 on Page 60. Give a complete and careful algebraic specification**

$$M_4 \quad = \quad (Q, \Sigma, \delta, q_0, F)$$

**for $M_4$.**

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**[25] 4. Textbook Problem 7(c) in B.2 on Pages 522 and 523.**

**Use a (fooling set)-plus-(Continuation Lemma) argument to prove that the following language is *not* Regular:**

$$L_5 \quad = \quad \{a^i b^j c^k \mid i = j \text{ OR } j = k\}.$$

**Your argument will be a proof by contradiction.**

**The Continuation Lemma is Lemma 3.2 on Page 57, while the fooling set kind of argument is first utilized on Pages 76 and 77.**

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