

**CS 4124**  
**Solutions to Homework Assignment 2**  
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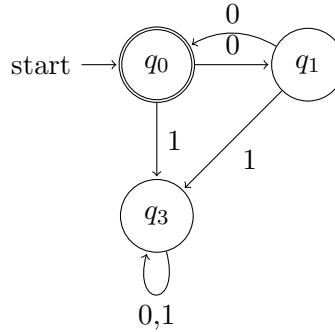
February 16, 2024

[50] 1. For each of the languages  $L_1$  and  $L_2$  below, decide whether the language is Regular or not Regular. If Regular, give an FA that recognizes the language. If not Regular, use a fooling set argument to demonstrate that there is no FA that recognizes the language.

$$\begin{aligned} L_1 &= \{0^i 0^j \mid i, j \geq 0 \text{ and } i = j\} \\ L_2 &= \{0^i 1^j 0^k \mid i, j, k \geq 0 \text{ and } i = k\} \end{aligned}$$

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The language  $L_1$  is regular as it is recognized by the FA.



For the language  $L_1 = \{0^i 0^j : i, j \geq 0 \text{ and } i = j\}$  we can simplify the set to just be  $L_1 = \{0^i 0^i : i \geq 0\} = \{0^{2i} : i \geq 0\}$ . Therefore we just have to distinguish between an odd and even number of 0's and go to a state which never accepts if it receives any 1's which this FA does.

$L_2$  is not regular.

*Proof.* We claim that  $F = \{0^i : i \geq 0\}$  is a fooling set for  $L_2$ . To see this consider the two arbitrary distinct words  $0^j, 0^n \in F$ , we have for  $m \geq 1$  that  $0^j 1^m 0^j \in L_2$  but  $0^n 1^m 0^j \notin L_2$  as  $n \neq j$  this implies  $0^j \not\equiv_{L_2} 0^n$ . As  $F$  is an infinite subset of  $\{0, 1\}^*$  and every distinct element is not in the same congruence class of  $\equiv_{L_2}$  then this implies that  $\equiv_{L_2}$  has infinite index by the Myhill-Nerode Theorem this implies  $L_2$  is not regular.  $\square$

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[50] 2. Textbook Problem 20, on Page 525, parts (a) and (b).

**Prove or disprove:**

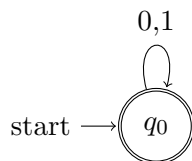
(a) Every subset of a Regular language is Regular.

(b) There exists a non-Regular language  $L$  such that  $L^*$  is Regular.

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*Proof.* (a) this is false

Consider the language  $L_2$  from problem 1 we have that  $L_2$  is not regular. However consider the language  $L'_2 = \{w \in \{0,1\}^*\}$  we have that  $L'_2$  is regular as it is recognized by the FA.



We have that  $L_2 \subseteq L'_2$  but  $L_2$  is not regular. Therefore we have a counter example to the claim.  $\square$

*Proof.* (b) this is true

We have the language  $L = \{a^{n^2} : n \in \mathbb{N}\}$  from page 90 is not regular. However  $L^* = \{\epsilon, a, aa, aaa, \dots\} = \{a\}^*$  this follows as 1 is a perfect square so we can have any finite length of  $a$ 's in  $L^*$ . Then by the definition of Kleene closure we have  $L \subset L^*$ . But as  $L^*$  is regular and  $L$  is not we have shown the existence of the claim which completes the proof.  $\square$

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