CS 4124
Solutions to Homework Assignment 2

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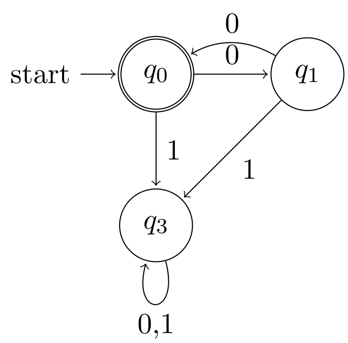
[50] 1. For each of the languages L_1 and L_2 below, decide whether the language is Regular or not Regular. If Regular, give an FA that recognizes the language. If not Regular, use a fooling set argument to demonstrate that there is no FA that recognizes the language.

$$\begin{aligned} L_1 &= \{0^i 0^j \mid i, j \geq 0 \text{ and } i = j\} \\ L_2 &= \{0^i 1^j 0^k \mid i, j, k \geq 0 \text{ and } i = k\} \end{aligned}$$

The language L_1 is regular as it is recognized by the FA.



For the language $L_1 = \{0^i 0^j : i, j \geq 0 \text{ and } i = j\}$ we can simplify the set to just be $L_1 = \{0^i 0^i : i \geq 0\} = \{0^{2i} : i \geq 0\}$. Therefore we just have to distinguish between an odd and even number of 0's and go to a state which never accepts if it receives any 1's which this FA does.

L_2 is not regular.

Proof. We claim that $F = \{0^i : i \geq 0\}$ is a fooling set for L_2. To see this consider the two arbitrary distinct words $0^j, 0^n \in F$, we have for $m \geq 0$ that $0^j 1^m 0^j \in L_2$ but $0^n 1^m 1^j \notin L_2$ as $n \neq j$ this implies $0^j \not\equiv_{L_2} 0^n$. As F is an infinite subset and every distinct element is not in the same congruence class of $\equiv_{L_2}$ then this implies that $\equiv_{L_2}$ has infinite index. By the Myhill-Nerode Theorem this implies L_2 is not regular. $\square$

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[50] 2. Textbook Problem 20, on Page 525, parts (a) and (b).

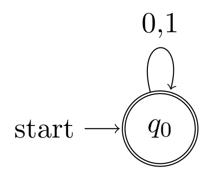
Prove or disprove:

(a) Every subset of a Regular language is Regular.

(b) There exists a non-Regular language L such that L^* is Regular.

Proof. (a) this is false

Consider the language L_2 from problem 1 we have that L_2 is not regular. However consider the language $L_2' = \{w \in \{0,1\}^*\}$ we have that L_2' is regular as it is recognized by the FA.



We have that $L_2 \subseteq L_2'$ but L_2 is not regular. Therefore we have a counter example to the claim. $\square$

Proof. (b) this is true

We have the language $L = \{a^{n^2} : n \in \mathbb{N}\}$ from page 90 is not regular. However $L* = \{\epsilon, a, aa, aaa, ...\} = \{a\}^*$ this follows as 1 is a perfect square so we can have any finite length of a's in L^*. Then by the definition of Kleene closure we have $L(M_5) \subset L(M_5)^*$. But as L^* is regular and L is not we have shown the existence of the claim which completes the proof. $\square$
