Math 4324 Test 2 Cover Page April 12, 2024 Haskell

This page is the cover page for the second test. The rest of the file is the test. Once you look beyond the cover page of the file, you have no more than 2 hours to finish the test. That time limit is 2 consecutive hours starting when you look beyond the cover page. Stretching 2 hours of work, interspersed with other activity, over a time period lasting longer than 2 hours is not permitted. During the two hours you are working on the test, you may not consult any person or other source. You must email your completed test to me no later than 5:00 pm on Friday, April 12. There will be no class on Friday, April 12.

Unless you're unusually good at typing and TeXing, I suggest that you handwrite your answers. You may use time beyond the 2 hours to put your answers into a TeX file, but don't change your answers during that transcription. A pdf file made from a TeX file is easiest for me to read, but I will also accept a scanned pdf file of your handwritten answers or, if necessary, a photo of your handwritten answers.

It is an honor issue that anyone who has seen the test must not risk communicating any information about the test to anyone who has not finished the test. This covers conversations about topology (including conversations that are overheard by others not in the conversation), leaving scratchwork where it can be seen by others, etc. If you have any questions about the honor expectations, please ask me before engaging in any questionable behavior.

People who have not started the test may engage in all the usual preparations for the test, including consulting books, notes, and other sources, and discussing the material with classmates, as long as all such discussions are not overheard by anyone currently taking the test.

The test will cover material related to that on assignments due from March 1 through April 5, inclusive, but dependence on topics represented on earlier assignments is unavoidable.

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Show all reasoning. Prove your assertions. If you use a counterexample in your answer, explain why it has the properties that make it a counterexample. Use no books, notes, or other sources. If a problem appears to repeat a statement from the book, from class, or from the homework, then you are being asked to repeat or develop an argument that proves the statement.

(10 pts) 1. Suppose that X is a compact topological space and $f : X \to \mathbb{R}$ is a function, not necessarily continuous, with the following property: for each $x \in X$, there is an open neighborhood U_x of x and a constant M_x such that, for all $z \in U_x$, $|f(z)| \leq M_x$. Show that there is a constant M such that, for all $y \in X$, $|f(y)| \leq M$.

(10 pts) 2. Suppose that X is a topological space, that C is a connected subset of X and that, for each α in some index set A, C_α is a connected subset of X. Suppose also that, for every $\alpha \in A$, $C_\alpha \bigcap C \neq \emptyset$. Show that $C \bigcup (\bigcup_{\alpha \in A} C_\alpha)$ is connected.

(10 pts) 3. Give $\mathbb{R}$ its standard topology and give its subsets their subspace topologies.

a. Let A be $\mathbb{R}$'s topological subspace $\{1/n : n \in \mathbb{N}\}$. Describe as explicitly as possible which subsets of A are closed subsets of A.

b. Let B be $\mathbb{R}$'s topological subspace $\{0\} \bigcup \{1/n : n \in \mathbb{N}\}$. Describe as explicitly as possible which infinite subsets of B are closed subsets of B.

(10 pts) 4. Give the natural numbers $\mathbb{N}$ the topology in which the open sets are: the empty set; all of $\mathbb{N}$; and, for each $k \in \mathbb{N}$, the set $U_k = \{n \in \mathbb{N} : n \leq k\}$. (You do **not** need to check that this is a topology.) Using this topology on both the domain and range space of $f : \mathbb{N} \to \mathbb{N}$, show that f is continuous if and only if, for all $m, n \in \mathbb{N}$ with $m < n$, $f(m) \leq f(n)$.

(10 pts) 5. Let X be a first countable topological space, let A be a subset of X, and let c be in A's closure. Show that there exists a sequence (a_n) of elements of A that converges to c.