Problem 1. Recall the following useful technique for computing the determinant of a matrix.

Theorem. (Cofactor Expansion, Laplace). Let A be an $n \times n$ matrix and let $M_{i,j}$ denote the $(n-1) \times (n-1)$ submatrix obtained by deleting Row i and Column j from A. The determinant of an $n \times n$ matrix A can be computed *along the i^{th} row* as the sum

$$\det A = \sum_{\text{col. } j} (-1)^{i+j} A_{i,j} \det(M_{i,j})$$

or *along the j^{th} column* as the sum

$$\det A = \sum_{\text{row. } i} (-1)^{i+j} A_{i,j} \det(M_{i,j})$$

Let $T \in \mathcal{L}(\mathbb{C}^4)$ be an operator with matrix (in the standard basis) given by

$$A = \begin{pmatrix} 1 & -1 & 1 & -2 \\ 0 & 0 & 0 & -1 \\ -1 & 1 & -1 & 2 \\ 0 & 1 & 0 & 2 \end{pmatrix}.$$

1. Find the characteristic polynomial for A.
2. Find a eigenvalues for A.
3. Find basis $\mathcal{G}$ for $\mathbb{C}^4$ so that $\mathcal{M}(T, \mathcal{G})$ is upper triangular with eigenvalues along the diagonal.
4. Is A diagonalizable? Why or why not?

Problem 2. Let $T \in \mathcal{L}(\mathbb{R}^4)$ be an operator whose matrix (in the standard basis) is given by

$$\begin{pmatrix} -2 & 1 & 0 & 3 \\ -2 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Find a basis $\mathcal{B}$ for $\mathbb{R}^4$ so that $\mathcal{M}(T, \mathcal{B})$ is block-diagonal. That is,

$$\mathcal{M}(T, \mathcal{B}) = \begin{pmatrix} * & * & 0 & 0 \\ * & * & 0 & 0 \\ 0 & 0 & * & * \\ 0 & 0 & * & * \end{pmatrix}$$

HINT: Given $S \in \mathcal{L}(\mathbb{C}^2)$ with

$$\mathcal{M}(S, \{\mathbf{b_1}, \mathbf{b_2}\}) = \begin{pmatrix} ke^{i\theta} & 0 \\ 0 & ke^{-i\theta} \end{pmatrix}$$

then

$$\mathcal{M}(S, \{\mathrm{Re}(\mathbf{b_1}), \mathrm{Im}(\mathbf{b_1})\}) = \begin{pmatrix} k\cos\theta & -k\sin\theta \\ k\sin\theta & k\cos\theta \end{pmatrix}.$$