CS 4124
Solutions to Homework Assignment 3

Collin McDevitt

March 15, 2024

[25] 1. Let

$$M_1 \quad = \quad (Q^{(\text{poll,acc})}, Q^{(\text{poll,rej})}, Q^{(\text{aut})}, \Sigma, \Gamma, \delta, q_0)$$

be the OTM with the following algebraic specification

$$\begin{aligned} Q^{(\text{poll,acc})} &= \{q_0\} \\ Q^{(\text{poll,rej})} &= \{q_1\} \\ Q^{(\text{aut})} &= \emptyset \\ \Sigma &= \{0,1\} \\ \Gamma &= \{0,1,\boxed{\text{B}}\}, \end{aligned}$$

where the specification of δ is given in a transition table; see Table 1.

A. On input $w = 01100$, give the computation (sequence of configurations) that M_1 goes through.

B. Let L_1 be the language accepted by M_1. What is L_1? Explain.

The sequence of configurations is

$$C_0^{M_1}(01100) = \langle \varepsilon, \boxed{\text{B}}q_0\boxed{\text{B}}\boxed{\text{B}}\rangle$$

$$C_1^{M_1}(01100) = \langle 0, \boxed{\text{B}}0q_1\boxed{\text{B}}\rangle$$

$$C_2^{M_1}(01100) = \langle 01, \boxed{\text{B}}01q_0\boxed{\text{B}}\rangle$$

$$C_3^{M_1}(01100) = \langle 011, \boxed{\text{B}}011q_1\boxed{\text{B}}\rangle$$

$$C_4^{M_1}(01100) = \langle 0110, \boxed{\text{B}}0110q_0\boxed{\text{B}}\rangle$$

$$C_5^{M_1}(01100) = \langle 01100, \boxed{\text{B}}01100q_1\boxed{\text{B}}\rangle$$

The language that M_1 accepts is given by

$$L_1 = \{w \in \{0,1\}^\star : w \text{ has even length}\}$$

q	$\delta((q,0),0)$	$\delta((q,1),0)$	$\delta((q,0),1)$	$\delta((q,1),1)$	$\delta((q,0),\boxed{\text{B}})$	$\delta((q,1),\boxed{\text{B}})$
q_0	$(q_1,0,R)$	$(q_1,1,R)$	$(q_1,0,R)$	$(q_1,1,R)$	$(q_1,0,R)$	$(q_1,1,R)$
q_1	$(q_0,0,R)$	$(q_0,1,R)$	$(q_0,0,R)$	$(q_0,1,R)$	$(q_0,0,R)$	$(q_0,1,R)$

Table 1: Transition table for Problem 1.

. We have no autonomous states and all the state transition functions only ever move right hence we only have to examine when the final state is q_0. Looking at the transition table we see that on any input and any tape symbol the state goes from q_0 to q_1 or from q_1 to q_0 informally the state flips at each step in the configuration. As the initial state is q_0 then for us to end up in q_0 again we need to have flipped an even number of times which implies that the word was even length.

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[25] 2. Let $L_2 \subseteq \{a, b\}^\star$ be the language

$$L_2 \quad = \quad \{(aab)^i \mid i \geq 0\}.$$

Three example strings in L_2 are ϵ, $aabaab$, and $aabaabaabaabaab$. Three example strings not in L_2 are baa, $aaaab$, and $aabaa$.

Design an OTM M_2 that accepts L_2. Give the complete algebraic specification for M_2:

$$M_2 \quad = \quad (Q^{(\text{poll,acc})}, Q^{(\text{poll,rej})}, Q^{(\text{aut})}, \Sigma, \Gamma, \delta, q_0).$$

You may want to put δ in a transition table; otherwise, you can specify all the transitions as equations in an `eqnarray*` or a `displaymath` environment.

HINT: **You will need a dead state that is in $Q^{(\text{poll,rej})}$.**

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[20] 3. Let D be a set of cards from a standard deck, 52 cards in all. Let C be a set of 50 pennies, each with a different year so they are easily distinguished.

Prove by contradiction that there is no injection

$$h : D \rightarrow C.$$

HINT: **Assume that such a injection h exists and derive a contradiction. Think about the Pigeonhole Principle.**

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[30] 4. Let A be any nonempty set. Let $\mathcal{P}(A)$ be the power set of A, the set of all subsets of A; see Page 492.

Prove by contradiction (diagonalization) that there is no bijection

$$h : A \rightarrow \mathcal{P}(A).$$

HINT: **Assume that such a bijection h exists and derive a contradiction. Be careful with your argument.**
