

CS 4124
Solutions to Homework Assignment 1
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[25] 1. Textbook Problem 5 in B.1 on Page 520.

Prove Lemma A.4: For all alphabets Σ and all languages $L \subseteq \Sigma^*$, the equivalence relation \equiv_L is right-invariant.

The lemma and the definition of the equivalence relation are on Page 505.

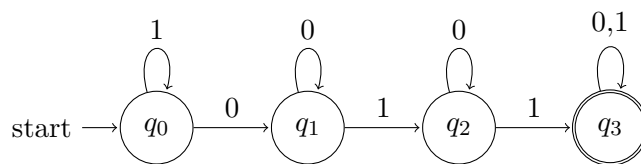
Proof. Let Σ be an alphabet and $L \subseteq \Sigma^*$ and \equiv_L be an equivalence relation as defined by (A.3). Let $x, y, z, w \in \Sigma^*$ and $x \equiv_L y$ then we have $xz, yz \in L$ as $zw \in \Sigma^*$ we have $xzw, yzw \in L$ which implies $xz \equiv_L yz$. \square

[25] 2. Textbook Problem 3(c) in B.2 on Page 522.

Design an FA M , with alphabet $\Sigma = \{0, 1\}$, that recognizes (c) the set of all strings that contain the string 011, in that order, but not necessarily consecutively.

Be certain you understand the set (language) of M before you start designing. Constructing some examples of strings both in $L(M)$ and not in $L(M)$ can be helpful.

You may use an algebraic specification or a transition diagram (labeled directed graph) specification to present your design for M . Be certain to explain why your design works.



This design for a **FA** works as reading through the input string we have it only enters state q_1 if it has read 0 and will loop otherwise. Then for each of the next states q_1, q_2 they will loop if they receive a 0 otherwise they will go to q_2 and q_3 respectively. q_3 is an accepting state and will loop for any input which ensures that the string only need to contain 011 not end in that.

[25] 3. Consider the OA M_4 in Figure 3.4 on Page 60. Give a complete and careful algebraic specification

$$M_4 = (Q, \Sigma, \delta, q_0, F)$$

for M_4 .

[25] 4. Textbook Problem 7(c) in B.2 on Pages 522 and 523.

Use a (fooling set)-plus-(Continuation Lemma) argument to prove that the following language is *not* Regular:

$$L_5 = \{a^i b^j c^k \mid i = j \text{ OR } j = k\}.$$

Your argument will be a proof by contradiction.

The Continuation Lemma is Lemma 3.2 on Page 57, while the fooling set kind of argument is first utilized on Pages 76 and 77.
