Concise Description of the Sequences of Conformal Iterates of the Unit Disk

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Theorem 1 (Iterates of $\mathbb{D}$)**.** *Let $f : \mathbb{D} \to \mathbb{D}$ be a biholomorphism, that is bijective and holomorphic (hence conformal) self map of $\mathbb{D}$, then f takes the form*

$$f(z) = e^{i\theta} \frac{z - a}{1 - \bar{a}z}$$

Where $a \in \mathbb{D}$ and $\theta \in \mathbb{R}$

We wish to fully explain the condition for convergence of the sequence of iterates of functions of this form. The reason for doing so, is due to the Riemann mapping theorem if we can solve this problem on this domain, we will be able to fully understand the iterates for any domain of $\mathbb{C}$.

Theorem 2 (Reimann Mapping Theorem)**.** *Let $\Omega \subset \mathbb{C}$ be a domain of $\mathbb{C}$, then for all $z_0 \in \Omega$ there exists some biholomorphism $F : \Omega \to \mathbb{D}$ such that $F(z_0) = 0$ and $F'(z_0) > 0$*

Corollary 3. *Any two domains of $\mathbb{C}$, say Ω_1 Ω_2 are conformally equivalent, meaning there exists some biholomorphism between them.*

To see why, let F_i be the biholomorphisms from Ω_i to $\mathbb{D}$, then clearly $F_2^{-1} \circ F_1 : \Omega_1 \to \Omega_2$ and clearly a biholomorphism