A Review of Cellular Automata-Based Approaches For Pedestrian & Spatio-Temporal modelling

Cristopher McIntyre Garcia
Department of Computer Science
University of Ottawa
Ottawa, ON K1N 6N5
cmcin019@uottawa.ca

Abstract

In this survey, we will delve into the concept of cellular automata as it relates to intelligent transportation systems. Specifically, we will explore how pedestrian spatio-temporal behaviours can be effectively modelled using state-of-the-art cellular automata techniques, which can ultimately help optimize our transportation systems. Throughout the report, we will provide an overview of important definitions for understanding the field, as well as examine various applications that have been proposed and explored. We will also delve into specific techniques and categorize them into tables based on relevant literature. While our main focus will be on techniques that impact pedestrian behaviour, we will also touch upon how these techniques are used to model vehicle behaviour. In addition, we will examine how interactions between pedestrians and vehicles can be modelled to create more realistic simulations. Finally, we will conduct an analysis of all the techniques discussed and discuss potential future directions for this exciting field of research. It is worth noting that this report mainly focuses on works proposed since 2019, with a few exceptions for fundamental works.

1 Introduction

Accurately modelling pedestrian dynamics is essential when designing infrastructure and transportation systems. Understanding the movements and behaviours of pedestrians as they interact with their environment is crucial for engineering optimal roads, buildings, and city expansions. With various components involved, including object avoidance, path prediction, flow interpretation, and more, it is vital to develop complex yet information-preserving models that can create realistic human movement in a variety of scenarios.

A wide range of approaches have been studied in the literature for modelling pedestrian behaviours and dynamics. These approaches can vary from heuristic-based to strictly defined, and in recent years, a great deal of attention has been given to data-driven, deep learning models. However, deep learning often requires large amounts of data that may be difficult to obtain and analyze, and privacy concerns may arise when collecting such data. Cellular automata (CA) is a type of technology that has been used in the past and is still being studied for pedestrian simulation purposes ([33], [9], [36], etc.). This technique is intuitive in its basic form and does not necessitate the vast amounts of data required by deep learning. That said, there have been studies that combine both CA and deep learning techniques [26]. In this text, we will explore the various applications of CA in the context of pedestrian modelling, compare similar methods, and evaluate the advantages and disadvantages of each approach.

While a recent review of CA was conducted by [15] in 2019, many new applications and novelties have emerged since then. Moreover, the focus of the previous review was on evacuation systems, whereas this text will consider systems directly related to pedestrians in transportation settings.

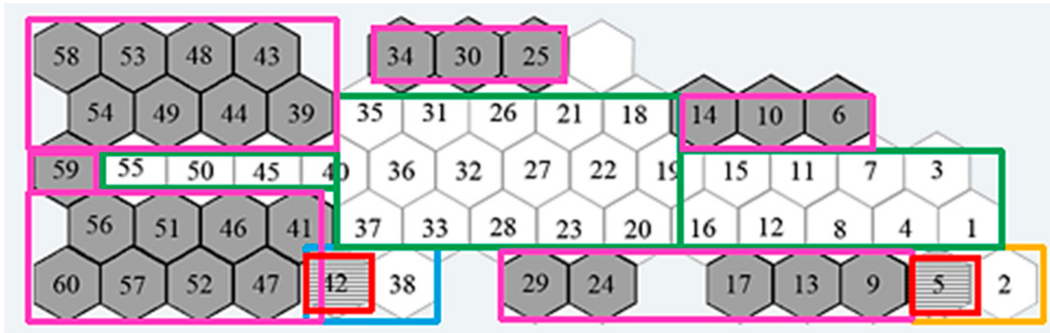


Figure 1: Bus interior modeled as with a CA

Although we will include evacuation systems, our emphasis will be on pedestrian-related systems. We will mainly present approaches proposed since 2019, with a few exceptions. This paper does not serve as a substitute for the aforementioned review; rather, it serves as an extension that features up-to-date technologies. We will address prior concerns and challenges in this work and discuss the methods and technologies that have successfully addressed them. We will also present new challenges and discuss those that have not been resolved as of the writing of this paper.

The purpose of this text is to present the reader with material that will allow them to better understand the powerful capacities of CA and their applicable usability to pedestrian dynamics. The reader should feel they have developed a light understanding how the technologies, and more importantly, their use-cases. The reader is encouraged to further research the papers presented that fit with their motivations and expand their understanding in doing so. CA have been utilized in several fields, and therefor there has been ample amount of research on them. We will provide a short introduction to the CA structure, as well as other concepts relating to pedestrian & spatio-temporal modelling.

The inclusion of spatio-temporal modelling is crucial due to the prevalence of CA structures in its implementation in various works ([9], [10], [11], etc.). It is also important to note that understanding the dynamics of expanding cities and areas of interest can lead to more powerful modelling capabilities for infrastructure that will eventually be used by pedestrians. Therefore, discourse surrounding the type of growth that will significantly impact pedestrian dynamics and modelling is also necessary. In earlier literature, the terms microscopic and macroscopic modelling were used to describe modelling individual agents and large groups, respectively (more on this in section 2). In this text, we argue that spatio-temporal modelling can be seen as super-macroscopic modelling due to its dynamic nature that can be both chaotic and structured.

This paper is structured into six sections. Following the introduction, we provide definitions for important terminology and explanations of key concepts. In section 3, we discuss various applications and techniques utilized in pedestrian modelling. These applications will be thoroughly analyzed and compared in section 4, empowering readers to select specific technologies for their desired tasks. Section 5 addresses open issues and future work required to enhance the results and overall pedestrian simulations. Finally, we draw our conclusions in section 6.

2 Definitions

In this section, we will provide definitions of key topics and technologies that will be discussed throughout this text. We will begin by introducing the Cellular Automata model and its computational power as a tool for simulating complex behaviours. Next, we will explain the different types of pedestrian behaviours and the goals that each try to achieve through algorithms. Then, we will discuss the different urban settings that have been prevalent in the literature. Finally, we will define spatio-temporal modelling, which we argue is a significant component for modelling future simulations with important setting behaviour knowledge.

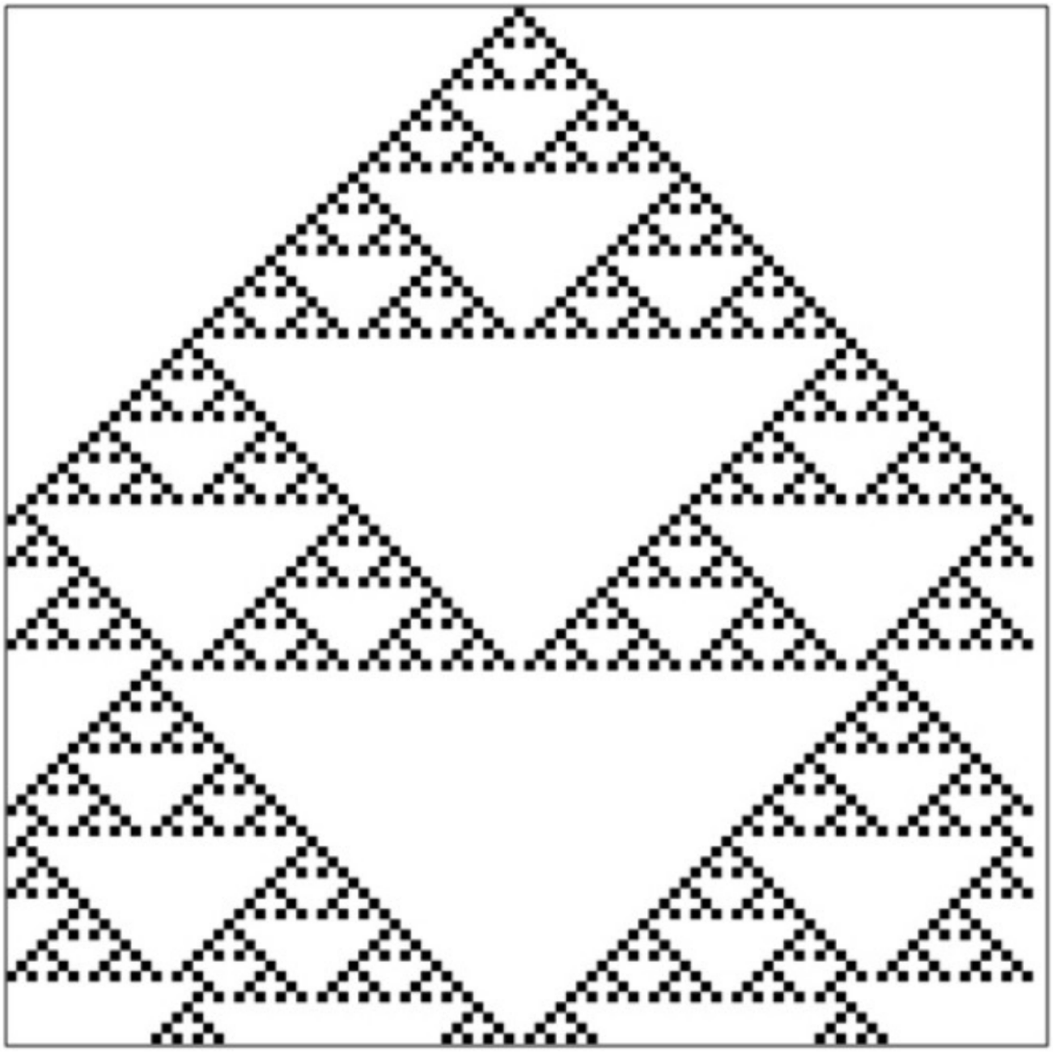


Figure 2: Complex 1-dimensional CA pattern

2.1 Cellular Automata

The concept of cellular automata was first introduced in 1963 by Neumann in [20], where it was defined as a discrete, two-dimensional system capable of performing parallel computations and emulating a universal Turing machine. Since then, many new systems with varying complexities and properties have been proposed. Almost all cellular automata systems can be defined by four parameters: a discrete, n-dimensional lattice of cells; discrete states that each cell may correspond to; local interactions between cells in a neighbourhood; and discrete dynamics, which are defined by a deterministic transition function that modifies the cells at each time step of a process. An example of a simple cellular automaton is shown in Figure 2, where a 1-dimensional lattice of square cells is used with two states (0 and 1). The neighbourhood of a cell includes the cells that are directly adjacent to it, and the transition rule is defined as "at each time step, a cell state is 1 if exactly one of the neighbouring cells was 1 at the previous time step, otherwise 0." [1]

Different classes of rules produce various computational patterns in CA systems. Class 1 rules quickly generate uniform configurations, while class 2 rules lead to a final pattern. Class 3 rules produce seemingly random patterns, with some patterns appearing occasionally, and class 4 rules result in complex patterns with chaotic behaviour. As information is propagated and undergoes transformations at each time step, the overall pattern is retained. Class 4 CA, due to their unpredictability while preserving pattern information, are the only CA capable of universal computation. They are also

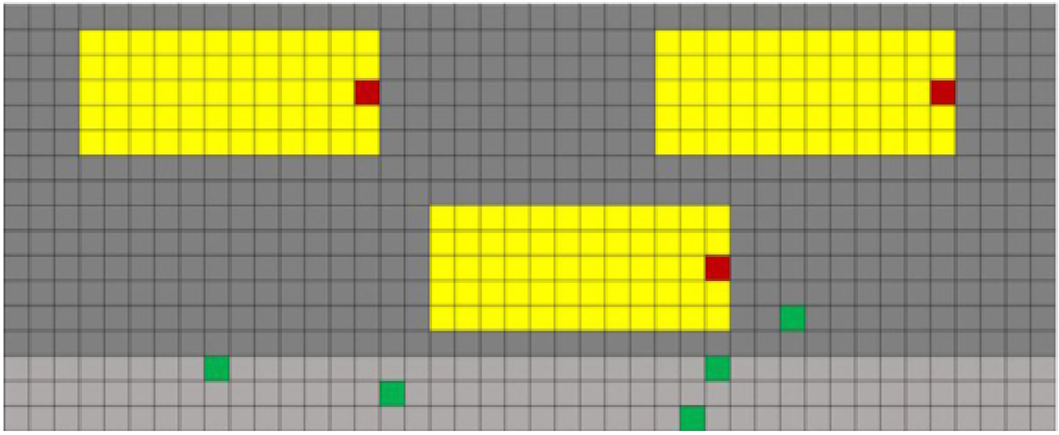


Figure 3: Agents and road CA model

linked to the concept of interesting complexity, which is observed in biological entities and their dynamics [1]. For that reason, these are the types of CAs used to model the behaviours of humans.

In the field of intelligent transportation systems (ITS), CA have been applied to model various simulation components such as roads, sidewalks, vehicles, pedestrians, and even infrastructure. By utilizing CA models, complex behaviours can be constructed that closely resemble real-life scenarios. However, the challenge lies in properly selecting transition rules that accurately model the movements of each component over time, with respect to their neighboring cells [33]. As the complexity of these rules increases, the behaviours of the agents become more realistic. A major strength of CA is the ability to assign different rules to different types of cells, allowing for diverse behaviours of the agents within the simulation and leading to a richer and more accurate model.

In this report, we will explore various CA models and their properties in the context of ITS. Through experimentation, researchers have been able to uncover important details that improve the efficiency and security of our transportation systems. One advantage of CA models is their ability to reproduce results seen with more complex techniques in a comprehensive and explainable manner, while outperforming some data-intensive models. Additionally, the simplicity of CA models allows for easy adaptation to different tasks with minimal modifications. These properties and more will be explored throughout the paper.

2.2 Pedestrian Behaviour

In ITS, pedestrian behaviour is an important aspect to model accurately because it can greatly impact the overall efficiency and safety of the transportation system. Pedestrians must navigate through a complex environment that includes not only other pedestrians, but also vehicles and infrastructure such as crosswalks, sidewalks, and traffic lights. Therefore, it is important to consider how pedestrians interact with each other and with their surroundings in order to create realistic and effective simulations. The level of detail provided about individual pedestrian movements can vary depending on the modelling approach chosen, with options ranging from microscopic to macroscopic models [51]. Both models have their own advantages and limitations, and the choice of which to use depends on the specific task.

Microscopic models provide a detailed view of pedestrian behaviour and can be used to understand the underlying mechanisms that lead to crowd phenomena such as congestion and bottleneck formation [51]. These models take into account the characteristics and goals of individual pedestrians, as well as their interactions with other individuals and the environment. However, in order to properly model the behaviour of individual pedestrians, complex transition functions with many parameters must be developed. This is because the model needs to provide a high level of detail about factors such as walking speed, direction, and individual behaviour. While this may seem daunting, having a variety of complex yet explainable functions with different parameters can create very realistic simulations that are able to model realistic scenarios. In addition, microscopic models can provide insights into the behaviour of individuals in specific situations and can be used to develop more efficient and effective pedestrian flow management strategies.

Macroscopic models focus on the overall behaviour of pedestrian flow, rather than individual movements [51]. These models are often based on flow models, where pedestrian movement is described by fluid dynamics equations that take into account factors such as density, velocity, and flow rate. These models are also useful in situations where there are too many pedestrians to model individually or when the level of detail required for individual pedestrian behaviour is not necessary. However, macroscopic models may not be able to capture the nuances of individual pedestrian behaviour, and may not be able to account for situations where individual behaviour can significantly impact the overall system behaviour. Sometimes, our focus may not be on the pedestrians, and rather the vehicles that surround them [33]. It is important to have a method to model the general behaviour of pedestrians and a more microscopic approach for modelling the vehicle behaviour in these cases. These types of models are usually a lot easier to define with transition functions, and can still have a good level of variety due to the definition of different parameter tuning.

In this work, we also introduce the notion of super macroscopic models. Super macroscopic models can provide a valuable tool for city planners to make informed decisions regarding the expansion of transportation infrastructure [38]. By focusing on the behaviour of large groups of pedestrians and analyzing changes in the spatio-temporal environment, these models can help identify areas of high pedestrian traffic and determine the most efficient ways to connect them with transportation services. In contrast to microscopic and macroscopic models, super macroscopic models simplify pedestrian behaviour even further, making them easier to model through transition functions. This allows for a greater focus on the modelling of new roads and infrastructure while still taking into account the needs of pedestrians in the context of the environment. Overall, super macroscopic models provide a unique perspective on pedestrian behaviour in the context of ITS and can aid in the development of more efficient and connected cities.

Some important factors regarding pedestrian behaviours are flow, trajectory, and collision avoidance [51]. Flow refers to the movement of pedestrians through a space and is influenced by factors such as density, trajectory and velocity. Pedestrian flow can be affected by various factors, such as the design of the environment, the presence of obstacles or bottlenecks, and the behaviour of individual pedestrians. Understanding how to properly model flow is important in the design of simulations to better model public spaces and transportation systems, as it can help ensure the safe and efficient movement of people.

Trajectory refers to the path that pedestrians take through a space, and is influenced by factors such as obstacles, destination proximity and social norms [51]. The latter usually refers to the fact that most people tend not to want to get too close to others. Trajectory can be complex to model with CA due to their limitation of being made up of a discrete grid. However, as we will see there are many methods for overcoming this limitation such as using lattice gas techniques and complex cell shapes. Overall, knowing how to properly model trajectory can provide insights into factors that influence pedestrian behaviour, such as crowd density, environmental features, and social interactions.

Collision avoidance refers to the ability of pedestrians to avoid collisions with each other and with obstacles, and is influenced by factors such as visibility and reaction time [51]. These are very important factors when discussing evacuation scenarios where the pedestrians may be in a hurry to get to somewhere safe while avoiding hazards. Collision avoidance has therefore been greatly studied in the context of evacuations, however many of these findings are also applicable to ITS. Namely, the avoidance of potholes or low visibility at night can cause different types of behaviours is pedestrians and drivers alike. We will see many evacuation techniques in this report that whose techniques are valid in the context of ITS to help improve pedestrian safety and reduce the likelihood of accidents in crowded areas, poor road conditions, and low visibility environments.

Overall, properly modelling pedestrian behaviour in simulations can serve as a powerful tool for comprehending the intricate interactions between them, vehicles and the surrounding environment. Through the use of both microscopic and macroscopic models, researchers can gain deeper insights into the underlying mechanisms that drive crowd phenomena and design pedestrian environments that are not only more efficient but also safer.

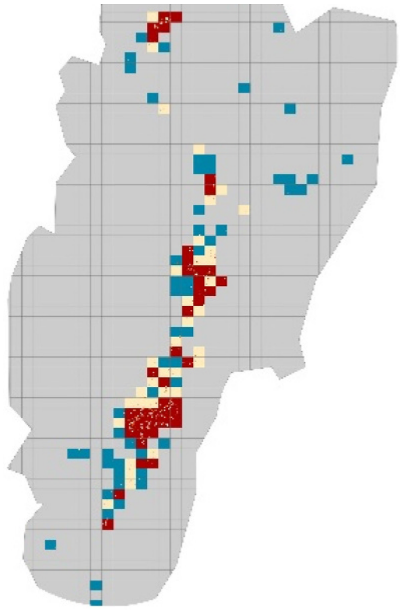


Figure 4: Density of the commercial units

2.3 Urban Settings

An urban setting is a geographical area characterized by a high population density and the presence of various infrastructure and public transportation systems [33]. Urban settings are typically associated with cities and are distinguished from rural areas by their concentration of people and the intensity of social activities that take place within them. In general, urban settings are dynamic and complex, and therefore, we need models that are capable of reaching the same levels of complexity when properly building simulations. Proper simulations include the design and layout of roads, sidewalks, and other transportation infrastructure, as well as the placement and design of buildings, parks, and other public spaces. Most urban areas have intricate road and parking structures such as intersections, roundabouts, bus stops, campus roads, and so on.

To replicate real-world urban environments, one also needs to include the complex behaviours of vehicles and pedestrians. Vehicles can range from cars and buses to unmanned aerial vehicles and boats [24] [28] [22]. These vehicles interact with the road infrastructure and with each other in ways that must be accurately modelled in order to create a realistic simulation. Pedestrians also interact with these vehicles in many different ways. Depending on the environment, the dynamics between the pedestrians and vehicles can differ greatly. It become a large research topic to determine the most optimal road and infrastructure layouts to protect all agents while keeping efficiency. Factors that can change the behaviours between pedestrians and vehicles include the time of day and conditions, types of roads, speed limits, group density, and so on.

In addition to transportation infrastructure and vehicles, urban settings also incorporate network communication. These settings can be incorporated into the simulations to predict the type of connectivity that may exist between agents. This includes the use of sensors, cameras, and other data collection tools to monitor traffic and the surrounding environment as a whole [51]. Smart urban settings can also incorporate advanced technologies such as artificial intelligence and machine learning to optimize traffic flow and reduce congestion. Overall, urban simulations provide a valuable tool for researchers and city planners to explore different scenarios and test potential solutions for improving the quality of life in urban areas.

2.4 Spatio-Temporal modelling

Spatio-temporal modelling is a powerful computational approach that enables the simulation and analysis of the movement and interaction of objects and individuals in both space and time [2]. This methodology finds particular application in the context of city planning and infrastructure, as it allows for the prediction of the future expansion of cities and the associated impacts on transportation networks and pedestrian traffic. By modelling land as a cellular automaton, one can take into account a wide range of factors, such as altitude, vegetation, water bodies, and infrastructure, to represent various features of urban settings. Cells within this model can be representative of diverse features, including residential areas, transportation networks, and pedestrian pathways, among others. As time progresses, the behaviour of large populations can have far-reaching consequences on the surrounding environment, necessitating sustainable methods for creating more efficient and accessible urban areas.

One key aspect of spatio-temporal modelling in city planning is the representation of roads and transportation networks [2]. By incorporating existing road networks, traffic patterns, and projected infrastructure projects into simulations, planners can analyze the potential impact of changes to the road network on traffic flow and congestion. This allows for the testing of various scenarios and the identification of optimal solutions for efficient and safe transportation. Effective spatio-temporal modelling can also facilitate early project planning, minimizing inconvenience for the growing population and promoting the expansion of urban areas to new regions.

In addition to roads, spatio-temporal modelling can also take into account wide pedestrian traffic and movement patterns. By simulating the movement of pedestrians at a large scale in and around urban areas, planners can identify potential bottlenecks and areas of congestion, as well as opportunities to improve walk ability and accessibility.

Overall, spatio-temporal modelling is a powerful tool for city planners and infrastructure designers. By simulating the movement and interaction of objects and individuals in both space and time, planners can better understand the impact of their decisions and make more informed choices about the future development of urban areas.

3 Applications

In this section, we will delve into the various techniques and applications that employ CA. We will also explore different variations and enhancements of CA, as well as its compatibility with other techniques, while highlighting their strengths and weaknesses. To aid the reader's comprehension, we will present examples that illustrate these concepts. Table 1 provides a visual representation of the different application categories, along with their respective methods and papers.

3.1 Evacuation

When designing infrastructure, it is important to take into consideration a good evacuation scheme for events of emergency. Many papers in the literature have suggested techniques that utilize the CA model to mimic humans as they exit buildings or large vehicles like plains, boats, or trains. Many of these techniques can also be translated to other simulations such as some meant to model the behaviours of pedestrians during rush hours, or on a school campus. Because CA are very flexible, tweaking the parameters to better suit the needs of other types of simulations is simple. In the following section, we will explore some of the tools and techniques that are available, and describe how they can be modified to better suit ITS simulations of pedestrians or even vehicles.

In [49], the authors propose the use of a continuous floor field CA where pedestrians are treated as particles in a continuous space instead of occupying cells. The interactions between pedestrians are modelled by transition functions that depend on the area in which they are placed, much like cells. This technique utilizes a smoother behaviour due to the continuous movement, however it also keeps computation low by using functions similar to those found in traditional CA models. This type of technique could be useful in modelling simulations where there are larger crowds of people who might be in a hurry and so do not care so much about keeping a large distance between them such as in a train station.

In [27], the authors were able to properly characterize impatience in pedestrians during evacuations. They use a CA and transition functions to model the level of impatience of individuals over time, with growth also happening through propagation among a crowd. To properly model the parameters of the pedestrians, a study of impatience was conducted to evaluate realistic behaviours in their simulation. They show how the different levels of inpatients can drastically change the evacuation time of a group. Once again, these are techniques that can be translated to many urban settings. As an example, these types of models would be we suited for vehicle stuck in traffic, as they keep waiting they may grow

Table 1: CA applications

Application	Method	Reference	Description
Evacuation	Floor field CA Impatience-determined Heterogeneous Game theory Dynamic Monte Carlo	[49] [27] [14] [8] [29] [47] [30]	Useful for rush hours or campuses
Flow	Collision avoidance Heterogeneity	[23] [52] [46] [5] [44]	Methods to simulate flow
Urban Cities	Traffic conflict Trajectories Crossing Bus	[33] [48] [12] [21] [16] [35] [17] [24]	Some works related to cities
Spatio-Temporal	Inner city dynamic City expansion	[9] [38] [10] [11]	Large changes in cities and expansion
Fuzzy-Theory	Fuzzy logic Social force model Fuzzy inference system	[18] [41] [45] [42] [13], [12]	Works that incorporate fuzzy theory
Sim Tools	Real-coded lattice gas Susceptible Infected Recovered Numerical Fine grid Inference model	[40], [50] [32] [43], [4] [25] [7]	Simulation with methods and results

more and more impatient and become riskier drivers. The dynamic level of impatience is a vary powerful idea that can be used in any stressful situation.

In [14], the authors model a CA simulation for pedestrians with a restriction to their vision of field. In the paper, the pedestrians are placed in a room that supposedly is either very dark or contains a lot of smoke. They do this by limiting the field of view of the pedestrians through the definition of their neighbourhood. In the text, the technique is used for evacuation purposes, however it is easy to see how this technique could also be used to model pedestrian and vehicle behaviours during days where vision might be impaired, such as foggy or snowy days.

In [8], the authors explore the notion of stampedes and try to create a realistic simulation for their behaviour. The model considers factors such as pedestrian crowding, support, and friction that may influence normal pedestrian falls, and examines the state of pedestrians after they have fallen to the ground. Stampedes are common in evacuation settings, but like the previous examples, there is also room for these types of tools in ITS. It is important to understand the density of the targeted groups, and the obstacles that may arise when people are moving. These types of simulations are good useful for determine where to place signs, estimate space, and methods for keeping the density of groups to a minimum.

Many of the literature related to CA models in simulations focus on the behaviours of agents during evacuations. That said, many of the techniques, and even the implementations, are still relevant to ITS. Because CAs are flexible in their implementations, it is possible to extend many of these ideas to settings where pedestrians may be in a hurry, impatient, unaware, or lacking proper vision. It is therefore important to understand the techniques that are also applicable to pedestrians. Many of

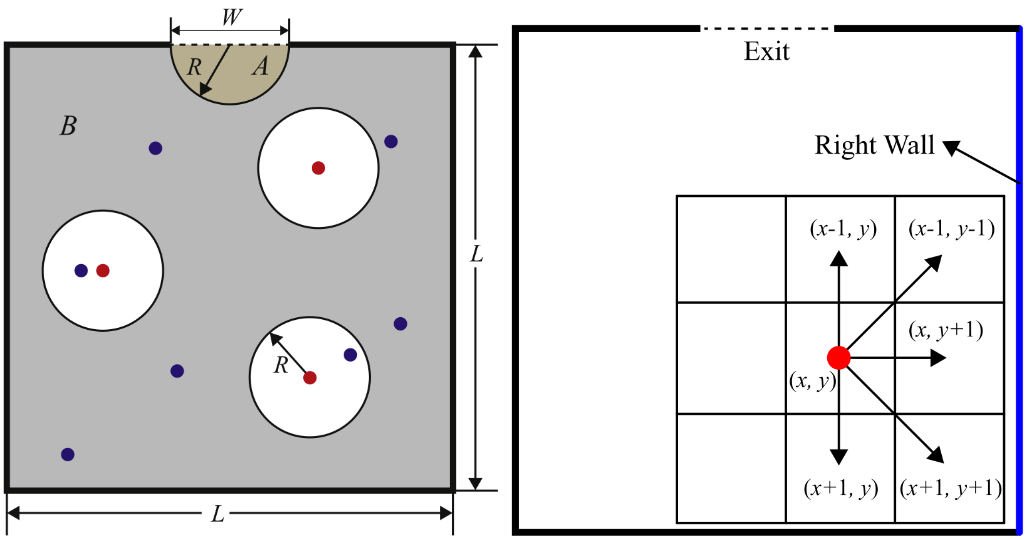


Figure 5: a) Limited view b) CA of escape room

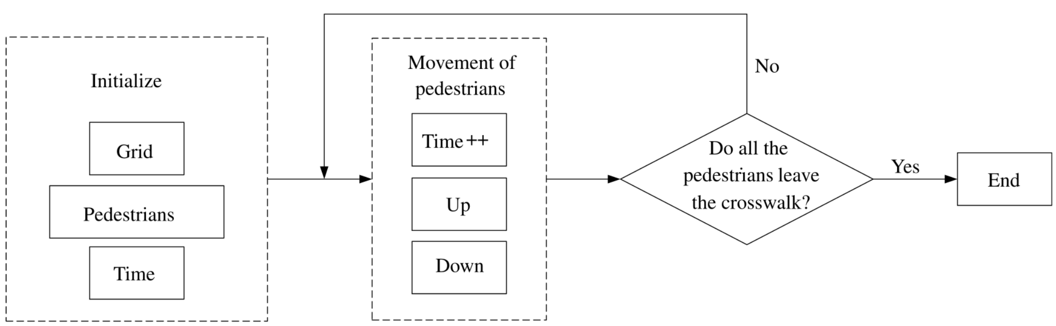


Figure 6: Flow operation process

these techniques can also be implemented as vehicles. More examples from the literature are given in table 1.

3.2 Flow

In [4], the authors developed a model that considers the characteristics of pedestrian crossing at signalized crosswalks. They use three parameters to model the flow of pedestrians. The first is the benefit parameter which dictates whether a pedestrian is willing to cross the road. The second is the attraction parameter which defines how likely the pedestrian is to follows others around them. The third parameter if the occupy parameter which does not allow multiple pedestrians to be in the same cell of the grid. Then the simulation follows the operation process in Figure 6. The parameters are fed into the transition function which is what dictates whether a pedestrian decides to leave the crosswalk or not. Once a pedestrian decides to cross, many others will soon follow thanks to the attraction parameter flow of pedestrians.

In [26], a two lane road is represented by a CA and a Long Short-Term Memory (LSTM) and Support Vector Machine (SVM) are oped for to learn vehicle behaviours. This is an example of using multiple techniques in one application. In this case DL is used to define the transition functions, and the layout of the environment is represented by a CA grid. These models can be used with several different road configurations because they were trained along with CA and so this makes it possible for their to be more flexibility in their inference. The flow of the pedestrians is continuous and relies on the past actions.

In [39], the authors model the interior of a bus using cells that are shaped like hexagons to model the flow dynamics of pedestrians. This allows for more verity in in movement for the pedestrians when boarding or alighting. Figure 1 shows the interior of the bus. Each of the cells has a classification which are dynamic. In this particular systems, the authors allow the passengers to share a cell for a brief moment to let one passenger cross between two other passengers standing in his way. The flow of the passengers is determined by a target cell which is at the back of the bus if they are boarding, or it is at the exit if they are alighting.

3.3 Urban Cities and Pedestrians

It is important to not only model pedestrians, but also the vehicles that surround them, network communication, streetlights, and every other aspect of urban cities. Many works in the literature focus on finding the most optimal ways of modelling specific scenarios in urban life, others focus more on the behaviours of pedestrians. In any case, these many techniques have been derived to create realistic simulations to study the behaviours of pedestrians and vehicles alike in urban settings. These techniques are specific, but they present ideas that are transferable to many other urban environments. In fact, many of these works stem from past works with modifications to the speed limits of the vehicles and the structure of the roads.

In [3], the authors present a CA model for pedestrian-vehicle interactions in areas with a zebra crossing. This is an older paper that represented an urban street as a CA and modelled the vehicle and pedestrian behaviours using transition functions. The agents react according to the cells that are in their environment. More specifically, the vehicles are modelled to give way to perceived pedestrians whenever they are able to safely brake. On the other hand, when the conditions of the zebra crossing does not seem safe, pedestrians yield the right of way to the vehicle which they perceive to be traveling a high speed. modelling both the pedestrian and vehicle behaviours is important when designing these systems, however it is equally as important to model the environment correctly. In doing so, we can evaluate the best properties that will allow for the safest measures when pedestrians interact with vehicles.

In [33], the authors model both the behaviour of pedestrians and vehicles on a campus road setting. This work builds off of other works that perform a similar task but with different scenarios such as right lanes or four way passes. They use techniques such as the Kerner-Klenov-Wolf model which is meant to simulate the behaviours of linearly moving atoms. They improve these methods by adding parameters that are modifiable to represent different types of drivers through acceleration and perceived safety speeds. Pedestrians are modelled in such a way that they are prone to walking on the road such as they would on a campus road. Their awareness is dimmed, so as to simulate someone that may not be fully paying attention. These systems help determine the speed limits, sidewalk widths, among other factors.

An example of the equations used to model a vehicle is seen in Figure 7. Here, the behaviour of the vehicles are decomposed into ten equations. The first equation is used to determine the acceleration (a) of a vehicle given its current speed (v_{tar}^t), the current speed of a leading vehicle or pedestrian (v_l^t), a max acceleration (a_{max}) and a max speed (v_{free}). The second equation defines the minimum safety speed of a vehicle and depends on the distance between it and the leading vehicle ($g_{c,front}$). To calculate the current adaption speed, we use the third equation which takes into account the speed of the vehicle, the speed of the leading vehicle or pedestrian, the current speed, and max speed, and the max acceleration. Also, some hyper parameters that can be modified to vary the behaviour types are considered (k_1 and k_2). The adaptation speed is then modified by adding or subtracting the acceleration computed earlier in the first equation, or not modified at all.

To add a stochastic element to the simulation, the authors also include equations to calculate the probability of whether a vehicle will slowdown or speed up and to what extent. The fifth and sixth equations are meant to model the acceleration probability of the vehicle, which is impacted by the current velocity, the velocity of the leading vehicle of pedestrian, and the maximum acceleration. The probabilities are also hyper parameters which add another layer of customized to the vehicle behaviours. The seventh, eighth, and ninth equations define the speed of the vehicle at the next time step and depend on the probabilities from the previous equation. In all, a vehicle will either accelerate and be around the speed limit if there are no nearby leading vehicles, or it will stay at a safe speed while behind a leading vehicle, or it will decelerate if it is coming up on a vehicle at a speed that is too fast. In Figure 8 there is a list of the variable used in these past equations. These are only

Determine the acceleration a of the target vehicle.

$$a = \begin{cases} a_{\min} & \text{for } abs\left(v_{tar}^t - v_l^t\right) < ceil(a_{\max}/2) \\ ceil\left(a_{\max} * \left(v_{tar}^t - v_l^t\right)/v_{free}\right) & \text{for } \left(v_{tar}^t - v_l^t\right) > ceil\left(v_{free}/2\right) \\ ceil(a_{\max}/2) & \text{otherwise} \end{cases}$$

Calculate safe driving speed of the target vehicle at t-th timestep (v_s^t).

$$v_s^t = g_{c,front}/\tau$$

Calculate the adaptation speed of the target vehicle at t-th timestep (v_a^t).

$$v_a^t = \begin{cases} v_{tar}^t + a\tau & \text{for } g_{c,front} > k_1 v_{tar}^t \tau \\ v_{tar}^t + a\tau & \text{for } k_2 v_{tar}^t \tau < g_{c,front} \le k_1 v_{tar}^t \tau \text{ and } v_{tar}^t - v_l^t < ceil(\lambda * v_{free}) \\ v_{tar}^t & \text{for } g_{c,front} \le k_2 v_{tar}^t \tau \text{ and } v_l^t < v_{tar}^t \le a_{\max} \\ v_{tar}^t + a\tau \,\mathrm{sgn}(v_l^t - v_{tar}^t) & \text{otherwise} \end{cases}$$

Calculate the speed of the dynamic part of the target vehicle at time t+1 ($\tilde{v}_{tar}^{t+1}$).

$$\tilde{v}_{tar}^{t+1} = \max(0, \min(v_{free}, v_s^t, v_a^t))$$

Determine the probability of target vehicle's acceleration p_a

$$p_a = \begin{cases} p_{a1} & \text{if } \tilde{v}_{tar}^{t+1} < v_p \\ p_{a2} & \text{if } \tilde{v}_{tar}^{t+1} \ge v_p \end{cases}$$

Determine the probability of target vehicle's deceleration p_b

$$p_b = \begin{cases} p_0 & \text{if } v_{tar}^t = 0 \\ 1 - p_a & \text{if } \tilde{v}_{tar}^{t+1} - v_l^t > a_{\max} \\ p_1 & \text{if } v_l^t < \tilde{v}_{tar}^{t+1} \le a_{\max} \\ p_2 & \text{otherwise} \end{cases}$$

Calculate the target vehicle's speed at time t+1 (v_{tar}^{t+1}).

$$\eta = \begin{cases} -1 & \text{if } rand < p_b \\ 1 & \text{if } p_b \le rand < p_b + p_a \\ 0 & \text{otherwise} \end{cases}$$

$$a' = \begin{cases} a & \text{if } v_{tar}^t = 0 \\ a_{\min} & \text{if } v_{tar}^t \ne 0 \end{cases}$$

$$v_{tar}^{t+1} = \max(0, \min(\tilde{v}_{tar}^{t+1} + a'\tau\eta, v_{tar}^t + a\tau, v_{\text{free}}, v_s^t))$$

Location update.

$$x_{tar}^{t+1} = x_{tar}^t + v_{tar}^{t+1}\tau \qquad ($$

Figure 7: Vehicle behaviour equations

a sub set of all the equations and variables that are defined to model the behaviours of the agents. Though daunting at first glance, these equations are very intuitive and easily understood, making the behaviour of the agents explainable.

In [48], the authors describe a technique for modelling urban road traffic while considering a network of vehicles to analyze their characteristics while an emergency vehicle is present. They use a synchronized traffic flow approach that considers the reaction time of the drivers and the vehicles around them. The rules that are used include methods for modelling perception, low reaction times (when a driver might be distracted or impaired), and their level of avoidance of other vehicles. Over fourteen parameters are used to modify the behaviours of each vehicle, making them varied and very flexible to the needs to the user. This study was able to suggest traffic capacities and average road speeds for more optimal urban traffic while emergency vehicles are present.

In [12], the authors describe a technique for modelling the behaviours of vehicles and pedestrians at a scenic spot with a two lane island roundabout. It is common for pedestrians to crowed the crosswalks that connect the island to the adjacent sidewalks, which can be lower the efficiency of the roundabout and be dangerous for the pedestrians if not properly planned. In this paper, the authors use a fuzzy CA (we will see more of these later) to model the decision-making process of the individual

List of variables in car-following rule.

a	Acceleration of the target vehicle.
v_s^t	Safe driving speed of the target vehicle at t-th timestep
v_a^t	Adaptation speed of the target vehicle at t-th timestep
v_{tar}^t	Target vehicle's speed at t-th timestep
v_l^t	Speed of the leader one (vehicle or pedestrian) at t-th timestep
v_{free}	Free flow velocity (maximum velocity)
$\tilde{v}_{tar}^t$	Speed of the dynamic part of the target vehicle at t-th timestep
η	Speed fluctuation
p_a	Probability of target vehicle's acceleration
p_b	Probability of target vehicle's deceleration
x_{tar}^t	Position of target vehicle at t-th timestep
$g_{c,front}$	Bumper-to-bumper space gap between the target vehicle and its leader one (vehicle or pedestrian) in the current lane

Figure 8: Vehicle behaviour variable

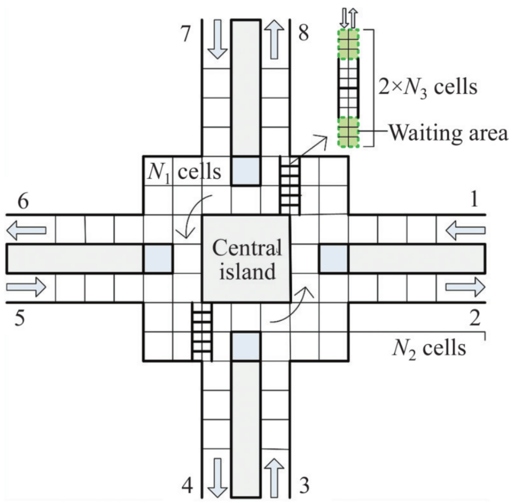


Figure 9: Roundabout CA grid

vehicles. They show that roundabouts are not suitable for high traffic areas because they are prone to congestion. The CA model for this work is shown in Figure 9. The cells clearly define a two lane road, and certain cells also make up crosswalks for pedestrian usage.

As shown in this section, there have been many papers focused on developing the best methods for modelling pedestrian and vehicle interactions. These methods explore different types of roads to find the most optimal and safest conditions. Because roads are different in many parts of the world, reusing these results for every country does not seem appropriate. That said, because we are using CA models, it is much easier to translate the techniques form these works to other scenarios. Especially because the traffic behaviours and shape of the road and sidewalks are so easily malleable, virtually any road can be modelled.

3.4 Spatio-Temporal

During the past years, the expansion of cities has begun to grow at an accelerated rate. As such, planning for new infrastructure has been a topic in resent discussions for lowering costs and having more sustainable transportation. One method of modelling cities is with the CA model and transition functions. I this section, we do not talk about pedestrians as individuals, or even as a group, but rather as many groups in a collective in the context of a large city. We model the behaviours of people in cities and the expansion of land use over many years. As such, the modelling of this type of agents can be referred to as super macroscopic modelling because we are working with even broader groups of people. This gives us the ability to focus on modelling other aspects such as infrastructure growth, residence expansion, and all types of transportation systems including trains and subways.

In [38], the authors model landscape dynamics with a grid-based graphic Information System (GIS). It was one of the first works to use the CA model to capture population dynamics. An example of these

Figure 10: Grid-based graphic Information System dynamics

is shown in Figure 10. Here the complexity of the CA is shown. With very simple transition rules, the authors are able to produce realistic expansion movement of a population. This type of technique is used all throughout these methods because of its accurate depiction of civilization expansion.

In [9], the authors propose that there needs to be better ways of predicting the expansion of cities to better prepare for the construction of infrastructure and transportation systems. By evaluating data from previous years, they are able to model a realistic behaviour of city expansion. In Figure 4, the authors show the density of the commercial units in the city. Though not directly modelling the behaviour of pedestrians, in modelling the city expansion and infrastructure, one can make better decisions when designing roads, tracks, subways, and other types of transportation systems that would be suit the landscape in the long run. Cities can have a more defined budget plan for future construction work which would lower costs and make the process more efficient.

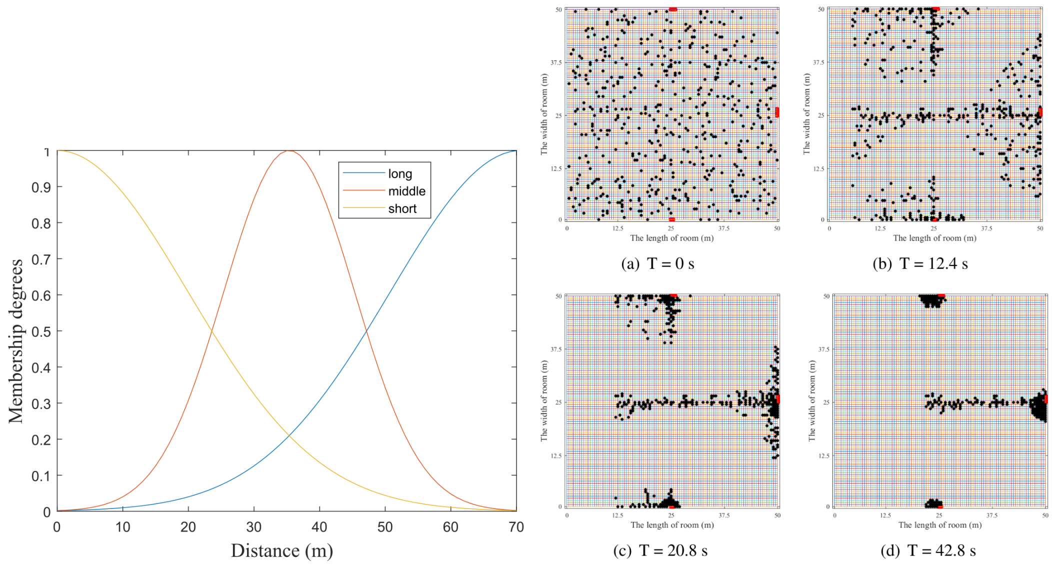
In [10], the authors focus on building new transportation systems for expanding cities due to growth in populations. They use the CA model to organize, understand and manage the urbanization of new areas. They organize their land predictions into seven types and they use a combination of the Land use Expansion Analysis Strategy model and a CA model based on the multi-class random patch seed. The latter model employs an decision tree as the transition function to predict the probability of growth and of which type. They use criteria such as GDP, subways, elevation, rivers, slope, population, schools, and hospitals to organise the patches of land into classes that make the most sense for future infrastructure and transportation. They are also able to evaluate the environmental impact of such expansion, and find ways of mitigating this by means of more sustainable, long term changes to the cities.

In [11], the authors explore the use of Markov chains when defining their CA model. More specifically, they use multi-temporal Landsat data and geographical information system techniques to predict changes in the spatio-temporal changes. The data is used to model the transition function of the CA which is modelled by a Markov chain. Image processing is used to select regions of interest meant for classification. The Markov chain is used to generate a transition probability matrix and a transition area matrix which are fed to the CA to predict the expansion of land usage. They classify the different types of terrain in terms of topology and land usage (urban area, vegetation, barren, water body).

As we have seen above, it is becoming more and more important for cities to predict the future of their expansion in terms of land, infrastructure and transportation. Because there are not many models that already exist for these tasks, it is hard to determine whether the CA model really works. That said, it is an intuitive and fully explainable model that does not demand a lot of data and will not over-fit to any city. This means that we are capable to taking the techniques used for one city and apply them to many other cities. Because of this, it is clear that the versatility of the CA model has an advantage when modelling the spatio-temporal expansion of cities and urban areas.

3.5 Fuzzy-Theory

Fuzzy-theory can be used as a technique to make the behaviour of agents more realistic by adding more complexity to the transition functions. In fuzzy logic, there are several aspects such as variables, fuzzification, rules, inference systems and defuzzification. Fuzzification entails a fuzzy set of classification for a given variable. in other words, an input variable can be part of multiple classes with a certain level of membership. An example of this would be treating Fridays as both 30 percent the weekend and 70 percent a week day. In many cases, this fuzzification is able to create ambiguity in the transition functions creating more realistic behaviours.



(a) T = 0 s (b) T = 12.4 s (c) T = 20.8 s (d) T = 42.8 s

Figure 11: a) Fuzzy rules b) Simulation

In [18], the authors use fuzzification to evaluate the distance between pedestrians and a given destination. At the same time, pedestrians are also aware of the distance between themselves. This creates a model for heards that are trying to find the shortest path to a given destination by also choosing to stay in less dense areas. An example of the fuzzy variable is given in Figure 11 where the distance can either be short, medium, or long to some degree of membership. Depending on the fuzzy outputs, the pedestrians gauge their behaviours by comparing potential paths with the probability of them reaching their destination in the shortest amount of time. An example to the simulation is given in Figure 11. Here the pedestrians start randomly dropped in a room. They then choose the paths that will lead them to the doors (the destination in this case) that are closest to them. If there are too many people in their way, they may opt for choosing another door.

In [42], the authors model the behaviour of pedestrians with a waiting area choice model for subway platforms. This model takes into account various factors, such as the distance of passengers from waiting areas, the number of passengers present in the waiting area, and the length of the remaining waiting area. The authors use a combination of fuzzy logic and a social force model along with cellular automata to simulate pedestrian behaviour. The waiting section on the grid is selected by the pedestrians based on the output of the fuzzy inference model. The social force model guides the pedestrians on how to reach their intended destination.

In [41], the authors propose a similar approach to that in [18] for evacuation. In this paper however, there is the addition of a guide who is helping the other pedestrians evacuate. This paper shows how there can be many different types of pedestrians who share the same goals but may have different priorities. This idea of a guide can be used in ITS as a cross walk guide at a school intersection. It can also be used to simulate a vehicle guide in a police chaise. The guides can either be chosen by the person implementing the simulation, or can be chosen by the simulation as it plays out.

3.6 Simulation Tools

In this section, we discuss the simulations that are available. These papers take approaches that we have either seen, or novel approaches and apply them to simulations for real world tasks. Many of these simulations are built specifically for certain layouts, however the approach can be used in any CA grid. Learning about the simulations that exist can help the reader be more informed about the steps needed to building a simulation with a CA model. Some of these methods even provide ready to use tools that the reader can use to build their own simulation. Most of these are meant for evacuation tasks, however as mentioned before, due to the flexibility of the CA model, we are able to modify the goals of the pedestrians quit easily in these simulations making it plausible to use in an ITS setting.

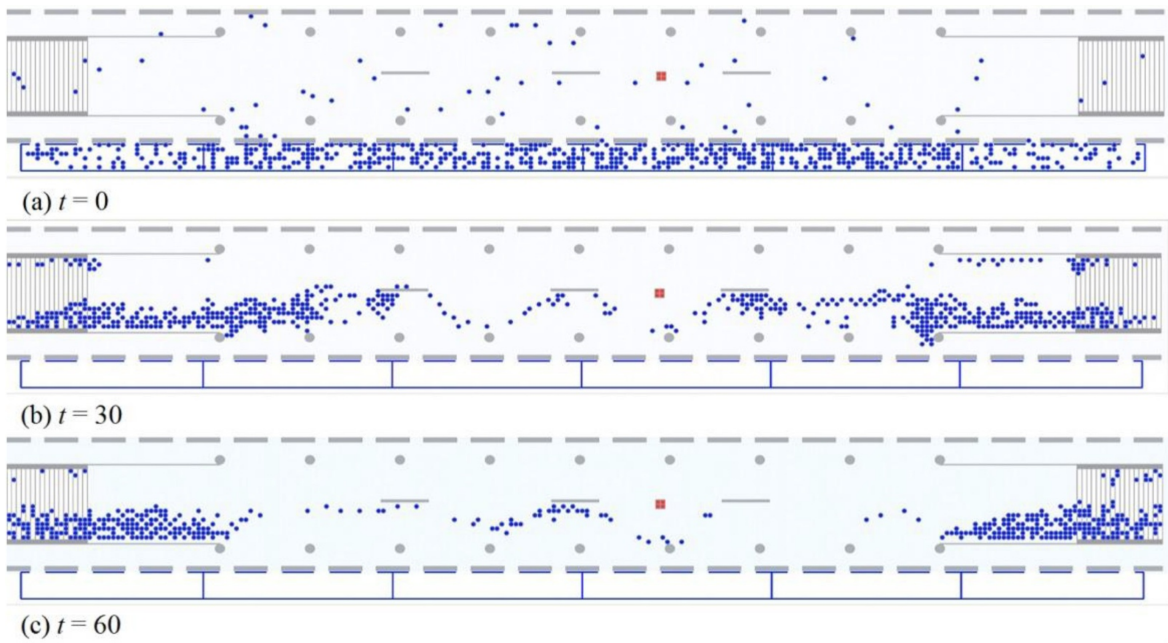


Figure 12: Flood evacuation simulation

In [40], the authors use a real-coded lattice gas model to develop a pedestrian behaviours dynamically. These models are used in simulations for modelling the behaviour of fluids. They show that the velocity and directions of pedestrians can be accurately modelled this way with a continuous distribution that models the Maxwell–Boltzmann distribution in the equilibrium state. This allows for the velocity and distributions to be real numbers as opposed to states. Pedestrians are therefor not constraint by the discrete grid, and rather can move linearly in any direction. This technique is capable of producing results that are similar to those seen in Figure 11. This work was one of the first to fully code a simulation using the lattice gas approach.

In [32], the authors introduce a model that takes the concept of panic transmission to describe the behaviour of pedestrians. At the same time, they include static hazards on the grid that the pedestrians must avoid when traversing. This simulation is meant to model the behaviour of pedestrians during evacuations, and the results show that when levels of panic are higher, the hazards become more dangerous. That said, they also show that when a larger crowd is evacuating, the hazards tend to be less dangerous due to the slower and more collected movements of the pedestrians. This kind of method can be used to model obstacles on roads such as potholes or debris from another vehicle. The limitation here is of course the fact that the hazard is static, however this simulation can easily be interpreted for ITS is a number of other ways where there are hazards that need to be avoid in a moving crowd.

In [50], the authors propose an evacuation simulation for underground floods. They model the water as gradually rising, which can impact the behaviours of the pedestrians trying to reach a safe point. They manage to model the pedestrians in such a way that includes holding objects for support, keeping out of dangerous scenarios and moving along with the flow of the group. The states of the cells are static, height, support, danger, and water flow floor field. The original implementation was meant to simulate a subway system being flooded, however there could be other applications such as a sinking ship or an underground garage. The simulation is visualized in Figure 12. In it you can see how the pedestrians attempt to leave the room while at the same time keeping distance between them.

In [7], the authors implement a simulation to model the pedestrian interference with vehicles in a drop-off area of railway stations. They aimed to determine the most optimal conditions for traffic efficiency and and safety. Pedestrians are modelled as a heard when deciding whether to cross, and vehicle are modelled to be lenient in giving way to those crossing. The results found in this study were then compared to real world scenarios, where they tended to perform similarly in terms of traffic efficiency. The authors were able to propose changes to the drop-off zone, making the outflow of vehicles increase by 40 percent. The details of their experiment mentions their results, the hyper parameters they chose for the vehicles, pedestrians and layout, and the implementation details.

As shown in the discussion above, there are many readily available tools that one can use as is or modify to better suit another task. The simulations are simple and light weight, and for the most part do not need any training whatsoever. These simulations methods can be combined with each-other or with other techniques that were discussed. Overall, the availability and ease of use of simulation tools, along with their potential to be combined with other techniques, make them an invaluable resource for anyone looking to model and understand behaviours as a whole.

4 Analysis

In this section we will analyse and compare the different technologies so that the reader is able to more thoroughly understand when to use what tools. We start by discussing the different types of pedestrian models, microscopic and macroscopic, and their purposes. Then we compare the applications that are used in urban settings, and finally we discuss the methods of validating the results.

4.1 Microscopic modelling

Using CAs to model the behaviours of microscopic agents has been explored in many of the papers seen previously. The environment is defined as a grid and every agent is placed on the grid to interact with the neighbouring agents. CAs are capable to modelling complex concepts such as flow and trajectory at the agent level making it an excellent choice for microscopic modelling. In many instances, the agents share the same transition functions, however their behaviours can vary by the hyper parameters chosen for each individual agent. This makes for a more chaotic simulation that is able to capture realistic interactions between pedestrians and vehicles of all kinds. Using probability, one can define the oscillation of a vehicle's movement, how aggressive they accelerate, how prone they are to be aware, or even modify their vision.

4.2 Macroscopic modelling

Using CAs to model the behaviours of macroscopic groups has also been explored in the literature, though not as much. Usually these types of models tend to focus more on the flow of the pedestrians as a whole when crossing the streets in large groups, or during a evacuation. These models can be helpful for developing realistic traffic simulations where there are many vehicles in a congested area. Having macroscopic transition functions alleviate the need to over define the behaviour of individual agents and can make simulations perform much faster.

4.3 Urban Settings

There are many papers discussing the interactions between pedestrians and vehicles in urban settings. These papers reconstruct an environment using cells and defining transition functions for both the pedestrians and the vehicles. That said, there are many road types that have not yet been explored such as boulevards, roads with speed bumps, slippery roads, roads with pothole, and many more. If one would like to further explore these topics, they could start with one of the papers focusing on vehicle-pedestrian behaviours to understand the transition functions and even re-use them.

4.4 Combinations

A combination of techniques can be used to improve the CA model by introducing either more suitable grid layouts or continuous layouts, and modelling the transition functions with sophisticated techniques. We have seen fuzzy logic being used in sever of the papers presented in this report. We have also seen a technique that uses a continuous layout with traditional transition functions. Probabilistic models can also be used to create more realistic agent behaviours. Usually these techniques work well to make the models more realistic, however they may also compromise the explainable functionality of the models and might need large amounts of data. Instead of processing data to learn a model, one can use the data to model a CA grid. This is a form of background knowledge which can be useful for creating more realistic simulations. Many of the papers seen

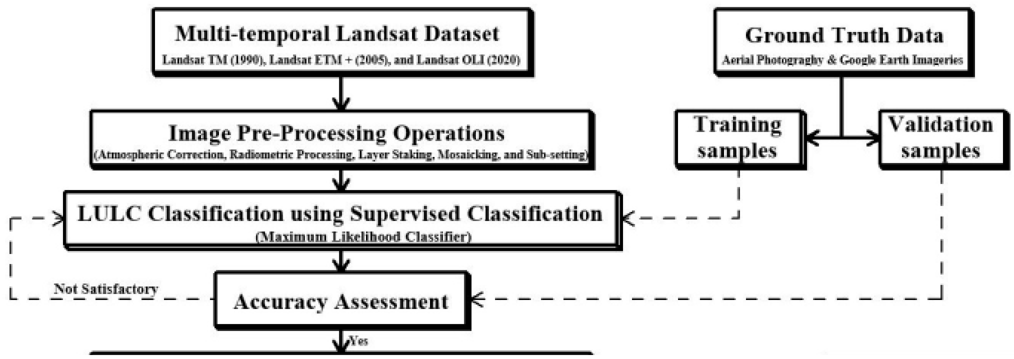


Figure 13: Combining image processing with a CA model

combine different approaches and cell layouts to better suit their tasks. This is easier done when more traditional transition functions are used because of their flexibility. It consists of simply adding constraints that better suit the task and agents of the simulation.

4.5 Validation

Validation of the CA model usually happens by comparing it to other types of models that are deemed to perform well, or it is compared to past data. Normally when compared to other models, usually these models are only good at performing the task that is chosen, and are not easily modified to fit other tasks or terrains. Because CA models are very flexible, they are able to be implemented with one task in mind and still work properly with other tasks. They are also able to be implemented fast and do can be more efficient than other data hungry methods. Because of this, it is possible to simply validate the CA models on one task by comparing it to other well performing methods, and then use the same CA model in many different installments for a divers range of tasks. Data can also be used to validate the model with needing to have trained a DL model specifically for the same task. In figure 13 we see that image pre-processing is used to manipulating data which is then classified and read by the transition functions. This is a more efficient method of using smaller amounts of data to validate the expansion of cities that is cost effective and sustainable.

The other method of validating is through data. Normally this is that is used when there is an abundance of data or there are no other models that are known to be well performing. This happens a lot in spatio-temporal behaviour modelling because there are many records from past expansions and cellular images that can be processed and used for validation. There are also not many deep learning models that are used for spatio-temporal modelling because it is too specific to the environment and so models tend to either over-fit because there is not enough verity for the models to train on. These are also valid reason as to why one would opt into using a CA approach rather than a deep learning approach when modelling these types of behaviours. That said, Deep learning can still play a large role in processing the data and finding good heuristics that aid with the performance of the transition functions.

5 Open Issues & Future Work

As the reader may have noticed, there is still a lot to be done surrounding the modelling of behaviours in simulations using CAs. There are many techniques that are readily available to be implemented into the domain of ITS that were introduced in other domains such as evacuation modelling. There is also room for combining techniques together such as fuzzy logic and machine learning using different types and shapes of CA cells. Because CAs are very flexible in their construction, there are many ways of approaching the same problems differently, and techniques that worked for one task, may as well work for another. While exploring this topic, I also developed some ideas that may be worth exploring but that were not prevalent in the literature. In Table 2 there are some of the topics that we found most interesting.

Table 2: Current and future work

Status	Idea	Reference
Not explored	Fuzzy + DL CA + DL Pedestrians + bike-lane Variety of road types Spatio-temporal (America)	N/A
Somewhat explored	Drones Fine grid Ships Spatio-temporal	[24] [25] [22] [23] [9] [38] [10] [11]
Explored	Lattice gas Fuzzy inference system	[40], [50] [13], [12]
Thoroughly explored	Evacuations Simulations Buses	[50] [32] [6] [34] [31] [39] [32] [43] [19] [37] [17]

6 Conclusion

In conclusion, this text has presented an overview of the applications of cellular automata in pedestrian and spatio-temporal modelling. We have discussed the importance of modelling dynamics accurately and the different approaches that have been studied in the literature, ranging from heuristic-based to data-driven models. While deep learning models have gained popularity in recent years, CA remains a useful and intuitive technology that does not require large amounts of data.

This text has focused on CA in settings where transportation is available, rather than just evacuation systems, which was the focus of previous reviews. We have presented a variety of applications proposed since 2019, with a few exceptions, and discussed the pros and cons of each method. We have also addressed previous concerns and challenges and the methods and technologies that have been able to overcome them. Additionally, we have presented new challenges and discussed those that have not yet been solved.

The purpose of this text was to provide the reader with material that will allow them to better understand the powerful capacities of CA and their applicable usability to pedestrian dynamics. We have provided a short introduction to the CA structure, as well as other concepts relating to pedestrian and spatio-temporal modelling. We have also highlighted the importance of spatio-temporal modelling for understanding the dynamics of expanding cities and areas of interest, which can have a significant impact on the dynamics of pedestrians and their modelling.

Moving forward, future work needs to be done to better the results of pedestrian simulations as a whole. In section 5, we discussed the open issues and challenges that need to be addressed, such as further modelling road structure yet to be explored, develop simulations for determining the expansion of cities in the Americas and continue to explore combinations of techniques. As researchers continue to develop new and innovative techniques for modelling pedestrian dynamics, it is important to keep in mind the potential implications for improving infrastructure and transportation systems in cities in economical and sustainable manners.

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