Hw1 Problem 1

$$f(x) = \sqrt{1+x} = (1+x)^{1/2}$$

use (1.28) in text book

$$f(x) \approx 1 + \frac{1}{2} \cdot x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \frac{7}{256}x^5 - \frac{21}{1024}x^6$$

at $x = \pi/4$

the answer is 1.34

$$f(x) = \cos(x)$$

use (1.27)

$$f(x) \approx 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$$

answer = 0.707

Hw1 Problem 4

$$\tan^{-1}(x) = \int_0^x \frac{1}{1+t^2}\, dt$$

use (1.28)

$$= \int_0^x (1 - t^2 + t^4 - t^6 + t^8 - t^{10} \cdots)\, dt$$

$$= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \frac{x^{11}}{11} \cdots$$

Using $\tan^{-1}(1)$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} \cdots$$

to obtain 10^{-15} accuracy, the last term should be $< 10^{-15}$

$$\Rightarrow \frac{1}{2n-1} < 10^{-15} \Rightarrow n \approx 5 \times 10^{+14}$$

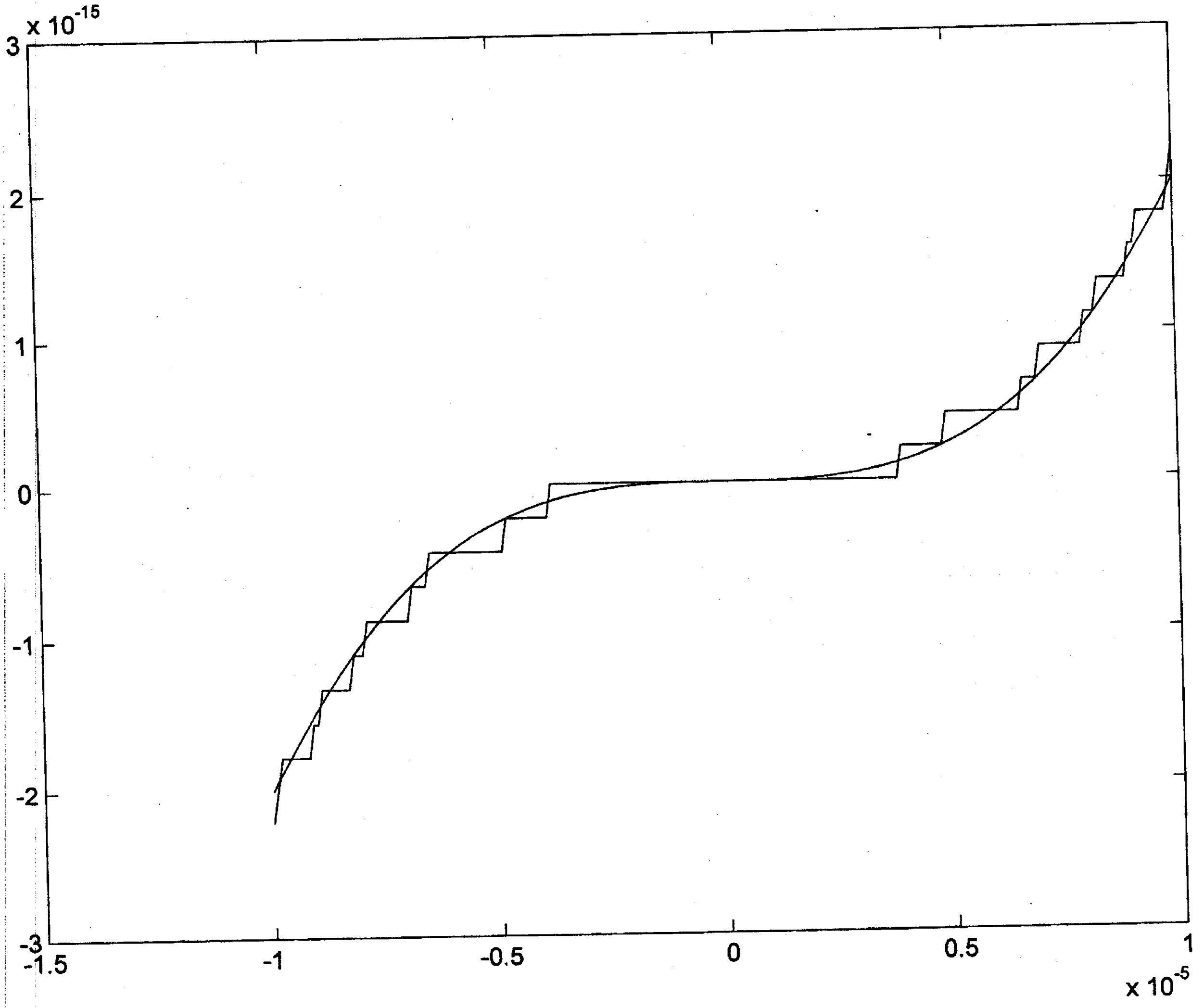
Using $\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{3})$

the last term $\approx \frac{1}{(2n-1)} \cdot \left(\frac{1}{2}\right)^{2n-1} < 10^{-15}$

$$\Rightarrow n \approx 23$$

Hw 1

Problem ~~2~~ 3 Rewrite as $\frac{2x^3}{1-x^6}$ to avoid loss-of-significance



Matlab code:

```
>>x = -0.00001 : 0.0000001 : 0.00001 ;
>>y = 1./(1-x.^3) - 1./(1+ x.^3);
>> z = (2.*(x.^3))./(1-x.^6);
>> plot (x,y,x,z);
```

 HW 1

Problem 2

```
>> format short
>> a=0;b=1;
>> c=(a+b)/2;
>> f=c-exp(-c);
>> while ((b-c)>0.005)
[a b c f]
if (f>0) b=c;
else a=c;
end
c=(a+b)/2;
f=c-exp(-c);
end
```

a	b	c	f(c)
0	1.0000	0.5000	-0.1065
0.5000	1.0000	0.7500	0.2776
0.5000	0.7500	0.6250	0.0897
0.5000	0.6250	0.5625	-0.0073
0.5625	0.6250	0.5938	0.0415
0.5625	0.5938	0.5781	0.0172
0.5625	0.5781	0.5703	0.0050

```
>> a=1;b=2;
>> c=(a+b)/2;
>> f=c^3-2*c-2;
>> while ((b-c)>0.005)
[a b c f]
if (f>0) b=c;
else a=c;
end
c=(a+b)/2;
f=c^3-2*c-2;
end
```

a	b	c	f(c)
1.0000	2.0000	1.5000	-1.6250
1.5000	2.0000	1.7500	-0.1406
1.7500	2.0000	1.8750	0.8418
1.7500	1.8750	1.8125	0.3293
1.7500	1.8125	1.7813	0.0891
1.7500	1.7813	1.7656	-0.0270
1.7656	1.7813	1.7734	0.0307

Hw 1

Problem 5 (a)

```
>> x=1;
>> x=x-(x^2-3)/(2*x)
x =
     2
>> x=x-(x^2-3)/(2*x)
x =
   1.750000000000000
>> x=x-(x^2-3)/(2*x)
x =
   1.732142857142857
>> x=x-(x^2-3)/(2*x)
x =
   1.732050810014728
>> x=x-(x^2-3)/(2*x)
x =
   1.732050807568877
>> x=x-(x^2-3)/(2*x)
```

~~x =~~
~~3.141592657879261~~
~~>> x=x-(x^2-9.869604401089358)/(2*x)~~
~~x =~~
~~3.141592653589793~~

~~[illegible]~~ Hw 1 #5(b)

```
>> x=2;
>> x=x-(x^3-3*x^2+3*x-1)/(3*x^2-6*x+3)
x =
   1.666666666666667
>> x=x-(x^3-3*x^2+3*x-1)/(3*x^2-6*x+3)
x =
   1.444444444444445
>> x=x-(x^3-3*x^2+3*x-1)/(3*x^2-6*x+3)
x =
   1.296296296296296
>> x=x-(x^3-3*x^2+3*x-1)/(3*x^2-6*x+3)
x =
   1.197530864197529
..........
>> for i=1:50
x=x-(x^3-3*x^2+3*x-1)/(3*x^2-6*x+3)
end
  .......
x =
   1.000026349609155
x =
   1.000017608139816
x =
   1.000011878818727
x =
   1.000007682559692
x =
   1.000002666445425
x =
   1.000002666445425
```

$f(x) = x^3 - 3x^2 + 3x - 1$

$f'(x) = 3x^2 - 6x + 3$

$$x \leftarrow x - \frac{x^3 - 3x^2 + 3x - 1}{3x^2 - 6x + 3}$$

$f(x)\big|_{x=1} = 0 \quad f'(x)\big|_{x=1} = 0 \quad f''(x)\big|_{x=1} = 0$

$f(x)$ has triple root at $x = 1$

Hence convergence is order 1

$\lambda = 2/3$.

It never quite goes to 1 because of errors in addition/subtraction, Loss of signi

Use $x \leftarrow x - \frac{(x-1)^3}{3(x-1)^2}$ instead. $\Rightarrow$ Converges $\underline{\underline{1.}}$

~~[illegible]~~ Hw 2 #3

```
>> x=1;
>> x=x-(x-sin(x))/(1-cos(x))
x =
   0.655145072042430
>> xnew=x-(x-sin(x))/(1-cos(x))
xnew =
   0.433590368363493
>> xnew/x
ans =
   0.661823444709375
>> x=xnew;
>> xnew=x-(x-sin(x))/(1-cos(x))
xnew =
   0.288148400892501
>> xnew/x
ans =
   0.664563657121962
>> x=xnew;
>> xnew=x-(x-sin(x))/(1-cos(x))
xnew =
   0.191832312150639
```

$\lambda \approx 0.6665 \approx 2/3$

$\Rightarrow m = 3$

$f(x) = x - \sin x \Rightarrow f(0) = 0$

$f'(x) = 1 - \cos x \Rightarrow f'(0) = 0$

$f''(x) = -\sin x \Rightarrow f''(0) = 0$

$f'''(x) = -\cos x \Rightarrow f'''(0) = -1$

$\Rightarrow x = 0$ is triple root

$m = 3$.

```
>> xnew/x
ans =
   0.665741373391154
>> x=xnew;
>> xnew=x-(x-sin(x))/(1-cos(x))
xnew =
   0.127809667560708
>> xnew/x
ans =
   0.666257243776243
>> x=xnew;
>> xnew=x-(x-sin(x))/(1-cos(x))
xnew =
   0.085183233602864
>> xnew/x
ans =
   0.666485057262221
>> x=xnew;
>> xnew=x-(x-sin(x))/(1-cos(x))
xnew =
   0.056781952786616
>> xnew/x
ans =
   0.666586021509134
==============================================================
```

Hw 2 Problem 1

~~$X = \cos X$~~ $X = \cos X$

here $g(X) = \cos X$

$\alpha \approx 0.74$ ← fixed point

$g'(x) = -\sin x$

$g'(\alpha) = -0.67 \Rightarrow |g'(\alpha)| < 1$

hence converges.

$X = \sin X$

$g'(x) = \cos X$

$\alpha = 0$

$g'(\alpha) = 1 \longrightarrow$ so cannot conclude!

but it converges practically

although convergence is very slow

bonus question:

$x = \sin x$

$\Rightarrow x \approx x - \frac{x^3}{6}$ ← using taylor's series

$x(n+1) = x(n) - \frac{x(n)^3}{6}$

$\Rightarrow \frac{dx}{dn} = -\frac{x^3}{6}$

$\Rightarrow \frac{6\,dx}{-x^3} = dn$

$\Rightarrow \int_{x_0}^{\delta} \frac{6}{-x^3}\,dx = \int_0^N dn$

$\Rightarrow \frac{18}{\delta^2} - \frac{18}{x_0^2} = N$

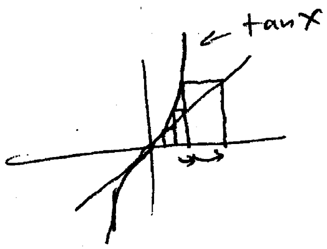
$\Rightarrow N = O\left(\frac{1}{\delta^2}\right)$

$x = \tan x$

again $g'(\alpha) = 1$

but in this case it diverges.

the divergence can be explained graphically.



Hw 2

Problem 2

$g(x) = x + c(x^2 - 9)$

$g'(\alpha) = 1 + 2c\alpha$ we need $-1 < g'(\alpha) < 1$

at $\alpha = 3$ $\Rightarrow$ $-1 < 1 + 6c < 1$ $\Rightarrow$ $-\frac{1}{3} < c < 0$ at $c = -\frac{1}{6}$ $g'(\alpha) = 0$ $\Rightarrow$ faster convergence

similarly for $\alpha = -3$ $0 < c < \frac{1}{3}$ & $c = \frac{1}{6}$ for faster convergence.