CS 2262: Numerical Methods
Mid-Semester Exam 1

Tuesday 3/3/09 3:10-4:30pm
Maximum Score 60
Answer any 3 questions

1. (20 points) Find the interpolating polynomial $P(x)$ passing through the points: $(1,3), (2,8), (3,9), (4,0)$. Use Newton's Method to find approximately a point c (upto two digit accuracy) such that $P(c) = 1$.

2. (a) (10 points) Obtain Taylor's series for the function

$$\log \sqrt{\frac{1+x}{1-x}}$$

   upto 4 terms. Find its approximate value at $x = 1/2$.
   (b) (10 points) For some Newton's Method, following are the iterative values of x_i's:
   $1.178138, 1.244435, 1.299317, 1.344843, 1.382663, ...$ What is the multiplicity m of this root? Also can you obtain an approximate value of the root (where the sequence will ultimately converge)?

3. (20 points) Let $f(x) = x^3 - 3x^2 + 4$. Find two of its roots using Newton's method. Start with $x_0 = -2$ and $x_0 = 1$ in each case and calculate upto 3 iterations. What is the order of convergence at each root?

4. (20 points) Consider fixed point iterations for $x_{n+1} = x_n^2 - 2 + 2\cos x_n$. That is $g(x) = x^2 - 2 + 2\cos x$. Find its order of convergence at the fixed point $\alpha = 0$. According to Equation (3.38) in Theorem (3.4.2) of the text book, for what (positive) starting values $x_0 > 0$, will this iteration converge? Can you obtain a broader range of starting values than what Equation (3.38) suggests?