

CSC 2262 Homework Assignment 4

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70 points Due: 4/1/09 in class

1: Simpson's Rule (20 points)

Evaluate following integrals using Simpson's Rule and Corrected Simpson's Rule(or Richardson Extrapolation in case the corrected Simpson's is not applicable). In each case, use $n = 2, 4, 8, 16, 32$. Tabulate your results, along with error. Use the ratios of successive errors to find p such that error is proportional to n^{-p} .

- (a) $\int_0^1 \frac{4}{1+x^2} dx$
(b) $\int_0^1 \sqrt{x} dx$

2: Numerical Integration (30 points)

Consider the following rule for evaluating integrals numerically. Given an integral $\int_a^b f(x)dx$, we divide range $[a, b]$ into even number of partitions, say n . Let $h = (b - a)/n$. Then, we choose x_i 's for $i \in [1, n]$ as follows: For odd value of i , $x_i = a + (i - 1/\sqrt{3})h$ and for even value of i , $x_i = a + (i - 1 + 1/\sqrt{3})h$. We define the integration rule as $N_n(f) = h \sum_{i=1}^n f(x_i)$.

- (a) Using this rule find $\int_0^1 \frac{1}{1+x} dx$. Use $n = 2, 4, 8, 16, 32$. Find p as in question 1.
(b) Apply Richardson Extrapolation (pg 210 eqn 5.42) to this formula to find more accurate value for the above integral. Call this corrected rule as $CN_n(f)$. Tabulate the values and errors. What is p after applying this correction? (i.e., p for this corrected rule.)

You will need to modify the trapez.m program given in class to convert it to program for this rule.

3: Numerical Differentiation(20 points)

Find a more accurate formula for $f'(t)$ using method of undetermined coefficient. Let's say formula is of the form

$$f'(t) = Af(t+2h) + Bf(t+h) - Bf(t-h) - Af(t-2h)$$

Find the values of A and B such that error is minimized. What power of h is the error proportional to? Use this formula to find $\frac{d}{dx} \sin(x)$ at $x = \pi/3$. Tabulate errors for different values of h .