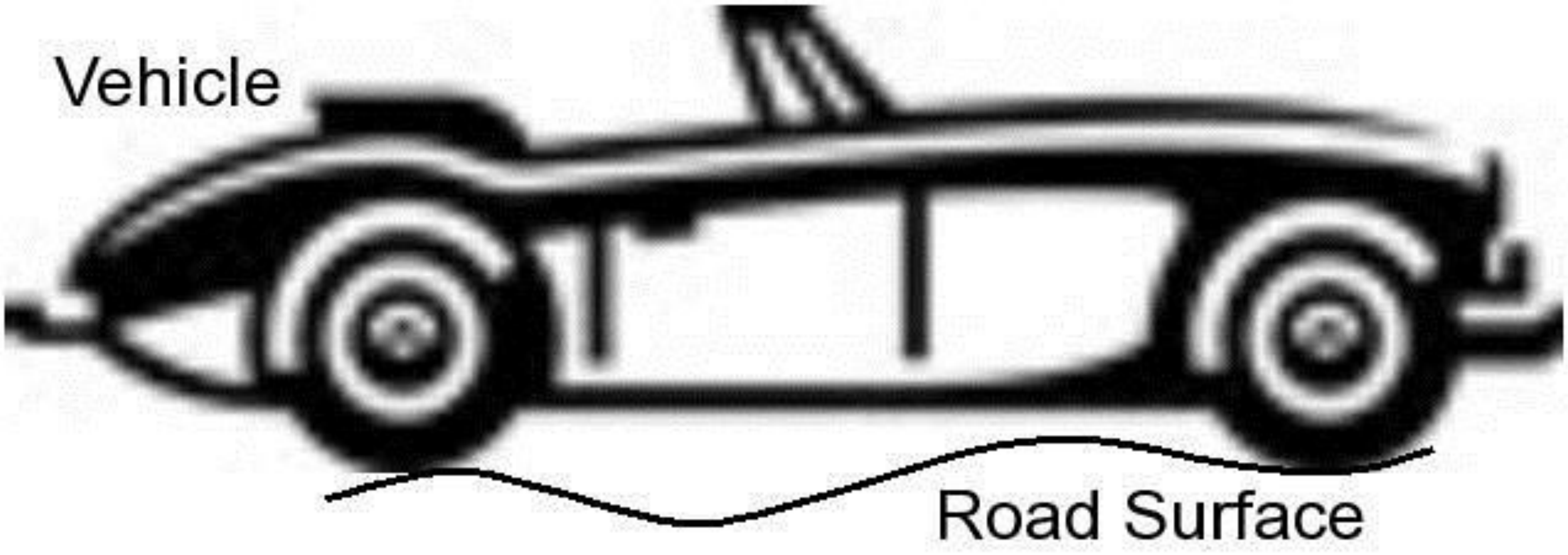
SIMULATION OF THE HALF-CAR MODEL


Vehicle
Road Surface

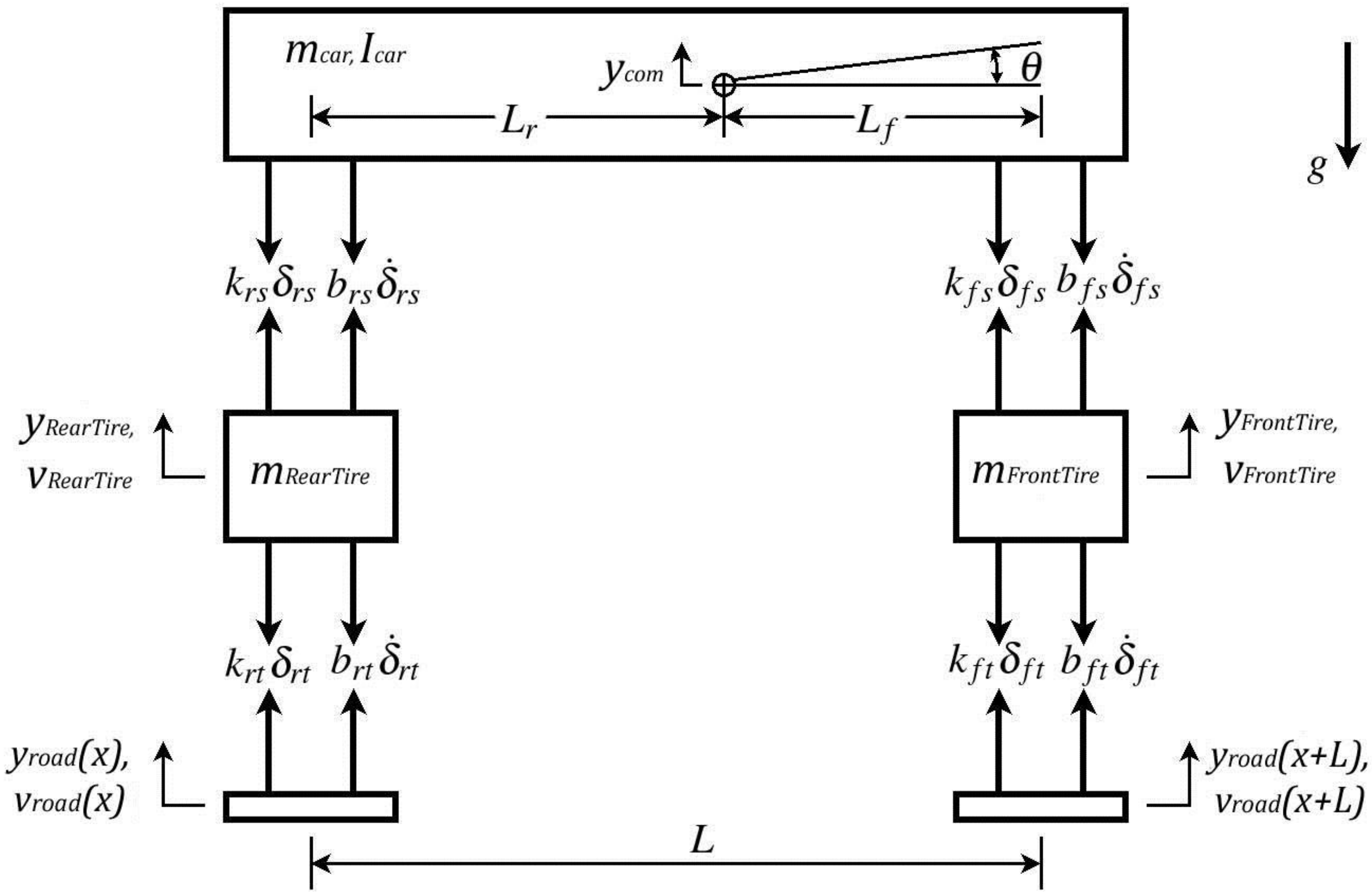
Describing the Half-Car

- Suspension: Springs and dampers
- Tires: Springs (wall stiffness) and dampers (air pressure)
- Second-order governing equations
- 8 state-variables: $\theta, y_{com}, y_{rt}, y_{ft}, \dot{\theta}, \dot{y}_{com}, \dot{y}_{rt}, \dot{y}_{ft}$

User Inputs

- road surface
- spring constants and damper coefficients
- free length of all springs
- tire masses
- car mass
- car length
- location of center of mass

Free Body Diagram



Governing Equations

- <u>Rear Tire</u>

$$\Sigma F = m_{rt}\ddot{y}_{rt} = -m_{rt}g + k_{rt}\delta_{rt} + b_{rt}\dot{\delta}_{rt} - k_{rs}\delta_{rs} - b_{rs}\dot{\delta}_{rs}$$

- $\delta_{rt} = y_{rt} - y_{road}(x) - fl_{rt}$,
- $\delta_{rs} = y_{com} - Lr * \sin(\theta) - y_{rt} - fl_{rs}$

  fl_{rt} is the ‘length’ of the tire under gravity

  fl_{rs} is the length of the rear spring under gravity

- <u>Front Tire</u>

$$\Sigma F = m_{ft}\ddot{y}_{ft} = -m_{ft}g + k_{ft}\delta_{ft} + b_{ft}\dot{\delta}_{ft} - k_{fs}\delta_{fs} - b_{fs}\dot{\delta}_{fs}$$

- $\delta_{ft} = y_{ft} - y_{road}(x + L) - fl_{ft} \quad ,$
- $\delta_{fs} = y_{com} + Lf * \sin(\theta) - y_{ft} - fl_{fs}$

  fl_{rt} is the 'length' of the tire under gravity

  fl_{rs} is the length of the front spring under gravity

- <u>Car Body</u>

$$
\begin{aligned}
\Sigma\tau = I_{car}\ddot{\theta} \\
&= \left(-k_{fs}\delta_{fs} - b_{fs}\dot{\delta}_{fs}\right)Lf\cos(\theta) \\
&+ \left(k_{rs}\delta_{rs} + b_{rs}\dot{\delta}_{rs}\right)Lr\cos(\theta)
\end{aligned}
$$

$$
I_{car} = \frac{1}{3L}(LrLf^2 + LfLr^2)m_{car}
$$

$$
\begin{aligned}
\Sigma F = m_{car}\ddot{y}_{com} \\
&= -m_{car}g - k_{fs}\delta_{fs} - b_{fs}\dot{\delta}_{fs} - k_{rs}\delta_{rs} - b_{rs}\dot{\delta}_{rs}
\end{aligned}
$$

What’s In the Simulation:

- Runge-Kutta4 algorithm solves the equations over discrete time steps
- Road and car are plotted over time
- Show demonstration

Test Calculation

- In the case of the car at rest and sitting on a flat surface, $\theta, y_{com}, y_{rt}, y_{ft}$ can be solved analytically (but not easily)
- Evaluate static equilibrium equations
  - set forces equal to zero, gives a system of equations
  - Easier to work with linear equations. To linearize, use $Lf\cos(\theta) = y_{FrontCar}$ and $Lr\cos(\theta) = y_{RearCar}$

$$\begin{bmatrix} -k_{fs} & -k_{rs} & k_{fs} & k_{rs} \\ -k_{fs}L & 0 & k_{fs}L & 0 \\ 0 & k_{rs} & 0 & -(k_{rt}+k_{rs}) \\ k_{fs} & 0 & -(k_{ft}+k_{fs}) & 0 \end{bmatrix} \begin{bmatrix} y_{FrontCar} \\ y_{RearCar} \\ y_{FrontTire} \\ y_{RearTire} \end{bmatrix}$$

$$= \begin{bmatrix} m_{car}g \ - k_{rs}fl_{rs} - k_{fs}fl_{fs} \\ m_{car}g \ - k_{fs}L\ fl_{fs} \\ m_{rt}g = k_{rt}fl_{rt} + k_{rs}fl_{rs} - k_{rt}y_{road}(x) \\ m_{ft}g - k_{ft}fl_{ft} + k_{fs}fl_{fs} - k_{ft}y_{road}(x+L) \end{bmatrix}$$

Test Calculation

- Drop car from height, wait for transient response to decay, observe values of $y_{rt}, y_{ft}, y_{fCar}, y_{rCar}$ and compare to equilibrium values

- Test the simulation

References

- http://mece.utpa.edu/Kypuros/teaching/mece-4305/notes/VehicleSuspensionModelingNotes.pdf

- Dixon, John. The Shock Absorber Handbook.

- http://www.library.cmu.edu/ctms/ctms/simulink/examples/susp/suspsim.htm