AATM 504 – Radiation Basics and Laws

Crizzia Mielle De Castro
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Homework 1

Problem 1

Questions

(a) Convert the following wavelengths to units of frequency (Hz): 1.0 nm, 1.0 μm, 10 mm, and 10 cm. You can use the approximate speed of light is $c = 3 \times 10^8\ m/s$.

(b) What is the energy in a photon of wavelength $0.45\mu m$? Of wavelength $10\mu m$? How many times more energy is there in a photon of $0.45\mu m$ compared to a photon of 10 μm?

(c) Given the emitted radiance of 50 $Wm^{-2}sr^{-1}$ integrated over the wavelength region from $8.3 - 9.1\ \mu m$ (i.e., $\int_{\lambda_1}^{\lambda_2} I_\lambda d\lambda = 50\ Wm^{-2}sr^{-1}$), what is the average spectral emitted radiance within this wavelength interval ($Wm^{-2}\mu m^{-1}sr^{-1}$)?

Solution.

(a) The speed of light is given by $c = \lambda f$, where λ is the wavelength and f is the frequency. Thus,

$$f = \frac{c}{\lambda} \tag{1.1}$$

Using our given wavelengths, we get the following.

$$f_1 = \frac{3 \times 10^8\ m/s}{1.0 \times 10^{-9} m} = 3 \times 10^{17} s^{-1} = 3 \times 10^{17}\ Hz \tag{1.2}$$

$$f_2 = \frac{3 \times 10^8\ m/s}{1.0 \times 10^{-6} m} = 3 \times 10^{14} s^{-1} = 3 \times 10^{14}\ Hz \tag{1.3}$$

$$f_3 = \frac{3 \times 10^8\ m/s}{10 \times 10^{-3} m} = 3 \times 10^{10} s^{-1} = 3 \times 10^{10}\ Hz \tag{1.4}$$

$$f_4 = \frac{3 \times 10^8\ m/s}{10 \times 10^{-2} m} = 3 \times 10^{9} s^{-1} = 3 \times 10^{9}\ Hz \tag{1.5}$$

(b) The energy of a photon is given by

$$E = h\frac{c}{\lambda} \tag{1.6}$$

where $h = 6.63 \times 10^{-34}\ J \cdot s$ is Planck's constant. Thus, we get the energy at $\lambda = 0.45\mu m$.

$$E_1 = (6.63 \times 10^{-34}\ J \cdot s)\frac{3 \times 10^8\ m/s}{0.45 \times 10^{-6} m} = 4.4 \times 10^{-19}\ J \tag{1.7}$$

At $\lambda = 10\ \mu m$,

$$E_2 = (6.63 \times 10^{-34}\ J \cdot s)\frac{3 \times 10^8\ m/s}{10 \times 10^{-6}\ m} = 2.0 \times 10^{-20}\ J \tag{1.8}$$

By getting the ratio between E_2 and E_1, we have

$$\frac{E_1}{E_2} = \frac{4.4 \times 10^{-19}\ J}{2.0 \times 10^{-20}\ J} = 22 \tag{1.9}$$

The energy at $\lambda = 0.45\ \mu m$ is 22 times the energy at $\lambda = 10\ \mu m$.

(c) Here, we get the average of a definite integral using the equation

$$\frac{1}{\lambda_2 - \lambda_1} \int_{\lambda_1}^{\lambda_2} I_\lambda d\lambda \tag{1.10}$$

We already know the value of our integral to be 50 $Wm^{-2}sr^{-1}$, so the average spectral emitted radiance is just

$$\frac{50\ Wm^{-2}sr^{-1}}{9.1\ \mu m - 8.3\ \mu m} = 62.5\ Wm^{-2}\mu m^{-1}sr^{-1} \tag{1.11}$$

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Problem 2

Solid angles: During a total solar eclipse, the moon entirely covers (almost exactly) the sphere of the sun.

(a) Write down the mathematical relation between the distances and sizes of the sun and moon and discuss the meaning of the relationship in your own words.

(b) Look up the radius of the Moon. Then measure the diameter of a coin (say a dime or a quarter) and apply the equations to estimate how far you would have to hold the coin in front of your eye (one eye view) to cover the moon. Is an arm length sufficient? Do you need any other information to solve the calculations?

(c) The moon drifts away from Earth at about 1.5 inches per year (according to NASA). Assuming that at present the moon has the same solid angle as the sun for a viewer on Earth, and all other things constant except R_{me} , how long would it take before the moon's solid angle would be only 50% of its current value?

Solution.

(a) The solid angle Ω is given by $\Omega = \frac{A}{r^2}$ where A is the area of the region of interest, and r is the distance between the reference point and the region of interest. In this problem, our regions of interest are the Moon and the Sun, and our reference point is an observer on Earth viewing a total solar eclipse. From the observer's POV, the Moon almost covers the Sun, so their solid angles must be equal. In other words,

$$\Omega = \frac{A_s}{R_{se}^2} = \frac{A_m}{R_{me}^2} \tag{2.1}$$

where A_s is the area of the Sun, A_m is the area of the Moon, R_{se} is the distance of the Earth to the Sun, and R_{me} is the distance of the Earth to the Moon. Here, we use the area of a circle ($2\pi r^2$) to calculate A_s and A_m. Thus,

$$\Omega = \frac{2\pi R_s^2}{R_{se}^2} = \frac{2\pi R_m^2}{R_{me}^2} \tag{2.2}$$

In our diagram, θ is just half of the solid angle, so

$$\theta = \frac{\pi R_s^2}{R_{se}^2} = \frac{\pi R_m^2}{R_{me}^2} \tag{2.3}$$

Rewriting this, we have

$$\frac{R_s^2}{R_{se}^2} = \frac{R_m^2}{R_{me}^2} \tag{2.4}$$

Simplifying further, we finally get the following relation

$$\frac{R_s}{R_{se}} = \frac{R_m}{R_{me}} \quad or \quad \frac{R_s}{R_m} = \frac{R_{se}}{R_{me}} \tag{2.5}$$

(b) The Moon's radius is about $1,737.5\ km$ or $1737.5 \times 10^3\ m$ (Dobrijevic and Sharp, 2022). On the other hand, the radius of a quarter is about $12.13\ mm$ or $12.13 \times 10^{-3} m$. For this problem, we also need the distance between the Earth and the Moon, which is about $384,400\ km$ or $384400 \times 10^3\ m$. By rewriting the final relation we obtained in (a), we have

$$\frac{R_q}{R_m} = \frac{R_{qe}}{R_{me}} \tag{2.6}$$

where R_q is the radius of a quarter, and R_{qe} is the distance between my eye and the quarter. We want to solve for R_{qe}, so

$$R_{qe} = R_q \frac{R_{me}}{R_m} \tag{2.7}$$

Using our given values, we get

$$R_{qe} = 12.13 \times 10^{-3}\ m \cdot \frac{384400 \times 10^3\ m}{1737.5 \times 10^3\ m} = 2.68\ m \tag{2.8}$$

Here, we notice that an arm length is not enough.

(c) From (a), the solid angle of the Moon is

$$d\Omega_m = \frac{2\pi R_m^2}{R_{me}^2} \tag{2.9}$$

At 50% of its original value, it is simply $\frac{\pi R_m^2}{R_{me}^2}$. At this point, the distance between the Earth and the Moon should be $\sqrt{2}R_{me}$, since

$$\frac{d\Omega_m}{2} = \frac{2\pi R_m^2}{(\sqrt{2}R_{me})^2} \tag{2.10}$$

Again, the current radius of the moon is $1737.5 \times 10^3\ m$, so the new radius is $1737.5\sqrt{2} \times 10^3\ m$ or about $2457 \times 10^3\ m$. The moon drifts away from the Earth at about $0.0254\ m$ per year. Thus,

$$t = 1737.5\sqrt{2} \times 10^3\ m \cdot \frac{1\ \text{year}}{0.0254\ m} = 96,740\ \text{years} \tag{2.11}$$

It would take about $96,740$ years for the moon's solid angle to be 50% of its current value.

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Problem 3

Calculations: Consider the surface of a blackbody with a temperature (T) of 300 K (e.g., Earth's surface temperature) and 6000 K (e.g., Sun's surface temperature), respectively (suggestion: using excel or coding):

(a) Calculate the spectral emitted radiance as a function of wavelength (using an interval of 0.02 μm from 0 to 4 μm) for the blackbody with $T = 6000\ K$ and plot the results (radiance as Y-axis and wavelength as X-axis).

(b) Calculate the spectral emitted radiance as a function of wavelength (using an interval of 0.5 μm from 0 to 100 μm) for the blackbody with $T = 300\ K$ and plot the results (radiance as Y-axis and wavelength as X-axis).

(c) Find the wavelength of maximum radiance for 1) and 2) from your plots and compare your estimates with those calculated from Wien's displacement law.

(d) Look up information online. Which frequency bands do the 5G mobile phone networks transmit on (in the United States). The highest frequency band that you can find, report frequency and wavelength. Where are these frequencies located relative to the spectral maximum of the 300 K blackbody emitter? Is it well below or above the 300 K maximum wavelength (see Wien's Law)?

Solution.

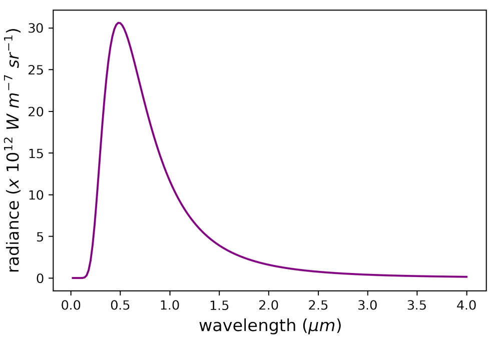
(a) From section 4.3.1 in Wallace and Hobbs, the spectral emitted radiance of a blackbody is given by

$$B_\lambda(T) = \frac{c_1 \lambda^{-5}}{\pi(e^{c_2/\lambda T} - 1)} \tag{3.1}$$

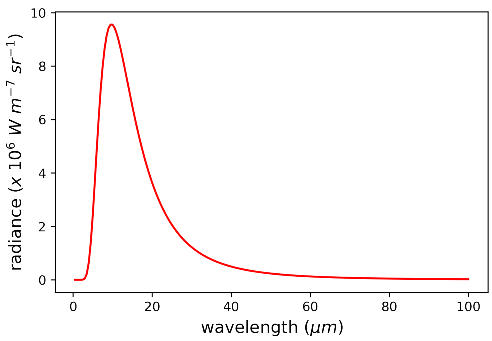
where $c_1 = 3.74 \times 10^{16}\ W m^2$ and $c_2 = 1.45 \times 10^{-2} m\ K$. At $T = 6000\ K$,

$$B_\lambda(T = 6000\ K) = \frac{c_1 \lambda^{-5}}{\pi(e^{c_2/\lambda \cdot 6000\ K} - 1)} \tag{3.2}$$

The plot of the results for (a) and (b) are shown below.



(a) radiance at $T = 6000\ K$



(b) radiance at $T = 300\ K$

(b) Using the same equation at $T = 300\ K$,

$$B_\lambda(T = 300\ K) = \frac{c_1 \lambda^{-5}}{\pi(e^{c_2/\lambda \cdot 300\ K} - 1)} \tag{3.3}$$

(c) Using *Python*'s *numpy.argmax* function, I estimated the wavelengths of maximum radiance. At $T = 6000\ K$, the wavelength at maximum is 0.48 μm, while at $T = 300\ K$, the wavelength at maximum is 9.5 μm. Wien's displacement law is given by

$$\lambda_m = \frac{2897}{T} \tag{3.4}$$

Using Wien's displacement law, the wavelength at maximum for $T = 6000\ K$ is 0.48 μm. On the other hand, the wavelength at maximum for $T = 300\ K$ is 9.66 μm. My estimates from the plot match well with the expected values using Wien's displacement law.

(d) The frequency bands used by the three big cellular networks for millimeter wave 5G are n260 (39 GHz) and n261 (28 GHz) (Coverage Critic, 2021). The following frequency bands are used for sub-6 5G: n5 (850 MHz), n41 (2500 MHz), n71 (600 MHz), and n77 (3700 MHz). Thus, the highest frequency band is at 39 GHz (n260). To get the wavelength, we use $\lambda = \frac{c}{f}$.

$$\lambda = \frac{3\ \times\ 10^8\ m/s}{39\ \times 10^9\ s^{-1}} = 0.0077\ m = 7700\ \mu m \tag{3.5}$$

By comparing this wavelength to our plots, we see that the highest 5G frequency band is well above the wavelength at maximum radiance for $T = 300\ K$.

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Problem 4

Readings and Questions: Earth's Atmosphere. Venus, Earth, Mars are sibling planets in our solar system. They all have atmospheres, weather surfaces, massive volcanoes, and chemically and thermally evolved interiors. Comparison of them can provide a context of how unique or not our planet Earth is. It also helps to have a basic knowledge of the properties of our two nearest neighboring planets. Please answer the following questions:

(a) **Atmospheric composition**: Summarize the major features of atmospheric composition of these three planets.

(b) Given that the**Top of Atmosphere (TOA) incoming solar irradiance** depends on the distance squared FTOA $1/r^2$, look up the planet's orbits and calculate their respective TOA values using Earth's value (1361 W/m^2).

(c) **Surface temperatures**: What is the mean surface temperature of these three planets? Mars and Venus have very similar types of greenhouse gases in their atmospheres. Explain why Venus has an extremely high surface temperature while Mars has a very low surface temperature.

(d) **Vertical temperature profile**: The vertical temperature profile below 100km is different among these three planets. Earth is the only planet with a unique temperature "bump" (i.e., temperature increases with height) for a stratosphere, while the temperature of Venus and Mars decreases with altitude. Explain what causes this difference and why this only happened in the Earth (use online research tools to find answers. Cite your resources, and try to validate your sources with at least one alternative independent online resource, or book etc.)?

Solution.

(a) Earth's atmosphere is primarily composed of nitrogen ($\sim 78\%$) and oxygen ($\sim 21\%$). Other gases comprise the remaining $\sim 1\%$. In fact, nitrogen, oxygen, and argon already comprise about 99.96% of the gases in the atmosphere, so the remaining gases are called trace gases. On the other hand, Venus and Mars have very similar primary atmospheric compositions: carbon dioxide (CO_2), and nitrogen. Venus is composed of $\sim 96\%$ CO_2 and $\sim 3.5\%$ nitrogen, while Mars is composed of $\sim 95\% CO_2$ and $\sim 2.7\%$ nitrogen. The remaining 1% is mainly composed of argon and water vapor. However, it's important to remember that Venus has a much thicker atmosphere than Earth (about $100\times$), while Mars has a much thinner atmosphere (about 1% of Earth's).

(b) The radius of Venus' orbit is about $108,200,00\ km$, Mars' orbit is about $228,000,000\ km$, and Earth's orbit is about $149,600,000\ km$. We first get the squared ratio between Earth's orbital radius and the other planets' orbital radius.

$$\left(\frac{r_e}{r_v}\right)^2 = \left(\frac{149,600,000\ km}{108,200,00\ km}\right)^2 = 1.91 \tag{4.1}$$

$$\left(\frac{r_e}{r_m}\right)^2 = \left(\frac{149,600,000\ km}{228,000,00\ km}\right)^2 = 0.431 \tag{4.2}$$

From here, we multiply these ratios to Earth's TOA incoming solar irradiance.

$$\text{TOA}_{\text{Venus}} = 1.91 \cdot 1361\ W/m^2 = 2602\ W/m^2 \tag{4.3}$$

$$\text{TOA}_{\text{Mars}} = 0.431 \cdot 1361\ W/m^2 = 586\ W/m^2 \tag{4.4}$$

As expected, Venus has the highest value, since it's closest to the Sun, and Mars has the lowest value, since it's farthest.

(c) The mean surface temperature on Venus is 467^oC, -46^oC on Mars, and 14^oC on Earth. Despite having almost the same chemical compositions, Venus is much hotter than Mars, because it has a much thicker atmosphere. In other words, there is a much higher amount of CO_2 in Venus than in Mars, which leads to a stronger greenhouse effect. The high amounts of CO_2 traps the outgoing radiation from Venus, thereby heating it up. On the other hand, the very low amounts of CO_2 in Mars can't trap enough outgoing radiation to heat its surface up.

(d) Earth is the only planet with a unique temperature "bump" below 100 km among the three, since it's the only one with an ozone layer within the stratosphere (Prinn and Fegley, 1987). The ozone layer in Earth's stratosphere absorbs UV solar radiation, which causes the increasing temperature and ultimately, the "bump" or maximum temperature at the stratopause(Strobel, 2022).

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References

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