AATM 504 – Radiation Laws, Atmospheric Scattering, Absorption and Refraction

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Homework 4

Problem 1

Questions

1. What's the unit mass extinction coefficient and unit volume extinction coefficient, respectively? What's their relationship?

2. a) What is the atmospheric *normal optical depth* (τ_λ)?
   b) Given the TOA and BOA irradiance at wavelength λ relative to local surface normal are denoted as $F_\lambda(\text{TOA})$, and $F_\lambda(\text{BOA})$, formulate the total atmospheric optical depth and transmissivity.
   c) If the incoming light has an elevation angle (θ) relative to surface horizon (not to surface normal!), what is the total atmospheric optical depth and transmissivity for the incoming light?

3. The Earth's atmosphere consists of air molecules, aerosols, and clouds, with the following total unit mass extinction coefficients at wavelength λ (k_{ext}, λ):

$$k_{\text{ext},\lambda}(\text{atm}) = k_{\text{ext},\lambda}(\text{molecules}) + k_{\text{ext},\lambda}(\text{aerosols}) + k_{\text{ext},\lambda}(\text{clouds}) \tag{1.1}$$

   And the optical depth can be defined similarly:

$$\tau_\lambda(\text{atm}) = \tau_\lambda(\text{molecules}) + \tau_\lambda(\text{aerosols}) + \tau_\lambda(\text{clouds}) \tag{1.2}$$

   For air molecules, we only consider CO_2, O_2, O_3 and H_2O and assume that their total atmospheric optical depths at wavelength λ at the BOA are τ_1, τ_2, τ_3, and τ_4. For aerosols, we only consider two types of aerosols and assume that their total atmospheric optical depths at wavelength λ in BOA are τ_5 and τ_6. For clouds, we consider three types of clouds and assume that their total atmospheric optical depths at wavelength λ in BOA are τ_7, τ_8 and τ_9.

   If the TOA solar irradiation is $F_\lambda(z = \infty)$ and the sun is over head, formulate the BOA solar irradiation $F_\lambda(z = 0)$ in terms of
   a) optical depths (denoted as τ_1 to τ_9)
   b) transmissivity (denoted as T_1 to T_9)

Solution.

1. The unit mass extinction coefficient dictates how much one unit mass of gas scatters or absorbs as radiation hits or passes through it. On the other hand, the volume extinction dictates how much one unit volume of gas scatters or absorbs as radiation hits or passes through it. The rate of scattering or absorption can be written as

$$dI_\lambda = -I_\lambda \rho r k_\lambda ds \tag{1.3}$$

where I_λ is incident radiation, ρ is the density of air, r is the mass of absorbing gas per unit mass of air, and k_λ is the mass extinction coefficient. The volume extinction coefficient is just the $\rho r k_\lambda$.

2. a) According to Wallace and Hobbs, the normal optical depth is given by

$$\tau_\lambda = \int_z^\infty k_\lambda \rho r dz \tag{1.4}$$

It measures the cumulative extinction a beam of radiation oriented downwards at solar zenith angle $\theta = 0$ experiences passing through a layer of the atmosphere.

b) The total atmospheric optical depth at altitude z is

$$\tau_\lambda(z) = \int_\infty^z \rho(z) k_{ext}(z) dz = ln(F_\lambda(\text{TOA})/F_\lambda(z)) \tag{1.5}$$

where $\rho(z)$ and $k_{\text{ext}}(z)$ are the density and unit mass extinction coefficients at altitude z. For the total atmosphere from the top to bottom, this becomes

$$\tau_\lambda(\text{BOA}) = ln(F_\lambda(\text{TOA})/F_\lambda(0)) = ln(F_\lambda(\text{TOA})/F_\lambda(\text{BOA})) \tag{1.6}$$

Here, at $z = 0$, we consider the irradiance at the bottom of the atmosphere. The total transmissivity of the atmosphere is given by

$$T_\lambda = e^{-\tau_\lambda(\text{BOA}) \sec\theta} \tag{1.7}$$

where θ is the solar zenith angle. Using our equation for the total optical depth for the whole atmosphere, we have

$$\begin{aligned} T_\lambda &= e^{-\sec\theta ln(F_\lambda(\text{TOA})/F_\lambda(\text{BOA}))} \\ T_\lambda &= e^{ln(F_\lambda(\text{TOA})/F_\lambda(\text{BOA}))^{-\sec\theta}} \\ T_\lambda &= \left[\frac{F_\lambda(\text{BOA})}{F_\lambda(\text{TOA})}\right]^{\sec\theta} \end{aligned} \tag{1.8}$$

c) If θ is the angle from the surface horizon, then $90 - \theta$ would be the solar zenith angle from the surface normal. Getting the cosine of the solar zenith angle, we get

$$\cos(90 - \theta) = \cos(90)\cos\theta + \sin(90)\sin\theta = \sin\theta \tag{1.9}$$

Thus, the total optical transmissivity becomes

$$T_\lambda = \left[\frac{F_\lambda(\text{BOA})}{F_\lambda(\text{TOA})}\right]^{\csc\theta} \tag{1.10}$$

The total atmospheric optical length remains unchanged.

$$\tau_\lambda(\text{BOA}) = ln(F_\lambda(\text{TOA})/F_\lambda(\text{BOA})) \tag{1.11}$$

3. a) The total optical path length for the whole atmosphere at BOA is given by

$$\tau_\lambda(BOA) = \tau_1 + \tau_2 + \tau_3 + \tau_4 + \tau_5 + \tau_6 + \tau_7 + \tau_8 + \tau_9 = \sum_{n=1}^{9} \tau_n \tag{1.12}$$

From our equation for the total atmospheric optical depth, we see that

$$\begin{aligned}
e^{\tau_\lambda(\mathrm{BOA})} &= e^{ln(F_\lambda(\mathrm{TOA})/(F_\lambda(\mathrm{BOA}))} \\
e^{\tau_\lambda(\mathrm{BOA})} &= F_\lambda(\mathrm{TOA})/(F_\lambda(\mathrm{BOA}) \\
F_\lambda(\mathrm{BOA}) &= F_\lambda(\mathrm{TOA})e^{-\tau_\lambda(\mathrm{BOA})}
\end{aligned} \tag{1.13}$$

Plugging in our equation for the total optical path length, we get

$$F_\lambda(\mathrm{BOA}) = F_\lambda(\mathrm{TOA})e^{-\sum_{n=1}^{9}\tau_n} \tag{1.14}$$

b) For this problem, the sun is overhead, so $\theta = 0$. Thus, $\sec\theta = 1$. Isolating T_λ from our equation for the total atmospheric transmissivity, we get

$$F_\lambda(\mathrm{BOA}) = F_\lambda(\mathrm{TOA})T_\lambda \tag{1.15}$$

The total transmissivity can also be summed similar to the optical path length

$$T_\lambda(BOA) = T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7 + T_8 + T_9 = \sum_{m=1}^{9} T_m \tag{1.16}$$

Plugging this in, we have

$$F_\lambda(\mathrm{BOA}) = F_\lambda(\mathrm{TOA})\sum_{m=1}^{9} T_m \tag{1.17}$$

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Problem 2

Derivations

1. The Beer's law for the well-mixed solution of Rhodamine 6B indicate that the transmission (T) is an exponential function of the product of unit mass absorption coefficient k_{abs}, density ρ of solution and light pathlength L:

$$F(z=L) = F(z=0)e^{-\rho k_{\text{abs}} L} \tag{2.1}$$

Now **if the solution is not well mixed**, i.e., the unit mass absorption coefficient $k_{\text{abs}}(z)$ and density $\rho(z)$ of solution vary along the path length L, **rewrite the above equation in terms of integrals**.

2. The following slide gives the approximate solution of a plan-parallel atmosphere for general atmospheric radiative transfer equation.

Explain the physical meaning of the three terms in the bottom equation, including the left-hand term, and the 1st and 2nd right-hand term.

3. For terrestrial radiation, we have the following Schwartzchild equation:

$$I_\nu(\tau_\nu, \mu) = I_\nu(0, \mu)e^{\tau_\nu(z)/\mu} + \int_0^{\tau_v(z)} \mu^{-1} B_v\left(T\left(\tau_v'\right)\right) e^{\{(\tau_v' - \tau_v(z))/\mu\}} d\tau_v' \tag{2.2}$$

Explain the physical meaning of the three terms (both left- and right-hand terms) and of the following variables.

Solution.

1. If the solution is not well mixed, then the exponent needs to be integrated over all path lengths. In other words,

$$F(z=L) = F(z=0)e^{-\int_0^L \rho(z)k_{\text{abs}}(z)dz} \tag{2.3}$$

2. The left hand side of the equation denotes the total radiation reaching height $z1$. Next, the first term at the right hand side of the equation denotes the initial radiation entering the atmosphere at height $z2$. Lastly, the second term at the right hand side denotes the source radiation from emission and scattering in at each point in the optical path length from height $z1$ to $z2$. The exponential factors are the transmissivities, which means that the initial radiaiton and the source radiation get attenuated along its path.

3. The left hand side of the equation denotes the total terrestrial radiation at some height z and zenith angle μ. Next, the first right hand side term denotes the initial terrestrial radiation entering the BOA (height $z = 0$) reduced by attenuation through absorption and scattering out. Lastly, the second right hand side term denotes the source radiation from emissions and scattering in at every point along the path from BOA to some height z. This is also attenuated through absorption and scattering.

   The term $I_\nu(0, \mu)$ denotes the emission at the Earth's surface. The factor $e^{\tau_\nu(z)/\mu}$ denotes the transmissivity of the whole atmosphere from BOA to height z. The exponent $\tau_\nu(z)/\mu$ denotes the total optical path length for the whole atmosphere from BOA to height z, where μ is dependent on the zenith angle. The term $T(\tau_v')$ denotes the temperature for each section of the atmosphere with path length τ_ν'. The function $B_v(T(\tau_v'))$ denotes the Planck equation at temperature $T(\tau_v')$. The term $e^{\{(\tau_v' - \tau_v(z))/\mu\}}$ denotes the transmissivity of each section of the atmosphere between optical path lengths τ_v' and τ_v. Lastly, the exponent $(\tau_v' - \tau_v(z))/\mu$ denotes the optical path length for each section of the atmosphere.

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Problem 3

Calculations

1. Assuming a uniform atmosphere with a total atmospheric optical depth, $\tau = 0.5,\ 0.75$ and 1.5, respectively, plot **direct beam solar irradiance** versus time of day (every three hours) for a flat land surface at $30^o N$ on the summer solstice (June 21).

   a) The following equation can be used to calculate the BOA surface irradiance with solar zenith angle (θ):

   $$F_{\text{BOA}} = \cos\theta F_{\text{TOA}} e^{-\frac{\tau}{\cos\theta}} = \mu F_{\text{TOA}} e^{-\frac{\tau}{\mu}} \tag{3.1}$$

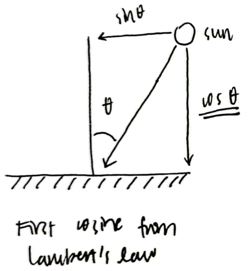
   Where F_{TOA} is the TOA solar irradiance (Wm^{-2}), $\mu = \cos\theta$, and θ is the solar zenith angle learned in class. Explain why $\mu[\mu = \cos\theta]$ appears twice and what's the physical meaning for each appearance?

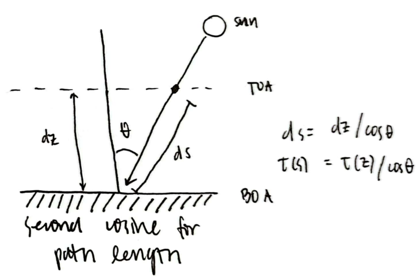
   b) Use the above equation to calculate the BOA surface irradiance as a function of solar zenith angle (θ_o). Don't worry about plotting time in terms of GMT, just plot time relative to solar noon (i.e. 0 corresponds to solar noon, 3 to 3 hours after solar noon and -3 to 3 hours before solar noon). For simplicity, assume the TOA solar irradiance is $1360 Wm^{-2}$ on June 21 (i.e., no need to calculate the TOA solar irradiance on June 21). Also, declination angle (δ) is 23.5^o on the summer solstice.

Solution.

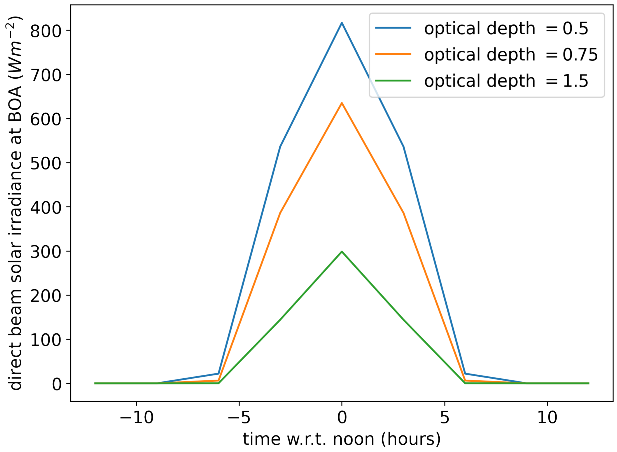
1. a) The first $\cos\theta$ accounts for Lambert's cosine law, where the fraction of irradiance hitting our surface of interest equals the projection of the irradiance along the normal direction. The amount of irradiance hitting the surface decreases with increasing zenith angle θ.

   In the equation, τ alone accounts for path length only along the surface normal. Thus, the second $\cos\theta$ accounts for a longer path length τ if the radiation arrives at a zenith angle of θ from the normal. The following diagrams illustrate the physical reasonings behind the two cosines.





   b) As given for simplicity, we set $I_\nu(0, \mu) = 1360\ Wm^2$ on June 21. The Jupyter notebook given to us has a function to calculate the declination angle given the location's latitude, and a function to calculate the hour angles given the hour of the day with respect to noon. Negative hours mean before noon, while positive hours mean after noon. The notebook also has a function for calculating the solar zenith angle given the latitude, declination, and hour angles. To avoid the values from blowing up, at solar zenith angles before sunrise and after sunset, the solar irradiance was set to zero. The final plot is shown below.



Here, we observe that the direct solar beam irradiance is highest at noon and decreases (almost like a normal distribution) hours before and after noon. A lower optical depth also results to higher solar irradiance, and vice versa.

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