AATM 504 – Radiative Fluxes and Thermodynamics

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Homework 5

Problem 1

Radiative Energy Budget Questions

1. We have the following figure to illustrate the surface longwave radiation.
   a) If the surface is assumed as a blackbody, write down the following radiation components in mathematical notation consistent with the symbols used in the above figure.
      i. the downward radiation absorbed by the surface
      ii. the downward radiation transmitted by the surface
      iii. the downward radiation reflected by the surface
      iv. the radiation emitted by the surface
      v. and the total upwelling radiation
   b) If the surface is assumed to be a graybody with an emissivity of ε_{sl}, write down the radiation components as in a)
2. For a single layer atmosphere, we have the following figure to illustrate the atmosphere longwave radiation budget. Note: Assume that the surface below is non-reflective (a blackbody emitter). Write down the following radiation components.
   a) the total longwave radiation absorbed by the atmosphere
   b) the total longwave radiation transmitted by the atmosphere
   c) the total longwave radiation reflected by the atmosphere
   d) the total downward radiation at bottom of the atmosphere
   e) the total upwelling radiation at top of the atmosphere

Solution.

1. a) For a blackbody, its emissivity and absorbance are both equal to 1, and its reflectance and transmissivity are both 0.
      i. A blackbody absorbs all radiation hitting it, so absorbed $= L_a \downarrow$.
      ii. The transmissivity is 0 at the surface so transmitted $= 0$.
      iii. A blackbody does not reflect any radiation hitting is, so reflected $= 0$.
      iv. A blackbody's emission is given by the Stefan-Boltzmann law: $L_s \uparrow = \sigma T_s^4$, where σ is the Stefan-Bolztmann constant.
      v. The total upwelling is just the emitted radiation, since blackbodies are non-reflective surface: $L_s \uparrow = \sigma T_s^4$.
   b) From Kirchoff's law, absorptivity is equal to the emissivity such that $\varepsilon_{sl} = \alpha_{sl}$. Since $1 = \alpha_{sl} + \tau_{sl} + \varepsilon_{sl}$, the reflectivity must be given by $r_{sl} = 1 - \alpha_{sl} = 1 - \varepsilon_{sl}$.
      i. The absorbed radiation for a graybody is $\alpha_{sl} L_a \downarrow = \varepsilon_{sl} L_a \downarrow$.

ii. The transmissivity is still 0 at the surface so transmitted $= 0$.

iii. The reflected radiation for a graybody is $r_{sl} L_a \downarrow = (1 - \varepsilon_{sl}) L_a \downarrow$.

iv. The emitted radiation for the surface is given by a portion of the Stefan-Boltzmann law: $L_s \uparrow = \varepsilon_{sl} \sigma T_s^4$.

v. The total upwelling is the sum of emitted and reflected radiation: $\varepsilon_{sl} \sigma T_s^4 + r_{sl} L_a \downarrow = \varepsilon_{sl} \sigma T_s^4 + (1 - \varepsilon_{sl}) L_a \downarrow$.

2. Again, from Kirchoff's law, $\varepsilon_{al} = \alpha_{al}$.

a) A single layer of the atmosphere absorbs a portion of all radiation incident on it: absorbed $= \varepsilon_{al}(L_s \uparrow + L_u \downarrow)$.

b) A single layer of the atmosphere transmits a portion of all radiation incident on it: transmitted $= \tau_{al}(L_s \uparrow + L_u \downarrow)$.

c) A single layer of the atmosphere reflects a portion of all radiation incident on it: reflected $= r_{al}(L_s \uparrow + L_u \downarrow)$.

d) The emission of the atmosphere (as a graybody) is given by $\varepsilon_{al} \sigma T_a^4$. The total downward radiation is a combination of emitted, reflected, and transmitted LW radiation: total downward $= \varepsilon_{al} \sigma T_a^4 + \tau_{al} L_u \downarrow + r_{al} L_s \uparrow$.

e) The total upwelling radiation is a combination of emitted, reflected, and transmitted LW radiation: total upwelling $= \varepsilon_{al} \sigma T_a^4 + \tau_{al} L_s \uparrow + r_{al} L_u \downarrow$.

$\square$

Problem 2

SW radiative flux budget analysis for a climate system with one atmospheric layer, and a solid ground layer.

Assuming that all sources of energy come initially into the system as SW radiative fluxes at the top of atmosphere (TOA), we want to know how much of the flux is net gained by the system (atmosphere plus ground surface), how much of the energy flux density is absorbed by the atmosphere, and how much is reflected back/transmitted through the atmosphere back to space (at the TOA).

Assumptions: The atmosphere absorbs parts of upwelling and downwelling SW fluxes but does not reflect any SW fluxes. The surface reflects parts of the incoming SW fluxes and absorbs the other part. Write down the terms/equation for the following.

(a) the total shortwave downward (downwelling) radiation absorbed by the atmosphere

(b) the total downwelling shortwave radiation transmitted by the atmosphere

(c) the total shortwave radiation absorbed by the surface

(d) the total shortwave radiation reflected by the surface

(e) the total upwelling shortwave radiation absorbed by the atmosphere

(f) the total upwelling shortwave radiation transmitted through the atmosphere

(g) the net SW flux absorbed in the atmosphere

(h) the net SW flux absorbed in the climate system

(i) check that your energy budget system is energy preserving: total downwelling flux at TOA must equal sum of what terms?

Solution.

(a) The total downwelling SW radiation absorbed by atmosphere: $a_{s1} S^{\downarrow}_{1+1/2}$.

(b) The total downwelling SW radiation transmitted by atmosphere: $\tau_{s1} S^{\downarrow}_{1+1/2} = (1 - a_{s1}) S^{\downarrow}_{1+1/2}$.

(c) The total SW radiation absorbed by surface: $a_{s0} S^{\downarrow}_{1-1/2}$.

(d) The total SW radiation reflected by surface: $\rho_{s0} S^{\downarrow}_{1-1/2} = (1 - a_{s0}) S^{\downarrow}_{1-1/2}$.

(e) The total upwelling SW radiation absorbed by atmosphere: $a_{s1} S^{\uparrow}_{1-1/2}$.

(f) The total upwelling SW radiation transmitted by atmosphere: $\tau_{s1} S^{\uparrow}_{1-1/2} = (1 - a_{s1}) S^{\uparrow}_{1-1/2}$.

(g) The net SW flux absorbed by the atmosphere is the sum of absorbed upwelling and downwelling radiation: $a_{s1}(S^{\downarrow}_{1+1/2} + S^{\uparrow}_{1-1/2})$.

(h) The net SW flux absorbed in the climate system is the sum of SW flux absorbed by the atmosphere and the Earth's surface: $a_{s1}(S^{\downarrow}_{1+1/2} + S^{\uparrow}_{1-1/2}) + a_{s0} S^{\downarrow}_{1-1/2}$

(i) Portions of the incoming SW flux gets absorbed by the atmosphere ($a_{s1}S^{\downarrow}_{1+1/2}$), absorbed by the surface ($a_{s0}S^{\downarrow}_{1-1/2}$), and reflected by the surface ($\rho_{s0}S^{\downarrow}_{1-1/2}$). Consequently, some of the reflected SW flux by the surface gets absorbed again by the atmosphere ($a_{s1}S^{\uparrow}_{1-1/2}$) and some get transmitted to the TOA ($\tau_{s1}S^{\uparrow}_{1-1/2}$). Thus, $\rho_{s0}S^{\downarrow}_{1-1/2} = a_{s1}S^{\uparrow}_{1-1/2} + \tau_{s1}S^{\uparrow}_{1-1/2}$. Looking at the energy budget, the total incoming SW flux must be the sum of all these: $S_{in} = a_{s1}S^{\downarrow}_{1+1/2} + a_{s0}S^{\downarrow}_{1-1/2} + a_{s1}S^{\uparrow}_{1-1/2} + \tau_{s1}S^{\uparrow}_{1-1/2}$.

$\square$

Problem 3

Thermodynamics: Application of ideal gas law for quantitative calculations

1. What is the amount of work done when blowing up a spherical balloon to a diameter of 8 inches? Assume standard sea level pressure (i.e., 1013.25 hPa) and ignore the work done in stretching the balloon's latex. Note $1\ Pa = 1\ kg\ m^{-1}s^{-2}$
2. At $T = 0^oC$ and $p = 1000\ mb$, $1\ g$ of dry air receives an amount of heat during an isochoric process. It is then observed that its pressure increases by $50mb$. What is the change in the temperature of the air and what is the amount of heat absorbed?
3. Commercial aircraft fly near 200 mb where typically the outside temperature is 60^oC. (1) Calculate the temperature of air if compressed adiabatically to the cabin pressure of 1000 mb. (2) Then, how much heat must be added or removed (isobarically) to maintain the cabin at 25^oC? Consider the air as dry air.
4. A sample of 100 g of dry air has an initial temperature of 270 K and pressure $900mb$. During an isobaric process heat is added and the volume expands by 20% of its initial volume. Estimate: (1) the final temperature of the air, (2) the amount of heat added, and (3) the work done against the environment.

Solution.

1. Work done is given by $W = \int_i^f pdV$. Since we're given discrete data, we can just write this as $W = p\Delta V$. The pressure is just given by 1013.25 hPa. Here, ΔV is just the final volume of the spherical balloon. The volume of the balloon (in terms of diameter) is

$$V = \frac{1}{6}\pi d^3 = \frac{1}{6}\pi \left[\left(8\ in \cdot \frac{0.0254\ m}{1\ in}\right)\right]^3 \tag{3.1}$$

Plugging this into our equation for work, we get

$$\begin{aligned} W &= \left[1013.25\ hPa \cdot \frac{100\ Pa}{1\ hPa} \cdot \frac{1\ kg\ m^{-1}s^{-2}}{1\ Pa}\right]\left[\frac{1}{6}\pi\left(8\ in \cdot \frac{0.0254\ m}{1\ in}\right)^3\right] \\ W &= 445\ kg\ m^2/s^2 = 445\ J \end{aligned} \tag{3.2}$$

2. According to Gay-Lussac's law, at constant volume, $\frac{P_1}{T_1} = \frac{P_2}{T_2}$. The new temperature of the air (assuming it behaves like an ideal gas) is

$$\begin{aligned} T_2 &= P_2\frac{T_1}{P_1} \\ &= 1050\ mb \cdot \frac{273.15\ K}{1000\ mb} \\ T_2 &= 286.8\ K \end{aligned} \tag{3.3}$$

Thus, the temperature increased by about 13.7 K or 13.7^oC. In other words, $\Delta T = 13.7\ K$.

Since we're looking at an isochoric process, then the first law of thermodynamics just becomes $dQ = dU$, where dQ is the change in heat, and dU is the change in internal energy. Under constant volume, the specific heat is given by

$$mc_v = \left(\frac{dU}{dT}\right)_V = \left(\frac{dQ}{dT}\right)_V \tag{3.4}$$

where m is the mass (in kg) of the air parcel, and dT is the change in temperature (in K). Rewriting this, we see that

$$\Delta Q = mc_v\ \Delta T \tag{3.5}$$

The specific heat at constant volume of dry air is 717 $J\ K^{-1}\ kg^{-1}$. Plugging in our given and calculated values, we get

$$\begin{aligned} \Delta Q &= 1\ g \cdot \frac{1\ kg}{1000\ g}(717J\ K^{-1}\ kg^{-1})(13.7\ K) \\ \Delta Q &= 9.8\ J \end{aligned} \tag{3.6}$$

Thus, the air parcel absorbed about 9.8 J heat to increase its temperature by 13.7 K.

3. Poisson's equation for the potential temperature is given by

$$\theta_{1000} = T_1 \left(\frac{1000\ mb}{p_1}\right)^{R_D/c_p} \tag{3.7}$$

In our problem, we want to go adiabatically from an initial temperature of $T_1 = -60^oC = 213.15\ K$ and initial pressure of $p_1 = 200\ mb$ to a pressure of 1000 mb and an unknown temperature $T_2 = \theta_{1000}$. The specific heat of dry air at constant pressure is $c_p = 1004\ J\ K^{-1}\ kg^{-1}$, and the dry air gas constant is $R_D = 287\ J\ K^{-1}\ kg^{-1}$. Plugging in the given values and constants,

$$\theta_{1000} = (213.15\ K)\left(\frac{1000\ mb}{200\ mb}\right)^{\frac{287\ J\ K^{-1}\ kg^{-1}}{1004\ J\ K^{-1}\ kg^{-1}}} = 337.7\ K = 64.5^oC \tag{3.8}$$

The temperature inside the cabin would be 64.5^oC. To get amount of heat needed to maintain the temperature at 25^oC, we use the thermodynamic equation for one unit mass of dry air. For an isobaric process, $dp = 0$, so it simplifies into

$$dq = c_p dT \qquad \text{or} \qquad \Delta q = c_p \Delta T \tag{3.9}$$

Plugging in our given values, we have

$$\Delta q = (1004\ J\ K^{-1}\ kg^{-1})(298.15\ K - 337.7\ K) = -39708.2\ J/kg = -39.7\ kJ/kg \tag{3.10}$$

About 39.7 kJ of heat must be removed for each 1 kg of air inside the cabin to maintain 25^oC.

4. According to Charles's Law, at constant pressure, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$. The new temperature of the air (assuming it behaves like an ideal gas) is

$$\begin{aligned} T_2 &= T_1 \frac{V_2}{V_1} \\ &= (270\ K)(1.2) \\ T_2 &= 324\ K \end{aligned} \tag{3.11}$$

Thus, $\Delta T = 54\ K$. Since the air expands to become 1.2 times its original volume, $\frac{V_2}{V_1} = 1.2$. For an isobaric process, we use the equation for the specific heat at constant pressure.

$$mc_p = \left(\frac{dU}{dT}\right)_p = \left(\frac{dQ}{dT}\right)_p \tag{3.12}$$

Rewriting this, we see that

$$\Delta Q = mc_p\ \Delta T \tag{3.13}$$

The specific heat at constant pressure of dry air is 1004 $J\ K^{-1}\ kg^{-1}$. Plugging in our given and calculated values, we get

$$\begin{aligned} \Delta Q &= 100\ g \cdot \frac{1\ kg}{1000\ g}(1004\ J\ K^{-1}\ kg^{-1})(54\ K) \\ \Delta Q &= 5421.6\ J = 5.42\ kJ \end{aligned} \tag{3.14}$$

Thus, the amount of heat added to the air parcel is about 5.42 kJ. To get the initial volume of the air parcel, we use the ideal gas law such that

$$\begin{aligned} V_1 &= \frac{mR_D T_1}{p} \\ &= \frac{0.1\ kg \cdot 287\ J\ K^{-1}\ kg^{-1} \cdot 270\ K}{900\ mb \cdot \frac{100\ Pa}{1\ mb}} \\ V_1 &= 0.0861\ m^3 \end{aligned} \tag{3.15}$$

Thus, $V_2 = 1.2V_1 = 0.10332\ m^3$. The change in volume is $\Delta V = 0.01722\ m^3$. Using our equation for work and repeating a similar process as question 1, we get

$$\begin{aligned} W &= \left(900\ mb \cdot \frac{100\ Pa}{1\ mb} \cdot \frac{1\ kg\ m^{-1}s^{-2}}{1\ Pa}\right)(0.01722\ m^3) \\ W &= 1550\ kg\ m^2/s^2 = 1550\ J = 1.55\ kJ \end{aligned} \tag{3.16}$$

The expanding air parcel does 1.55 kJ work against the environment.

□

Problem 4

Derivations

Derive specific heat at constant volume for moist air (C_{vm}) in terms of specific heat at constant volume for dry air (C_{vd}) and specific humidity (q). Refer to our derivations in class for the specific heat at constant pressure for moist air (C_{pm}).

Solution.

The relationship between between heat change and temperature change for dry air $Q_{v,d}$ and water vapor $Q_{v,v}$ are given by

$$\begin{aligned} Q_{v,d} &= m_d c_{v,d} (T_f - T_i) \\ Q_{v,v} &= m_v c_{v,v} (T_f - T_i) \end{aligned} \tag{4.1}$$

where m_d and m_v are the masses of dry air and water vapor, respectively, and $c_{v,d}$ and $c_{v,v}$ are the specific heat capacities for dry air and water vapor, respectively. Similarly, we can also write an expression for heat change for moist air $Q_{v,m}$.

$$Q_{v,m} = m_m c_{v,m} (T_f - T_i) \tag{4.2}$$

where the $m_m = m_d + m_v$ is the mass of moist air and $c_{v,m}$ is its specific heat capacity at constant volume. Adding the heat changes for dry air and water vapor to get the heat change for moist air, we have

$$\begin{aligned} Q_{v,d} + Q_{v,v} = Q_{v,m} &= m_d c_{v,d} (T_f - T_i) + m_v c_{v,v} \\ Q_{v,m} &= (m_d c_{v,d} + m_v c_{v,v}) (T_f - T_i) \end{aligned} \tag{4.3}$$

Comparing our two equations for $Q_{v,m}$, we see that

$$m_m c_{v,m} = (m_d + m_v) c_{v,m} = (m_d c_{v,d} + m_v c_{v,v}) \tag{4.4}$$

Isolating $c_{v,m}$, we get

$$c_{v,m} = \frac{m_d c_{v,d} + m_v c_{v,v}}{m_d + m_v} \tag{4.5}$$

Since the specific humidity is given by $q = \frac{m_v}{m_d + m_v}$, then

$$c_{v,m} = (1 - q) c_{v,d} + q c_{v,v} \tag{4.6}$$

At constant volume, the specific heat capacity of dry air is $c_{v,d} = 717 J kg^{-1} K^{-1}$ and the specific heat capacity of water vapor is $c_{v,v} = 1407 J kg^{-1} K^{-1}$. Thus, $c_{v,v} = 1.96 c_{v,d}$. Plugging this in, we get

$$c_{v,m} = (1 - q) c_{v,d} + 1.96 c_{v,d} q \tag{4.7}$$

Simplifying this, we get our final expression as

$$c_{v,m} = (1 + 0.96 q) c_{v,d} \tag{4.8}$$

□

Problem 5

Feedback

Please give us feedback on the homework assignment:

1. How much time you spend on doing this homework?
2. How clear are the instructions written? Is it confusing or easy to understand you are supposed to do?
3. Is the homework helping you to develop a better understanding of the global radiative energy fluxes and processes in the atmosphere? In either case, please explain why.
4. For the Thermodynamics part, is the lecture material/reading material helpful?
5. Any other comments?

Solution.

1. If I were to accumulate all the spread out hours I used, I spent around 1 day answering this homework.
2. The instructions were clear enough, but some of the notations in the questions on radiative energy fluxes are a little confusing. Most of the notations look alike and it's a little difficult to differentiate and consolidate them to what we used in class.
3. Yes, the homework helped me better understand the global energy fluxes and processes, since I have more time to review my answers and compare my understanding with my classmates. In class, some of the material can go by pretty fast and I need a little more time to fully understand everything. Sitting down and answering simple step-by-step questions helped guide me to understand the material better.
4. The lecture and reading material are somewhat helpful, but I still needed to do some online research and review of my previous thermodynamics classes to be able to answer all the questions. Some parts of the lecture and reading materials also needed to be expounded on or clarified. Specially for problem 4, since this wasn't discussed or expounded on in class.
5. Giving us two full weeks to finish the HW is an improvement from just giving us a week, since we also have ATM500 (Dynamics) HWs to finish. I wish the fourth problem was discussed further in class, since it took me and my classmates more time to answer it. The office hours with Tyler greatly helped. Other than that, all the other questions were more or less pretty straightforward.

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