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## AATM 504 – Radiation Laws, Atmospheric Scattering, Absorption and Refraction

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Homework 3

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### Problem 1

#### Questions on atmospheric scattering

1. Rayleigh Scattering
  - a) If the atmosphere did not scatter light, what would you see when you looked at the daytime sky (except the sun) and explain why?
  - b) For Rayleigh scattering, the effectiveness (amount) of scattering is inversely related to the wavelength of radiation ( $\lambda$ ):  $I_{\text{sca}} \propto \lambda^{-4}$ . Calculate the scattering ratio for blue and ultraviolet light relative to red light.
2. The following image shows the atmospheric layer with different colors. Explain why the sky changes from black at the top of atmosphere (TOA) to blue at upper atmospheres and to colorful at the bottom of atmosphere (BOA) using what you learned in class about the three types of scattering.
3. The following image shows the TOA and BOA solar irradiance as a function of wavelength. Tell what each line represents. Explain this figure in terms of atmospheric scattering and absorption, and atmospheric windows. Combined with what we learned about absorption and scattering in Earth's lower atmosphere, also explain why we need to only consider atmospheric scattering for solar radiation (with wavelengths of  $0.3 - 4.0 \mu\text{m}$ ) and atmospheric absorption for terrestrial radiation (with wavelengths of  $3.5 - 50 \mu\text{m}$ ).

*Solution.*

#### 1. Rayleigh Scattering

- a) If the atmosphere did not scatter light, then the daytime sky would just look black. All the wavelengths of light the sun emits would just directly hit the Earth's surface, and no wavelengths would scatter in the sky. The absence of any wavelength of light in the atmosphere would cause a dark sky.
- b) Blue light has a wavelength around  $450 \text{ nm}$ , ultraviolet light has a wavelength around  $200 \text{ nm}$ , and red light has a wavelength around  $650 \text{ nm}$ . For blue versus red light, the scattering ratio would be

$$\frac{\lambda_b^{-4}}{\lambda_r^{-4}} = \frac{(450 \text{ nm})^{-4}}{(650 \text{ nm})^{-4}} = 4.35 \quad (1.1)$$

For ultraviolet versus red light, the scattering ratio would be

$$\frac{\lambda_{uv}^{-4}}{\lambda_r^{-4}} = \frac{(200 \text{ nm})^{-4}}{(650 \text{ nm})^{-4}} = 112 \quad (1.2)$$

2. At the top of the atmosphere, we have a vacuum containing no air particles. There are no particles present that can scatter any wavelength of light, causing the camera (used to take the photograph) to detect the absence of light.

At the upper atmosphere, sunlight begins to encounter air molecules, which can scatter certain wavelengths of light. Air molecules are very small, so Rayleigh scattering can only occur at lower wavelengths: blue to purple to UV light. The lowest wavelength scattered at the upper atmosphere is blue light, while the much lower wavelengths are typically absorbed.

Once sunlight reaches the lower atmosphere (stratosphere to troposphere), it encounters more particles with varying sizes such as cloud droplets, aerosols, precipitation, and dust. These particles are larger than air molecules so they can scatter higher sunlight wavelengths. Here, the light undergoes Mie scattering and/or fall under geometric optics. More wavelengths going into the orange to red range are scattered, so we see a more colorful picture. These observations are based on the scattering size parameter plot shown in problem C.

3. The solar irradiance curve outside the atmosphere corresponds to the TOA solar irradiance, while the solar irradiance curve at sea level corresponds to the BOA solar irradiance. We notice that the TOA solar irradiance follows closer to the blackbody curve at 6000  $K$ , while the BOA solar irradiance is more varied.

At TOA, there are fewer air molecules so less solar radiation absorption and/or scattering happens. Minimal absorption and scattering only occurs at lower wavelengths. At the BOA, more particles and air molecules are present, so more absorption and scattering are present. The shaded regions in the figure show dips where  $H_2O$ ,  $CO_2$ ,  $O_3$ , and  $O_2$  in the atmosphere absorb specific solar irradiance wavelengths very well. On the other hand, the bumps (or maximums) in the BOA solar irradiance indicate the atmospheric window. The atmospheric window are wavelength regions where minimal solar radiation absorption occurs, so more flux reaches the Earth's surface.

Based on the scattering size parameter  $x = 2\pi r/\lambda$ , lower wavelengths are the easiest to scatter. On the other hand, higher wavelengths are more difficult to scatter, so mostly absorption occurs instead. The solar radiation primarily has shortwave (short wavelengths) radiation, while terrestrial radiation primarily has longwave (long wavelength) radiation. Thus, we consider only scattering for shortwave radiation, while we only considered absorption for longwave radiation.

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## Problem 2

### Work with Equations

Albedo ( $\alpha$ ) is a variable commonly used in meteorology. It is defined as the ratio of the incoming solar radiant flux density (i.e., irradiance,  $W/m^2$ ) to the reflected solar radiant flux density (i.e., exitance,  $W/m^2$ ) integrated over the entire wavelength spectrum for a unit surface area:

$$\alpha = \frac{\text{Reflected solar radiant flux density } (W/m^2)}{\text{Incident solar radiant flux density } (W/m^2)} = \frac{\text{Exitance } (W/m^2)}{\text{Irradiance } (W/m^2)} \quad (2.1)$$

As illustrated below, for a unit surface area, the total incident solar irradiance ( $W/m^2$ ) is the wavelength-integrated sum of the direct beam solar irradiance plus the wavelength-integrated sum of the diffuse solar irradiance that is integrated over all the solid angles from the entire upper hemisphere.

Now assume that the surface is a Lambertian, with an isotropic reflectance of  $r_\lambda$ , which is defined as the fraction of incident solar radiation (i.e., irradiance) versus the reflected solar radiation (i.e., exitance) at a given wavelength ( $\lambda$ ). In other words,  $r_\lambda$  is similar to albedo ( $\alpha$ ) but is a function of  $\lambda$ . At a given wavelength  $\lambda$ , the incoming direct beam solar irradiance ( $Wm^{-2}\mu m^{-1}$ ) is  $S_\lambda(\theta_0)$ , where  $\theta_0$  is the solar zenith angle, and the incoming diffuse solar radiance ( $Wm^{-2}\mu m^{-1}sr^{-1}$ ) is  $I_\lambda(\omega)$  or  $I_\lambda(\theta, \phi)$ , where  $\omega$  represents any solid angle of the incoming diffuse radiation, described by zenith angle ( $\theta$ ) and azimuth angle ( $\phi$ ).

Use integrals to express the surface albedo ( $\alpha$ ) as a function of  $S_\lambda(\theta_0)$ ,  $I_\lambda(\theta, \phi)$ , and  $r_\lambda$  in terms of wavelength ( $\lambda$ ), zenith ( $\theta$ ) and azimuth ( $\phi$ ) angles.

*Solution.*

First, we determine the total integrated solar irradiance. We have discussed in class that the total incoming diffuse solar radiance integrated over all solid angles (in the upper hemisphere) is given by

$$F_\lambda[Wm^{-2}\mu m^{-1}] = \int_0^{2\pi} \int_0^{\pi/2} I_\lambda(\theta, \phi)[Wm^{-2}\mu m^{-1}sr^{-1}] \cos \theta \sin \theta d\theta d\phi \quad (2.2)$$

Thus, the total wavelength-integrated diffuse solar irradiance becomes

$$F_\lambda[Wm^{-2}] = \int_0^\infty \int_0^{2\pi} \int_0^{\pi/2} I_\lambda(\theta, \phi)[Wm^{-2}\mu m^{-1}sr^{-1}] \cos \theta \sin \theta d\theta d\phi d\lambda \quad (2.3)$$

Similarly, the wavelength-integrated direct beam solar irradiance is just

$$S[Wm^{-2}] = \int_0^\infty S_\lambda(\theta_0)[Wm^{-2}\mu m^{-1}] d\lambda \quad (2.4)$$

The total irradiance is the sum of the total wavelength-integrated direct beam solar irradiance and the total wavelength-integrated solar diffusion. Thus,

$$\text{irradiance} = \int_0^\infty S_\lambda(\theta_0) d\lambda + \int_0^\infty \int_0^{2\pi} \int_0^{\pi/2} I_\lambda(\theta, \phi) \cos \theta \sin \theta d\theta d\phi d\lambda \quad (2.5)$$

We know that  $r_\lambda = \frac{\text{exitance}}{\text{irradiance}}$ , so the total exitance must be

$$\text{exitance} = \int_0^\infty r_\lambda S_\lambda(\theta_0) d\lambda + \int_0^\infty r_\lambda \int_0^{2\pi} \int_0^{\pi/2} I_\lambda(\theta, \phi) \cos \theta \sin \theta d\theta d\phi d\lambda \quad (2.6)$$

The reflectance has to remain inside the integral, since it's wavelength dependent. Combining our two final equations for total irradiance and exitance, the albedo becomes

$$\alpha = \frac{\int_0^\infty r_\lambda \left[ S_\lambda(\theta_0) d\lambda + \int_0^{2\pi} \int_0^{\pi/2} I_\lambda(\theta, \phi) \cos \theta \sin \theta d\theta d\phi \right] d\lambda}{\int_0^\infty \left[ S_\lambda(\theta_0) d\lambda + \int_0^{2\pi} \int_0^{\pi/2} I_\lambda(\theta, \phi) \cos \theta \sin \theta d\theta d\phi \right] d\lambda} \quad (2.7)$$

□

### Problem 3

#### Calculations

1. The following figure shows the type of atmospheric scattering as a function of the particle size and the radiation wavelength ( $\lambda$ ) taught in class.

If you have six types of objects on Earth, five in the atmosphere (air molecules, large aerosols, cloud droplets, raindrops and hailstones) and one on land surface such as a lake with a radius of 10 km. The typical size of five atmospheric particles is listed in next table. Take the upper limit if the particle has a size range.

- a) Calculate the size parameter for scattering ( $x = 2\pi r/\lambda$ ) and determine which type of scattering will happen for the above six objects. Consider solar radiation at  $0.5 \mu m$  (where the solar radiation peaks), terrestrial radiation at  $10 \mu m$  (where terrestrial emission peaks), and terrestrial radiation at  $10^5 \mu m$  (where terrestrial emission also occurs at a smaller magnitude and is used in weather radars).
- b) For radiative sensors at the top of atmosphere (TOA) measuring the reflected solar radiation ( $\lambda: < 4 \mu m$ ) and emitted longwave radiation ( $\lambda: 4 - 20 \mu m$ ) by land surface objects, the measured radiation is strongly impacted by the presence of clouds and aerosols in the atmosphere (i.e., sensors cannot see through clouds and fogs). Clear-sky images are often needed for monitoring land surface objects, please explain why.
- c) Microwave radiation ( $\lambda: 103 - 106 \mu m$  or  $1 mm - 1 m$ ) can pass through clouds, aerosols and air molecules, but not raindrops, graupels and hailstones ranging from  $0.1 mm$  to  $1 cm$  in size. Note that weather radars emit microwave radiation to the atmosphere and measure the back-scattered radiation (i.e. radiation scattered back by the particles to the radars). In other words, weather radars can detect the presence of raindrops, graupels and hailstones but not clouds, aerosols, and air molecules (because the sensor can see through clouds and fogs). They are often referred to as all-weather capacity remote sensors, please explain why.

*Solution.*

- (a) The table below summarizes the size parameter of scattering for the different particles and different wavelengths.

	radius ( $\mu m$ )	size parameter		
		solar radiation ( $\lambda = 0.5 \mu m$ )	terrestrial radiation ( $\lambda = 10 \mu m$ )	weather radar ( $\lambda = 10^5 \mu m$ )
air molecules	$10^{-4}$	0.001	$6.28 \times 10^{-5}$	$6.28 \times 10^{-9}$
large aerosols	1	12.6	0.628	$6.28 \times 10^{-5}$
cloud droplets	50	628	31.4	0.003
raindrops	$3 \times 10^3$	$3.77 \times 10^4$	$1.88 \times 10^3$	0.188
hailstones	$1 \times 10^6$	$1.26 \times 10^7$	$6.28 \times 10^5$	62.8
lake	$1 \times 10^{10}$	$1.26 \times 10^{11}$	$6.28 \times 10^9$	$6.28 \times 10^5$

- (b) Here, we are interested in wavelengths  $20 \mu m$  and below. Based on the plot of the particle radius versus the radiation wavelength, we can see that cloud droplets cause emitted longwave radiation to undergo Mie scattering. Solar radiation would then fall under geometric optics. On the other hand, depending on the size of aerosols, they can cause the measured radiation to undergo Rayleigh or Mie scattering. Without clear skies, sensors would receive incorrect radiation measurements that have already been scattered by atmospheric particles.

- (c) Microwave radars can detect different kinds of precipitation, including drizzles, ice crystals, rain drops, graupels, and hailstones. Now, depending on the kind of precipitation present in its region of interest, radars can differentiate precipitation based on how well they scatter the radars emitted radiation. Radars can also determine the absence of precipitation (when no precipitation particles back-scatter).

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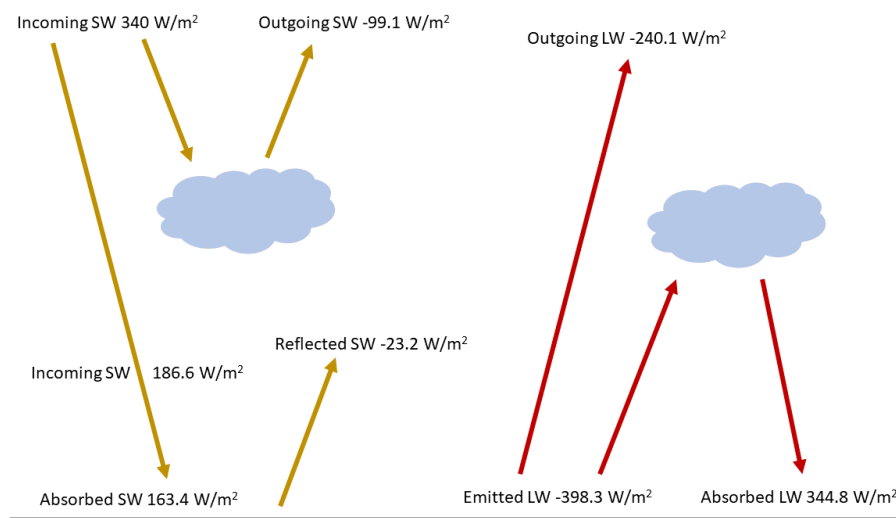
#### Problem 4

##### Applications and Questions

Use the provided scripts and CERES data to create a similar schematic figure but only consider the global annual mean radiative fluxes ( $W/m^2$ ) at BOA and TOA for the years 2005-2015. Please compare your values and discuss any differences that are noteworthy, or comment on the level of agreement. Also calculate the planetary TOA and BOA albedos and explain why they can differ from other numbers that are reported on web pages or in textbooks. Note that the albedo of TOA and BOA is defined as the ratio of incoming solar flux to the reflected solar flux at TOA and BOA, respectively.

- Calculate the global annual mean TOA shortwave and longwave irradiances with the help of the provided Jupyter Notebook. What is the planetary albedo, and what value do the CERES data give as an estimate? Calculate the global annual mean BOA shortwave and longwave irradiances with the same Notebook. What is the surface albedo? Why do the planetary and surface albedo values differ?
- Calculate and plot the seasonal cycle of global mean values of (i) TOA net shortwave radiation (i.e., incoming minus outgoing shortwave) and TOA net longwave radiation (i.e., outgoing longwave; note that incoming longwave is zero at TOA), (ii) BOA net shortwave radiation (i.e., incoming minus outgoing shortwave) and BOA net longwave radiation (i.e., outgoing minus incoming longwave), and (iii) TOA and BOA albedo, as a function of month in separate figures.
- Discuss the BOA net longwave radiation global annual mean. What does the value tell you? Is the net longwave irradiance close to zero? Further, what is the total SW and LW net budget for the surface? Near zero, or a strong net energy surplus or deficit? Explain what the number means.

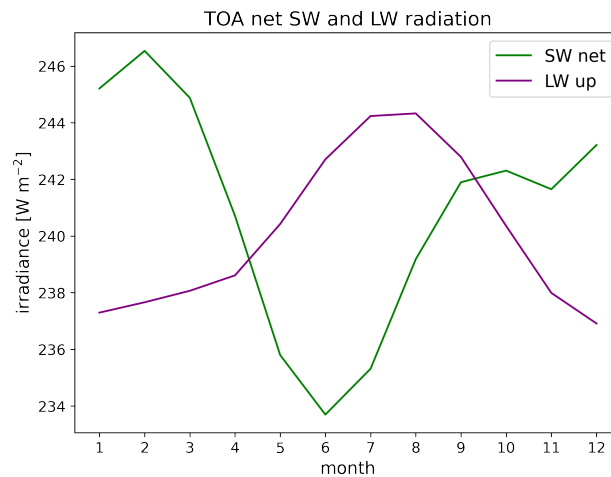
*Solution.* The figure below shows a similar schematic as the given figure, but the values shown were calculated from the CERES data. The negative values indicate radiation pointing away from the Earth's surface, and vice versa.



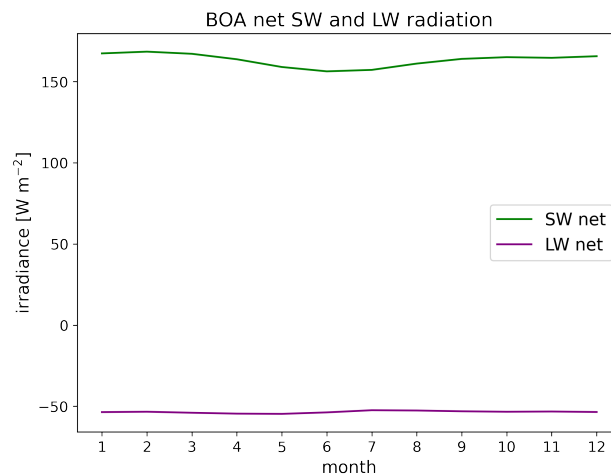
The calculated data from CERES match very well with the given figure, with only small decimal values differentiating the two. The TOA albedo is 0.29, while the BOA albedo is 0.12. CERES's calculated TOA albedo is different from the usual reported 0.3 in webpages or textbooks possibly

due to decreased cloud coverage or land use changes (more farmlands and urban areas, and less ice).

- (a) Based on the Jupyter Notebook given to us, the net shortwave and longwave TOA irradiances are  $240.9 \text{ W/m}^2$  and  $-240.1 \text{ W/m}^2$ , respectively. The CERES data estimates the planetary albedo as 0.29. Again, based on the Jupyter Notebook given to us, the net shortwave and longwave BOA irradiances are  $163.4 \text{ W/m}^2$  and  $-53.53 \text{ W/m}^2$ , respectively. The CERES data estimates the surface albedo as 0.12. The planetary albedo considers solar radiation reflected by the Earth's atmosphere and surface. On the other hand, the surface albedo considers the solar radiation reflected by the Earth's surface only. Thus, the planetary albedo is higher than the surface albedo.
- (b) The figure below shows the top of atmosphere net shortwave and longwave radiation.

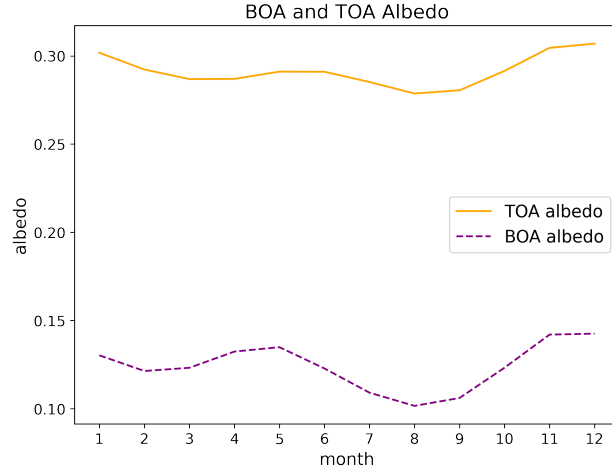


The figure below shows the bottom of atmosphere net shortwave and longwave radiation.



The figure below shows the top of atmosphere and bottom of atmosphere albedo.





- (c) Some of the LW radiation emitted by the Earth's surface ( $-398.3 \text{ W/m}^2$ ) gets reflected by (clouds in) the atmosphere back to the surface, while some energy reach past the atmosphere. The reflected LW radiation gets absorbed by the Earth's surface ( $344.8 \text{ W/m}^2$ ). The Earth's surface emits more energy than it absorbs back from reflected LW radiation. This leads to a BOA net outgoing LW radiation at  $-53.53 \text{ W/m}^2$ .

The net shortwave and longwave radiation at the surface is at  $109.84 \text{ W/m}^2$ , indicating a strong net energy surplus. Based on the CERES data, the net shortwave radiation absorbed by the Earth's surface is at  $163.4 \text{ W/m}^2$ , while the net longwave radiation it emits is at  $-53.53 \text{ W/m}^2$ . Here, the shortwave radiation absorbed by the Earth's surface is higher than the longwave radiation it emits. Based on the diagram, sensible and latent heat released during the upward convection of water vapor also contribute to the energy emitted by the Earth's surface.

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