AATM 529 – Air Sea Interactions

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Homework 8

Problem 1

Estimate the 10 m drag (C_D) and heat transfer (C_H) coefficients over land, given $z_o = 1\ cm$, and the Obukhov length $= 20\ m$. How would they differ if the static stability were neutral?

Solution. Since the Obukhov length is positive, I assumed statically stable conditions. The drag coefficients during statically stable conditions is given by

$$C_D = k^2 \left[ln\left(\frac{z}{z_o}\right) - \Psi_{\mathrm{M}}\left(\frac{z}{L}\right)\right]^{-2} \tag{1.1}$$

where $\Psi_{\mathrm{M}}\left(\frac{z}{L}\right) = \frac{4.7z}{L}$ for statically stable conditions. I assumed $k = 0.35$ as suggested by Businger, et al. for using this equation of Ψ_{M}. Plugging in the given values for $z_o = 0.01\ m$, $L = 20\ m$, and $z = 10\ m$, we get

$$\mathbf{C_D} = (0.35)^2 \left[\ln\left(\frac{10\ m}{0.01\ m}\right) - \frac{4.7(10\ m)}{20\ m}\right]^{-2} = \mathbf{0.0059} \tag{1.2}$$

On the other hand, the heat transfer coefficient is given by

$$\mathrm{C_H} = \mathrm{k}^2 \left[\ln\left(\frac{\mathrm{z}}{\mathrm{z_o}}\right) - \Psi_{\mathrm{M}}\left(\frac{z}{L}\right)\right]^{-1} \left[\ln\left(\frac{\mathrm{z}}{\mathrm{z_o}}\right) - \Psi_{\mathrm{H}}\left(\frac{z}{L}\right)\right]^{-1} \tag{1.3}$$

To solve for Ψ_{H}, I integrated $\int \frac{\phi_{\mathrm{H}}}{z} dz$. For statically stable conditions, $\phi_{\mathrm{H}} = 0.74 + \frac{4.7z}{L}$. Thus,

$$\int \frac{\phi_{\mathrm{H}}}{z} dz = 0.74 \ln\left(\frac{z}{z_o}\right) + \frac{4.7z}{L} \tag{1.4}$$

Rewriting this, we get

$$\int \frac{\phi_{\mathrm{H}}}{z} dz = \ln\left(\frac{z}{z_o}\right) + \Psi_{\mathrm{H}} \tag{1.5}$$

where $\Psi_{\mathrm{H}} = \frac{4.7z}{L} - 0.26 \ln\left(\frac{z}{z_o}\right) = \Psi_{\mathrm{M}} - 0.26 \ln\left(\frac{z}{z_o}\right)$. Thus, for stable conditions, the heat transfer coefficient is given by

$$\mathrm{C_H} = \mathrm{k}^2 \left[\ln\left(\frac{\mathrm{z}}{\mathrm{z_o}}\right) - \Psi_{\mathrm{M}}\left(\frac{z}{L}\right)\right]^{-1} \left[1.26 \ln\left(\frac{\mathrm{z}}{\mathrm{z_o}}\right) - \Psi_{\mathrm{M}}\left(\frac{z}{L}\right)\right]^{-1} \tag{1.6}$$

Plugging in the given values again, we get

$$\mathrm{C_H} = (0.35)^2 \left[\ln\left(\frac{10\ m}{0.01\ m}\right) - \frac{4.7(10\ m)}{20\ m}\right]^{-1} \left[1.26 \ln\left(\frac{10\ m}{0.01\ m}\right) - \frac{4.7(10\ m)}{20\ m}\right]^{-1} = \mathbf{0.0042} \tag{1.7}$$

During neutral conditions, we use the same equation for C_D, except $\Psi_{\mathrm{M}}\left(\frac{z}{L}\right) = 0$. Again, plugging in the given values, we get

$$\mathbf{C_{DN}} = (0.35)^2 \left[\ln\left(\frac{10\ m}{0.01\ m}\right)\right]^{-2} = \mathbf{0.0026} \tag{1.8}$$

For neutral conditions, it is often assumed that $C_{HN} = C_{EN} = C_{DN}$. Thus, $\mathbf{C_{HN} = 0.0026}$. □

Problem 2

Given a 1 km constant thickness boundary layer with initial $\theta = 10^o\ C$ flowing at M = 10 m/s over land, where the land has the same surface temperature as that of the air near the surface. At some point, the air leaves the land and flows over the ocean, where the ocean sea surface temperature is $20^o\ C$ and the pressure is 100 kPa. Assume that the boundary layer is well mixed. Calculate and plot the heat flux Q_H and the boundary layer temperature as a function of distance from the shoreline.

Solution. The sensible heat flux is given by $Q_H = \left(\overline{w'\theta'}\right)_s$, and $\left(\overline{w'\theta'}\right)_s$ can be parameterized as $\overline{(w'\theta')}_s = -C_H M\left(\overline{\theta} - \theta_G\right)$. Thus, the sensible heat flux can be parameterized as

$$Q_H = -C_H M\left(\overline{\theta} - \theta_G\right) \tag{2.1}$$

where θ_G is the surface potential temperature and $\overline{\theta}$ is the potential temperature at some height above the surface (usually 10 m). Assuming a well mixed 1 km boundary layer, the potential temperature should be constant throughout the 1 km. Initially over land, we are given that $\overline{\theta} = \theta_G$, so the initial sensible heat flux $Q_{H0} = 0$ at the shore.

The conservation of heat equation is given by

$$\frac{\partial\overline{\theta}}{\partial t} + \overline{U}_j\frac{\partial\overline{\theta}}{\partial x_j} = -\frac{1}{\overline{\rho}C_p}\left[L_v E + \frac{\partial \overline{Q}_j^*}{\partial x_j}\right] - \frac{\partial\left(\overline{u_j'\theta'}\right)}{\partial x_j} \tag{2.2}$$

Since not much else information was given, I assumed no latent heat release, no radiation, and steady state conditions, which simplifies the heat conservation equation into

$$\overline{U}_j\frac{\partial\overline{\theta}}{\partial x_j} = \frac{\partial\left(\overline{u_j'\theta'}\right)}{\partial x_j} \tag{2.3}$$

Looking only along the x direction since we're asked to calculate values as a function of the distance from the shoreline, this becomes

$$M\frac{\partial\overline{\theta}}{\partial x} = -\frac{\partial Q_H}{\partial z} \tag{2.4}$$

Here, $Q_H = \left(\overline{w'\theta'}\right)_s$. Integrating both sides and plugging in our equation for the sensible heat flux, we get

$$M\frac{\partial\overline{\theta}}{\partial x} = -\left[-C_H M\left(\overline{\theta} - \theta_G\right)\right] \tag{2.5}$$

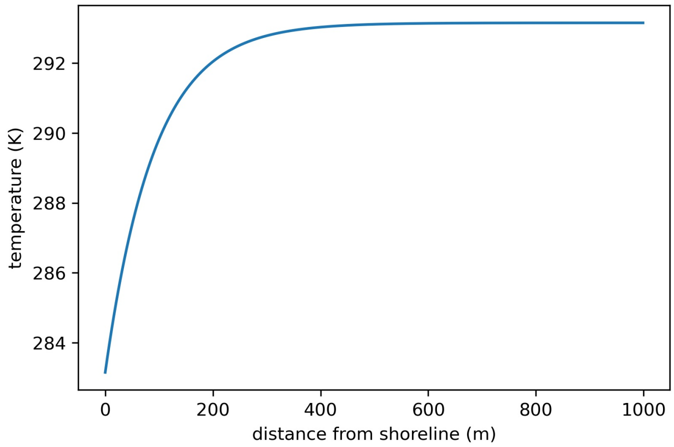
Solving for $\overline{\theta}$ in this differential equation, we get

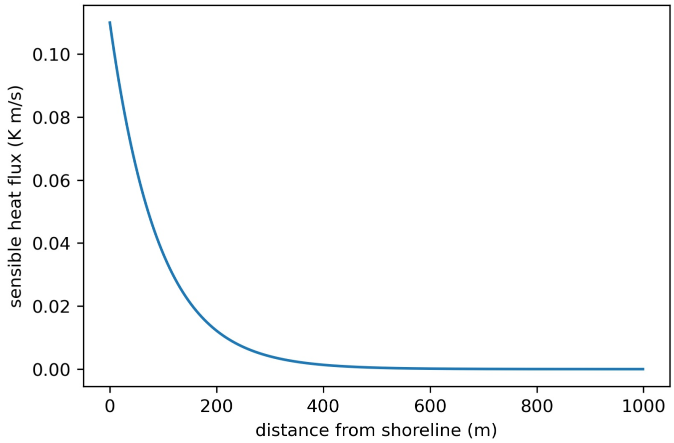
$$\begin{aligned}\ln(\overline{\theta} - \theta_G) &= C_H M x + C \\ \overline{\theta}(x) &= \theta_G + e^{C_H M x + C}\end{aligned} \tag{2.6}$$

Given that $\overline{\theta}(x=0) = \overline{\theta}_o$, then $C = \ln(\overline{\theta}_o - \theta_G)$. In the end, the potential temperature is given by

$$\overline{\theta}(x) = \theta_G + (\overline{\theta}_o - \theta_G)e^{C_H M x} \tag{2.7}$$

Again, the sensible heat flux is just $Q_H = -C_H M\left(\overline{\theta}(x) - \theta_G\right)$. The plots for potential temperature and sensible heat flux are shown below. Here I assumed $\overline{\rho} = 1.204\ kg/m^3$, $C_H = 1.1\times10^{-3}$, and $C_p = 1004.67\ J\ kg^{-1}K^{-1}$.





□

Problem 3

What is the roughness length, z_o, over the ocean for a wind speed of 40 m/s ?

Solution. We are given that the wind speed $u_* = 40\ m/s$ over the ocean, and asked to find the roughness length z_o. Charnock's relation over the ocean is given by

$$z_o = 0.015\frac{u_*^2}{g} \tag{3.1}$$

The surface stress over the ocean is given by

$$u_*^2 = \left(4.4 \times 10^{-4}\overline{\mathrm{M}}^{2.55}\right) \tag{3.2}$$

Plugging in our equation for u_* and the given $\overline{M}$, we get

$$\mathbf{z_o} = 0.015\frac{\left(4.4 \times 10^{-4}(40\ m/s)^{2.55}\right)}{9.8\ m/s^2} = 0.008\ m \tag{3.3}$$

□