AATM 529 – Air Sea Interactions

Crizzia Mielle De Castro Oct. 19 , 2023
Homework 7

Problem 1

Given the following data:

$$\begin{aligned}
&\overline{\mathrm{w}'\theta'} = 0.2 \text{ K m/s} \\
&u_* = 0.2 \text{ m/s} \\
&\mathrm{z_i} = 500 \text{ m} \\
&\mathrm{k} = 0.4 \\
&\mathrm{g}/\overline{\theta} = 0.0333 \text{ m s}^{-2} \text{ K}^{-1} \\
&\mathrm{z} = 6 \text{ m} \\
&\mathrm{z_o} = 0.01 \text{ m} = \text{ roughness length} \\
&\text{ no moisture}
\end{aligned} \tag{1.1}$$

Find

(a) L
(b) z/L
(c) w_*
(d) θ_*
(e) static stability
(f) R_f at 6 m
(g) Ri at 6 m
(h) dynamic stability
(i) flow state (turbulent or not)

Solution.

(a) The Obukhov length is given by

$$L = \frac{-\overline{\theta_v} u_*^3}{kg\left(\overline{w'\theta_v'}\right)_s} \tag{1.2}$$

Assuming $\overline{\theta} = \overline{\theta_v}$ because of no moisture and plugging in the given values, we get

$$\mathbf{L} = \frac{-(0.2\ m/s)^3}{(0.4)(0.0333\ m\ s^{-2}\ K^{-1})\,(0.2\ K\ m/s)} = \mathbf{-3.003\ m} \tag{1.3}$$

(b) Using the previously calculated Obukhov length and the given $z = 6\ m$, we get

$$\frac{\mathbf{z}}{\mathbf{L}} = \frac{6\ m}{-3.003\ m} = \mathbf{-2} \tag{1.4}$$

(c) The convective velocity scale is given by

$$w_* = \left[\frac{gz_i}{\overline{\theta_v}}\left(\overline{w'\theta_v'}\right)_s\right]^{1/3} \tag{1.5}$$

Assuming $\overline{\theta} = \overline{\theta_v}$ and plugging in the given values, we get

$$\mathbf{w_*} = \left[(500\ m)(0.0333\ m\ s^{-2}\ K^{-1})\,(0.2\ K\ m/s)\right]^{1/3} = \mathbf{1.49\ m/s} \tag{1.6}$$

(d) The temperature scale is given by

$$\theta_* = \frac{\left(\overline{w'\theta_v'}\right)_s}{w_*} \tag{1.7}$$

Using the previously solved for w_* and the given $\left(\overline{w'\theta_v'}\right)_s$, we get

$$\theta_* = \frac{(0.2\ K\ m/s)}{1.49\ m/s} = \mathbf{0.13\ K} \tag{1.8}$$

(e) The buoyancy flux $\left(\overline{w'\theta_v'}\right)_s$ is positive and z/L is negative so the BL is **statically unstable**.

(f) The flux Richardson number assuming homogeneity and neglecting subsidence is given by

$$R_f = \frac{\left(\frac{g}{\overline{\theta}_v}\right)\left(\overline{w'\theta_v'}\right)}{\left(\overline{u'w'}\right)\frac{\partial \overline{U}}{\partial z} + \left(\overline{v'w'}\right)\frac{\partial \overline{V}}{\partial z}} \tag{1.9}$$

Using the law of the wall for an unstable BL, the vertical wind profile is given by

$$\frac{d\overline{U}}{dz} = \frac{u_*}{kz}\left(1 - 15\frac{z}{L}\right)^{-1/4} \tag{1.10}$$

Assuming the momentum flux is approximately constant with height in the surface layer, $-\overline{u'w'} = u_*^2$. Finally, assuming $\overline{V}$ doesn't change with height and using the law of the wall, this simplifies into

$$R_f = \frac{\left(\frac{g}{\overline{\theta}_v}\right)\left(\overline{w'\theta_v'}\right)}{\frac{-u_*^3}{kz}\left(1 - 15\frac{z}{L}\right)^{-1/4}} \tag{1.11}$$

Plugging in the given values and the previously solved for z/L, we get

$$\mathbf{R_f} = \frac{\left(0.0333\ m\ s^{-2}\ K^{-1}\right)(0.2\ K\ m/s)}{\frac{-(0.2\ m/s)^3}{0.4(6\ m)}\left(1 - 15\,(-2)\right)^{-1/4}} = \mathbf{-4.71} \tag{1.12}$$

(g) The gradient Richardson number is given by

$$\mathrm{Ri} = \frac{\frac{g}{\overline{\theta}_v}\frac{\partial \overline{\theta}_v}{\partial z}}{\left[\left(\frac{\partial \overline{U}}{\partial z}\right)^2 + \left(\frac{\partial \overline{V}}{\partial z}\right)^2\right]} \tag{1.13}$$

Using the same assumptions as R_f and using the law of the wall wind profile again for an unstable BL, this simplifies into

$$\mathrm{Ri} = \frac{\frac{g}{\overline{\theta}_v}\frac{\partial \overline{\theta}_v}{\partial z}}{\left(\frac{u_*}{kz}\left(1 - 15\frac{z}{L}\right)^{-1/4}\right)^2} \tag{1.14}$$

Since it's statically unstable, I assumed a negative dry lapse rate of about $-9.8\ K/km$ or $-0.0098\ K/m$ at the surface layer. Plugging this in together with the other given values, we get

$$\mathbf{Ri} = \frac{(0.0333\ m\ s^{-2}\ K^{-1})(-0.0098\ K/m)}{\left(\frac{0.2\ m/s}{0.4(6\ m)}\left(1 - 15\,(-2)\right)^{-1/4}\right)^2} = \mathbf{-0.26} \tag{1.15}$$

(h) Since $\mathrm{Ri} < R_c$ where $R_c = 0.25$ and $R_f < 1$, it is **dynamically unstable** at $z = 6\ m$.

(i) Using the same reasoning as the dynamic stability above, the flow is **turbulent** at $z = 6\ m$.

□

Problem 2

Given the following sounding in the morning boundary layer. Determine whether each layer is stable or unstable (in both the static and dynamic sense), and state if the flow is turbulent.

Solution. Since no information is given on $\overline{V}$, I just assumed that it doesn't change with height. Using discrete values, the Richardson number is then given by

$$\mathrm{Ri} = \frac{\frac{g}{\overline{\theta_v}} \frac{\Delta\overline{\theta_v}}{\Delta z}}{\left(\frac{\Delta\overline{U}}{\Delta z}\right)^2} \tag{2.1}$$

I based the static stability of each layer on the vertical profile of $\overline{\theta_v}$. On the other hand, the dynamic stability and turbulence depend on Ri. The summary of calculated values and characteristics per layer starting from the very top (layer 1) is shown below. Layer 5 has an undefined $\left(\frac{0}{0}\right)$ Ri, but since it's statically unstable, it is also dynamically unstable.

Layer	$\overline{\theta_v}$ (K)	$\Delta\overline{\theta_v}/\Delta z$ (K/m)	$\Delta\overline{U}/\Delta z$ (s^{-1})	Ri	Static Stability	Dynamic Stability	Turbulent? (Y/N)
1	293.5	0.002	0.002	16.7	stable	stable	N
2	292.5	0.01	0.04	0.21	stable	unstable	Y
3	292	0	0.0025	0	neutral	unstable	Y
4	291	0.01	0.01	3.37	stable	stable	N
5	290	0	0	undefined	unstable	unstable	Y
6	290.5	-0.01	0.02	-0.84	unstable	unstable	Y

□

Problem 3

The dissipation rate of TKE is sometimes approximated by $\varepsilon = \overline{e}^{3/2}/l$, where l is the ***dissipation length scale***. It is often assumed that $l = 5z$ in statically neutral conditions (Louis, et al., 1983). If the TKE shown in Fig 2.9b is assumed as an initial condition, there is no shear or buoyancy production or loss, and no redistribution nor turbulent transport, then at z = 100m:

(a) What is the initial value of the dissipation rate?

(b) How long will it take the TKE to decay to 10% of its initial value?

Solution.

(a) Initially, at $z = 100\ m$, TKE$/m = \overline{e_o} = 3.3\ m^2/s^2$. Plugging this into the given equation for the dissipation rate, we get the dissipation rate

$$\varepsilon = \frac{\left(3.3\ m^2/s^2\right)^{3/2}}{5(100\ m)} = \mathbf{0.012\ m/s^2} \tag{3.1}$$

(b) The simplified TKE equation is given by

$$\frac{\partial\overline{e}}{\partial t} = \frac{g}{\overline{\theta}_v}\left(\overline{w'\theta_v'}\right) - \overline{u'w'}\frac{\partial\overline{U}}{\partial z} - \frac{\partial\left(\overline{w'e}\right)}{\partial z} - \frac{1}{\overline{\rho}}\frac{\partial\left(\overline{w'p'}\right)}{\partial z} - \varepsilon \tag{3.2}$$

Neglecting shear, buoyancy, redistribution, and turbulent transport, TKE will only depend on the dissipation rate such that

$$\frac{\partial \overline{e}}{\partial t} \equiv \frac{\Delta \overline{e}}{\Delta t} = -\varepsilon \tag{3.3}$$

We are asked to solve for Δt for the TKE to decay to 10% its initial value, so

$$\Delta t = -\frac{0.1\overline{e_o} - \overline{e_o}}{\varepsilon} = \frac{0.9\overline{e_o}}{\varepsilon} \tag{3.4}$$

Plugging in the initial TKE $\overline{e_o}$ and the previously calculated ε, we get

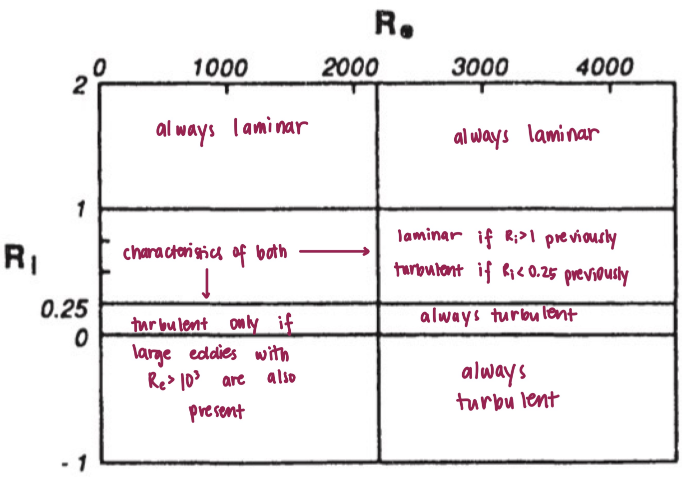
$$\Delta t = \frac{0.9\left(3.3\ m^2/s^2\right)}{0.012\ m/s^2} = \mathbf{247.7\ s} \equiv 4.13\ mins. \tag{3.5}$$

□

Problem 4

Indicate the nature of the flow (laminar or turbulent) for each cell in the table below.

Solution.



□