AATM 529 – Air Sea Interactions

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Homework 5

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Problem 1

Given values for the viscous dissipation rate of velocity variance (see section 4.3.1), express that rate as a heating rate $\partial\overline{\theta}/\partial t$ for air, and compare its magnitude with the magnitudes of other terms in (3.4.5b). Hint, remember that viscosity dissipates turbulent motions into heat.

Solution. The dissipation rate of velocity variance is given by

$$\varepsilon = v\overline{\left(\frac{\partial u_i'{}_i}{\partial x_j}\right)^2} \tag{1.1}$$

It's magnitude is around $10^{-4}\ m^2/s^3$, but it increases to $10^{-2}\ m^2/s^3$ near the surface during smaller scales of motion. By dividing this rate by the specific heat of air C_p, we can get a heating rate for air.

$$\frac{v}{C_p}\overline{\left(\frac{\partial u_i'{}_i}{\partial x_j}\right)^2} \equiv \left[\frac{1}{J\ kg^{-1}K^{-1}}\frac{m^2}{s^3}\right] \equiv \left[\frac{1}{kg\ m^2\ s^{-2}\ kg^{-1}\ K^{-1}}\frac{m^2}{s^3}\right] \equiv \left[\frac{K}{s}\right] \tag{1.2}$$

The magnitude of C_p is around $10^3\ J\ kg^{-1}K^{-1}$, so the magnitude of this heating rate is around $10^{-7}\ m^2/s^3$ within the mixing layer and around $10^{-5}\ m^2/s^3$ near the surface. The equation for the conservation heat is given by

$$\frac{\partial\overline{\theta}}{\partial t} + \frac{\overline{U}_j\partial\overline{\theta}}{\partial x_j} = \frac{v_\theta\partial^2\overline{\theta}}{\partial x_j^2} - \frac{1}{\overline{\rho}C_p}\frac{\partial\overline{Q}_j^*}{\partial x_j} - \frac{L_vE}{\overline{\rho}C_p} - \frac{\partial\left(\overline{u_j'\theta'}\right)}{\partial x_j} \tag{1.3}$$

Here, the advection term (2nd), radiation divergence term (4th), latent heat release term (5th), and the divergence of turbulent heat flux term (6th) are all roughly the same magnitude at around $10^{-4}\ K/s$. On the other hand, the viscosity term or molecular conduction of heat (3rd) has a much smaller magnitude at around $10^{-11}\ K/s$, so it can be neglected when considering mean temperatures in turbulent flow.

The magnitude of the heating rate due to velocity variance dissipation within the mixing layer at $10^{-7}\ K/s$ is then much smaller compared to the other terms except for the conduction of heat. However, nearer to the surface, since the heating rate due to velocity variance dissipation increases to $10^{-5}\ K/s$, it's contributions may be more important.

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Problem 2

In Fig 3.1a of chapter 3 are plotted two data points at each height. One data point represents heat flux and one represents moisture flux. Using the values from this figure , calculate $\overline{w'\theta'_v}$ for each of those heights, and plot the result. Do NOT normalize your results by the surface value.

Solution.

From Eqn. 4.4.5d in Stull, the buoyancy flux can be estimated in terms of the heat and moisture flux using the following equation.

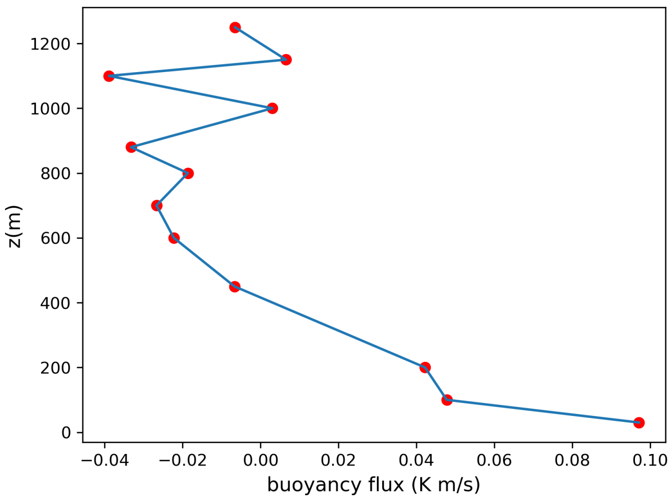
$$\overline{w'\theta'_v} \cong \left(\overline{w'\theta'}\right)[1 + 0.61\overline{q}] + 0.61\overline{\theta}\left(\overline{w'q'}\right) \tag{2.1}$$

The values for $\overline{w'\theta'}$ and $\overline{w'\rho'_v}$ at each height are given in Fig. 3.1a, and the mean potential temperatures are given in Fig. 3.4a. Since, not much additional information was provided, I just assumed the mean specific humidity q can be estimated from Fig. 3.6.

The following table summarizes the values given in Fig. 3.1a, Fig. 3.4a, Fig. 3.6, and $\overline{w'\theta'_v}$ using the above equation. I also just divided $\overline{w'\rho'_v}$ by the density of air $\rho_{air} = 1.225\ kg/m^3$ to match the units. I also converted all instances of q from g/kg to g/g to cancel out the units and ultimately leave us with $K\ m/s$ after solving the final equation.

z (m)	$\overline{w'\theta'}$ $(K\ m/s)$	$\overline{w'\rho'_v}(g/m^3\ m/s)$	$\overline{\theta}$ (K)	$\overline{q}$ (g/kg)	$\overline{w'\theta'_v}$ $(K\ m/s)$
30	0.08	0.11	301.3	12.5	0.1
100	0.04	0.05	300.8	12.5	0.048
200	0.03	0.08	300.9	12.5	0.042
450	-0.02	0.09	300.9	12.7	-0.007
600	-0.04	0.13	302	12	-0.022
700	-0.025	-0.01	302.8	12.5	-0.027
800	-0.02	0.01	302.3	12	-0.019
880	-0.045	0.08	302.3	10	-0.033
1000	0	0.02	303.8	9	0.003
1100	-0.05	0.075	304.5	8	-0.039
1150	0.005	0.01	305	7	0.007
1250	-0.005	-0.01	305.5	7	-0.007

When plotted, the buoyancy flux looks like the following.



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