AATM 529 – Air Sea Interactions

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Homework 6

Problem 1

Given the following wind speeds measured at various heights in the boundary layer. Assume that the potential temperature increases with height at the constant rate of 6 K/km. Calculate the bulk Richardson number for each layer and indicate the static and dynamic stability of each layer. Also, show what part of the atmosphere is expected to be turbulent in these conditions.

Solution. The bulk Richardson number R_B is given by

$$R_B = \frac{g\Delta\overline{\theta_v}\Delta z}{\overline{\theta_v}\left[(\Delta\overline{U})^2 + (\Delta\overline{V})^2\right]} \tag{1.1}$$

Since no information is given on V, I just assumed it's zero or it doesn't change with height, so the bulk Richardson number is just

$$R_B = \frac{g\Delta\overline{\theta_v}\Delta z}{\overline{\theta_v}(\Delta\overline{U})^2} \tag{1.2}$$

I also assumed the virtual potential temperature to be 300 K at the surface. We are also given that $\Delta\overline{\theta}_v/\Delta z = 6$ K/km. Lastly, applying the no-slip boundary condition at the surface, $\overline{U} = 0$ at $z = 0$ m. The summary table for all calculated values is shown below.

<table>
<tr><th>z (m)</th><th>U (m/s)</th><th>$\overline{\theta}_v$</th><th>Δz</th><th>ΔU</th><th>$\Delta\overline{\theta}_v$</th><th>$R_B$</th><th>turbulent? (Y/N)</th><th>dynamic stability</th></tr>
<tr><td>2000</td><td>10</td><td>312</td><td>1000</td><td>0</td><td>6</td><td>∞</td><td rowspan="5">N</td><td rowspan="5">stable</td></tr>
<tr><td>1000</td><td>10</td><td>306</td><td>500</td><td>0.5</td><td>3</td><td>192</td></tr>
<tr><td>500</td><td>9.5</td><td>303</td><td>200</td><td>0.5</td><td>1.2</td><td>31</td></tr>
<tr><td>300</td><td>9</td><td>301.8</td><td>200</td><td>1</td><td>1.2</td><td>7.79</td></tr>
<tr><td>100</td><td>8</td><td>300.6</td><td>50</td><td>0.6</td><td>0.3</td><td>1.36</td></tr>
<tr><td>50</td><td>7.4</td><td>300.3</td><td>30</td><td>0.9</td><td>0.18</td><td>0.218</td><td rowspan="5">Y</td><td rowspan="5">unstable</td></tr>
<tr><td>20</td><td>6.5</td><td>300.12</td><td>10</td><td>0.7</td><td>0.06</td><td>0.04</td></tr>
<tr><td>10</td><td>5.8</td><td>300.06</td><td>6</td><td>0.8</td><td>0.036</td><td>0.01</td></tr>
<tr><td>4</td><td>5</td><td>300.024</td><td>3</td><td>1.3</td><td>0.018</td><td>0.001</td></tr>
<tr><td>1</td><td>3.7</td><td>300.006</td><td>1</td><td>3.7</td><td>0.006</td><td>8.6×10^{-8}</td></tr>
</table>

Assuming $R_C = 0.25$ and $R_T = 1$, if $R_B < R_C$, the flow becomes turbulent, and if $R_B > R_T$, the flow becomes laminar. Looking at the vertical profile of $\overline{\theta}_v$, it is increasing constantly with height, so **all layers are statically stable**.

□

Problem 2

Given the following TKE equation:

$$\underset{\text{A}}{\frac{\partial \overline{e}}{\partial t}} + \underset{\text{B}}{\overline{U}_j \frac{\partial \overline{e}}{\partial x_j}} = \underset{\text{C}}{-\overline{u'w'}\frac{\partial \overline{U}}{\partial z}} + \underset{\text{D}}{\frac{g}{\overline{\theta}_v}\left(\overline{w'\theta_v'}\right)} - \underset{\text{E}}{\frac{\partial\left(\overline{w'e}\right)}{\partial z}} - \underset{\text{F}}{\frac{1}{\overline{\rho}}\frac{\partial\left(\overline{w'p'}\right)}{\partial z}} - \underset{\text{G}}{\varepsilon} \tag{2.1}$$

(a) Which terms are always loss terms?

(b) Which terms, neither create nor destroy TKE?

(c) Which terms can be either production or loss?

(d) Which terms are due to molecular effects?

(e) Which production terms are largest on a cloudy, windy day?

(f) Which production terms are largest on a calm sunny day over land?

(g) Which terms tend to make turbulence more homogeneous?

(h) Which terms tend to make turbulence less isotropic?

(i) Which terms describe the stationarity of the turbulence?

(j) Which terms describe the kinetic energy lost from the mean wind?

Solution.

(a) The viscous dissipation ε (**term G**) is always a loss of TKE into heat.

(b) The turbulent transport (**term E**) and pressure correlation (**term F**) can neither create nor destroy TKE, only redistribute it.

(c) The buoyancy (**term D**) can either be production or loss depending on whether the heat flux is positive or negative. The mechanical shear (**term C**) can also be either production or loss, but $\overline{u'w'}$ is usually opposite the sign of the mean wind shear, so the shear term usually results to TKE production. Lastly, the turbulent transport (**term E**) can either be production or loss *locally* depending on flux convergence or divergence. However, looking at the overall boundary layer, it neither creates or destroys just redistributes TKE.

(d) The viscous dissipation ε (**term G**) is due to molecular effects.

(e) The shear production (**term C**) has the strongest magnitude during a windy overcast day.

(f) The buoyancy production (**term D**) has the strongest magnitude during a calm sunny day over land.

(g) The turbulent transport (**term E**) and pressure correlation (**term F**) tend to make turbulence more homogenous by redistributing TKE within the BL via turbulent eddies and pressure perturbations, respectively.

(h) The mechanical shear (**term C**) and buoyancy (**term D**) are both anisotropic. The mechanical shear makes turbulence less isotropic mostly along the horizontal, while buoyancy makes turbulence less isotropic along the vertical.

(i) The TKE storage (**term A**) describes the stationarity (statistically not changing over time) of the turbulence.

(j) The mechanical shear (**term C**) describes kinetic energy lost from the mean flow.

□

Problem 3

Very briefly define the following, and comment or give examples of their use in micrometeorology.

(a) inertial subrange

(b) friction velocity

(c) Obukhov length

(d) return-to-isotropy term

(e) static stability

(f) convective velocity scale

(g) Reynold's stress

(h) turbulence closure problem

(i) Richardson number

(j) TKE

Solution.

(a) The inertial subrange is the middle portion of the atmospheric turbulence spectrum. TKE production from buoyancy and shear feeds into larger sized eddies (wavenumbers 0.001-0.1), while dissipation acts on smaller sized eddies (wavenumbers 100-1000). Within the middle wavenumbers, the rate of transfer of inertia from larger eddies to smaller eddies is equal to the rate of dissipation ε. It is essential in determining the cascade of energy from larger to smaller scales.

(b) The friction velocity is a velocity scale used when wind shear generates or modulates turbulence near the surface. It uses the magnitude of the surface Reynolds' stress τ_{R} as a scaling variable such that

$$u_*^2 \equiv \left[\overline{u'w'_s}^2 + \overline{v'w'_s}^2\right]^{1/2} = \left|\tau_{\mathrm{R}}\right| / \bar{\rho} \tag{3.1}$$

(c) The Obukhov length is proportional to the altitude where buoyancy first dominates mechanical shear. It is used as a scaling parameter of turbulence in the surface layer. As an equation, it is defined as

$$L = \frac{-\bar{\theta}_v u_*^3}{kg\left(\overline{w'\theta_v'}\right)_s} \tag{3.2}$$

(d) The pressure redistribution term is known as the return-to-isotropy term. It does not contribute to TKE production or loss, but it tends to take energy from components with high energy and redistribute said energy into components with less energy.The return-to-isotropy term is essential in studying the dynamics of small versus large eddies, since smaller sized eddies are more isotropic.

(e) Static stability is a measure of the capability of buoyant convection, a flow independent of wind. Measurements of the turbulent buoyancy flux or the whole vertical profile of $\overline{\theta}_v$ determine static stability. Static stability is one of the factors that determine the presence of convective circulations such as thermals that *push* buoyant air towards the top of an unstable layer.

(f) The convective velocity scale removes the nonstationary effects of the strong diurnal cycle of solar heating on the buoyancy flux. It uses the buoyancy flux at the surface $\overline{w'\theta_v'}$ and the thermal length scale z_i such that

$$w_* = \left[\frac{g z_i}{\overline{\theta_v}} \left(\overline{w'\theta_v'} \right)_s \right]^{1/3} \tag{3.3}$$

(g) Reynolds stress is turbulent momentum flux that deforms bodies of fluids. Reynolds stress can be used to describe eddies mixing surrounding airflow towards our fluid of interest.

(h) In the turbulence closure problem, the number of unknowns in the set of equations for turbulent flow is larger than the number of equations. New equations formulated to solve for these unknown variables include even more unknown variables, which presents an infinite loop. Simply, the total statistical description of turbulence requires an infinite set of equations.

(i) The Richardson number is the ratio between the buoyancy term and the mechanical shear term in the TKE budget equation. The Richardson number determines the presence of turbulent versus laminar flow, which can indicate the height of the stable boundary layer.

(j) In micrometeorology, turbulent kinetic energy is a measure of the intensity of turbulence, and its budget is dictated by momentum, heat, and moisture transport through the boundary layer. It is defined as $\overline{e} = 0.5 \left(\overline{u'^{\,2}} + \overline{v'^{\,2}} + \overline{w'^{\,2}} \right)$, half the sum of the velocity variances.

□

Problem 4

What is the Reynolds stress? Why is it called a stress? How does it relate to u_*?

Solution. Reynolds stress is momentum flux only present in turbulent motion. The Reynolds stress considers three further directions of deformations across each Cartesian direction, resulting into a tensor that has nine components. It causes deformations across a body of fluid, thus it is classified as type of stress. The Reynolds stress at the surface τ_R acts as a velocity scaling variable for the friction velocity u_* such that

$$u_*^2 = \left| \tau_R \right| / \bar{\rho} \tag{4.1}$$

□

Problem 5

Define the following types of convection. Under what weather conditions is each type of convection most likely? What term in the TKE equation is small under each condition?

(a) Free convection

(b) Forced convection

Solution.

(a) During free convection, the buoyancy term is much larger then the mechanical shear term. The buoyancy term tends to have higher magnitudes during sunny calm days over land or cold air advecting over a warmer surface, since active thermal convection is associated with more positive values of buoyancy. Turbulency is primarily produced in the vertical during free convection.

(b) During forced convection, the mechanical shear term is much larger then the buoyancy term. The mechanical shear term tends to have higher magnitudes during cloudy wind days, and at night over land (or anytime the ground is colder than the air). Turbulence is primarily produced in the horizontal during forced convection.

□

Problem 6

If the TKE at 10 m is at steady state, and if $\varepsilon = 0.01\ m^2s^{-3}$, then is the transport term supplying or removing TKE from the air at $z = 10\ m$?

Solution. The simplified TKE equation at steady state $\left(\frac{\partial \overline{e}}{\partial t} = 0\right)$ is given by

$$0 = \frac{\mathrm{g}}{\bar{\theta}_{\mathrm{v}}}\left(\overline{w'\theta_v'}\right) - \overline{u'w'}\frac{\partial \overline{\mathrm{U}}}{\partial z} - \frac{\partial\left(\overline{w'e}\right)}{\partial z} - \frac{1}{\bar{\rho}}\frac{\partial\left(\overline{w'p'}\right)}{\partial z} - \varepsilon \tag{6.1}$$

Rearranging this to focus on the transport term, we get

$$\frac{\partial\left(\overline{w'e}\right)}{\partial z} = \frac{\mathrm{g}}{\bar{\theta}_{\mathrm{v}}}\left(\overline{w'\theta_v'}\right) - \overline{u'w'}\frac{\partial \overline{\mathrm{U}}}{\partial z} - \frac{1}{\bar{\rho}}\frac{\partial\left(\overline{w'p'}\right)}{\partial z} - \varepsilon \tag{6.2}$$

From here, we see that the dissipation ε has a negative effect on the transport. We are given that $\varepsilon = 0.01\ m^2s^{-3}$, so the transport term must be removing TKE from the air at $z = 10\ m$. □

Problem 7

What is the static stability of each of the layers in the diagram?

Solution. Because of the negative virtual potential gradient near the surface and a positive virtual potential gradient towards the top of the BL, **A and B are statically unstable** and **C is statically stable**.

Because of the positive virtual potential gradient near the surface and towards the top of the BL, **D and F are statically stable**. Since the virtual potential gradient is almost zero in the middle, **E is neutral**.

□