Language Modeling and N-Grams

Ling 282/482: Deep Learning for Computational Linguistics

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Language Modeling (task)

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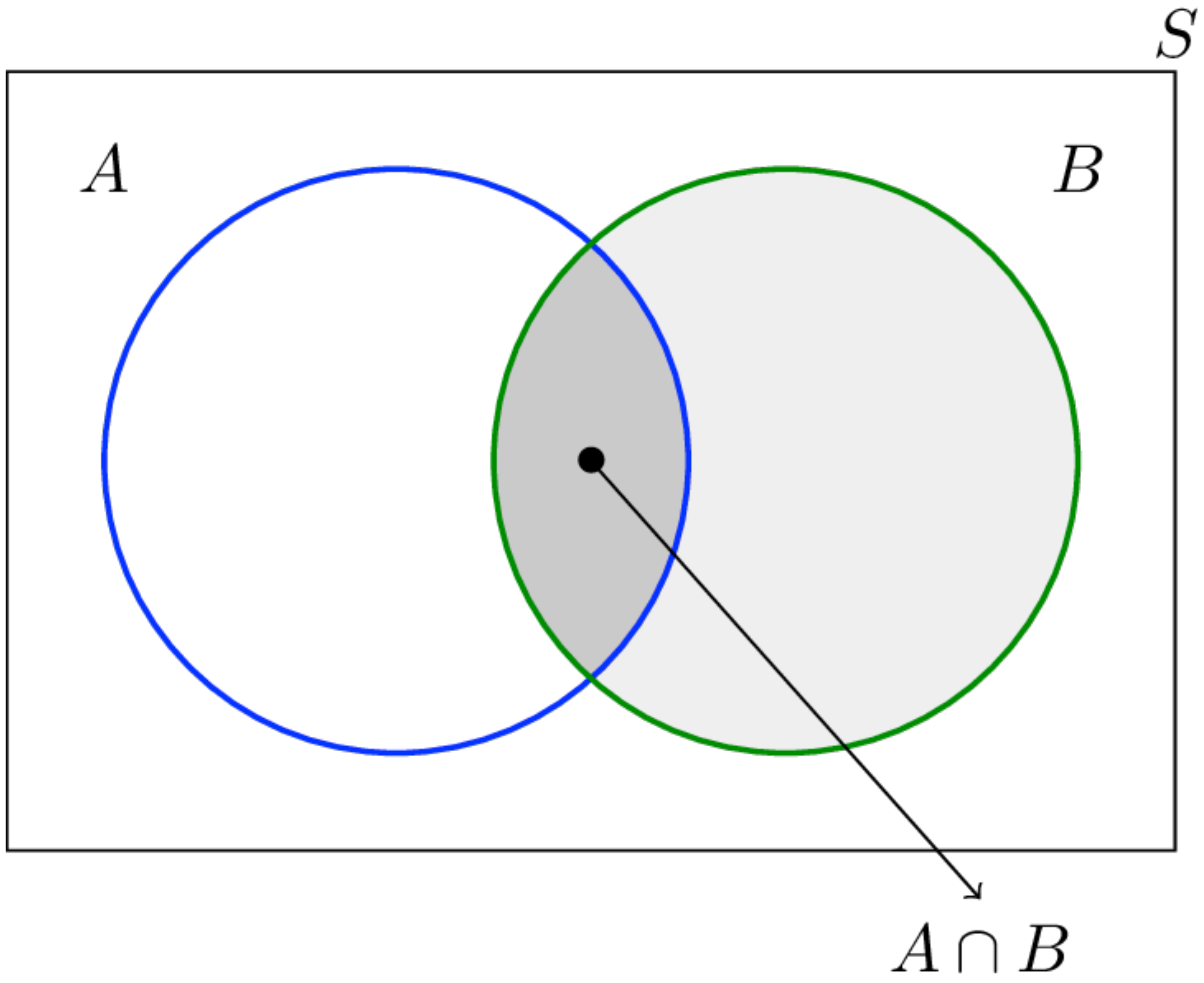
    - This is shorthand for the **joint probability** of seeing these words **together and in this order**
    - This is a simplification of notation, as you might notice if taking LING 214. More precise would be something like $P(X_1 = w_1, X_2 = w_2 \ldots X_k = w_k)$

Briefly: Conditional Probability



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

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- For Language Modeling: what is the probability of the next word **given the previous words**

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- But how do we get these **conditional probabilities?**

Counting-based Language Models

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- **But...** how many times can we actually expect to encounter that prefix?
  - Not many! This probably gives a **poor estimation** of the conditional prob.

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N-Grams

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- **Markov assumption:** probabilities are only conditioned on a **finite history**
  - i.e. **don't look too far** into the past
- For LMs, often called the **n-gram assumption**
  - Only use **a few previous words** to predict the current word!

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- Any other size would be called 4-gram, 5-gram, etc.
- The **value of n** is left as an **engineering choice**
  - **However,** for large n we run into the **exact same rarity problem** as before

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  - Simple: we'll bring back **counting occurrences!**
  - The prob. of **all possible following** w_i will **sum to 1.0**

$$P(w_i \mid w_{i-1}) = \frac{C(w_{i-1}w_i)}{C(w_{i-1})}$$

Practice Example

- What are all the **bigram probabilities** in the following dataset?
  - (<s> and </s> are special symbols meaning **beginning/end of sequence**)

```
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$$P(\texttt{I}|\texttt{<s>}) = \frac{2}{3} = 0.67 \qquad P(\texttt{Sam}|\texttt{<s>}) = \frac{1}{3} = 0.33 \qquad P(\texttt{am}|\texttt{I}) = \frac{2}{3} = 0.67$$

$$P(\texttt{</s>}|\texttt{Sam}) = \frac{1}{2} = 0.5 \qquad P(\texttt{Sam}|\texttt{am}) = \frac{1}{2} = 0.5 \qquad P(\texttt{do}|\texttt{I}) = \frac{1}{3} = 0.33$$

Real Bigram Counts

	i	**want**	**to**	**eat**	**chinese**	**food**	**lunch**	**spend**
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Figure 3.1 Bigram counts for eight of the words (out of $V = 1446$) in the Berkeley Restaurant Project corpus of 9332 sentences. Zero counts are in gray. Each cell shows the count of the column label word following the row label word. Thus the cell in row **i** and column **want** means that **want** followed **i** 827 times in the corpus.

Real Bigram Probabilities

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Figure 3.2 Bigram probabilities for eight words in the Berkeley Restaurant Project corpus of 9332 sentences. Zero probabilities are in gray.