Variables and Descriptive Statistics

Ling250: Data Science for Linguistics
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Variable types and scales

- A variable is some measurable attribute of data
 - Sometimes comes from instrumentation or observation (e.g. vowel formants are measured with a microphone and audio software)
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- Variables are not all on the same scale of measurement
 - Can't directly compare birth year, eye color, and vowel formants

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- The scale of variables tells us how we should analyze them
- We'll go into each of these in more detail

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- I will usually refer to these as categorical variables

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- Lack of zero means intervals are not meaningful to multiply & divide

- Ratio variables have ordering, meaningful differences, and a true zero
 - Examples: race finish time, weight, age, sports scores (usually)
 - Meaningful to say a 30 y/o is 5 years older than a 25 y/o (30 25)
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- These are what you might think of as "regular" numbers

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- Nominal and ordinal variables are discrete by definition

	continuous	discrete	
nominal			- -
ordinal			LSR (p.17)
interval			
ratio			

Exercise

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 - Sex: nominal/categorical
 - Height: ratio

Briefly: factors in R

- Sometimes a categorical variable is coded as a number, e.g. in a CSV
 - A column for "sex" might use 0 for male, 1 for female
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- Data can be converted to factors with the as.factor() function
 - This prevents them being manipulated in inappropriate ways

```
> sex = c(0, 1, 0, 0, 1, 0, 0, 0)
> sex + 1
[1] 1 2 1 1 2 1 1 1
> sex = as.factor(sex)
> sex
[1] 0 1 0 0 1 0 0 0
Levels: 0 1
> sex + 1
[1] NA NA NA NA NA NA NA NA
Warning message:
In Ops.factor(sex, 1) : '+' not meaningful for factors
```

Factor levels

Factor levels

- The same numbers might be used for different factor variables
 - This also causes unintended
 equivalence between these
 variables

```
> sex = as.factor(c(0, 1, 0, 0, 1, 0, 0, 0))
> placebo_group = as.factor(c(1, 0, 0, 1, 1, 0, 1, 1))
> sex == placebo_group
[1] FALSE FALSE TRUE FALSE TRUE TRUE FALSE FALSE
> levels(sex) = c("male", "female")
> levels(placebo_group) = c("placebo", "drug")
> sex
[1] male female male male female male
                                             male
[8] male
Levels: male female
> placebo_group
[1] drug placebo placebo drug
                                           placebo
                                   drug
[7] drug
           drug
Levels: placebo drug
> sex == placebo_group
Error in Ops.factor(sex, placebo_group) :
 level sets of factors are different
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- The levels (values) of a factor can be given names with levels ()
 - This can disambiguate mixups

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Descriptive statistics

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- Statistics is usually divided into descriptive and inferential versions
 - Descriptive: goal is to concisely and meaningfully summarize some data
 - Inferential: goal is to well... make inferences from some data (more in a week or two)
- Two broad types of descriptive statistics are **central tendencies** (mean, median, mode, etc.) and **variability** (variance, standard deviation, etc.)
 - Central tendency: where the "middle" of the data is
 - Variability: how much the data deviates from the central tendency

Central tendencies

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- (\overline{X} is the mean, X is the vector of all datapoints, X_i is a particular datapoint, N is the total number of datapoints)
- Can be calculated easily in R with mean()

```
> mean(c(1, 2, 3, 4, 5))
[1] 3
```

- The median is the middle datapoint, when arranged in order
 - 3 is the median of **both** [1, 2, 3, 4, 5] and [1, 2, 3, 8, 10]
 - If there's an **even** number of datapoints, it's the **average of the middle two** (the median of [1, 2, 3, 4] is 2.5

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- Ralso has a median () function

Mean vs. median

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- Advantages of using the median:
 - It is **resistant to outlier datapoints** (thus why most people refer to "median salary" or "median home price"; ultra-high earners skew the mean)
 - It is appropriate for ordinal data whereas mean is not!

Mean vs. median intuition

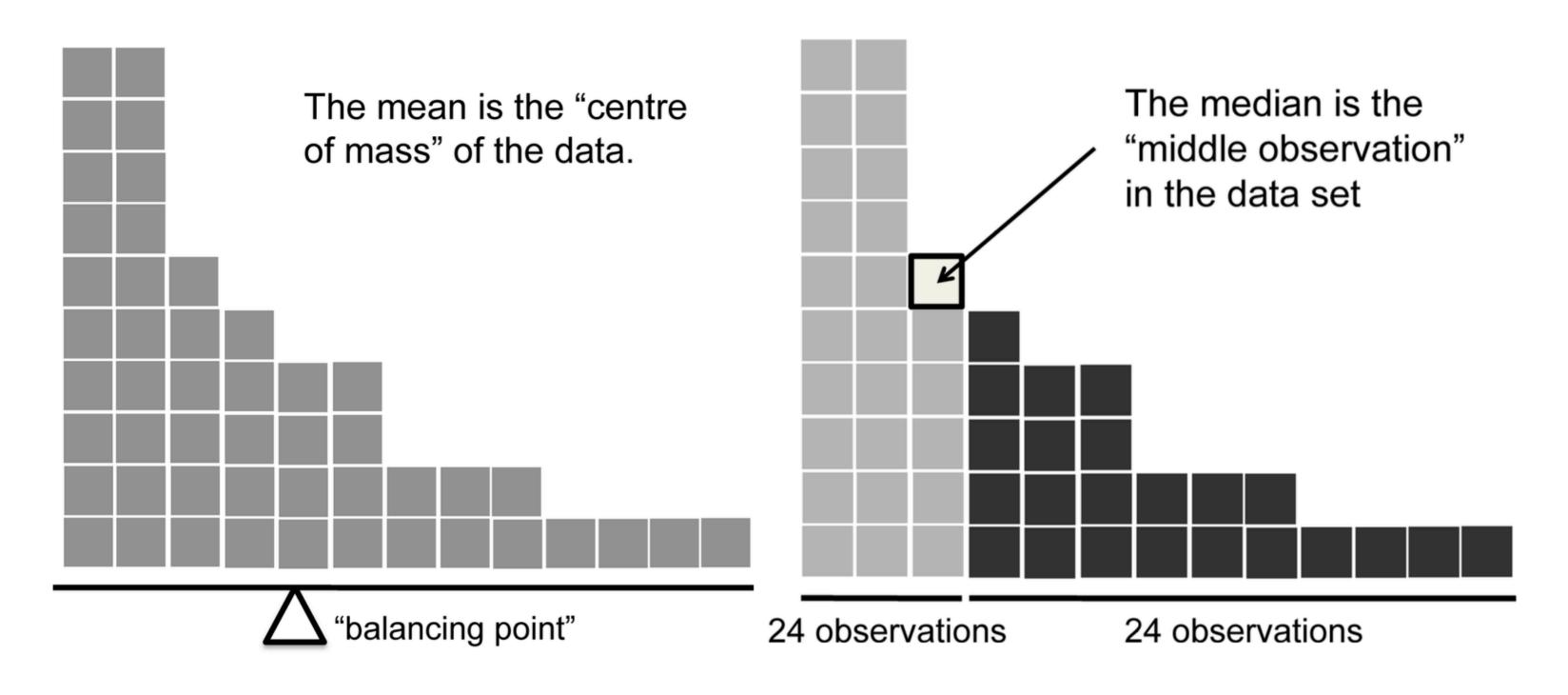


Figure 5.2: An illustration of the difference between how the mean and the median should be interpreted. The mean is basically the "centre of gravity" of the data set: if you imagine that the histogram of the data is a solid object, then the point on which you could balance it (as if on a see-saw) is the mean. In contrast, the median is the middle observation. Half of the observations are smaller, and half of the observations are larger.

Variability

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- Mean Absolute Deviation: the **average deviation** between the mean and all datapoints $\frac{1}{N} \sum_{i=1}^{N} |X_i \overline{X}|$
- This measure is **easy to understand**, but **not used much** (to my knowledge)

Variance is can also be defined as the mean squared deviation

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- R command: var()

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• Standard Deviation is defined as the square root of the variance:

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- R command: sd()

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 - The median is the same thing as the 50th percentile
- Can view quantiles in R with quantile()

```
> quantile(vowels$HEIGHT, probs=0.5)
50%
169
> quantile(vowels$HEIGHT, probs=c(0.25, 0.5, 0.75))
    25%    50%    75%
163.00 169.00 175.25
```

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- The idea is this shows how wide the middle 50% of the data is
 - More useful in comparison to the total range
- Can view in R with IQR()

```
> IQR(vowels$HEIGHT)
[1] 12.25
> range = max(vowels$HEIGHT) - min(vowels$HEIGHT)
> range
[1] 36
```

Summary statistics

 R has a handy function to get several summary statistics in a single command, and can be called on either a vector or a data frame

```
> summary(vowels$HEIGHT)
  Min. 1st Qu. Median
                        Mean 3rd Qu.
                                       Max.
 157.0 163.0
                169.0
                       169.9 175.2
                                      193.0
> summary(vowels)
   SPEAKER
                     WORD
                                      VOWEL
                              Length:484
            Length:484
Length:484
                                                    Min. : 177.6
Class:character Class:character Class:character
                                                    1st Qu.: 447.0
                                                    Median : 549.2
Mode :character
                 Mode :character
                                   Mode :character
                                                     Mean : 581.3
                                                     3rd Qu.: 710.2
                                                           :1059.1
                                                     Max.
      F2
                  SEX
                                   HEIGHT
              Length:484
                               Min. :157.0
Min.
     : 379
                               1st Qu.:163.0
1st Qu.:1256
              Class :character
Median :1541
                               Median :169.0
              Mode :character
Mean :1576
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Max. :2875
                                Max. :193.0
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Notice the summary of the character columns isn't very useful

Max. :2875

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