Three Factor Seasonal Commodity Price Process

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1 Introduction

This paper presents a specific set for parameters for the multi-factor model presented in Fowler (2020) such that the model should have similar statistical properties to the three-factor spot price model presented in Boogert and de Jong (2011) the model used in the commercial KyStore gas storage valuation model.

2 Forward Price SDE

The starting point is the SDE (stochastic differential equation) for the forward price process:

$$\frac{dF(t,T)^l}{F(t,T)^l} = \sum_{i=1}^{n^l} \sigma_i^l(T) e^{-\alpha_i^l (T-t)} dz_i^l(t) \tag{1}$$

$$\alpha_i^l \in \mathbb{R}_{\geq 0}$$
$$t \in \mathbb{R}_{\geq 0}$$
$$T \in \{T_0^l, T_1^l, T_2^l, \ldots | T_j^l \geq t\}$$
$$\sigma_i^l : \mathbb{R}_{\geq 0} \to \mathbb{R}$$
$$l \in [1, m]$$

Where $z_i^l(t)$ follow correlated Wiener processes with correlation $\rho_{i,j}^{x,y}$, i.e.

$$\mathbb{E}[dz_i^x(t) dz_j^y(t)] = \rho_{i,j}^{x,y} dt \tag{2}$$

$F(t, T_j)^l$ is the forward price observed at time t, for delivery over the time interval $[T_j, T_{j+1})$ of the l^{th} of m commodity underlyings.

3 Critique of Three Factor Seasonal Model

The strength of the three-factor seasonal model is it's parsimony. Being able to specify the gas price dynamics using only four parameters is of great help in allowing users to intuitively see what is driving the extrinsic value of gas storage being valued. Traders can easily adjust the input parameters based on their view of the market. For example if a trader take a view that the future summer-winter spread volatility is going to be higher than in the historical period used to calibrate the parameters they can easily bump up the seasonal volatility parameter when valuing a potential storage deal. For a non-parsimonious model with many parameters some of which are completely abstract (for example correlation between factors), such usage would not be practical.

Another example of the is that risk managers can create scenario matrices containing storage facility (or portfolio) P&L based on scenarios applied to any of the four parameters.

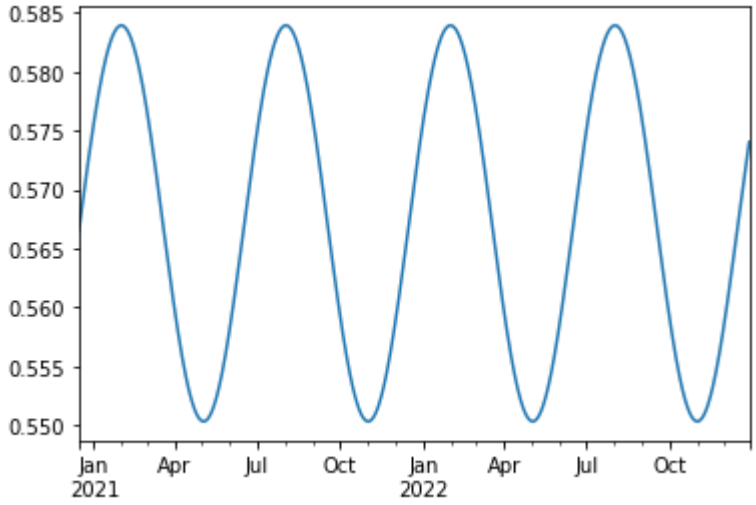

Figure 1: Forward Volatility By Delivery Date

It also allows for a relatively simple and intuitive way of calibrating model parameters from historic spot and forward prices. The KyStore product is clearly a popular one and The Author believes that this is at least partially due to the parsimony of the underlying price process model.

The big downside of this model is that the forward volatility seasonality structure is unrealistic. It is well known that, adjusting for time-to-maturity effects, the volatility of winter periods will generally be higher than those in summer. Figure 1 plots the forward volatility implied by the model, with mean reversion set to zero in order to remove any t time-to-maturity effect. This chart shows that the model implies two volatility peaks a year, once in February, as expected, but the other around August.

References

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