Three Factor Seasonal Commodity Price Process

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1 Introduction

This paper presents a specific set for parameters for the multi-factor model presented in Fowler (2020) such that the model should have similar statistical properties to the three-factor spot price model presented in Boogert and de Jong (2011) the model used in the commercial KyStore gas storage valuation model.

2 Forward Price SDE

The starting point is the SDE (stochastic differential equation) for the forward price process:

$$\frac{dF(t,T)^l}{F(t,T)^l} = \sum_{i=1}^{n^l} \sigma_i^l(T) e^{-\alpha_i^l(T-t)} dz_i^l(t) \tag{1}$$

$$\alpha_i^l \in \mathbb{R}_{\geq 0}$$

$$t \in \mathbb{R}_{\geq 0}$$

$$T \in \{T_0^l, T_1^l, T_2^l, \ldots | T_j^l \geq t\}$$

$$\sigma_i^l : \mathbb{R}_{\geq 0} \to \mathbb{R}$$

$$l \in [1, m]$$

Where $z_i^l(t)$ follow correlated Wiener processes with correlation $\rho_{i,j}^{x,y}$, i.e.

$$\mathbb{E}[dz_i^x(t) dz_j^y(t)] = \rho_{i,j}^{x,y} dt \tag{2}$$

$F(t, T_j)^l$ is the forward price observed at time t, for delivery over the time interval $[T_j, T_{j+1})$ of the l^{th} of m commodity underlyings.

3 Three Factor Seasonal Form

The three factor seasonal parameters clearly has parameter $n = 3$. The first factor is the spot or short-term factor. For this factor there is a constant volatility and mean reversion, i.e. $\sigma_1(T) = \sigma_{spot}$ and as this is the only factor with non-zero mean reversion $\alpha_1 = \alpha$. This factor has the heuristic interpretation as random shocks which move the forward curve in an exponentially decaying (as function of time to maturity) manner, with the biggest effect being on the spot price. It can also be interpreted as driving random shocks to the spot price which then mean revert.

As mentioned above the remaining two factors have zero mean reversion, i.e. $\alpha_2 = \alpha_3 = 0$. TODO seasonal factor

The third factor, the long-term factor, has constant volatility, $\sigma_2 = \sigma_{long}$, and represents parallel movement which effect the whole forward curve in a maturity independent manner.

Finally, it is assumed that the three Brownian Motions are independent. Putting this together, and using the subscripts *spot*, *seas* and *long* the Brownian Motions of the spot, seasonal, and long-term factors we get the following.

$$\frac{dF(t,T)}{F(t,T)} = \sigma_{spot} e^{-\alpha(T-t)} dz_{spot}(t) + \sigma_{seas}(T) dz_{seas}(t) + \sigma_{long} dz_{long}(t) \tag{3}$$

Refer back to Fowler (2020) for the statistical properties of the forward and spot price assumed by just substituting the parameters in Fowler (2020) with the specific one given above.

4 Critique of Three Factor Seasonal Model

The strength of the three-factor seasonal model is it's parsimony. Being able to specify the gas price dynamics using only four parameters is of great help in allowing users to intuitively see what is driving the extrinsic value of gas storage being valued. Traders can easily adjust the input parameters based on their view of the market. For example if a trader take a view that the future summer-winter spread volatility is going to be higher than in the historical period used to calibrate the parameters they can easily bump up the seasonal volatility parameter when valuing a potential storage deal. For a non-parsimonious model with many parameters some of which are completely abstract (for example correlation between factors), such usage would not be practical.

Another example of the is that risk managers can create scenario matrices containing storage facility (or portfolio) P&L based on scenarios applied to any of the four parameters.

It also allows for a relatively simple and intuitive way of calibrating model parameters from historic spot and forward prices. This is of particular importance for gas storage where there is generally a lack of liquid traded instruments which can be used to calibrate model volatility and correlation parameters to [1].

The KyStore product is clearly a popular one and The Author believes that this is at least partially due to the parsimony of the underlying price process model.

The big downside of this model is that the forward volatility seasonality structure is unrealistic. It is well known that, adjusting for time-to-maturity effects, the volatility of winter periods will generally be higher than those in summer. Figure 1 plots the forward volatility implied by the model, with mean

[1] For many natural gas markets, there are European options traded, for which the implied volatility can be used to calibrate price dynamics. However, the extrinsic value of storage is mostly derived from the relative movement of different delivery points on the forward curve, i.e. by calendar spreads. The European option volatility curve does convey some information about calendar spread volatility (e.g. a big difference in implied vol for two forward contracts implies that one contract will move by much more than another, hence higher calendar spread volatility) but not enough to fully calibrate the joint price dynamics of the whole forward curve.

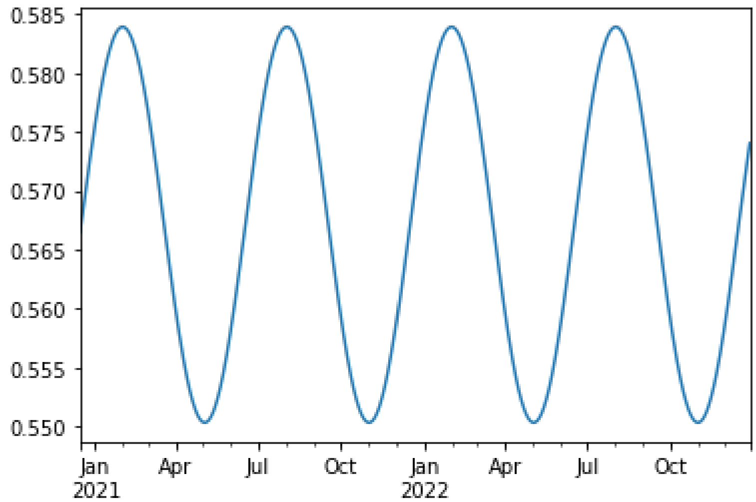

Figure 1: Forward Volatility By Delivery Date

reversion set to zero in order to remove any t time-to-maturity effect. This chart shows that the model implies two volatility peaks a year, once in February, as expected, but the other around August.

References

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