

Multi-Factor Commodity Price Process: Spot and Forward Price Simulation

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1 Introduction

This document describes a very general multi-factor price process, which is suitable use in simulating the spot and forward prices of a commodity under the risk-neutral probability measure.

$$\frac{dF(t, T)}{F(t, T)} = \sum_{i=1}^n \sigma_i(T) e^{-\alpha_i(T-t)} dz_i(t) \quad (1)$$

Where $F(t, T)$ is the forward price observed at time t , for delivery over the period starting at time T .

$$F(t, T) = \quad (2)$$

2 Appendices

2.1 Appendix I - Integration of Forward Price SDE

This appendix gives the detailed step for integrated the forward price process SDE, hence getting from 1 to 2.

Using Ito's Lemma to calculate the stochastic differential of the natural logarithm of the forward price.

$$d \ln(F(t, T)) = \frac{1}{F(t, T)} dF(t, T) - \frac{1}{2} \frac{1}{F(t, T)^2} (dF(t, T))^2 \quad (3)$$

Using the properties of Brownian Motion $(dz(t))^2 = dt$ and $dt dz(t) = 0$ the forward price differential squared can be evaluated as follows.

$$(dF(t, T))^2 = F(t, T)^2 \sum_{i=1}^n \sum_{j=1}^n \sigma_i(T) \sigma_j(T) e^{-(\alpha_i + \alpha_j)(T-t)} \rho_{i,j} dt \quad (4)$$

Substituting for $(dF(t, T))^2$ and $dF(t, T)$ in 3.

$$d \ln(F(t, T)) = \sum_{i=1}^n \sigma_i(T) e^{-\alpha_i(T-t)} dz_i(t) - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \sigma_i(T) \sigma_j(T) e^{-(\alpha_i + \alpha_j)(T-t)} \rho_{i,j} dt \quad (5)$$

The purpose of finding the stochastic differential of $\ln(F(t, T))$ is to remove the $F(t, T)$ coefficient of the Brownian motions, leaving an SDE which can be integrated to a form where the Ito Integrals have non-stochastic integrands, and hence have a normal distribution with known mean and variance. Integrating 5:

$$\ln(F(t, T)) = \ln(F(0, T)) + \quad (6)$$