Multi-Factor Commodity Price Process: Spot and Forward Price Simulation

Jake C. Fowler

January 2020

1 Introduction

This document describes a very general multi-factor price process, which is suitable use in simulating the spot and forward prices of a a commodity under the risk-neutral probability measure.

$$\frac{dF(t,T)}{F(t,T)} = \sum_{i=1}^{n} \sigma_i(T) e^{-\alpha_i(T-t)} dz_i(t) \tag{1}$$

Where $F(t,T)$ is the forward price observed at time t, for delivery over the period starting at time T.

$$F(t,T) = \tag{2}$$

2 Appendices

2.1 Appendix I - Integration of Forward Price SDE

This appendix gives the detailed step for integrated the forward price process SDE, hence getting from 1 to 2.

Using Ito's Lemma to calculate the stochastic differential of the natural logarithm of the forward price.

$$d\ln(F(t,T)) = \frac{1}{F(t,T)} dF(t,T) - \frac{1}{2}\frac{1}{F(t,T)^2}(dF(t,T))^2 \tag{3}$$

Using the properties of Brownian Motion $(dz(t))^2 = dt$ and $dtdz(t) = 0$ the forward price differential squared can be evaluated as follows.

$$(dF(t,T))^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_i(T)\sigma_j(T) e^{-(\alpha_i+\alpha_j)(T-t)} \rho_{i,j} dt \tag{4}$$